Analysis of Channeling Experiments in Diamond and Silicon Single Crystals with the Aid of the Fokker-Planck Equation

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Charged and Neutral Particles Channeling Phenomena
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Outline

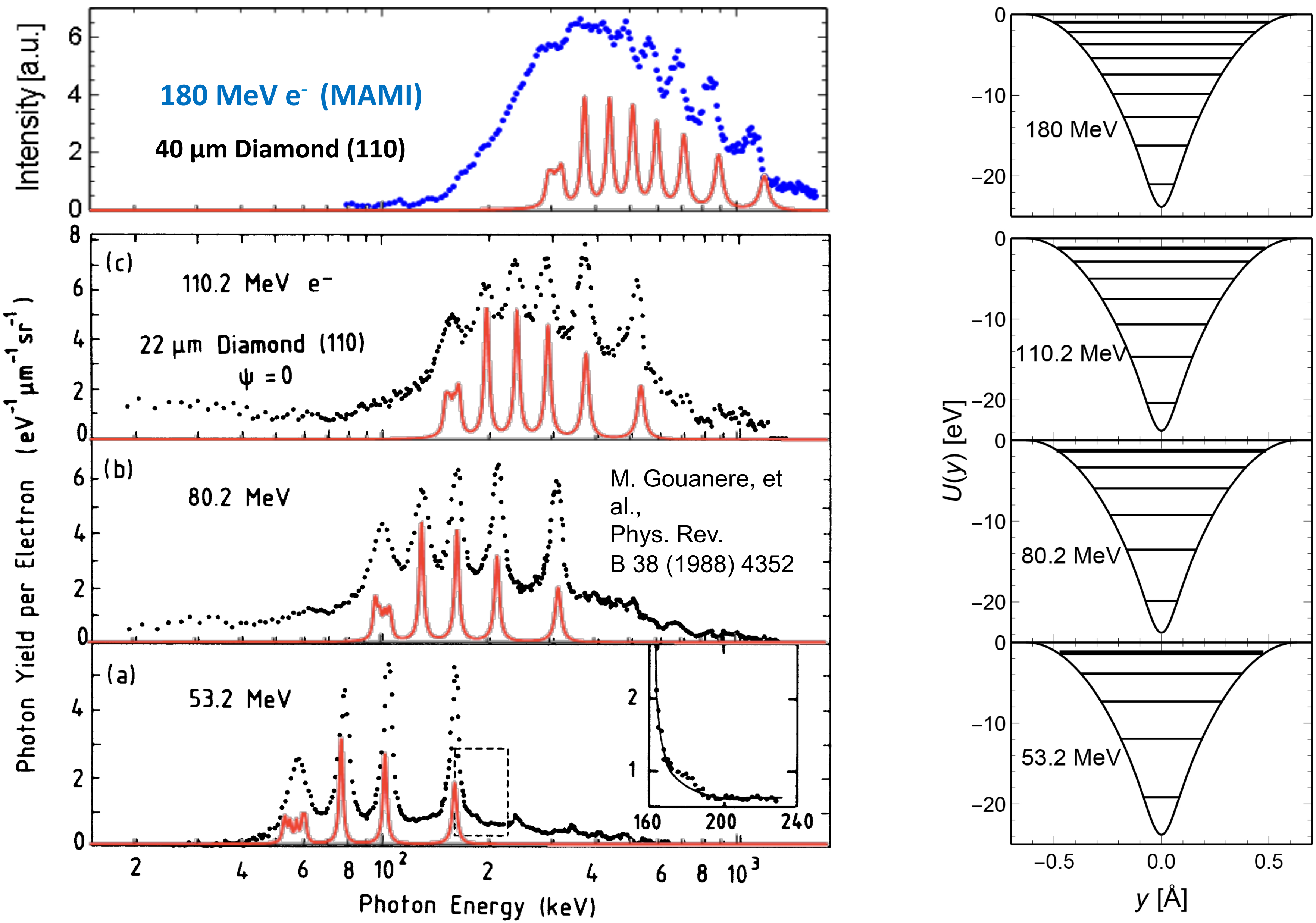
1. Motivation
2. MAMI Experiments on Diamond
3. Analysis of MAMI Experiments on Diamond with Solutions of a Fokker-Planck Equation
4. Results from a Modified Fokker-Planck Equation for (111) Channeling of Electrons in a Bent Silicon Single Crystal (SLAC Experiment)
5. Conclusions

1. Motivation

Is an intense photon source with **micro-undulators** and **electrons** feasible?

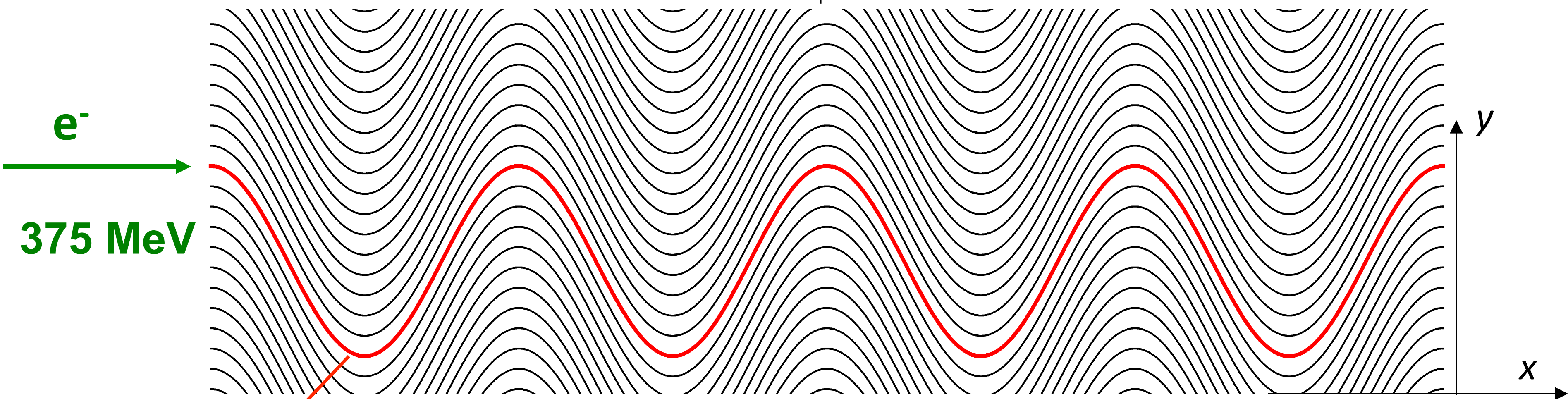
W. Greiner, A.V. Solov'yov, and A.V. Korol et al.

Results: Planar Channeling of Electrons in Diamond



Motivation: Micro-Undulator

(110) $U_0 = 22$ eV → $?_U = 9.9\ \mu\text{m}$ ← $y \approx A \cdot \cos\left(\frac{2\pi}{\lambda_U} x\right)$ $A = 4.64$ Å



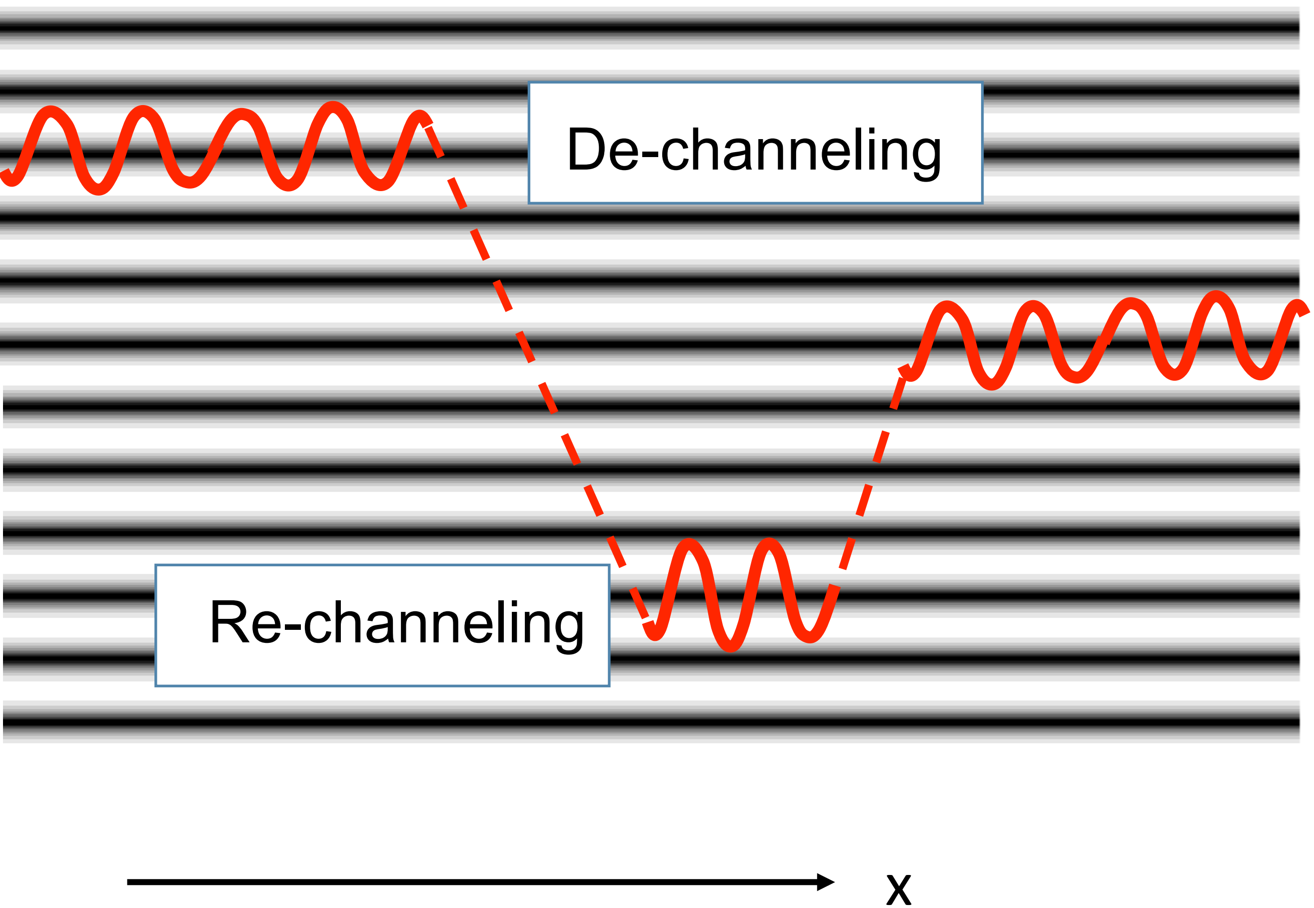
Coherent undulator radiation emission

$R = 6.60$ mm $B_{equiv} = \frac{pv}{e\,R\,c} = 223$ Tesla $K = \gamma \cdot A \cdot \frac{2\pi}{\lambda_U} = 0.216$

Photon energy $$\hbar\omega = k \frac{4\pi \cdot \gamma^2 \hbar c}{\lambda_U (1 + K^2/2 + \gamma^2(\theta_x^2 + \theta_y^2))} = 131.6 \text{ keV}$$

at $\theta_x = \theta_y = 0$, and first order $k = 1$

How long remains an electrons in a channel?
(Classical picture)



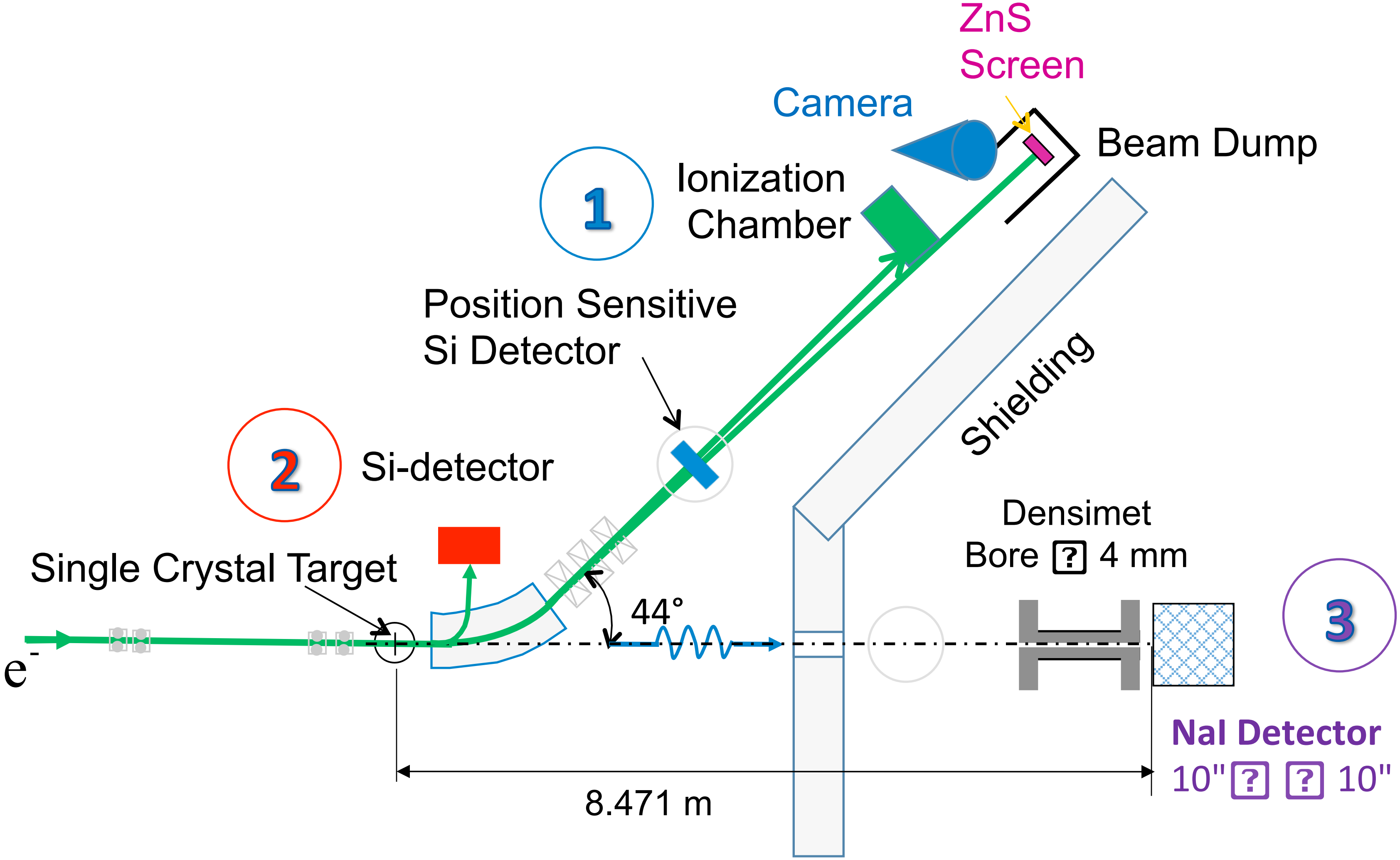
De-channeling rate [?]

for straight and bent crystals

Re-channeling rate [?]

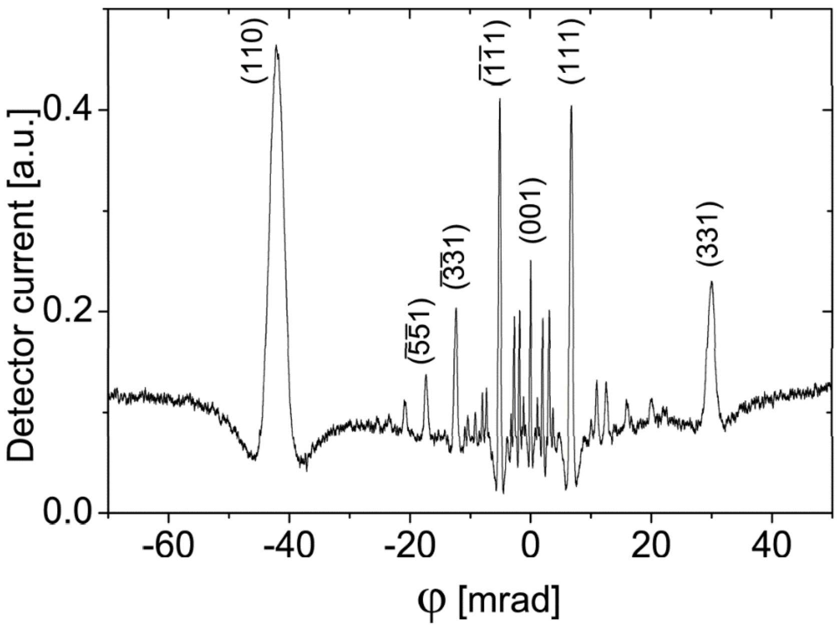
2. MAMI Experiments on Diamond

Basic Setup for the De-channeling Measurement with Electrons at MAMI



Planar Channeling Characteristics
Angular ? - Scans at 855 MeV

Flat crystal



2

High energy loss of electron after emission of a Bremsstrahlung photon with $\hbar$? ? 855 MeV/2

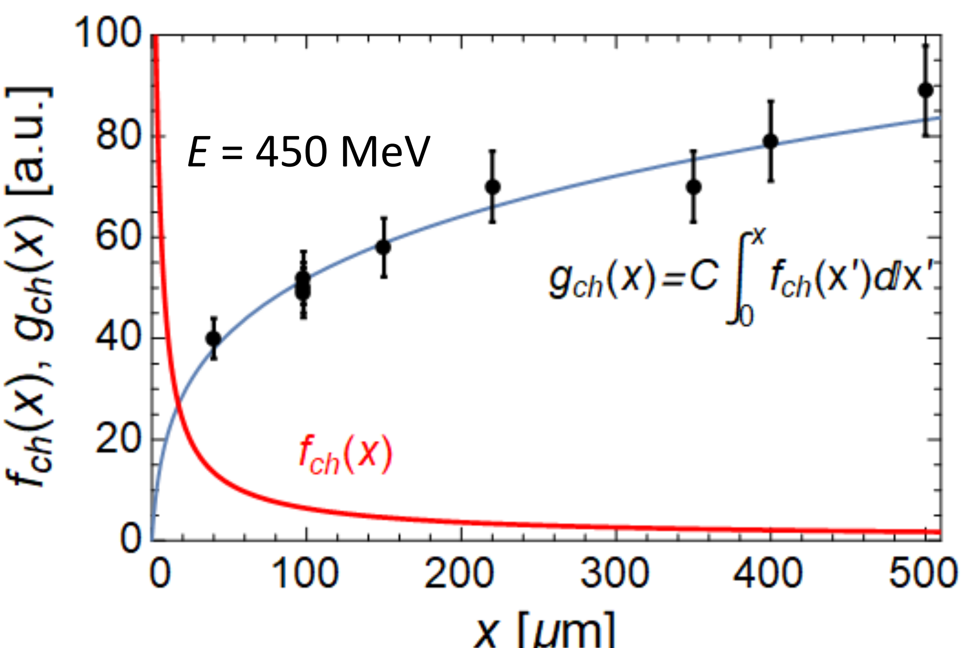
Analysis of De-channeling Length Measurements for Diamond (110)

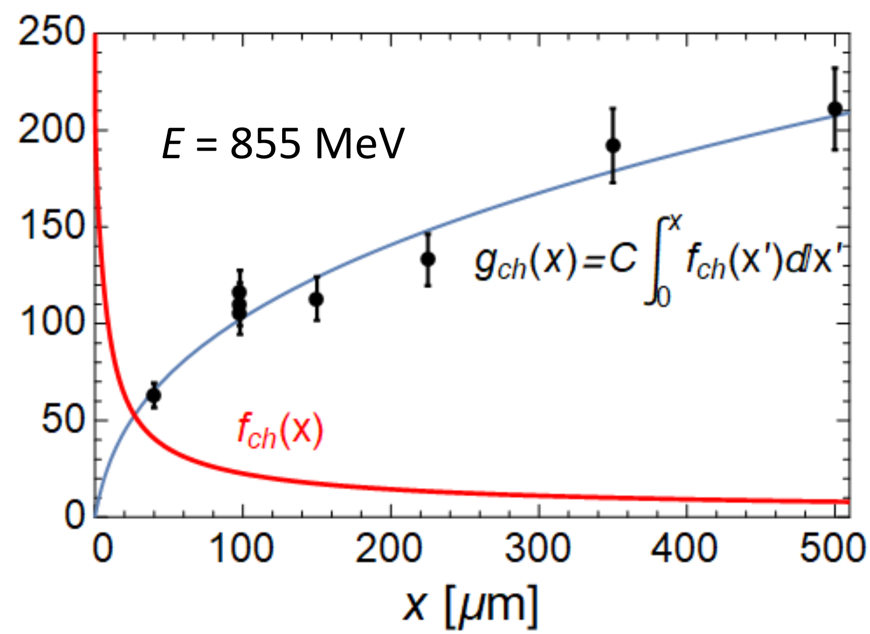
$$f_{ch}'(x) + f_{ch}(x) A \cdot \left[\lambda_{de}(x) - \lambda_{re}(x)\right] = 0 \qquad g_{ch}(x) = C \int_0^x f_{ch}(x')\, dx'$$

Crucial point: $\left[\lambda_{de}(x) - \lambda_{re}(x)\right]$ required for analysis

A and *C* are fit parameters
leading to the asymptotic de-channeling length

$$L_{de}^{x\to\infty} \simeq \frac{1}{A \cdot \lambda_{de}(x = 500\ \mu\text{m})}$$





3. Analysis of MAMI Experiments on Diamond with Solutions of a Fokker-Planck Equation

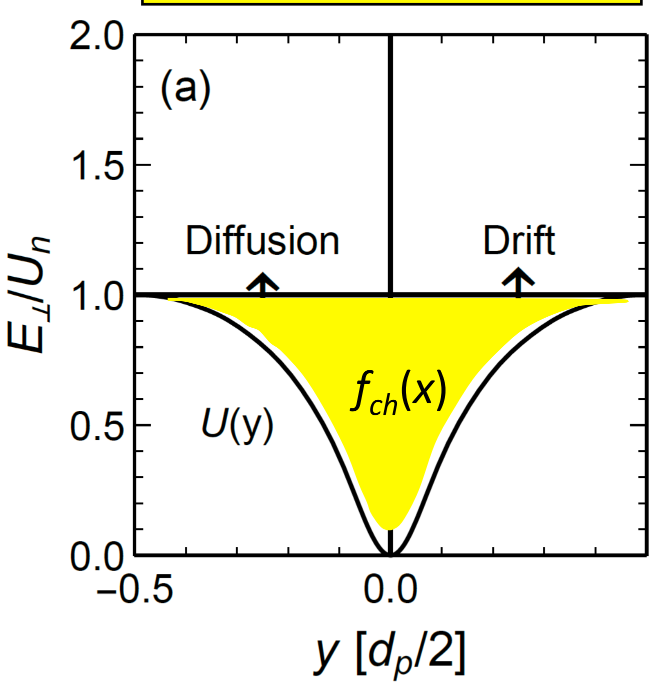
The Fokker-Planck equation

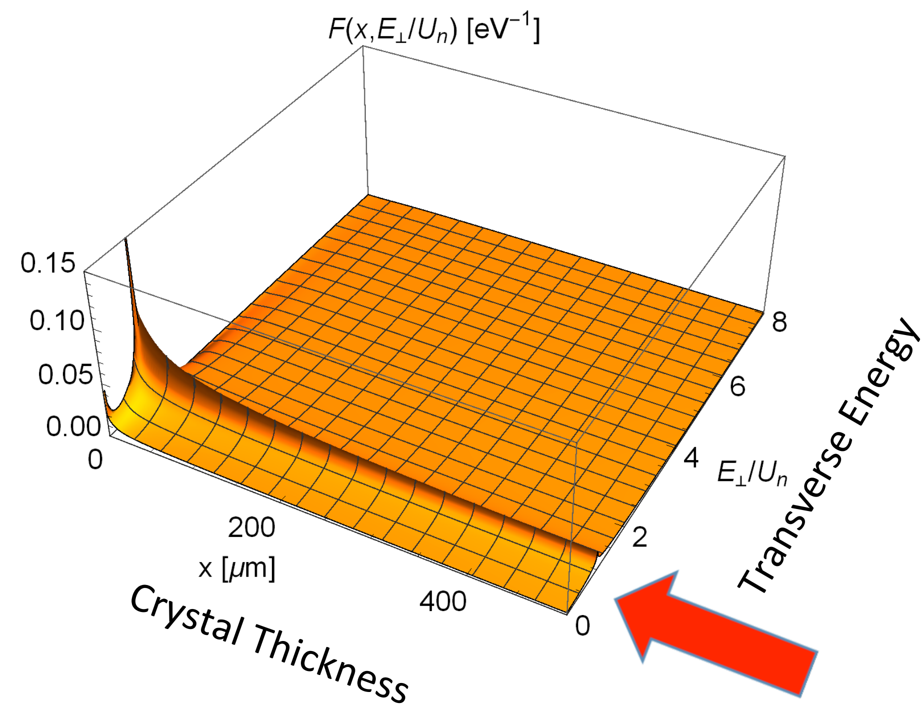
$$\frac{\partial F(x,E_\perp)}{\partial x} + \frac{\partial J(x,E_\perp)}{\partial E_\perp} = 0$$

continuity equation with density F and current J

$$J(x,E_\perp) = -\frac{\partial}{\partial E_\perp}\left[D_e^{(2)}(E_\perp)\,F(x,E_\perp)\right] + D_e^{(1)}(E_\perp)\,F(x,E_\perp) = J_{diff}(x,E_\perp) + J_{drift}(x,E_\perp)$$

$$f_{ch}(x) = \int_{E_\perp=0}^{U_n} F(x,E_\perp)\,dE_\perp$$

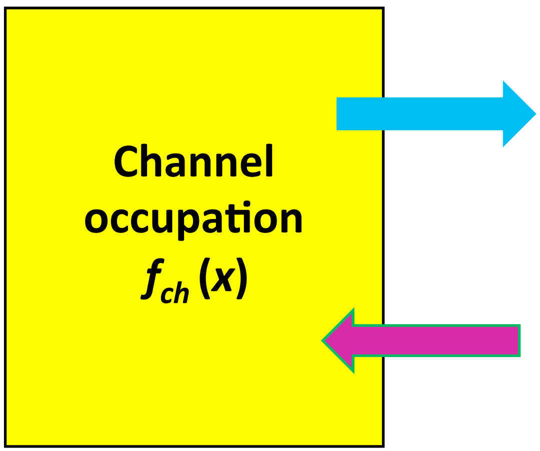




Channel with $E_\perp / U_n \leq 1$

De- and Re-channeling Rates $?_{de}$ and $?_{re}$ De- and Re-channeling Lengths L_{de} and L_{re}

Definition of de- and re-channeling rates ?



$$\lambda_{de}(x) = \frac{\text{outward current}}{\text{channel occupation}} = \frac{J_{\uparrow}(x, E_{\perp} = U_n)}{f_{ch}(x)}$$

$$\lambda_{re}(x) = \frac{\text{inward current}}{\text{channel occupation}} = -\frac{J_{\downarrow}(x, E_{\perp} = U_n)}{f_{ch}(x)}$$

Definition of de-and re-channeling lengths L

$$L_{de}(x) = \frac{1}{\lambda_{de}(x)} \qquad L_{re}(x) = \frac{1}{\lambda_{re}(x)}$$

Essential Ingredients of the Fokker-Planck Equation

Drift coefficient
(Kitagava-Ohtsuki)

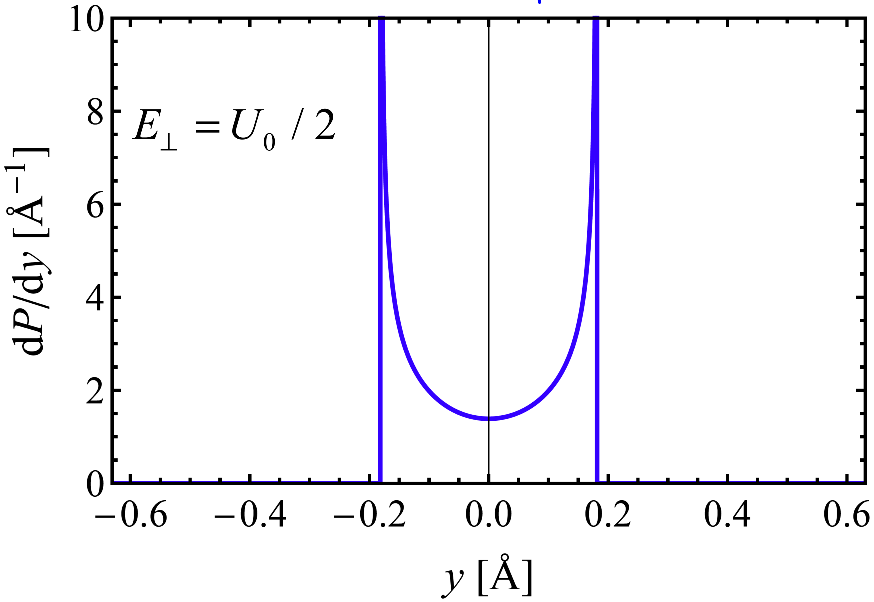
$$D_e^{(1)}(E_\perp) = \frac{E_s^2}{2pvX_0} \int_{y_{\min}}^{y_{\max}} \frac{dP}{dy}(E_\perp, \eta) \frac{d_p}{\sqrt{2\pi} u_1} \exp[-\eta^2 / 2u_1^2] d\eta$$

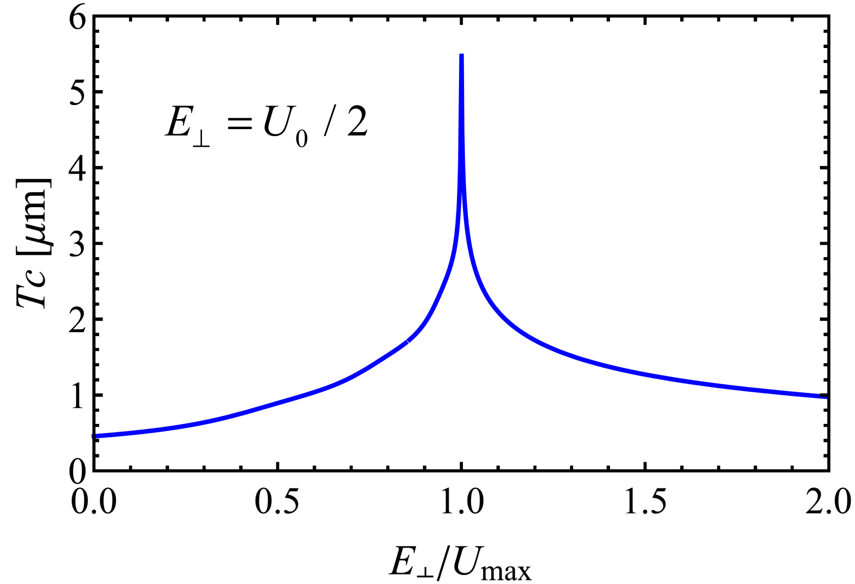
X_0 = 0.1213 m is radiation length, u_1 = 0.04226 Å thermal vibration amplitude,
E_s = 15.0 MeV standard scattering parameter, pv = 855 MeV, U_0 = 22.34 eV potential depth

Standard probability for a quasi-periodic transverse motion

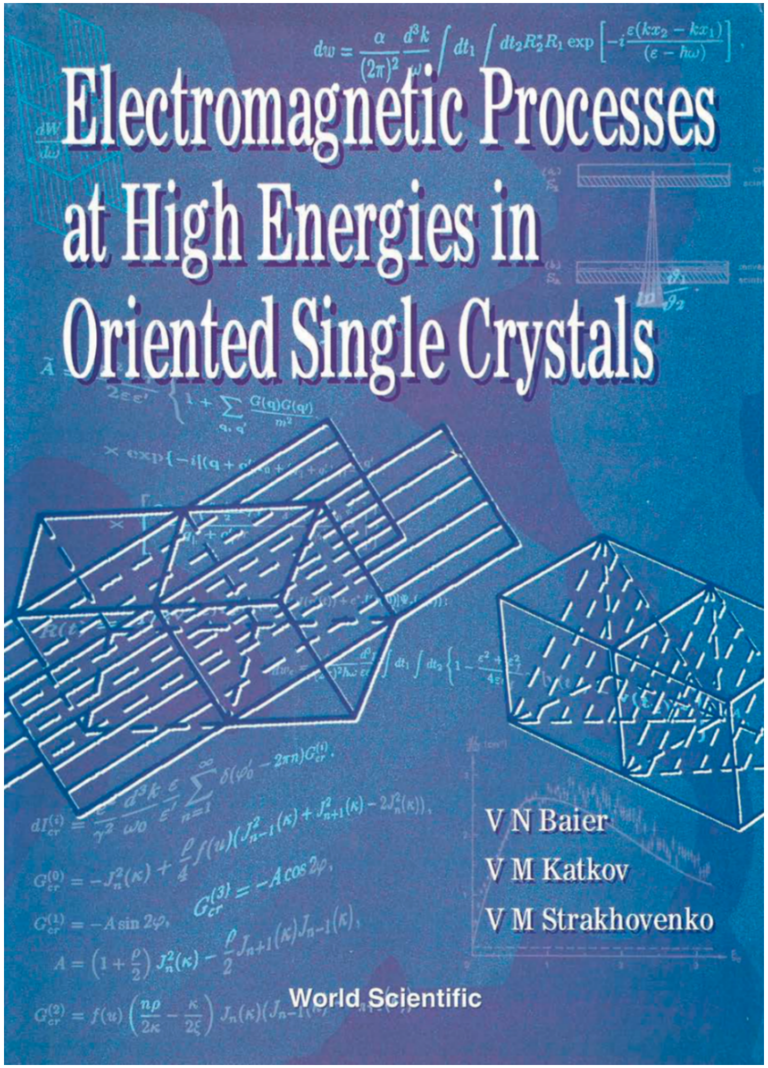
$$\frac{dP}{dy}(E_\perp, y) = \frac{2}{T(E_\perp)c} \sqrt{\frac{\gamma m_e c^2}{2(E_\perp - U(y))}}$$

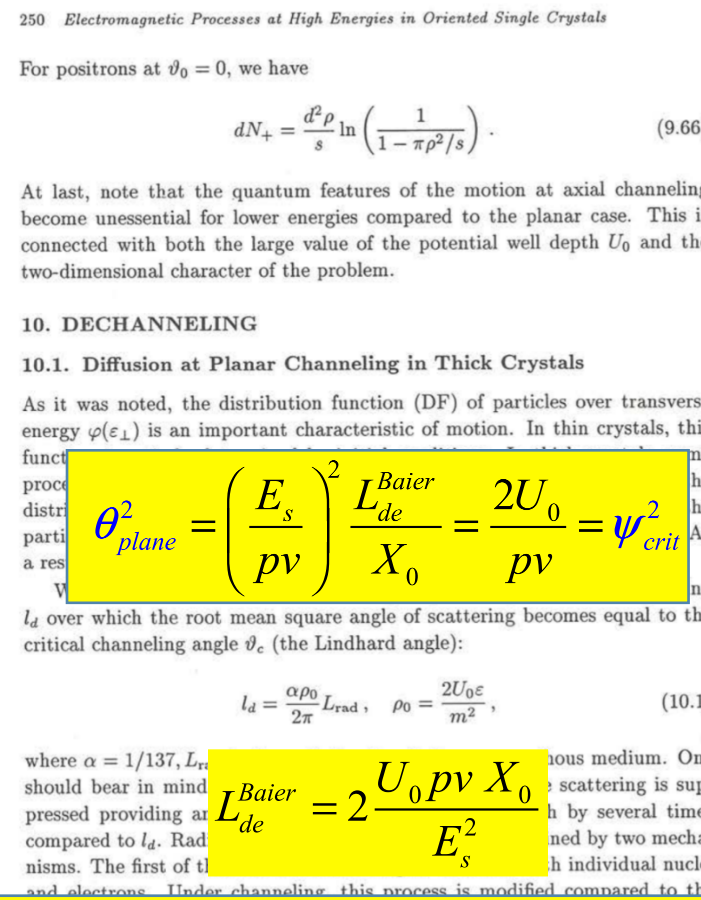
$$T(E_\perp)c = 2 \int_{y_{\min}}^{y_{\max}} \sqrt{\frac{\gamma m_e c^2}{2(E_\perp - U(\eta))}} dy$$





Evaluation of the Scattering Parameter E_s





For positrons at $\vartheta_0 = 0$, we have

$$dN_+ = \frac{d^2\rho}{s} \ln\left(\frac{1}{1 - \pi\rho^2/s}\right) . \qquad (9.66)$$

At last, note that the quantum features of the motion at axial channeling become unessential for lower energies compared to the planar case. This is connected with both the large value of the potential well depth U_0 and the two-dimensional character of the problem.

10. DECHANNELING

10.1. Diffusion at Planar Channeling in Thick Crystals

As it was noted, the distribution function (DF) of particles over transverse energy $\varphi(\varepsilon_\perp)$ is an important characteristic of motion. In thin crystals, this funct ... ne proc ... he distri ... he parti ... As a res ...

W ... ng l_d over which the root mean square angle of scattering becomes equal to the critical channeling angle ϑ_c (the Lindhard angle):

$$l_d = \frac{\alpha\rho_0}{2\pi} L_{\rm rad} , \quad \rho_0 = \frac{2U_0\varepsilon}{m^2} , \qquad (10.1)$$

where $\alpha = 1/137$, $L_{\rm r}$ ... nous medium. One should bear in mind ... scattering is suppressed providing ar ... h by several times compared to l_d. Rad ... ned by two mechanisms. The first of t ... h individual nuclei and electrons. Under channeling this process is modified compared to the

$$\theta^2_{plane} = \left(\frac{E_s}{pv}\right)^2 \frac{L^{Baier}_{de}}{X_0} = \frac{2U_0}{pv} = \psi^2_{crit}$$

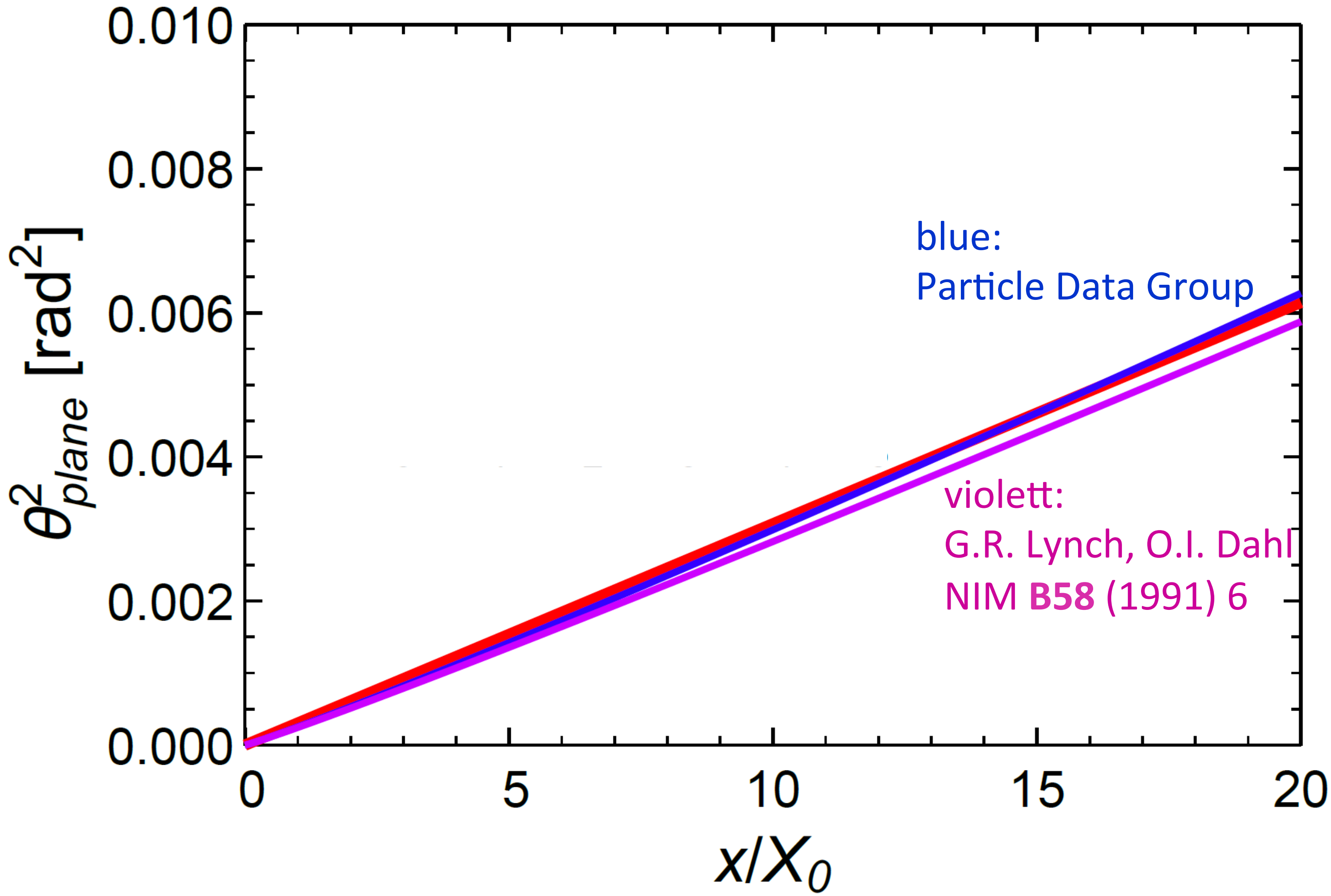
$$L^{Baier}_{de} = 2\frac{U_0 pv\, X_0}{E_s^2}$$

$$E^2_{s,Baier} = \frac{2\pi}{e^2} m^2 \Rightarrow \frac{2\pi}{\alpha}\left(m_e c^2\right)^2 = (15.0\ \mathrm{MeV})^2$$

see also: B. Rossi and K. Greisen, Cosmic-ray theory, Rev. Mod. Phys. **13** (1941) 240
§ 22. *Multiple Scattering. Calculation of the Mean Square Angle of Scattering*

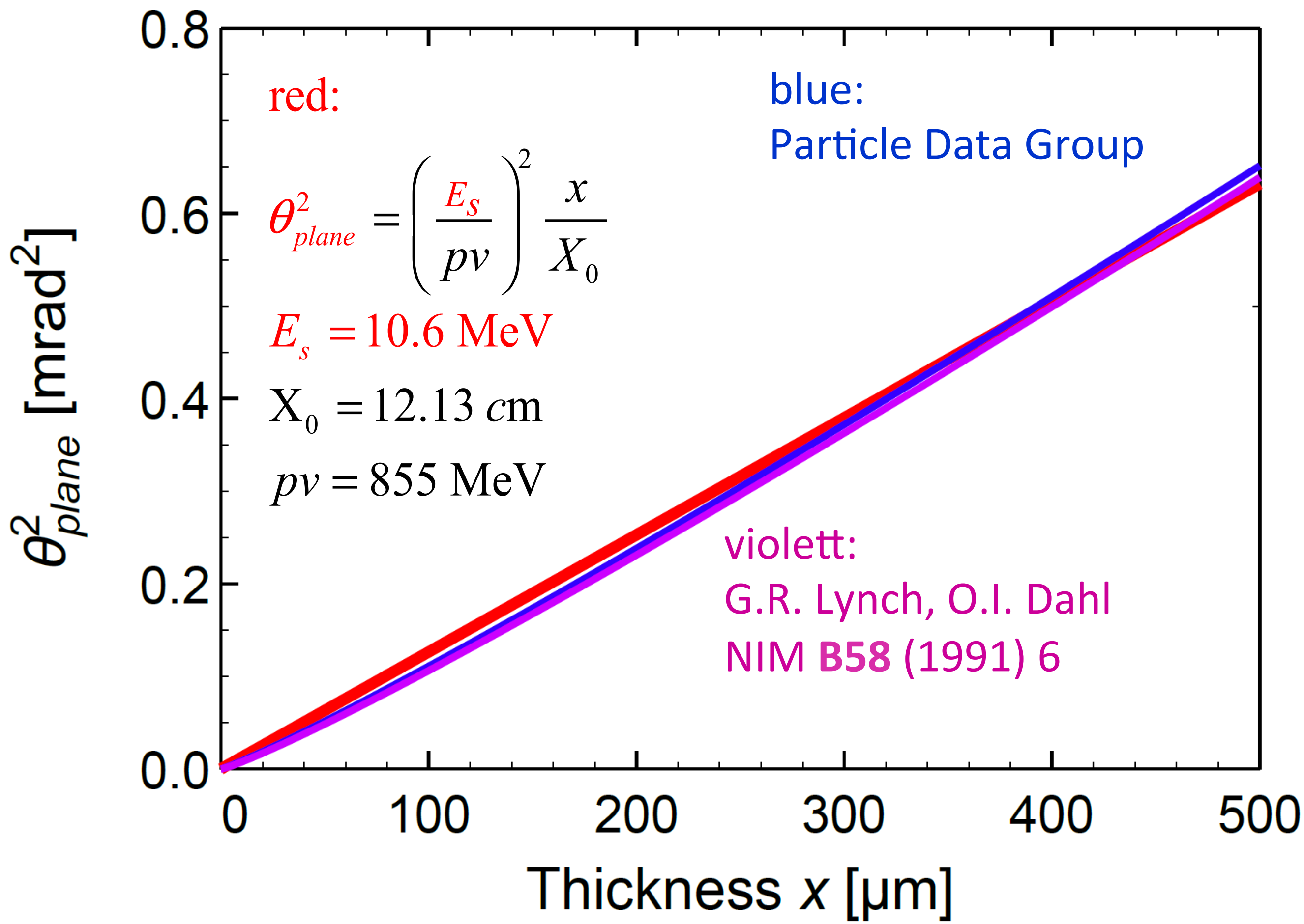
Variance of Scattering Angle according to Baier et al. for Diamond at 855 MeV

$$\theta^2_{plane} = \left(\frac{E_{s,Baier}}{pv}\right)^2 \frac{x}{X_0} \qquad E^2_{s,Baier} = \frac{2\pi}{e^2} m^2 \Rightarrow \frac{2\pi}{\alpha}\left(m_e c^2\right)^2 = (15.0\ \text{MeV})^2$$



Modified Variance of Scattering Angle at Small Thicknesses for Diamond at 855 MeV

Conclusion: E_s = 10.6 MeV has to be used always instead of E_s = 15.0 MeV



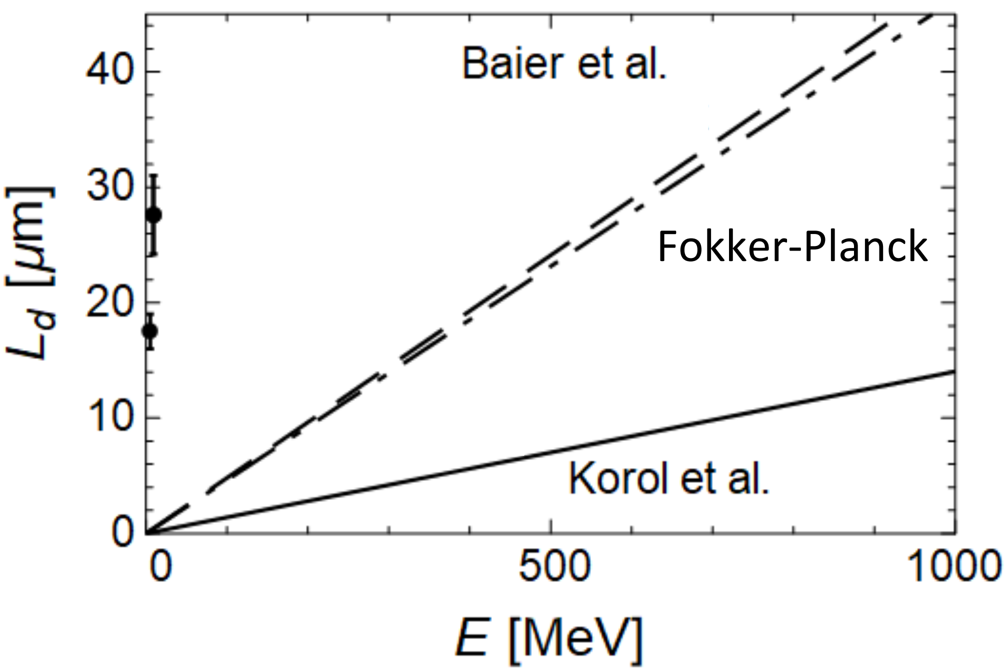
Various De-channeling Lengths for Diamond (110)

$$L_{de}^{Baier} = \frac{2U_0 pvX_0}{E_s^2}$$

with E_s = 10.6 MeV instead of 15.0 MeV

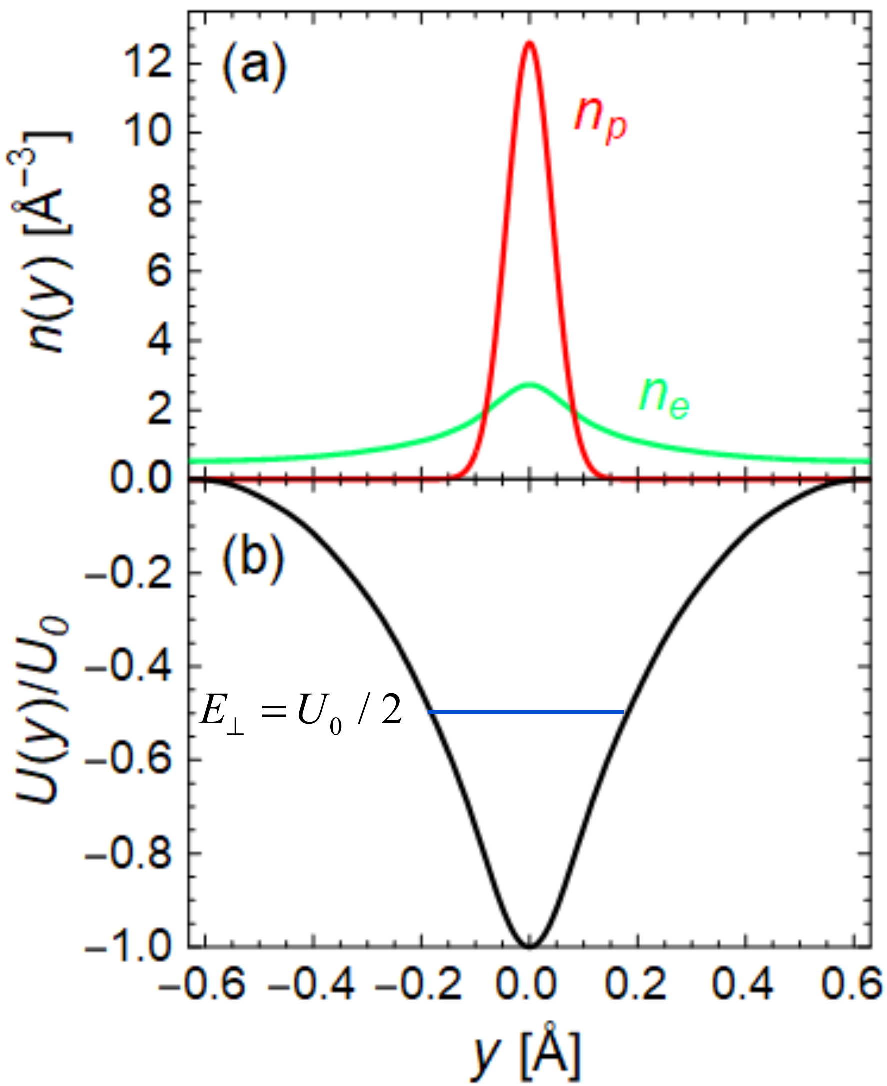
Drift coefficient
(Kitagava-Ohtsuki)

$$D_e^{(1)}(E_\perp) = \frac{E_s^2}{2pvX_0} \int_{y_{min}}^{y_{max}} \frac{dP}{dy}(E_\perp, \eta) \frac{d_p}{\sqrt{2\pi}u_1} \exp[-\eta^2 / 2u_1^2] d\eta$$



Simulation calculation with MBN explorer software package
A.V. Korol et al.,
Eur. Phys. J. **D 71** (2017) 174

Modelling the Fokker-Planck Equation



n_p is distribution of the positive charge

n_e is distribution of the positive charge (neglected)

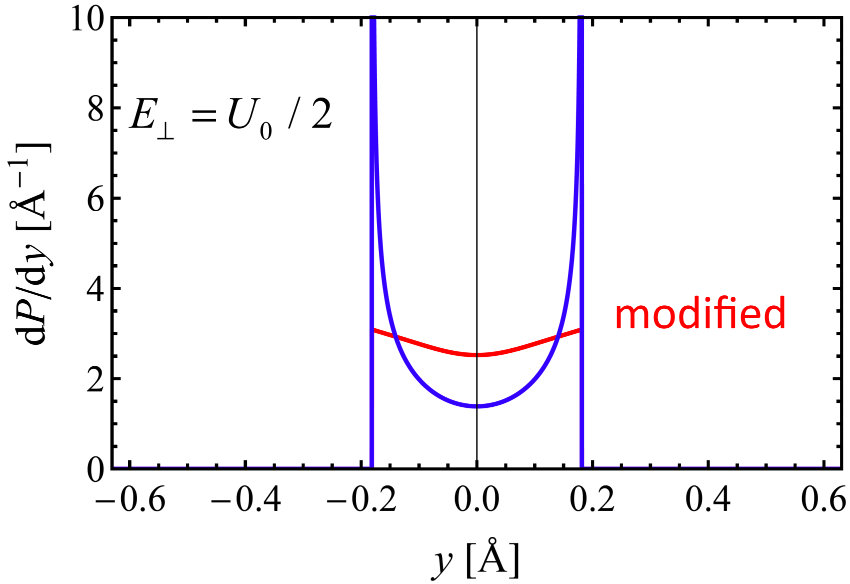
Potential calculated in Molière approximation

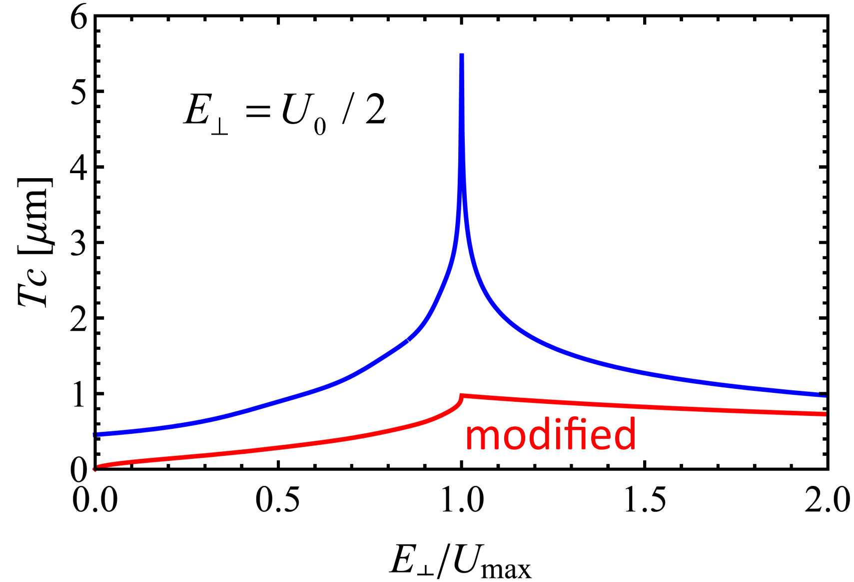
Modified Channel Occupation Probability *dP/dy*

Modification by introducing heuristically an ? = 22.3 eV in

$$\frac{dP}{dy}(y, E_\perp) = \frac{2}{T(E_\perp)}\sqrt{\frac{\gamma m_e}{2\left(E_\perp + \varepsilon - U(y)\right)}}$$

$$T(E_\perp) = 2\int_{y_{\min}}^{y_{\max}} \sqrt{\frac{\gamma m_e}{2\left(E_\perp + \varepsilon - U(y)\right)}}\, dy$$

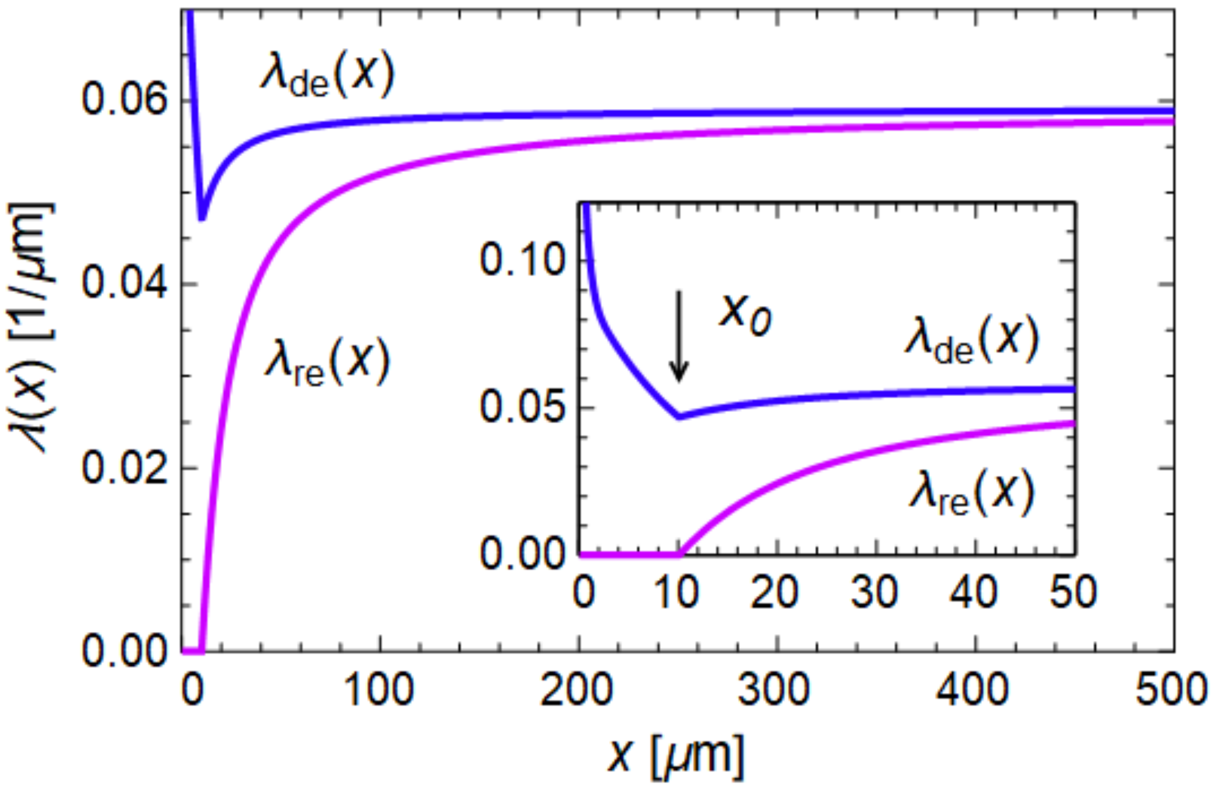




De-channeling Lengths for Diamond (110) with Results of the Modified Drift Coefficient

Obtain $[\lambda_{de}(x) - \lambda_{re}(x)]$ from solution of modified Fokker-Plank equation and solve

$$f_{ch}'(x) + f_{ch}(x) A \cdot [\lambda_{de}(x) - \lambda_{re}(x)] = 0$$



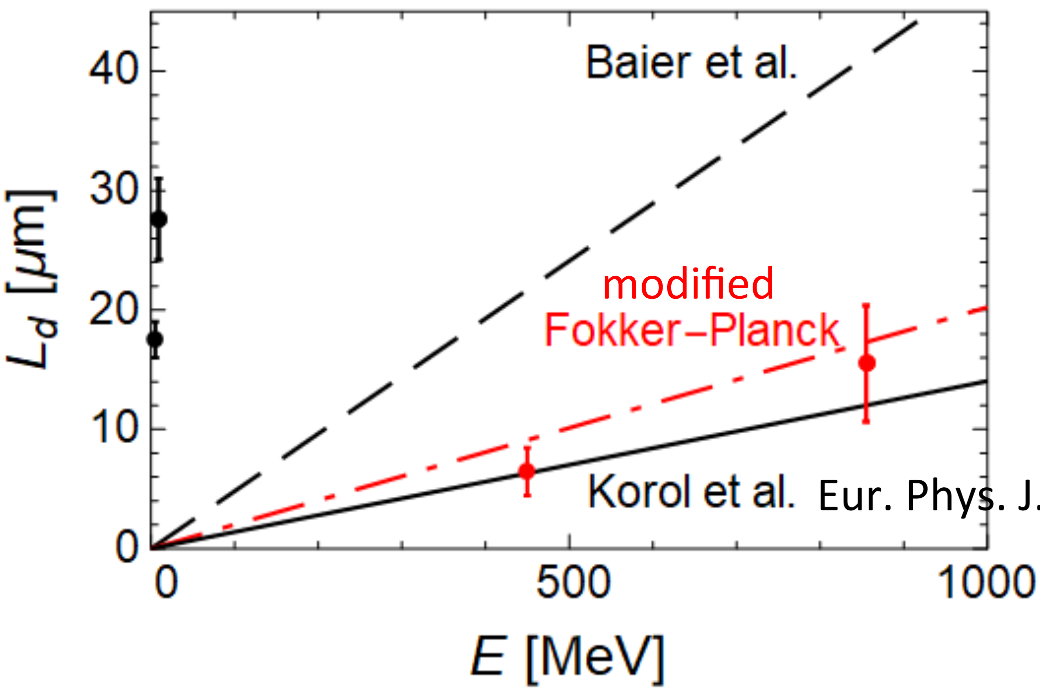
$$L_{de}^{x \to 500\ \mu\text{m}} = \frac{1}{\lambda_{de}(x \to 500\ \mu\text{m})}$$

$$= 17.0\ \mu\text{m}$$

Corresponds in a good approximation to the asymptotic de-channeling length

De-channeling Lengths for Diamond (110) with Results of the Modified Fokker-Planck Equation

Difference $[\lambda_{de}(x) - \lambda_{re}(x)]$ is taken from solution of the Fokker-Planck equation with E_s = 10.6 MeV and modified dP/dy.



MAMI experiment

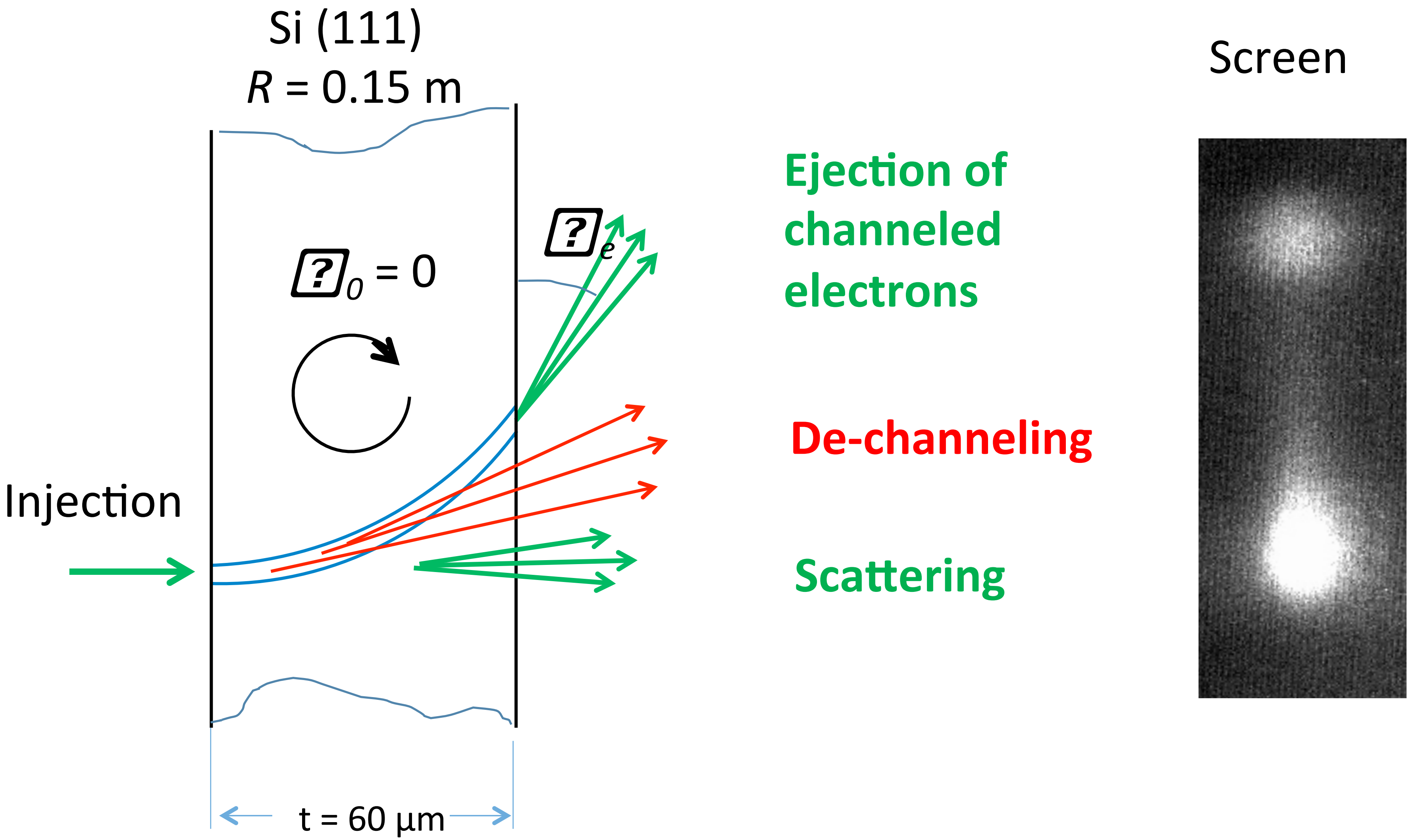
Eur. Phys. J. **D 71** (2017) 174

4. Results from a Modified Fokker-Planck Equation for (111) Channeling of Electrons in a Bent Silicon Single Crystal (SLAC Experiment)

T. N. Wistisen, U. I. Uggerhøj, U. Wienands,
T.W. Markiewicz, R. J. Noble, B. C. Benson, T. Smith,
E. Bagli, L. Bandiera, G. Germogli, V. Guidi,
A. Mazzolari, R. Holtzapple, and S. Tucker

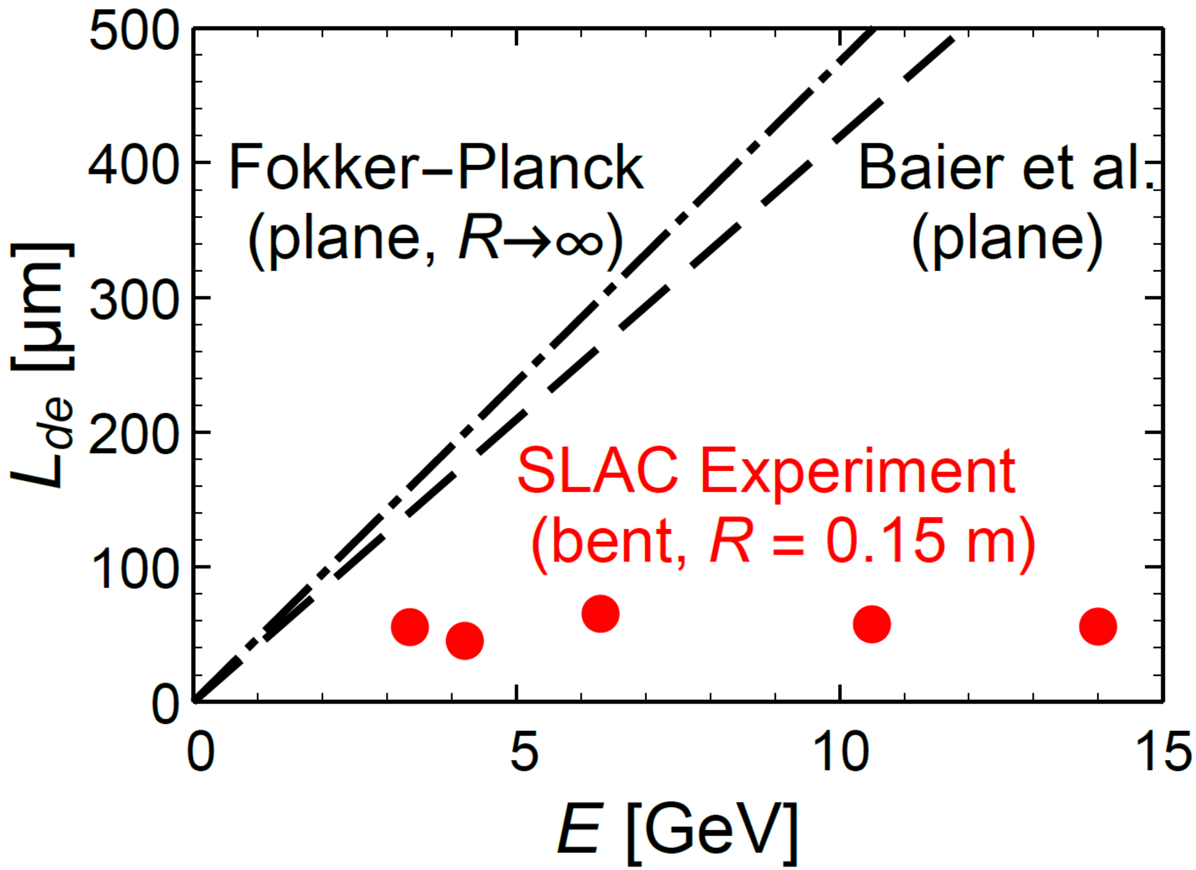
Phys. Rev. ST-AB 19, 071001 (2016)

Target Setup and Intensity Distribution on a Luminescence Screen for Channeling of Electrons in a Bent Silicon Single Crystal at FACET (SLAC)



De-channeling Length Measurements at (111) Bent Silicon Single Crystal (SLAC)

T. N. Wistisen, U. I. Uggerhøj, U. Wienands,et al., Phys. Rev. ST-AB 19, 071001 (2016)



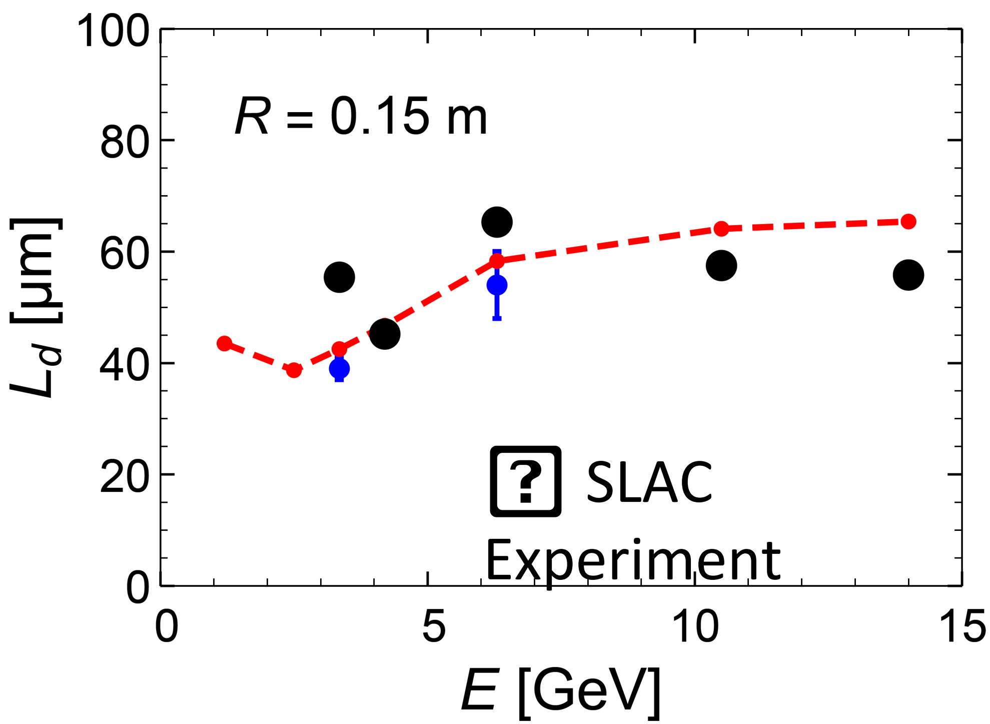
Modification of Fokker-Planck Equation in Bent Crystal

$$\frac{\partial F(x,E_\perp)}{\partial x}+\frac{\partial J(x,E_\perp)}{\partial E_\perp}=0$$

continuity equation with density *F* and current *J*

$$J(x,E_\perp)=-\frac{\partial}{\partial E_\perp}\left[D_e^{(2)}(E_\perp)\,F(x,E_\perp)\right]+D_e^{(1)}(E_\perp)\,F(x,E_\perp)$$

Solution of Modified Fokker-Planck Equation for (111) Bent Silicon Single Crystal with R = 0.15 m



Blue: MBN Explorer calculations of G.B. Sushko, A.V. Korol, A.V. Solov'yov NIM **B 355** (2015) 39

5. Conclusion

1. De-channeling lengths were measured for straight crystals and compared with results from the Fokker-Planck equation. A modified scattering parameter E_s = 10.6 MeV must be used.

2. The results are also sensitive on the probability dP/dy (y) across the channel coordinate y.

3. Our analysis of the de-channeling length is model dependent! Difference $[?_{de}(x)$ - $?_{re}(x)]$ is taken from solutions of the Fokker-Planck equation!

4. Results from simulation calculations of $[?_{de}(x)$ - $?_{re}(x)]$ as well as dP/dy (y) would help to improve our analysis.

5. The de-channeling length measurements at (111) bent silicon single crystal (SLAC) were calculated with a modified Fokker-Planck equation with additional drift term due to centrifugal force.

6. For further details see recent references quoted in the abstract.

H. Backe
W. Lauth
Thu Nhi Tran Thi

Institute for Nuclear Physics
University of Mainz
X-ray Optics Group, ESRF, Grenoble

References to presented work:

H. Backe, W. Lauth, and Thu Nhi TRAN THI, Channeling experiments at planar diamond and silicon single crystals with electrons from the Mainz Microtron MAMI, JINST **13** (2018) C04022

H. Backe, Electron channeling experiments with bent silicon single crystals - a reanalysis based on a modified Fokker-Planck equation, JINST **13** (2018) C02046

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