First observation of the Grating Diffraction Radiation

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Content

- Introduction
- Experimental set-up
- Coherent SPR
- Coherent GDR
- Summary
• Linac-based THz sources provide sub-ps ~μJ radiation pulses with continuous spectrum;
• To obtain narrow-band spectral line with a possibility of the frequency tuning one should use a monochromator;
• Other possibility is to use SPR source with spectral line adjustment changing emission angle;
• Source based on Grating Transition Radiation (GTR) can provide line adjustment for fixed emission angle.
Smith-Purcell radiation (resonant diffraction radiation)

First observed by S.J. Smith and E.M. Purcell, Phys. Rev. Lett. 92, 1069 (1953)

Far-field zone:

\[ R >> \gamma^2 \lambda \]

\[ R >> Nd \]

\[ N \text{ – number of periods} \]

Smith-Purcell dispersion relation:

\[ \lambda_n = \frac{d}{n} \left( \frac{1}{\beta} - \cos \eta \right) \]
SPR investigations at LUCX (KEK, Japan)

Michelson interferometer, see M. Shevelev et al. // NIM A 771, 126 (2015)
A. Aryshev et al. // PR-AB 20, 024701 (2017)

Beam energy, typ. 7 MeV
Micro-bunch charge $Q$ and stability 25 pC, 6 %
Number of micro-bunches, 1.2, 4
Bunch length, for given $Q$ 250 fs
Normalized emittance, $\epsilon_x \times \epsilon_y$ $1.5 \times 1 \, \mu\text{mm mrad}$

Top: experimental schematics
Bottom: photograph of the experimental station
Right: SPR target

Abbreviations: M1 — fixed interferometer mirror, M2 — movable interferometer mirror, BS — splitter, PM — off-axis parabolic mirror

Detectors:
SBD 60-90 ($\nu = 60 \div 90$ GHz)
SBD 320-460 ($\nu = 320 \div 460$ GHz)
Smith-Purcell geometry

Smith Purcell geometry: $\theta = 0, \eta = 93.5^\circ$

Interferograms

SBD_60-90

SBD_320-460
Coherent SPR spectral lines

Line broadening is due to finite aperture

SPR spectral measurements results

\[ \frac{\Delta \nu_1}{\nu_1} \approx 9\% \quad \frac{\Delta \nu_5}{\nu_5} \approx 4\% \]

Frequency resolution of Michelson interferometer:

\[ \Delta \nu_{int} \sim \frac{c}{2L_{int}} \sim 5 \text{ GHz} \]
Reflection of the EM by a grating

$$\lambda_n = \frac{d}{n} \left( \cos \Theta - \cos \left( \Theta - \eta \right) \right)$$
Grating Transition Radiation

Dispersion relation:

\[ \lambda_n = \frac{d}{n} \left[ \cos \eta - \frac{1}{\beta} \right] \]

\[ \lambda_n = \frac{d}{n} \left[ \cos (\eta - \theta) - \frac{\cos \theta}{\beta} \right] \]


GTR monochromaticity is defined by overlapping of the Coulomb field and the grating

\[ \frac{\Delta \lambda}{\lambda} \sim \frac{1}{N_{\text{eff}}} \approx \frac{d \cdot \sin \theta}{\gamma \lambda}, \text{ if } N_{\text{eff}} \gg 1 \]
GTR Interferogram

SBD_60-90

SBD_320-460
Spectra reconstruction (SBD 320-460)

1 bunch
SBD 320-460
SBD att = 1 dB
θ = 10°

1 bunch
SBD 320-460
SBD att = 5 dB
θ = 15°
Confirmation of the dispersion relation

Frequency shift of GTR lines from the Smith–Purcell frequency in comparison with an estimate from the dispersion relation

$\eta = 93.5^\circ$
Coherent Diffraction Radiation

- Diagram showing a target with diffraction patterns at angles θ.
- Graphs comparing CDR and CTR with simulation and experiment data.

**Graph Details:**
- X-axis: θ (degree)
- Y-axis: SBD 320-460 (arb. units)
- Graphs for CDR and CTR with simulation and experimental data.
Geometry of the experiment
GDR monochromatisty

\[
\frac{\Delta \lambda}{\lambda} : \frac{L_{\text{eff}}}{d} = \frac{d \sin \theta}{\gamma \lambda}
\]
GDR Spectral lines

- **a**: CTR
- **b**: SPR, HP
- **c**: GDR, HP, $\eta = 5^\circ$
- **d**: CTR, SPR, HP, GDR, HP, GDR, VP, $\eta = 0^\circ$

Normalized intensity (arb. units)

$M_2$ displacement (mm) vs Frequency (GHz)
GDR Spectral lines

Zoom-in of the typical measured auto-correlation curves of: a - CTR, b - SPR, c - GDR, horizontal polarization and d - reconstructed spectra

GDR dispersion relation:

$$\lambda_n = \frac{d}{n} \left[ \cos(\eta - \theta) - \frac{\cos \theta}{\beta} \right]$$
Peak Position

Line Width
GDR Polarization components

\[ \Delta \nu = 320 - 460 \text{ GHz} \]

Normalized intensity (arb. units)

Orientation angle \( \eta \) (deg.)
Calculation method

We used generalized surface current method to simulate GDR from the striped grating

\[ E_R^D(r_D, \lambda) = \frac{1}{2\pi} \int \left[ [n(r_T), E_e^T(r_T, \lambda)], \nabla G(r_T, r_D, \lambda) \right] dS_T \]

\[ E_e^T(r_T, \lambda) = \frac{2e}{\beta^2 c^2 \gamma \lambda} \cdot e^{\frac{k}{\beta^2 \gamma \lambda}} \cdot \left\{ x \sqrt{\frac{x_T^2 + y_T^2}{x_T^2 + y_T^2}} \right\} \]

\[ \nabla G(r_T, r_D, \lambda) = \frac{r_D - r_T}{|r_D - r_T|^2} \cdot e^{ik|r_D - r_T|} \cdot \left( \frac{1}{|r_D - r_T|} - ik \right) \]

\[ n(r_T) = A(\psi). \{0,0,1\} \]

\[ \frac{d^2 W_e}{d\omega d\Omega} = cr^2 |E_R^D(r_D, \lambda)|^2 \]

Where \( r_T = \{x_T, y_T, z_T\} \) and \( r_D = \{x_D, y_D, z_D\} \) are the coordinate on the target and detector surface respectively, \( \lambda \) is the radiation wavelength, \( k = 2\pi/\lambda \) is the wave number, \( |r_D - r_T| \) is the distance between the points on the detector and the target, \( A(\psi) \) is the rotation matrix for the normal vector at \( \psi \) angle for each strip in the target, \( S_T \) is the target surface and \( E_e^T \) is the electron coulomb field, \( E_R^D \) is the radiation field.

Simulation scheme

I. Number of strips (From 1 to 15 strips in “0” for $\lambda$)

II. Different polarization components of GDR

III. Observation point $\rightarrow$ detector aperture (Different obs. point $Z = -5, 0$ and 5 mm)

IV. Spatial distributions (3D distribution and 2 sections (along Z and Y) for $\lambda$)

V. Target tilting angle (From $-25^\circ$ to $60^\circ$ in “0” for $\lambda$ and spectra)

VI. Target tilting angle for two detectors (From $-5^\circ$ to $29^\circ$ in “0”)

VII. Electron energy $\rightarrow$ energy spread (Spectra for 8.25 MeV $\pm$ 1% (8.1675 MeV, 8.3325 MeV))

VIII. Bunch length $\rightarrow$ coherence (Single case)

IX. Micro-train (for 2 bunches with different distance between them)
Spectrum simulation

Peaks positions correspond to the dispersion relation.

\[
\lambda_m = \frac{d}{m} \left( \frac{\cos \theta}{\beta} - \cos (\eta - \theta) \right)
\]

<table>
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<th>m</th>
<th>(\sim\lambda, \text{mm})</th>
<th>(\sim\nu, \text{GHz})</th>
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<tr>
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</tbody>
</table>

8.25 MeV
\(\theta = 0^\circ\)
\(\eta = 90^\circ\)
Z, Y = 0
15 strips
d = 4 mm

Spectrum in terms of wave length and frequency at 90° to the target.
The comparison of peak position which derived from the simulation and dispersion relation is presented. We may see that the comparison is in good agreement. Main reason of discrepancies between simulation and dispersion relation is that the relation was obtained in the far field approximation when simulation was performed for certain distance to the observation point.
SUMMARY

- We have observed GDR in the sub-THz range and confirmed the dispersion relation depending on two angular variables (θ, η);
- Monochromaticity of GDR \( \frac{\Delta \nu_k}{\nu_k} \) depends on the diffraction order \( k \) and coincides practically with the SPR monochromaticity;
- Intensity of GDR is comparable with intensity of SPR for the grating rotation angles \( \theta = 0 \div 20^\circ \) and spectral line shift \( \sim 20 \div 25\% \).
- In contrast with conventional SPR there exist two polarization components in GDR
Thanks for attention!