On a Zone Structure for Channeled Particles in Optical Lattices

Dik A.V., Frolov E.N., Dabagov S.B.

23 - 28 September, Channeling 2018

Contents

Optical Lattice

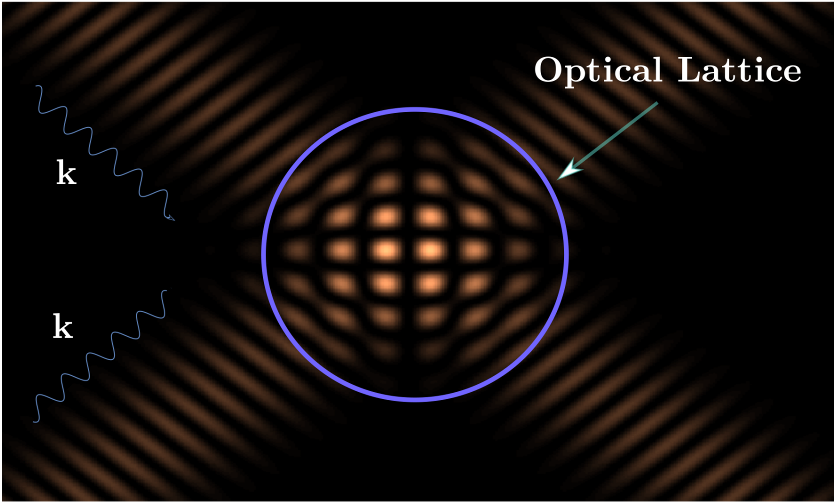

Figure: Two crossed laser beams forming Optical Lattice (OL)

Typical trajectory of classical particle in OL

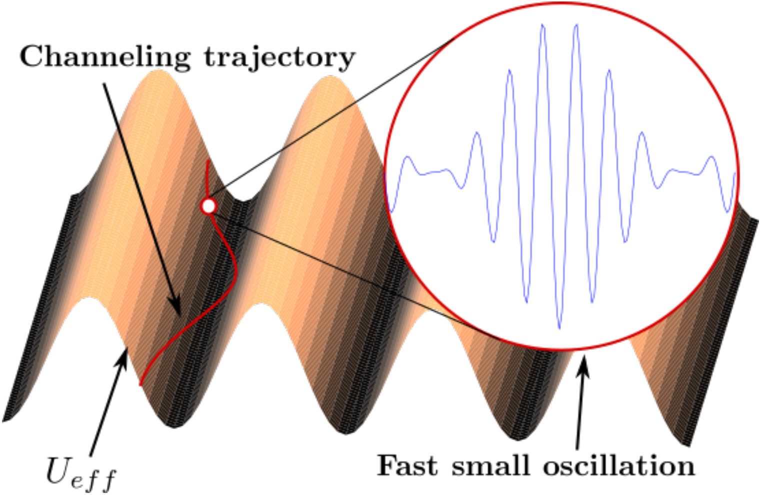

Figure: Typical channeled particle trajectory in OL

Scalar quantum particle in nonuniform field of EM wave

General equation for scalar particle in EM field

$$\left[\left(i\hbar\partial_\mu + \frac{e}{c}\mathcal{A}_\mu\right)\left(i\hbar\partial^\mu + \frac{e}{c}\mathcal{A}^\mu\right) - m^2c^2\right]\Psi = 0$$

In high frequency EM field $\mathcal{A} = (0, \mathbf{A})$, wave function can be presented as $\Psi = \bar{\psi}\chi e^{i(\mathbf{q}_\parallel \mathbf{r}_\parallel - \omega_\parallel t)}$ (suppose $\hbar\omega_\parallel \gg |e\mathcal{A}|$, $\hbar\omega_\parallel = c\sqrt{(\hbar q_\parallel)^2 + (mc)^2}$ - longitudinal energy)

Equation for particle wave function in EM field with constant amplitude

$$i\hbar\frac{\partial\chi}{\partial t} = \left(e\beta_\parallel A_\parallel - \frac{i\hbar e}{\gamma_\parallel mc}\mathbf{A}\nabla\ln\bar{\psi} + \frac{e^2A_1^2}{2\gamma_\parallel mc^2}\right)\chi$$

here $\beta_\parallel = \dfrac{q_\parallel c}{\omega_\parallel}$ - longitudinal velocity, $\gamma_\parallel = \dfrac{\hbar\omega_\parallel}{mc^2}$ - longitudinal Lorentz-factor, A_1^2 - time-oscillating part of the square of the field $\mathbf{A}$

Scalar quantum particle. Slow motion equation

Equation for slow motion particle wave function

$$i\hbar\frac{\partial\bar{\psi}}{\partial t} = \left(-\frac{\hbar^2}{2\gamma_{\parallel} m}\nabla^2 + U_{eff}\right)\bar{\psi}$$

Effective potential[1]

$$U_{eff} = \frac{e^2 A_2^2}{4\gamma_{\parallel} mc^2} - \frac{\hbar^2}{2\gamma_{\parallel} m}\overline{(\nabla\ln\chi)^2} - \frac{i\hbar e}{\gamma_{\parallel} mc}\overline{(\mathbf{A},\nabla\ln\chi)} +$$
$$+\frac{\hbar^2}{2\gamma_{\parallel} mc^2}\left[\overline{\left(\frac{\partial}{\partial t}\ln\chi\right)^2} - 2\beta_{\parallel} c\,\overline{\frac{\partial}{\partial t}\ln\chi\frac{\partial}{\partial\zeta}\ln\chi} + \beta_{\parallel}^2 c^2\overline{\left(\frac{\partial}{\partial\zeta}\ln\chi\right)^2}\right]$$

here $\zeta = z - \beta_{\parallel} ct$, $\omega = \omega_0 - \beta_{\parallel} k_z$ - oscillation frequency of χ, A_2 - slowly changing term of the field $\mathbf{A}$, U_{eff} - complex function, $\mathrm{Im}\,U_{eff} \sim \frac{e^2 A^2}{(\gamma_{\parallel} mc^2)^2}\left(\nabla\bar{\psi}\right)^2$

[1] A.V. Dik, E.N. Frolov, S.B. Dabagov, *JINST*, **Vol. 13**, 2017

Scheme of particle interaction with nonuniform field of EM wave

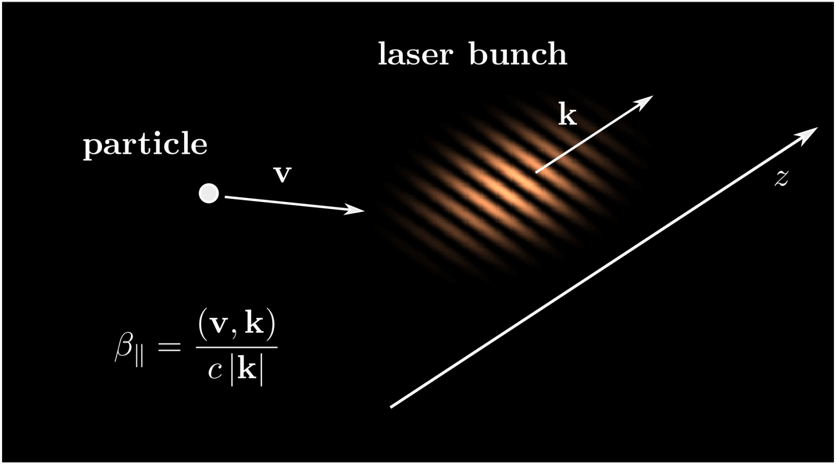

Figure: Nonuniform laser bunch and charge particle

Scheme of particle interaction with OL

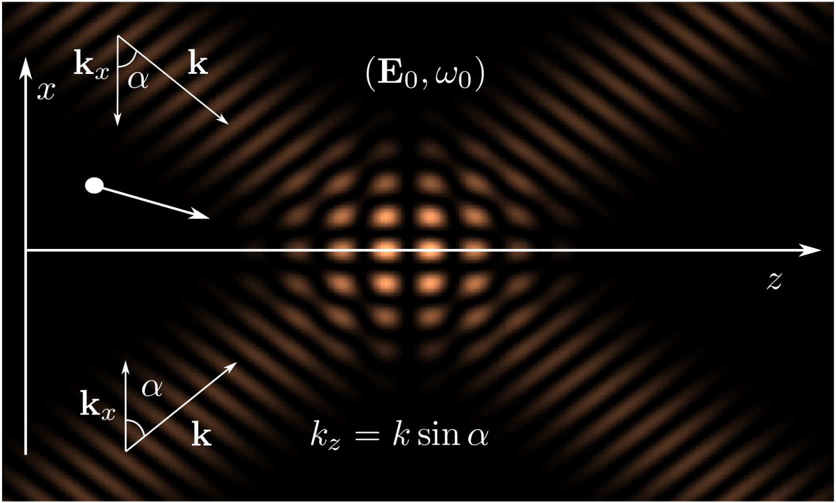

Figure: P-polarized laser field characteristic and geometry

OL formed by two crossed plane EM waves

Vector potential of two crossed plane waves forming OL

$$\mathbf{A} = A_z \sin(\omega_0 t - kz \sin\alpha)\mathbf{e}_z + A_x \cos(\omega_0 t - kz\sin\alpha)\mathbf{e}_x$$

$$A_z = -2\frac{E_0}{k}\cos\alpha \sin(kx\cos\alpha), \quad A_x = 2\frac{E_0}{k}\sin\alpha\cos(kx\cos\alpha)$$

Effective potential corresponding slow/channeling motion[2]

$$U_{eff}(x) = U_0 - \frac{e^2 E_0^2}{\gamma_{||} m\omega_0^2} f(\beta_{||})\cos(2kx\cos\alpha)$$

$$f(\beta_{||}) = \left(\frac{\beta_{||} - \sin\alpha}{1 - \beta_{||}\sin\alpha}\right)^2 - \frac{1}{2}$$

[2] S.B. Dabagov, A.V. Dik, E.N. Frolov, *PRST-AB*, **Vol. 18**, 2015

Potential inversion points

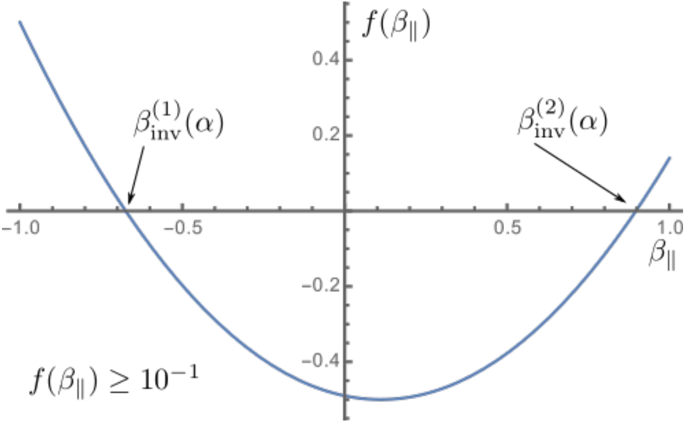

Figure: The characteristic behavior function $f(\beta_{||})$ as a function of $\beta_{||}$. The inversion points determine the sign of the amplitude of the effective potential. At the points of inversion the effective potential equal zero.

Single Channel Potential

Dimensionless equation for planar channeling

$$\frac{d^2\bar{\psi}}{d\xi^2} + \Omega^2 \cos^2(\xi)\bar{\psi} = \lambda\bar{\psi}$$

here $\Omega^2 = \frac{64 I_0 e^2 d_{ch}^2}{\pi \hbar^2 c \omega_0^2} f(\beta_{||})$ - dimensionless amplitude of effective potential (well depth).

$\lambda = \frac{2\gamma_{||} m E_{\perp} d_{ch}^2}{\pi^2 \hbar^2}$ - dimensionless transverse energy

$\xi = \frac{x}{d_{ch}}$ - dimensionless transverse coordinate

$d_{ch} = \frac{\pi}{k \cos \alpha}$ - channel width

Single Channel. Effective potential

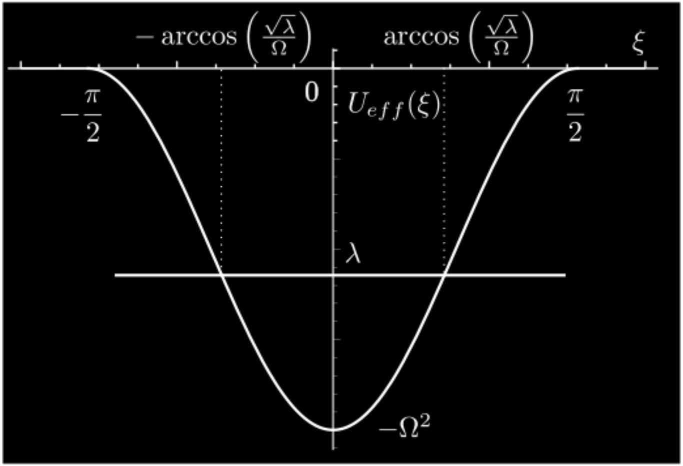

Figure: Dimensionless effective potential of single channel

Single Channel. Solution of transverse motion equation

Solution in quasi-classical approximation

$$\bar{\psi} = \frac{C}{\sqrt{\sigma'}} e^{\pm i\sigma}, \quad \sigma = \int \sqrt{\lambda - \Omega^2 \cos^2 \xi} d\xi = \Omega F \left(\frac{\lambda}{\Omega^2} \right)$$

here $F(\zeta) = 2\sqrt{1-\zeta} E \left(\arccos \sqrt{\zeta}, \frac{1}{1-\zeta} \right)$, $E\,(..)$ - incomplete elliptic integral of the second kind.

equation for energy levels

$$F \left(\frac{\lambda_n}{\Omega^2} \right) = \frac{\pi}{\Omega} \left(n + \frac{1}{2} \right)$$

maximum number of energy levels

$$N_{\max} = \left[\frac{\Omega}{\pi} F \left(\frac{\lambda_{N_{\max}}}{\Omega^2} \approx 1 \right) - \frac{1}{2} \right] \approx 2 \left[\frac{\Omega}{\pi} \right]$$

Single Channel. Dependence potential depth on laser field parameters

$$\Omega^2 = 6.3 \cdot 10^{54} \frac{I_0}{\omega_0^4}$$

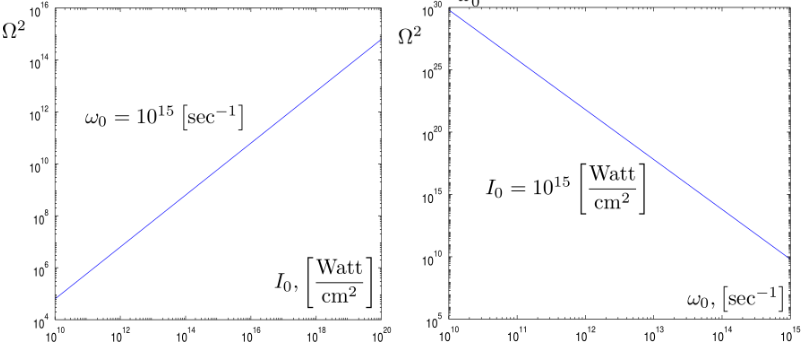

Figure: Dependence of potential well depth on laser field intensity I_0 (Watt/cm^2) and frequency ω_0 (sec^{-1})

Single Channel. Dependence function F on dimensionless energy

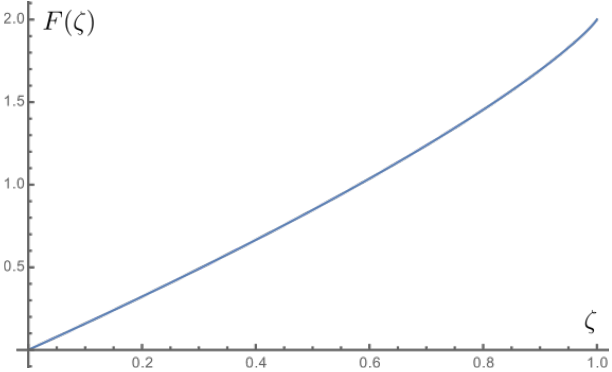

Figure: Dependence of dimensionless function F, which determined particle's energy spectrum, on dimensionless energy $\zeta = \lambda/\Omega^2$

Single Channel. Energy levels spacing

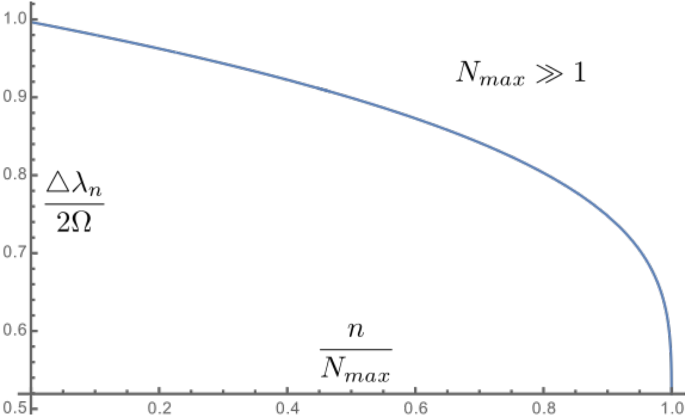

Figure: Dependence of dimensionless energy levels spacing $\triangle\lambda_n = \lambda_{n+1} - \lambda_n$ on normalizing state number n. Near the bottom of the channel energy levels spacing is similar to harmonic oscillator. And for the lower levels is almost doubled compared to the highest levels.

Periodical Potential. Zone Structure

Dispersion equation for zone structure

$$\cos\left(qd_{ch}\right) = 2\cosh\left(\frac{\tau(\lambda)}{2}\right)e^{\tau(\lambda)/2}\cos\left(\sigma(\lambda)\right)$$

here $\tau = 2\sqrt{\lambda}E\left(\frac{\pi}{2} - \arccos\sqrt{\frac{\lambda}{\Omega^2}}, \frac{\Omega^2}{\lambda}\right)$, $E(..)$ - incomplete elliptic integral of the second kind

$\sigma = \Omega F\left(\frac{\lambda}{\Omega^2}\right)$, q - quasi-momentum

d_{ch} - channel width

Zone Structure.

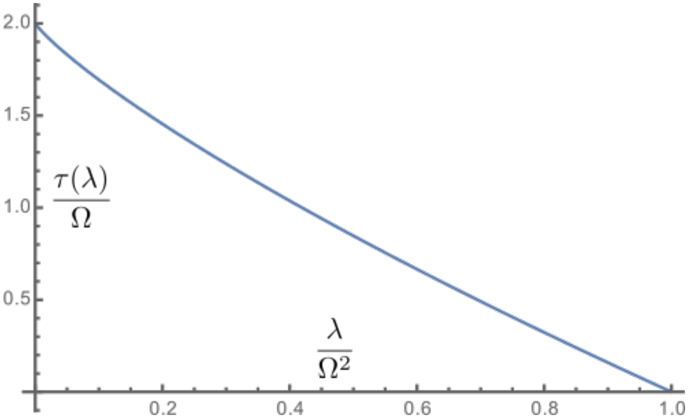

Figure: Dependence of τ on dimensionless energy λ/Ω^2. The value of τ determines the transmission coefficient D through the potential barrier $D \sim e^{-\tau}$.

Zone Structure. Zone width

As follows from the dependence $\tau(\lambda)$, the width of the bands is determined basically Ω and for the intensities of the laser $I_0 \geq 10^{10}$ Watt/cm^2 we can determine the width of the n-th zone as $\triangle\lambda_{nq} \approx \Omega^2 e^{-\tau(\lambda_n)}$

Equation defining the equality of the width of the zone with the distance between levels in an isolated channel

$$\sqrt{\lambda_n} E\left(\frac{\pi}{2} - \arccos\sqrt{\frac{\lambda_n}{\Omega^2}}, \frac{\Omega^2}{\lambda_n}\right) \approx \frac{\ln\Omega}{2\Omega}$$

Zone Structure. Influence of potential periodicity

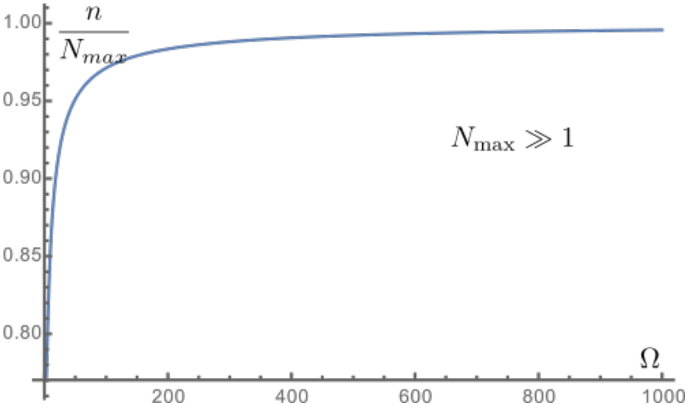

Figure: Dependence of normalizing state *n*, which corresponds to the equality of the distance between levels in an isolated channel with a band width, on dimensionless frequency Ω

Zone structure

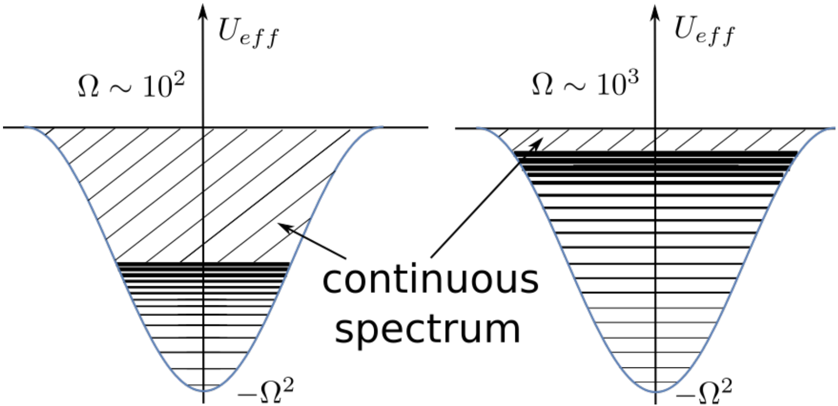

Figure: Schematic plot of the zone structure for different values of dimensionless frequence. For large values of the dimensionless frequency, the continuous spectrum forms high-lying levels

Condition for the applicability of quasi-classical description

Solution in quasi-classical approximation

$$\bar{\psi} = Ce^{\pm i\sigma}, \quad i\sigma'' - (\sigma')^2 + \Omega^2 \cos^2 \xi = \lambda$$

Representation of the wave function phase in the form of decreasing series far from turning points

$$\sigma = \sigma_0 + \sigma_1 + \sigma_2 + \sigma_3 + \ldots \sim \Omega + \ln \Omega + \frac{\ln \Omega}{\Omega} + \frac{\ln \Omega}{\Omega^2} + \ldots$$

For a microwave laser with intensity $I_0 \geq 10^{10}$ Watt/cm^2 the series for wave function phase converges very fast. Hence the presented description is accurate within $\lesssim \frac{\ln \Omega}{\Omega} \approx \frac{3 \ln 10}{10^3}$

Conclusion

1. The states number of channeled particle in OL in region far from the potential inversion points depends mainly on the field parameters, and in case s-polarized laser waves it depends only on the charge of the channeled particle and the field parameters
2. In the field of a microwave laser with intensity $I_0 \geq 10^{10}$ Watt/cm^2 the states number of the channeled particle $N_{\max} \gtrsim 10^3$, and far from the channel bottom the quasi-classical approximation may be used
3. For case $N_{\max} \gg 1$ distances between energy levels decreases by half with increasing state number, and the maximum value corresponds to the distance between the levels of harmonic oscillator
4. For $N_{\max} \gg 1$ the width of the energy zones is significant only for the highest-lying states

Thank you for attention

Deflection angle

Maximum particle deflection angle

$$\alpha_{max} = \frac{\alpha_0}{\gamma_{\|}} \left| \frac{q}{e} \right| \frac{m_e}{m} \frac{\omega_0}{\omega} \sqrt{\frac{I}{I_0}}$$

here e, m_e - electron charge and mass, q, m - charge and mass of deflection particle, $\gamma_{\|}$ - longitudinal Lorentz factor of deflecting particle, I, ω - laser field intensity and frequency.

maximum deflection angle of a nonrelativistic electron

$$\alpha_0 = \alpha_{max} \left(\gamma_{\|} \sim 1, I_0, \omega_0 \right) = \sqrt{\frac{8\pi e^2}{m_e^2 c^3}} \sqrt{\frac{I_0}{\omega_0^2}} = 0.05[rad]$$

α_0 - maximum deflection angle of nonrelativistic electron in field of OL formed by two plane laser waves with intensity $I_0 = 10^{15}$ Watt/cm^2 and frequency $\omega_0 = 10^{15}$ sec^{-1}.

Deflection angle for electron

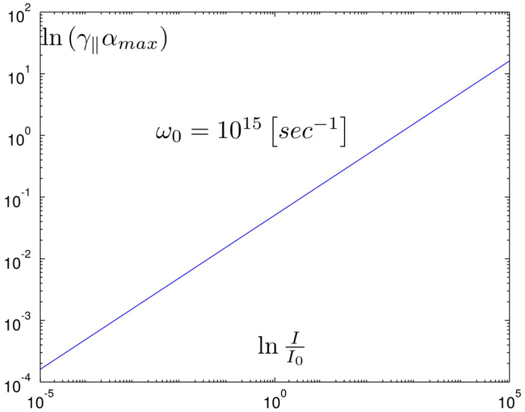

Figure: Dependence of electron angle deflection $\gamma_{\|}\alpha_{max}$ on laser field intensity I for frewuency of laser field $\omega_0 = 10^{15}$ sec^{-1}.

Deflection angle for electron

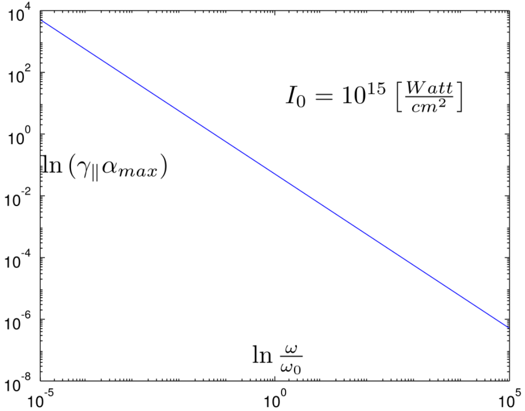

Figure: Dependence of electron angle deflection $\gamma_{\|}\alpha_{max}$ on laser field frequency ω for intensity of laser field $I = 10^{15}$ Watt/cm^2.

Depth of a potential well

depth of a potential well

$$U_{max} = \frac{q^2}{e^2}\frac{m_e}{m}\frac{I}{I_0}\frac{\omega_0^2}{\omega^2}\frac{U_0}{\gamma_{||}}$$

here e, m_e - electron charge and mass, q, m - charge and mass of channeled particle, $\gamma_{||}$ - longitudinal Lorentz factor of channeled particle, I, ω - laser field intensity and frequency.

depth of a potential well for a nonrelativistic electron/positron

$$U_0 = U_{max}\left(\gamma_{||} \sim 1, I_0, \omega_0\right) = \frac{8\pi e^2 I_0}{m_e c \omega_0^2} = 1.3[KeV]$$

U_0 - depth of a potential well for nonrelativistic electron in field of OL formed by two plane laser waves with intensity $I_0 = 10^{15}$ Watt/cm^2 and frequency $\omega_0 = 10^{15}$ sec^{-1}.