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X-Ray Optics Lab

Influence of uncertainties on optimized parameters in X-ray differential phase contrast imaging system

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Outline

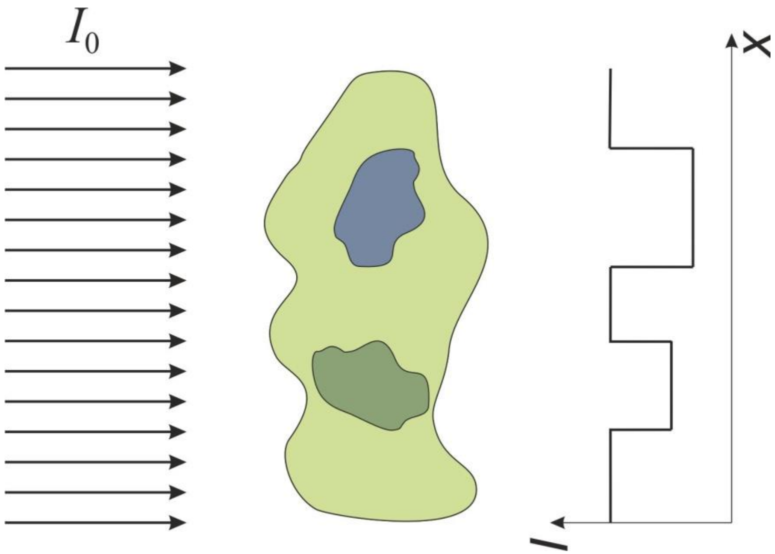
15 slides

1. X-ray imaging and tomography
2. Talbot effect as the engineering basics
3. Deterministic design
4. Uncertainty quantification
5. Uncertainty based design
6. Comparison of uncertainty and deterministic strategy
7. Conclusion: examples of parameters shift

X-ray imaging techniques

Approaches to X-ray imaging

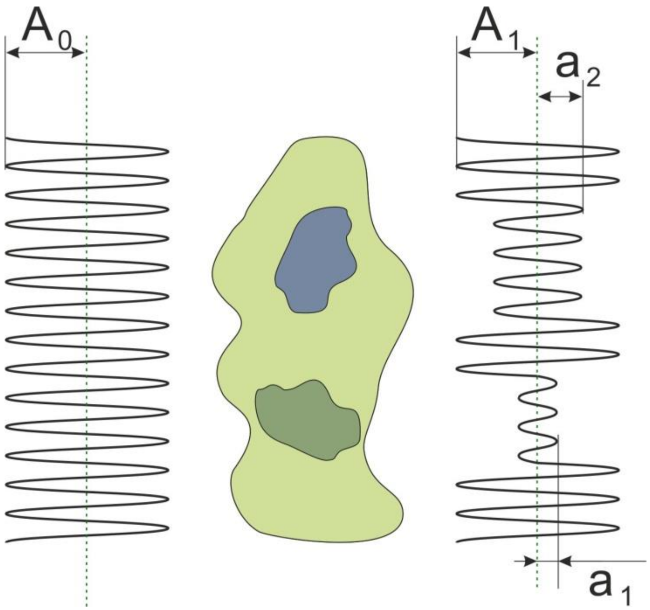
Absorption based technology



X-ray imaging techniques

Approaches to X-ray imaging

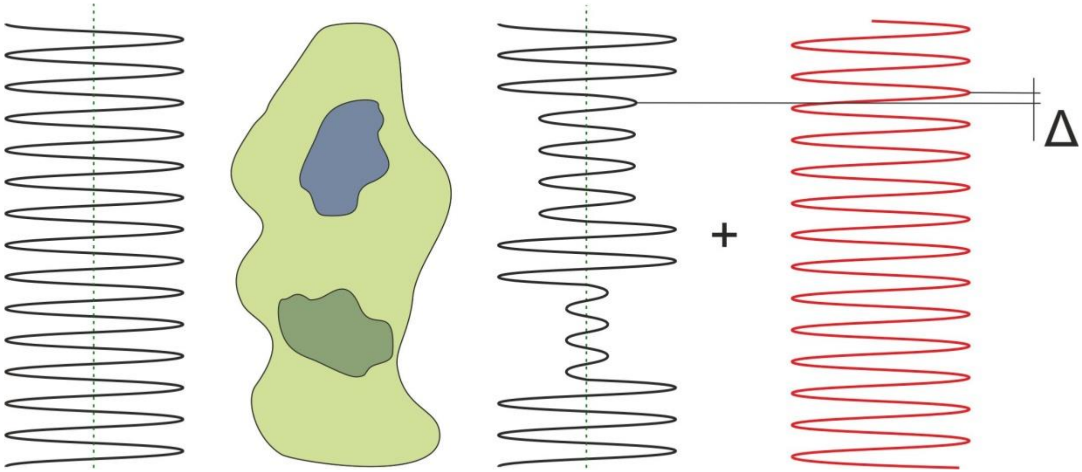
Scattering based technology



X-ray imaging techniques

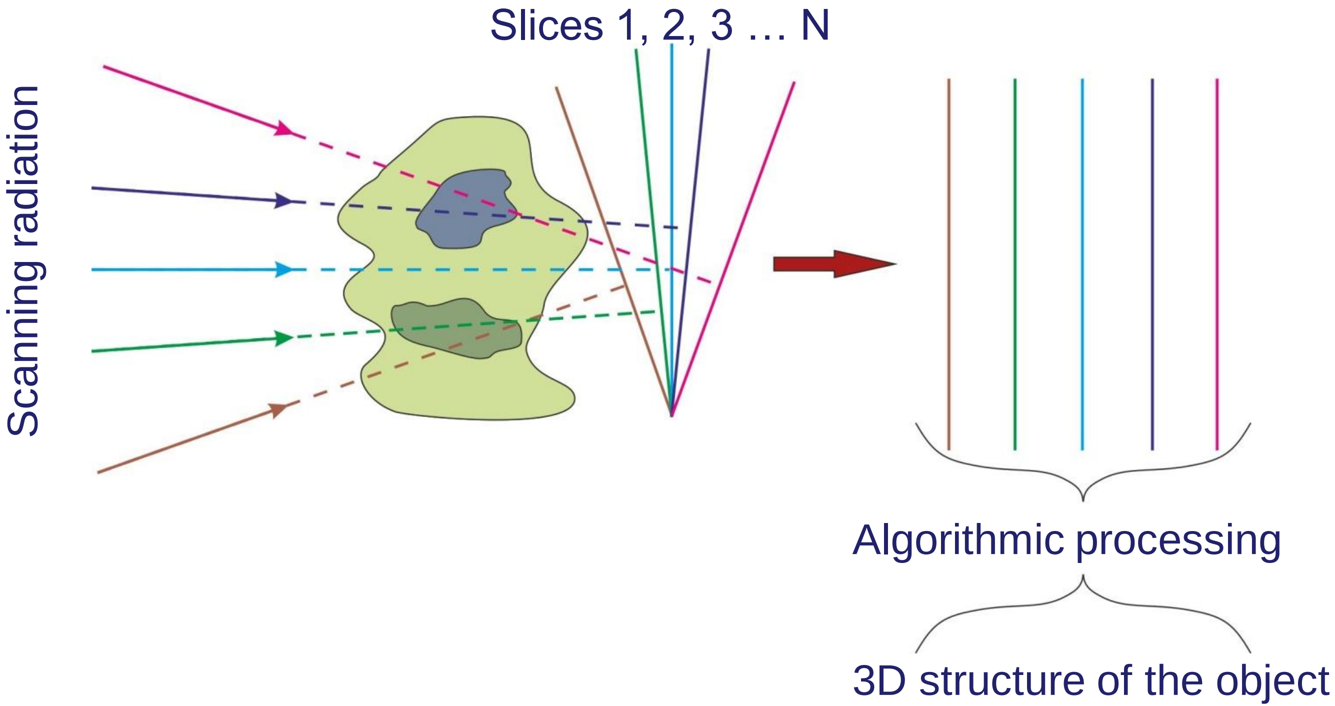
Approaches to X-ray imaging

Interference based technology



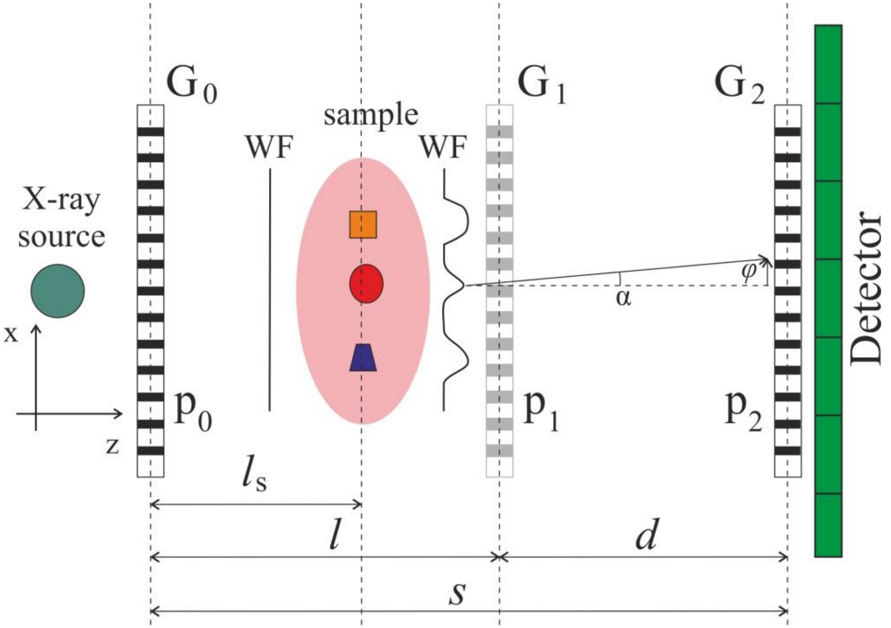
Tomography principles

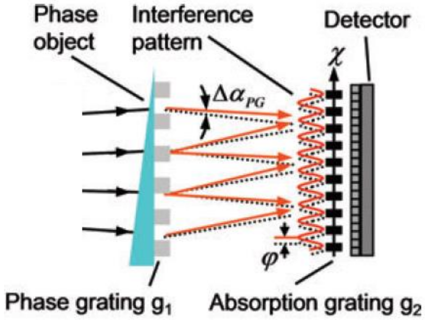
No matter which method of imaging used – principles are similar



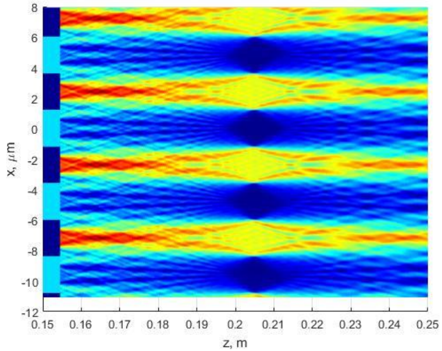
Engineering of basics

Design of a technical supporting system to provide a better characteristics





Talbot effect



Deterministic design

Some details of optimization techniques

The transmission functions of the phase grating

$$T_1(x) = \exp(i\varphi U_1(x))$$

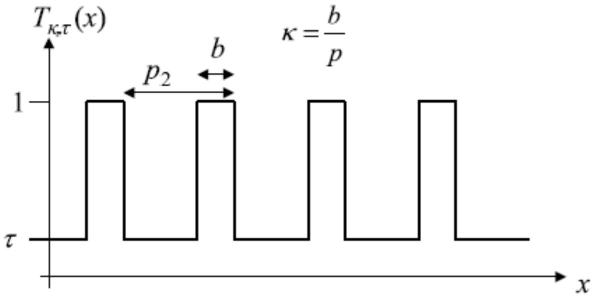
The wave field downstream

$$\psi_z(x) = I_0(T_1(x) * P_z(x))$$

$$P_z(x) = \frac{1}{i\lambda z}\exp(i\pi\frac{x^2}{\lambda z})$$

The projected source intensities for Talbot (T) and Talbot-Lau (TL)

$$S'_T(x) = \frac{1}{\sigma'_s\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma'^2_s}\right) \qquad S'_{TL}(x) = T_{\kappa_0,\tau_0}(x)$$



The fringe visibility of the phase-stepping curve

$$V = 2\frac{|\hat{I}_P[1]|}{|\hat{I}_P[0]|} \qquad V^*_{S,TL} = \frac{\kappa_0(1-\tau_0)\,\mathrm{sinc}(\kappa_0)}{\tau_0 + \kappa_0(1-\tau_0)}$$

$$V^*_{S,T} = \exp\left(-2\left(\pi\frac{d}{l}\frac{\sigma_s}{p_2}\right)^2\right)$$

The smallest detectable refraction angle

$$\alpha_{\min,T} = \frac{1}{2\pi}\frac{p_2^2}{V_T(p_2 - p_{11})}\frac{\sqrt{2}}{a\sqrt{I_0 D t_{\exp}}} \qquad \alpha_{\min,TL} = \frac{1}{2\pi}\frac{p_0 + p_2}{V_{TL}}\frac{\sqrt{2}}{a\sqrt{I_0 D t_{\exp}}}$$

Thüring T, Stampanoni M 2014 *Phil. Trans. R. Soc. A* **372** 20130027

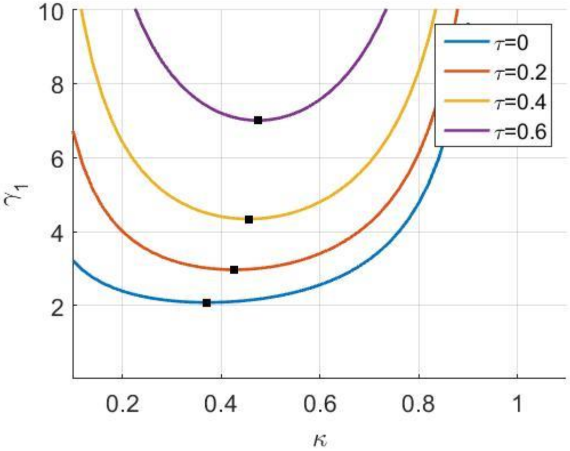
Deterministic design

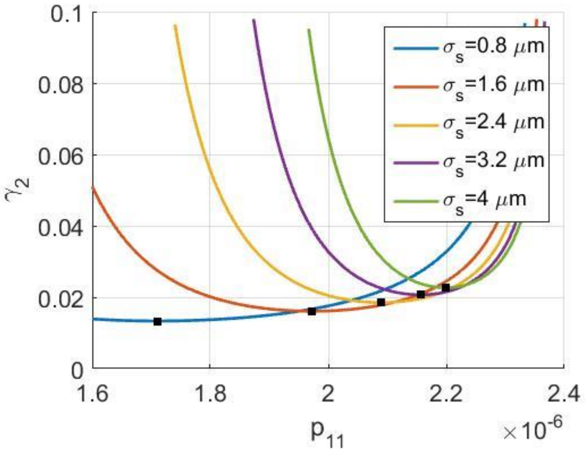
Optimization of the geometry

Minimization of standard deviation of the refraction angle

$$\sigma_{\alpha,\mathrm{T}} = \frac{\gamma_1(\kappa_2,\tau_2)\cdot\gamma_2(p_1,p_2,\sigma_s)}{4a\sqrt{2t_{\mathrm{exp}}}} \qquad \sigma_{\alpha,\mathrm{TL}} = \frac{\gamma_1(\kappa_0,\tau_0)\cdot\gamma_1(\kappa_2,\tau_2)\cdot\gamma_3(p_0,p_2)}{4a\sqrt{2I_0 t_{\mathrm{exp}}}}$$

$$\gamma_1(\kappa,\tau) = \frac{\sqrt{\tau+\kappa(1-\tau)}}{\kappa(1-\tau)\,\mathrm{sinc}(\kappa)} \qquad \gamma_2(p_{11},p_2,\sigma_s) = \frac{p_2^2}{p_2-p_{11}}\,\frac{\exp\left(2\left(\pi\sigma_s\frac{p_2-p_{11}}{p_{11}p_2}\right)^2\right)}{\sqrt{\varrho\sigma_s}} \qquad \gamma_3(p_0,p_2) = p_0+p_2$$

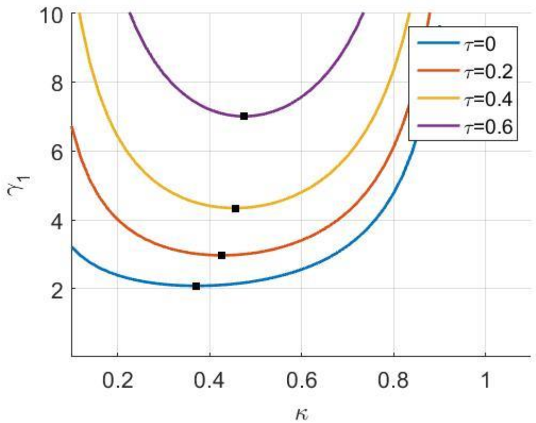


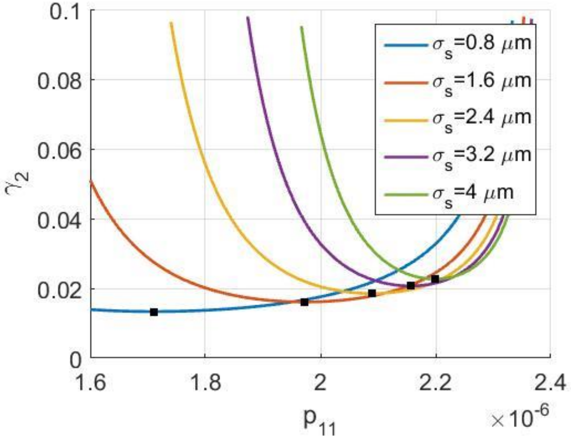


Deterministic design

Thüring T, Stampanoni M 2014 *Phil. Trans. R. Soc. A* **372** 20130027

Optimization of the geometry





Parameters of G2 grating

p_2, μm	τ_2	κ_2	$\min \gamma_1$
2.4	0	0.371	2.0822
	0.2	0.426	2.9677
	0.4	0.455	4.3387
	0.6	0.474	7.0023

Period of G1 grating and std source size

p_{11}, μm	σ_s, μm	$\min \gamma_2$
1.711	0.8	0.0133
1.973	1.6	0.0161
2.09	2.4	0.0185
2.156	3.2	0.0207
2.199	4	0.0227

→ Maximum visibility

Uncertainty quantification

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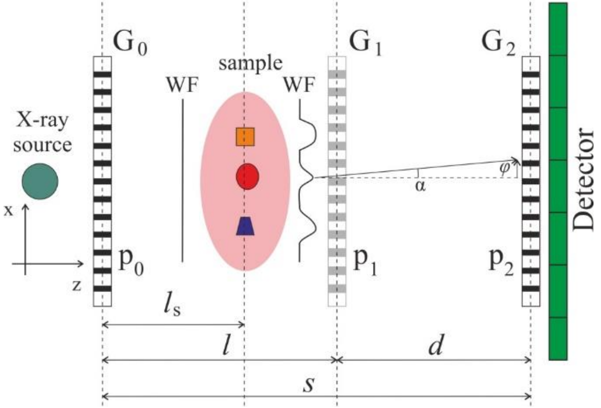
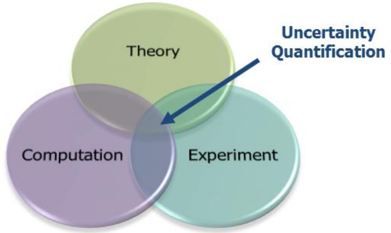
The interference based technology is very sensitive to deviations

A lot of inputs with uncertainties



UQ is the end-to-end study of the impact of all forms of error and uncertainty in the models arising in the applications



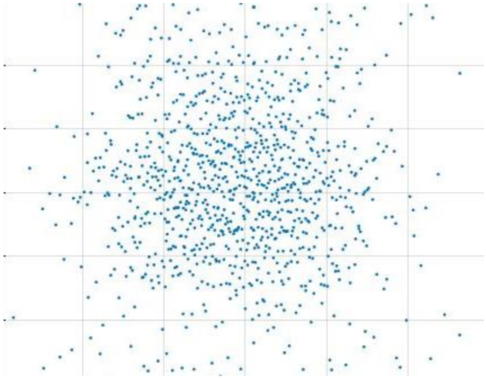
Multidimensional problems – “curse of dimensionality”

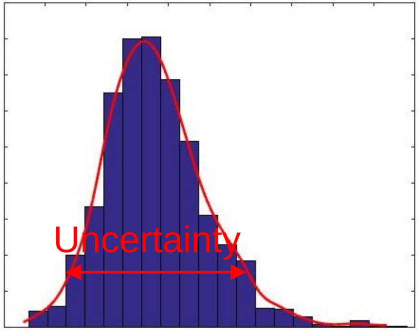
Uncertainty-based design

Parameters with uncertainties of Talbot interferome

G_2 grating	Duty cycle	$\kappa_2 \sim Normal(\mu_{\kappa 2}, 5\%), \mu_{\kappa 2} \in [0.02, 0.5]\ \mu m$
	Transparent coef.	$\tau_2 \sim Normal(\mu_{\tau 2}, 5\%), \mu_{\tau 2} \in [0.02, 0.5]\ \mu m$
	Period	$p_2 \sim Normal(2.4\ \mu m, 5\%)$
G_1 grating	Period	$p_{11} \sim Normal(\mu_{p2}, 5\%), \mu_{p2} \in [1.2, 2.4)\ \mu m$
Std of Source size		$\sigma_s \sim Normal(0.8, 5\%)$

Uncertainty Quantification





Std of the refraction angle

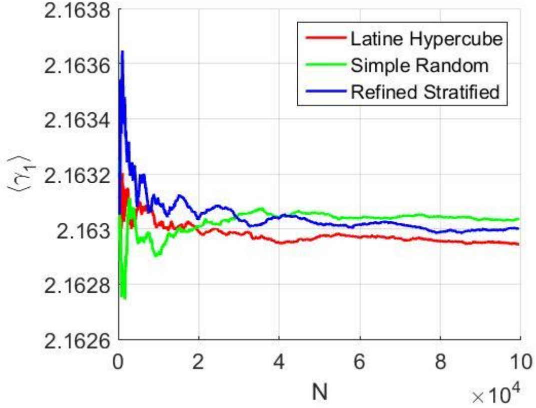
Refined Stratified sampling

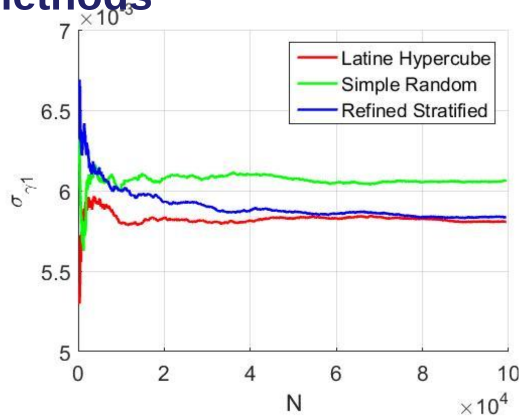
Latin hypercube sampling

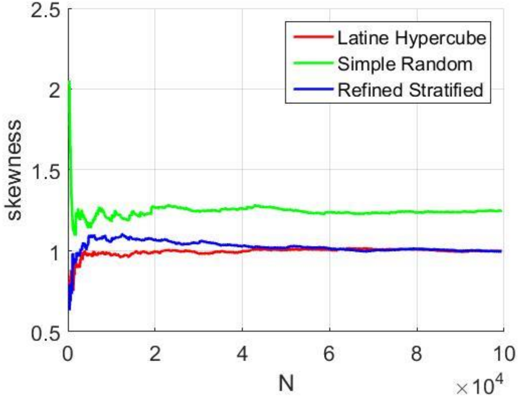
Simple random sampling

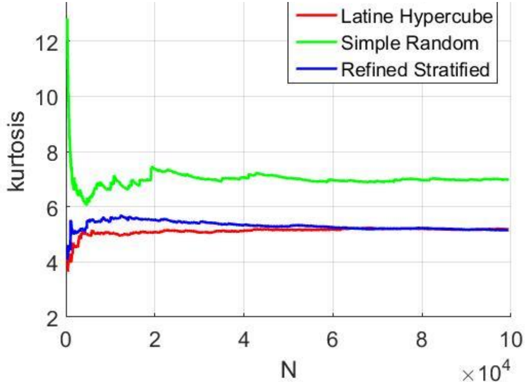
Uncertainty-based design

Comparison of different methods









Statistics	Simple Random sampling	Latin hypercube sampling	Refined Stratify Sampling
$\langle\gamma 1\rangle$	2.1630	2.1629	2.1630
$\sigma_{\gamma 1}$	0.061	0.058	0.058
Skewness	1.2442	0.99	0.9953
Kurtosis	6.9594	5.1841	5.1421

Uncertainty-based design

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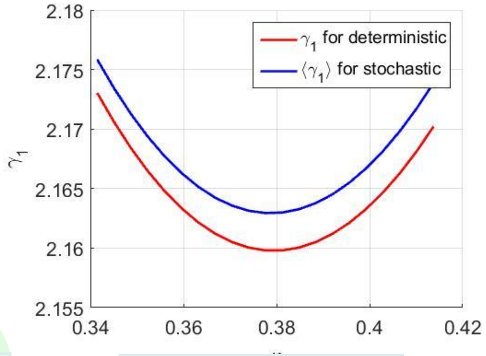
UQ

Std of Source size $\sigma_s \sim Normal(0.8, 5\%)$

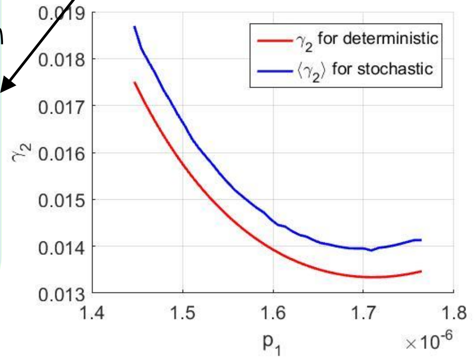
G_2 grating period $p_2 \sim Normal(2.4\ \mu m, 5\%)$

Design G1 parameters to maximize visibility

std of source size	Exact solution		Uncertainty solution		
σ_s, μm	p_{11}, μm	min γ_2	$\langle p_{11}\rangle$, μm	σ_{p11}, μm	min$\langle \gamma_2\rangle$
0.8	1.7105	0.0133	1.7075	5%	0.0138
1.6	1.973	0.0161	1.9457	5%	0.0173
2.4	2.0897	0.0185	2.0121	0.15%	0.0223
3.2	2.1561	0.0207	-	-	-
4	2.1979	0.0227	-	-	-



4% difference



21% difference

Instead of conclusion: an example

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Redesign parameters to recover initial visibility

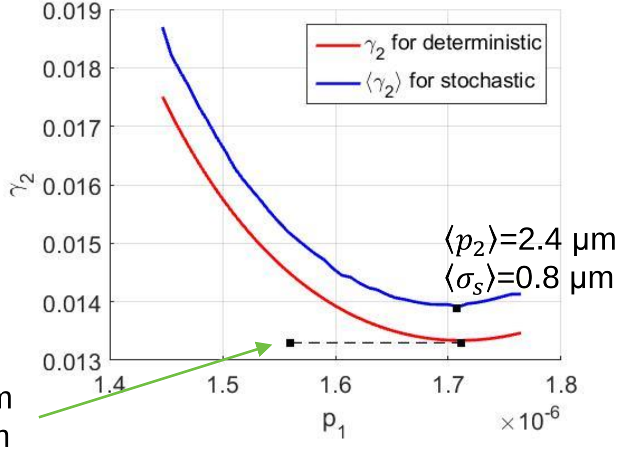
Std of source size	$\sigma_s \sim Normal(0.8\ \mu m, 5\%)$
Period G_2	$p_2 \sim Normal(2.4\ \mu m, 5\%)$

Solution with uncertainty / Deterministic solution

$\langle p_{11} \rangle$, μm	$\min\langle \gamma_2 \rangle$	p_{11}, μm	$\min \gamma_2$
1.7075	0.0138	1.7105	0.0133

Redesigned parameters to rich initial visibility

$\langle p_{11} \rangle$, μm	$\langle p_2 \rangle$, μm	$\langle \sigma_s \rangle$, μm	$min\langle \gamma_2 \rangle$
1.56	2.16	0.77	0.01334



Waiting for experiments…

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The very last slide

Thanks for attention!