Proton-boron fusion reactions in solid matter, laser-plasmas and quantum plasmas

The 8th International Conference
Channeling 2018

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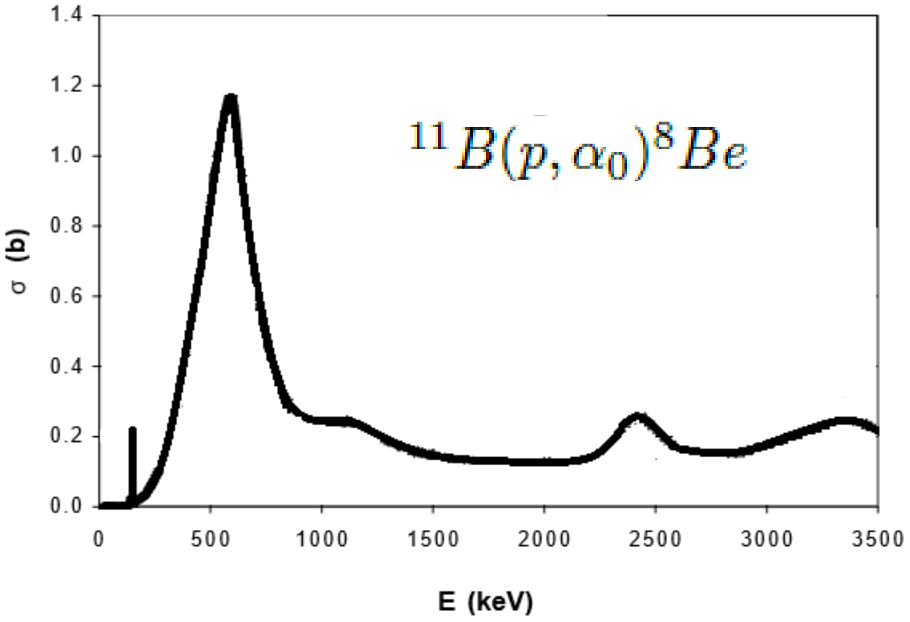
Summary

The proton-Boron fusion reaction is interesting in the framework of the production of nuclear energy for civil purposes, because aneutronic.

We refer to proton-Boron collisions, not to thermonuclear reactions.
Nuclear fusions can be produced during the interaction of pulsed lasers with plasmas.
Schemes for proton-Boron collisions in laser-plasmas have been successfully tested already.
Same reactions induced in solid B targets by proton beams as well.

Here we present a summary of theoretical concepts useful to answer the question:
When is it more advantageous to induce proton-boron reactions in laser-plasmas than in solid targets by means of proton beams?

Proton-boron fusion cross-section



The fusion cross section is often parametrized as:

$$\sigma(E) = \frac{S(E)}{E} \exp(\gamma) \tag{1}$$

where E is the center of mass energy and γ is the Gamow factor defined as:

$$\gamma = \frac{1}{\hbar} \int_{r_0}^{b} \sqrt{2M\,(V(r) - E)}dr \tag{2}$$

Eq. 2 makes use of the assumption of central potential (both for electromagnetic and nuclear interactions), $r_0 = 1.07A^{1/3}F$ is the nuclear radius (with $A = A_1 + A_2$ the sum of the mass numbers of the colliding nuclei; $F = 10^{-15}m$ is the Fermi unit) and b is the minimum impact parameter for an electromagnetic collision between two charged particles (here we are interested only in non-relativistic collisions).

$$b = max\left\{\frac{Z_1 Z_2 e^2}{2\pi\varepsilon_0 M\beta^2 c^2}, \frac{\hbar}{2\gamma M\beta c}\right\}$$

The minimum impact parameter is chosen as the max between the turning-point distance and the De Broglie wavelength (approximately)

Enhancement factor to the cross-section

$$\sigma(E) \rightarrow f \times \sigma(E)$$

$$f = \exp\left[2\pi\eta\left(1 - \frac{\gamma_{p,t}}{\gamma_0}\right)\right]$$

Sommerfeld Parameter

$$\eta = \alpha Z_1 Z_2 \sqrt{Mc^2/E}$$

In plasmas:

Screened Coulomb Potential

$$V(r) = \frac{Z_2 e}{4\pi\varepsilon_0 r}\exp(-r/\lambda_D)$$

Debye length

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{n_e < Z > e^2}}$$

Plasma Gamow Factor

$$\gamma_p \sim \gamma_0\left(1 - \frac{r_0}{2\lambda_D}\right)$$

Gamow integral in vacuum

$$\gamma_0 = \frac{2\sqrt{2M}Z_1 Z_2 e^2}{4\pi\varepsilon_0 \hbar\sqrt{E}} g\left(\frac{r_0}{b}\right)$$

Some considerations and calculations

When considering a boron plasma of temperature 1 keV and density comparable to the solid density of the solid boron in normal conditions, we get $\lambda_D \sim 2.9$ Å and $r_0 \sim 2.5$ F, then the screening effects are completely negligible and no fusion enhancement occurs ($f \sim 1$). The situation in matter is completely different since the screening factor for a proton energy $E \sim 675$ keV, close to the resonance for fusion as in Fig. 1, can be estimated $f \sim 4$. In other words the minimum impact parameter in solid state matter is shorter by a factor $1 - U_e/2E$ than in a laser-produced plasma, increasing the rate of nuclear collisions and fusion reactions by a factor 4.

In solids

Constant Atomic Screening

$$V(r) - U_e$$

Following (Barker, 2002) the screening potential of ^{11}B is $U_e \sim 225$ eV.

Solid Gamow Factor

$$\gamma_t = \frac{1}{\hbar}\int_{r_0}^{b}\sqrt{2M\left(V(r) - U_e - E\right)}dr \sim \gamma_0\left(1 - \frac{U_e}{2E}\right)$$

Proton range and fusion probability

Fermi formula for energy losses

$$-\frac{dE}{dx} = -\frac{2}{\pi}\frac{Z_1^2\alpha\hbar}{\beta^2 c} Re\left[\int_0^\infty i\omega\kappa^* b K_1(\kappa^* b) K_0(\kappa b)\left(\frac{1}{\varepsilon(\omega)} - \beta^2\right) d\omega\right]$$

Plasma environment

Dielectric function

Solid environment

$$\psi = \nu_{ei}/\omega$$

$$\varepsilon_p(\omega) = 1 - \frac{\omega_p^2}{\omega^2}\left(\frac{1}{1+i\psi} + \sum_j \frac{\omega^2 f_j}{\omega_j^2 - \omega^2 - i\omega\Gamma_j}\right)$$

Proton losses in plasmas

$$-\left(\frac{dE}{dx}\right)_p = \frac{2Z_1^2\alpha\omega_p^2\hbar}{\pi\beta^2 c}\left[ln\left(\frac{2\beta c}{b\omega_p}\right) - \gamma_E\right] + 4\pi\frac{n_e Z_1^2\alpha^2\hbar^2}{m_e\beta^2} ln\left[\frac{2\beta^2 m_e c^2}{< I >}\right]$$

Proton losses in solid targets

$$-\left(\frac{dE}{dx}\right)_t = 4\pi\frac{n_t Z_1^2\alpha^2\hbar^2}{m_e\beta^2} ln\left[\frac{2\beta^2 m c^2}{< I >}\right]$$

Proton range in plasmas

$$R_p(\beta) = \int_E^0 \frac{dE}{-\left(\frac{dE}{dx}\right)_p} \sim \frac{\exp(4\gamma_E)\pi M b^4\omega_p^2}{32 c Z_1^2\alpha\hbar} Ei\left[-4\left(\gamma_E + ln\left(\frac{b\omega_p}{2\beta c}\right)\right)\right]$$

Proton range in solid targets

$$R_t(\beta) \sim \frac{M < I >^2}{32\pi\alpha^2 m_e c^2\hbar^2 n_t Z_1^2} Ei\left[2 ln\left(\frac{2 m_e\beta^2 c^2}{< I >}\right)\right]$$

Fusion probability comparison

Enhancement factor → $f_{p,t}(\beta)$

Fusion cross section → $\sigma(\beta)$

Boron number density → $n_{p,t}$

Proton range → $R_{p,t}(\beta)$

$$P_{p,t}(\beta) = n_{p,t} \int_E^0 \frac{dE f_{p,t}(\beta)\sigma(\beta)}{-\left(\frac{dE}{dx}\right)_p}$$

The number of fusion reactions over a length corresponding to the proton range

Plasma temperature $1\ keV$

Proton energy $E \sim 675\ keV$

We consider proton beams impinging to a plasma or a solid-target slab

Both plasma and solid at solid-state density (of Boron)

Proton Range in Boron plasmas

$$R_p \sim 1.7\ \mu m$$

Proton Range in solid Boron

$$R_t \sim 5.3\ \mu m$$

Fusion Probablity in Boron plasmas

$$P_p(675\ keV) \sim 2.2 \times 10^{-5}$$

Fusion Probability in solid Boron

$$P_t(675\ keV) \sim 3 \times 10^{-4}$$

Relativistic quantum plasmas

Dielectric function to be inserted into the Fermi formula…

$$\varepsilon_p(\omega) = \frac{2c^2}{\omega(\omega + i\nu)\lambda_c^2}\left(\sqrt{(1+\beta_F^2)^2 + \frac{(i\nu\omega + \omega^2 - \omega_p^2)\lambda_c^2}{c^2}} - (1+\beta_F^2)\right)$$

…together with the general loss rate ν, giving the width of the plasma resonance.

We consider a plasma temperature $10\ eV$

Electron plasma density
10^3 greater than the solid-boron density

Proton Range

$R_p \sim 2\ nm$

Fusion Probability

$P_p(675\ keV) \sim 5 \times 10^{-5}$

Electron plasma density
10^7 greater than the solid-boron density

Proton Range

$R_p \sim 50\ F$

Fusion Probability

$P_p(675\ keV) \sim 0.001$

Conclusions

It was already clear that p-B fusion reactions, induced by proton beams impinging on Boron are favored when Boron is solid, due to atomic screening, lost in plasmas.
We wanted especially to investigate when the opposite scenario becomes true.

We summarized few concepts and showed few calculations helping to find the answer.
These concepts could be important both for fusion reactions occurring in laser-plasmas as well as in astrophysical plasmas (nuclear photosynthesis).

Only within extreme plasma environments the Debye shielding becomes effective.

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Thanks for your attention

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