Diffraction radiation for 2D transverse beam size diagnostics

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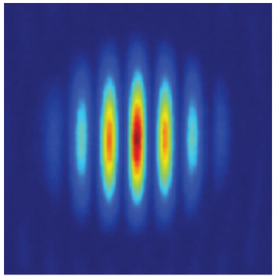
"…splitting two polarization components it becomes possible to measure vertical and horizontal beam sizes independently."

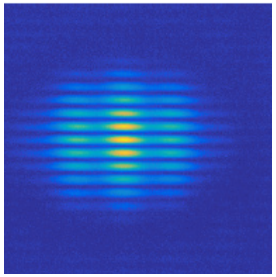
P. Karataev, et al., Diffraction radiation from a charged particle moving through a rectangular hole in a rectangular screen, NIM B (2005)

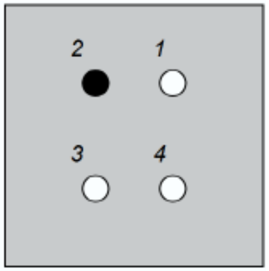
"The beam size and beam position affect the distribution distortion in a similar way…"

P. Karataev, et al., Beam-Size Measurement with Optical Diffraction Radiation at KEK Accelerator Test Facility, PRL (2004)

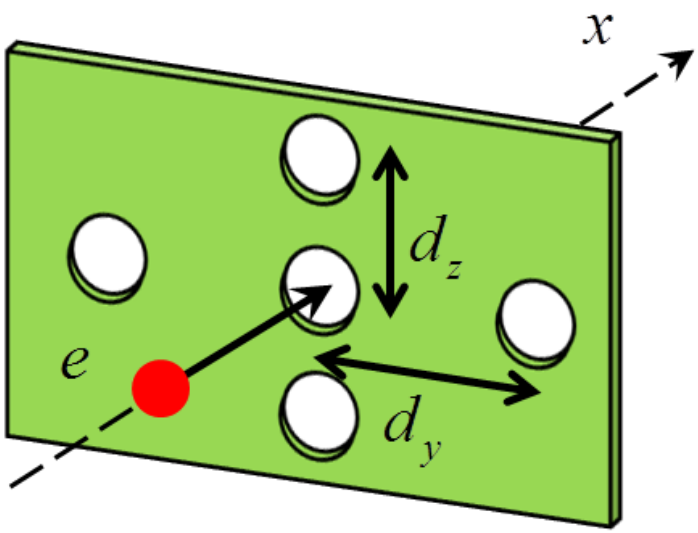
- Two periods ⇨ two beam sizes
- Synchrotron radiation
- Full 2D fit with theory is needed







A.I. Novokshonov, A.P. Potylitsyn, G. Kube, Proc. of IPAC (2017)



What is supposed:

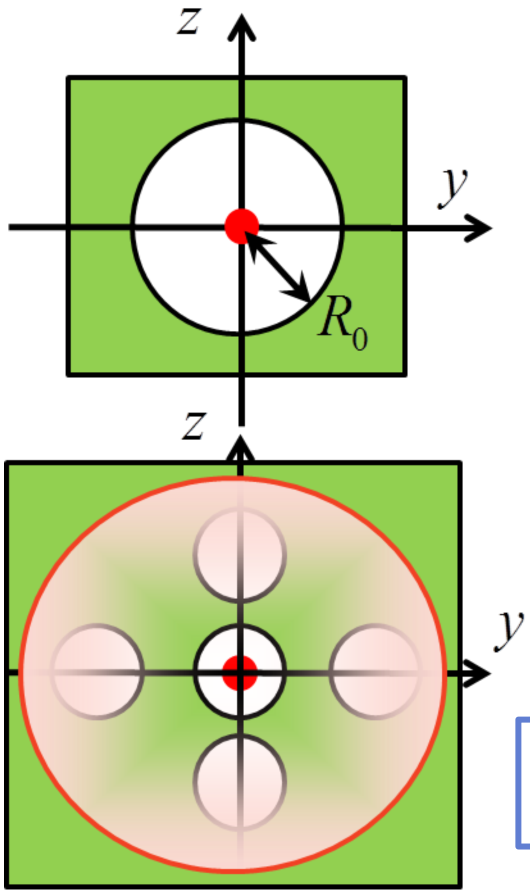
- d_y and d_z are different
- Interference
- *R_0 (central) and R_s (side) could be different*
- *Non-central motion*

What else should be considered:

- Circular currents
- Beam
- Beam divergence
- Metal structure

Fast estimation

$$\mathbf{E}_{rad}(\mathbf{r},\omega) = -\frac{2e}{c\gamma}\frac{e^{ikr}}{r}\frac{\mathbf{e}_y\cos\theta + \mathbf{e}_z\sin\theta}{1-\beta^2\cos^2\theta}\left[R_0 k_\perp J_0(R_0 k_\perp) K_1\left(\frac{R_0\omega}{v\gamma}\right) + \frac{R_0\omega}{v\gamma} J_1(R_0 k_\perp) K_0\left(\frac{R_0\omega}{v\gamma}\right)\right]$$



$$\lambda\beta\gamma >> 2\pi d_{y,z}$$

$$\Downarrow$$

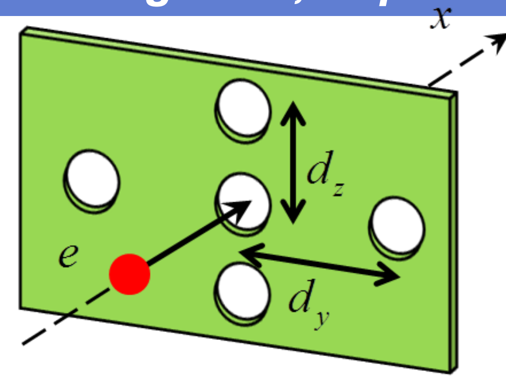
$$\mathbf{E}_{tot}(\mathbf{r},\omega) = \mathbf{E}_0(\mathbf{r},\omega) + \sum_{s=1}^{4}\exp[i\delta_s]\mathbf{E}_0(\mathbf{r},\omega)$$

$$\delta_s = \pm k_{y,z} d_{y,z}$$

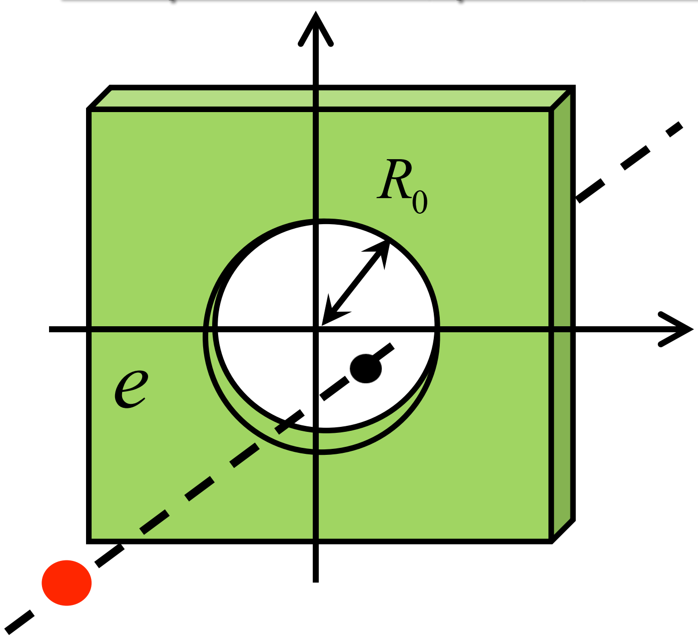
$$\mathbf{E}_{tot}(\mathbf{r},\omega) = \mathbf{E}_0(\mathbf{r},\omega)\left(1 + 2\cos\left(k_y d_y\right) + 2\cos\left(k_z d_z\right)\right)$$

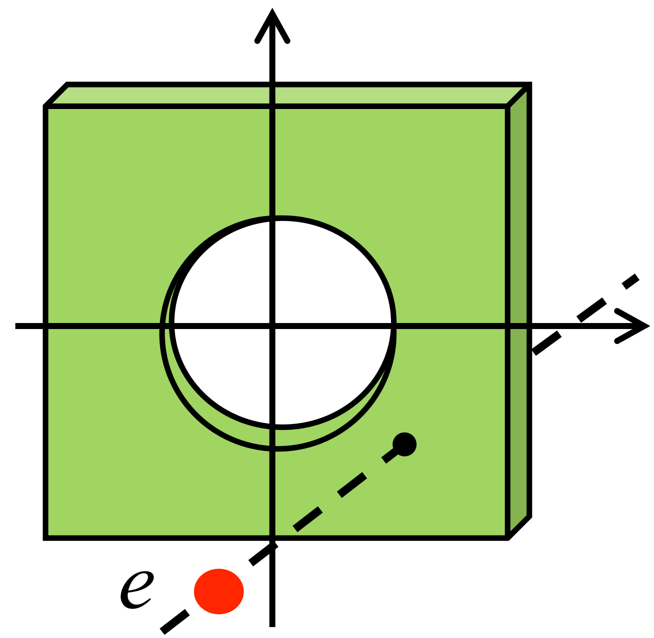
A.P. Potylitsyn, M.I. Ryazanov, M.N. Strikhanov, A.A. Tishchenko, Diffraction radiation from relativistic partciles, Springer, 2011.

Arbitrary lengths d



The problem is splited into two:



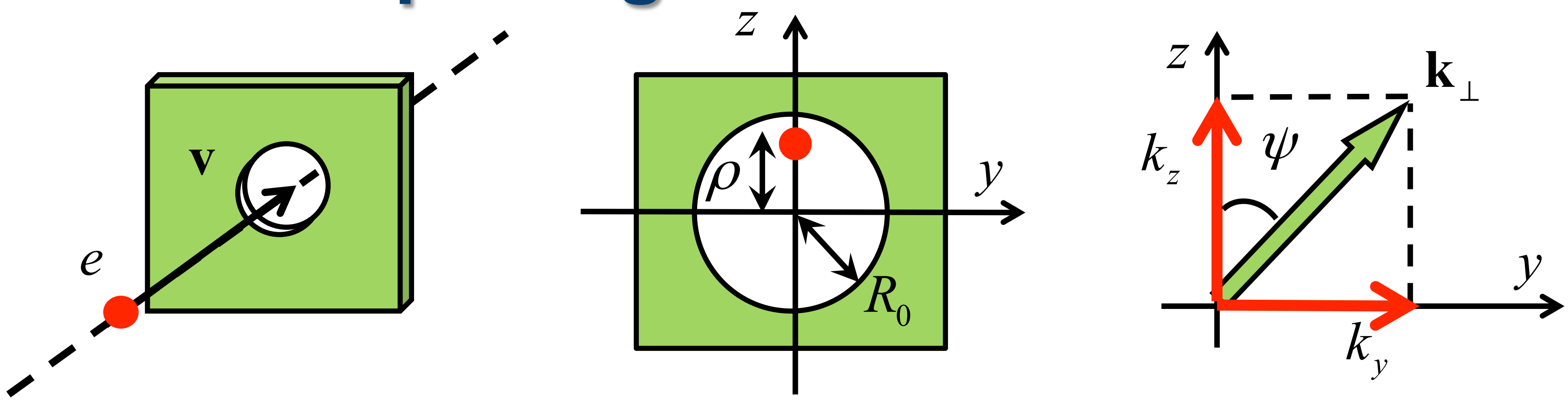


Non-central moving

For $R_0 \rightarrow 0$ goes to TR

M.L. Ter-Mikaelyan, High-Energy Electromagnetic Processes in Condensed Media. Wiley-Interscience, New York, NY (1972).

Non-central passage

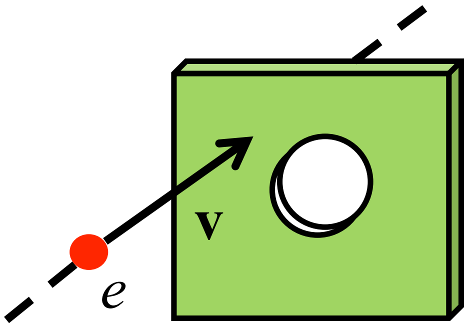


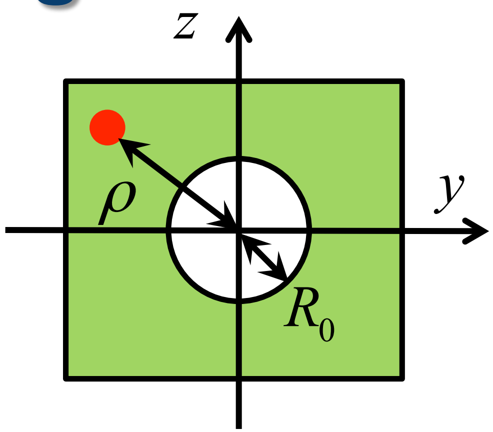
$$\mathbf{E}_0(\mathbf{r},\omega) = -\frac{e(\varepsilon(\omega)-1)}{2\pi v}\frac{e^{ikr}}{r}\frac{2\sin\left(\frac{a\varphi}{2}\right)}{\varphi}\frac{\sum_{m=0}^{+\infty} C_m\left[\mathbf{n}',\left[\mathbf{n}',B_m\hat{\mathbf{L}}\,I_m\left(\frac{\omega}{c\beta\gamma}\rho\right)\right]\right]\cos(m\psi)}{n_y^2+n_z^2+\beta^{-2}\gamma^{-2}}$$

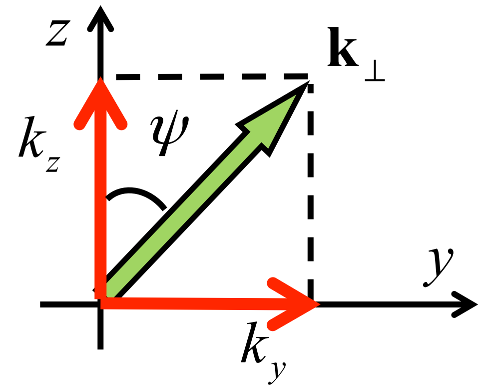
$$B_m = R_0 k_\perp J_{m-1}(R_0 k_\perp) K_m\left(\frac{\omega}{c\beta\gamma}R_0\right) + \frac{\omega R_0}{c\beta\gamma}K_{m-1}\left(\frac{\omega}{c\beta\gamma}R_0\right) J_m(R_0 k_\perp)$$

$$\hat{\mathbf{L}} = i\frac{\omega}{c\beta\gamma^2}\mathbf{e}_x + \frac{1}{\rho}\frac{\partial}{\partial\psi}\mathbf{e}_y - \frac{\partial}{\partial\rho}\mathbf{e}_z \qquad C_m = \begin{cases} 2(-i)^m, & m \neq 0 \\ 1, & m = 0 \end{cases} \qquad \varphi = \frac{\omega}{c}\left(\frac{1}{\beta} - \sqrt{\varepsilon - 1 + n_x^2}\right)$$

Out-of-hole passage







$$\mathbf{E}_s(\mathbf{r},\omega) = -\frac{e(\varepsilon(\omega)-1)}{2\pi v}\frac{e^{ikr}}{r}\frac{2\sin\left(\frac{a\varphi}{2}\right)}{\varphi}\frac{\sum_{m=0}^{+\infty} C_m\left[\mathbf{n}',\left[\mathbf{n}', A_m\hat{\mathbf{L}}K_m(\mu\rho)+D_m\hat{\mathbf{L}}I_m(\mu\rho)\right]\right]\cos(m\psi)}{n_y^2+n_z^2+\beta^{-2}\gamma^{-2}}$$

$$A_m = -\mu\rho J_m(k_\perp\rho)I_{m-1}(\mu\rho)+k_\perp\rho I_m(\mu\rho)J_{m-1}(k_\perp\rho)+ \\ +\mu R_0 J_m(k_\perp R_0)I_{m-1}(\mu R_0)-k_\perp R_0 I_m(\mu R_0)J_{m-1}(k_\perp R_0)$$

$$k_\perp = \frac{\omega}{c}\sqrt{n_y^2+n_z^2} \qquad \mu = \frac{\omega}{c\beta\gamma}$$

$$\sqrt{\varepsilon}\mathbf{n}' = \left(\sqrt{\varepsilon-1+n_x^2}, n_y, n_z\right)$$

$$D_m = k_\perp\rho J_{m-1}(k_\perp\rho)K_m(\mu\rho)+\mu\rho K_{m-1}(\mu\rho)J_m(k_\perp\rho)$$

$$\hat{\mathbf{L}} = i\frac{\omega}{c\beta\gamma^2}\mathbf{e}_x+\frac{1}{\rho}\frac{\partial}{\partial\psi}\mathbf{e}_y-\frac{\partial}{\partial\rho}\mathbf{e}_z \qquad C_m = \begin{cases}2(-i)^m, & m\neq 0\\ 1, & m=0\end{cases} \qquad \varphi = \frac{\omega}{c}\left(\frac{1}{\beta}-\sqrt{\varepsilon-1+n_x^2}\right)$$

Total field

$$\mathbf{E}_{tot}\left(\mathbf{r},\omega\right)=\mathbf{E}_{0}\left(\mathbf{r},\omega\right)+\sum_{s=1}^{4}\mathbf{E}'_{s}\left(\mathbf{r},\omega;\rho=d_{y,z}\right)\exp\left[\pm ik_{y,z}d_{y,z}\right]$$

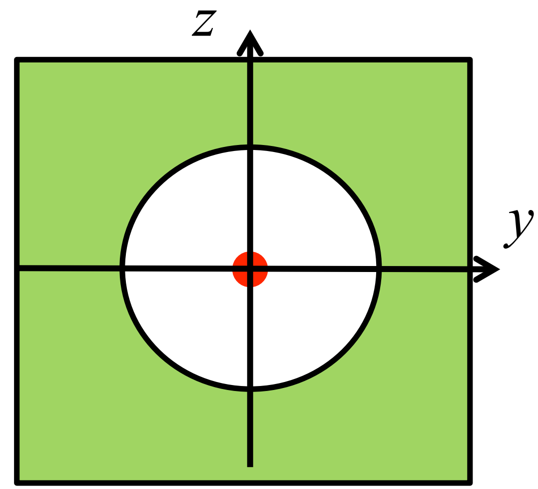
$$\mathbf{E}_{tot}\left(\mathbf{r},\omega\right)=-\frac{e\left(\varepsilon\left(\omega\right)-1\right)}{2\pi v}\frac{e^{ikr}}{r}\frac{2\sin\left(a\varphi/2\right)}{\varphi}\frac{\sum_{m=0}^{+\infty}C_{m}\left[\mathbf{n}',\left[\mathbf{n}',\hat{\mathbf{P}}\right]\right]\cos\left(m\psi\right)}{n_{y}^{2}+n_{z}^{2}+\beta^{-2}\gamma^{-2}}$$

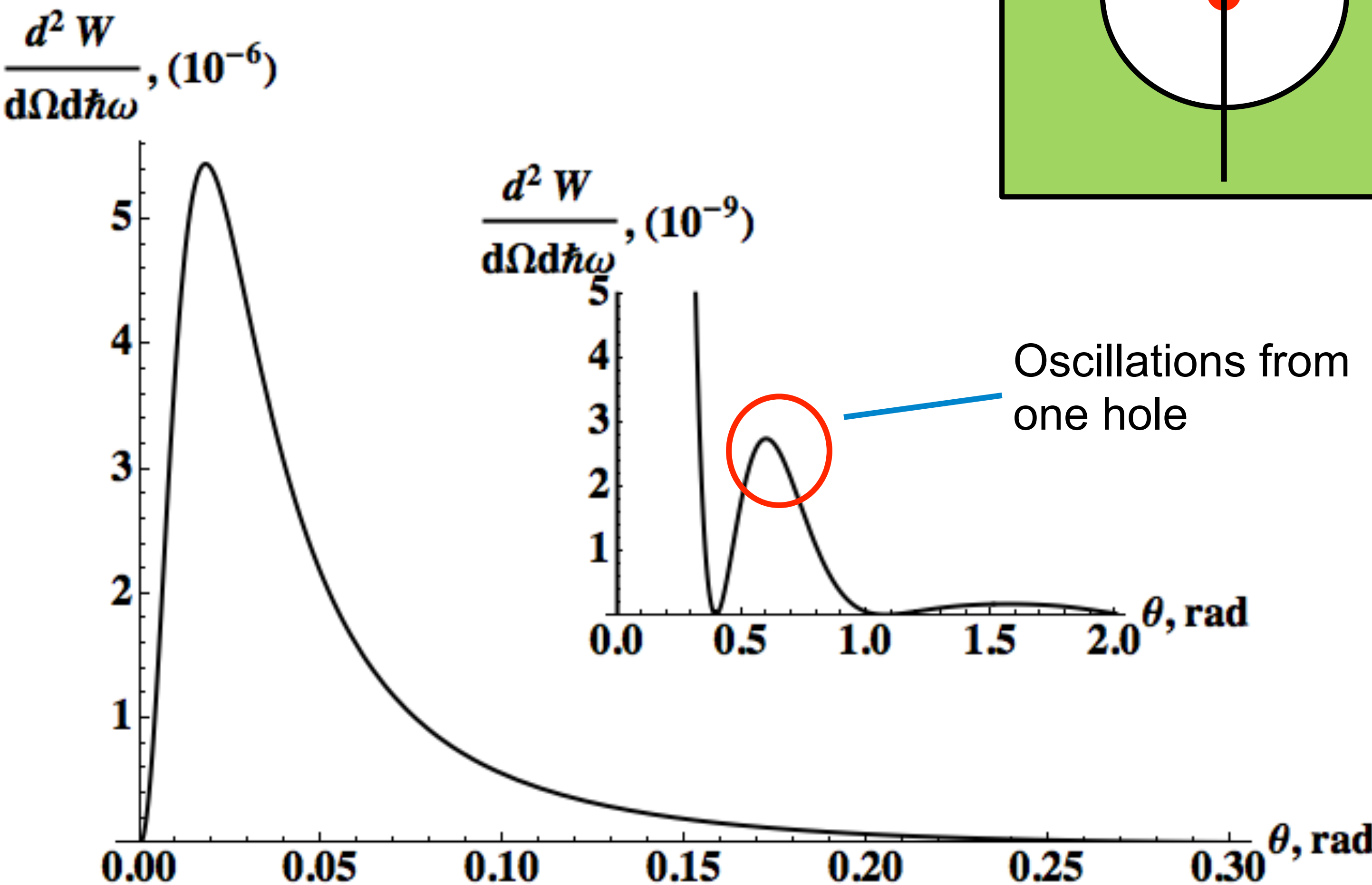
$$\hat{\mathbf{P}}=B_{m}\hat{\mathbf{L}}I_{m}\left(\rho\mu\right)+\sum_{s=1}^{4}Q_{m}\hat{\mathbf{L}}K_{m}\left(\mu d_{y,z}\right)\exp\left[\pm ik_{y,z}d_{y,z}\right]$$

$$B_{m}=\mu R_{0}J_{m}\left(k_{\perp}R_{0}\right)K_{m-1}\left(\mu R_{0}\right)+k_{\perp}R_{0}J_{m-1}\left(k_{\perp}R_{0}\right)K_{m}\left(\mu R_{0}\right)$$

$$Q_{m}=\mu R_{0}J_{m}\left(k_{\perp}R_{0}\right)I_{m-1}\left(\mu R_{0}\right)-k_{\perp}R_{0}J_{m-1}\left(k_{\perp}R_{0}\right)I_{m}\left(\mu R_{0}\right)$$

One central hole





$\gamma = 55$

$\varepsilon = 2$

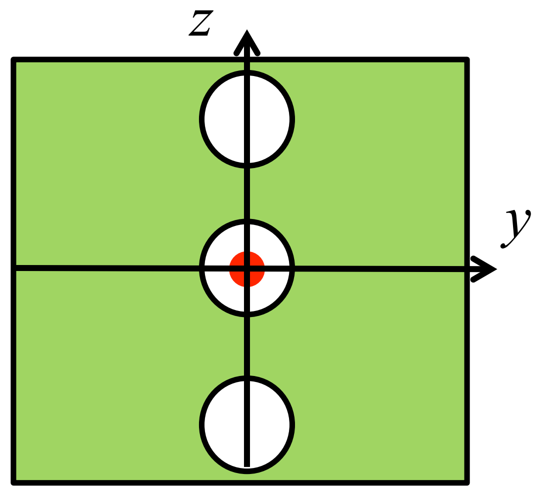
$R_0 = 0.5\ mm$

$a = 1\ \mu m$

$\phi = 0$

$$\mathbf{n} = (\cos\theta, \sin\theta\sin\phi, \sin\theta\cos\phi)$$

Three holes



$\gamma = 55$

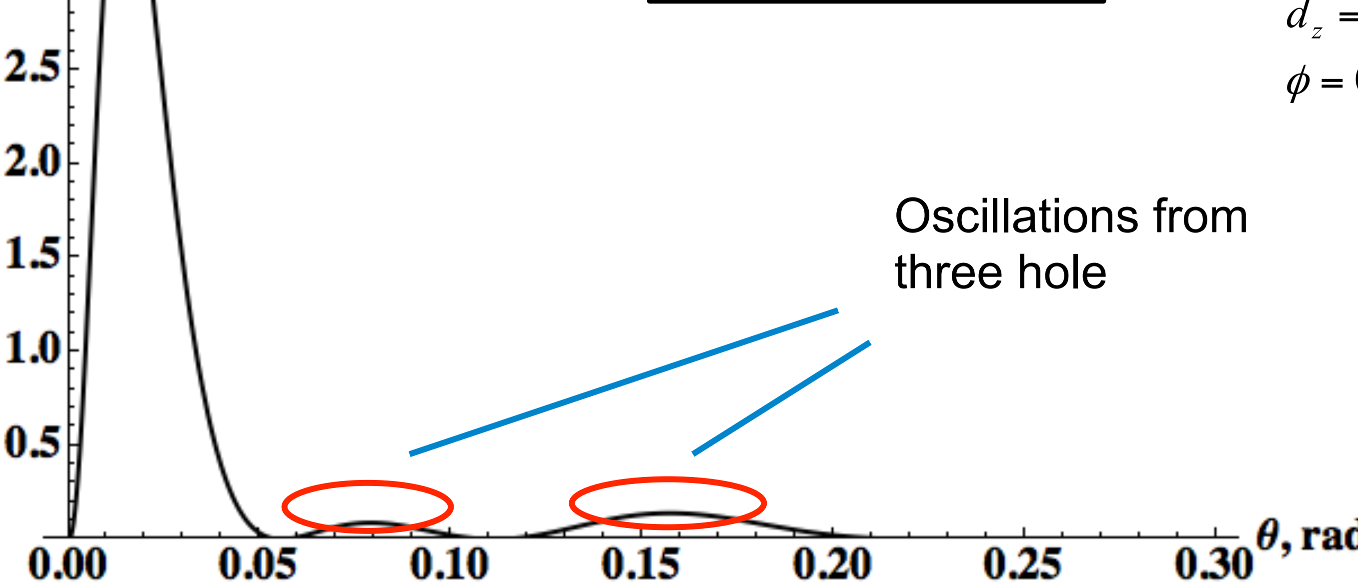
$\varepsilon = 2$

$R_0 = 0.5\ mm$

$a = 1\ \mu m$

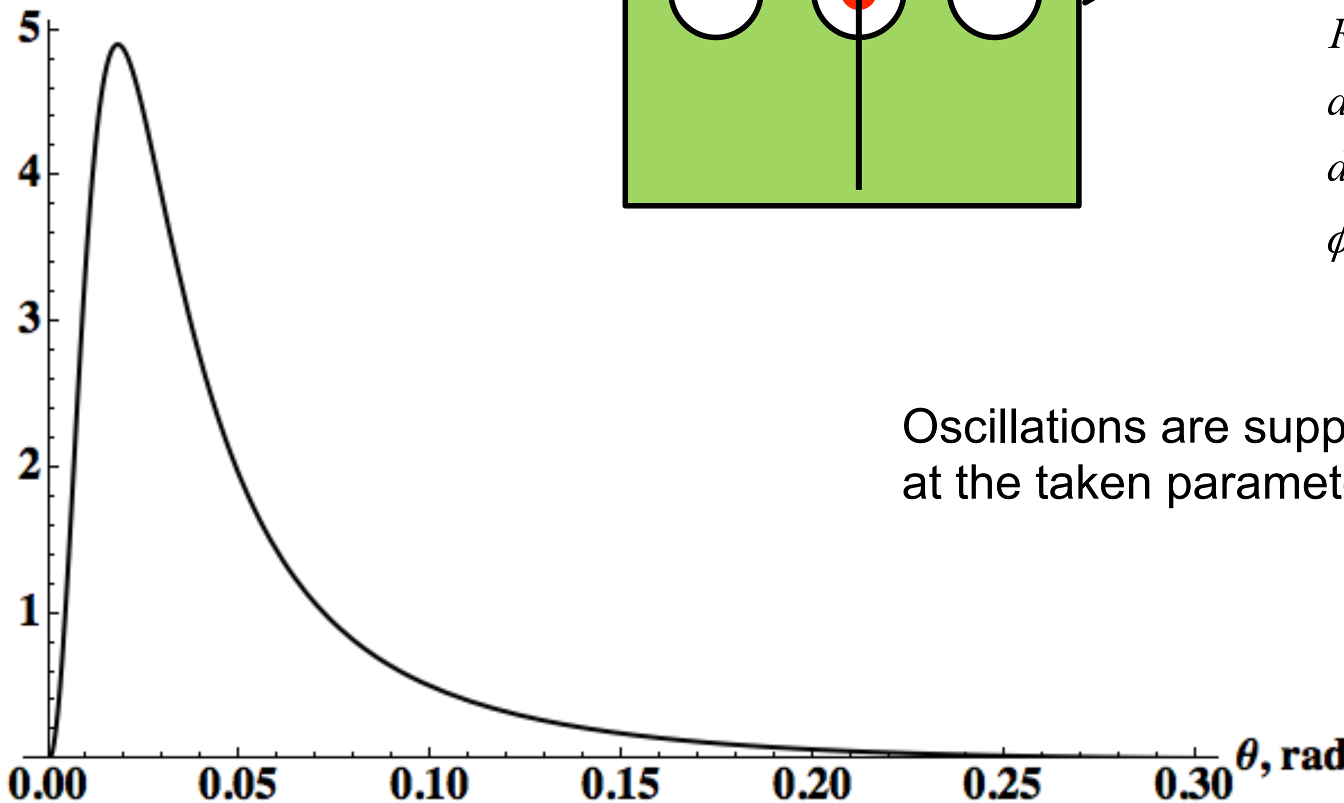
$d_z = 6\ mm$

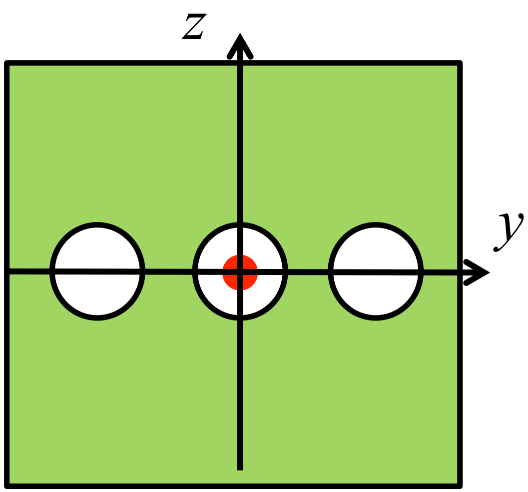
$\phi = 0$



$$\mathbf{n} = \left(\cos\theta, \sin\theta\sin\phi, \sin\theta\cos\phi\right)$$

Three holes





$\gamma = 55$

$\varepsilon = 2$

$R_0 = 0.5\,mm$

$a = 1\ \mu m$

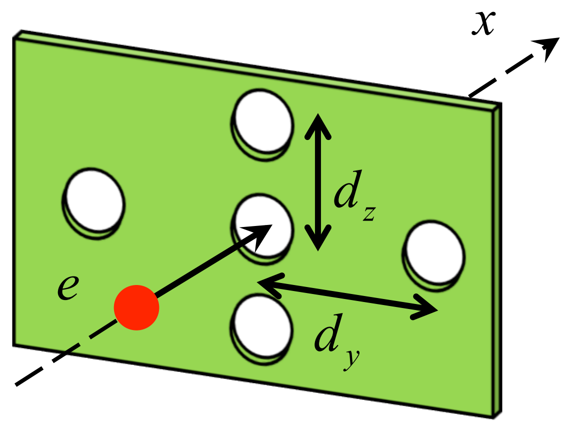
$d_y = 1\,mm$

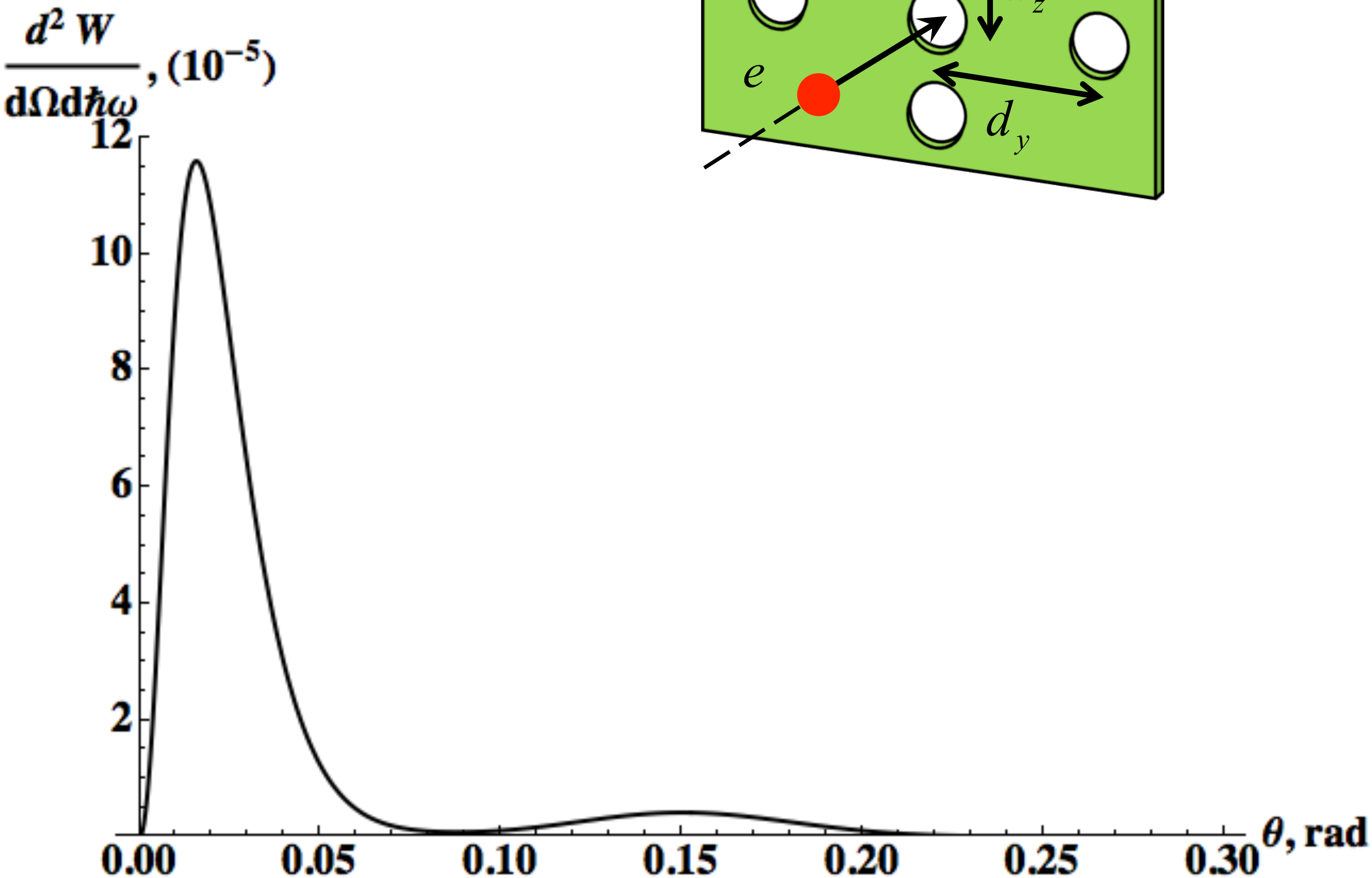
$\phi = 0$

Oscillations are suppressed /
at the taken parameters/

$$\mathbf{n} = (\cos\theta, \sin\theta\sin\phi, \sin\theta\cos\phi)$$

Five holes





$\gamma = 55$

$\varepsilon = 2$

$R_0 = 0.5\ mm$

$a = 1\ \mu m$

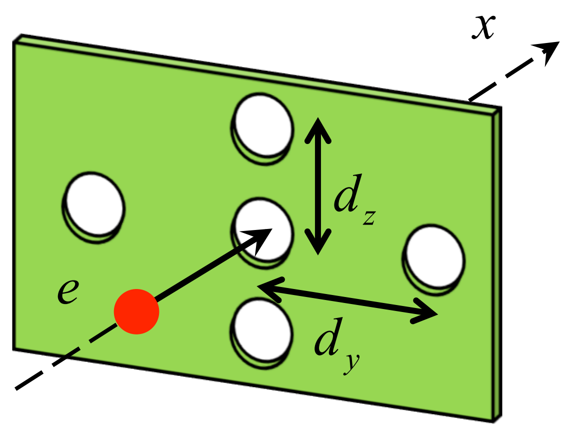
$d_y = 1\ mm$

$d_z = 6\ mm$

$\phi = 0$

$\mathbf{n} = (\cos\theta, \sin\theta\sin\phi, \sin\theta\cos\phi)$

Five holes



$\gamma = 55$

$\varepsilon = 2$

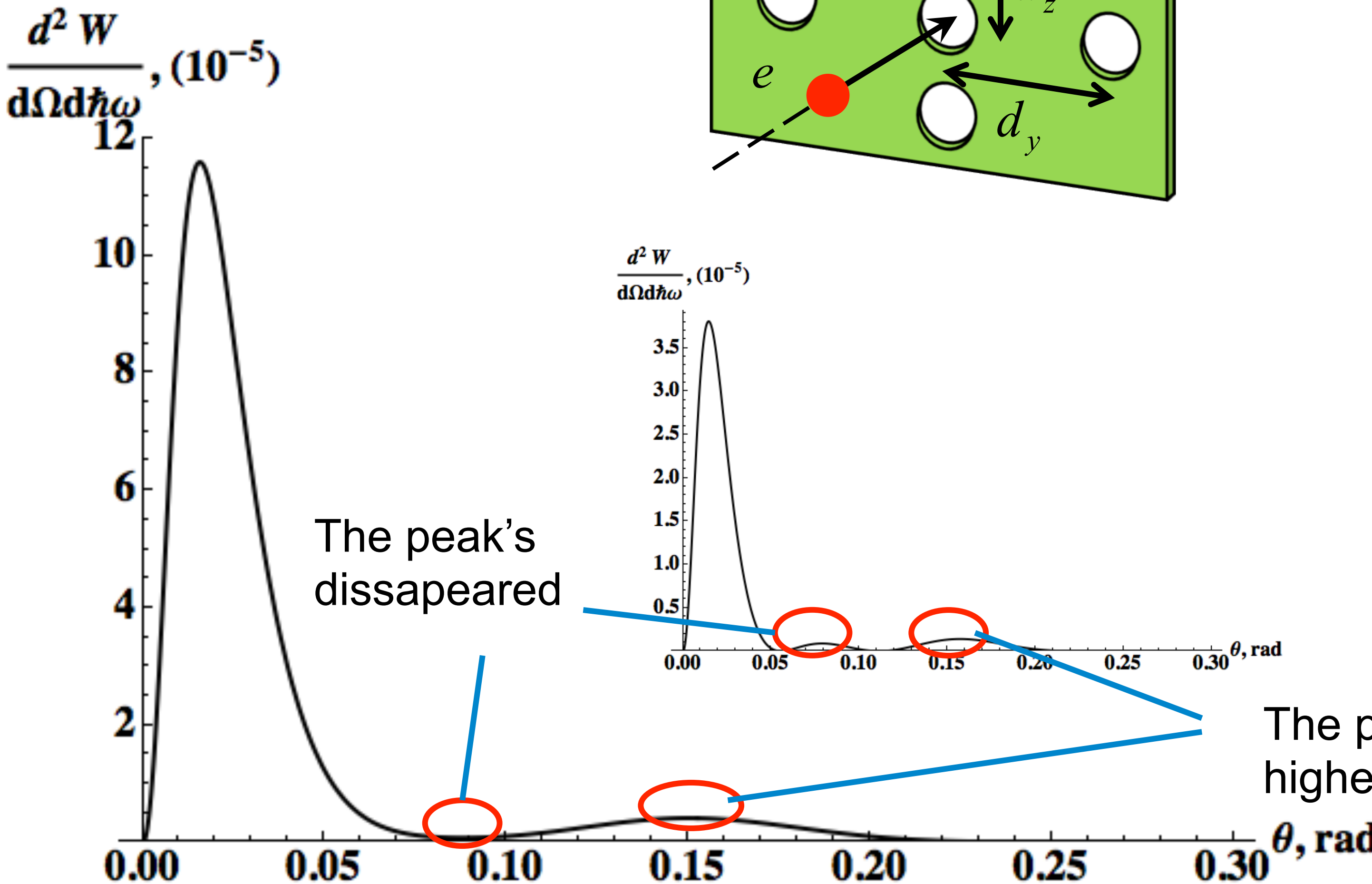
$R_0 = 0.5\ mm$

$a = 1\ \mu m$

$d_y = 1\ mm$

$d_z = 6\ mm$

$\phi = 0$

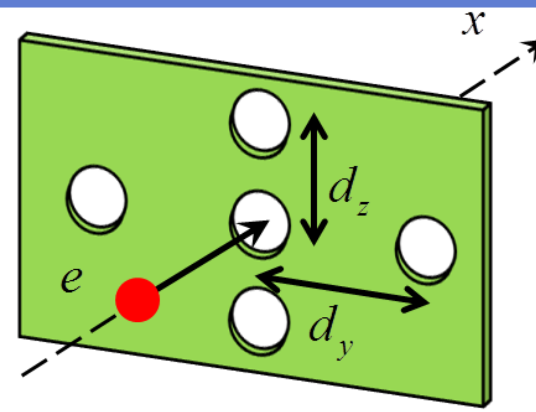


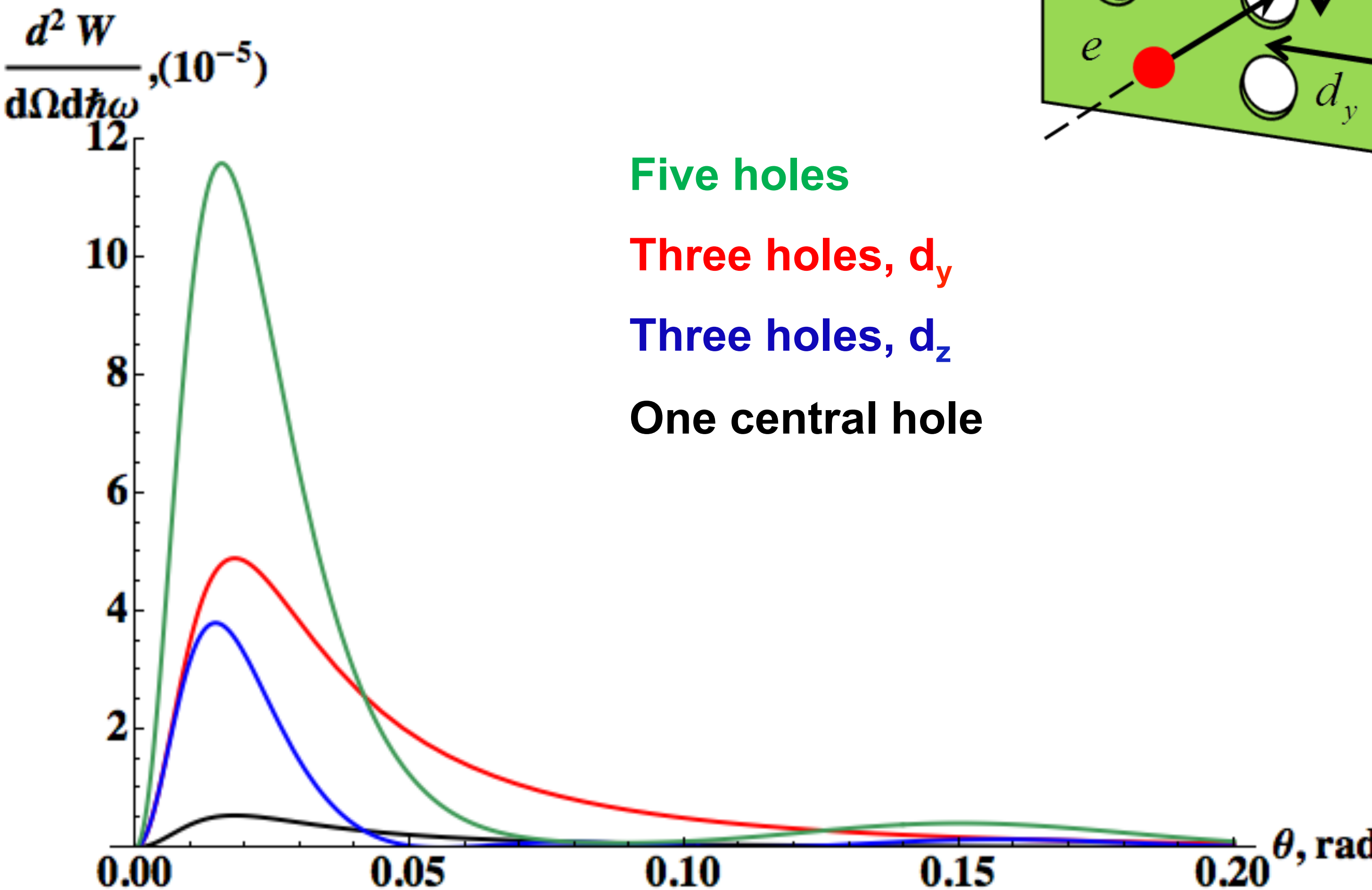
The peak's dissapeared

The peak's become higher

Interference -> is expected to contribute to coupling

$$\mathbf{n} = (\cos\theta, \sin\theta\sin\phi, \sin\theta\cos\phi)$$





$\gamma = 55$

$\varepsilon = 2$

$R_0 = 0.5\ mm$

$a = 1\ \mu m$

$d_y = 1\ mm$

$d_z = 6\ mm$

$\phi = 0$

$$\mathbf{n} = (\cos\theta, \sin\theta\sin\phi, \sin\theta\cos\phi)$$

$\gamma = 55$

$\varepsilon = 2$

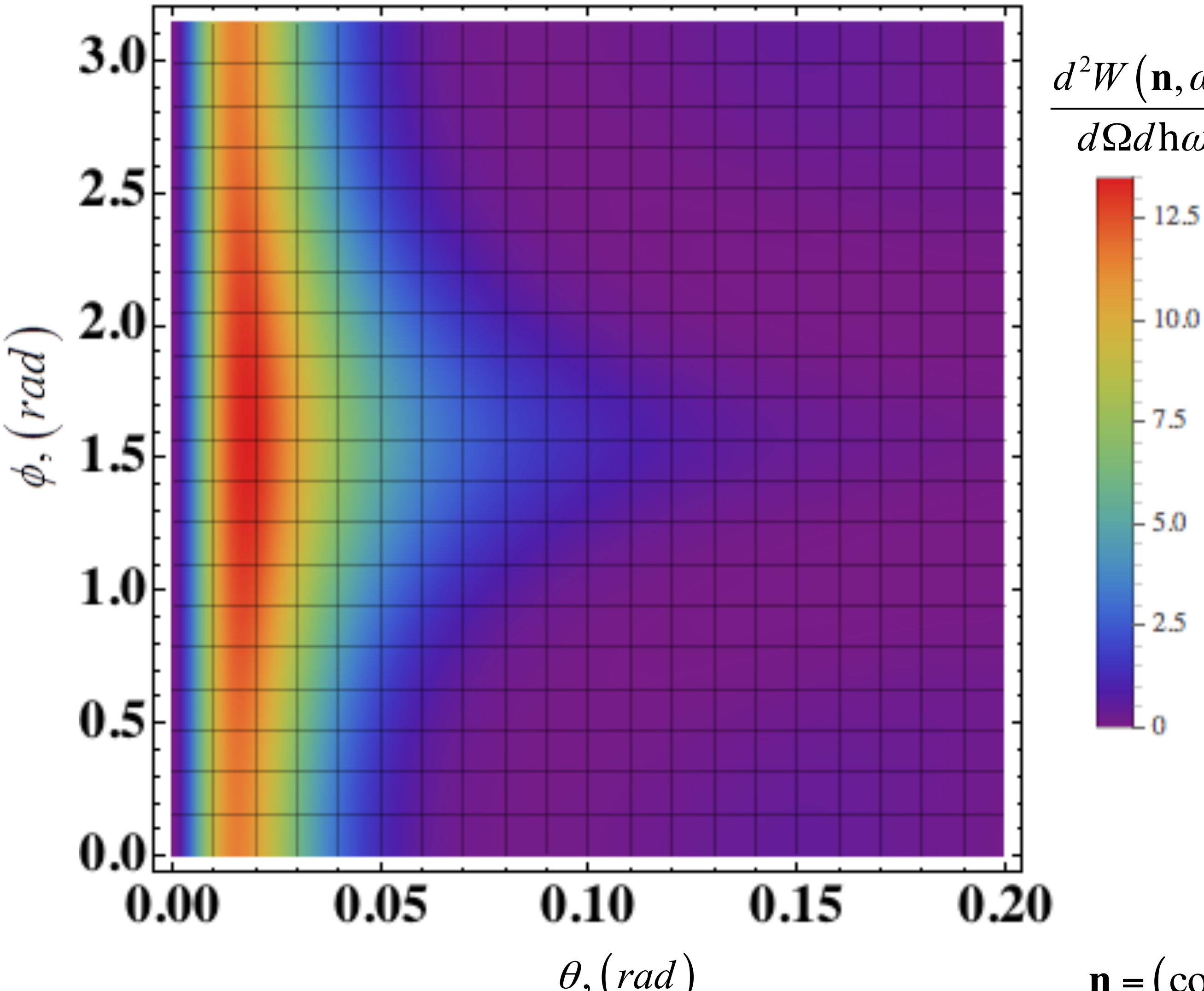
$R_0 = 0.5\ mm$

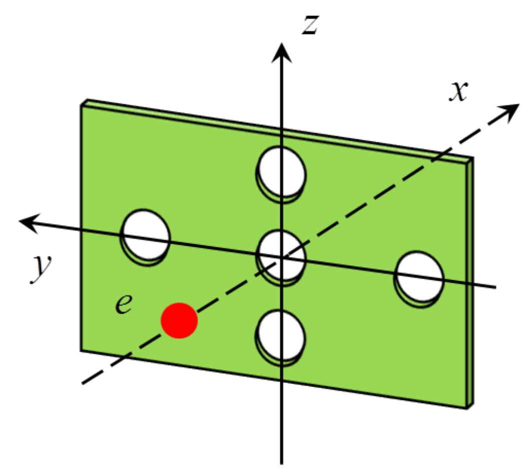
$a = 1\ \mu m$

$d_y = 1\ mm$

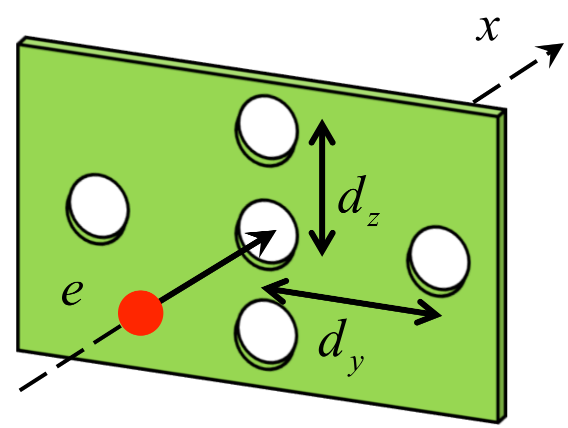
$d_z = 6\ mm$

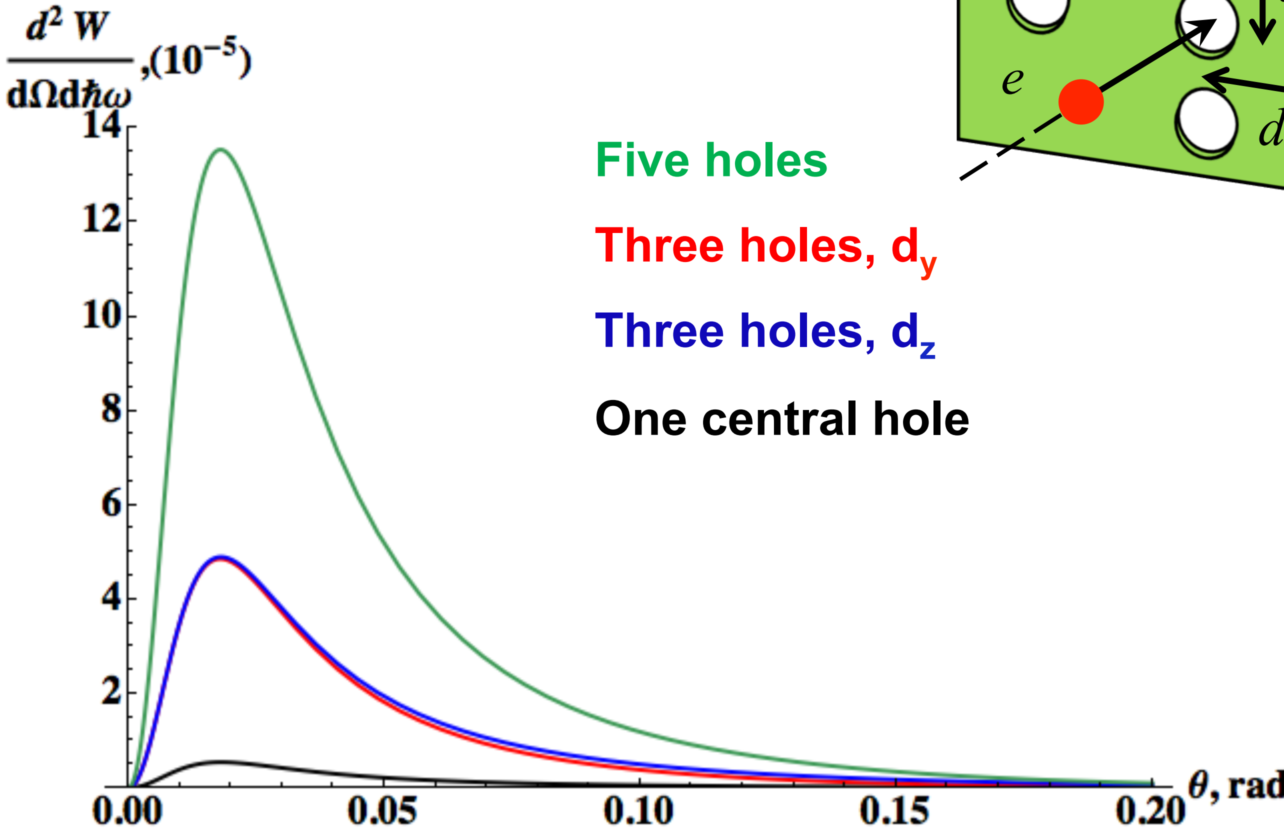
$$\frac{d^2W(\mathbf{n},\omega)}{d\Omega d\hbar\omega}, (10^{-5})$$





$$\mathbf{n} = (\cos\theta, \sin\theta\sin\phi, \sin\theta\cos\phi)$$





$\gamma = 55$

$\varepsilon = 2$

$R_0 = 0.5\ mm$

$a = 1\ \mu m$

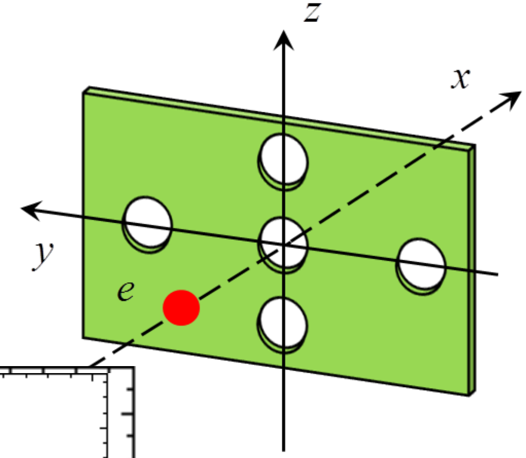
$d_y = 1\ mm$

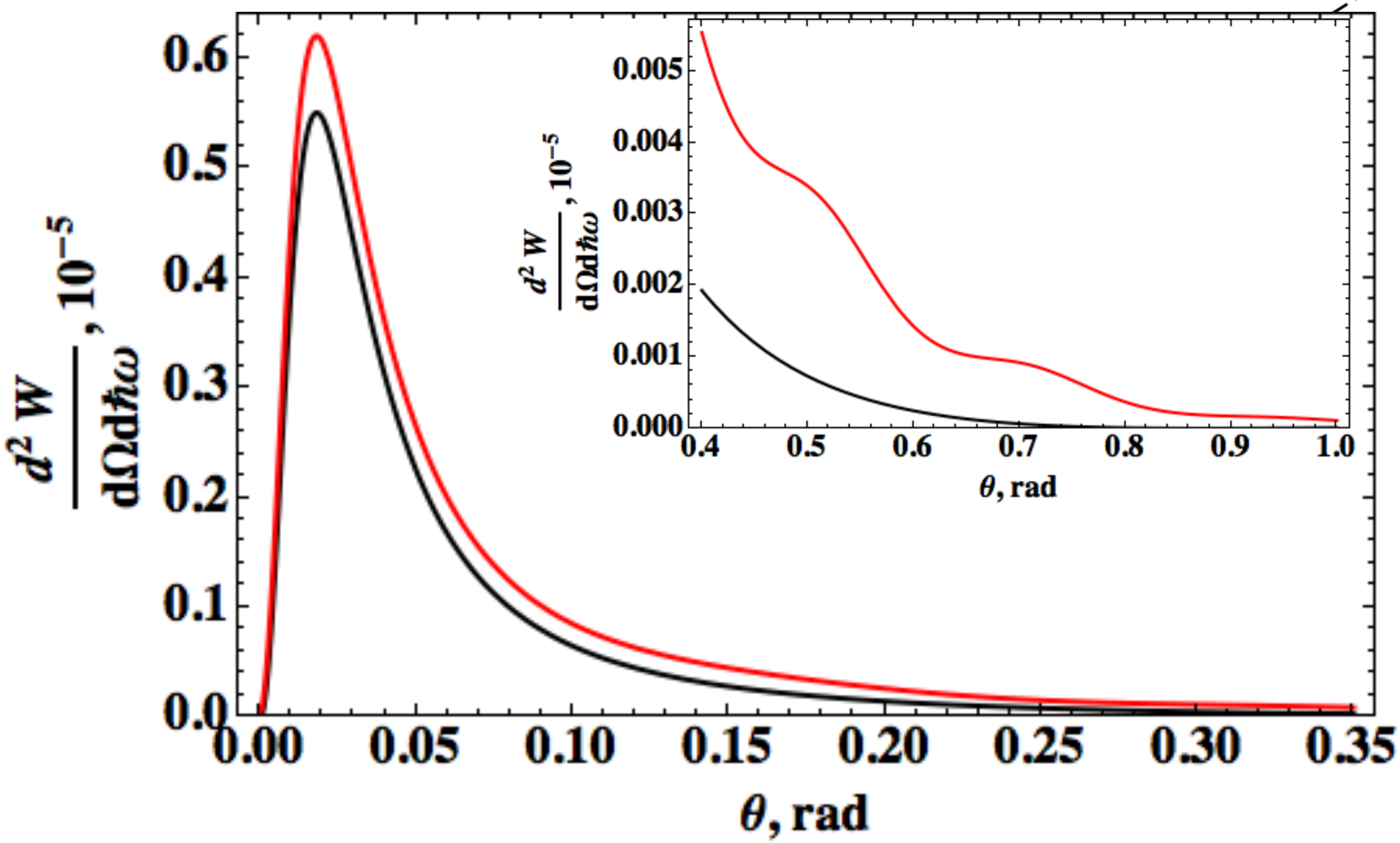
$d_z = 6\ mm$

$\phi = 90^0$

Five holes

One central hole





$\gamma = 55$

$\varepsilon = 2$

$R_0 = 0.5\ mm$

$a = 1\ \mu m$

$d_y = 1\ mm$

$d_z = 6\ mm$

exact

$$\mathbf{n} = (\cos\theta, \sin\theta\sin\phi, \sin\theta\cos\phi)$$

Conclusion

- ✓ We suggest new target, and he theory is constructed, both qualitative and exact, including polarisation of the radiation field.
- ✓ The results turn into those by Ter-Mikaelyan for the infinitely thin and ideally conducting screen.
- ✓ The interference effects can be defining. /which will supplement the coupling problem with complexity/

Future

- ✓ Develop theory for the case of bunch, including strongly asymmetric one.
- ✓ Defining the practical way of extracting the information about beam parameters.

Thank you
for your attention!