Dispersion relations for different types of radiation from periodic structures: similarities and differences

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Outline

1. Types of radiation in periodic structures.
2. Role of frequency dependent permittivity.
3. Small-angle X-ray TR from periodic stacks, end/butt TR.
4. About one asymptotic expression widely-used in periodic structures.

Function of dielectric permittivity

$$\varepsilon(\omega) = 1 + \int_0^{\infty} dt\ f(t) \exp(i\omega t)$$

X-ray: $$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} + i\chi(\omega)$$

$$\varepsilon(\omega) = 1 + \chi'(\omega) + i\chi''(\omega)$$

Absorption – other sign –
different consideration for X-ray CR needed?

L.D. Landau, et al., Electrodynamics of Continuous Media (Second Edition), V. 8 in Course of Theoretical Physics, 1984, Pergamon Press Ltd.

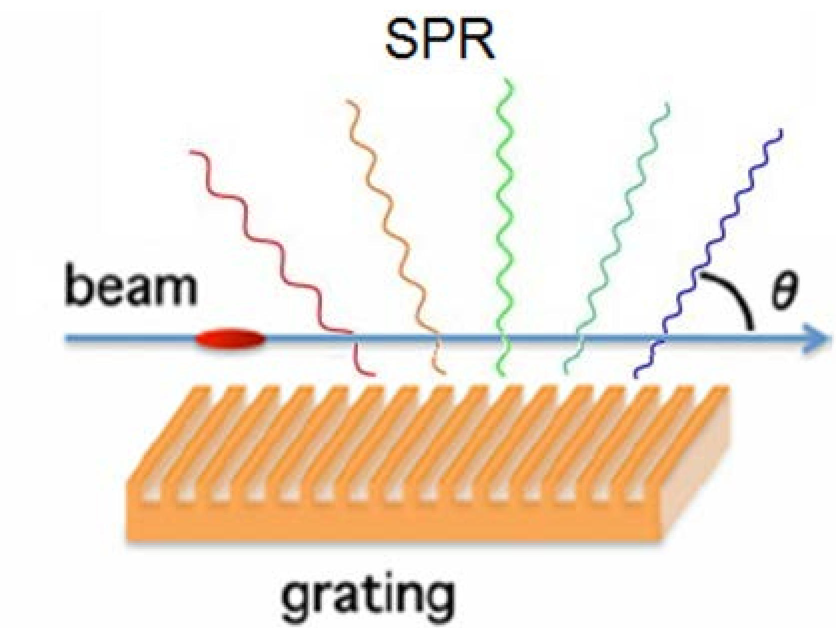
Smith-Purcell radiation

Smith-Purcell dispersion relation

$$\boxed{\lambda = \frac{d}{m}\left(\frac{1}{\beta} - \cos\theta\right), \quad m = 1, 2, \ldots}$$

Electron in vacuum ➡ NO $\varepsilon(\omega)$ in dispersion relation

$$\boxed{\omega d\left(\gamma^{-2} + \theta^2\right) = 4\pi c m, \quad m = 1, 2, \ldots}$$



$\theta << 1$

$\gamma >> 1$

One frequency ω_1 at one angle θ_1

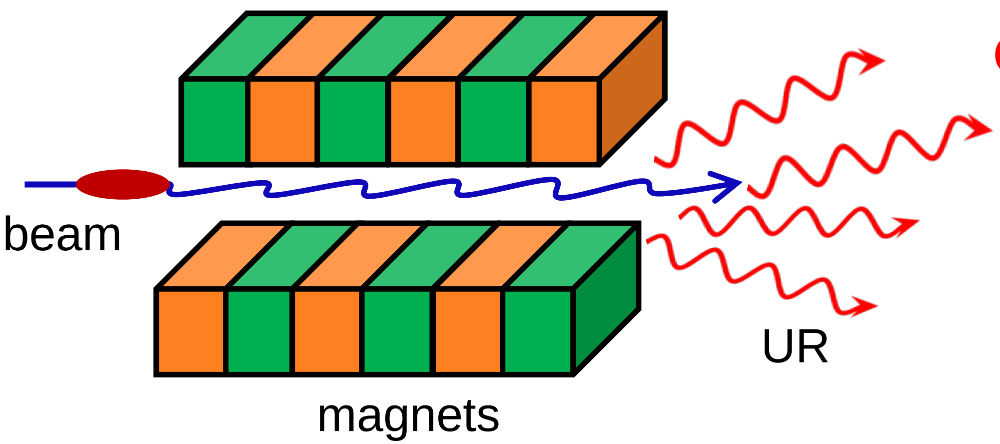
S.J. Smith, E.M. Purcell, Physical Review (1953)

Undulator radiation

Dispersion relation
(ultrarelativistic)

$$\boxed{\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)}$$

Electron in vacuum ➡ NO $\varepsilon(\omega)$ in dispersion relation



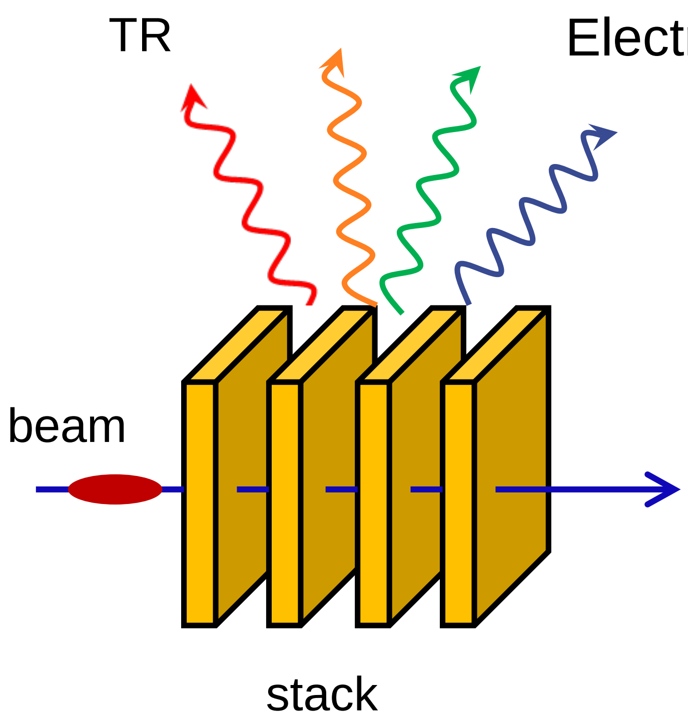
One frequency ω_1 at one angle θ_1

A.P. Potylitsyn, Electromagnetic Radiation of Electrons in Periodic Structures, Springer, 2011

Same about Thomson/Compton X-ray source?

Resonant transition radiation

Resonant TR dispersion relation $\boxed{\omega d\left(\gamma^{-2}+\theta^{2}\right)+a\omega\frac{\omega_p^2}{\omega^2}=4\pi c m,\ m=1,2,\dots}$



Electron in matter ➡ $\varepsilon(\omega)$ in dispersion relation

Two frequencies $\omega_{1,2}$ at one angle θ_1

G.M. Garibian and C. Yang, X-ray Transition Radiation, Yerevan: AN Arm. SSR Press, 1983

A.P. Potylitsyn, Electromagnetic Radiation of Electrons in Periodic Structures, Springer, 2011

Parametric X-ray radiation

PXR dispersion relation

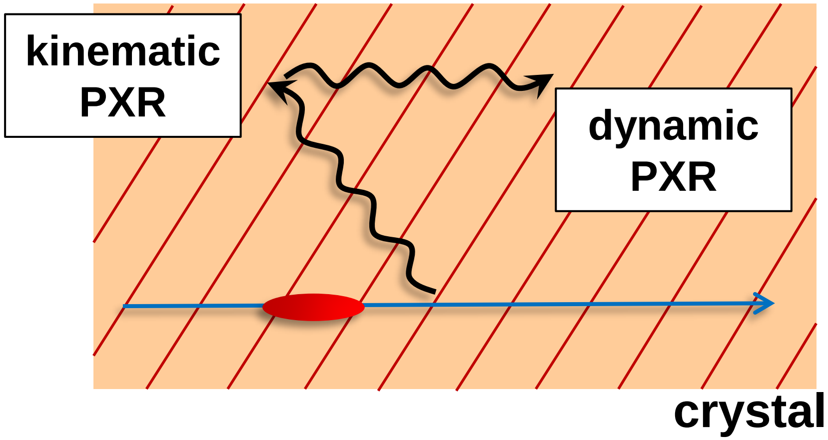
$$\boxed{\frac{\omega}{c}\beta\left(\frac{1}{\beta}-\cos\theta\sqrt{\varepsilon(\omega)}\right)=\mathbf{\beta g}}$$

Electron in matter ➡ $\varepsilon(\omega)$ in dispersion relation

$$\boxed{\omega^2\left(\gamma^{-2}+\theta^2\right)-2\omega\frac{\mathbf{vg}}{\beta}+\omega_p^2=0}$$

$\theta << 1$

$\gamma >> 1$



Two frequencies $\omega_{1,2}$ at one angle θ_1

M.L.Ter-Mikhaelyan, Physics – Uspekhi (2001)

Spectral distribution

$$\frac{d^2W(\mathbf{n},\omega)}{d\Omega\,d\omega}=\frac{d^2W_1(\mathbf{n},\omega)}{d\Omega\,d\omega}\frac{\sin^2(Nd\varphi/2)}{\sin^2(d\varphi/2)}$$

dispersion relation

$$\frac{\sin^2(d\varphi N/2)}{\sin^2(d\varphi/2)}\xrightarrow{N>>1}2\pi N\sum_m\delta(d\varphi-2\pi m)$$

$$\frac{dW(\omega)}{d\omega}=2\pi N\int d\theta\frac{d^2W_1(\mathbf{n},\omega)}{d\Omega\,d\omega}\sum_m\delta(d\varphi-2\pi m)$$

NO frequency dispersion

$$\theta_1=\sqrt{\frac{4\pi cm}{d\omega}-\gamma^{-2}}$$

Frequency dispersion

$$\theta_1=\sqrt{\frac{4\pi cm}{\omega d}-\gamma^{-2}-\frac{a}{d}\frac{\omega_p^2}{\omega^2}}$$

$$\boxed{\frac{dW(\omega)}{d\omega}=A\sum_m\left.\frac{d^2W_1(\mathbf{n},\omega)}{d\Omega\,d\omega}\right|_{\theta=\theta_1}}$$

no qualitative difference

Angular distribution

$$\frac{dW(\theta)}{d\theta} = 2\pi N \int d\omega \frac{d^2 W_1(\mathbf{n},\omega)}{d\Omega\, d\omega} \sum_m \delta(d\varphi - 2\pi m)$$

dispersion relation

NO frequency dispersion

$$\omega_1 = \frac{4\pi c m}{d\left(\gamma^{-2} + \theta^2\right)}$$

Frequency dispersion

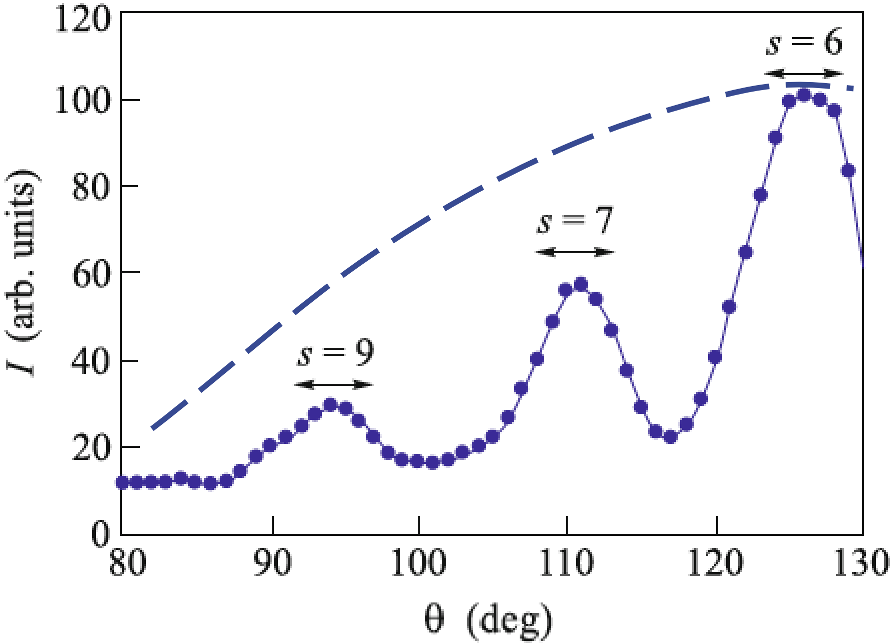
$$\omega_1 = \frac{2\pi c m + \sqrt{(2\pi c m)^2 - a d \omega_p^2 \left(\theta^2 + \gamma^{-2}\right)}}{d\left(\theta^2 + \gamma^{-2}\right)}$$

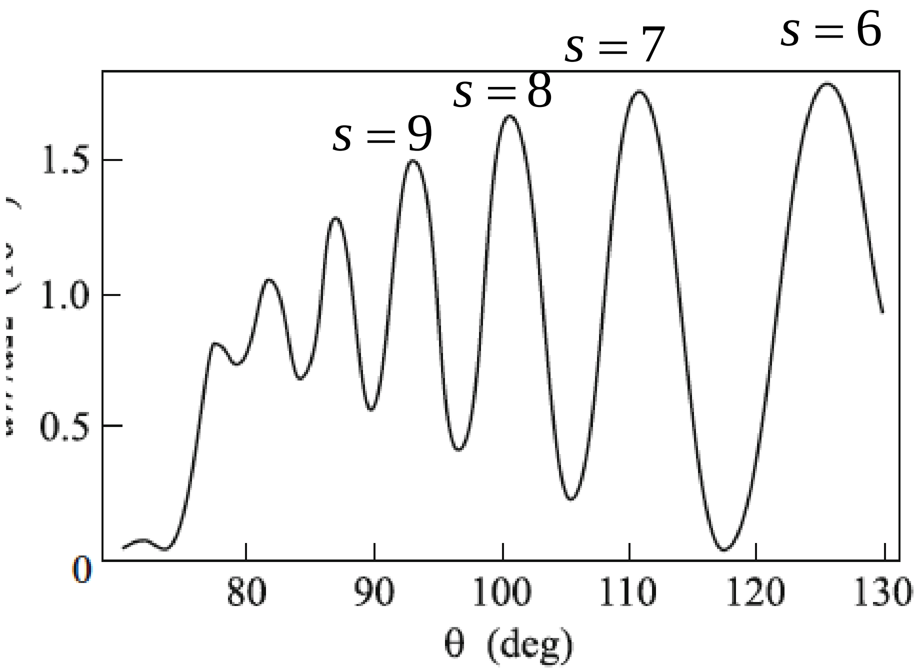
$$\omega_2 = \frac{2\pi c m - \sqrt{(2\pi c m)^2 - a d \omega_p^2 \left(\theta^2 + \gamma^{-2}\right)}}{d\left(\theta^2 + \gamma^{-2}\right)}$$

$$\boxed{\frac{dW(\theta)}{d\theta} = A \sum_m \left(\left.\frac{d^2 W_1(\mathbf{n},\omega)}{d\Omega\, d\omega}\right|_{\omega=\omega_1} + \left.\frac{d^2 W_1(\mathbf{n},\omega)}{d\Omega\, d\omega}\right|_{\omega=\omega_2} \right)}$$

qualitative difference !

***angular dispersion would lead to similar effect in spectral distribution**





Experiment

Theory

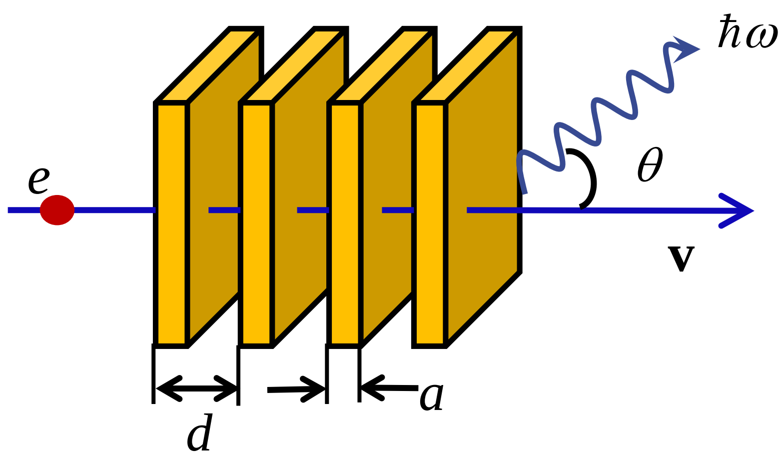
G.A. Naumenko et al.,
JETP Letters (2017)

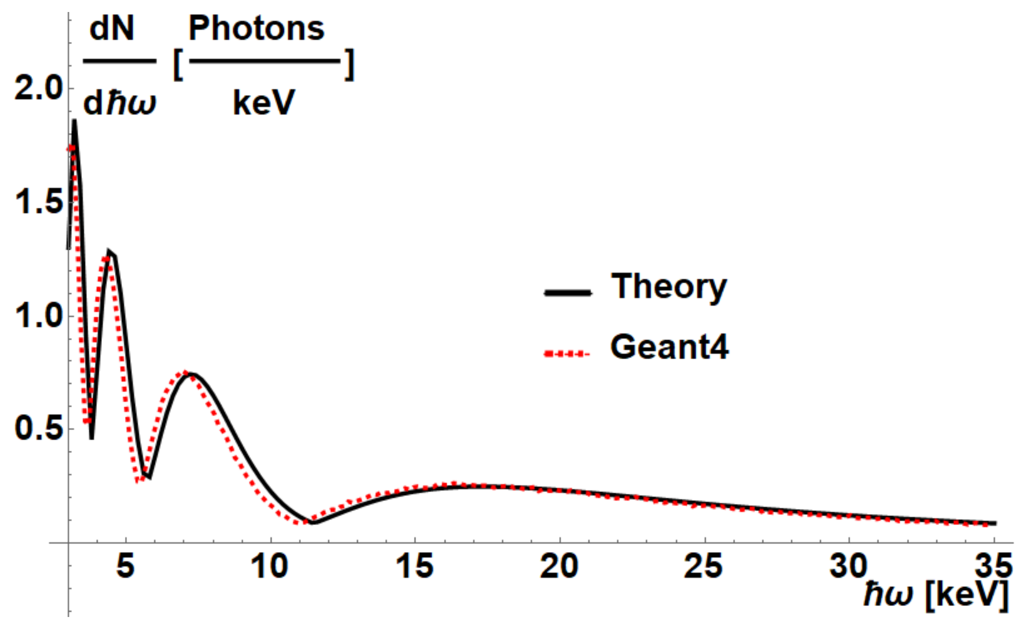
«frequency locked» SPR

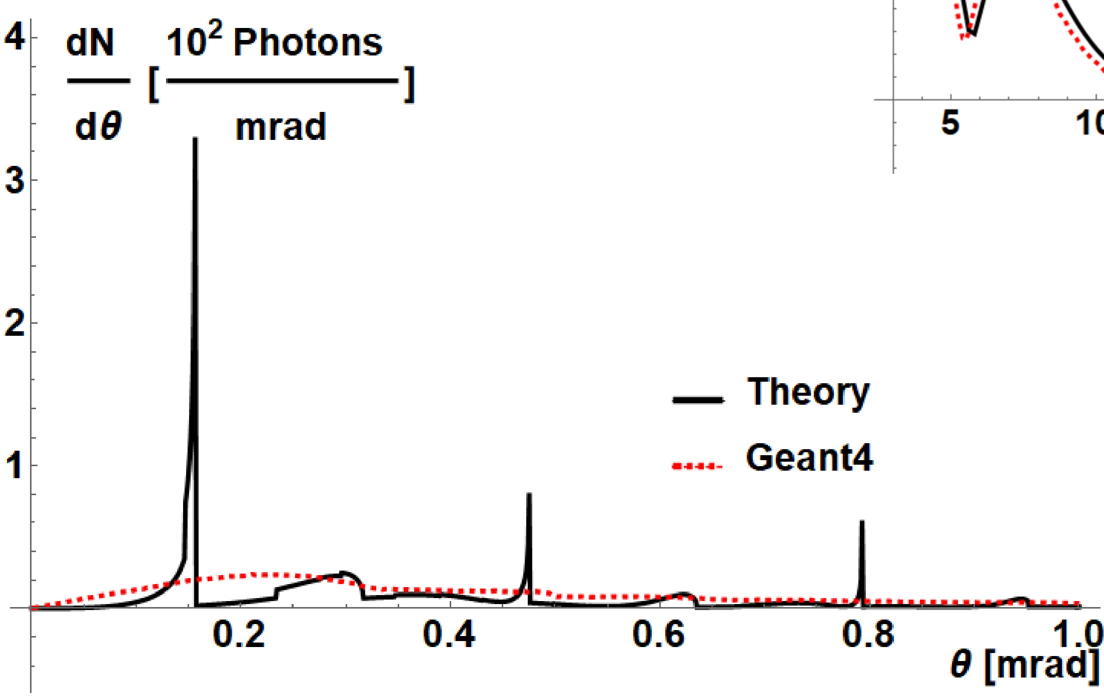
S.E. Korbly, A.S. Kesar, J.R. Sirigiri, R.J. Temkin, Observation of frequency-locked coherent terahertz Smith-Purcell radiation, PRL (2005).

G.A. Naumenko, A.P. Potylitsyn, P.V. Karataev, et al., Spectrum of coherent transition radiation generated by a modulated electron beam, JETP Letters (2017)

X-ray transition radiation







$$d = a + 2\ mm$$

$$a = 62\ \mu m$$

$$\gamma = 4 \cdot 10^4 \left(20\ GeV\right)$$

150 polypropylene foils

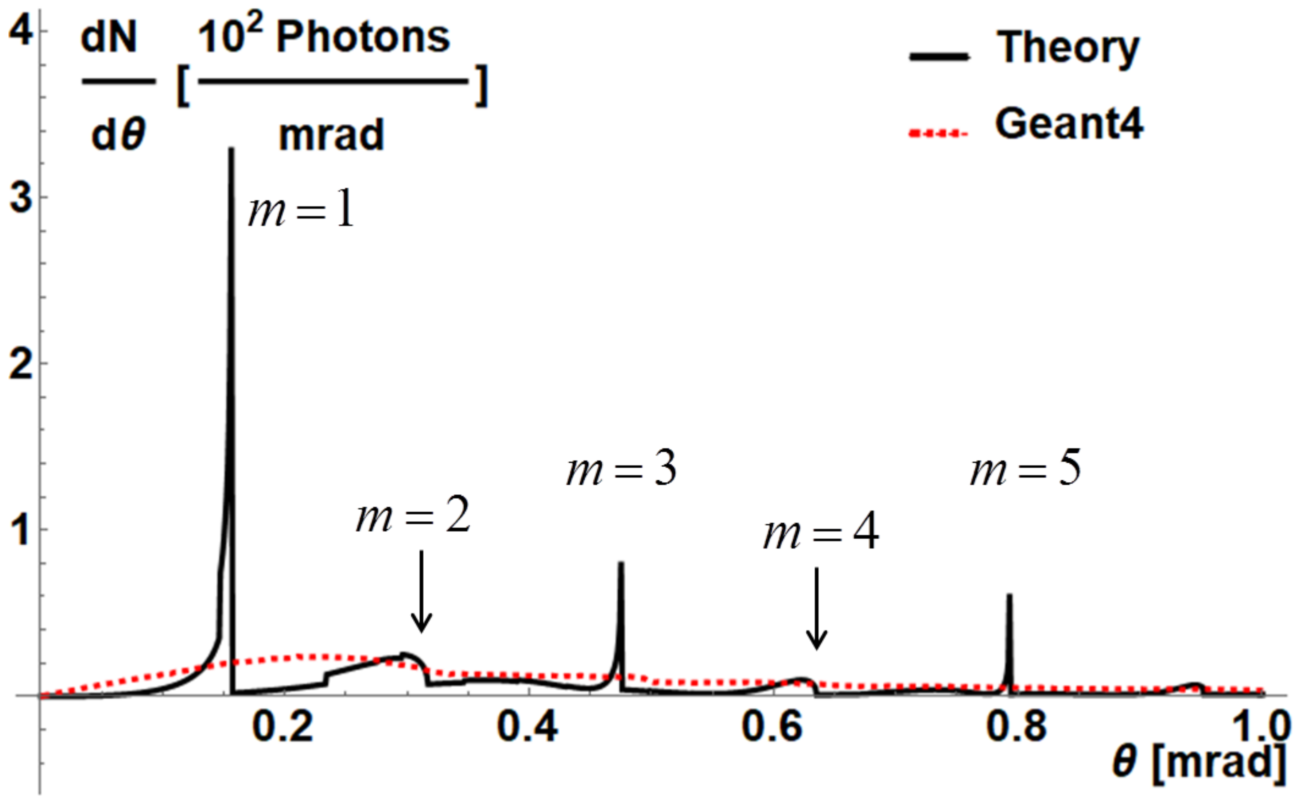
Courtesy A. Savchenko

Frequency dispersion

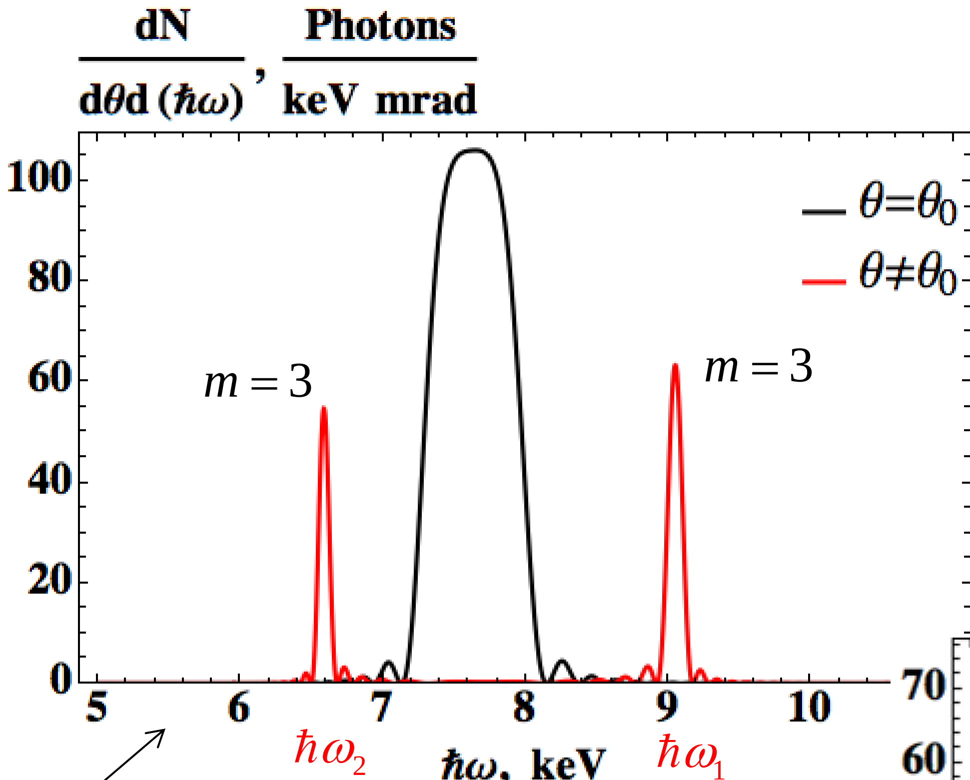
$$\omega_1 = \frac{2\pi cm + \sqrt{(2\pi cm)^2 - ad\omega_p^2\left(\theta^2 + \gamma^{-2}\right)}}{d\left(\theta^2 + \gamma^{-2}\right)}$$

$$\omega_2 = \frac{2\pi cm - \sqrt{(2\pi cm)^2 - ad\omega_p^2\left(\theta^2 + \gamma^{-2}\right)}}{d\left(\theta^2 + \gamma^{-2}\right)}$$

$\omega_1 = \omega_2$ at $\theta_0 = \sqrt{\dfrac{4\pi^2 c^2 m^2}{a\omega_p^2 (a+b)} - \gamma^{-2}}$



H. Berger, Complex Doppler effect in dispersive media, American Journal of Physics **44**, 851 (1976).



$$\theta_0 = \sqrt{\frac{4\pi^2 c^2 m^2}{a\omega_p^2(a+b)} - \gamma^{-2}}$$

$\theta_0 = 0.48\ mrad$

$\theta = 0.47\ mrad$

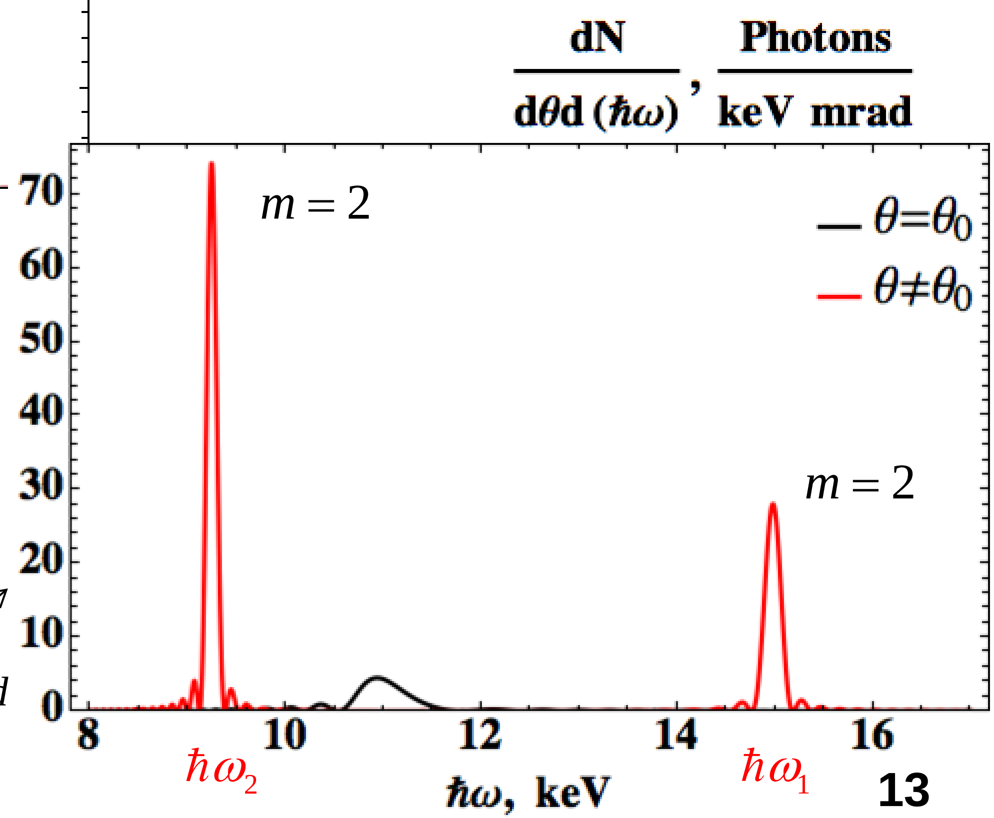
$d = a + 2\ mm$

$a = 62\ \mu m$

$\gamma = 4 \cdot 10^4\ (20\ GeV)$

$\theta_0 = 0.32\ mrad$

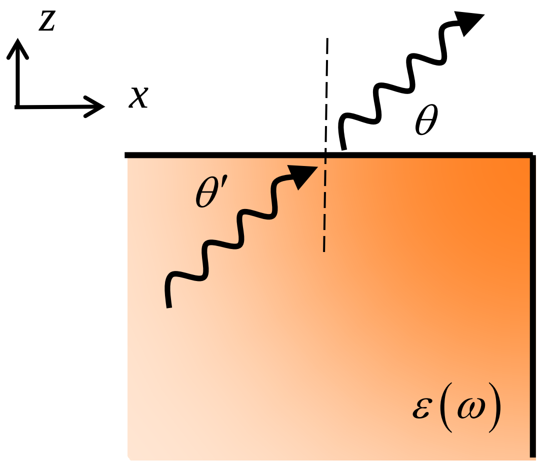
$\theta = 0.31\ mrad$



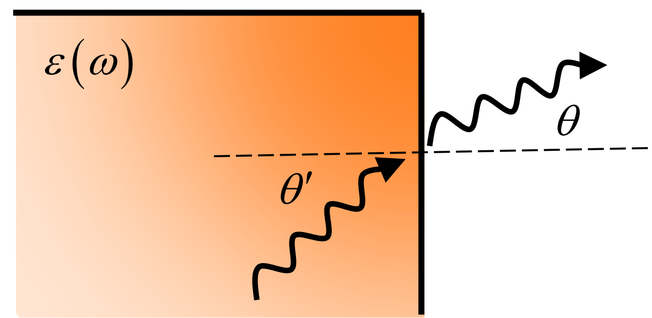
150 polypropylene foils

Role of surface

Often we consider the radiation process in an infinite media/structure.
But what of the surface? how does it change the radiation characteristics?



$$\lambda = \frac{d}{m}\left(\frac{1}{\beta} - \sqrt{\varepsilon}\cos\theta'\right)$$



$$\sqrt{\varepsilon(\omega)}\cos\theta' = \cos\theta$$

$$\boxed{\lambda = \frac{d}{m}\left(\frac{1}{\beta} - \cos\theta\right)}$$

$$\lambda = \frac{d}{m}\frac{1}{2}\left(\gamma^{-2} + \theta^2\right)$$

$$\sqrt{\varepsilon(\omega)}\cos\theta' = \sqrt{\underline{\varepsilon(\omega)} - \sin^2\theta}$$

$$\boxed{\lambda = \frac{d}{m}\left(\frac{1}{\beta} - \sqrt{\varepsilon(\omega) - \sin^2\theta}\right)}$$

$$\lambda = \frac{d}{m}\frac{1}{2}\left(\gamma^{-2} + \theta^2 + \frac{\omega_p^2}{\omega^2}\right)$$

D.V. Karlovets, JETP (2011)
D.Yu. Sergeeva, et al., PR STAB (2015)

End/butt radiation

$$\frac{d^2W(\mathbf{n},\omega)}{d\Omega\, d\omega} = \frac{d^2W_1(\mathbf{n},\omega)}{d\Omega\, d\omega}\frac{\sin^2(Nd\varphi/2)}{\sin^2(d\varphi/2)} = 2\pi N\sum_m I_1(\theta,\omega)\delta\left(d\varphi(\theta,\omega)-2\pi m\right)$$

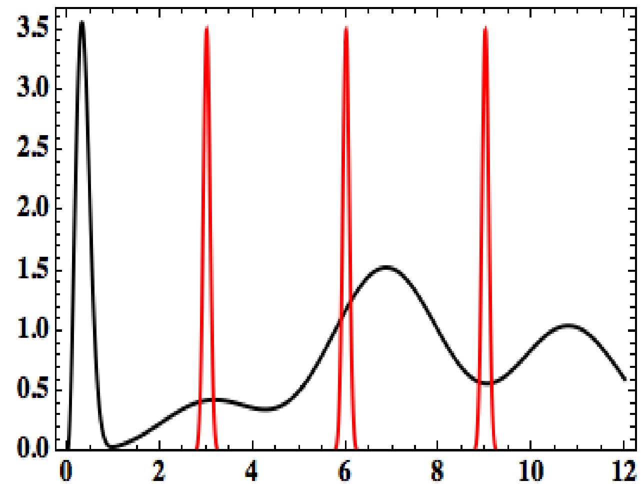
$$\frac{\sin^2(d\varphi N/2)}{\sin^2(d\varphi/2)} \xrightarrow{N>>1} 2\pi N\sum_m \delta\left(d\varphi-2\pi m\right)$$

$$\frac{dW(\omega)}{d\omega} = 2\pi N\sum_m \int d\theta\, I_1(\theta,\omega)\delta\left(d\varphi(\theta,\omega)-2\pi m\right)$$

The periodic target = equivalent plate

$$a\frac{\omega}{4c}\left(\gamma^{-2}+\theta^2+1-\varepsilon(\omega)\right)<<1$$

$$b\frac{\omega}{4c}\left(\gamma^{-2}+\theta^2\right)<<1$$



$$\frac{dW(\omega)}{d\omega} = 2\pi N\sum_m \int_{\theta_c}^{\infty} d\theta\, I_1(\theta,\omega)\delta\left(d\varphi(\theta,\omega)-2\pi m\right) + \quad \propto \sum_m I_1(\theta_m,\omega) - \text{resonant radiation}$$

analytically

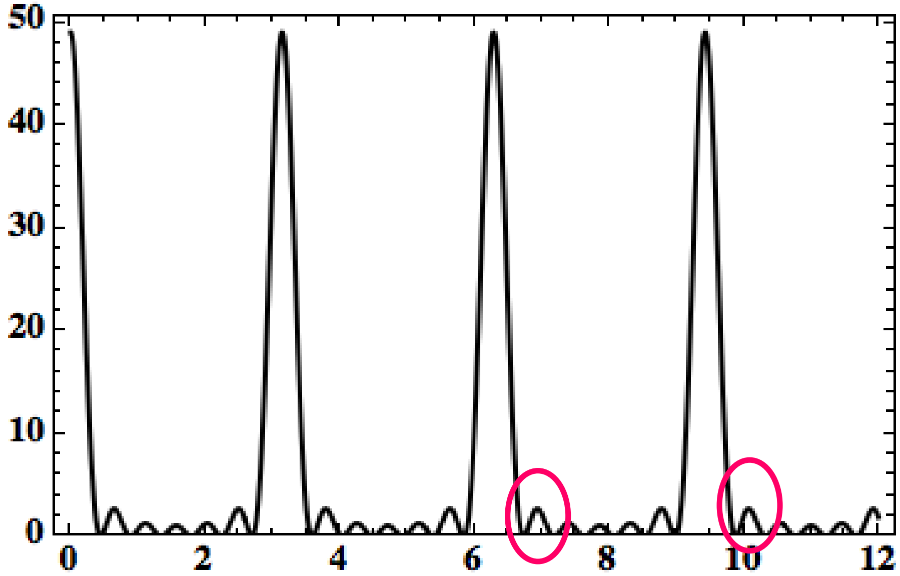
numerically, saddle-point method

$$+\int_0^{\theta_c} d\theta\, I_1(\theta,\omega) 2\pi N\sum_m \delta\left(d\varphi(\theta,\omega)-2\pi m\right) \quad \propto I_1\left(\theta\simeq\gamma^{-1};\omega\right) - \textbf{end / butt radiation}$$

Garibian, Yan Shi, 1983

Side maxima

$$\frac{\sin^2(d\varphi N/2)}{\sin^2(d\varphi/2)} \xrightarrow{N>>1} 2\pi N \sum_m \delta(d\varphi - 2\pi m)$$

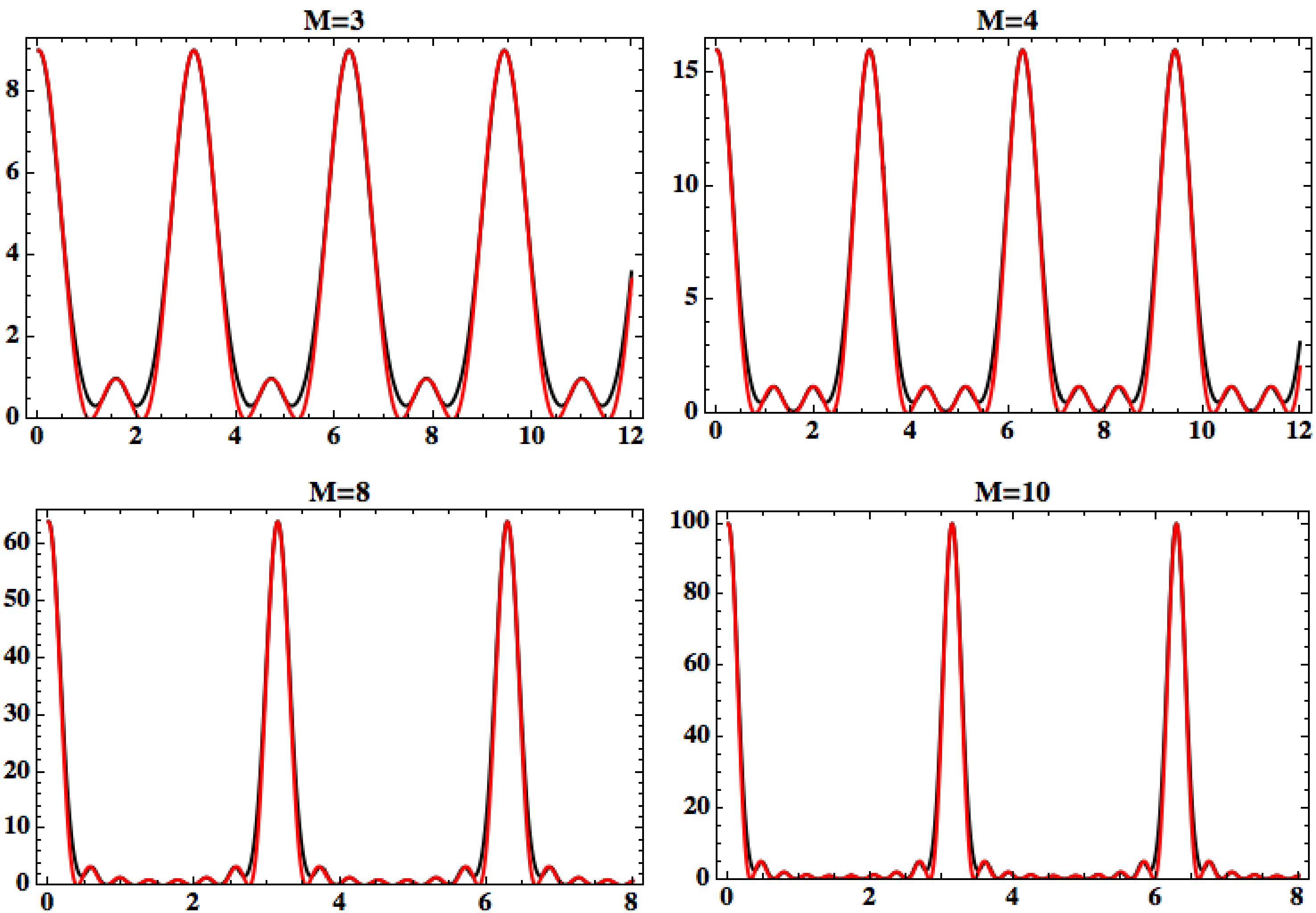


$$\frac{\sin^2(d\varphi N/2)}{\sin^2(d\varphi/2)} = 2\pi N \sum_m \delta(d\varphi - 2\pi m) + \sum_m \sum_{p=1}^{N-2} \frac{\exp\left(-\frac{1}{\pi}\left[N(d\varphi - 2\pi m) - \pi(2p+1)\right]^2\right)}{\sin^2\left(\pi m + \pi\frac{2p+1}{2N}\right)}$$

Side maxima

$$\left(\frac{\sin(Mg)}{\sin(g)}\right)^2 \approx \sum_{r=-\infty}^{+\infty}\left[M^2\exp\left(-\frac{(g-\pi r)^2}{\pi M^{-2}}\right)+\sum_{p=1}^{M-2}\frac{1}{\sin^2(g_p)}\exp\left(-\frac{(g-g_p)^2}{\pi M^{-2}/4}\right)\right]$$

$$g_p=\pi\left(r+\frac{2p+1}{2M}\right)$$



Red curve - exact $\left(\frac{\sin(Mg)}{\sin(g)}\right)^2$

Black curve – approximate one

for different number of periods M

Conclusions

- Frequency dependent of dielectric function can play an important role in radiation in periodic media/structures: it leads to new roots of dispersion relation, i.e. new types of radiation waves.
- For example, the effect is responsible for the sharp spikes in angular X-ray transition radiation.
- The role of the surface is considerable: radiation characteristics differ for radiation at small angles and large ones.
- In the case of Smith-Purcell effect in forward direction, we expect that its dispersion relation will contain the properties of the material of the target – like it happens in transition radiation.
- The “thin” structure of the widely used asymptotic expression is defined, the simple and easy-to-use expression is suggested. It must play a key part for the phenomena similar to “end transition radiation”, in all types of radiation in periodic media, and can be responsible for the fine structure of the radiation peaks, or even their existence when the main ones are suppressed.

Thank you
for your attention!