Charge Radiation in the Presence of Conical and Prismatic Dielectric Objects: Far-Field Area

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Supported by grant from Russian Science Foundation No. 18-72-10137.

Introduction

Radiation of charged particles in the presence of dielectric objects is of interest for applications in accelerator and beam physics. As a rule, complex geometry of the problem does not allow obtaining rigorous expressions for electromagnetic field.

At the same time, the size of the target is frequently much more than the wavelength under consideration, and this fact complicates computer calculations. However, it gives us an obvious small parameter of the problem and allows development of approximate methods of analysis.

We develop two methods which are applicable for objects having large size in comparison with wavelengths under consideration. One of them can be named "ray-optical technique", other of them can be named "aperture technique".

Ray-optical technique:

1. E.S. Belonogaya, A.V. Tyukhtin, S.N. Galyamin, *Phys. Rev. E*, vol. 87, p. 043201, 2013.
2. E.S. Belonogaya, S.N. Galyamin, A.V. Tyukhtin, *J. Opt. Soc. Am. B*, vol.32, p.649, 2015.

Aperture technique:

3. S.N. Galyamin and A.V. Tyukhtin, *Phys. Rev. Lett.*, vol. 113, p. 064802, 2014.
4. S.N. Galyamin, A.V. Tyukhtin, S. Antipov, S.S. Baturin, *Optics Express,* vol.22, p.8902, 2014.
5. A.V. Tyukhtin, V.V. Vorobev, S.N. Galyamin, E.S. Belonogaya, Journal of Instrumentation, 2018, vol.13, C02033.

Ray-optical and aperture methods

d – a size of an object

λ - wavelength under consideration

R – distance from the object to the observation point

The main condition for both methods: $d >> \lambda$

More exactly, we assume that:

- **the size of the external boundary of the object illuminated by Cherenkov radiation (the "aperture") is much more than the wavelength;**

- **the main part of the aperture is far from the path of the charge (in the wavelength scale).**

Ray-optical and aperture methods

The 1st and 2nd stages are the same for both methods.

1. We solve certain "etalon problem" which does not take into account "external" boundary of the target. For example, if the charge moves in the vacuum channel inside the target then we consider a channel border but do not take into account external boundaries of the object. In other words, initially we consider the problem for infinite medium with the boundary nearest to the charge trajectory and obtain the field inside the bulk of the target. This field can be called as "incident" field.

2. We select a part of the external surface of the object which is illuminated by Cherenkov radiation (the "aperture"). Using the fact that the object is large in comparison with the wavelengths we obtain asymptotic of the incident field (which is a wave field), and present it in the form of wave of two polarizations (vertical and horizontal). Further we calculate the field at the external surface of the aperture using the Snell's and Fresnel's laws.

The 3rd stage for ray-optical technique.

Calculation of the wave field outside the object using the ray-optics laws .

The 2nd stage for aperture technique.

Calculation of the field outside the object using Stratton-Chu formulae ("aperture integrals").

Ray-optical and aperture methods

The 3rd stage for ray-optical technique.

Calculation of the wave field outside the object using the ray-optics laws .

Advantage: Analytical formulae obtained by this method are not laborious for further computations.

Disadvantages: essential additional limitations.
- The distance from the aperture to the observation point should not be very large, i.e. so-called "wave parameter" should be small.
- The observation point cannot be close to focuses and caustics.

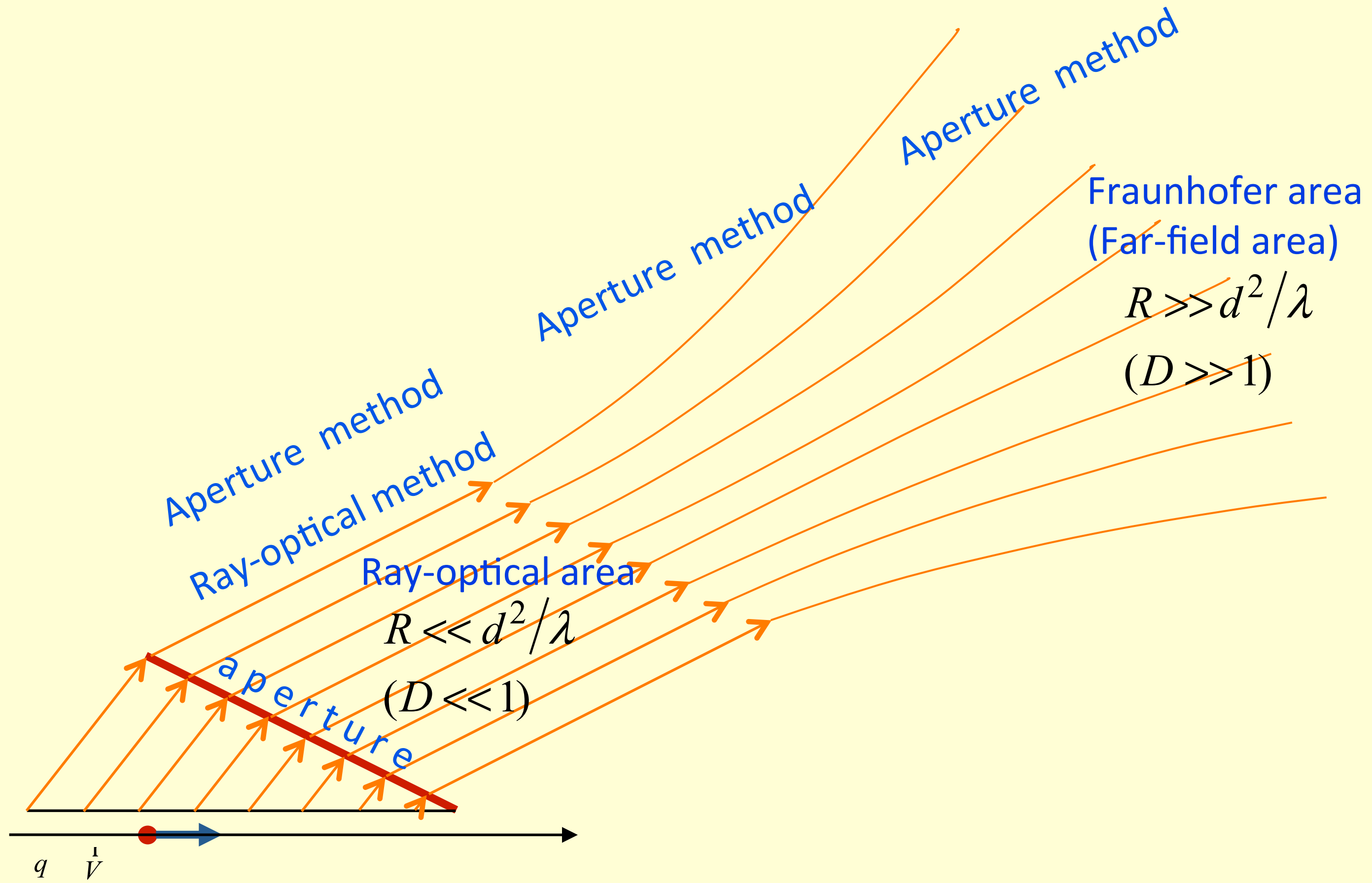
The 3rd stage for aperture technique.

Calculation of the field outside the object using Stratton-Chu formulae ("aperture integrals").

Advantage: No additional limitations. The method is valid for observation point with arbitrary wave parameter including Fraunhofer area, as well as in neighborhoods of focuses and caustics.

Disadvantage: It is necessary to compute complex double integrals.

Ray-optical and aperture methods



d is a size of the aperture

λ is a wavelenth

R is a distance from the aperture to the observation point

$D \sim \lambda R/d^2$ is a wave parameter

Aperture integrals (Stretton-Chu formulae)

General form

$$\vec{E}\left(\vec{R}\right)=\vec{E}^{(h)}\left(\vec{R}\right)+\vec{E}^{(e)}\left(\vec{R}\right),$$

$$\vec{E}^{(h)}\left(\vec{R}\right)=\frac{ik}{4\pi}\int_{\Sigma}\left\{\left[\vec{n}'\times\vec{H}\left(\vec{R}'\right)\right]G\left(\left|\vec{R}-\vec{R}'\right|\right)+\frac{1}{k^2}\left(\left[\vec{n}'\times\vec{H}\left(\vec{R}'\right)\right]\cdot\nabla'\right)\nabla' G\left(\left|\vec{R}-\vec{R}'\right|\right)\right\}d\Sigma',$$

$$\vec{E}^{(e)}\left(\vec{R}\right)=\frac{1}{4\pi}\int_{\Sigma}\left[\left[\vec{n}'\times\vec{E}\left(\vec{R}'\right)\right]\times\nabla' G\left(\left|\vec{R}-\vec{R}'\right|\right)\right]d\Sigma',$$

Σ is an aperture square,

$k=\omega/c$ is a wave number of the outer space,

$\vec{n}'$ is a unit normal to the aperture ,

$G\left(\vec{r}\right)=\dfrac{\exp\left(i\vec{k}\cdot\vec{r}\right)}{\left|\vec{r}\right|}$ is a Green function,

$$\nabla'=\vec{e}_{x'}\frac{\partial}{\partial x'}+\vec{e}_{y'}\frac{\partial}{\partial y'}+\vec{e}_{z'}\frac{\partial}{\partial z'}$$

Aperture integrals (Stretton-Chu formulae)

Approximate form for Fraunhofer (far-field) area

$$D \sim \lambda R \big/ d^2 \gg 1 \;\; \Rightarrow \;\; R \gg d \cdot d/\lambda \ggg d$$

$$\vec{E}^{(h)}\left(\vec{R}\right) \approx \frac{ik\exp\left(ikR\right)}{4\pi R} \int_{\Sigma} \left\{ \left[\vec{n}' \times \vec{H}\left(\vec{R}'\right)\right] - \vec{e}_R \left(\vec{e}_R \cdot \left[\vec{n}' \times \vec{H}\left(\vec{R}'\right)\right]\right) \right\} \exp\left(-ik\vec{e}_R \vec{R}'\right) d\Sigma',$$

$$\vec{E}^{(e)}\left(\vec{R}\right) \approx \frac{ik\exp\left(ikR\right)}{4\pi R} \int_{\Sigma} \left[\vec{e}_R \times \left[\vec{n}' \times \vec{E}\left(\vec{R}'\right)\right]\right] \exp\left(-ik\vec{e}_R \vec{R}'\right) d\Sigma',$$

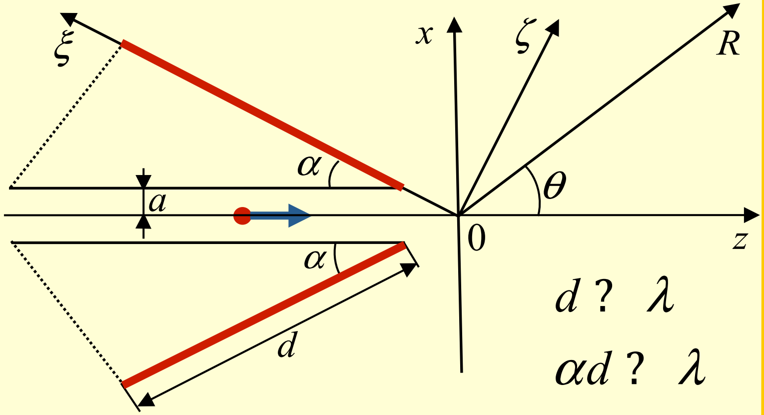
Σ is an aperture square,

$k = \omega / c$ is a wave number of the outer space,

$\vec{n}'$ is a unit normal to the aperture ,

$\vec{e}_R = \vec{R}/R$,

Conical target

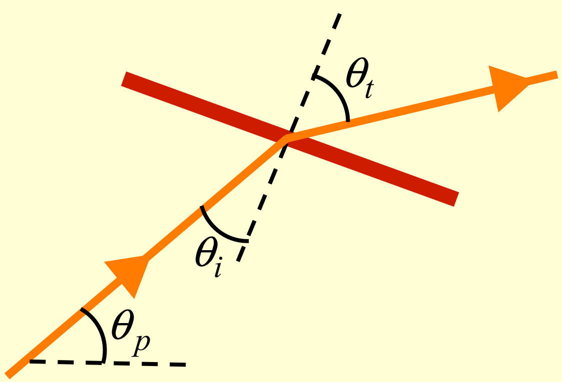


d ? λ

αd ? λ

Charge velocity: $\vec{V} = c\beta\vec{e}_z$

Refractive index: $n = \sqrt{\varepsilon\mu}$



Cherenkov angle: $\theta_p = \arccos\left(1/(n\beta)\right)$

Angle of incidence: $\theta_i = \dfrac{\pi}{2} - \alpha - \theta_p$

Angle of refraction: $\theta_t = \arcsin\left(n\sin\theta_i\right)$

The wave incident on aperture (Fourier transform):

$$H_\varphi^{(i)} \approx \frac{iq}{c}\eta\sqrt{\frac{s}{2\pi r}}\exp\left\{i\left(sr + \frac{\omega}{V}z - \frac{3\pi}{4}\right)\right\}$$

$$\eta = -\frac{2i}{\pi a}\left[\kappa\frac{1-n^2\beta^2}{\varepsilon\left(1-\beta^2\right)}I_1(\kappa a)H_0^{(1)}(sa) + sI_0(\kappa a)H_1^{(1)}(sa)\right]^{-1}$$

$$s(\omega) = \frac{\omega}{V}\sqrt{n^2\beta^2 - 1} \qquad \kappa(\omega) = \frac{\omega}{V}\sqrt{1-\beta^2}$$

Fourier-transform of the field
on outer surface of the aperture:

$$H_\varphi(\vec{R}) \approx \frac{T_{\mathrm{v}} q\eta\sqrt{s}}{c\sqrt{2\pi\xi\sin\alpha}}\exp\left\{i\frac{\omega}{V}\left(\sqrt{n^2\beta^2-1}\sin\alpha - \cos\alpha\right)\xi - \frac{i\pi}{4}\right\}$$

$$T_{\mathrm{v}} = 2\sqrt{\mu/\varepsilon}\cos\theta_i \Big/ \left(\sqrt{\mu/\varepsilon}\cos\theta_i + \cos\theta_t\right)$$

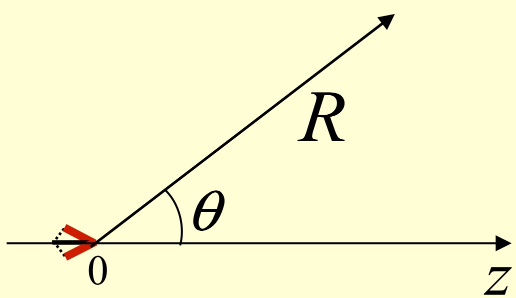
Conical target

After transformation, one can write the field in far-field zone in the form:

$$E_R = E_\varphi = H_R = H_\theta = 0$$

$$E_\theta = H_\varphi = -\frac{ke^{ikR}\sin\alpha}{2R}\int\limits_{\xi_1}^{\xi_2}\xi' H_{\varphi'}(\vec{R}')\Big[\big(\cos\theta_t + \sin\alpha\cos\theta\big)J_1\big(k\xi'\sin\alpha\sin\theta\big) +$$

$$+ i\cos\alpha\sin\theta\, J_0\big(k\xi'\sin\alpha\sin\theta\big)\Big]\exp\big(ik\xi'\cos\alpha\cos\theta\big)\,d\xi'.$$



Conditions of applicability:

1) $d >> \lambda, \quad \alpha d >> \lambda$

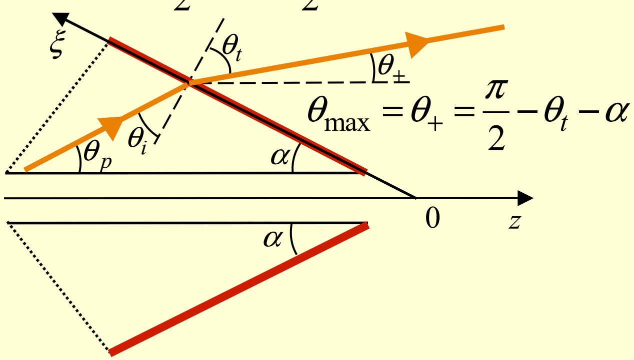
2) $D \sim \lambda R / d^2 >> 1 \;\Rightarrow\; R >> d \cdot d/\lambda >>> d$

Conical target

1) If angles $|\theta_\pm|$ are not small $\left(\left|\frac{\pi}{2}-\theta_t-\alpha\right| >> \frac{1}{kdsin\alpha}\right)$, then :

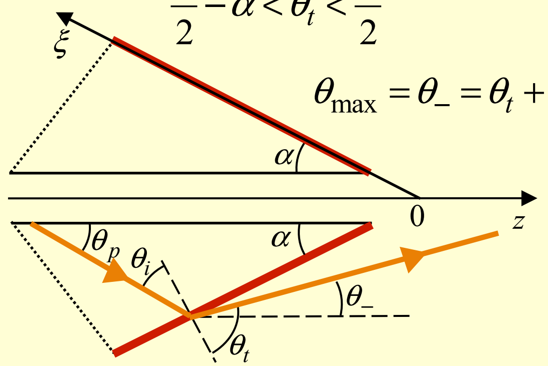
$$\theta_{\max} = \left|\frac{\pi}{2}-\theta_t-\alpha\right| \qquad E_{\theta\max} = \begin{Bmatrix} -i \text{ for } \theta_t < \pi/2-\alpha \\ 1 \text{ for } \theta_t > \pi/2-\alpha \end{Bmatrix} \frac{q\eta T_v \sqrt[4]{n^2\beta^2-1}\cos(\theta_t)}{2\pi\sqrt{\beta\left|\cos(\alpha+\theta_t)\right|}} \frac{kd\exp(ikR)}{cR}$$

$-\frac{\pi}{2} < \theta_t < \frac{\pi}{2}-\alpha$



$\frac{\pi}{2}-\alpha < \theta_t < \frac{\pi}{2}$

$\theta_{\max} = \theta_- = \theta_t + \alpha - \frac{\pi}{2}$



The angular half-width of the main lobe of the radiation pattern:

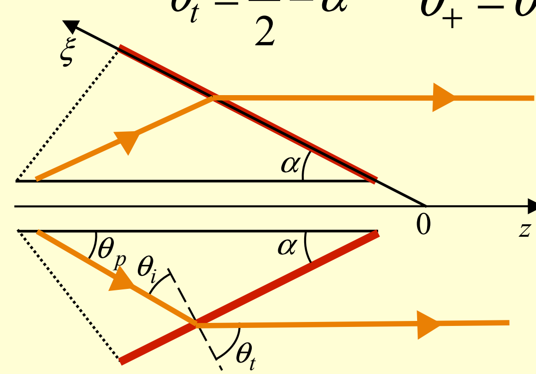
$$\delta\theta \approx \frac{2\pi}{k\,d\cos\theta_t}$$

2) If $\theta_t = \frac{\pi}{2}-\alpha$, then $\qquad \theta_{\max} \approx \frac{2.56}{kd\sin\alpha}$

$$\left|E_{\theta\max}\right| \approx 0.127\cdot\frac{q\eta T_v\sqrt[4]{n^2\beta^2-1}\,(kd\sin\alpha)^{3/2}}{cR\sqrt{\beta}}$$

"Cherenkov spotlight"

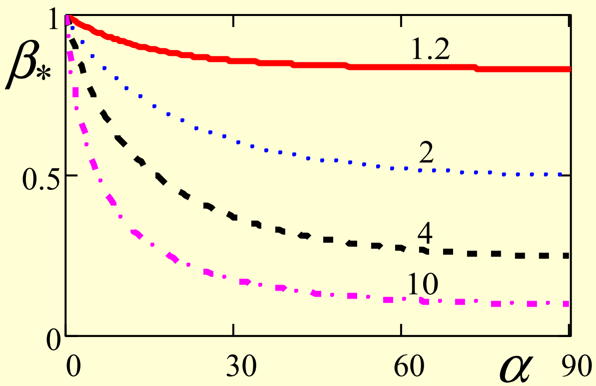
$\theta_t = \frac{\pi}{2}-\alpha \qquad \theta_+ = \theta_- = 0$



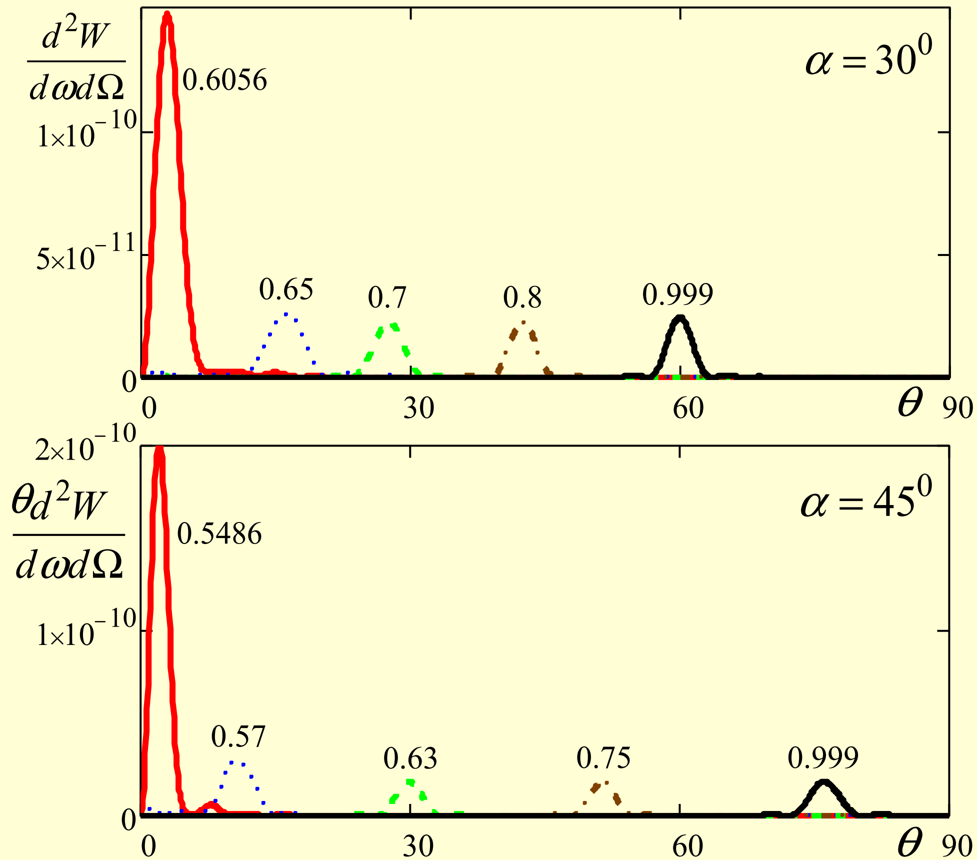
Conical target

"Cherenkov spotlight"

$$\beta = \beta_* = \frac{\sin\alpha\sqrt{n^2 - \cos^2\alpha} - \cos^2\alpha}{n^2\sin^2\alpha - \cos^2\alpha}$$



Dependence of $\beta_*(\alpha)$ for different refractive indexes $n = \sqrt{\varepsilon\mu}$

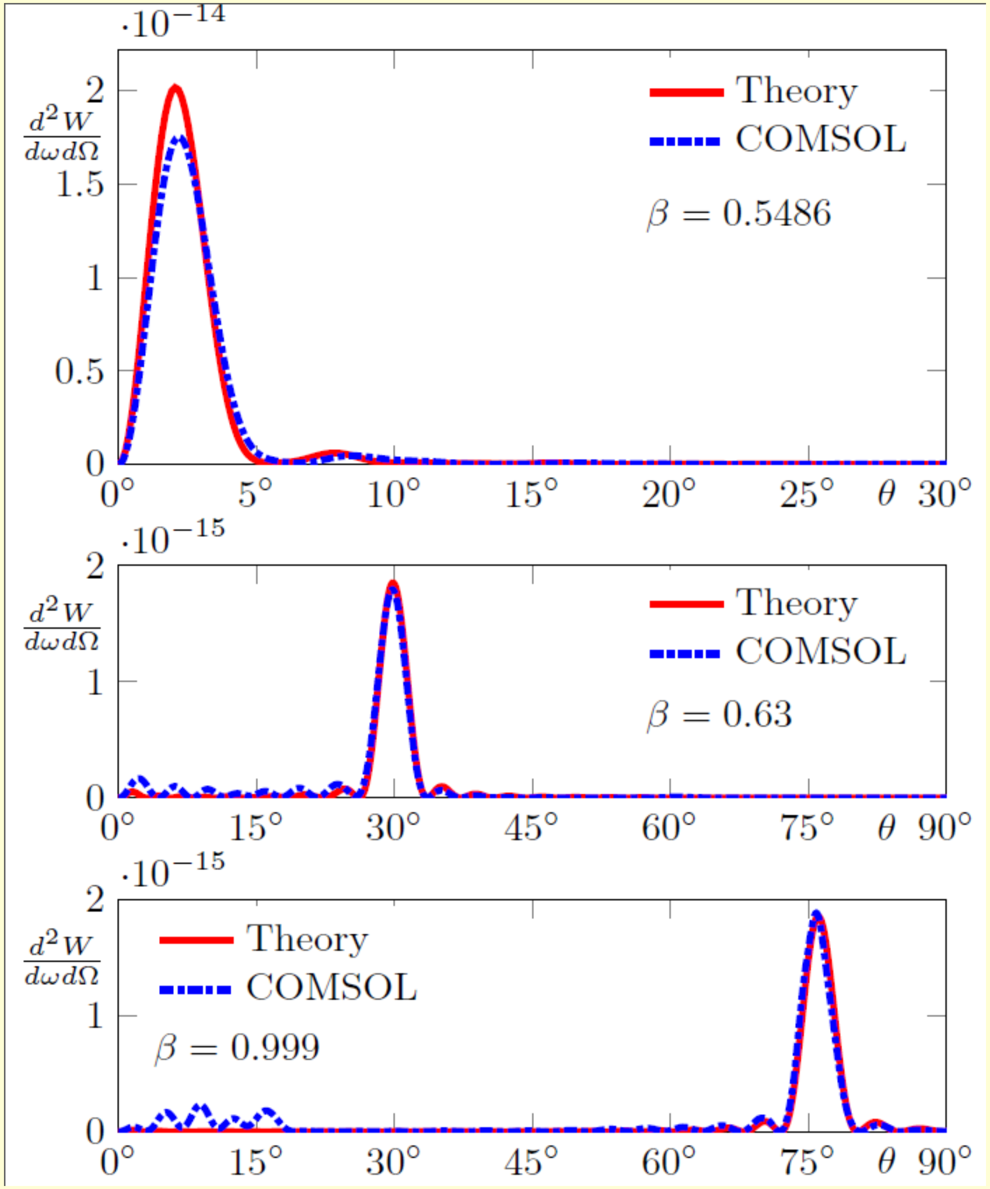


Frequency-angular density of the radiation power $(\mathrm{W\cdot s/rad^2})$ depending on the angle θ

$q = 1\mathrm{nC},\ \varepsilon = 4,\ \mu = 1,\ d\omega/c = 100,\ a\omega/c = 1.$

Conical target

"Cherenkov spotlight"



$\alpha = 45^0$

$q = 1\text{nC},$

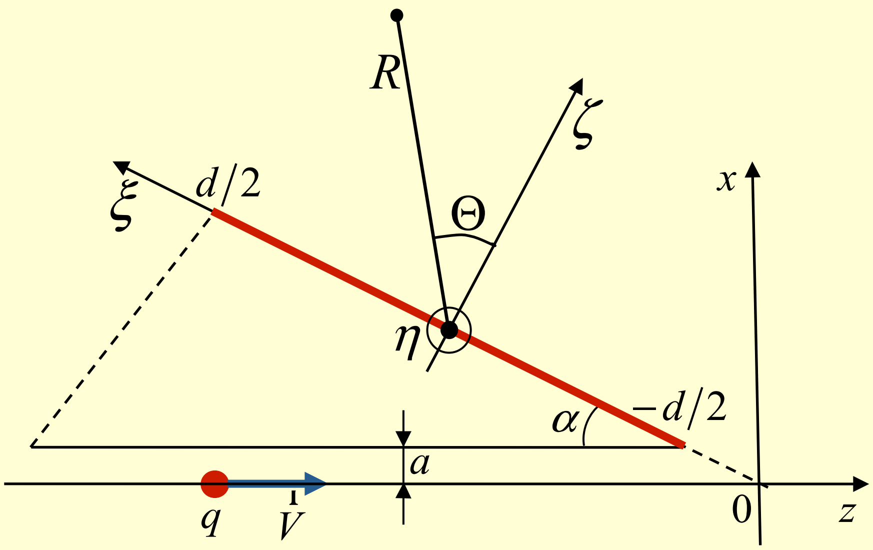
$\varepsilon = 4,\ \mu = 1,$

$d\omega/c = 100,$

$a\omega/c = 1.$

Prismatic target

Geometry of the problem

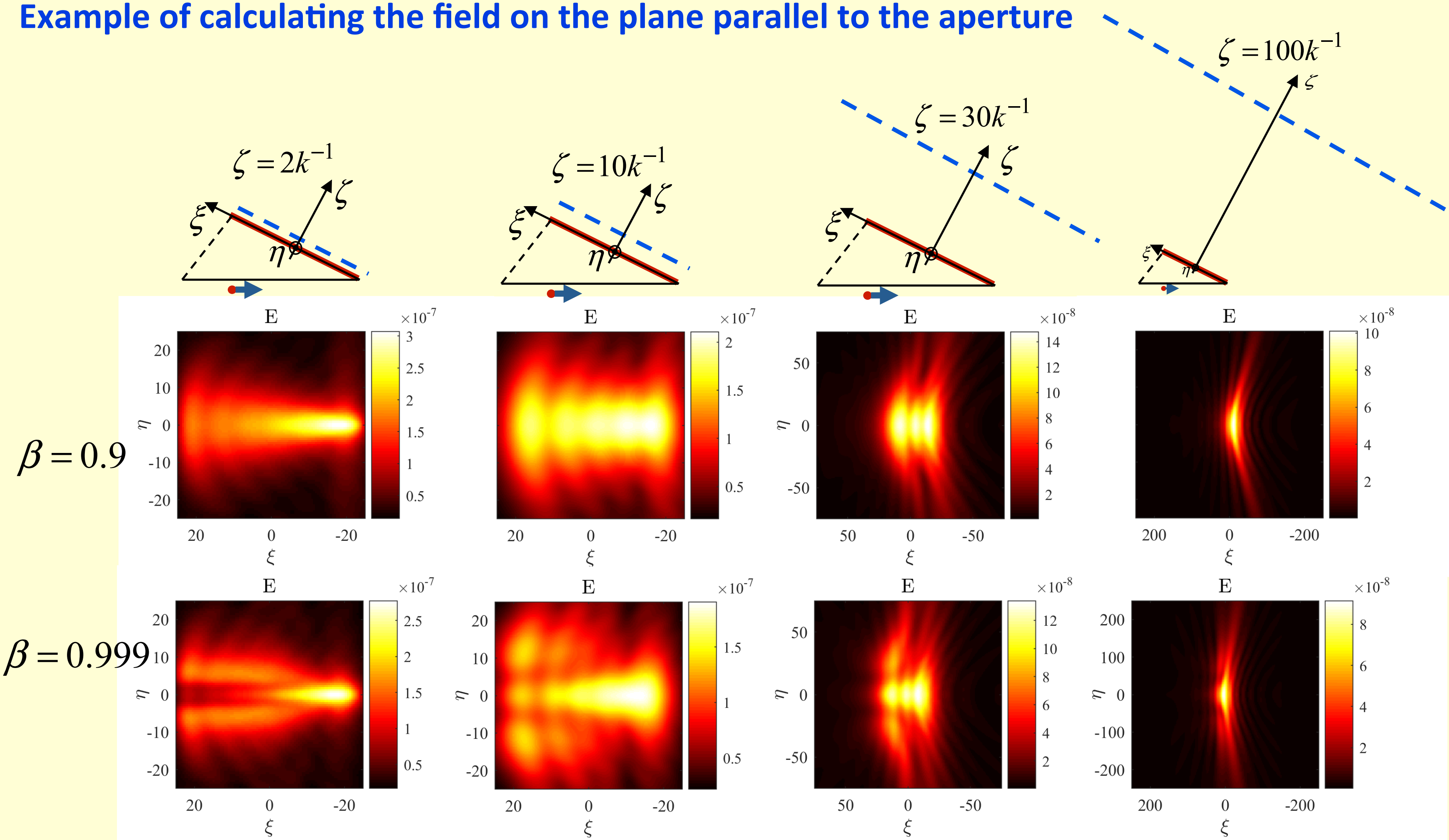


Aperture has the size
d (along *ξ*) and *b* (along *η*).

$$\xi = R\sin\Theta\cos\Phi,$$
$$\eta = R\sin\Theta\sin\Phi,$$
$$\zeta = R\cos\Theta.$$

Prismatic target

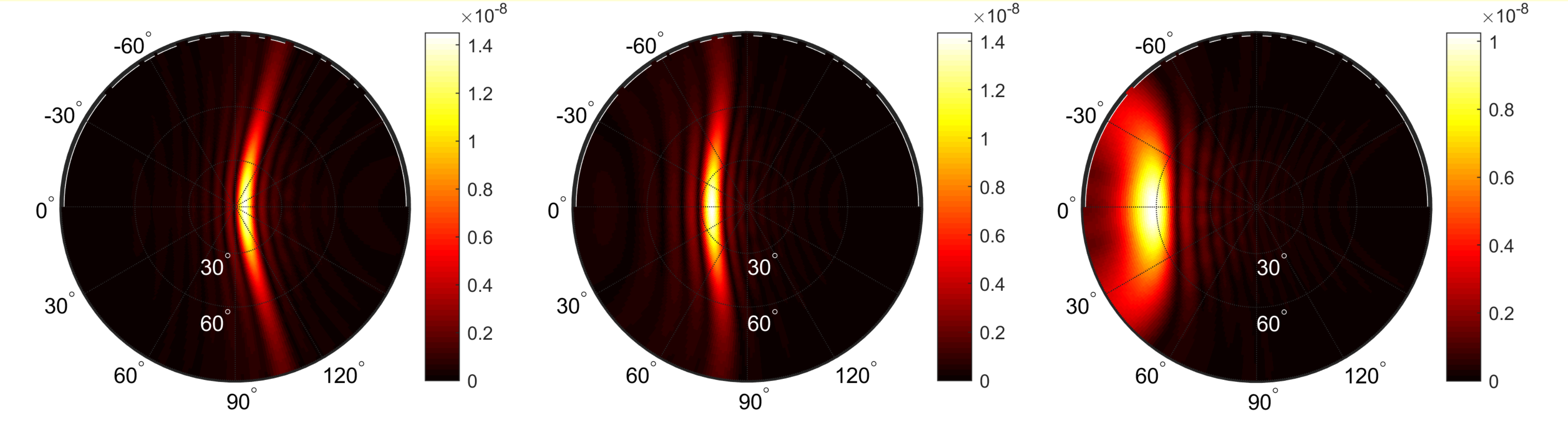
Example of calculating the field on the plane parallel to the aperture



Electric field magnitude $|\vec{E}|$ in the plane ξ, η (parallel to the aperture) for the following parameters: $\varepsilon = 4,\ \mu = 1,\ a = 1,\ d = b = 50,\ \alpha = 30^0.$

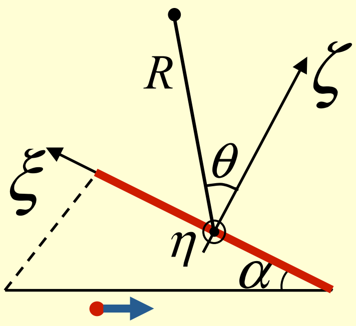
Prismatic target

Example of radiation pattern in the far-field (Fraunhofer) area



$\alpha = 30^0$ $\qquad \alpha = 45^0$ $\qquad \alpha = 60^0$

Electric field magnitude $|\vec{E}|$ in the far field zone.
Spherical coordinate system is used (with respect to normal ζ).
Radial axis is θ, polar axis is φ.
Parameters: $\varepsilon = 4,\ \mu = 1,\ a = 1,\ d = b = 50,\ \beta = 0.9$



Conclusion

The developed method based on the use of the aperture integrals allows calculating radiation from dielectric objects with a size of several wavelengths or more. This method is analogues to Kirchhoff method which has the widest distribution in optics, radiophysics and other “wave sciences”.

Analytical und numerical investigation of radiation in the cases of cone and prism have been performed. The characteristic distributions of the radiation field have been given.

For the conical object we found the phenomenon which can be called the effect of Cherenkov spotlight.

Thank you for attention!