Radiation of twisted photons in periodical structures in the classical regime

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Reference

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Outline

1. Twisted photons: generation, applications, observations.
2. Problem statement and results.
3. Probability of radiation of twisted photons by classical currents.
4. Undulators:
    - Dipole regime.
    - Wiggler.
5. Conclusion.

Twisted photons: generation, applications, observations

Definition

The twisted photons are the states of a free electromagnetic field with the definite energy k_0, the longitudinal projection of momentum k_3, the projection of the total angular momentum m, and the helicity s.

Generation

1. Spatial light modulators and digital micro-mirror devices (optical range and below) [e.g., A. Jesacher *et al.*, Opt. Express **15**, 5801 (2007)].
2. Spiral phase plates and diffraction gratings with ℓ-pronged forks dislocations (x-ray range and below) [M.W. Beijersbergen *et al.*, Opt. Commun. **112**, 321 (1994); V.Y. Bazhenov *et al.*, JETP Lett. **52**, 429 (1990)].
3. Helical undulators (x-ray range and below) [S. Sasaki, I. McNulty, Phys. Rev. Lett. **100**, 124801 (2008); E. Hemsing *et al.*, Nature Phys. **9**, 549 (2013); J. Bahrdt *et al.*, Phys. Rev. Lett. **111**, 034801 (2013)].
4. Inverse Compton scattering of low energy twisted photons and channeling (hundreds of MeVs and below) [U.D. Jentschura, V.G. Serbo, Phys. Rev. Lett. **106**, 013001 (2011)].

Applications

1. High-density information transfer.
2. Manipulation of the rotational degrees of freedom of irradiated objects: microstructures, molecules, atoms, nuclei, hadrons.
3. High-contrast microscopy.
4. Particle trapping and optical tweezers.

Observation of twisted photons at a single-photon level

1. Dove-prism interferometers [J. Leach *et al.*, Phys. Rev. Lett. **88**, 257901 (2002)].
2. Spatial light modulator based devices: holograms, optical transformation approaches [G.C.G. Berkhout *et al.*, Phys. Rev. Lett. **105**, 153601 (2010); G. Rufato *et al.*, Sci. Rep. **8**, 10248 (2018)].

Issues we are going to address

- To obtain the general formula for the radiation probability of twisted photons by classical currents.
- To develop theory of generation of twisted photons by undulators.
- To find the IR asymptotics of radiation of twisted photons. The IR asymptotics of radiation of photons is universal for any QED process [F. Bloch, A. Nordsieck, Phys. Rev. **52**, 54 (1937) and, e.g., S. Weinberg, Phys. Rev. **140**, B 516 (1965)].
- To find the selection rules and symmetry properties of radiation and absorbtion probabilities of twisted photons.

Results

1. The general formula for the probability of radiation of a twisted photon by a classical current is derived.
2. The general theory of generation of twisted photons by undulators is developed.
3. The explicit formulas for the average number of twisted photons generated by undulators in both dipole and wiggler regimes are obtained.
4. The exact formula for the radiation probability of twisted photons in the far infrared is obtained.
5. The main characteristics of the radiation twist – the differential asymmetry, the average projection of the total angular momentum of radiation, and the projection of the total angular momentum per photon – are found.
6. The main properties of the radiation probability of twisted photons by ultrarelativistic charged particles are established. In particular, the probability of radiation of twisted photons has a sharp peak near $n_\perp \approx \beta_\perp$, where $n_\perp = k_\perp/k_0$ and $k_\perp := \sqrt{k_0^2 - k_3^2}$.

Results (continuation)

7. Symmetries and selection rules:
   - For the forward radiation of an ideal right-handed helical undulator the harmonic number n of a twisted photon coincides with its projection of the total angular momentum m. As for the ideal left-handed helical undulator, $m = -n$.
   - The forward radiation of twisted photons by a planar undulator obeys the selection rule that $n + m$ is an even number.
   - The average number of twisted photons produced by the undulator and detected off the undulator axis is a periodic function of m in a certain spectral band of the quantum number m.
   - The symmetry property of the average number of twisted photons produced by a charged particle moving along a planar trajectory is found in the case when the detector axis lies in the orbit plane.
   - The general symmetry property of the average number of twisted photons produced in any QED process in the far infrared is established:

$$dP(s, m, k_3, k_\perp) = dP(-s, -m, k_3, k_\perp). \tag{1}$$

Results (continuation)

8. Selection rules for symmetric sources:
   - If the current density $j^i(t,\mathbf{x})$ is invariant under the rotation by an angle of $2\pi/r$, $r \in \mathbb{N}$, around the detector axis for all t, then
   $$dP(s,m,k_3,k_\perp)=0, \qquad m\neq lr, \quad l\in\mathbb{Z}. \tag{2}$$
   - If the trajectories of identical charged particles are obtained from one trajectory by the rotation by an angle of φ_k, the translation along the detector axis by x_3^k, and the translation in time x_0^k, where
   $$\varphi_k=\frac{2\pi k}{r}, \qquad x_3^k=\frac{\lambda_0}{2\pi}\varphi_k, \qquad x_0^k=\frac{\lambda_0}{2\pi\beta_{\|}}\varphi_k, \qquad k=\overline{1,r}, \tag{3}$$
   and λ_0 and $\beta_{\|}$ are some fixed parameters, then there is the selection rule
   $$m=\operatorname{sgn}(\lambda_0)n+lr, \qquad k_0=k_0^n:=\frac{2\pi n}{|\lambda_0|(\beta_{\|}^{-1}-n_3)}, \qquad n\in\mathbb{N},\ l\in\mathbb{Z}, \tag{4}$$
   $n_3:=k_3/k_0$, at the sharp maxima of radiation probability of twisted photons.
   - If the initial and final velocities and the detector axis lie in one plane, the initial and final energies and the velocity projections to the detector axis are the same, then m is an odd number in the far infrared.

Basis

$$\mathbf{e}_\pm := \mathbf{e}_1 \pm i\mathbf{e}_2. \tag{5}$$

$\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ is a right-handed orthonormal triple.
$\mathbf{e}_3$ is directed along the the detector axis.

Vector decomposition

$$\mathbf{x} = \frac{1}{2}(x_-\mathbf{e}_+ + x_+\mathbf{e}_-) + x_3\mathbf{e}_3. \tag{6}$$

$x_\pm = (\mathbf{x}, \mathbf{e}_\pm)$ and $x_3 = (\mathbf{x}, \mathbf{e}_3)$.

Process of radiation of photons

$$0 \to \gamma_\alpha + X. \tag{7}$$

0 is the vacuum state.
γ_α is the photon in the state α recorded by the detector.
X is for the other created photons.

Probability of the inclusive process (7)

$$w_{incl}(\alpha; 0) = 1 - e^{-n(\alpha;0)} \approx n(\alpha; 0). \tag{8}$$

$n(\alpha; 0)$ is the average number of photons in the state α created by the current $j_\mu(x)$.

Density of the average number of twisted photons created by the current of N particles

[O.V. Bogdanov *et al.*, Phys. Rev. A **97**, 033837 (2018)]

$$dP(s,m,k_3,k_\perp) = \Big| \sum_{l=1}^{N} e_l \int d\tau_l e^{-i[k_0 x_l^0(\tau_l) - k_3 x_{l3}(\tau_l)]} \Big\{ \frac{1}{2} \big[\dot{x}_{l+}(\tau_l) a_-(s,m,k_3,k_\perp;\mathbf{x}_l(\tau_l)) + \dot{x}_{l-}(\tau_l) a_+(s,m,k_3,k_\perp;\mathbf{x}_l(\tau_l)) \big] + \dot{x}_{l3}(\tau_l) a_3(m,k_\perp;\mathbf{x}_l(\tau_l)) \Big\} \Big|^2 \Big(\frac{k_\perp}{2k_0}\Big)^3 \frac{dk_3 dk_\perp}{2\pi^2}. \quad (9)$$

k_0 is the photon energy. $k_\perp := \sqrt{k_0^2 - k_3^2}$.

$s = \pm 1$ is the photon helicity.

$m \in \mathbb{Z}$ is the projection of the total angular momentum of a photon onto the detector axis.

The origin of the reference frame is taken on the line passing through the detector axis.

Mode functions

[e.g., R. Jáuregui *et al.*, Phys. Rev. A **71**, 033411 (2005)]

$$\begin{aligned} a_3(m,k_\perp;\mathbf{x}) &= \frac{x_+^{m/2}}{x_-^{m/2}} J_m(k_\perp x_+^{1/2} x_-^{1/2}) =: j_m(k_\perp x_+, k_\perp x_-), \\ a_\pm(s,m,k_3,k_\perp;\mathbf{x}) &= \frac{ik_\perp}{sk_0 \pm k_3} j_{m\pm1}(k_\perp x_+, k_\perp x_-). \end{aligned} \quad (10)$$

Differential asymmetry

$$A(s,m,k_3,k_0) := \frac{dP(s,m,k_3,k_0) - dP(s,-m,k_3,k_0)}{dP(s,m,k_3,k_0) + dP(s,-m,k_3,k_0)}. \tag{11}$$

Projection of the total angular momentum per photon

$$\ell(s,k_3,k_0) := \frac{dJ_3(s,k_3,k_0)}{dP(s,k_3,k_0)} \tag{12}$$

$$dJ_3(s,k_3,k_0) := \sum_{m=-\infty}^{\infty} m dP(s,m,k_3,k_0), \quad dP(s,k_3,k_0) = \sum_{m=-\infty}^{\infty} dP(s,m,k_3,k_0). \tag{13}$$

Integrated projection of the total angular momentum per photon

$$\begin{gathered} \ell(s,k_0) := \frac{dJ_3(s,k_0)}{dP(s,k_0)}, \\ dJ_3(s,k_0) = \int_0^{k_0} dk_3 \frac{dJ_3(s,k_3,k_0)}{dk_3}, \quad dP(s,k_0) = \int_0^{k_0} dk_3 \frac{dP(s,k_3,k_0)}{dk_3}. \end{gathered} \tag{14}$$

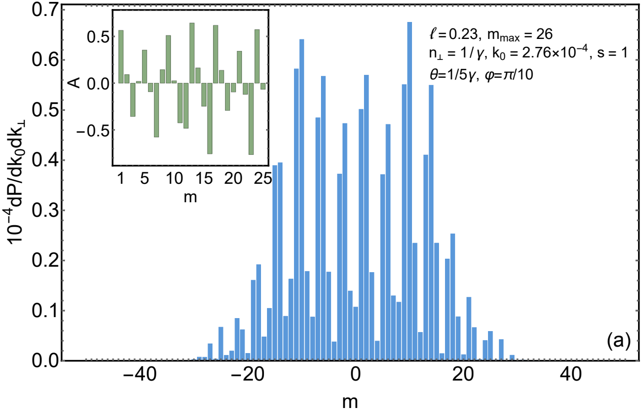

Figure: Trajectory: $a_z = a_y = 0$, $a_x = K\lambda_0/(\sqrt{2}\pi\gamma)$, where $K = 0.03$ is the undulator strength parameter, $\lambda_0 = 1$ cm is the length of the undulator section, and $\gamma = 10^3$ is the Lorentz-factor of the electron. The number of undulator sections $N = 40$. The energy of photons is measured in the units of the electron rest energy 0.511 MeV and $n_\perp := k_\perp/k_0$. (a) The distribution over m, the asymmetry, and the angular momentum projection per photon. The period of oscillations $T_m = 4$.

Undulators: Dipole regime for a planar undulator at the first harmonic

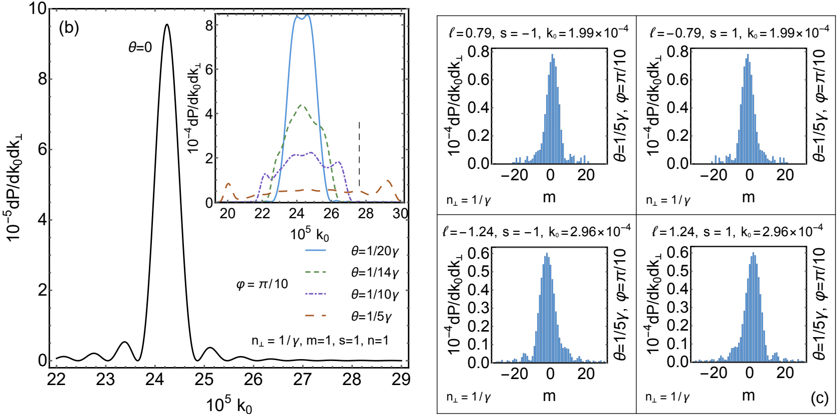

Figure: (b) The density of the average number of twisted photons against k_0 for different observation angles. The dashed vertical line in the inset depicts the photon energy used in the plot (a) on the previous slide. (c) The density of the average number of twisted photons against m at the left and right peaks appearing in the distribution over the photon energy for $\theta = 1/(5\gamma)$ (see the inset in the plot (b)). The average number of photons obeys the symmetry relation (1).

Undulators: Dipole regime for a helical undulator at the first harmonic

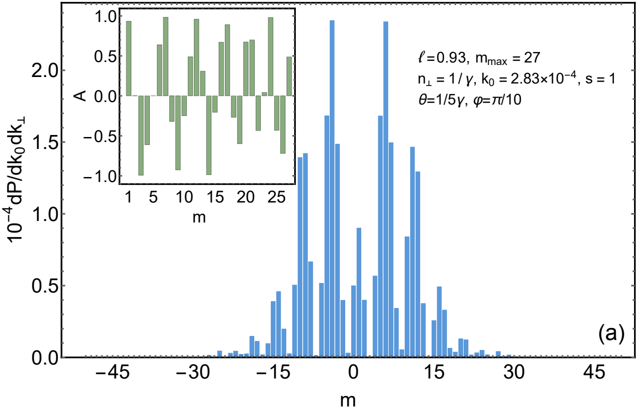

Figure: Trajectory: $K = 4.6 \times 10^{-3}$ is the undulator strength parameter, $\lambda_0 = 1$ cm is the length of the undulator section, and $\gamma = 10^3$ is the Lorentz-factor of the electron. The number of the undulator sections $N = 40$. The energy of photons is measured in the units of the electron rest energy 0.511 MeV. (a) The distribution over m, the asymmetry, and the angular momentum projection per photon. The period of oscillations $T_m = 5$.

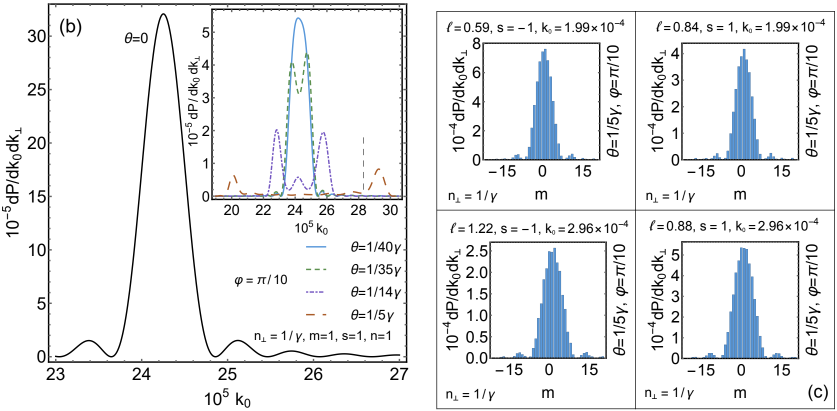

Figure: (b) The density of the average number of twisted photons against k_0 for different observation angles. The dashed vertical line in the inset depicts the photon energy used in the plot (a) on the previous slide. (c) The density of the average number of twisted photons against m at the left and right peaks appearing in the distribution over the photon energy for $\theta = 1/(5\gamma)$ (see the inset in the plot (b)).

Undulators: Forward radiation in the dipole regime at the first harmonic

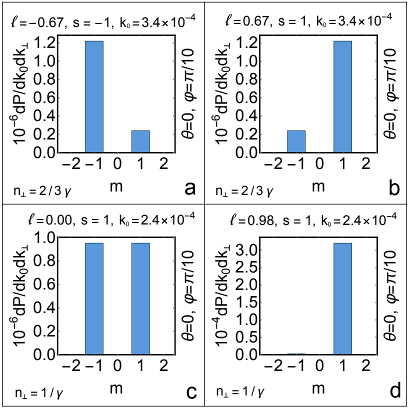

Figure: The distribution over m. The forward radiation of the planar undulator obeys the symmetry relation (1) and the selection rule $m + n$ is an even number. The forward radiation of the right-handed helical undulator obeys the rule $m = n$. The insets (a), (b), (c) are for the planar undulator, and (d) is for the helical one. The trajectories of the electron are the same as on the previous slides. The small contribution at $m = -1$ for the helical undulator is a consequence of deviation of the trajectory from an ideal right-handed helix.

Wigglers: Planar wiggler at the fifth harmonic

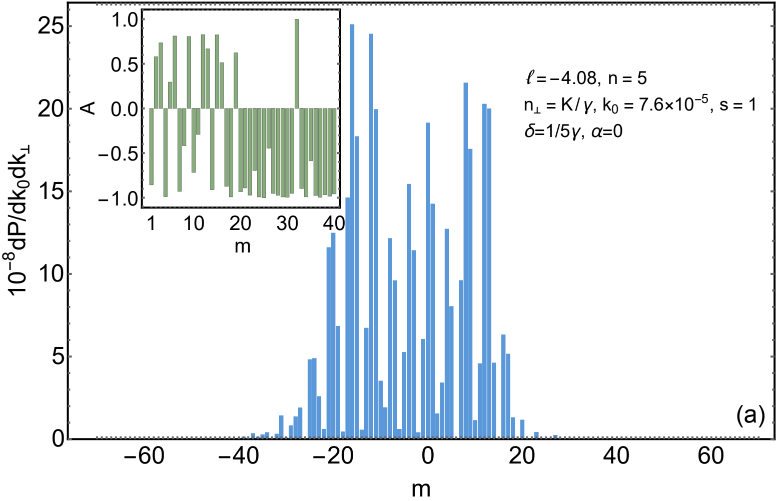

Figure: Trajectory: $K = 4$, $\omega = 2\pi\beta_{\parallel}/\lambda_0$, where $\lambda_0 = 1$ cm, $\gamma = 10^3$, and $N = 40$. (a) The distribution over m, the asymmetry, and the angular momentum projection per photon. The period of oscillations $T_m = 4$.

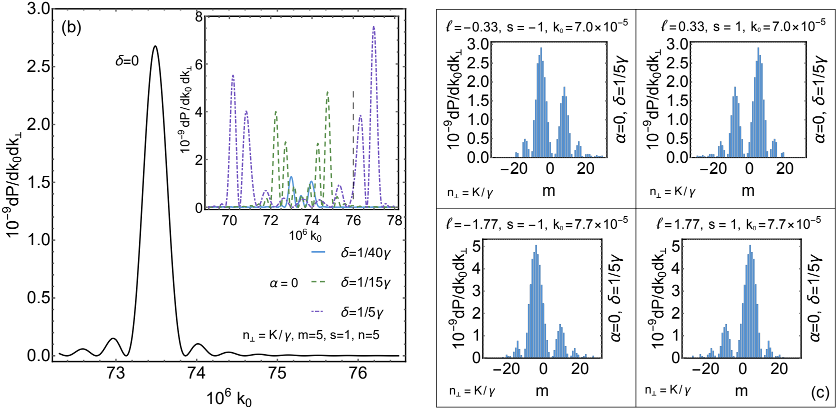

Figure: (b) The density of the average number of twisted photons against k_0 for different observation angles. The dashed vertical line in the inset depicts the photon energy used in the plot (a) on the previous slide. (c) The density of the average number of twisted photons against m at the left and right peaks appearing in the distribution over the photon energy for $\theta = 1/(5\gamma)$ (see the inset in the plot (b)). As long as the observation angle $\alpha = 0$, the distributions over m have the symmetry property (1).

Wigglers: Helical wiggler at the fifth harmonic

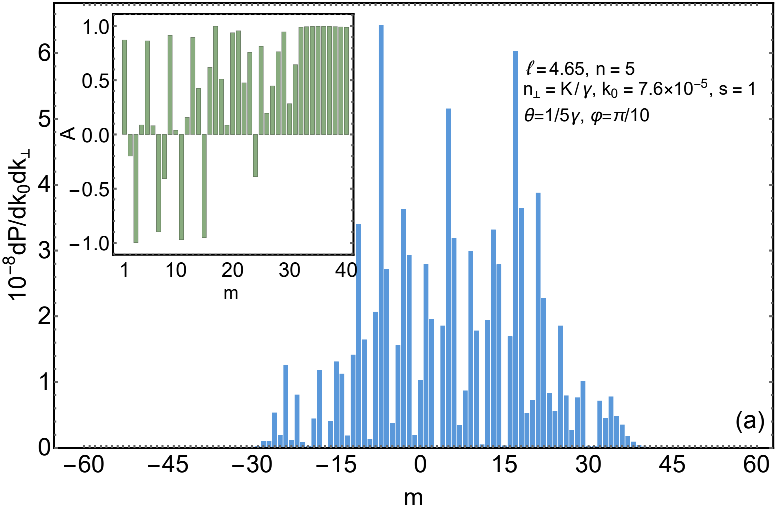

Figure: Trajectory: $K = 4$, $\omega = 2\pi\beta_{||}/\lambda_0$, where $\lambda_0 = 1$ cm, $\gamma = 10^3$, and $N = 40$. (a) The distribution over m, the asymmetry, and the angular momentum projection per photon. The period of oscillations $T_m = 4$.

Wigglers: Helical wiggler at the fifth harmonic

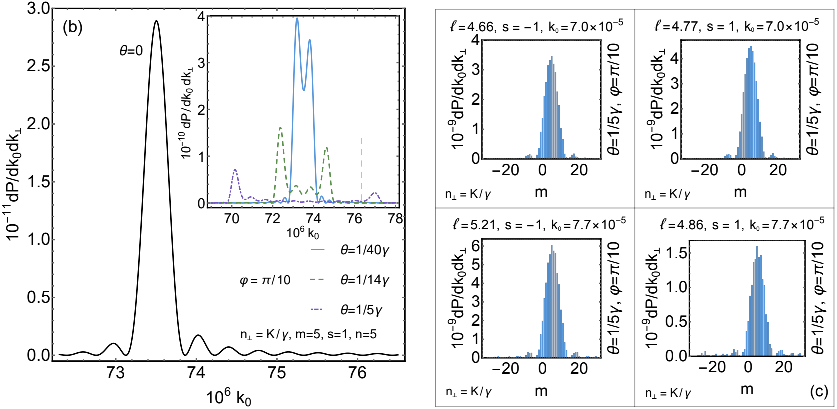

Figure: (b) The density of the average number of twisted photons against k_0 for different observation angles. The dashed vertical line in the inset depicts the photon energy used in the plot (a) on the previous slide. (c) The density of the average number of twisted photons against m at the left and right peaks appearing in the distribution over the photon energy for $\theta = 1/(5\gamma)$ (see the inset in the plot (b)).

Wigglers: Forward radiation at the fifth harmonic

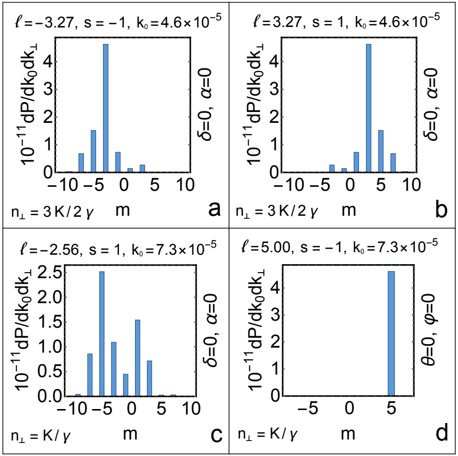

Figure: The forward radiation of the planar undulator obeys the symmetry relation (1) and the selection rule that $m+n$ is an even number. The forward radiation of the right-handed helical undulator obeys the rule $m=n$. The insets (a), (b), (c) are for the planar wiggler, and (d) is for the helical one. The trajectories of the electron are the same as on the previous slides.

Conclusion (see for more pra 97, 033837 (2018) and arXiv:1803.03447)

Synopsis of the results

1. The theory of radiation of twisted photons by classical currents is constructed.
2. The undulator radiation is studied in detail.
3. The IR asymptotics is investigated.
4. Several selection rules and symmetries for the radiation probability of twisted photons are established.