Cherenkov-Transition Radiation in a Waveguide with a Semibounded Strongly Magnetized Plasma

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CONTENTS

- ***Formulation of the problem***
- ***Methods of analysis***
- ***Cherenkov Radiation in a Strongly Magnetized Plasma***
- ***Cherenkov Transition Radiation (CTR) in vacuum and in the Plasma***
- ***Conclusion***

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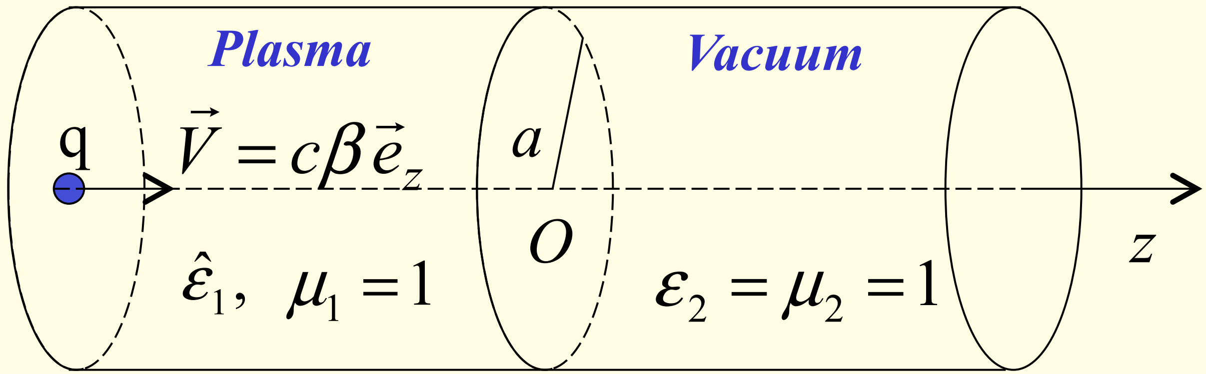
Formulation of the problem



Strongly magnetized plasma:

$$\hat{\varepsilon}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}, \qquad \varepsilon_z = \varepsilon_z(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + 2i\omega\omega_d}, \qquad (1)$$

$\omega_p = \sqrt{\dfrac{4\pi n_e e^2}{m_e}}$ - plasma frequency, ω_d - losses, $\omega_d \to +0$

The gyration parameter is neglected.

The bunch is characterized by a distribution along the - axis and a negligible thickness. The charge density is

$$\rho = q\delta(r)\eta(\zeta)/(2\pi r), \qquad \int_{-\infty}^{\infty} \eta(\zeta)d\zeta = 1, \qquad \zeta = z - ct\beta.$$

For the bunch with the Gaussian distribution

$$\eta(\varsigma) = \exp\left(-\zeta^2/\left(2\sigma^2\right)\right)/\left(\sqrt{2\pi}\sigma\right)$$

Formulation of the problem

$$\vec{H}_{1,2} = \vec{H}^{q}_{1,2} + \vec{H}^{b}_{1,2} \qquad (2)$$

The "forced" field (called by V.L. Ginzburg [*]) is the field of the bunch in a regular waveguide. It can contain **Cherenkov radiation (CR)** if the condition $n^2_{1,2}(\omega)\beta^2 > 1$ is fulfilled.

$$H^{q}_{\varphi 1,2} = \frac{2q}{\pi c a^3}\sum_{n=1}^{\infty}\frac{\chi_n J_1(\chi_n r/a)}{J_1^2(\chi_n)}\int_{-\infty}^{+\infty} d\omega \frac{\tilde{\eta}(\omega)\exp[\mathrm{i}\,\zeta/c\beta]}{k^2_{z1,2}-\omega^2\left(\beta^2c^2\right)^{-1}} h^{q}_{\varphi 1,2}, \qquad (3)$$

$$h^{q}_{\varphi 1} = (\varepsilon_z)^{-1}, \quad h^{q}_{\varphi 2} = 1, \qquad k_{z1} = \frac{\omega}{c}\sqrt{\frac{\omega^2-\omega_n^2-\omega_p^2}{\omega^2-\omega_p^2}}, \qquad k_{z2} = \frac{\sqrt{\omega^2-\omega_n^2}}{c}$$

$\tilde{\eta}(\omega) = \exp\left(-\frac{\omega^2\sigma^2}{2\beta^2c^2}\right)$ - for the bunch with the Gaussian distribution

$\omega_n = \frac{\chi_n c}{a}$, χ_n is the n^{th} zero of the Bessel function $\left(J_0(\chi_n) = 0\right)$

** V.L. Ginzburg and V.N. Tsytovich, "Transition Radiation and Transition Scattering", Hilger, London, p. 445 (1990).*

Formulation of the problem

The "free" field is connected with the influence of the boundary. It includes **transition radiation (TR)** and can includes **Cherenkov Transition Radiation (CTR)** under certain conditions.

$$H^{b}_{\varphi 1,2} = \frac{2q\beta}{\pi a^3} \sum_{n=1}^{\infty} \frac{\chi_n J_1(\chi_n r/a)}{J_1^2(\chi_n)} \int_{-\infty}^{+\infty} B_{n1,2} \tilde{\eta}(\omega) \exp\left[i\left(k_{z1,2}|z| - \omega t\right)\right] d\omega, \qquad (4)$$

$$B_{n1} = \frac{1}{k_{z1} + k_{z2}} \left[\left(c\beta k_{z2} + \omega\right)^{-1} + \frac{\left(\omega - c\beta k_{z2}\right)}{\varepsilon_z \left(c^2 \beta^2 k_{z1}^2 - \omega^2\right)} \right]$$

$$B_{n2} = \frac{1}{\varepsilon_z \left(k_{z1} + k_{z2}\right)} \left[\left(c\beta k_{z1} - \omega\right)^{-1} - \frac{\left(\omega \varepsilon_z - c\beta k_{z1}\right)}{\left(c^2 \beta^2 k_{z2}^2 - \omega^2\right)} \right]$$

$$k_{z1} = \frac{\omega}{c} \sqrt{\frac{\omega^2 - \omega_n^2 - \omega_p^2}{\omega^2 - \omega_p^2}}, \qquad k_{z2} = \frac{\sqrt{\omega^2 - \omega_n^2}}{c} \qquad \left(\mathrm{Im}\, k_{z1,2} > 0\right) \qquad (5)$$

$$\tilde{\eta}(\omega) = \exp\left(-\frac{\omega^2 \sigma^2}{2\beta^2 c^2} \right)$$ - for the bunch with the Gaussian distribution

Methods of analysis

We investigate the exact solution with analytical and computational methods.

The analytical investigation is based on the complex variable function theory.

Computations are based on an algorithm using certain transformation of the initial integration path. Such approaches were applied as well in some papers concerning both boundless homogenous media and problems with interface between two media

A.V. Tyukhtin and S.N. Galyamin Phys. Rev. E 77 (2008) 066606,
S.N. Galyamin.and A.V. Tyukhtin Phys. Rev. B 81 (2010) 35134

including the case of the waveguide partially filled with an isotropic cold plasma

T.Yu. Alekhina and A.V. Tyukhtin, Phys. Rev. E 83 (2011) 066401.

Note that CR is not generated in an isotropic cold plasma as opposed to the case considered here.

CR in the strongly magnetized plasma

The forced field in the plasma has two parts:
a quasi-Coulomb field and CR - $H_{\varphi 1}^{CR}$:

$$H_{\varphi 1}^{CR} = \frac{q}{a^2} \sum_{n=1}^{\infty} \frac{\chi_n J_1(\chi_n r/a)\eta(\omega_{0n})}{J_1^2(\chi_n)} A_n^{CR} Sin\left[\omega_{0n}\left(\frac{z}{c\beta} - t\right)\right]\theta(\beta ct - z), \qquad (6)$$

where $\theta(x)$ is the Heaviside step function,

$$A_n^{CR} = \frac{4\beta^2}{x_{0n} x_p (1-\beta^2)} \qquad (7)$$

$$\omega_{0n} = \omega_p x_{0n}, \qquad x_{0n} = \left[1 - \frac{y_n^2 \beta^2}{1-\beta^2}\right]^{1/2} \qquad (8)$$

The frequencies of the radiated waves is always less than the plasma frequency.

$$\boxed{\omega_{0n} < \omega_p}$$

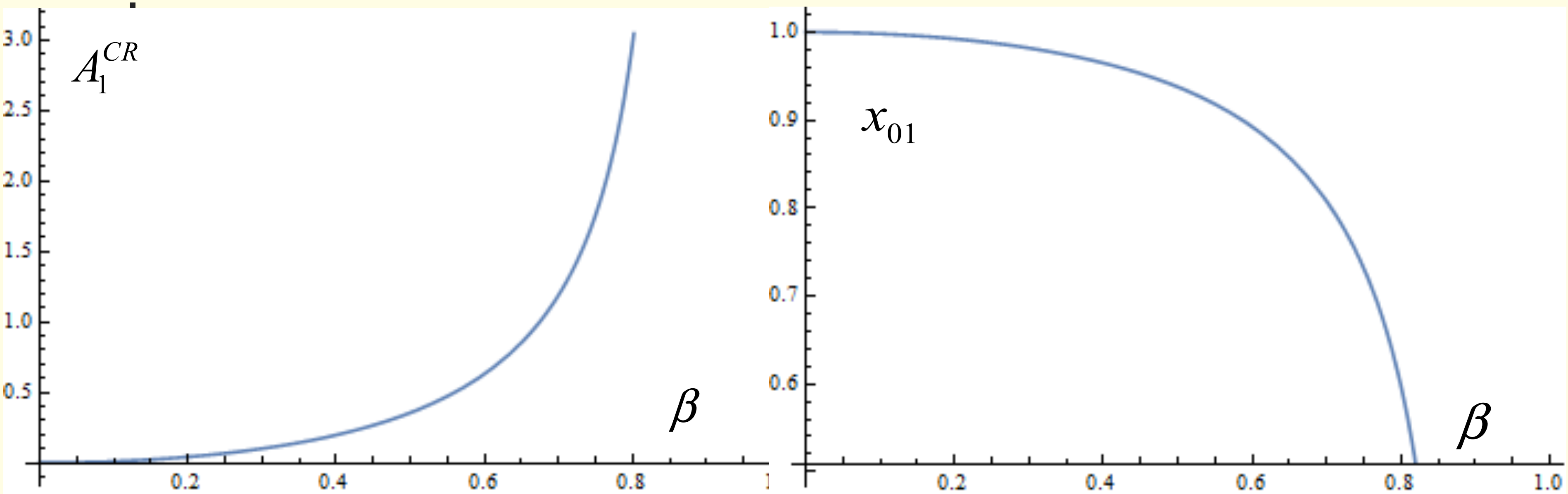
CR is generated if the velocity of the bunch motion is under condition

$$\boxed{\beta < \beta_{1n} = \left(1 + y_n^2\right)^{-1/2},} \qquad (9)$$

$$y_n = \omega_n/\omega_p = \chi_n/x_p, \quad x_p = \omega_p a/c,$$

χ_n is the n^{th} zero of the Bessel function $\left(J_0(\chi_n) = 0\right)$

The amplitude and the frequency of the first mode of CR:



$$x_p = 4, \quad x_p = a\omega_p / c$$

$$x_{0n} = \omega_{0n} / \omega_p$$

If $\beta << 1$, then

$$x_{0n} = 1 - \frac{1}{2} y_n^2 \beta^2 + O\left(\beta^4\right)$$

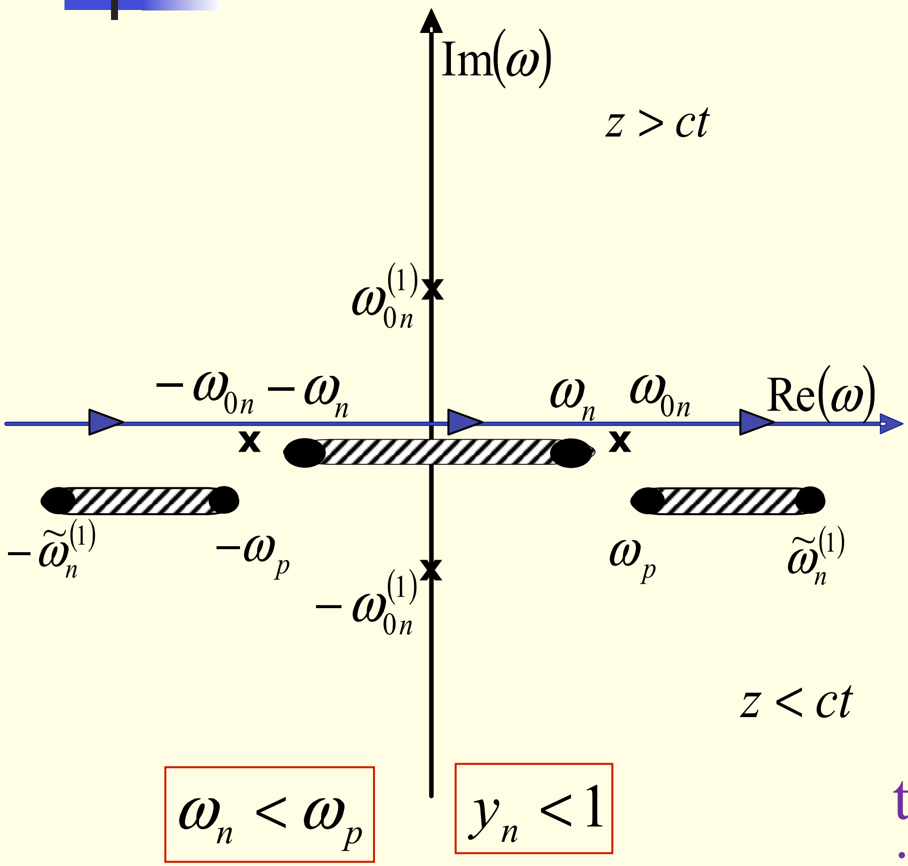
$$A_n^{CR} = 4\beta^2 x_p^{-1}\left[1 + O\left(\beta^2\right)\right]$$

If $\beta \approx \beta_{0n} - \delta^2$, then

$$x_{0n} = \delta\left[1 + \frac{\left(1 + y_n^2\right)}{y_n}\right]\left[1 + O\left(\delta^2\right)\right]$$

$$A_n^{CR} = \frac{4}{\delta\, y_n x_p\left(1 + y_n^2\right)}\left[1 + O\left(\delta^2\right)\right]$$

Vacuum: analysis of B_{n2} in a complex plane of (ω)



Poles: $\pm\omega_{0\mathrm{n}}^{(1)} = \pm\mathrm{i}\omega_{\mathrm{p}} x_{0\mathrm{n}}^{(1)}$,

$\pm\omega_{0n} = \pm\omega_p x_{0n}$, $\boxed{x_{0n} < 1}$

The branch points of the radical k_{z2}:

$\pm\omega_n - \mathrm{i}0$.

The branch points of the radical k_{z1}:

$\pm\omega_{\mathrm{p}} - \mathrm{i}0;\quad \pm\tilde{\omega}_n^{(1)} - \mathrm{i}0,$

$$\tilde{\omega}_n^{(1)} = \sqrt{\omega_p^2 + \omega_n^2},$$

Branch cuts: $\mathrm{Re}\, k_{z1} = 0,$

$$\mathrm{Re}\sqrt{\omega^2 - \omega_n^2} = 0$$

The poles contributions give the transmitted wave of CR (the CTR) in vacuum if $\boxed{\omega_n < \omega_{0n}}$ $\boxed{y_n < x_{0n}}$

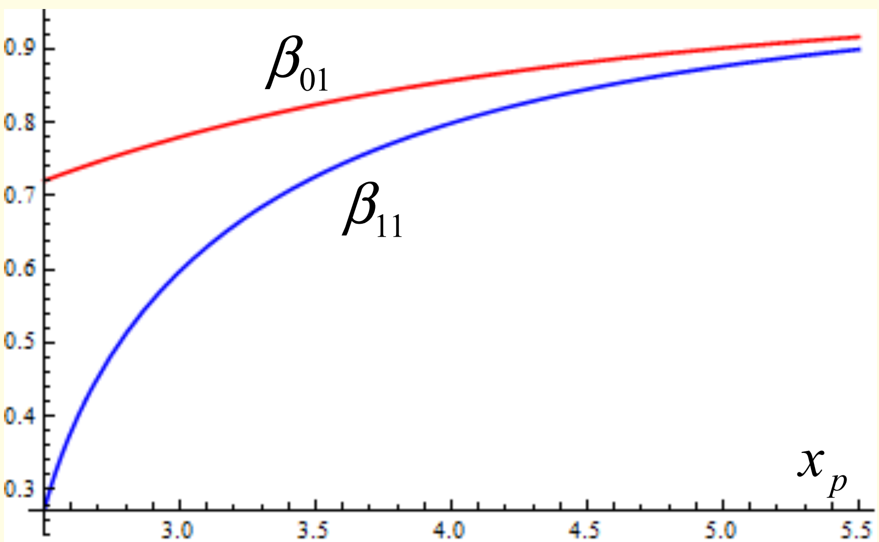
The condition for the single mode CTR in vacuum

$$y_n = \omega_n/\omega_p = \chi_n/x_p,$$

The number of propagating modes (which compose the CTR) is always finite.

If $\omega_n < \omega_p \; (y_n < 1)$, then the CTR with the number n exist in some domain in the vacuum area at the condition $\beta < \beta_{1n} = \sqrt{1 - y_n^2}$.

For $n = 1$ $\chi_1 = 2.405 < x_p < \chi_2 = 5.52,$ (10)



$$\beta_{1n} < \beta_{0n} = 1/\sqrt{1 + y_n^2}, \quad x_p = a\omega_p/c$$

If $\omega_p = 2\pi \cdot 9\text{GHz}, \left(n_{pe} = 10^{12}\,cm^{-3}\right)$

then $1.3 < a < 3$ (11)

(in centimeters).

The threshold value β_{1n} is explained by the total internal reflection of CR off the boundary.

CTR in vacuum and in the plasma

$$H_{\varphi 1,2}^{\mathrm{CTR}} = \frac{q}{a^2}\sum_{n=1}^{\infty}\frac{\chi_n J_1(\chi_n r/a)}{J_1^2(\chi_n)}\eta(\omega_{0n})I_{n1,2}^{\mathrm{CTR}} \tag{12}$$

$$I_{n1}^{CTR} = A_{n1}^{CTR}\sin\left[\omega_{0n}\left(z\beta^{-1}c^{-1}+t\right)\right]\theta\left(z_{f1}-z\right)$$

$$I_{n2}^{CTR} = A_{n2}^{CTR}\sin\left[\omega_{0n}\left(\Psi_{\mathrm{n}} zc^{-1}-t\right)\right]\theta\left(z_{f2}-|z|\right)$$

$$\Psi_n = \sqrt{1-y_n^2(x_{0n})^{-2}}\,, \tag{13}$$

Plasma ***Vacuum***

$$A_{n1}^{CTR} = A_n^{CR}\frac{1-\beta\Psi_{\mathrm{n}}}{1+\beta\Psi_{\mathrm{n}}},\qquad A_n^{CR} = \frac{4\beta^2}{x_{0n}x_p\left(1-\beta^2\right)},\qquad A_{n2}^{CTR} = A_n^{CR}\frac{2}{1+\beta\Psi_{\mathrm{n}}}, \tag{14}$$

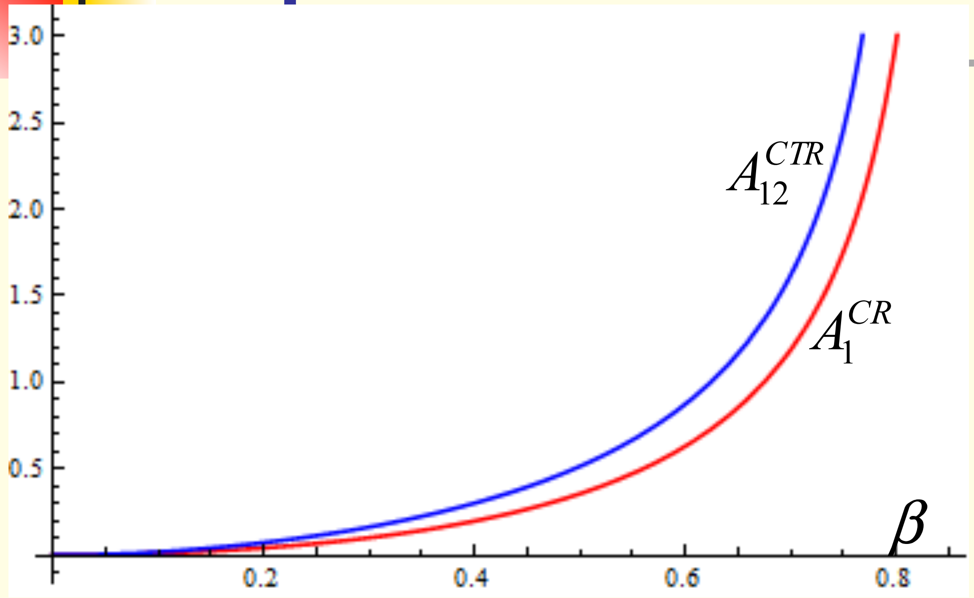
The CTR in vacuum exists in some domain

$$z < z_{f2},\qquad z_{f2} = ct\sqrt{\frac{1-\beta^2-y_n^2}{1-\beta^2-\beta^2 y_n^2}}$$

The front of the CTR propagates in vacuum with the group velocity:

$$V_g = c\Psi_{\mathrm{n}}$$

The first modes of the CR and CTR amplitude



$$x_p = 4, \quad x_p = a\omega_p / c$$

If $\beta << 1,$ then

$$A_n^{CR} = 4\beta^2 x_p^{-1}\left[1 + O\left(\beta^2\right)\right]$$

$$A_{n2}^{CTR} = 8\beta^2 x_p^{-1}\left[1 + O\left(\beta^2\right)\right]$$

If $\beta \approx \beta_{1n} - \delta^2$, then

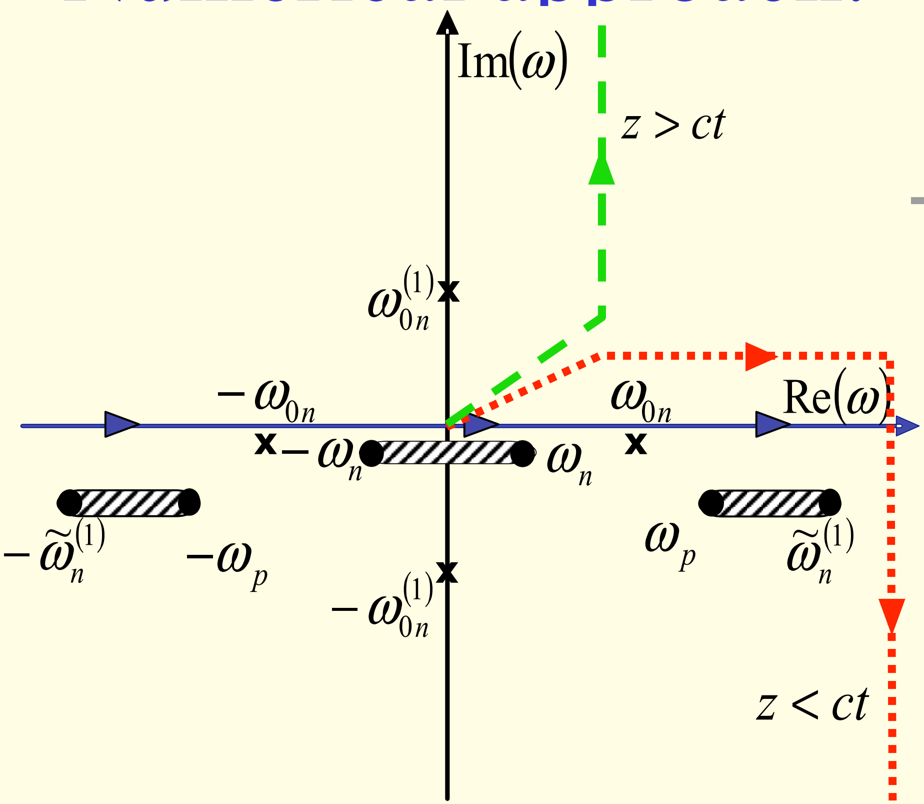
$$A_n^{CR} = \frac{4\left(1 - y_n^2\right)}{y_n^3 x_p}\left[1 + O\left(\delta^2\right)\right]$$

$$A_{n2}^{CTR} = \frac{8\left(1 - y_n^2\right)}{y_n^3 x_p}\left[1 + O(\delta)\right]$$

The CTR is effective for generation of radiation from nonrelativistic bunches.

CTR in vacuum and in the plasma

Numerical approach:



$$I_{1,2}^{b} = \int_{-\infty}^{+\infty} d\omega B_{n1,2} \exp\left[i\left(k_{z1,2}|z| - \omega t\right)\right]$$

Integrands $B_{n1,2}$ decrease in the I and II quadrants of a complex plane (ω) at $|z| < ct$ and in the III and IV quadrants – at $|z| > ct$

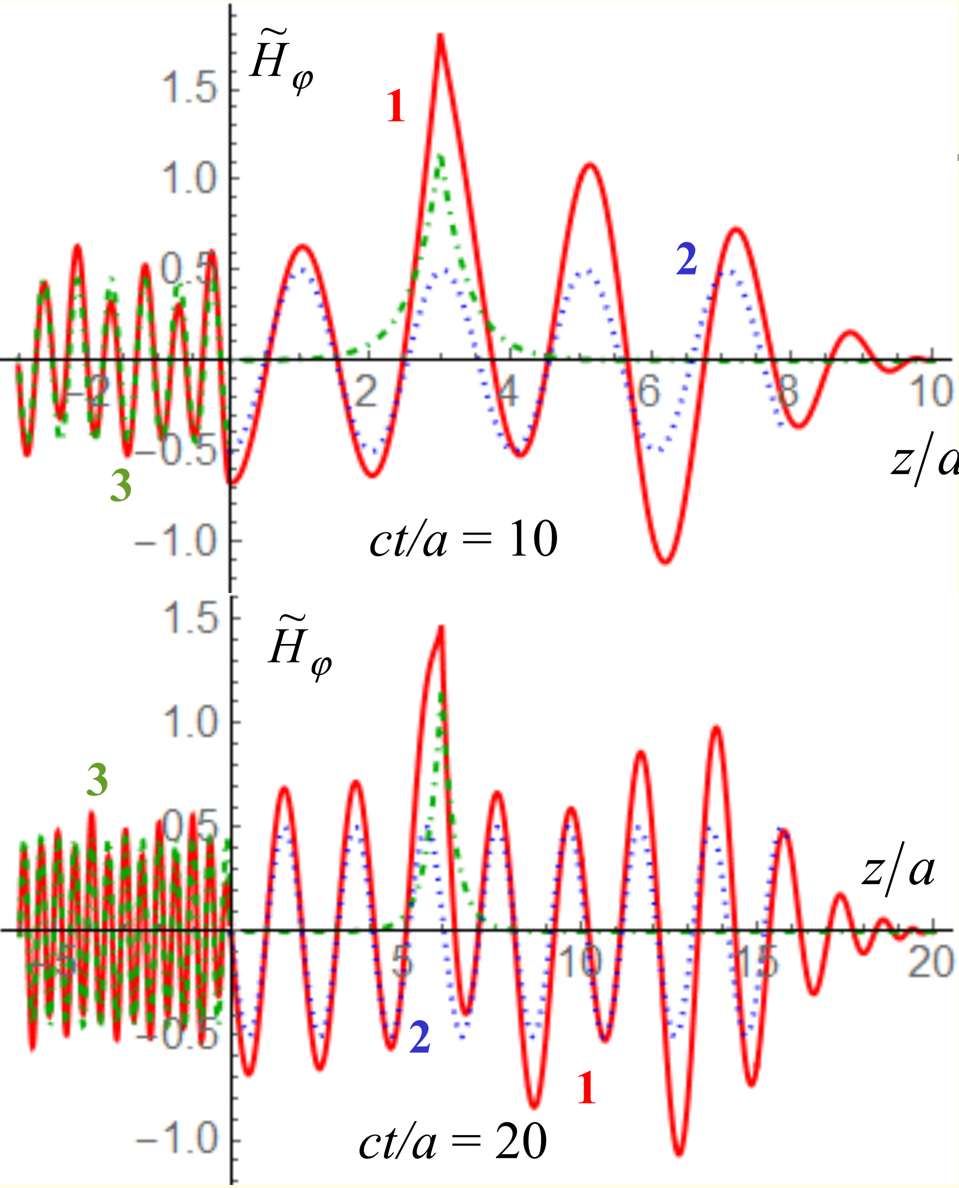
Transformation of the initial contour is in an upper half-plane into a **green** contour before "wave front" and in a lower half-plane into a **red** contour behind "wave front".

Single mode condition: $\chi_1 = 2.405 < x_p < \chi_2 = 5.52, \quad x_p = a\omega_p / c$

at $\omega_p = 2\pi \cdot 9\mathrm{GHz}, \left(n_{pe} = 10^{12}\,cm^{-3}\right)$ $\quad 1.3 < a < 3$ (in centimeters)

CTR in vacuum and in the plasma

Dependence of the first mode of the field $\tilde{H}_\varphi$ on *z/a* at different time moments *ct/a*.



$$\boxed{\beta = 0.3}$$

$$\tilde{H}_\varphi = H_\varphi \frac{a^2}{q}$$

$n = 1$ $\qquad \beta_{01} = 0.856$

$r/a = 0.5,$ $\qquad \beta_{11} = 0.8$

$x_\mathrm{p} = 4 \qquad \sigma = 0$

$\omega_p = 2\pi \cdot 9\mathrm{GHz}$

$a = 2.1\mathrm{cm},$

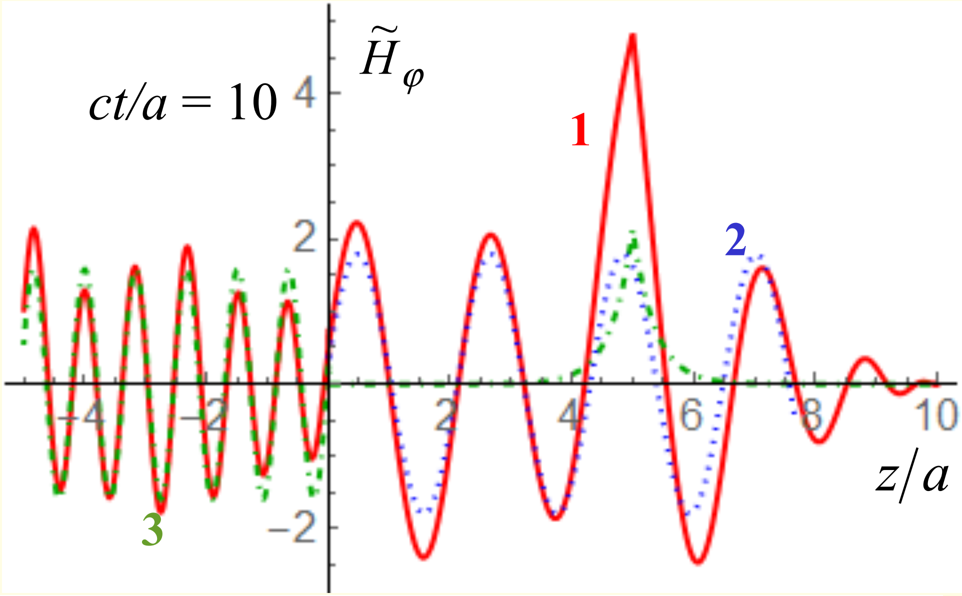
$$y_1 \equiv \frac{\omega_1}{\omega_p} \equiv \frac{\chi_1}{x_p} = 0.6$$

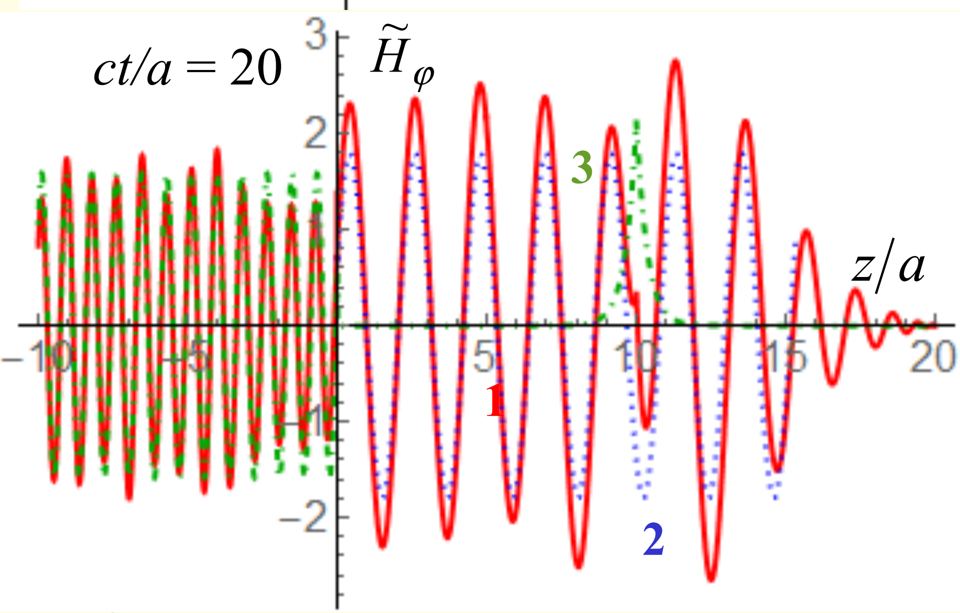
$y_1 < 1 \qquad \omega_{01} = 0.98\omega_\mathrm{p}$

1 – the total field, 2 – the CTR, 3- the forced field

CTR in vacuum and in the plasma

Dependence of the first mode of the field $\tilde{H}_\varphi$ on z/a at different time moments ct/a.





$$\tilde{H}_\varphi = H_\varphi \frac{a^2}{q}$$

$\boxed{\beta = 0.5}$

$n = 1$

$r/a = 0.5$

$x_p = 4 \quad \sigma = 0$

$\omega_p = 2\pi \cdot 9\text{GHz}$

$a = 2.1\text{cm},$

$\omega_{01} = 0.94\omega_p$

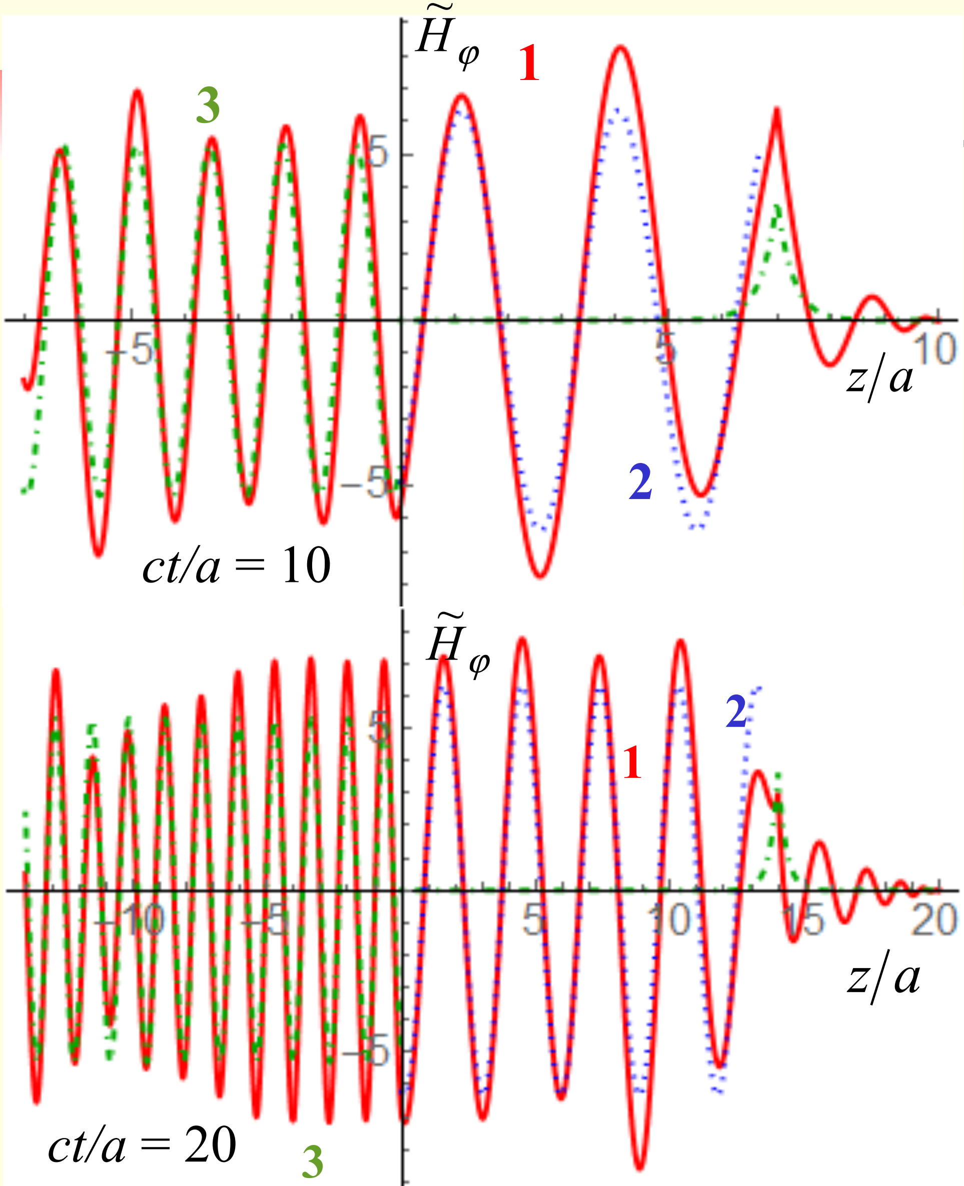
$\beta_{01} = 0.856$

$\beta_{11} = 0.8$

1 – the total field, 2 – the CTR, 3- the forced field

CTR in vacuum and in the plasma

Dependence of the first mode of the field $\tilde{H}_\varphi$ on *z/a* at different time moments *ct/a*.



$$\tilde{H}_\varphi = H_\varphi \frac{a^2}{q}$$

$$\boxed{\beta = 0.7}$$

$n = 1$

$r/a = 0.5$

$\beta_{01} = 0.856$

$\beta_{11} = 0.8$

$x_p = 4 \quad \sigma = 0$

$\omega_p = 2\pi \cdot 9\text{GHz}$

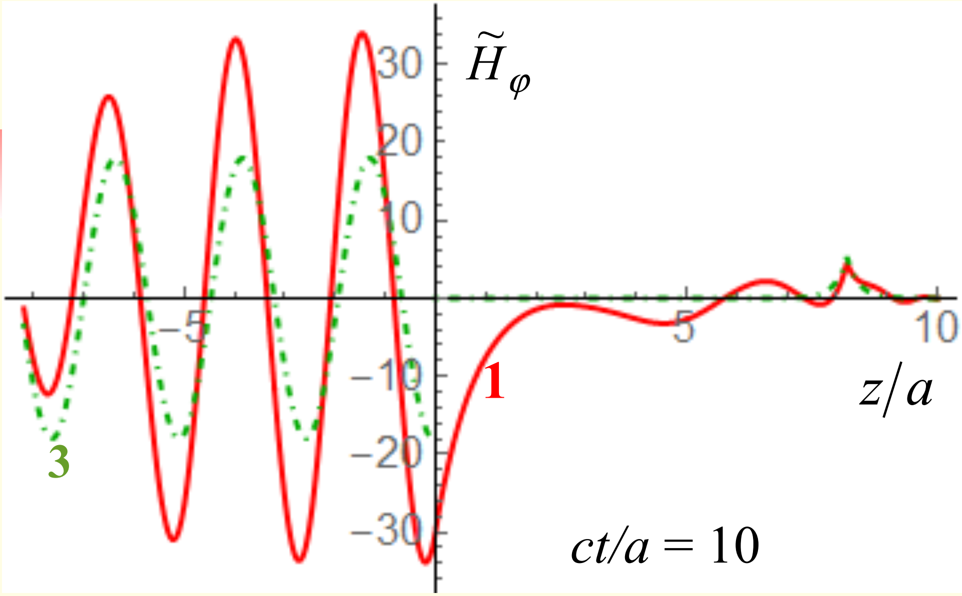
$a = 2.1\text{cm},$

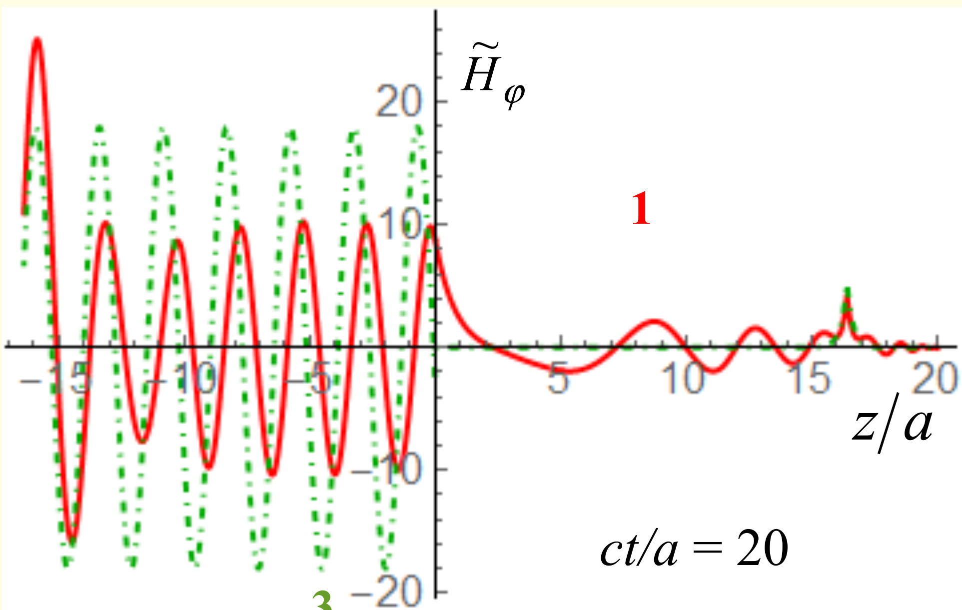
$\omega_{01} = 0.81\omega_p$

1 – the total field, 2 – the CTR,
3- the forced field

CTR in vacuum and in the plasma

Dependence of the first mode of the field $\tilde{H}_\varphi$ on *z/a* at different time moments *ct/a*.





$$\tilde{H}_\varphi = H_\varphi \frac{a^2}{q}$$

$$\boxed{\beta = 0.82}$$

$n = 1$

$r/a = 0.5$

$\sigma = 0$

$x_p = 4$

$\omega_p = 2\pi \cdot 9\text{GHz}$

$a = 2.1\text{cm},$

$\beta_{01} = 0.856$

$\beta_{11} = 0.8$

$$\boxed{\beta_{1n} < \beta < \beta_{0n}}$$

$\omega_{01} = 0.51\omega_p$

1 – the total field, 3- the forced field

Conclusion

- The CTR can be the main part of the wave field in some domain in the vacuum area of the waveguide under certain conditions.
- The CTR mode amplitude in vacuum may exceed the CR mode amplitude in the plasma.
- The CTR mode frequency is always less than the plasma frequency.
- The CTR is effective for generation of radiation from nonrelativistic bunches.
- The CTR effect can be used for the generation of single-mode monochromatic radiation.

Thanks for your attention!