X-RAY EMISSION BY A HIGH-ENERGY “HALF-BARE” ELECTRON IN ULTRA-THIN CRYSTALS

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PXR AND DTR IN THICK CRYSTAL ($\gamma \gg \omega_p/\omega$)

**PXR frequency:**

$$\omega_{\text{PXR}} = \frac{|g \cdot v|}{1 - (v/c)\sqrt{\epsilon} \cos \phi}$$

- $\theta_B$ – Bragg angle
- $\omega_p$ – plasma frequency of the crystal
- $g$ – reciprocal lattice vector
- $\gamma$ – electron Lorenz-factor

**DTR frequency:**

$$\omega_{\text{Bragg}} = c g / (2\sqrt{\epsilon} \sin \phi)$$

- Reflex center: $\phi \approx 2\theta_B$

**Graphical Representation:**

- Reflex angular distribution
- PXR
- DTR
COHERENT X-RAY EMISSION FROM ULTRATHIN CRYSTAL 
\((\gamma \gg \omega / \omega_p)\)

**spectral distribution:**

\[ \Delta \omega \sim 1/L \]

\[ \Delta \omega \gg \omega_{\text{Bragg}} - \omega_{\text{PXR}} \]

\[ l_c \sim \frac{2}{\omega \left( \gamma^{-2} + \omega_p^2 / \omega^2 + b_\perp^2 / \omega^2 \right)} \]

for \( \omega \sim 1 \text{ keV} \) \( \rightarrow l_c \sim 1 \mu\text{m} \)

\[ b = k - g \]

\( k \) – radiation wave vector

ELECTRON “UNDRESSING” BY A THICK TARGET

Field around the electron for $z \to 0$: 

$$E_\omega (\rho) = 2e \sqrt{\frac{\omega^2}{\gamma^2} + \omega_p^2} K_1 \left( \rho \sqrt{\frac{\omega^2}{\gamma^2} + \omega_p^2} \right) e^{i \frac{\omega}{\gamma} z}$$

frequencies $\omega < \gamma \omega_p$ are suppressed comparing to the Coulomb field.

electron is “half-bare”

$K_1(x)$ – Macdonald function
COHERENT X-RAY RADIATION BY “HALF-BARE” ELECTRON

\[ \omega_p \] – crystal plasma frequency

\[ \eta_p \] – amorphous target plasma frequency
TREATMENT IN THE FRAMEWORK OF KINEMATIC THEORY

Dielectric permittivity of crystal:

\[ \varepsilon(\omega, \mathbf{r}) = \varepsilon_0(\omega) + \varepsilon'(\omega, \mathbf{r}) \]

\[ \left\{ \begin{aligned} 
\varepsilon_0(\omega) &= 1 - \omega_p^2 / \omega^2 \\
\varepsilon'(\omega, \mathbf{r}) &= \sum_{\mathbf{g}} n_\mathbf{g} e^{i\mathbf{g}\cdot\mathbf{r}} 
\end{aligned} \right. \]

Where:

\[ \text{k} \] – radiation wave-vector
\[ \text{g} \] – reciprocal lattice vector
\[ \omega_p \] – plasma frequency of the crystal

PXR field (NLO-solution):

\[ \mathbf{E}_{\omega}^{(1)}(\mathbf{r}) = -\frac{e^{ikr}}{4\pi r} \sum_{\mathbf{g}} n_\mathbf{g} \int d^3 r' \mathbf{k} \times \left[ \mathbf{k} \times \mathbf{E}_{\omega}^{(0)}(\mathbf{r'}) \right] e^{i(g-k)r'} \]

electron's field in the medium with \( \varepsilon = \varepsilon_0 \) (LO-solution)
Field inside ultra-thin crystal:

$$E_\omega^{(0)}(r) = -\frac{ie}{\pi} \int d^2 q q e^{iq\rho} \left\{ Q(q) e^{i\omega z/v} + \right.$$ 

$$+ \left[ R(q) e^{i\omega l/v} - R'(q) e^{il\sqrt{\omega^2 - q^2}} \right] e^{i(z-l)\sqrt{\omega^2 \varepsilon_0 - q^2}} \right\}$$

Where:

$$\begin{cases} 
    b = k - g \\
    Q(q) = (q^2 + \omega_p^2 + \omega^2 / \gamma^2)^{-1} \\
    R(q) = (q^2 + \omega^2 / \gamma^2)^{-1} - Q(q) \\
    R' = R \text{ with substitution } \omega_p \rightarrow \eta_p
\end{cases}$$

Field of radiation by “half-bare” electron:

$$E_\omega^{(1)}(r) = ie \frac{e^{ikr}}{r} \sum g n_g k \times [k \times b_\perp] \left\{ Q(b_\perp) S_c e^{i(l+L/2)q_c} + 
$$

$$+ S_f e^{iLq_f/2 - ilb_z} \left[ R(b_\perp) e^{i\omega l/v} - R'(b_\perp) e^{il\sqrt{\omega^2 - b_{\perp}^2}} \right] \right\}$$

$$S_{c,f} = \frac{\sin(Lq_{c,f} / 2)}{q_{c,f} / 2}$$

$$q_c = \omega / v - b_z$$

$$q_f = \sqrt{\omega^2 \varepsilon_0 - b_{\perp}^2} - b_z$$
EVOLUTION OF RADIATION ANGULAR DISTRIBUTION

\[
\frac{dW}{do} = \frac{e^2 L |n_g|^2 |\mathbf{n} \times \mathbf{b}_\perp|^2}{2\pi(1 - \sqrt{\varepsilon_0 n_v})} \left[ F^2 + G^2 + 2FG \cos(l/l_v) \right]
\]

Where:

\[
\begin{align*}
\mathbf{b} &= \mathbf{k} - \mathbf{g} = n \omega \sqrt{\varepsilon_0} - \mathbf{g} \\
F(\mathcal{G}) &= (\mathcal{G}^2 + \gamma^{-2})^{-1} \\
G(\mathcal{G}) &= (\mathcal{G}^2 + \omega_p^2 / \omega^2 + \gamma^{-2})^{-1} - F(\mathcal{G})
\end{align*}
\]

\(\gamma\) – electron Lorenz-factor
\(\mathcal{G}\) – angle counted from the reflex center

distribution for e\(^{-}\) with Coulomb field

averaging over oscillations for \(l \gg l_F\)

\(l = 0.1l_F\)

\(l = 0.2l_F\)

\(l = 2.5l_F\)

\(dW/do\)

\(\vartheta\)
EVOLUTION OF ANGULAR DISTRIBUTION MAXIMUM (E=10 GeV)

For $\omega / \omega_p^2 \ll l \ll l_F$:

$$
\frac{dW}{do} = \frac{2e^2 L |n_g|^2 |n \times b_{\perp}|^2}{\pi(1 - \sqrt{\varepsilon_0 n v})} \frac{\sin^2(\omega \gamma^2 l / 4)}{\mathcal{G}^4}
$$

With the increase of $l$ from $l=0$ to $l \sim \gamma^2 / \omega$ the angle, corresponding to the reflex maximum, changes from $\omega_p / \omega$ (as for PXR in thick crystals) to $\gamma^{-1}$ (as in ultrathin crystals)
At moderately high energies both signal increase and decrease are possible with the increase of $l$:

$$W_{\text{int}} = \int_{0}^{\theta_0} d\theta \, 2\pi \theta dW / d\theta$$

At sufficiently high energies, when $\theta_0 \gg 1/\gamma$, the signal logarithmically grows with the increase of $l$:

$$W_{\text{int}} \approx 0.5 e^2 \omega^2 L |n_g|^2 \times \left[ 0.577 + \ln(\omega \theta_0^2 l / 2) \right]$$

Asymptote for $l \gg l_F$ exceeds the signal value for electron with Coulomb field.
INFLUENCE OF THE FINITE TRANSVERSAL SIZE OF THE CRYSTAL (return to the Coulomb value)

Finite size is manifested for \( l \gg l_F \sim \gamma^2/\omega \) and \( R < l/\gamma \) (which is \( \gg \gamma/\omega \))

Angular distribution for \( R \neq \infty \):

\[
\frac{dW}{do} = \frac{e^2 L |n_g|^2 |n \times b_\perp|^2}{2\pi(1-\sqrt{\varepsilon_0 n v})} \left\{ F^2 + G^2 |S(R)|^2 \right\}
\]

\[
S(R) = R \int_0^\infty du J_1(uR) J_0(u l_0) e^{-iu^2 l/(2\omega)} \approx \begin{cases} 
1 & \text{for } l < R/\gamma \\
0 & \text{for } l > R/\gamma 
\end{cases}
\]

Intensity decrease for \( l > R/\theta_0 \)

\( 2R=0.5 \text{ cm} \)
\( E=200 \text{ MeV} \)
\( 2\theta_0 = 2/\gamma = 5 \text{ mrad} \)
DTR BY “HALF-BARE” ELECTRON IN THICK CRYSTAL

Change of DTR angular distribution and intensity with the increase of $l$ from $l=0$ to $l \sim \gamma^2 / \omega$
CONCLUSIONS

- Characteristics of coherent X-ray radiation by “half-bare” electron in ultrathin crystal dramatically differ from the characteristics typical both for thick and ultrathin crystals.

- Smooth change of reflex angular width from $\sim \omega_p/\omega$ to $\sim \gamma^{-1}$ with the increase of the distance $l$ between the amorphous target and the crystal.

- Possibility of both radiation yield increase and decrease with the increase of $l$.

- At large $l$ finite transversal size of the crystalline target causes return of the radiation yield to the value typical for electron with Coulomb field.

- Possibility of the discussed effects for DTR in thick crystals.