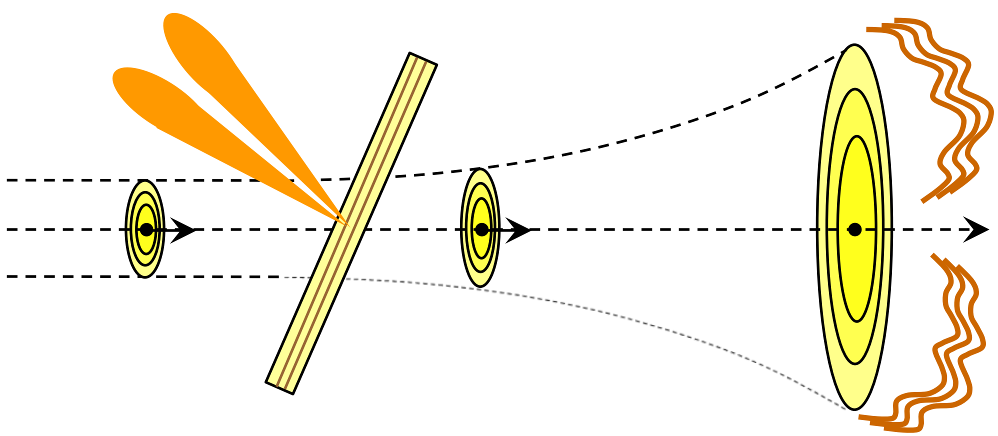
X-RAY EMISSION BY A HIGH-ENERGY "HALF-BARE" ELECTRON IN ULTRA-THIN CRYSTALS

S.V. Trofymenko, N.F. Shul'ga

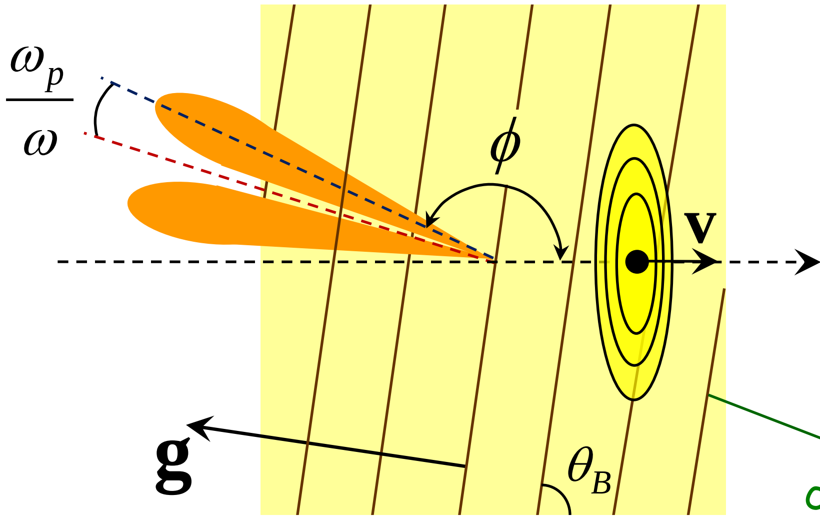
NSC Kharkiv Institute of Physics and Technology,
Karazin Kharkiv National University
Kharkiv, Ukraine



Phys. Rev. A ***98*** *(2018) 023813*

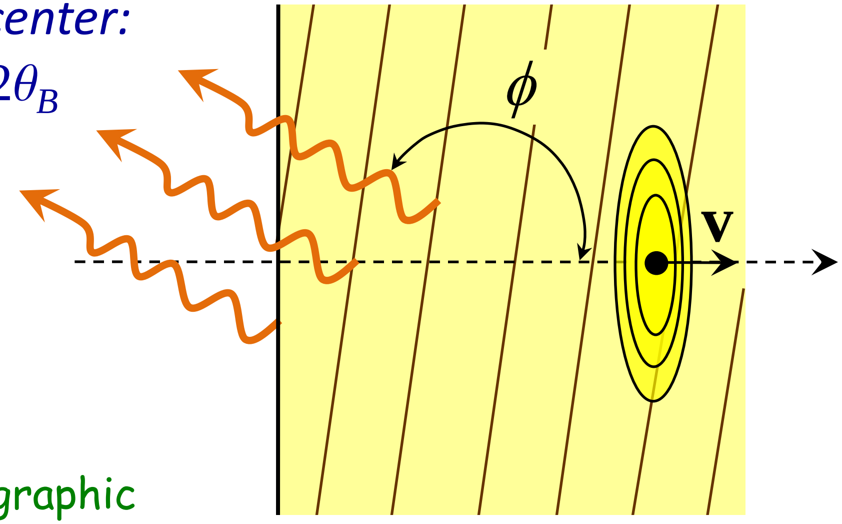
8th International Conference Channeling 2018 – Charged & Neutral Particles Channeling Phenomena, Ischia, Italy, September 23-28, 2018

PXR AND DTR IN THICK CRYSTAL ($\gamma \gg \omega_p/\omega$)



reflex center:

$\phi \approx 2\theta_B$



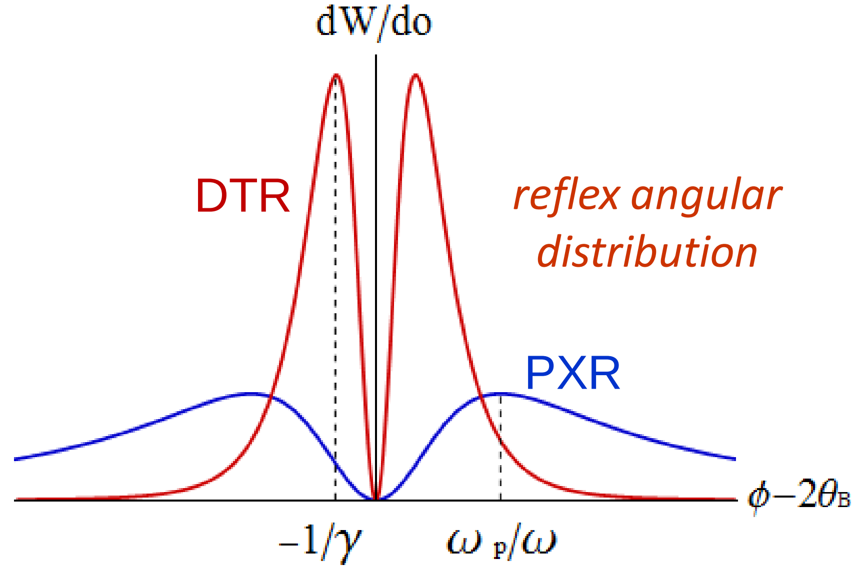
crystallographic planes

PXR frequency:

$$\omega_{PXR} = \frac{|\mathbf{g} \cdot \mathbf{v}|}{1 - (\mathrm{v}/c)\sqrt{\varepsilon}\cos\phi}$$

DTR frequency:

$$\omega_{Bragg} = c\,\mathrm{g}/(2\sqrt{\varepsilon}\sin\phi)$$



θ_B – Bragg angle

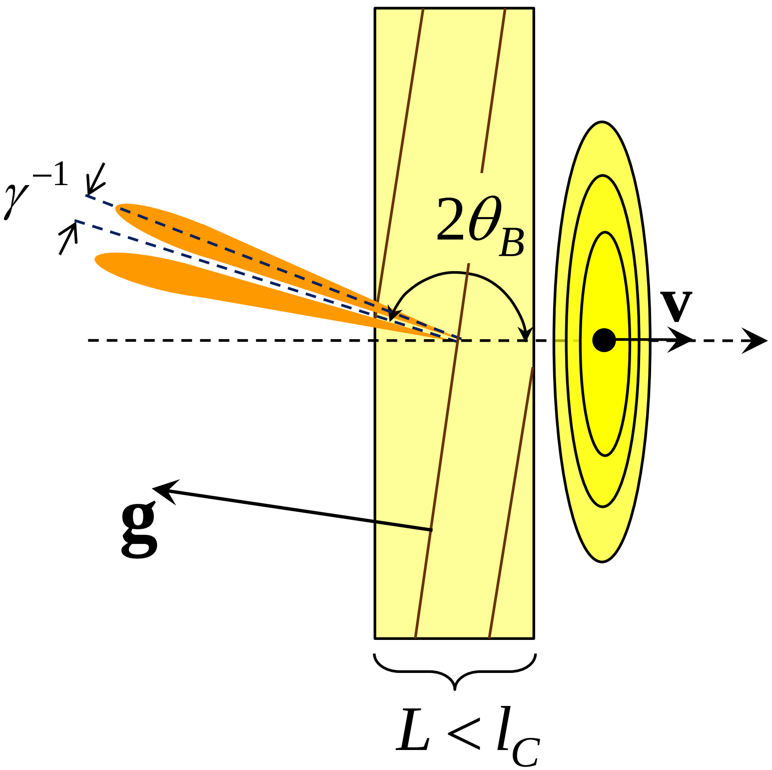
ω_p – plasma frequency of the crystal

$\mathbf{g}$ – reciprocal lattice vector

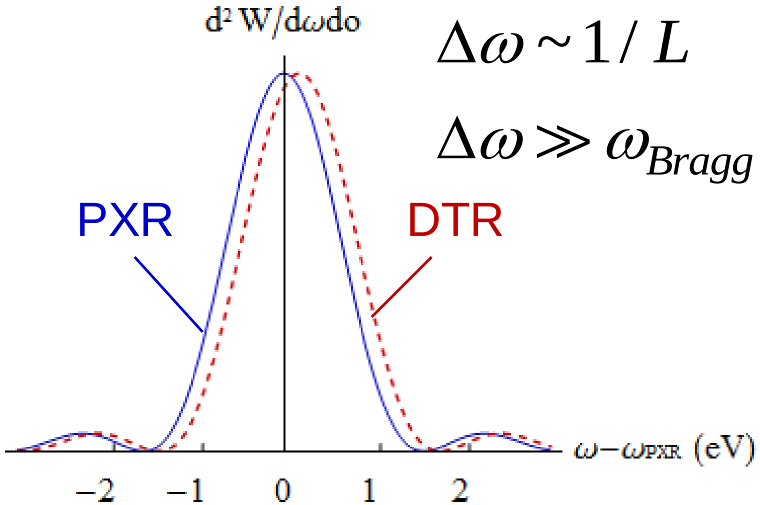
γ – electron Lorenz-factor

COHERENT X-RAY EMISSION FROM ULTRATHIN CRYSTAL

$(\gamma \gg \omega/\omega_p)$



spectral distribution:



$$\Delta\omega \sim 1/L$$

$$\Delta\omega \gg \omega_{Bragg} - \omega_{PXR}$$

$$l_C \sim \frac{2}{\omega\left(\gamma^{-2} + \omega_p^2/\omega^2 + b_\perp^2/\omega^2\right)}$$

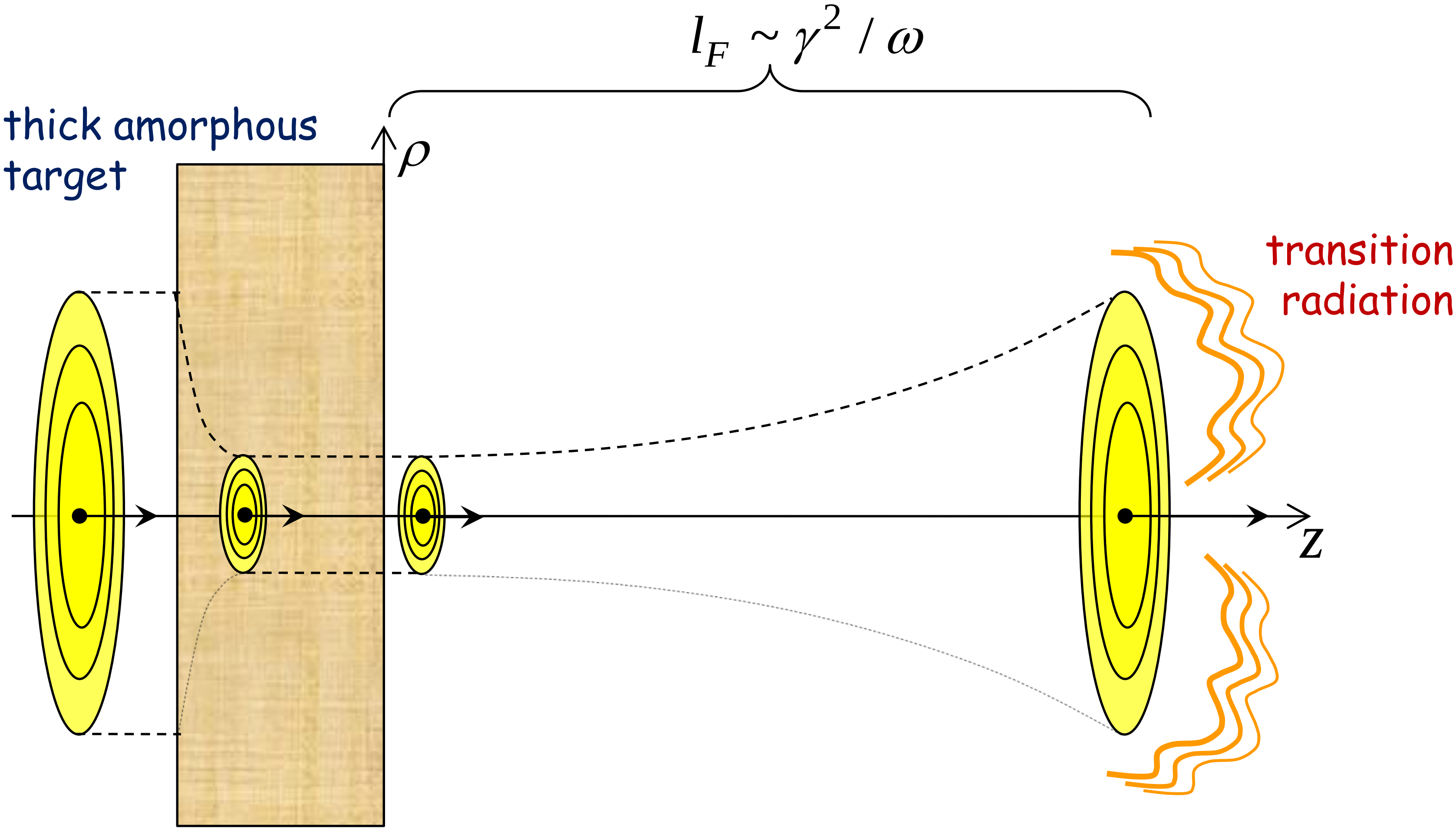
for $\omega \sim 1$ keV $\longrightarrow$ $l_C \sim 1\ \mu$m

$$\mathbf{b} = \mathbf{k} - \mathbf{g}$$

$\mathbf{k}$ – radiation wave vector

N. N. Nasonov // Phys. Lett. A ,1998

ELECTRON "UNDRESSING" BY A THICK TARGET



Field around the electron for $z \longrightarrow 0$:

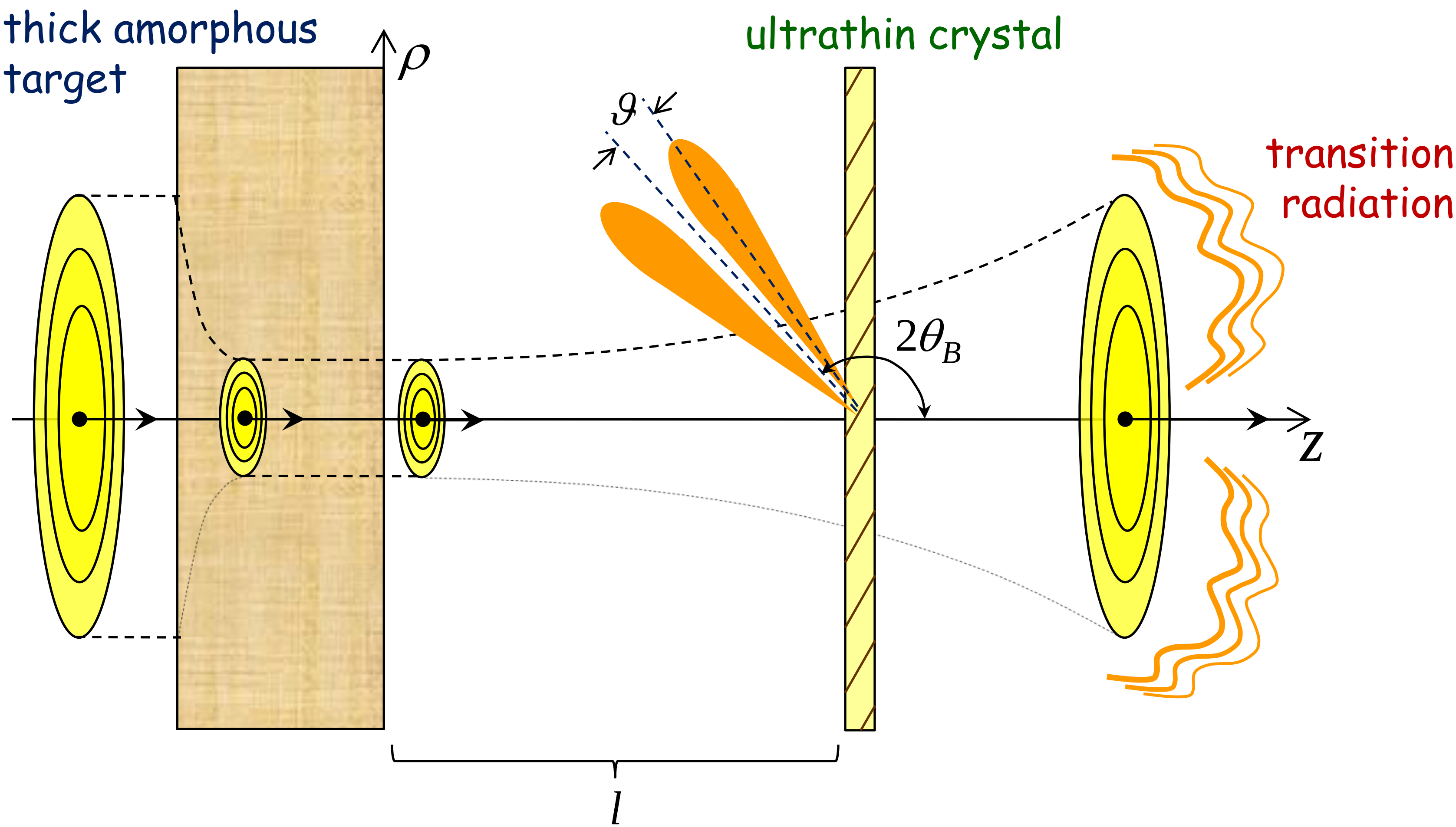
$$E_\omega(\rho) = 2e\sqrt{\frac{\omega^2}{\gamma^2} + \omega_p^2}\, K_1\left(\rho\sqrt{\frac{\omega^2}{\gamma^2} + \omega_p^2}\right) e^{i\frac{\omega}{\mathrm{v}}z}$$

frequencies $\omega < \gamma\omega_p$ are suppressed comparing to the Coulomb field

electron is "half-bare"

$K_1(x)$ – Macdonald function

COHERENT X-RAY RADIATION BY "HALF-BARE" ELECTRON



ω_p – crystal plasma frequency

η_p – amorphous target plasma frequency

TREATMENT IN THE FRAMEWORK OF KINEMATIC THEORY

M.L. Ter-Mikaelyan, High-Energy Electromagnetic Processes in Media,1969

Dielectric permittivity of crystal:

$$\varepsilon(\omega,\mathbf{r}) = \varepsilon_0(\omega) + \varepsilon'(\omega,\mathbf{r})$$

Where:
$$\begin{cases} \varepsilon_0(\omega) = 1 - \omega_p^2 / \omega^2 \\ \varepsilon'(\omega,\mathbf{r}) = \sum_{\mathbf{g}} n_{\mathbf{g}} e^{i\mathbf{g}\mathbf{r}} \end{cases}$$

$\mathbf{k}$ – radiation wave-vector

$\mathbf{g}$ – reciprocal lattice vector

ω_p – plasma frequency of the crystal

PXR field (NLO-solution):

$$\mathbf{E}_{\omega}^{(1)}(\mathbf{r}) = -\frac{e^{ikr}}{4\pi r}\sum_{\mathbf{g}} n_{\mathbf{g}} \int d^3 r' \mathbf{k} \times \left[\mathbf{k} \times \mathbf{E}_{\omega}^{(0)}(\mathbf{r}')\right] e^{i(\mathbf{g}-\mathbf{k})\mathbf{r}'}$$

electron's field in the medium with $\varepsilon = \varepsilon_0$ (LO-solution)

Field inside ultra-thin crystal:

$$\mathbf{E}_{\omega}^{(0)}(\mathbf{r}) = -\frac{ie}{\pi}\int d^2\mathrm{q}\,\mathbf{q}e^{i\mathbf{q}\boldsymbol{\rho}}\left\{Q(q)e^{i\omega z/\mathrm{v}} + \right.$$

$$\left. + \left[R(q)e^{i\omega l/\mathrm{v}} - R'(q)e^{il\sqrt{\omega^2 - q^2}}\right]e^{i(z-l)\sqrt{\omega^2\varepsilon_0 - q^2}}\right\}$$

Where:

$$\begin{cases} \mathbf{b} = \mathbf{k} - \mathbf{g} \\ Q(q) = (\mathrm{q}^2 + \omega_p^2 + \omega^2/\gamma^2)^{-1} \\ R(q) = (\mathrm{q}^2 + \omega^2/\gamma^2)^{-1} - Q(q) \\ R' = R \text{ with substitution } \omega_\mathrm{p} \longrightarrow \eta_\mathrm{p} \end{cases}$$

$$S_{c,f} = \frac{\sin(Lq_{c,f}/2)}{q_{c,f}/2}$$

$$q_c = \omega/\mathrm{v} - \mathrm{b}_z$$

$$q_f = \sqrt{\omega^2\varepsilon_0 - \mathrm{b}_\perp^2} - \mathrm{b}_z$$

Field of radiation by “half-bare” electron:

$$\mathbf{E}_{\omega}^{(1)}(\mathbf{r}) = ie\frac{e^{ikr}}{r}\sum_{\mathbf{g}} n_{\mathbf{g}}\mathbf{k}\times[\mathbf{k}\times\mathbf{b}_\perp]\left\{Q(\mathrm{b}_\perp)S_c e^{i(l+L/2)q_c} + \right.$$

$$\left. + S_f e^{iLq_f/2 - il\mathrm{b}_z}\left[R(\mathrm{b}_\perp)e^{i\omega l/\mathrm{v}} - R'(\mathrm{b}_\perp)e^{il\sqrt{\omega^2 - \mathrm{b}_\perp^2}}\right]\right\}$$

EVOLUTION OF RADIATION ANGULAR DISTRIBUTION

$$\frac{dW}{do}=\frac{e^2L\,|n_{\mathbf{g}}|^2|\mathbf{n}\times\mathbf{b}_\perp|^2}{2\pi(1-\sqrt{\varepsilon_0}\,\mathbf{nv})}\left[F^2+G^2+2FG\cos(l/l_{\mathrm{v}})\right]$$

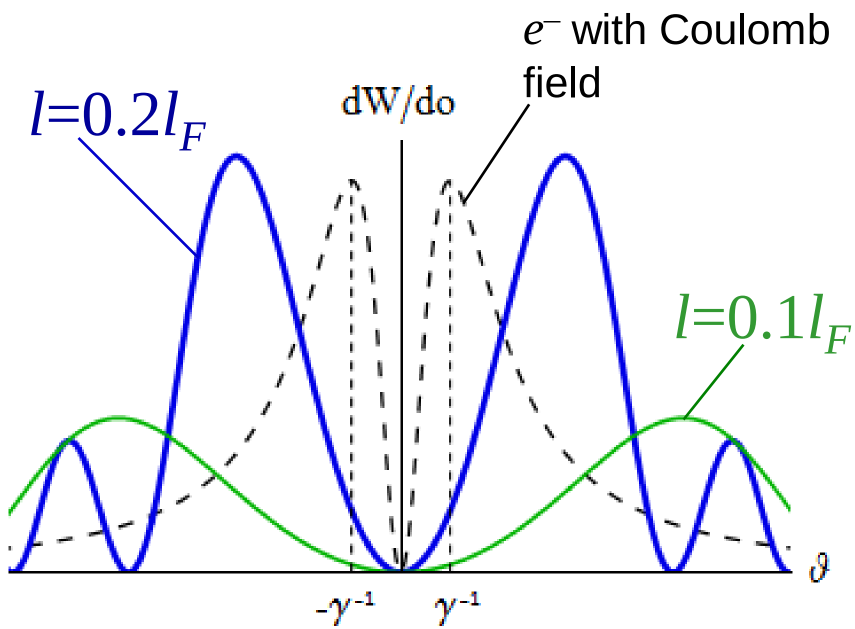
Where:

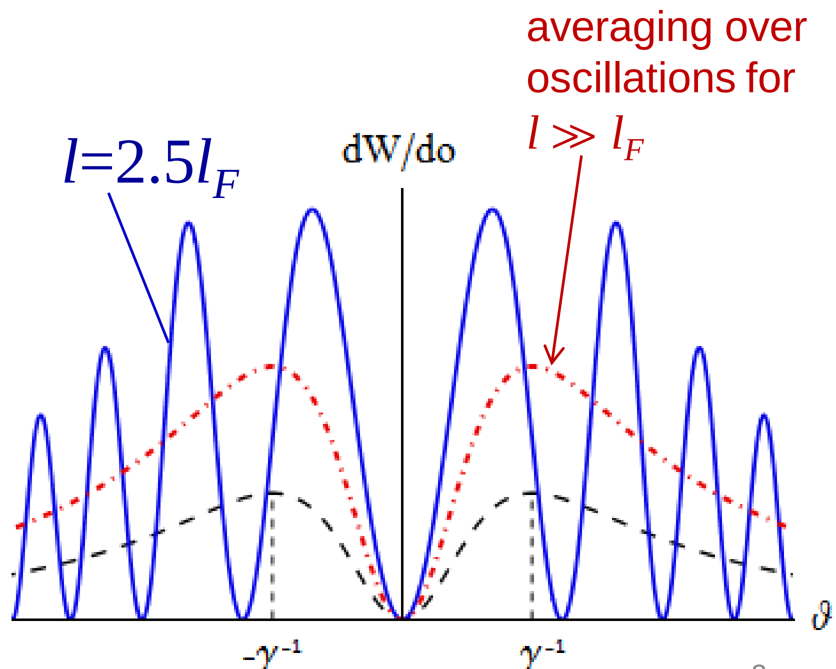
$$\begin{cases}\mathbf{b}=\mathbf{k}-\mathbf{g}=\mathbf{n}\omega\sqrt{\varepsilon_0}-\mathbf{g}\\ F(\vartheta)=(\vartheta^2+\gamma^{-2})^{-1}\\ G(\vartheta)=(\vartheta^2+\omega_p^2/\omega^2+\gamma^{-2})^{-1}-F(\vartheta)\end{cases}$$

γ – electron Lorenz-factor

ϑ – angle counted from the reflex center

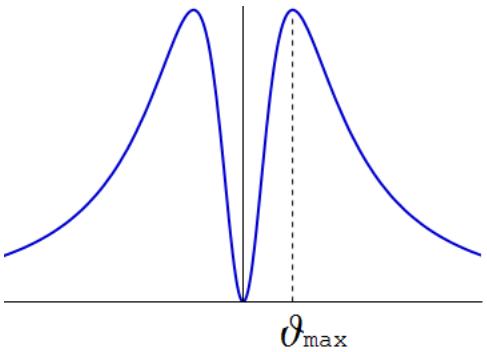
- - - - distribution for e^- with Coulomb field





EVOLUTION OF ANGULAR DISTRIBUTION MAXIMUM (E=10 GeV)

For $\omega / \omega_p^2 \ll l \ll l_F$:

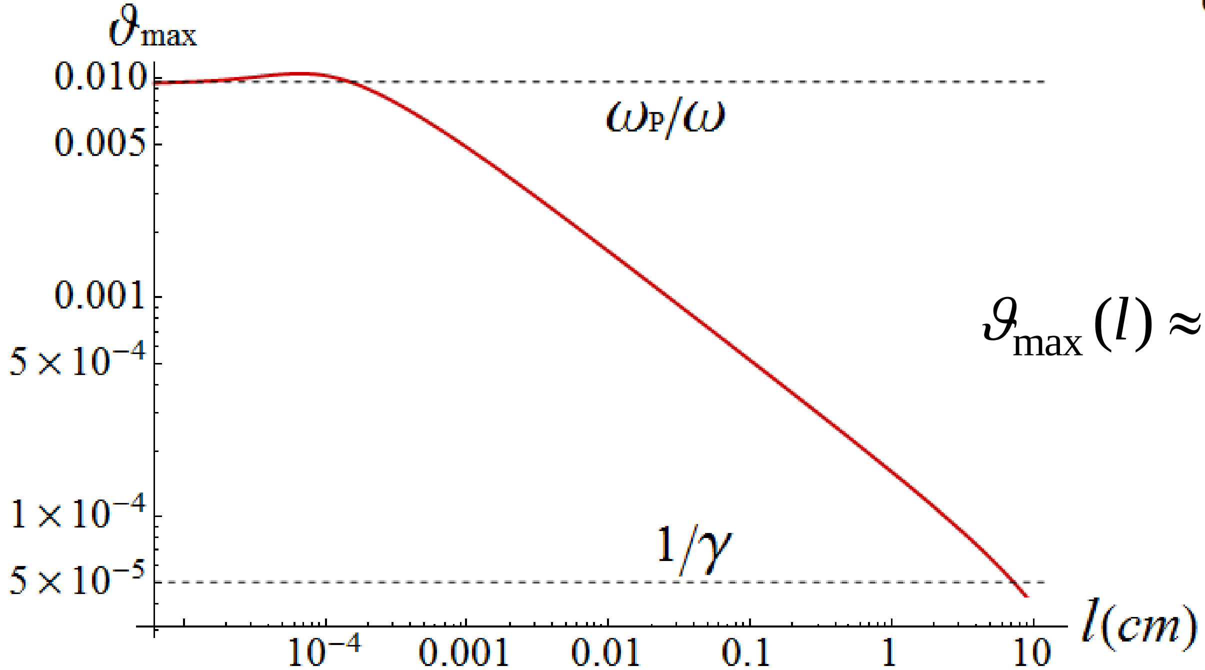


$$\frac{dW}{do} = \frac{2e^2 L |n_{\mathbf{g}}|^2 |\mathbf{n}\times\mathbf{b}_\perp|^2}{\pi(1-\sqrt{\varepsilon_0}\,\mathbf{nv})} \frac{\sin^2(\omega\vartheta^2 l/4)}{\vartheta^4}$$

$\gamma = 2{\cdot}10^4$

$l_F \sim \gamma^2/\omega \sim 5$ cm

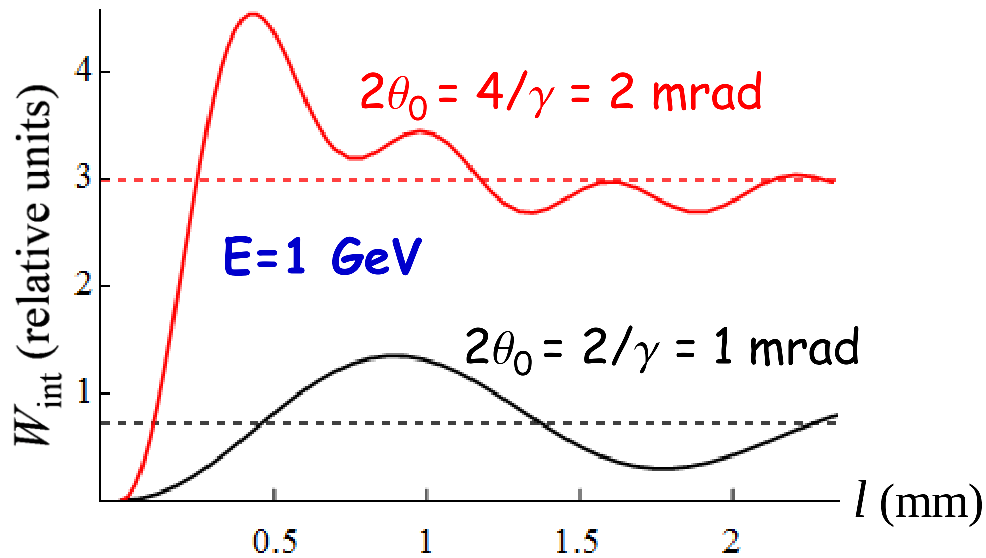
$\omega \approx 3$ keV



$$\vartheta_{\max}(l) \approx \sqrt{\frac{4.66}{\omega l}}$$

With the increase of l from $l=0$ to $l \sim \gamma^2/\omega$ the angle, corresponding to the reflex maximum, changes from ω_p/ω (*as for PXR in thick crystals*) to γ^{-1} (*as in ultrathin crystals*)

EVOLUTION OF RADIATION YIELD

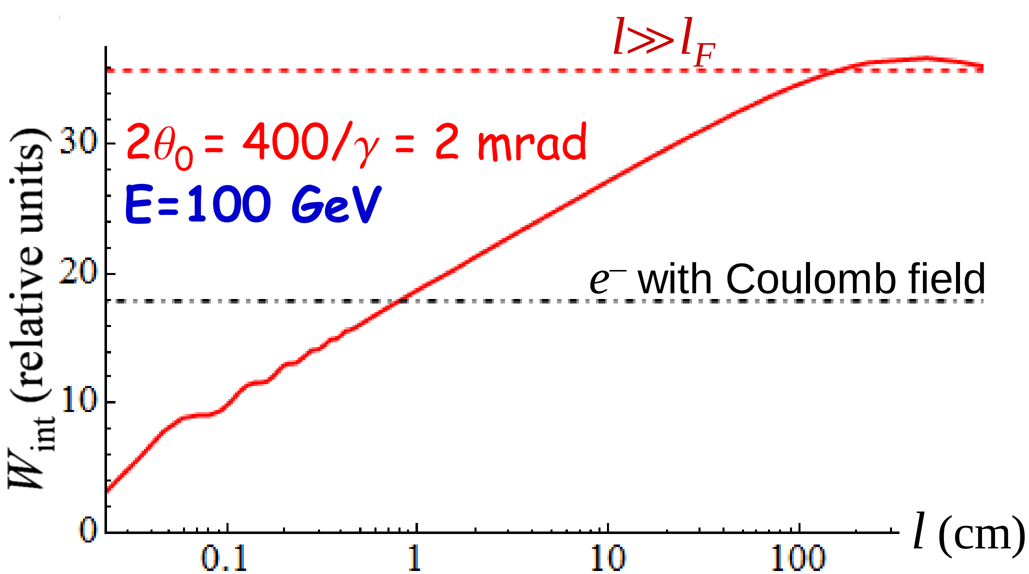


$$W_{int} = \int_0^{\theta_0} d\vartheta\, 2\pi\vartheta dW / do$$

At moderately high energies both signal increase and decrease are possible with the increase of l

At sufficiently high energies, when $\theta_0 \gg 1/\gamma$, the signal logarithmically grows with the increase of l :

$$W_{int} \approx 0.5 e^2 \omega^2 L \,|\, n_{\mathbf{g}} \,|^2 \times$$
$$\times \left[0.577 + \ln(\omega \theta_0^2 l / 2) \right]$$



Asymptote for $l \gg l_F$ **exceeds** the signal value for electron with Coulomb field

INFLUENCE OF THE FINITE TRANSVERSAL SIZE OF THE CRYSTAL (return to the Coulomb value)

Finite size is manifested for $l \gg l_F \sim \gamma^2/\omega$ and $R < l/\gamma$ (which is $\gg \gamma/\omega$)

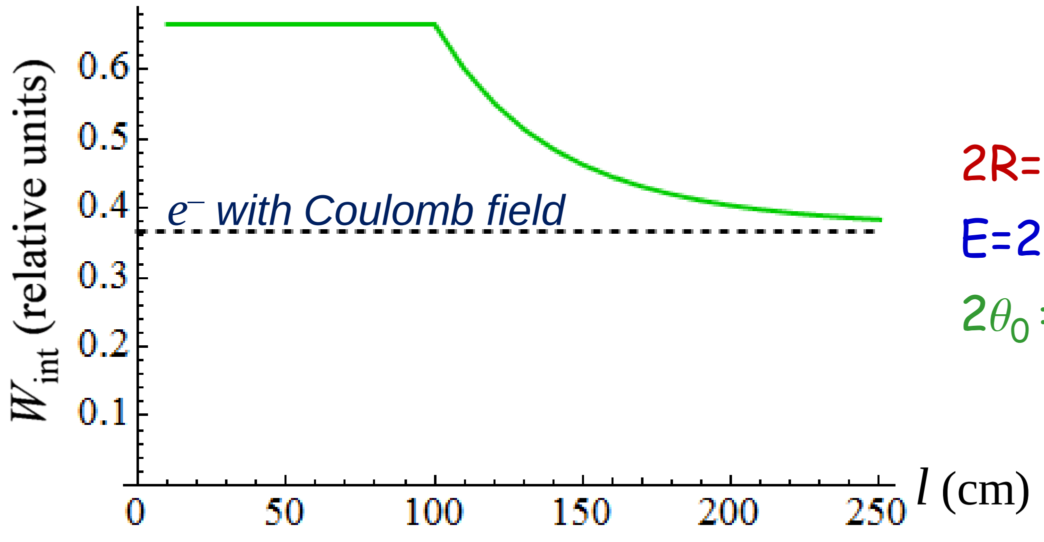
Angular distribution for $R \neq \infty$:

$$\frac{dW}{do} = \frac{e^2 L\,|n_{\mathbf{g}}|^2\,\left|\mathbf{n}\times\mathbf{b}_\perp\right|^2}{2\pi(1-\sqrt{\varepsilon_0}\,\mathbf{nv})}\left\{F^2 + G^2\left|S(R)\right|^2\right\}$$

$J_{0,1}$ – Bessel functions

$$S(R) = R\int_0^\infty du J_1(uR) J_0\left(ul\vartheta\right) e^{-iu^2 l/(2\omega)} \approx \begin{cases} 1 \text{ for } l < R/\vartheta \\ 0 \text{ for } l > R/\vartheta \end{cases}$$

Intensity decrease for $l > R/\theta_0$

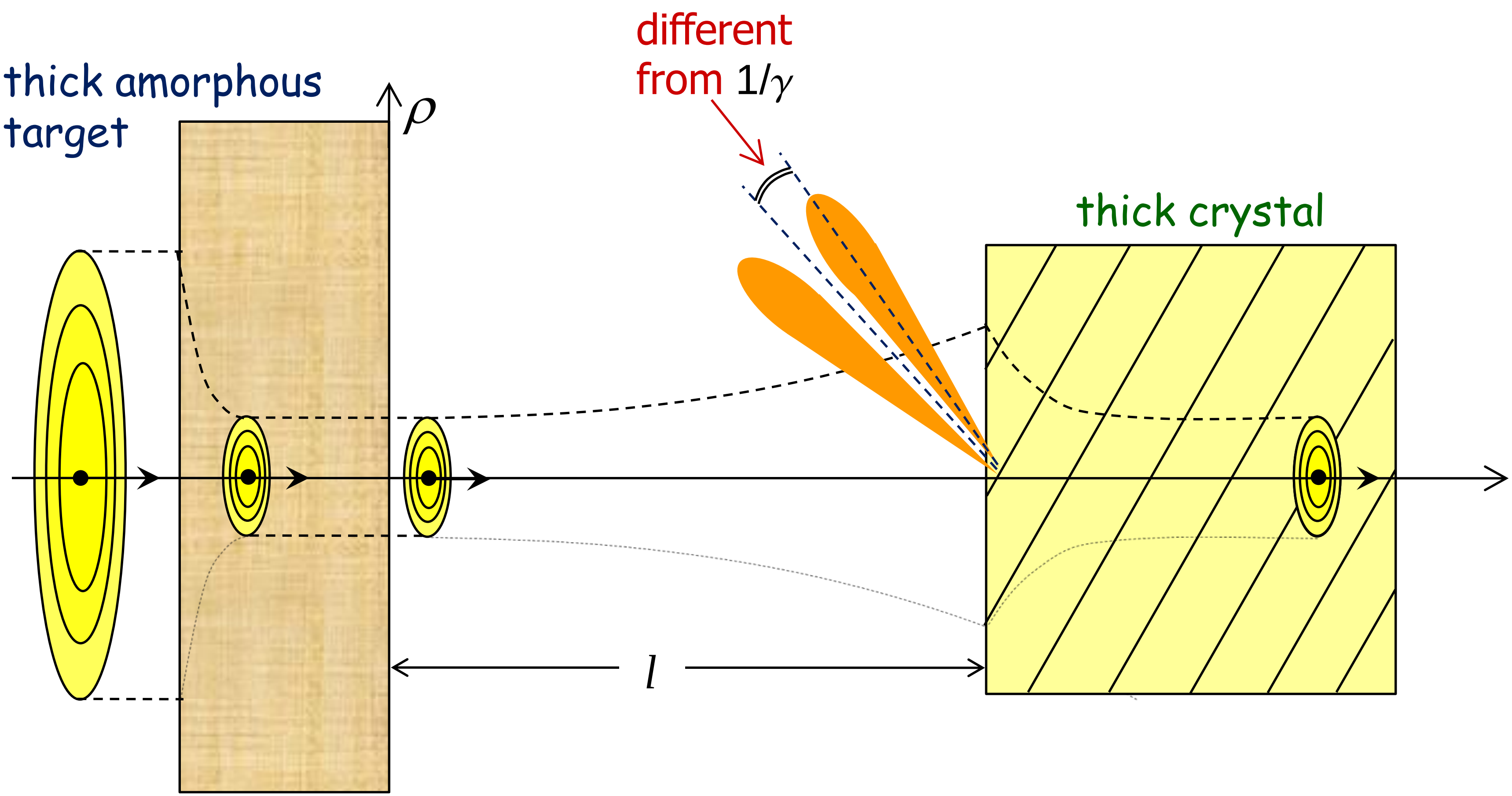


2R=0.5 cm

E=200 MeV

$2\theta_0 = 2/\gamma = 5$ mrad

DTR BY "HALF-BARE" ELECTRON IN THICK CRYSTAL



Change of DTR angular distribution and intensity with the increase of l from $l=0$ to $l \sim \gamma^2/\omega$

CONCLUSIONS

- Characteristics of coherent X-ray radiation by "half-bare" electron in ultrathin crystal dramatically differ from the characteristics typical both for thick and ultrathin crystals

- Smooth change of reflex angular width from $\sim\omega_p/\omega$ to $\sim\gamma^{-1}$ with the increase of the distance l between the amorphous target and the crystal

- Possibility of both radiation yield increase and decrease with the increase of l

- At large l finite transversal size of the crystalline target causes return of the radiation yield to the value typical for electron with Coulomb field

- Possibility of the discussed effects for DTR in thick crystals