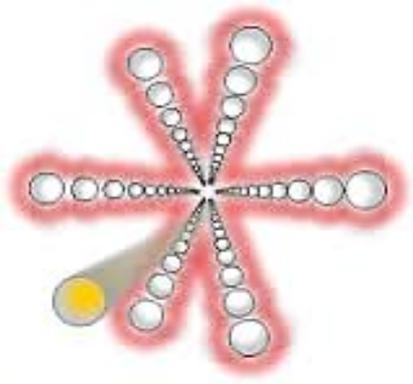


RADIATION OF A CHARGE MOVING IN A WIRE STRUCTURE



Sergey N. Galyamin,
Andrey V. Tyukhtin,
Victor V. Vorobev

Andrey Benediktovitch,
Stasis Chuchurka



Saint Petersburg
State University



Motivation

PHYSICAL REVIEW LETTERS **120**, 164801 (2018)

Experimental Characterization of Electron-Beam-Driven Wakefield Modes in a Dielectric-Woodpile Cartesian Symmetric Structure

P. D. Hoang,^{1,*} G. Andonian,¹ I. Gadjev,¹ B. Naranjo,¹ Y. Sakai,¹ N. Sudar,¹ O. Williams,¹ M. Fedurin,²

K. Kusche,² C. Swinson,² P. Zhang,³ and J. B. Rosenzweig¹

¹Department of Physics and Astronomy, University of California, Los Angeles, California 90095-1547, USA

²Accelerator Test Facility, Brookhaven National Laboratory, Upton, New York 11973, USA

³School of Physical Electronics, University of Electronic Science and Technology of China, Chengdu, 610054, China

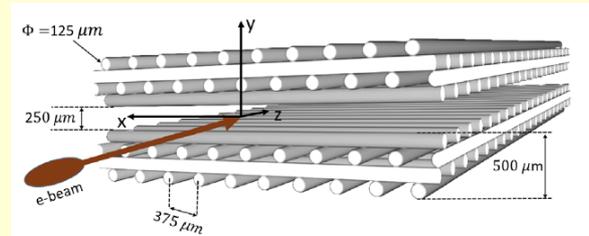


FIG. 1. Computer rendered model of the woodpile structure with relevant dimensions.



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Cherenkov and parametric (quasi-Cherenkov) radiation produced by a relativistic charged particle moving through a crystal built from metallic wires



V.G. Baryshevsky, E.A. Gurnevich*

Research Institute for Nuclear Problems, Belarusian State University, Bobruiskaya 11, 220030 Minsk, Belarus

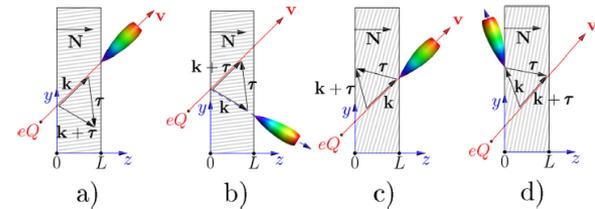


Fig. 2. Parametric radiation for the case of two-wave diffraction in Laue (a, b) and Bragg (c, d) geometries. Wire axes are perpendicular to the figure's plane.

PRL **108**, 184801 (2012)

PHYSICAL REVIEW LETTERS

week ending
4 MAY 2012

Nondivergent Cherenkov Radiation in a Wire Metamaterial

Viktor V. Vorobev and Andrey V. Tyukhtin

Physical Faculty of St. Petersburg State University, St. Petersburg 198504, Russia

(Received 11 January 2012; published 1 May 2012)

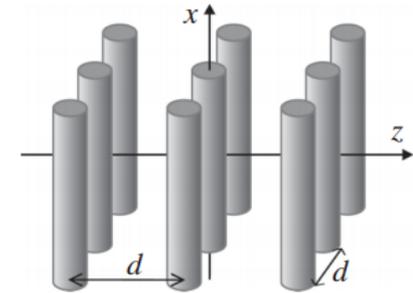
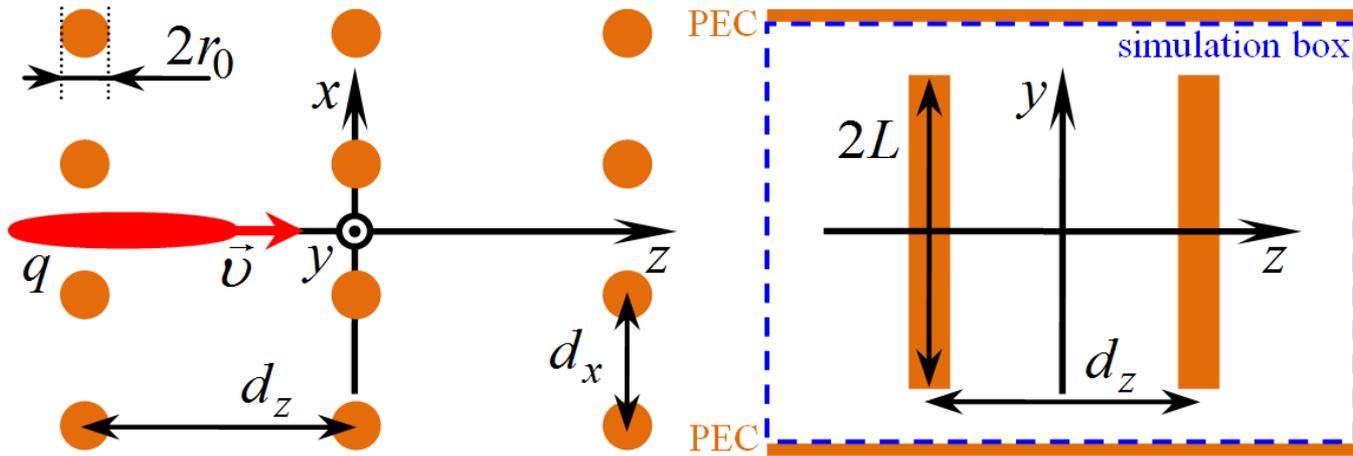


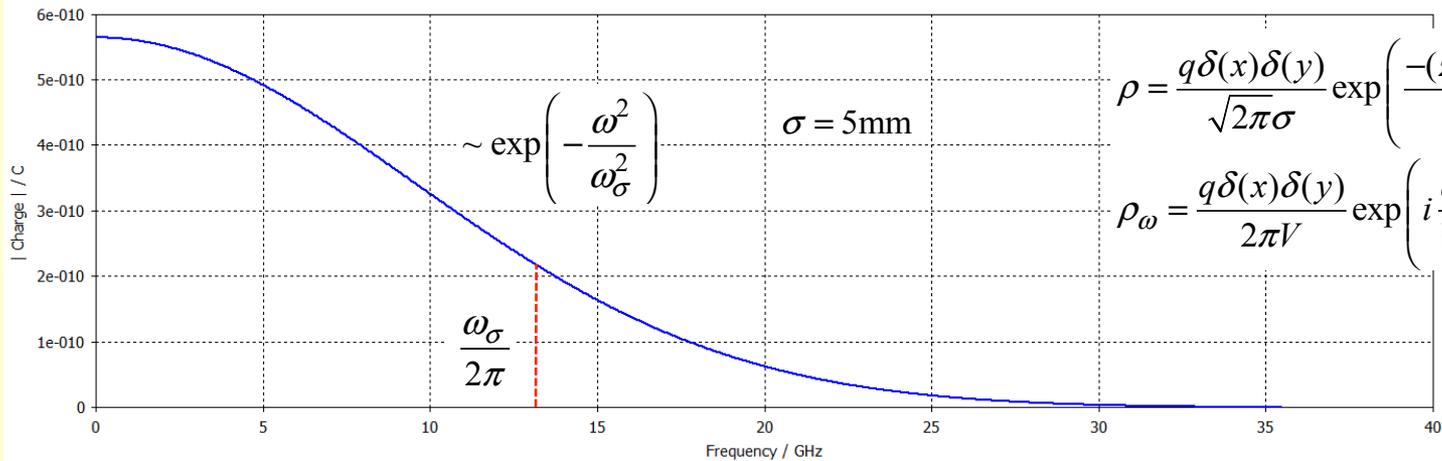
FIG. 1. Scheme of the structure.

CST Particle Studio simulations

CST model



Charge distribution amplitude spectrum



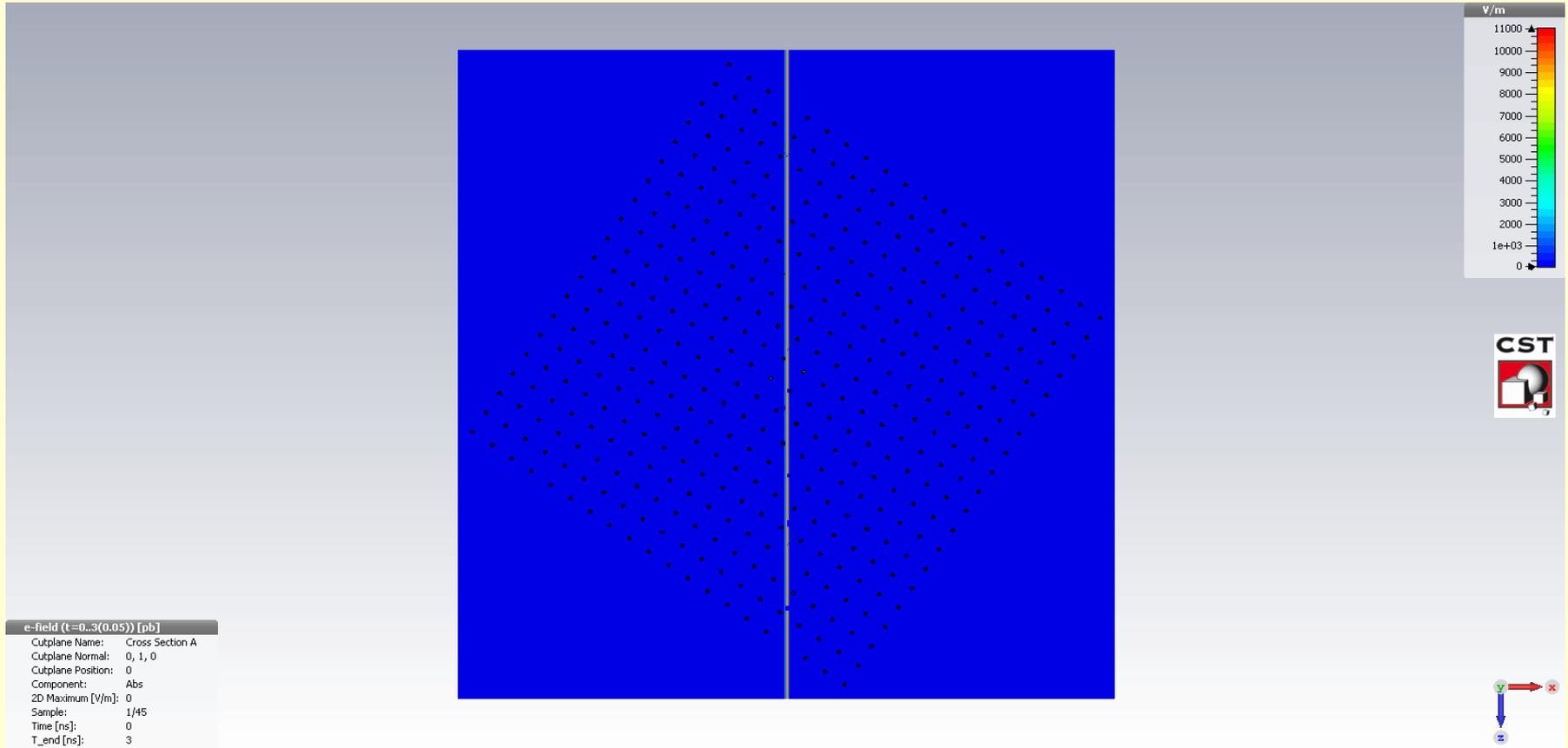
$$\rho = \frac{q\delta(x)\delta(y)}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(z-Vt)^2}{2\sigma^2}\right)$$

$$\rho_\omega = \frac{q\delta(x)\delta(y)}{2\pi V} \exp\left(i\frac{\omega}{V}z - \frac{\omega^2}{\omega_\sigma^2}\right), \quad \omega_\sigma = \frac{\sqrt{2}V}{\sigma}$$

CST Particle Studio simulations

CST result

20x20 structure



$$L = 20\text{cm}$$

$$r_0 = 0.5\text{mm}$$

$$\alpha = 55.4^\circ$$

$$\frac{L}{r_0} = 400$$

$$\sigma = 3\text{mm}$$

$$\omega_\sigma = 2\pi \cdot 22\text{GHz}$$

$$d_x = d_z = 8\text{mm}$$

“Long-wave” response

Effective medium approach

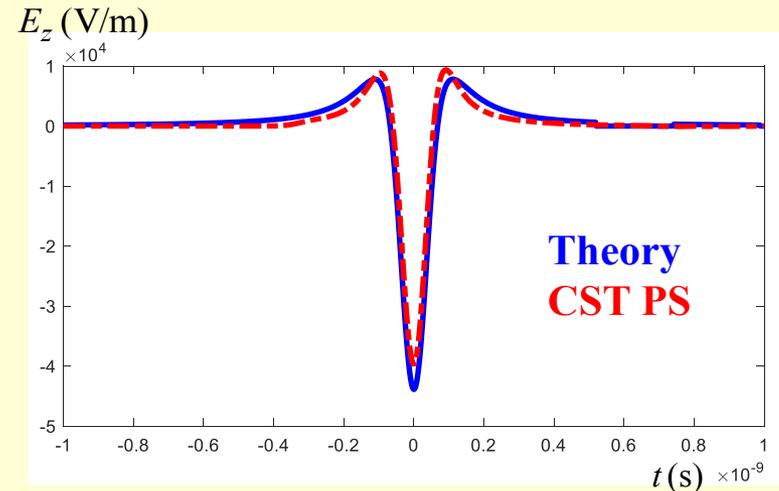
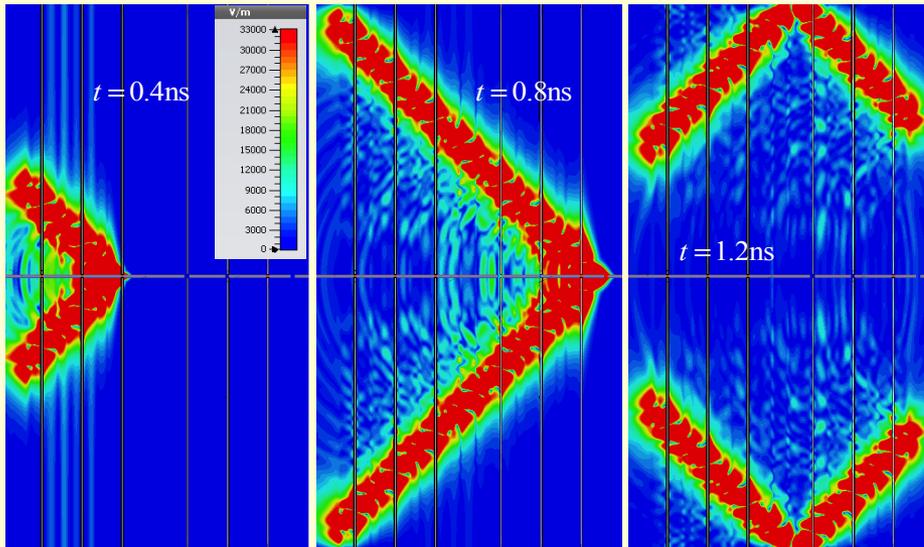
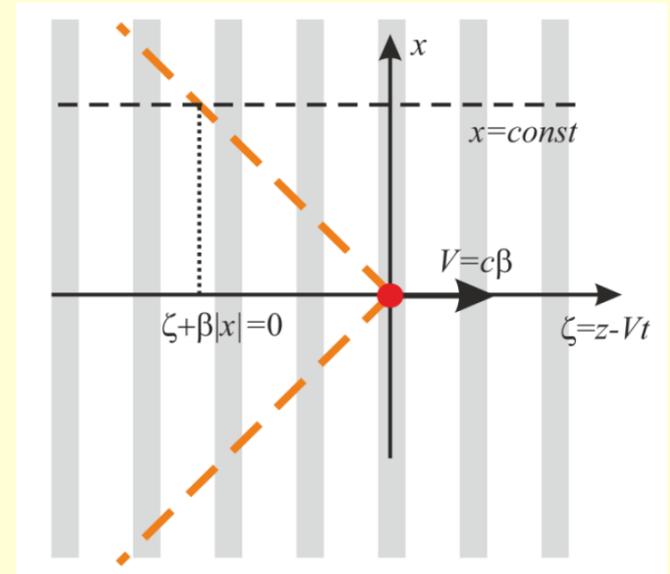
$$d_x, d_z \ll \Lambda, \lambda$$

$$\hat{\epsilon} = \begin{pmatrix} \epsilon_p & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \epsilon_p(\omega, k_y) = 1 - \frac{\omega_p^2}{\omega^2 - c^2 k_y^2 - 2i\omega\omega_d},$$

$$\omega_p^2 = \frac{2\pi c^2}{d_x d_z u}, \quad \omega_d \approx \frac{a^2 \omega_p^2}{8\pi^2 r_0^2 \sigma_{\text{wire}}}$$

Square lattice: $d_x = d_z = a \Rightarrow u = \ln(a/r_0) - C$

$C \approx 1.0487$ Tyukhtin A. V., Doilnitsina E. G. J. Phys. D: Applied Phys. 2011. V. 44. N. 26. 265401.



$dx = dz = s = 1\text{cm}, r_0 = 1\text{mm}, q = 1\text{ nC},$
 $x = 10\text{cm}, z = b/2, y = 0, b = 0.9999.$

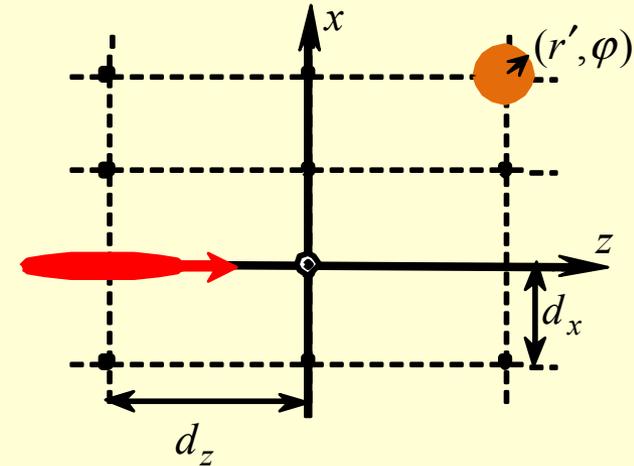
Vibrator antenna approach

“zero-order” approximation:

$$x = x_{lm} = ld_x \quad \beta \rightarrow 1 \quad L/r_0 \gg 1$$

$$z = z_{lm} = md_z$$

$$E_{\omega ylm}^{(i)} = E_{\omega r}^{(i)} \frac{y}{r_{lm}} = \frac{q}{\pi c} \exp\left(i\frac{\omega}{c}z_{lm} - \frac{\omega^2}{\omega_\sigma^2}\right) \frac{y}{y^2 + x_{lm}^2}$$



Hallen's problem:

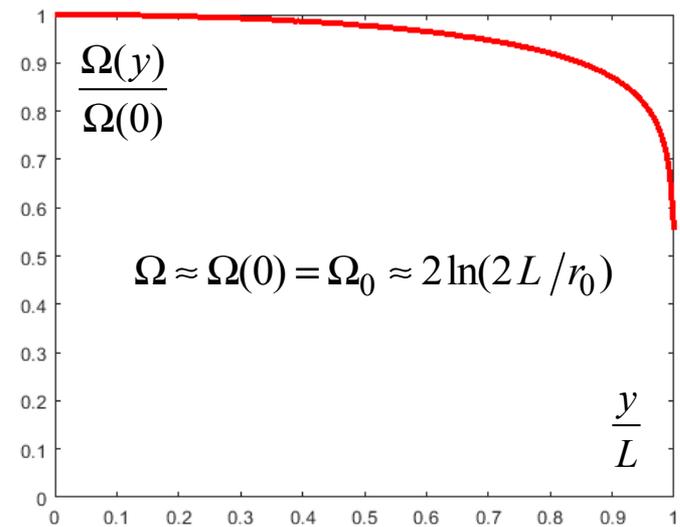
$$\left(\frac{d^2}{dy^2} + k_0^2\right)U(y) = 2ik_0 E_{\omega y}^{(i)}, \quad k_0 = \omega/c$$

$$U(y) = 2A_{\omega y} \Big|_{r'=r_0}$$

$$\frac{c}{2\mu}U(y) = I(y)\Omega(y) - \int_{-L}^L \left[I(y) - I(\xi)e^{ik_0|y-\xi|} \right] K_1(y-\xi)d\xi$$

$$K_1(y-\xi) = \frac{1}{2\pi} \int_{-L}^L \frac{d\varphi}{\sqrt{(y-\xi)^2 + 4r_0^2 \sin^2(\varphi/2)}}$$

$$\Omega(y) = \int_{-L}^L K_1(y-\xi)d\xi$$



Vibrator antenna approach

General solution for longitudinal potential

$$U(y) = A \sin(k_0 y) + B \cos(k_0 y) + C_1(y) \sin(k_0 y) + C_2(y) \cos(k_0 y)$$

$$C_1(y) = \frac{2iC_0}{\pi k_0} \int_0^y \frac{\cos(k_0 \xi) \xi}{\xi^2 + x_{lm}^2} d\xi, \quad C_2(y) = \frac{2C_0}{\pi i k_0} \int_0^y \frac{\sin(k_0 \xi) \xi}{\xi^2 + x_{lm}^2} d\xi$$

$$C_0 = q k_0 c^{-1} \exp(ik_0 z_{lm} - \omega^2 / \omega_\sigma^2)$$

Boundary condition

$$I_{lm}(\pm L) = 0$$

$$\frac{c}{2\mu} U(y) = I(y) \Omega(y) - \int_{-L}^L \left[I(y) - I(\xi) e^{-k_0 |y-\xi|} \right] K_1(y-\xi) d\xi$$

$$B = 0, \quad A = -\frac{C_1(L) \sin(k_0 L) + C_2(L) \cos(k_0 L)}{\sin(k_0 L)}$$

$$j_{lm}^{\text{surf}}(y) = \frac{I_{lm}(y)}{2\pi r_0}, \quad A_{\omega y}^{(lm)} = \frac{\mu}{c} \int_0^{2\pi} d\phi r_0 \int_{-L}^L d\xi j_{lm}^{\text{surf}}(\xi) \frac{\exp(ik_0 R_{lm})}{R_{lm}}$$

$$R_{lm} = \sqrt{\rho_{lm}^2 + r_0^2 - 2r_0 \rho_{lm} \cos \phi + (y - \xi)^2}, \quad \rho_{lm} = \sqrt{(z - z_{lm})^2 + (x - x_{lm})^2}$$

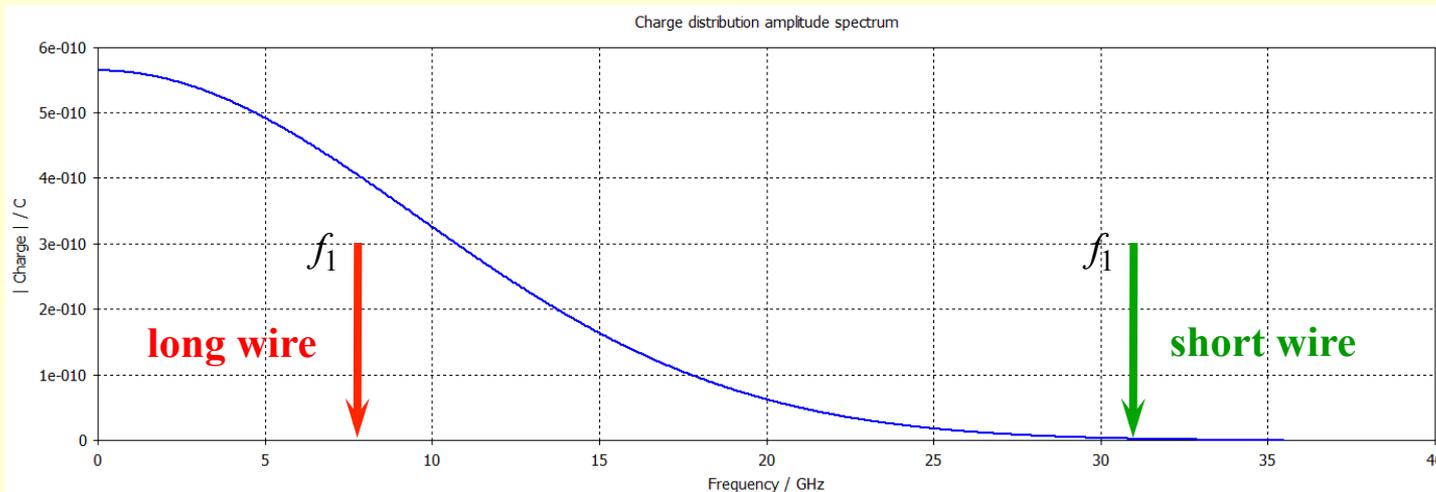
Vibrator antenna approach

General solution for surface current

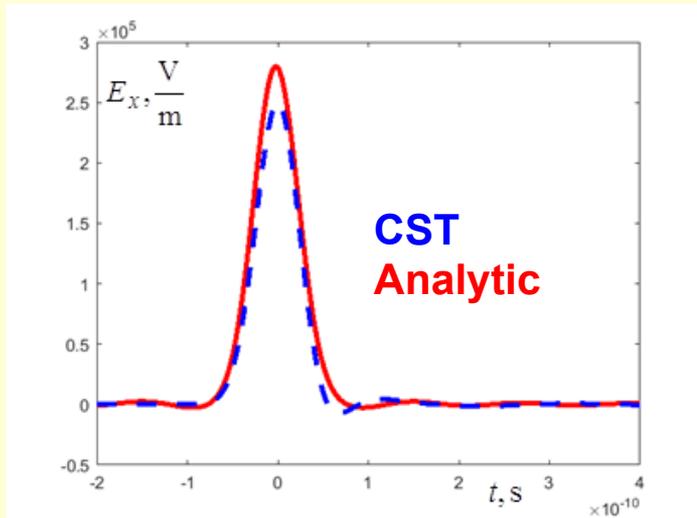
$$I(y) = \frac{c}{2\mu\Omega_0} \left[-\frac{C_1(L)\sin(k_0L) + C_2(L)\cos(k_0L)}{\sin(k_0L)} \sin(k_0y) + C_1(y)\sin(k_0y) + C_2(y)\cos(k_0y) \right]$$

Resonant frequencies

$$k_0L = \pm\pi m, \quad m = 1, 2, \dots \Rightarrow \omega_m = \frac{\pi c}{L} m \Rightarrow f_m = \frac{c}{2L} m.$$



Numerical results



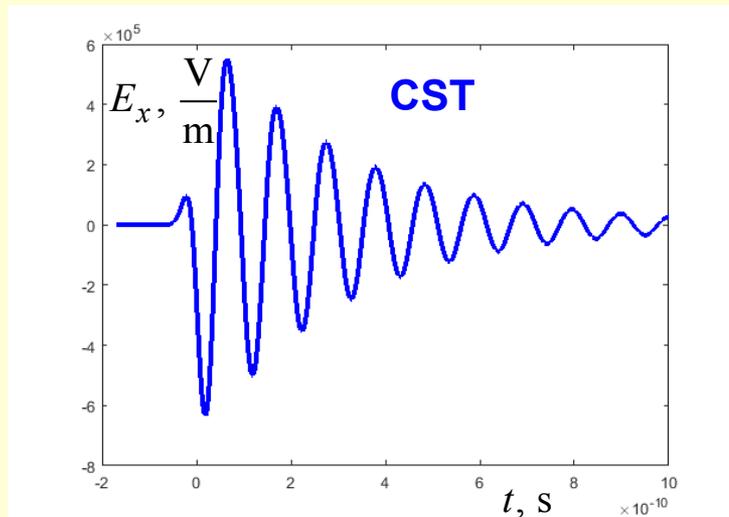
short wire

$$z_{lm} = 0, x_{lm} = d_x$$

$$d_x = 1.5\text{mm}, \sigma = 7\text{mm}, L = 4\text{mm}, r_0 = 0.05\text{mm}$$

$$q = 1\text{ nC}, \beta = 0.9999 (34\text{ MeV})$$

$$y = z = 0, x = 1.5d_x$$



long wire

$$\sigma = 5\text{mm}, L = 15\text{mm}$$

Resonant response results in radiation

$$f_1 \approx 10\text{GHz}$$

Thank you for your attention!