Local field effect in Smith-Purcell radiation from dotted grating

***D.Yu. Sergeeva*, A.A. Tishchenko, M.N. Strikhanov**

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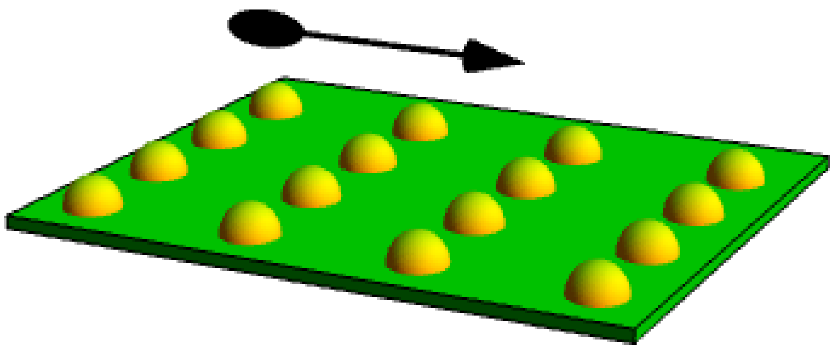
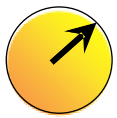
What for we investigate





$r_0 << \lambda$

$\alpha(\omega)$

- Amplification of the radiation because of…
  - … localized surface plasmon resonance in every particle

- *D.Yu. Sergeeva, A.A. Tishchenko, M.N. Strikhanov, NIMB (2017)*
- *D.Yu. Sergeeva, A.A. Tishchenko, A.S. Aryshev, M.N. Strikhanov, JINST (2018)*

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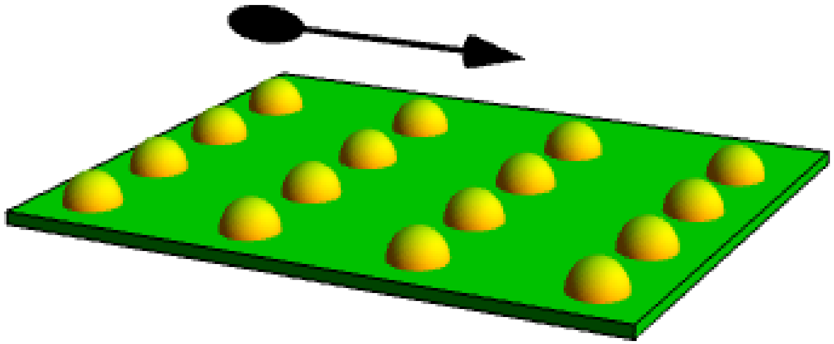

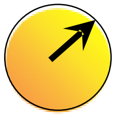

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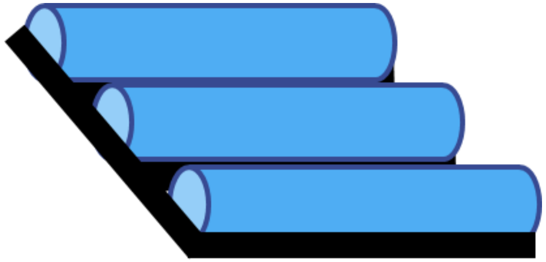


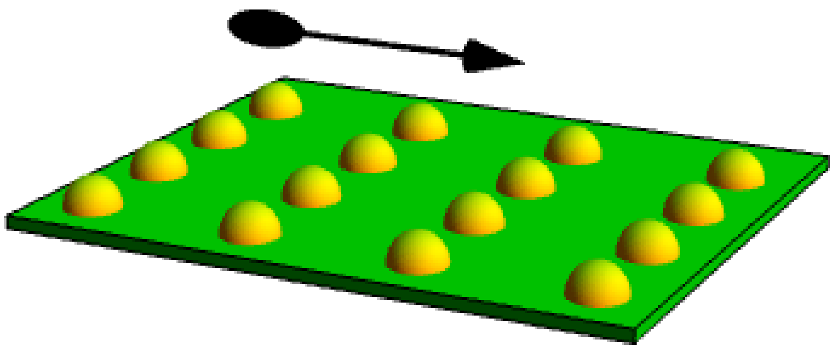
$$d^2W(\mathbf{n},\omega) \propto |\alpha(\omega)|^2$$

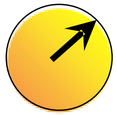
$$\alpha(\omega) = r_0^3 \frac{\varepsilon(\omega)-1}{\varepsilon(\omega)+2}$$

$$\omega^*: \quad \varepsilon(\omega^*)+2=0$$

- *D.Yu. Sergeeva, A.A. Tishchenko, M.N. Strikhanov, NIMB (2017)*
- *D.Yu. Sergeeva, A.A. Tishchenko, A.S. Aryshev, M.N. Strikhanov, JINST (2018)*
- *N.K. Zhevago, V.I. Glebov, Modified theory of SPR, NIM A (1994)*

What for we investigate





$r_0 << \lambda$

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- Amplification of the radiation because of…
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  - … local field effects in the system of interacting particles

- *D.Yu. Sergeeva, A.A. Tishchenko, M.N. Strikhanov, NIMB (2017)*
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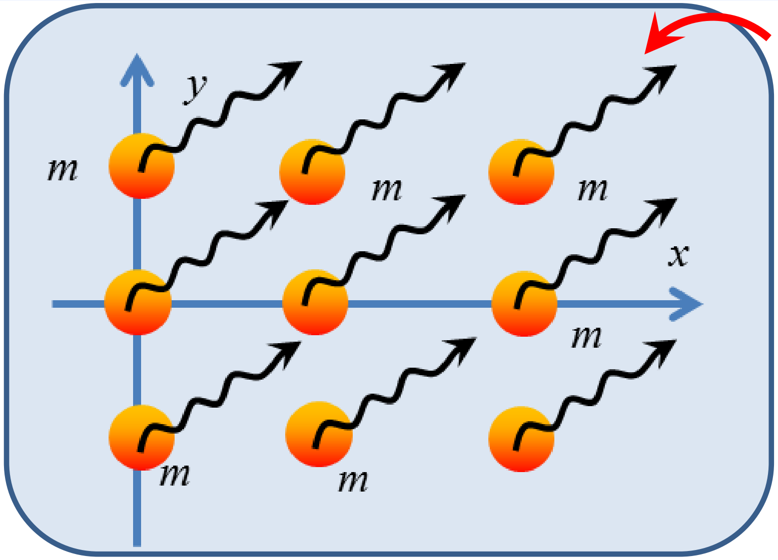
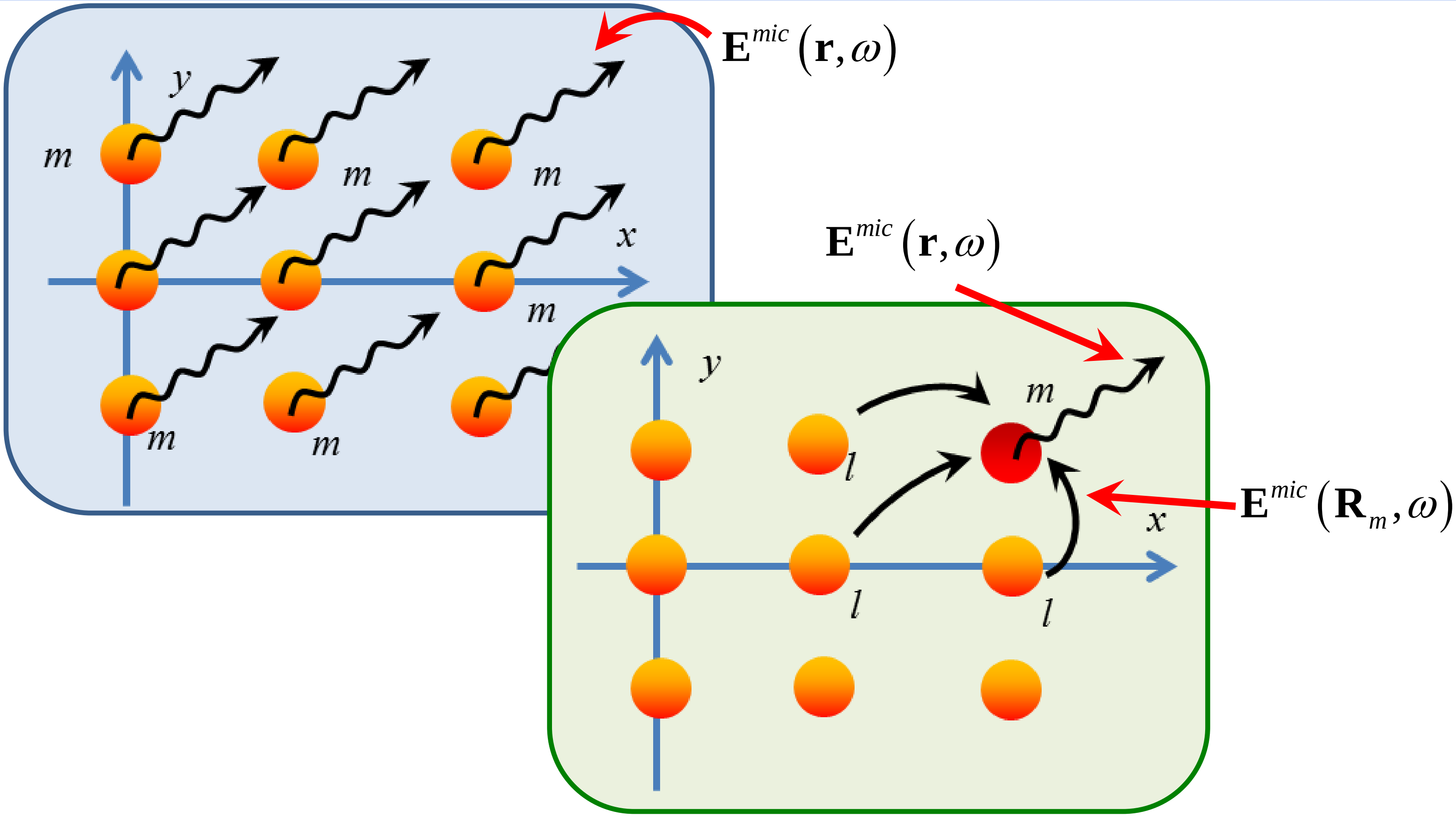
Interacting particles

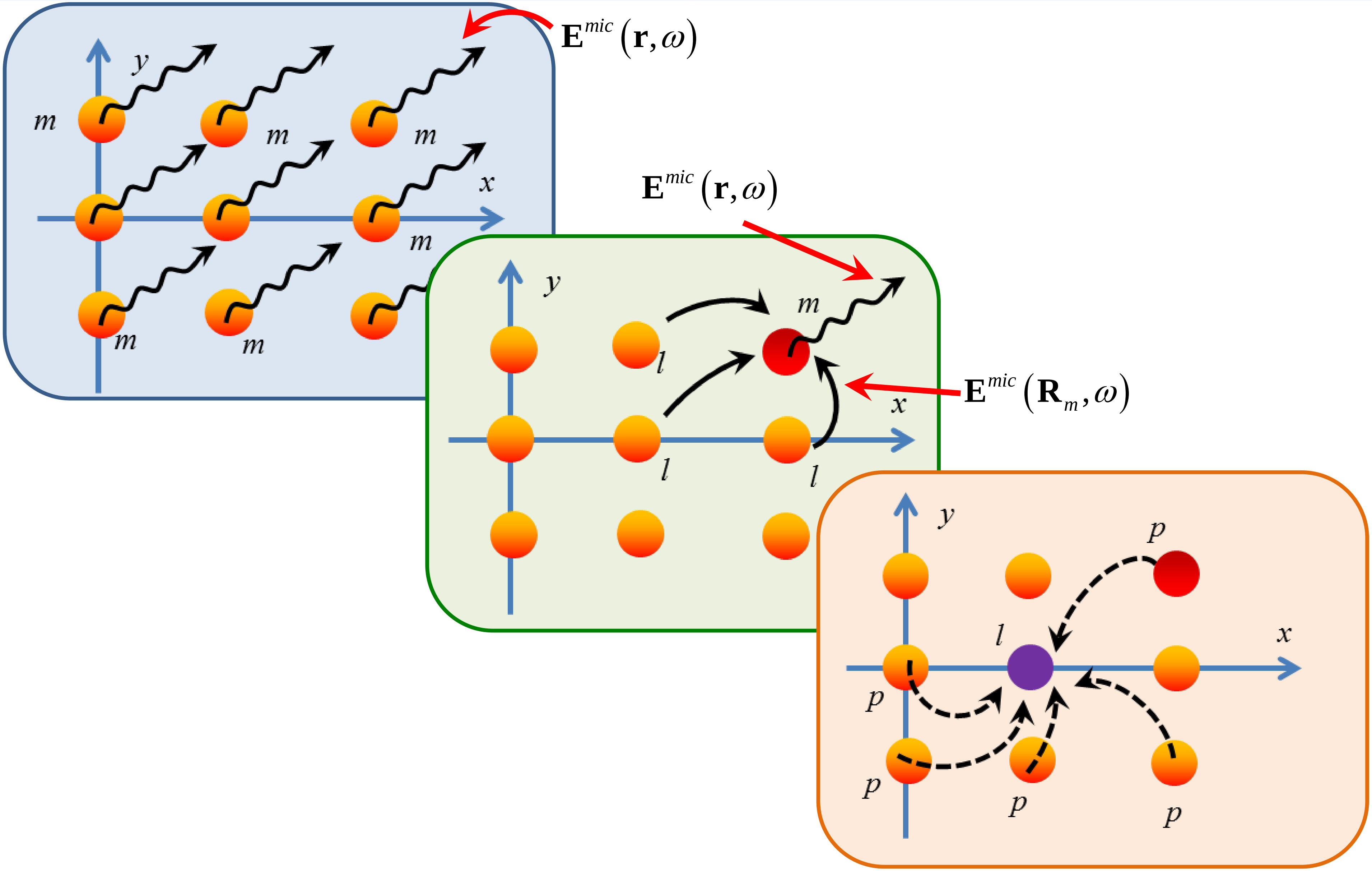


$\mathbf{E}^{mic}(\mathbf{r},\omega)$

Interacting particles



Interacting particles



Interacting particles

Maxwell equations

$$\left[\mathbf{q},\mathbf{H}^{mic}(\mathbf{q},\omega)\right]=-\frac{4\pi i}{c}\left(\mathbf{j}^{0}(\mathbf{q},\omega)+\mathbf{j}(\mathbf{q},\omega)\right)-\frac{\omega}{c}\mathbf{E}^{mic}(\mathbf{q},\omega)$$

$$\left[\mathbf{q},\mathbf{E}^{mic}(\mathbf{q},\omega)\right]=\frac{\omega}{c}\mathbf{H}^{mic}(\mathbf{q},\omega)$$

Electron and particle properties

$$\mathbf{j}^{0}(\mathbf{q},\omega)=\frac{e\mathbf{v}}{(2\pi)^{3}}e^{-iq_z h}\delta(\omega-\mathbf{q}\mathbf{v})$$

$$\mathbf{j}(\mathbf{r},\omega)=-i\omega\alpha(\omega)\sum_{m=1}^{N}\mathbf{E}^{mic}(\mathbf{R}_m,\omega)\delta(\mathbf{r}-\mathbf{R}_m)$$

Radius-vector of the particle

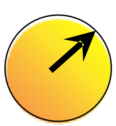
Exact solution

Field of radiation

$$E_i^{mic}(\mathbf{r},\omega)=E_i^{0}(\mathbf{r},\omega)+\frac{\alpha(\omega)}{2\pi^{2}}\int d^{3}q S_{ij}(\mathbf{q},\omega)\sum_{m=1}^{N}E_j^{mic}(\mathbf{R}_m,\omega)e^{i\mathbf{q}(\mathbf{r}-\mathbf{R}_m)}\delta(\mathbf{r}-\mathbf{R}_m)$$

$$E_i^{mic}(\mathbf{r},\omega)\Big|_{\mathbf{r}=\mathbf{R}_m}=\mathbf{E}^{0}(\mathbf{R}_m,\omega)+\sum_{\substack{l=1\\ l\neq m}}^{N}\mathbf{E}_l(\mathbf{R}_m,\omega)$$

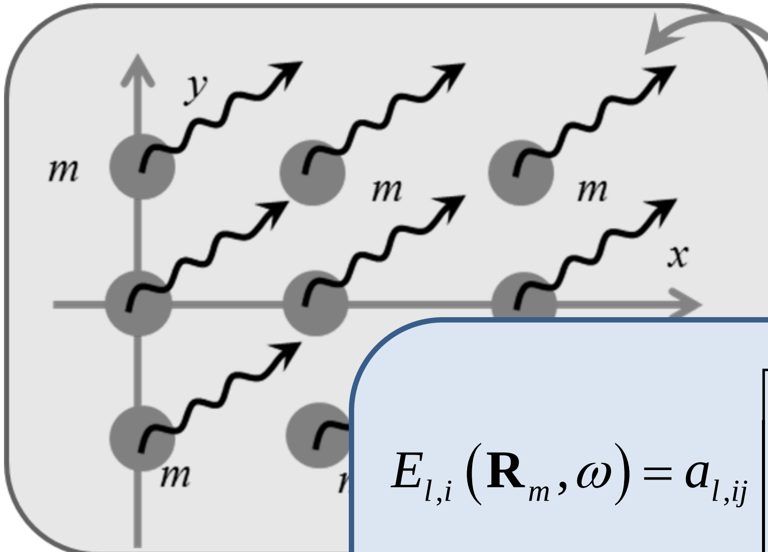
$\mathbf{R}_m$ → $\mathbf{E}_l(\mathbf{r},\omega)=f\left(\mathbf{E}^{0}\right)$ → $\mathbf{E}(\mathbf{r},\omega)$ → $\dfrac{d^{2}W(\mathbf{n},\omega)}{d\omega d\Omega}$



$r_0 << \lambda$

$\alpha(\omega)$

Interacting particles



$\mathbf{E}^{mic}(\mathbf{r},\omega)$

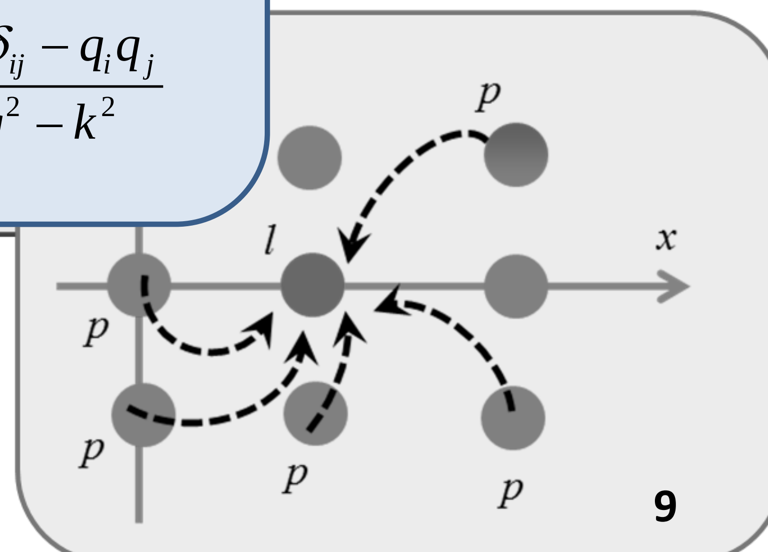
$\mathbf{E}^{mic}(\mathbf{r},\omega)$

$$E_{l,i}(\mathbf{R}_m,\omega)=a_{l,ij}\left[\mathbf{E}^{0}(\mathbf{R}_l,\omega)+\sum_{\substack{p=1\\p\neq l}}^{N}\mathbf{E}_p(\mathbf{R}_l,\omega)\right]_j$$

$$a_{l,ij}=\frac{\alpha_l(\omega)}{2\pi^2}\int d^3q e^{i\mathbf{q}\mathbf{R}_m}e^{-i\mathbf{q}\mathbf{R}_l}\frac{k^2\delta_{ij}-q_iq_j}{q^2-k^2}$$

$(\mathbf{R}_m,\omega)$

$N(N-1)$ self-consistent equations

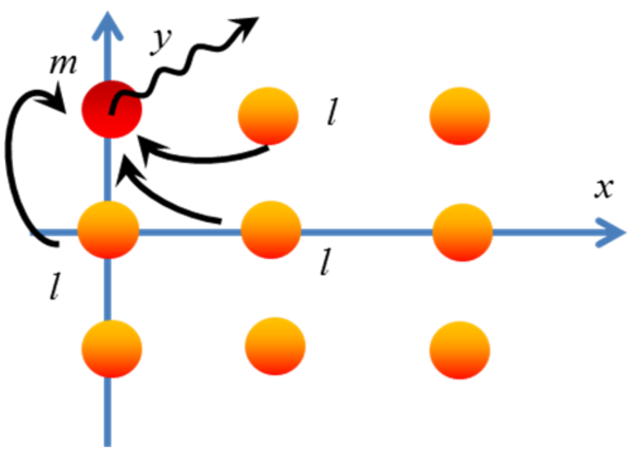


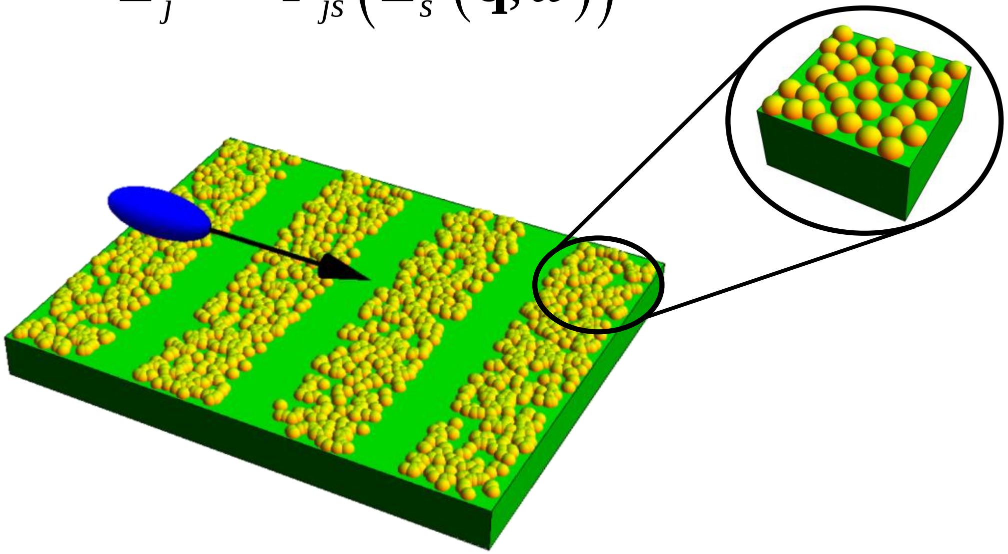
Local field effects

- Replacing the exact acting field by the averaged local field
- Averaging over positions of all *N* the particles

$$E_j^{mic}(\mathbf{r},\omega)\Big|_{\mathbf{r}=\mathbf{R}_a} \simeq E_j^{loc}(\mathbf{r},\omega)$$

- Averaging over positions of the rest particles relative to the given one
- Calculating the local field as a function of the Coulomb field $E_j^{loc}(\mathbf{q},\omega)$
- Calculating the field of radiation $E_j^{rad} = \widehat{F}_{js}\left(E_s^0(\mathbf{q},\omega)\right)$





M.I. Ryazanov, A.A. Tishchenko, JETP ***103*** *(2006) 539*

Local field effects

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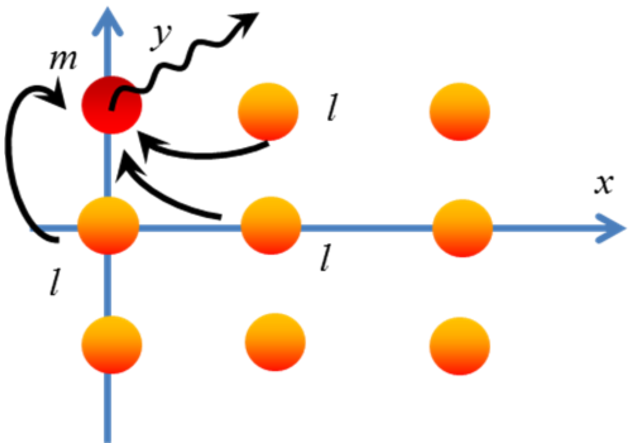
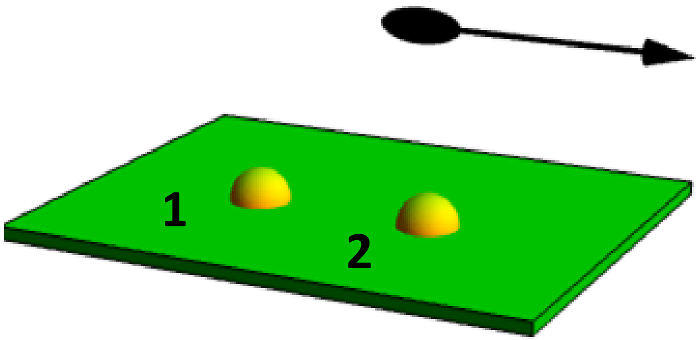


- N is not large
- The structure is finite, so the last particles in each row are influenced differently than the middle ones

Exact solution of $N(N-1)$ self-consistent equations should be found!

Two interacting particles



$$\frac{dW(\mathbf{n},\omega)}{d\omega d\Omega} = cr^2 \left|\mathbf{E}^{rad}(\mathbf{r},\omega)\right|^2$$

interaction

$$\mathbf{E}^{rad}(\mathbf{r},\omega) = -\alpha(\omega)\exp(-i\mathbf{k}\mathbf{R}_1)\frac{\exp(ikr)}{r}\left[\mathbf{k},\left[\mathbf{k},\mathbf{E}^0(\mathbf{R}_1,\omega)+\mathbf{E}_2(\mathbf{R}_1,\omega)\right]\right] - \\ -\alpha(\omega)\exp(-i\mathbf{k}\mathbf{R}_2)\frac{\exp(ikr)}{r}\left[\mathbf{k},\left[\mathbf{k},\mathbf{E}^0(\mathbf{R}_2,\omega)+\mathbf{E}_1(\mathbf{R}_2,\omega)\right]\right]$$

$$\alpha(\omega) = r_0^3\frac{\varepsilon(\omega)-1}{\varepsilon(\omega)+2}$$

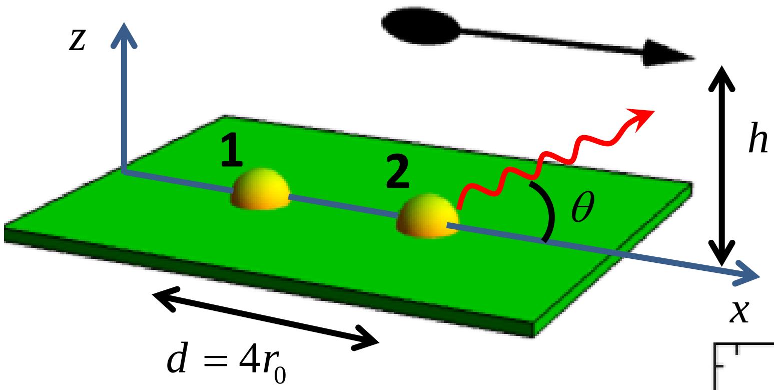
$$\mathbf{E}_{1,2}(\mathbf{R}_{2,1},\omega) = \frac{f\left(\mathbf{E}^0(\mathbf{R}_1,\omega),\mathbf{E}^0(\mathbf{R}_2,\omega)\right)}{\left(1-\alpha^2(\omega)e^{2iRk}B^2\right)\left(1-\alpha^2(\omega)e^{2iRk}(A+B)^2\right)}$$

resonance because of every particle

resonance because of interaction

$$\varepsilon(\omega) = -2 \qquad 1 = \mathrm{Re}\left[\alpha^2(\omega)e^{2iRk}B^2\right] \qquad 1 = \mathrm{Re}\left[\alpha^2(\omega)e^{2iRk}(A+B)^2\right]$$

Radiation

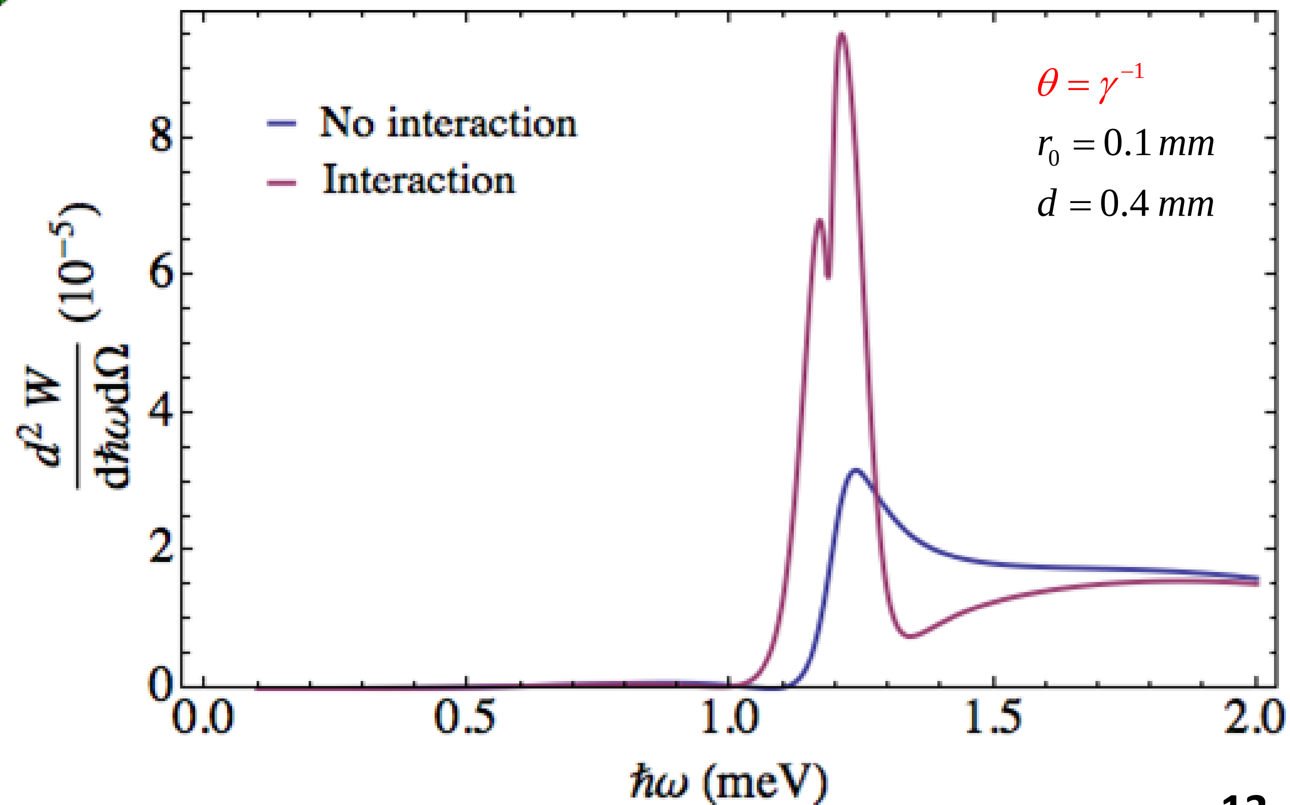


$$\alpha(\omega) = r_0^3 \frac{\omega_0^2}{\omega_0^2 - \omega^2 + i\Gamma\omega}$$

$\gamma = 12$

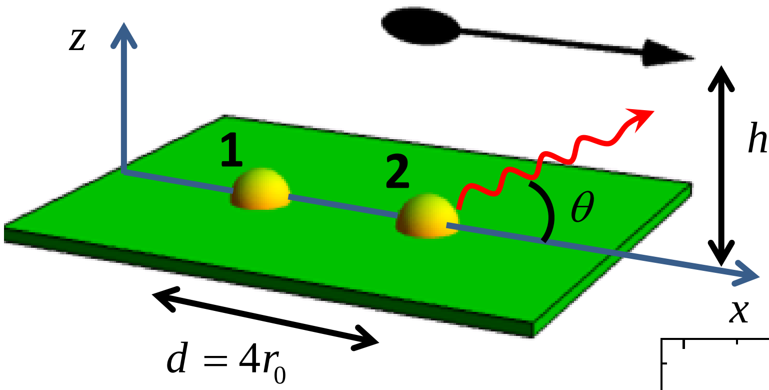
$h = 0.3\ mm$

$\omega_0 = 1.2\ meV$



Lorentz model

Radiation



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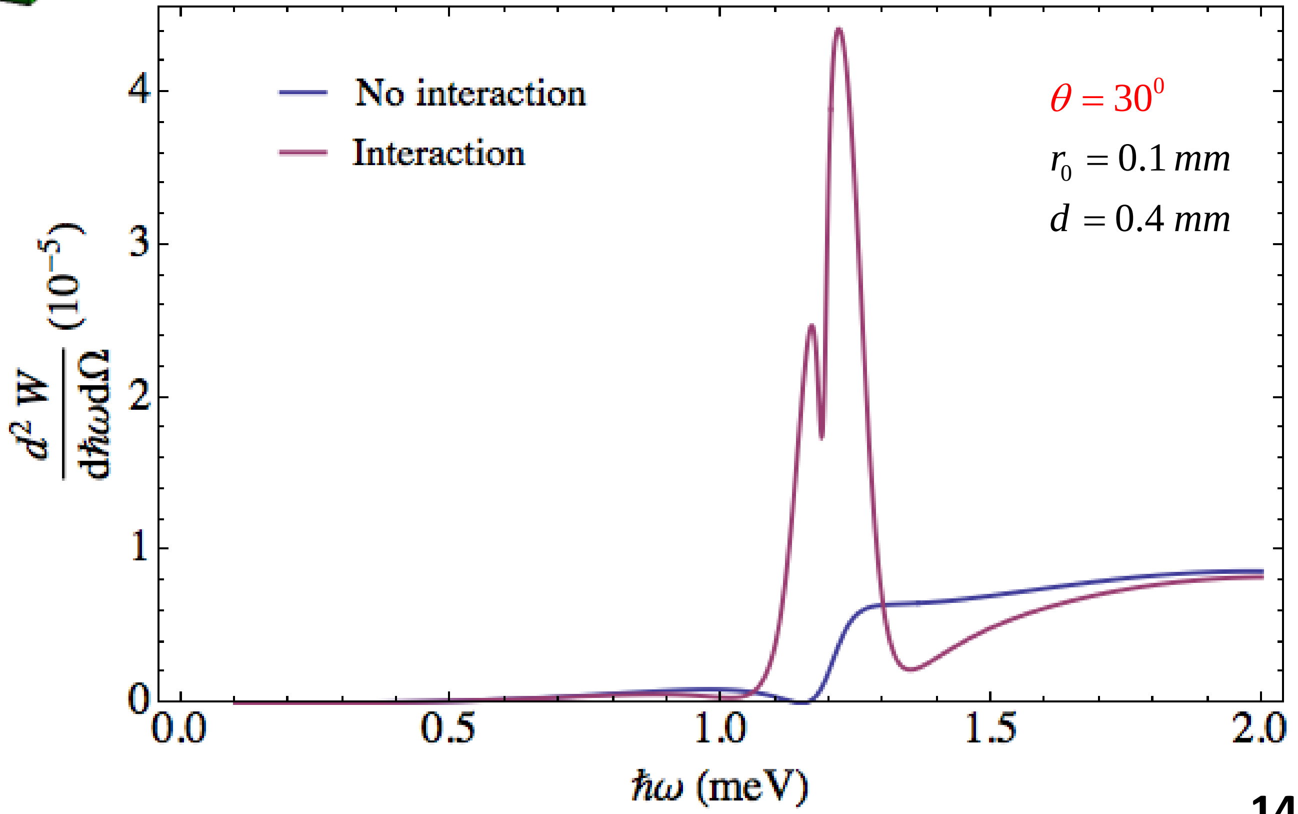

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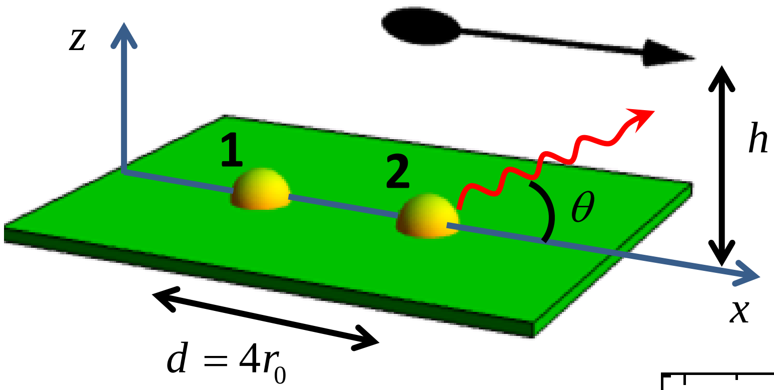

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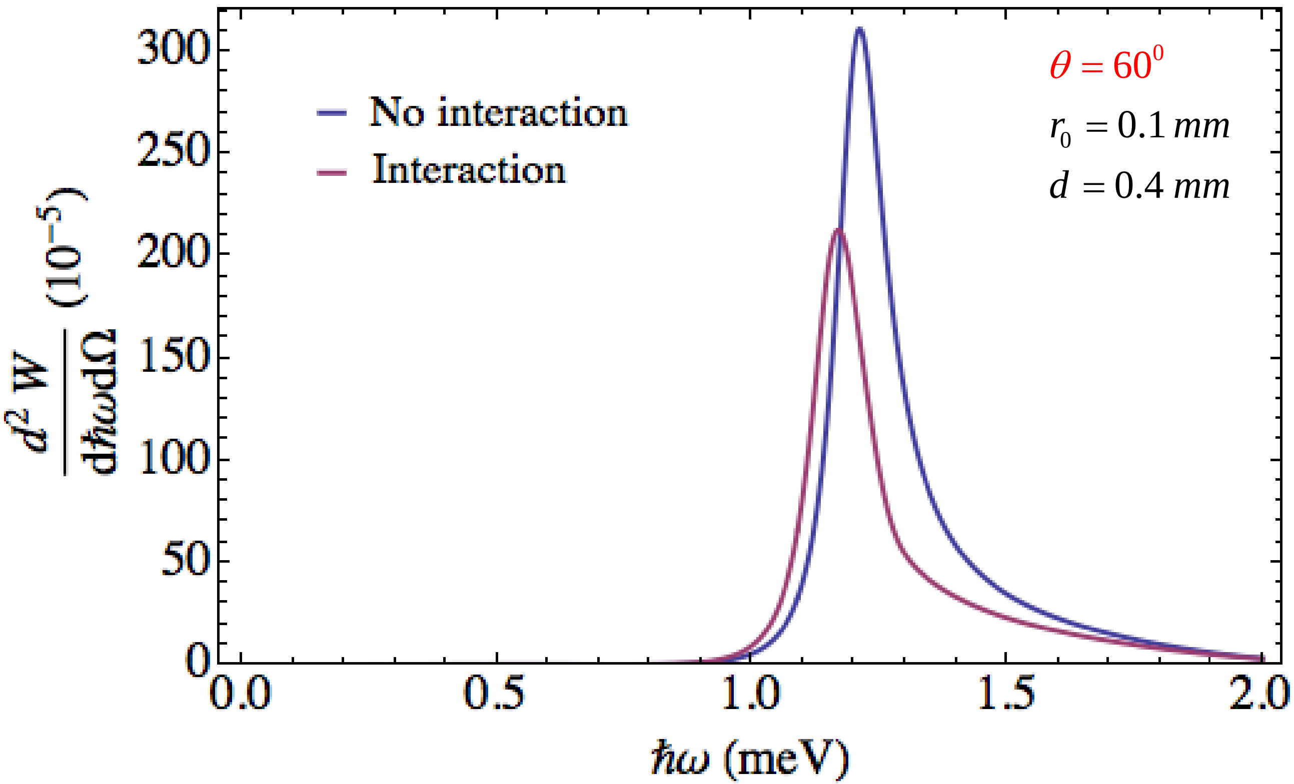

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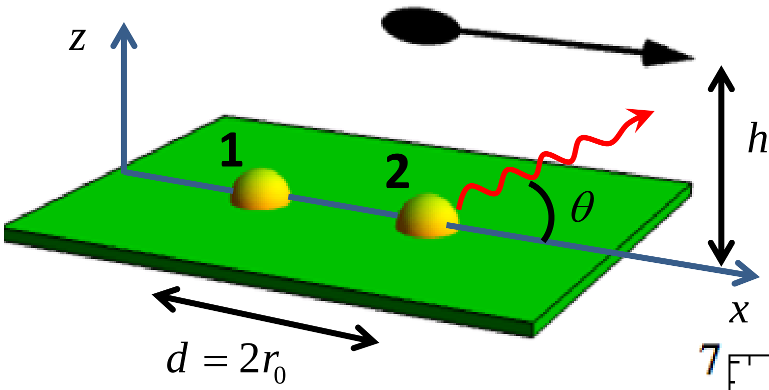

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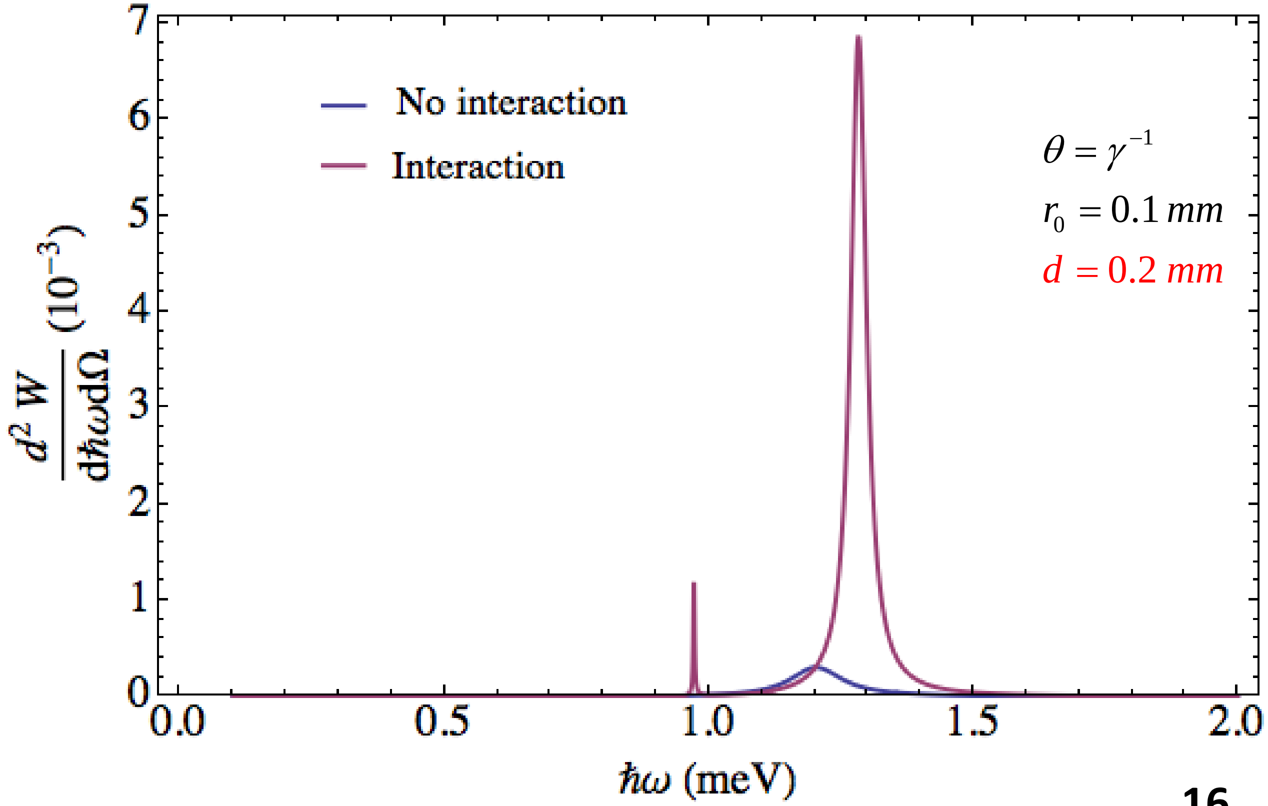
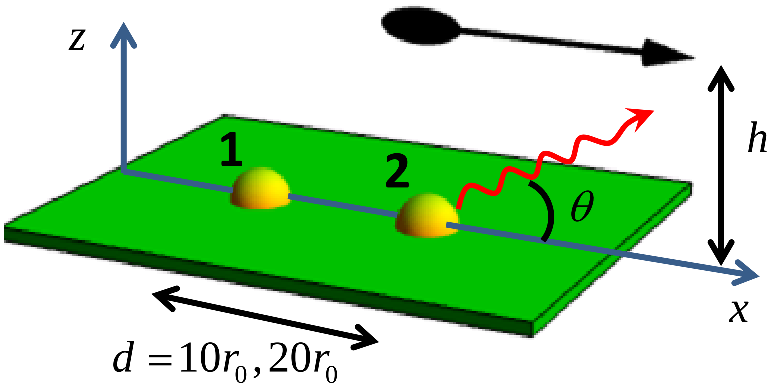


Radiation



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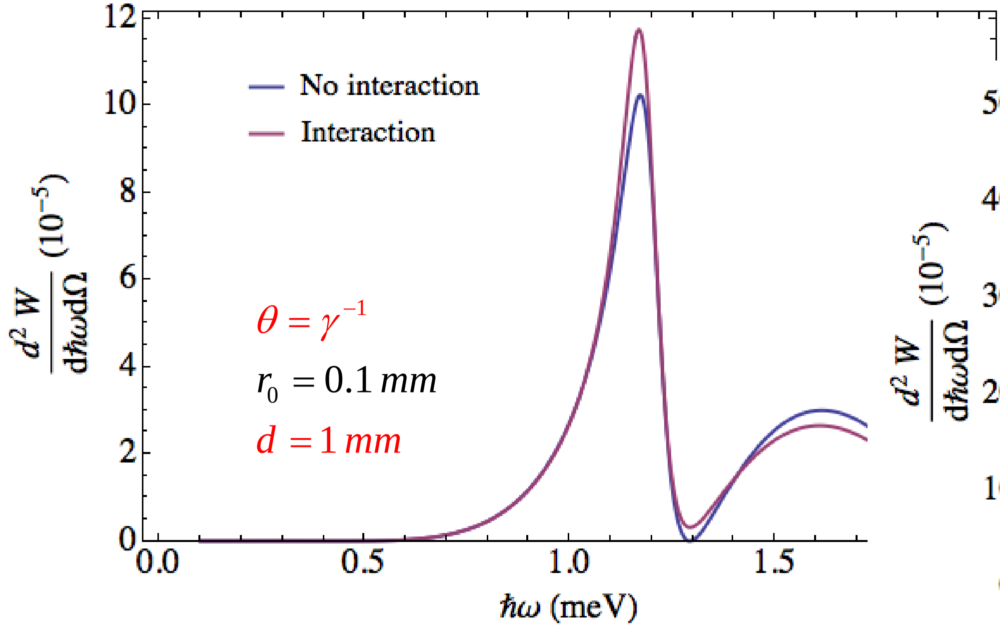

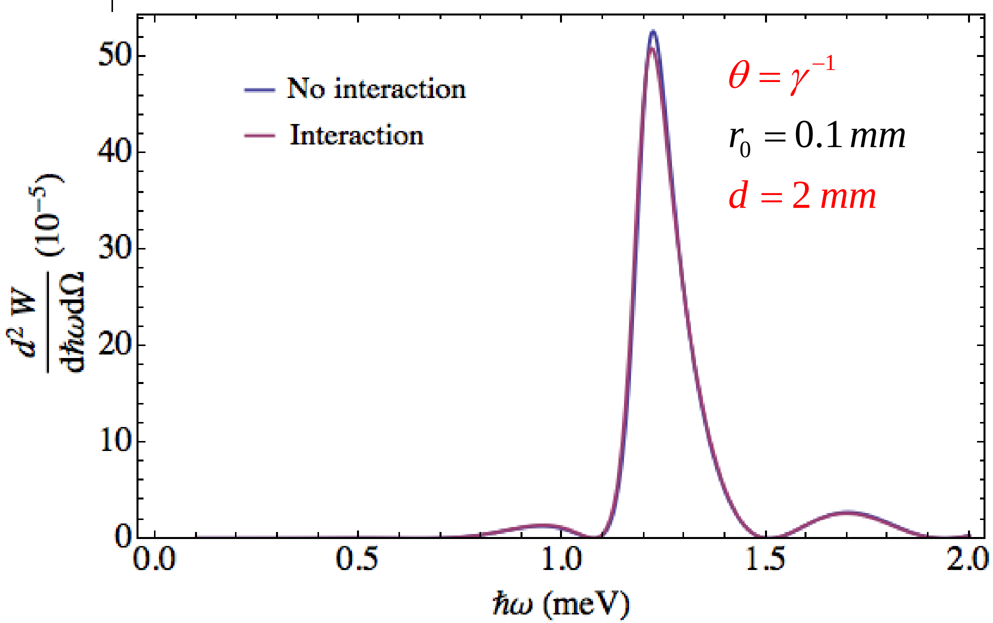

Summary

- Microscopic theory of radiation from 2D photonic grating consisting of interacting particles (needs calculation for *N>2* particles)
- Strong amplification of radiation is possible (needs optimization)
- Interaction leads to arising double or even triple maxima at relatively small angles

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Thank you!

Interacting particles

$$E_{1,2,s}\left(\mathbf{R}_{2,1},\omega\right)=-\alpha_{1,2}\left(\omega\right)e^{iRk}\frac{1}{V}\left(B\delta_{sj}-\frac{\alpha_1\left(\omega\right)\alpha_2\left(\omega\right)e^{2iRk}AB\left(A+B\right)+A}{W}\frac{R_sR_j}{R^2}\right)E_j^0\left(\mathbf{R}_{1,2},\omega\right)+$$

$$+\alpha_1\left(\omega\right)\alpha_2\left(\omega\right)e^{2iRk}\frac{1}{V}\left(B^2\delta_{sk}-\frac{A\left(A+2B\right)}{W}\frac{R_sR_k}{R^2}\right)E_k^0\left(\mathbf{R}_{2,1},\omega\right)$$

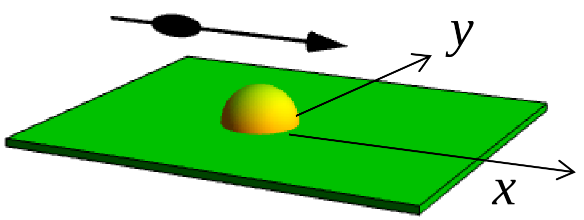
$$V=1-\alpha_1\left(\omega\right)\alpha_2\left(\omega\right)e^{2iRk}B^2$$

$$W=\left[\alpha_1\left(\omega\right)\alpha_2\left(\omega\right)e^{2iRk}\left(A+B\right)^2-1\right]$$

$$A=\frac{k^2R^2+3ikR-3}{R^3}$$

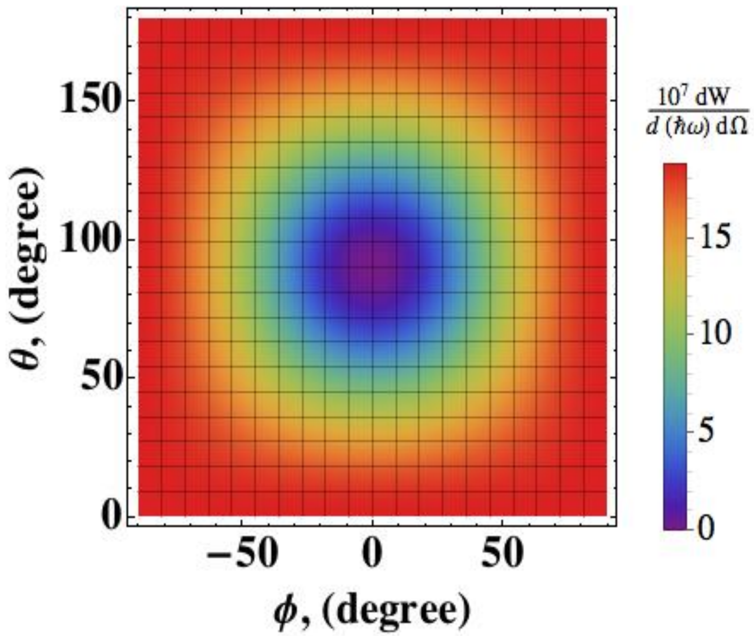
$$B=-\frac{k^2R^2+ikR-1}{R^3}$$

Одна частица



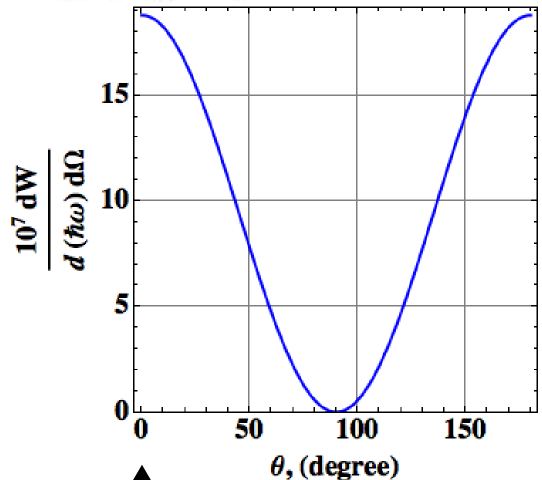
Парафин, полиэтилен

$$R = 0.1\,mm$$
$$\varepsilon = 2$$
$$E_e = 15\,MeV$$
$$\gamma = 30$$
$$h = 1\,mm$$
$$\lambda = 0.4\,mm$$



Ширина пика ДИ γ^{-1}

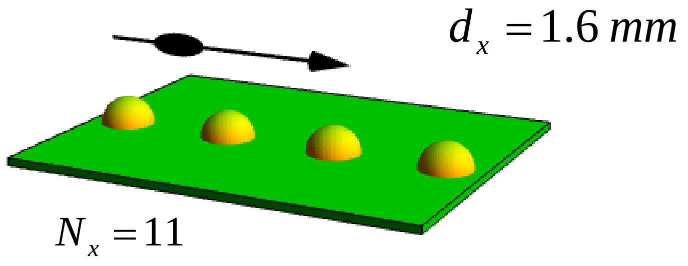
- Нерелятивистские связанные электроны
- Нет зависимости от энергии, размера, материала
- Усреднение? Когерентность $\left(\theta^2 + \gamma^{-2}\right)^{-2}$



$$n_x = \cos\theta$$
$$n_y = \sin\theta\sin\phi$$
$$n_z = \sin\theta\cos\phi$$

М.И. Рязанов, Электродинамика конденсированного вещества (1984)

Продольная цепочка частиц



$R = 0.1\ mm$ $\quad \gamma = 30$

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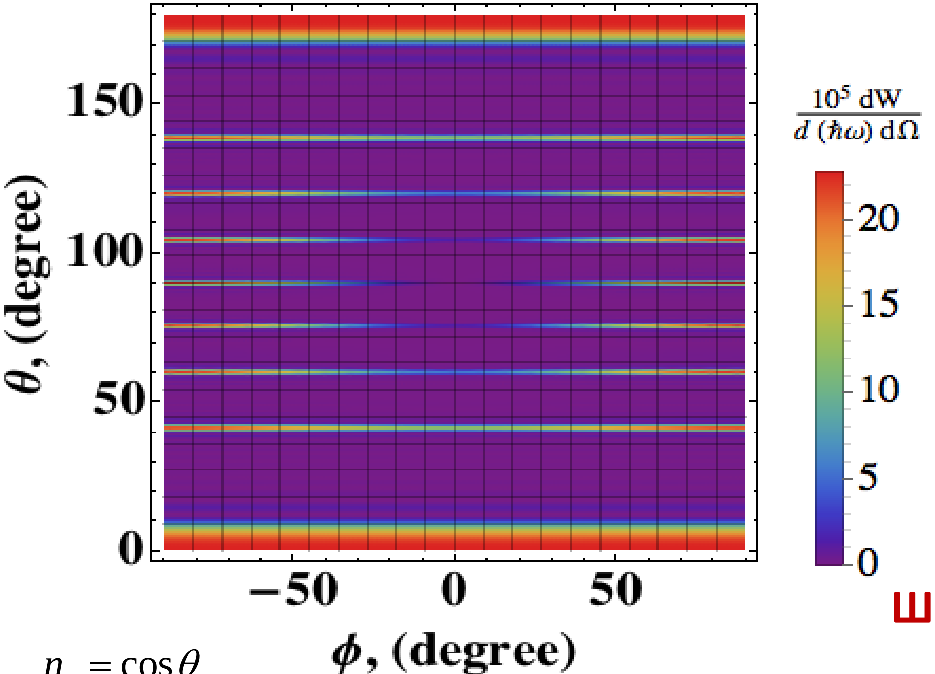

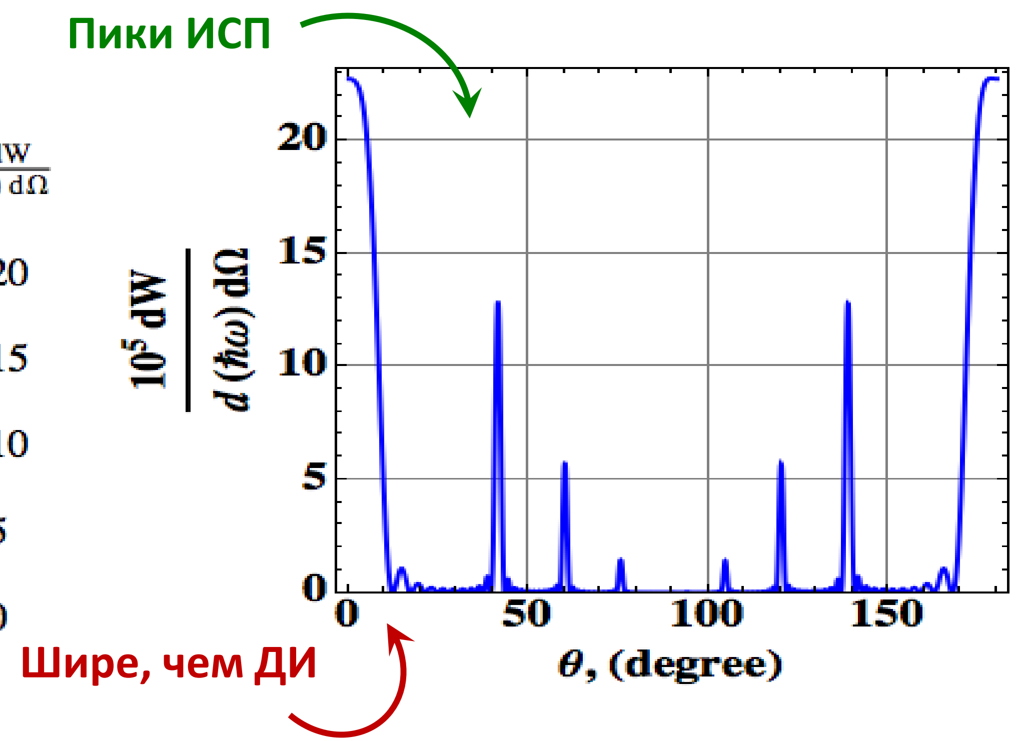

$n_x = \cos\theta$

$n_y = \sin\theta \sin\phi$

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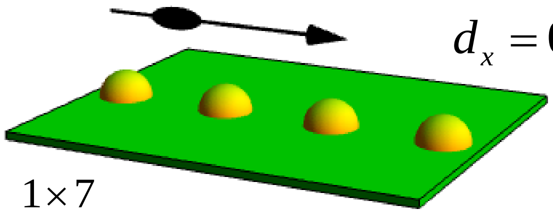
K.V. Lekomtsev, et al., J. Phys.: Conf. Ser. (2010)

Цепочки частиц

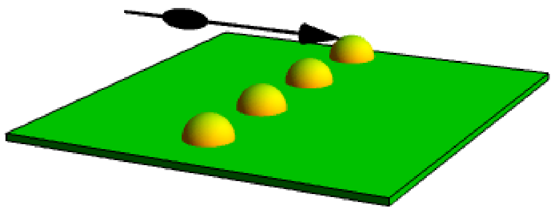


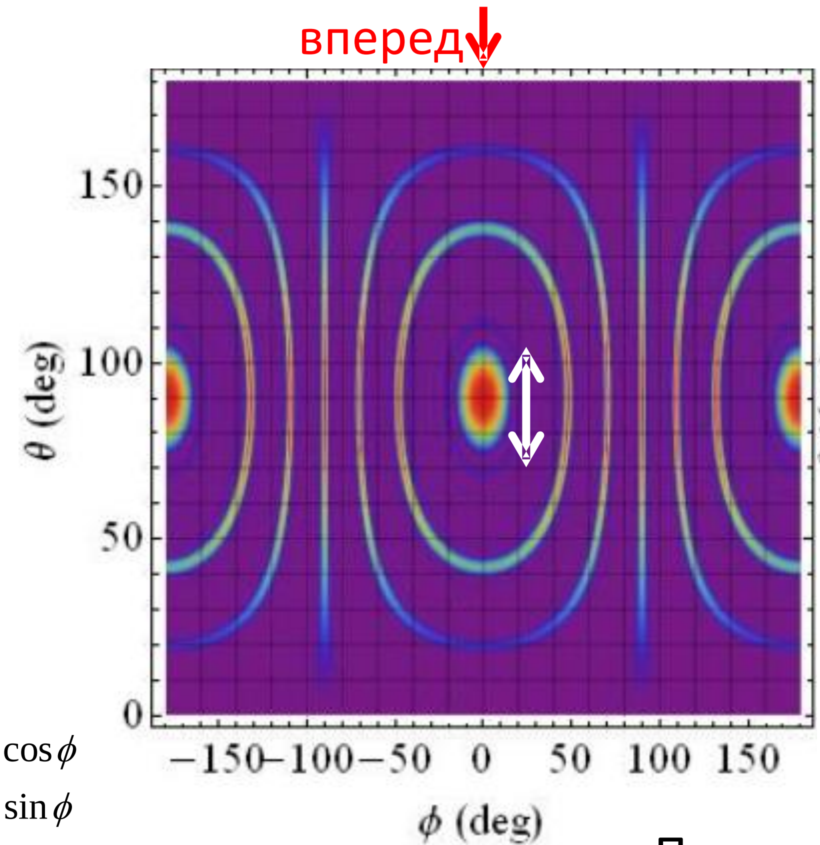
$\gamma = 55$

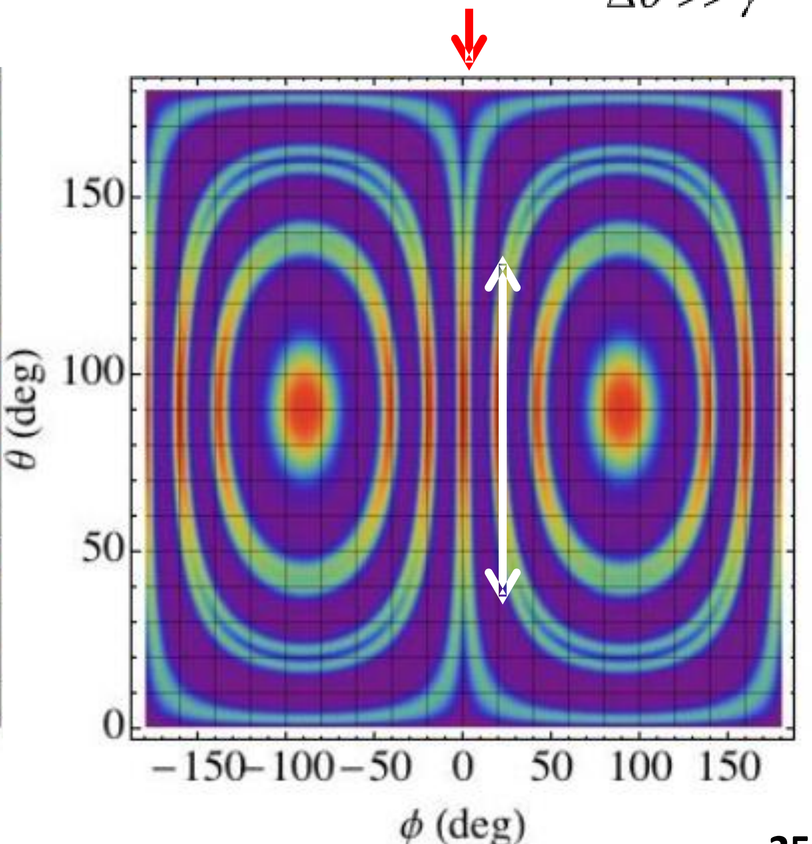
$\varepsilon = 2$

$h = 0.3\,mm$

$\lambda = 10R = 0.1\,mm$







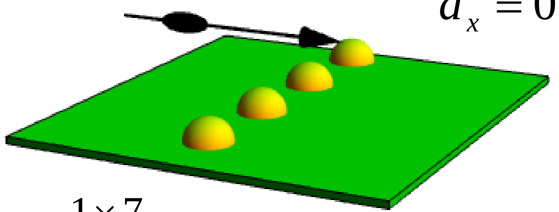
$n_x = \sin\theta\cos\phi$

$n_y = \sin\theta\sin\phi$

$n_z = \cos\theta$

Пики сдвинуты и шире

Поперечная цепочка частиц



$d_x = 0.3\,mm$

1×7

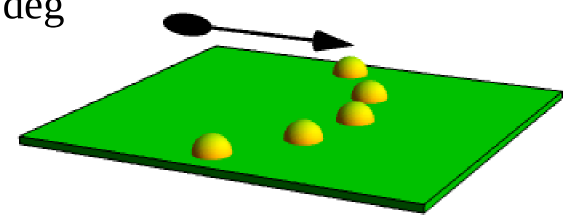
$\gamma = 55$

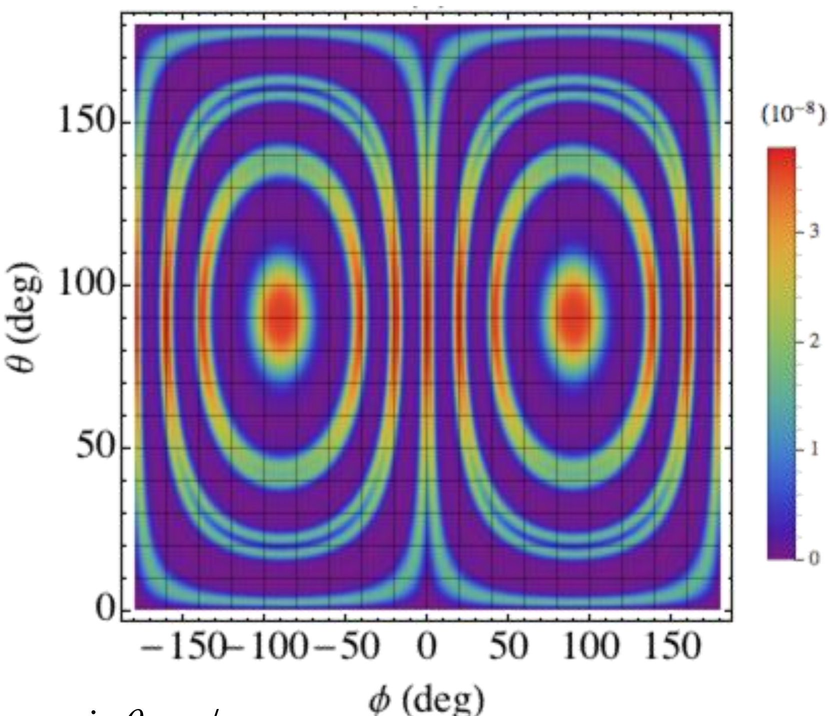
$\varepsilon = 2$

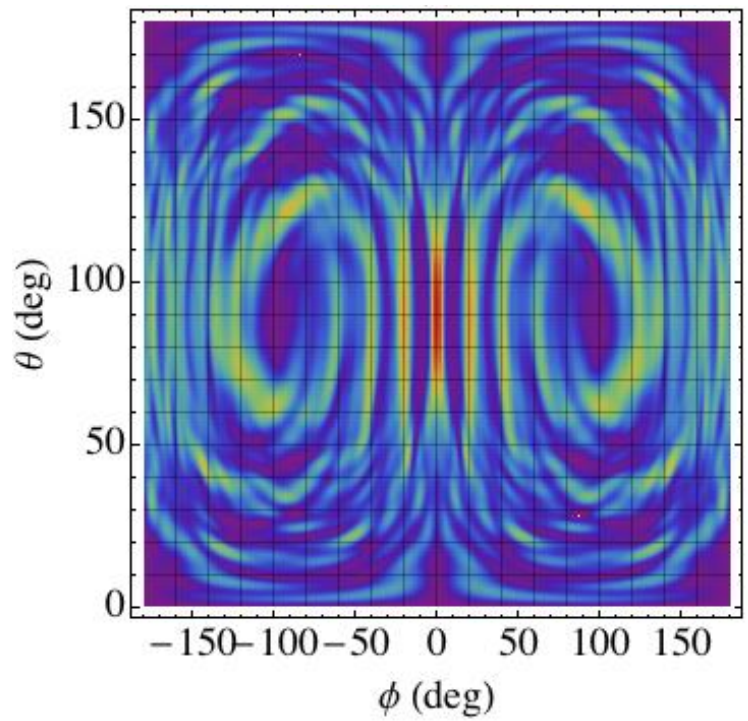
$h = 0.3\,mm$

$\lambda = 10R = 0.1\,mm$

100 deg







$n_x = \sin\theta\cos\phi$

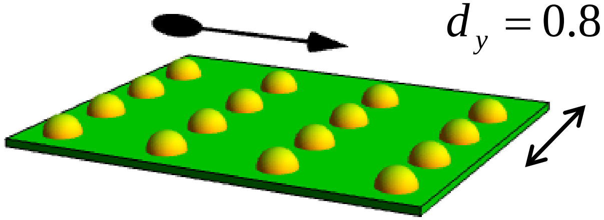
$n_y = \sin\theta\sin\phi$

$n_z = \cos\theta$

$$n_y = \frac{\lambda}{d_y} l, \quad l = 0, \pm 1, \pm 2, \ldots,$$

?

Массив частиц



$d_y = 0.8\,mm$

$R = 0.1\,mm$

$\varepsilon = 2$

$E_e = 15\,MeV$

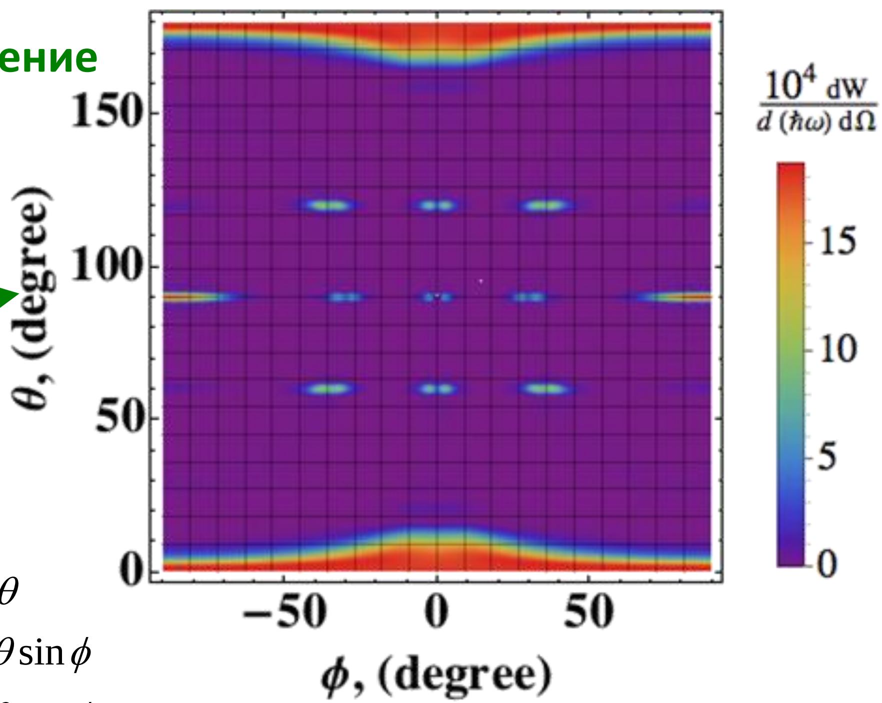
$\gamma = 30$

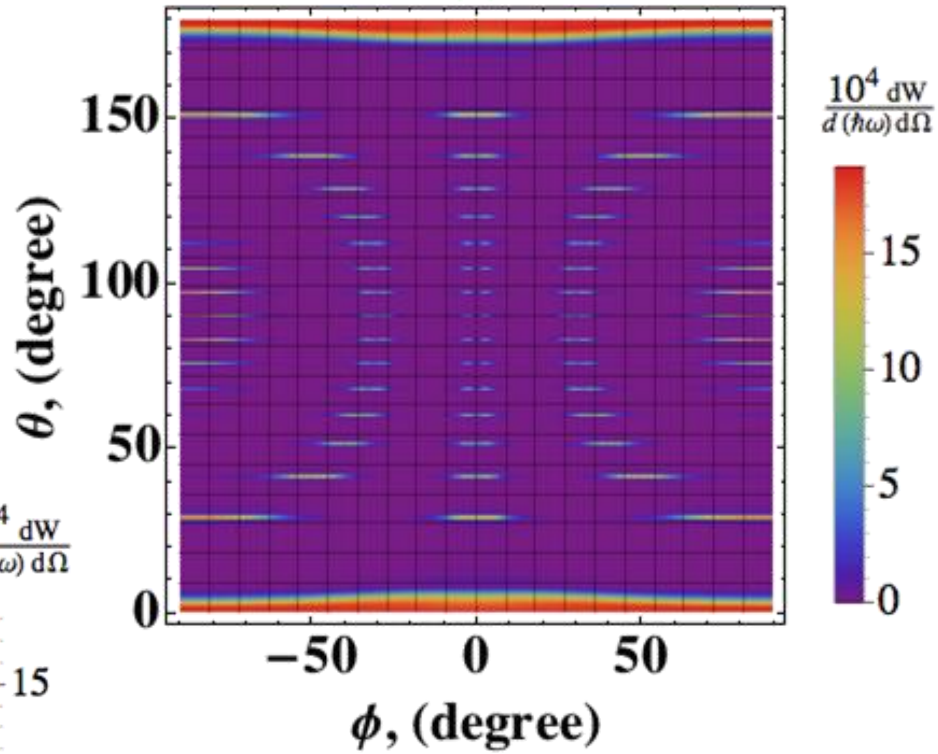
$h = 1\,mm$

$\lambda = 0.4\,mm$

$d_x = d_y = 0.8\,mm$

Излучение вбок





$d_x = 4d_y = 3.2\,mm$

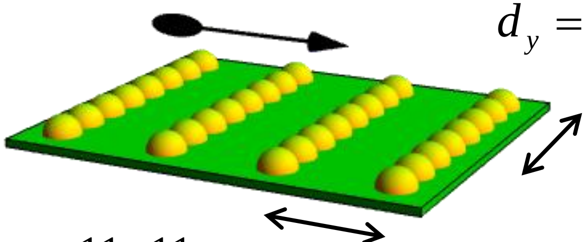
- **Строгий порядок**
- **Конические поверхности**

$n_x = \cos\theta$

$n_y = \sin\theta\sin\phi$

$n_z = \sin\theta\cos\phi$

Стриповая решетка



$d_y = 0.2\,mm$

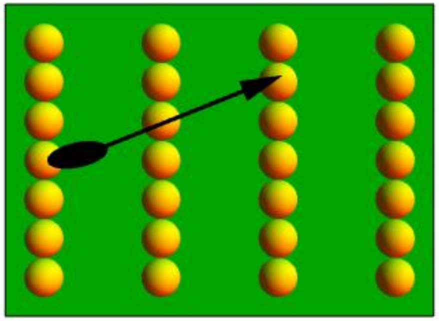
11×11

$d_x = 1.6\,mm$

$R = 0.1\,mm$ $\gamma = 30$

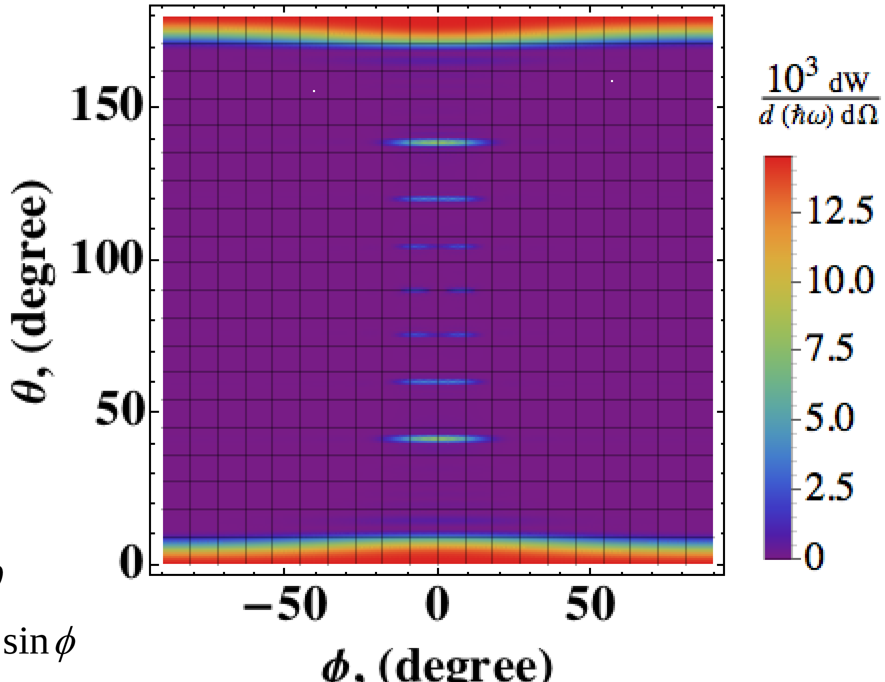
$\lambda = 0.4\,mm$ $E_e = 15\,MeV$

$\varepsilon = 2$ $h = 1\,mm$

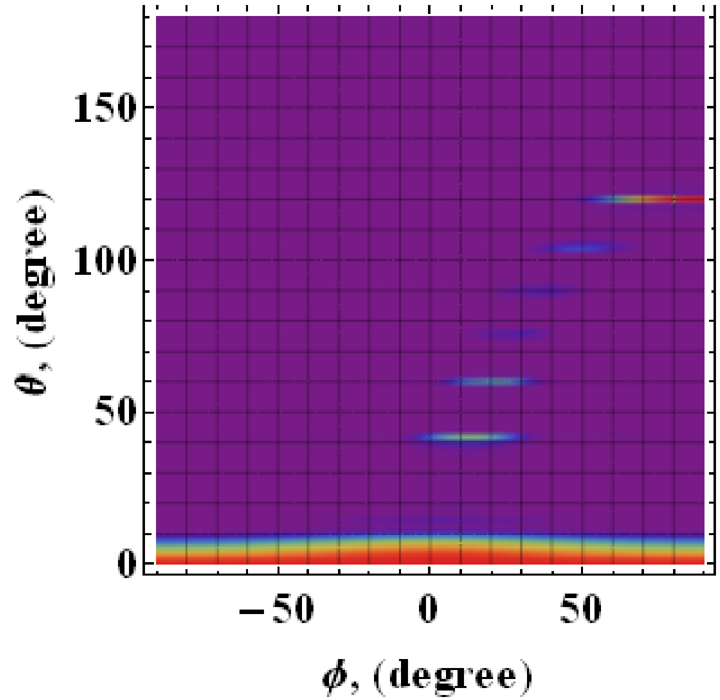


- **ИСП от стриповой решетки**
- **Конический эффект**

$\alpha = 0$



$\alpha = 60$ deg

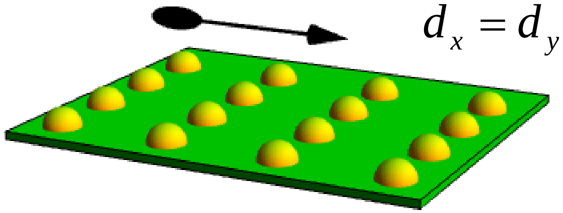


$n_x = \cos\theta$

$n_y = \sin\theta \sin\phi$

$n_z = \sin\theta \cos\phi$

Массивы частиц



7×7

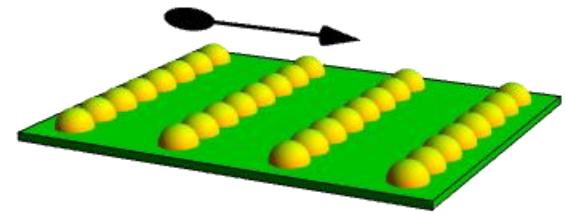
$d_x = d_y = 0.3\,mm$

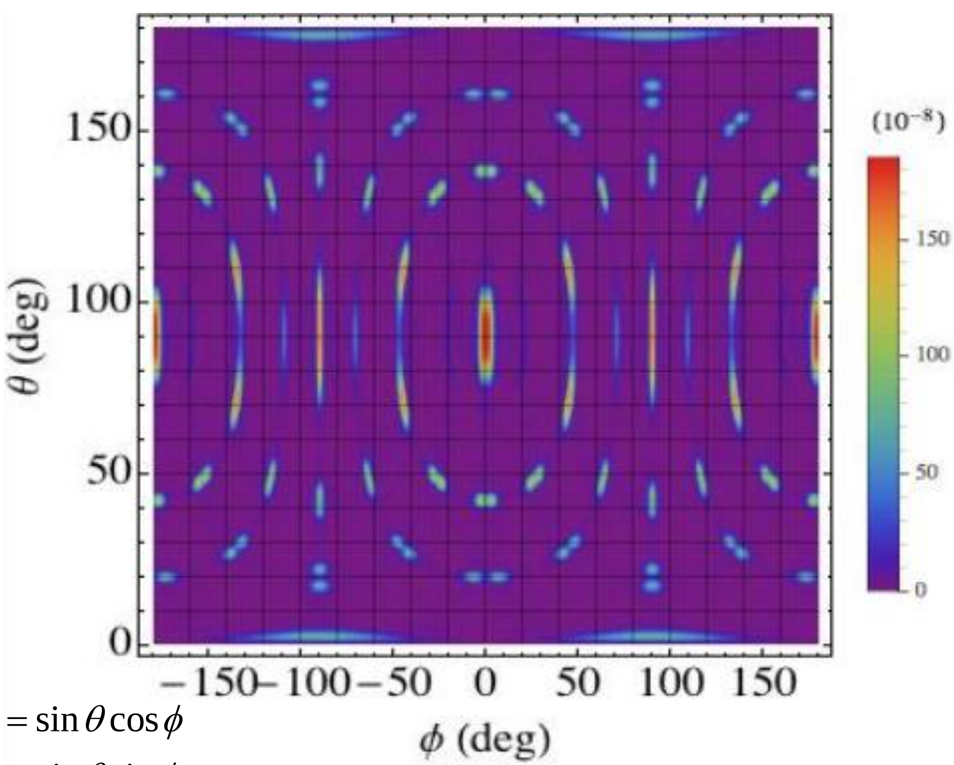
$\gamma = 55$

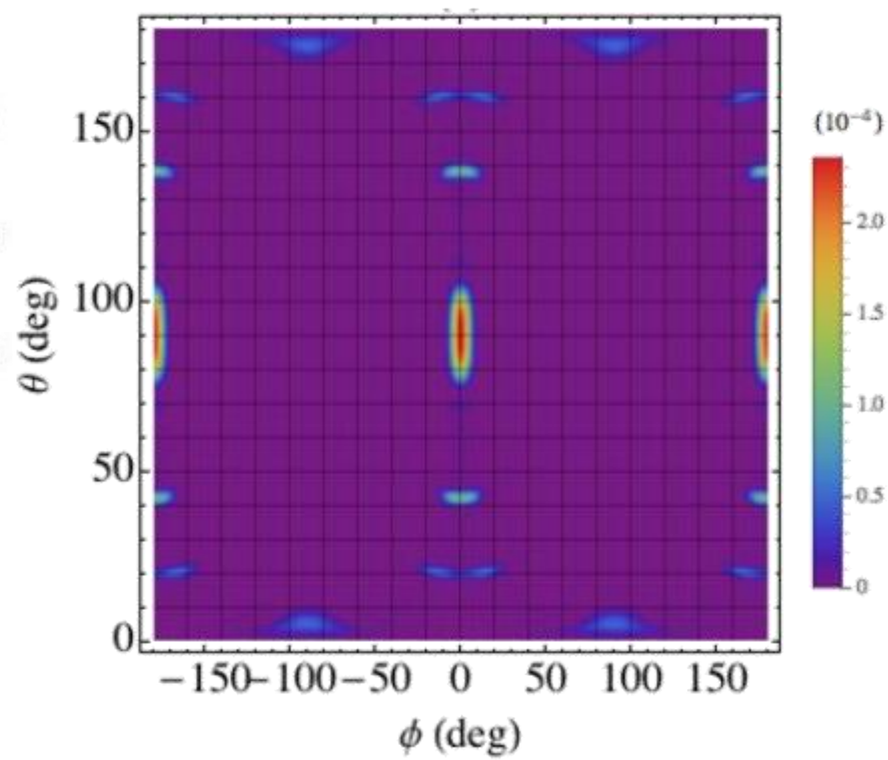
$\varepsilon = 2$

$h = 0.3\,mm$

$\lambda = 10R = 0.1\,mm$







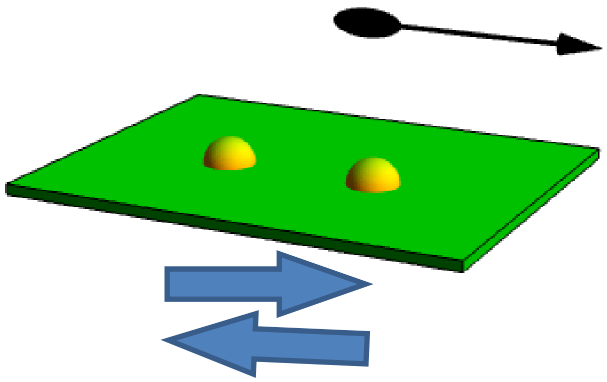
$n_x = \sin\theta\cos\phi$

$n_y = \sin\theta\sin\phi$

$n_z = \cos\theta$

Дополнительные пики

Эффекты локального поля



$$E_j^{loc}(\mathbf{q},\omega)$$

Фактор усиления $\sim 10^3$!

- M.I. Ryazanov, A.A. Tishchenko, Clausius-Mossotti-Type relation for planar monolayers, JETP **103** (2006)
- M.I. Ryazanov, M.N. Strikhanov, A.A. Tishchenko, Local field effect in diffraction radiation from a periodical system of dielectric spheres, NIM B **266** (2008)

Эффекты локального поля

$$E_i^{mic}(\mathbf{r},\omega) = E_i^0(\mathbf{r},\omega) + \widehat{A}_{ij} E_j^{mic}(\mathbf{r},\omega)$$

$$E_j^{mic}(\mathbf{r},\omega)\Big|_{\mathbf{r}=\mathbf{R}_a} \simeq E_j^{loc}(\mathbf{r},\omega)$$

$$E_i^{mic}(\mathbf{r},\omega) = E_i^0(\mathbf{r},\omega) + \widehat{A}_{ij} E_j^{loc}(\mathbf{r},\omega)$$

$$E_j^{loc}(\mathbf{q},\omega)\ ?$$

усреднение

По всем координатам N частиц

$$E_i(\mathbf{r},\omega) = \int d^3R_b\, g(\mathbf{R}_b) E_i^{mic}(\mathbf{r}-\mathbf{R}_b,\omega)$$

$$E_i = E_i^0 + \widehat{B}_{ij} E_j^{loc}$$

По координатам всех остальных частиц относительно данной

$$E_i^{loc}(\mathbf{R}_a,\omega) = \int \prod_{s=1}^{N-1} d^3R_{an_s}\, w(\mathbf{R}_{an_s}) E_i^{mic}(\mathbf{R}_a;\mathbf{R}_n,\omega)$$

$$w(\mathbf{R}_{ab}) = \frac{1}{S}\delta(Z_a - Z_{ab})\left[1 - f(X_{ab},Y_{ab})\right]$$

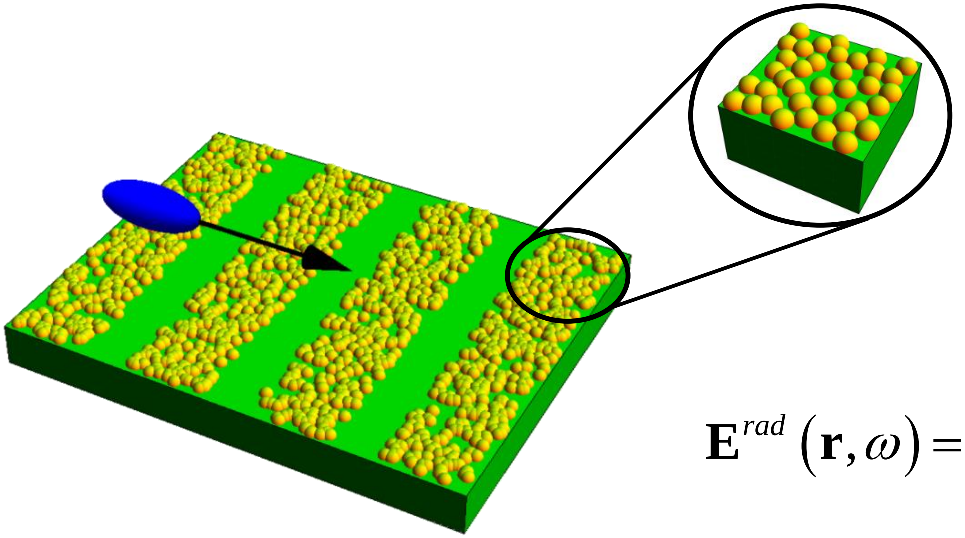
$$\mathbf{R}_n = (\mathbf{R}_{n_1},\ldots,\mathbf{R}_{n_{N-1}})$$

$$E_i^{loc} = E_i^0 + \widehat{C}_{ij} E_j^{loc}$$

$$E_i^{loc} = \hat{f}(E_i^0)$$

$$E_j^{rad} = \widehat{F}_{js}\left(E_s^0(\mathbf{q},\omega)\right)$$

1D фотонный кристалл



$$E_i^{loc}(\mathbf{r},\omega) = A_{ij}(\omega)E_j^0(\mathbf{r},\omega)$$

$$A_{ij}(\omega) = \frac{\delta_{ij} - e_i e_j}{1 - \pi n_1 \xi \alpha_\parallel(\omega)} + \frac{e_i e_j}{1 + 2\pi n_1 \xi \alpha_\perp(\omega)}$$

$$\mathbf{E}^{rad}(\mathbf{r},\omega) = \frac{e^{ikr}}{r} e^{-h\rho} \frac{\sin(\varphi L/2)}{\varphi} \sum_{s=0}^{N-1} e^{i\varphi s d} \frac{2ien_1}{v_x \rho} [\mathbf{k},[\mathbf{k},\mathbf{B}]]$$

n_1 поверхностная плотность частиц

$\mathbf{e}$ нормаль к поверхности

$$\frac{d^2W(\mathbf{n},\omega)}{d\omega d\Omega} = \frac{e^2}{c} e^{-2h\rho} \frac{\sin^2(L\varphi/2)}{\varphi^2} \frac{\sin^2(d\varphi N/2)}{\sin^2(d\varphi/2)} \frac{4n_1^2}{\beta_x^2 \rho^2} \frac{\omega^2}{c^2} \left| \frac{[\mathbf{k},\mathbf{k}_\perp - \mathbf{v}\omega/c^2]}{\alpha_\parallel^{-1}(\omega) - \pi n_1 \xi} + \frac{[\mathbf{k},\mathbf{e}]\rho}{\alpha_\perp^{-1}(\omega) + 2\pi n_1 \xi} \right|^2$$

$$\mathbf{B} = \frac{\mathbf{k}_\perp - \mathbf{v}\omega/c^2}{\alpha_\parallel^{-1}(\omega) - \pi n_1 \xi} + \frac{\mathbf{e}\rho}{\alpha_\perp^{-1}(\omega) + 2\pi n_1 \xi}$$

$$\xi = \int d^2\mathbf{p}_\parallel f(\mathbf{p}_\parallel) p_\parallel$$

*M.I. Ryazanov, A.A. Tishchenko, JETP **103** (2006) 539*

1D фотонный кристалл

$$\frac{d^2W(\mathbf{n},\omega)}{d\omega d\Omega}=\frac{e^2}{c}e^{-2h\rho}\frac{\sin^2(L\varphi/2)}{\varphi^2}\frac{\sin^2(d\varphi N/2)}{\sin^2(d\varphi/2)}\frac{4n_1^2}{\beta_x^2\rho^2}\frac{\omega^2}{c^2}\left|\frac{\left[\mathbf{k},\mathbf{k}_\perp-\mathbf{v}\omega/c^2\right]}{\alpha_\parallel^{-1}(\omega)-\pi n_1\xi}+\frac{[\mathbf{k},\mathbf{e}]\rho}{\alpha_\perp^{-1}(\omega)+2\pi n_1\xi}\right|^2$$

$$\rho_{\min}=\frac{\omega}{c\beta\gamma}$$

$$n_y=\beta_y/\beta^2$$

Конический эффект

$$\varphi=(1-n_x\beta_x-n_y\beta_y)\omega/(c\beta_x)$$

$$\frac{d}{\lambda}\frac{1}{\beta_x}(1-n_x\beta_x-n_y\beta_y)=m,\quad m=1,2....$$

Излучение Смита-Парселла

$$\begin{cases}\mathrm{Re}\left\{\alpha_\perp^{-1}(\omega_1)\right\}=-2\pi n\xi\\ \mathrm{Re}\left\{\alpha_\parallel^{-1}(\omega_2)\right\}=\pi n\xi\end{cases}$$

Эффекты локального поля

$$\alpha(\omega)=\frac{l^3}{8}\frac{\varepsilon(\omega)-1}{\varepsilon(\omega)+2}$$