Coherent radiation and channeling study

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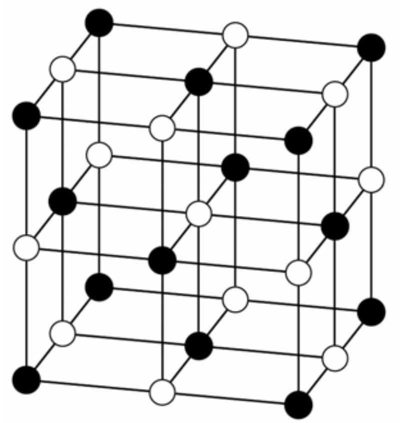
- X-Rays — particles or waves
- coherent processes in crystals at high energies (how it had started)
- channeling and coherent scattering in thin and ultrathin crystals
- geometrical optics method (ray optics)
- σ_{tot} for scattering in transition region of thickness to the channeling regime
- coherent radiation in thin crystals and channeling
- coherent radiation at beam-beam collisions
- …….

Ischia, September 22-29, 2018

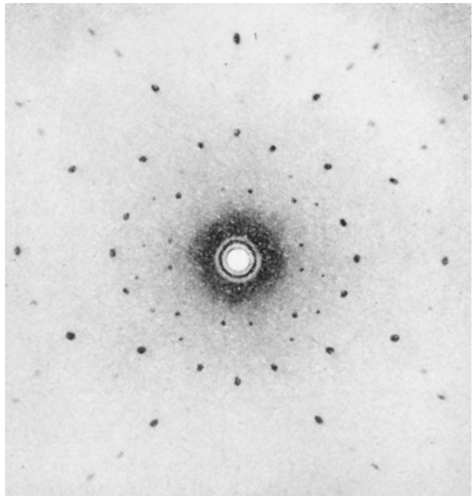
Interaction of particles and waves with crystals

Problem: *the nature of the Röntgen rays – particles or waves*

M. von Laue



NaCl



From: John C. Kendrew. The Thread of Life. London, G. Bell&Sons LTD, 1964.

$$\lambdabar = \frac{\hbar}{p} \geq a$$

H. Bethe, Ann. Phys. 4 (1928) 55,87.
F. Bloch, Z. Phys Bd 81 (1933) 363.

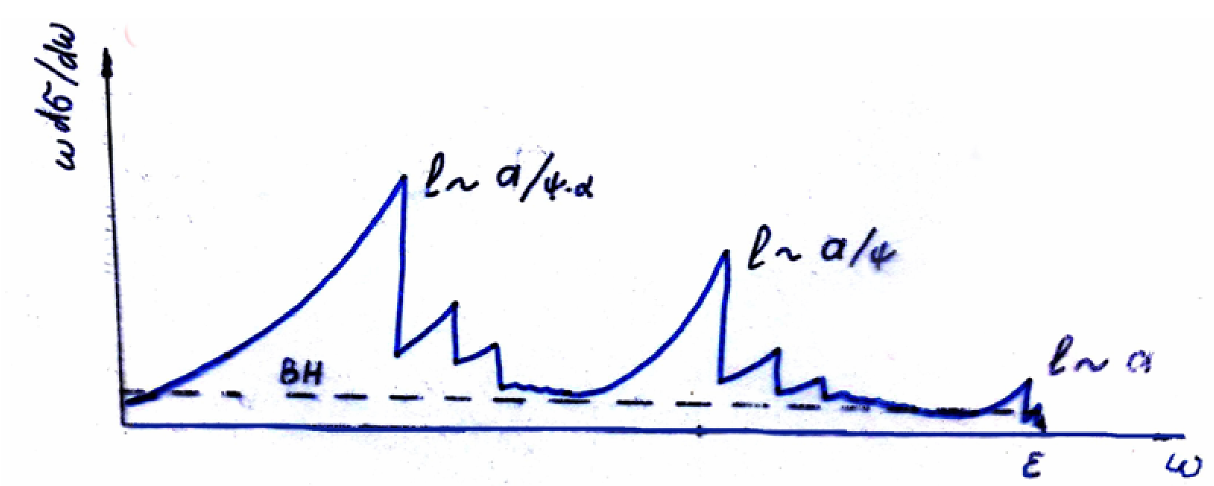
Coherent Bremsstrahlung in Born Approximation

(Ferretti 1950, Ter-Mikaelian 1952, Überall 1960)

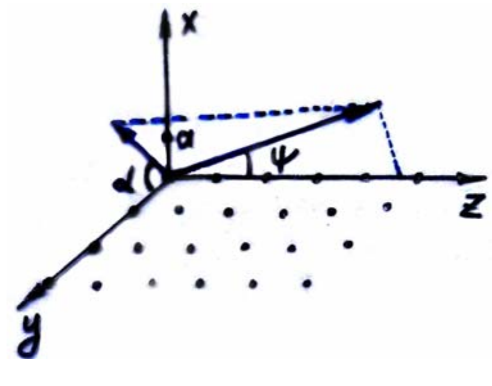
$$d\sigma = d\sigma_{ncoh} + \sigma_{coh}$$

$$\omega \frac{d\sigma_{coh}}{d\omega} = \frac{2e^2\delta}{m^2\Delta} \frac{\varepsilon'}{\varepsilon} \sum_{\vec{g}} \frac{g_\perp^2}{g_\parallel^2} \left[1 + \frac{\omega^2}{2\varepsilon\varepsilon'} - 2\frac{\delta}{g_\parallel}\left(1 - \frac{\delta}{g_\parallel}\right)\right] |U_g|^2 e^{-g^2\overline{u^2}}$$

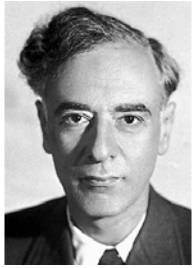
$$q_\parallel \geq \delta = \omega m^2 / 2\varepsilon\varepsilon', \qquad g_\parallel = g_z + \psi\left(g_y \cos\alpha + g_x \sin\alpha\right) \geq \delta$$



Akhiezer, Shul'ga Sov.Phys.Usp. 1982 v.25 p.54



Discussion: E.Feinberg and M.Ter- Mikaelian with L.Landau and I. Pomeranchuk (1952)

T. - M. – Interference radiation by ultrarelativistic electrons in crystals.

Landau – That is impossible because the interference effect is possible only for

$$\lambda = \frac{\hbar}{p} \geq a \quad , \qquad \textbf{but not for} \quad \lambda << a$$

The discussion was stopped

Later L. Landau had agreed that such an effect exists but it should go in a different way (*from recollections by A.I. Akhiezer*)

Coherent length

In the theory of high energy electrons' radiation besides the length $\lambda \sim \hbar / p$ there exists another length responsible for the radiation,

$$l_c = \frac{2\varepsilon\varepsilon'}{m^2\omega}$$

$$\begin{cases} \varepsilon = \varepsilon' + \omega \\ \mathbf{p} = \mathbf{p}' + \mathbf{k} + \mathbf{q} \end{cases}$$

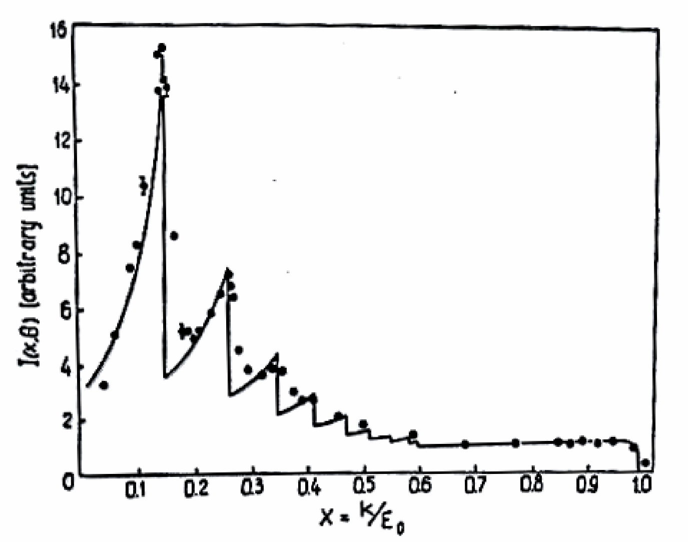
$$q_{\min} = \frac{m^2\omega}{2\varepsilon\varepsilon'}$$

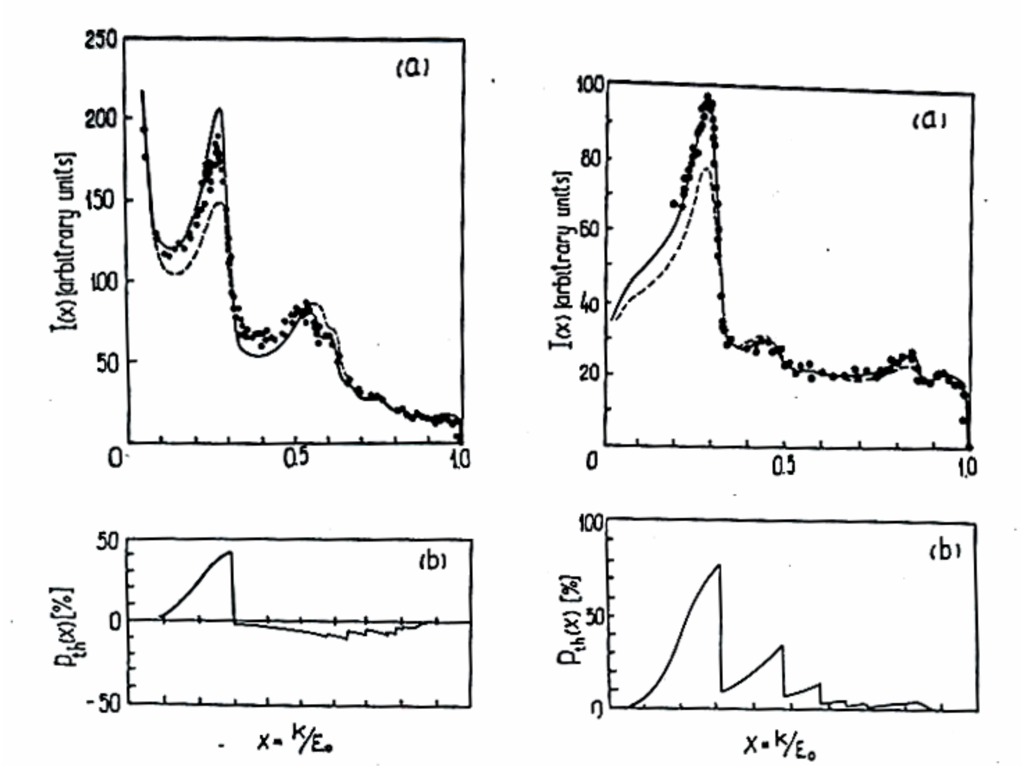
Interpretations of l_c

• Ter-Mikaelian (1952):	It is based on the first Born Approximation
• Landau, Pomeranchuk (1953):	It is based on classical electrodynamics
• Frish, Olsen (1959), Akhiezer, Shul'ga (1982)	It is based on the behavior of the wave packets
• Feinberg (1966) Akhiezer, Shul'ga, Fomin (1982)	Development of the process of radiation in space and time

Experiment $\varepsilon \sim 1 - 5$ GeV (1962 - 1965)

Frascati, DESY, Kharkov, Protvino, Tomsk, Yerevan, SLAC, …





Frascati
ε=1 GeV, θ =4,6 mrad

DESY
ε=4,8 GeV, θ =3,4 mrad

Higher Born approximation in the CB theory

A. Akhiezer, P. Fomin, N. Shul'ga, V. Boldyshev (1970-1975)

$$\frac{Ze^2}{\hbar c} \ll 1 \quad \rightarrow \quad N_{coh}\frac{Ze^2}{\hbar c} \sim \frac{R}{\psi a}\frac{Ze^2}{\hbar c} \ll 1$$

Quickly destroys for $\psi \rightarrow 0$

$$N_{coh} \sim \min\left(\frac{l_{coh}}{a}, \quad \frac{R}{\psi_a}\right)$$

PARADOX

This condition was practically not fulfilled at experiments (1960-1970) on verification of F – T – Ü theoretical results. But the experiments were in good agreement with this theory !!!

Why ???

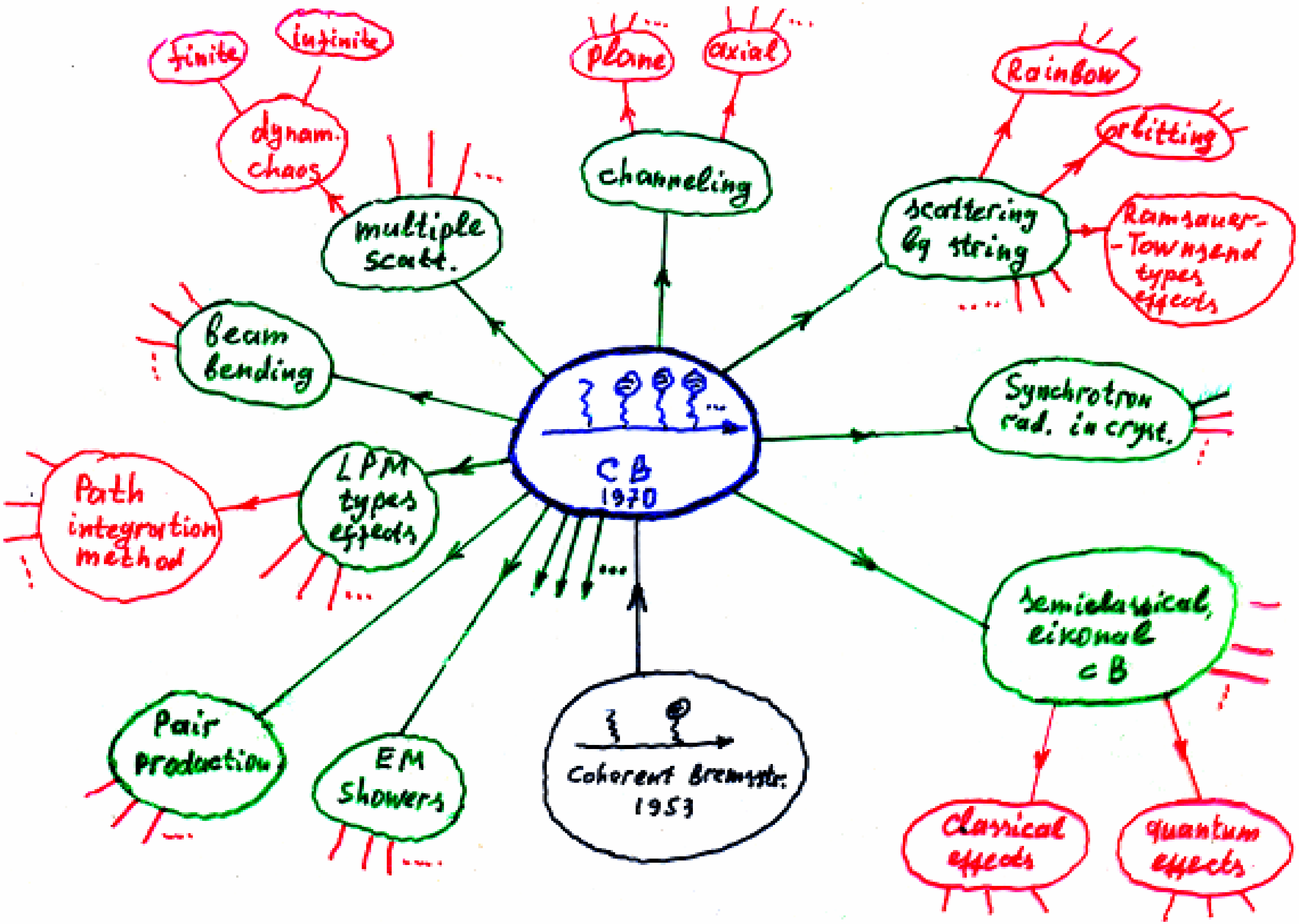
New area of research

The interaction of high-energy particles with matter in conditions of effectively strong interaction of the particle with atoms of media (semiclassical, classical approximations)

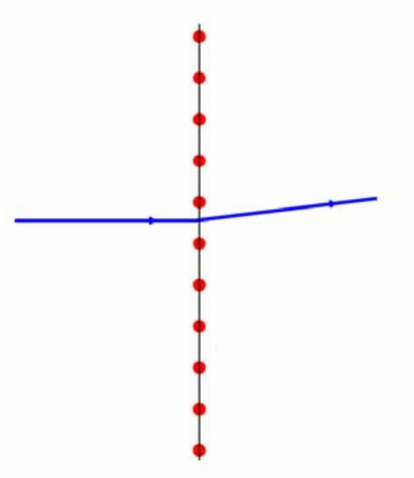
$$N_c \frac{Ze^2}{\hbar c} >> 1$$

- Radiation is determined by the classical trajectory !!!
- It is necessary to know the types of particles' motion in crystal
- Same methods for description of CB and LPM effects !!!

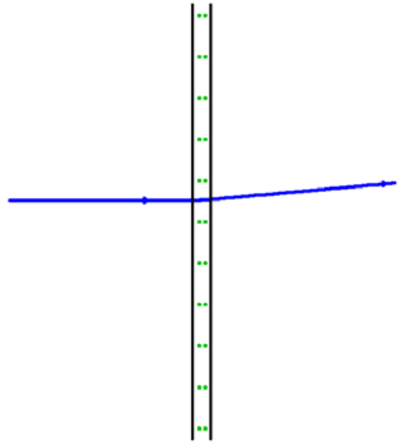
Problems generated by the theory of coherent radiation in crystals (state for 1995)



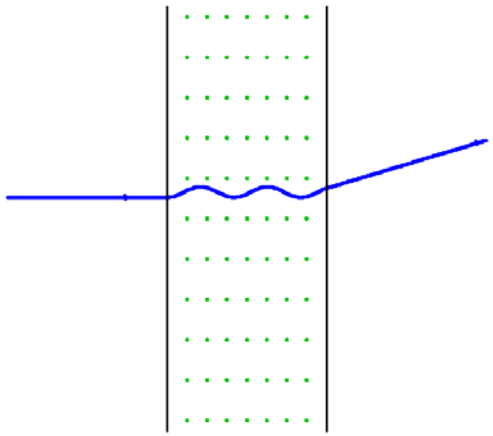
Quantum and Classical effects at high energy electrons scattering in ultrathin crystals



Graphene



Ultrathin crystal coherent effects



Channeling

Classical Mechanics

$$\ddot{\boldsymbol{\rho}} = -\frac{1}{\varepsilon}\nabla U(x,y)$$

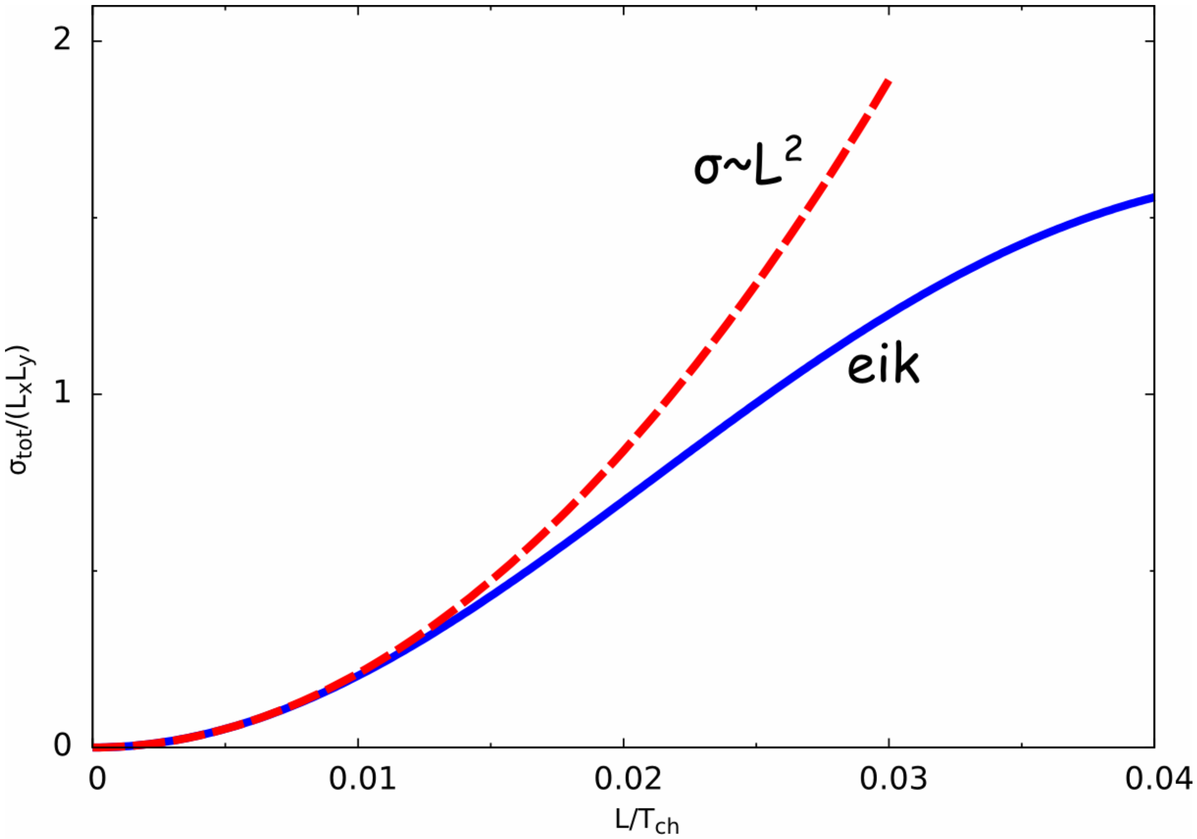
$$d\sigma_{cl}(\vartheta) = \sum_n d^2 b_n(\vartheta) = \sum_n \frac{1}{\left|\partial\vartheta/d\mathbf{b}\right|_n}\Bigg|_{\mathbf{b}=\mathbf{b}_n(\vartheta)}$$

N.F. Shul'ga, S.N. Shulga. Phys. Let. B 769 (2017) 141,
S.N. Shulga, N.F. Shul'ga, S. Barsuk, I. Chaikovska, R. Chehab. NIM B 402 (2017) 16.

Scattering in the Born and Eikonal approximations

$$a^{eik}(\mathbf{q}_\perp) = \frac{i\,p}{2\pi\hbar}\int d^2\rho\, e^{i\mathbf{q}\boldsymbol{\rho}/\hbar}\left(1 - e^{\frac{i}{\hbar}\chi_0(\boldsymbol{\rho},L)}\right), \qquad \chi_0 = \frac{4U_0}{v}\frac{b^2}{a^2}L$$

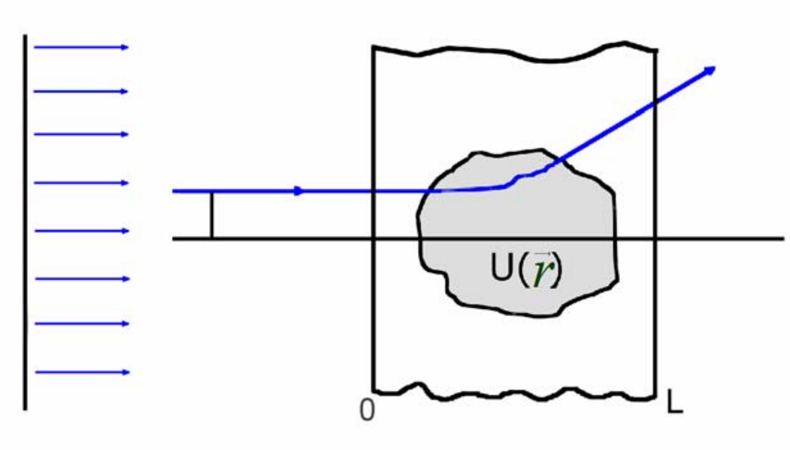
$$\frac{d\sigma_q}{do} = \left|a(\vartheta)\right|^2 \qquad \sigma_{tot} = \frac{4\pi\hbar}{p}\,\mathrm{Im}\,a(\vartheta)\Big|_{\vartheta=0}$$



N. Kalashnikov, E. Koptelov, M. Ryazanov. JETP Lett. 15 (1972) 82.
A. Akhiezer N. Shul'ga. Quantum Electrodynamics of High Energies in Matter, G&B, 1996.

Gauss Theorem in Quantum Scattering Theory

N. Bondarenco, N. Shul'ga Phys. Lett. B 427 (1998) 114



$$\psi = \varphi(\mathbf{r}) e^{i\mathbf{pr}} u_p$$

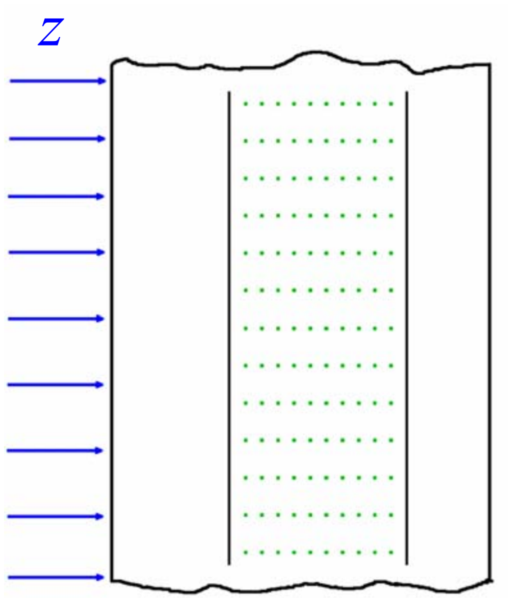
$$\mathbf{q} = \mathbf{p} - \mathbf{p}'$$

$$a(\vartheta) = -\frac{1}{4\pi}\int_V d^3r\, e^{-i\mathbf{p}'\mathbf{r}}\,\overline{u'}\gamma_0 U(\mathbf{r})\psi(\mathbf{r}) = -\frac{1}{4\pi}\int_V d^3r\ \mathrm{div}\left[\overline{u'}\boldsymbol{\gamma}\psi(\mathbf{r})e^{-i\mathbf{p}'\mathbf{r}}\right] =$$

$$= -\frac{i}{4\pi}\oint d\mathbf{S}\,\overline{u'}\boldsymbol{\gamma}\psi(\mathbf{r})e^{-i\mathbf{p}'\mathbf{r}} = -\frac{i}{4\pi\hbar}\oint d^2\rho\, e^{i\mathbf{qr}}\,\overline{u'}\gamma_z\psi(\boldsymbol{\rho}, z)e^{-i\mathbf{p}'\mathbf{r}}\Big|_{z=0}^{z=L}$$

$$\frac{d\sigma_q}{do} = |a(\vartheta)|^2 \qquad \sigma_{tot} = \frac{4\pi\hbar}{p}\mathrm{Im}\, a(\vartheta)\Big|_{\vartheta=0}$$

Operator method



$\psi_{in} = e^{ipz}$ $\psi = e^{i\mathbf{p}\mathbf{r}}\varphi(\mathbf{r})$

$$\psi = e^{i(pz-\varepsilon t)}\varphi(\boldsymbol{\rho}, z)$$

$$i\hbar v\partial_z\varphi(\boldsymbol{\rho}, z) = \left(\frac{\hat{\mathbf{p}}_\perp^{\ 2}}{2\varepsilon} + U(\boldsymbol{\rho})\right)\varphi(\boldsymbol{\rho}, z) =$$

$$= \left(\hat{H}_0 + U(\boldsymbol{\rho})\right)\varphi$$

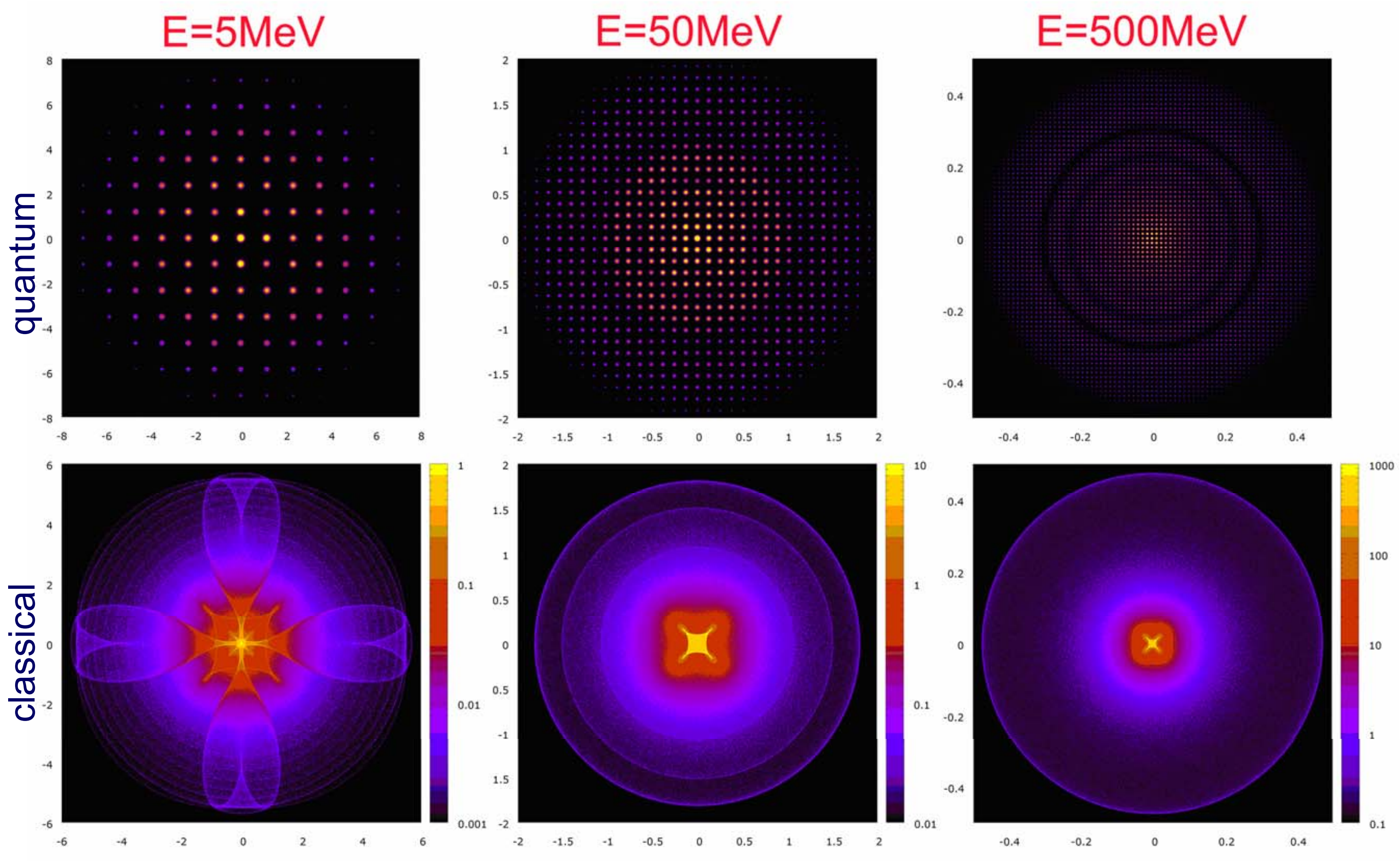
wave function $$\varphi(\boldsymbol{\rho}, z+\Delta z) = e^{-\frac{i}{\hbar}\left(\hat{H}_0 + U(\boldsymbol{\rho})\right)\Delta z}\varphi(\boldsymbol{\rho}, z)$$

$+$ iteration procedure

M. Feit, J. Fleck et al., J. Comput. Phys. 47 (1982) 412,
S. Dabagov, L. Ognev, NIM B 30 (1988) 185,
N. Shul’ga, S. Shul’ga Phys. Lett. B 769 (2017) 141.

Quantum and classical angular distributions of electrons in 1000Å Si <100>

N. Shul'ga, S. Shul'ga Phys. Lett. B 769 (2017) 141



Comparison with the Electron Microscopy

Electron microscopy | **Proposed theory**

Energy:

$\varepsilon \leq$ several MeV | $\varepsilon \geq$ MeV

Methods of description:

Two-wave, …many-wave formalisms

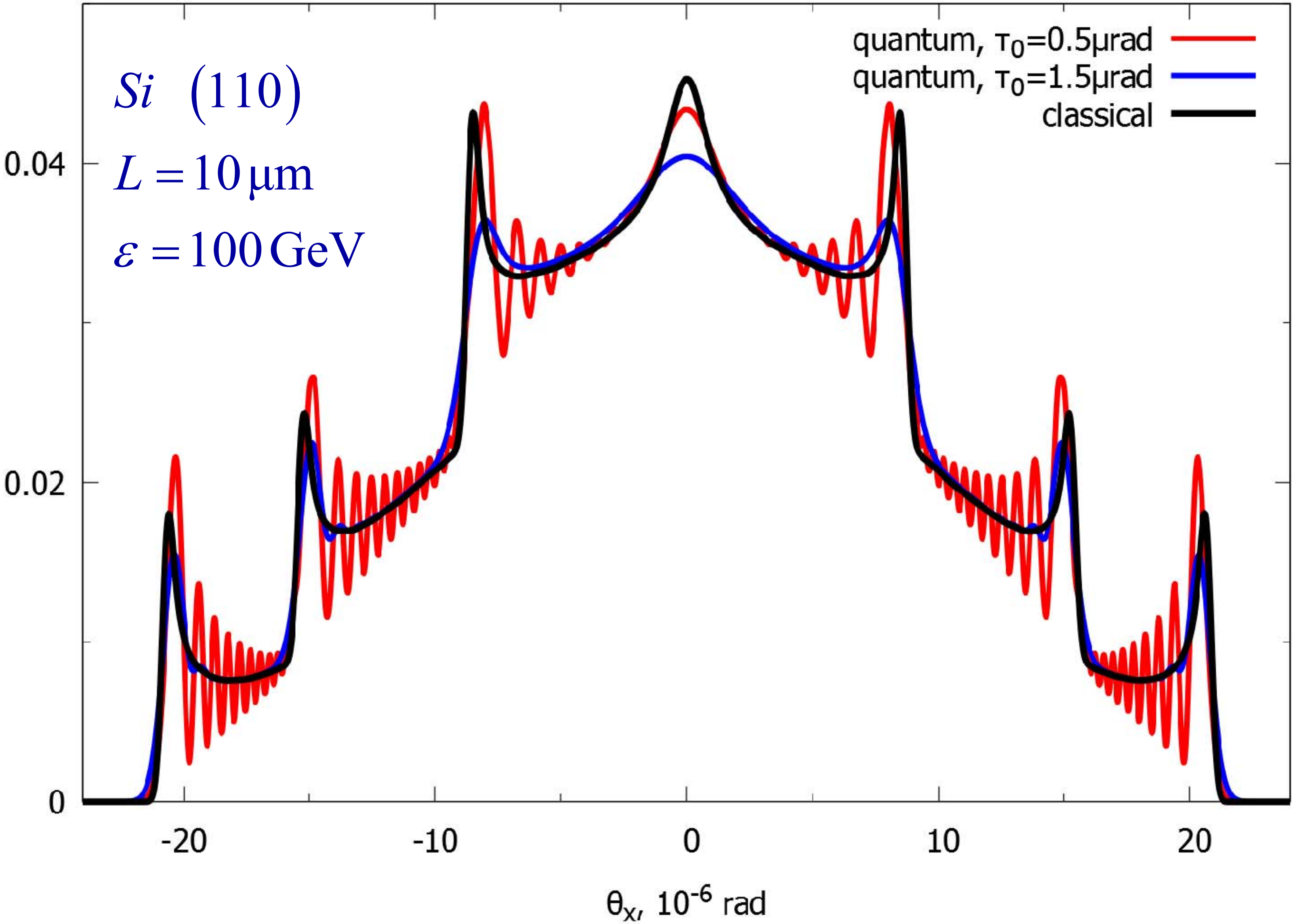
P.B. Hirsch et al. Electron Microscopy of thin crystals. Butterworths, 1965.
Y.-H. Ohtsuki. Charged Beam Interaction with Solids. T&F., 1983.

Operator Method: (direct wave function calculation)

M. Feit et al. J. Comp. Phys. 47 (1982) 412.

- The method can be also implemented to the electron microscopy
- Possibility of passage to the classical mechanics
- Possibility of description of scattering of other particles (protons, …), radiation and other effects

Rainbow scattering in the field of ultrathin Si (110) crystal planes



S. Shulga, N. Shul'ga, S. Barsuk, I. Chaikovska, R. Chehab NIM B 402 (2017) 16

Semiclassical approximation for the wave function

$$\left[\left(\varepsilon-U\right)^2-\left(i\hbar\nabla\right)^2-m^2+i\hbar\gamma_0\boldsymbol{\gamma}\nabla U\right]\psi=0$$

$$\psi^{WKB}\left(\boldsymbol{\rho},z\right)=\sqrt{f\left(\boldsymbol{\rho},z\right)}\;\exp\left(\frac{i}{\hbar}S\left(\boldsymbol{\rho},z\right)\right),\qquad S=\mathbf{pr}+\chi\left(\mathbf{r}\right)$$

$$\begin{cases}\left(\varepsilon-U\left(\mathbf{r}\right)\right)^2=\left(\nabla S\right)^2+m^2\\ \nabla S\,\nabla f+\left(\nabla^2 S\right)f=0\end{cases}$$

$$-v\,\partial_z\chi=U_c\left(\boldsymbol{\rho}\right)+\frac{1}{2\varepsilon}\left(\nabla_\perp\chi\left(\boldsymbol{\rho},z\right)\right)^2$$

Geometrical optics (ray optics)

$$\psi\left(\boldsymbol{\rho}\left(\mathbf{b},z\right),z\right)=\frac{1}{\sqrt{|D|}}\exp\left\{ipz+i\chi\left(\boldsymbol{\rho}\left(\mathbf{b},z\right),z\right)-i\frac{\pi}{2}\mu\right\}$$

(μ – Maslov-Morse index)

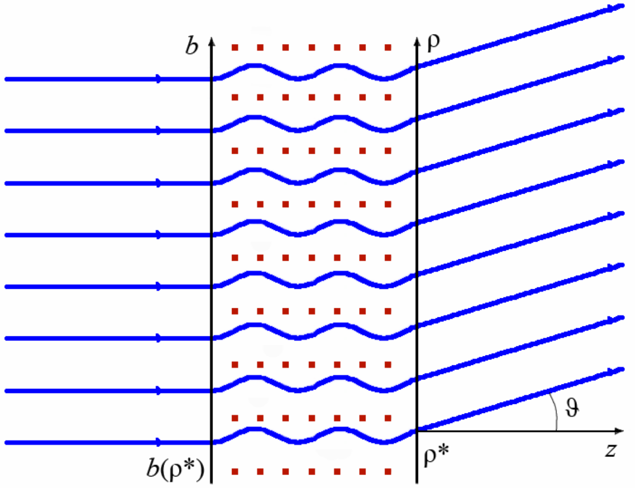
$$\frac{d^2\rho}{dz^2}=-\frac{c^2}{\varepsilon}\frac{\partial}{\partial\boldsymbol{\rho}}U\left(\boldsymbol{\rho},z\right)\qquad\rightarrow\qquad\boldsymbol{\rho}=\boldsymbol{\rho}\left(\mathbf{b},z\right)\qquad\mathbf{b}=\boldsymbol{\rho}\Big|_{z=0}$$

$$\chi\left(\boldsymbol{\rho}\left(\mathbf{b},z\right),z\right)=-\frac{1}{v}\int_0^z dz'\left[2U_c\boldsymbol{\rho}\left(\mathbf{b},z'\right)-\varepsilon_\perp\right]$$

$$D=\frac{\partial\left(x,y,z\right)}{\partial\left(b_x,b_y,\tau\right)}=\det\begin{pmatrix}\partial_{b_x}x & \partial_{b_y}x & \partial_\tau x\\ \partial_{b_x}y & \partial_{b_y}y & \partial_\tau y\\ \partial_{b_x}z & \partial_{b_y}z & \partial_\tau z\end{pmatrix}$$

Y. Kravtsov, Y. Orlov, Geometrical optics of in homogeneous media, Springer-Verlag, Berlin, 2011.
V. Arnold, Mathematical methods in classical mechanics, Springer-Verlag, NY, 1989.
V. P. Maslov, M. V. Fedoriuk Semi-classical approximation in quantum mechanics, D.R., Holland, 1981.

Wave function in geometrical optics approximation for plane channeled positrons



$$\ddot{x} = -\frac{1}{\varepsilon}\frac{\partial}{\partial x}U_c(x)$$

$$U_c(x) = U_0\frac{x^2}{(a/2)^2}, \qquad |x| \le a/2$$

$$\ddot{x} + \Omega^2 x = 0 \qquad \Omega = \sqrt{\frac{8U_0}{\varepsilon a^2}} = \frac{2\theta_p}{a}$$

$$x = b\cos\Omega z$$

$$\chi(x(b,z),z) = -\frac{1}{v}\frac{2b^2}{a^2}\frac{U_0}{\Omega}\sin 2\Omega z$$

$$D = \cos\Omega z$$

$$\psi(x,z) = \frac{1}{\sqrt{\cos\Omega z}}\exp\left\{ipz - i\frac{2U_0}{\Omega}\frac{b^2}{a^2}\sin 2\Omega z\right\}$$

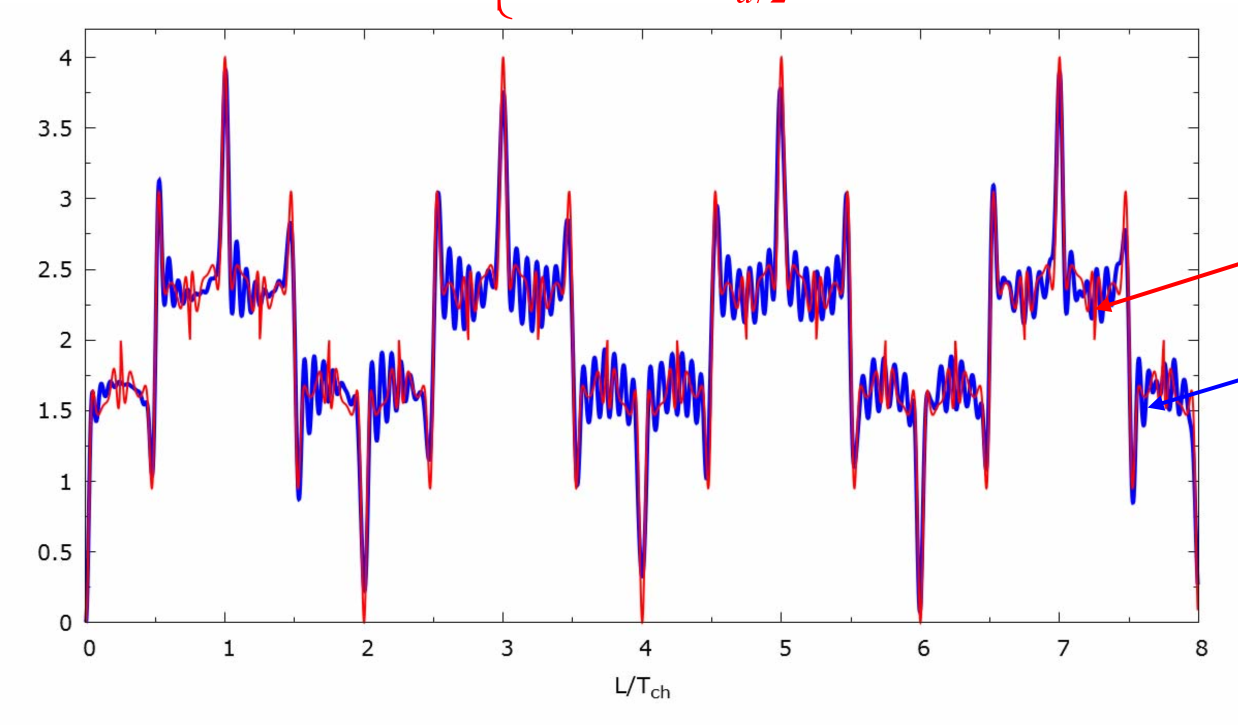
Total scattering cross-section

(*S. Shulga, N. Shul'ga, arxiv:1809:07522*)

$$\sigma_t = \frac{4\pi\hbar}{p} \operatorname{Im} a\Big|_{\theta=0} = 2L_y L_x \left(1 - \frac{1}{a}\operatorname{Re}\int_{x_{\min}}^{x_{\max}} dx(b,L)\,\varphi\big(x(b,L),L\big)\right)$$

$$dx = \left|\frac{dx}{db}\right| db = \left|\cos\Omega L\right| db$$

$$\sigma_t^{WKB} = 2L_y L_x \left\{1 - \frac{1}{a}\operatorname{Re}\int_{-a/2}^{a/2} db\left|\cos\Omega L\right|^{1/2} e^{-\frac{i}{\hbar v}\frac{2U_0}{\Omega}\frac{b^2}{a^2}\sin 2\Omega L - i\frac{\pi}{2}\mu}\right\}$$



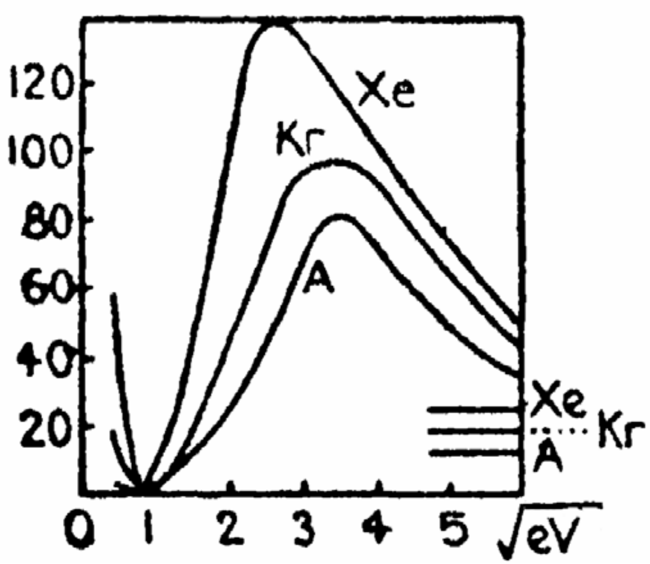
quasiclassical estimation (σ_t^{WKB})

calculated basing upon the opeator method (σ_t)

Ramsauer-Townsend effect

(*the scattering of electrons on Xe, Kr, Ar at ε~0.7 eV***)**

P. Ramsauer, Ann. d. Phys. 64 (1921) 513,
J. Townsend, V. Bailey. Phil. Mag. (1922) 593.



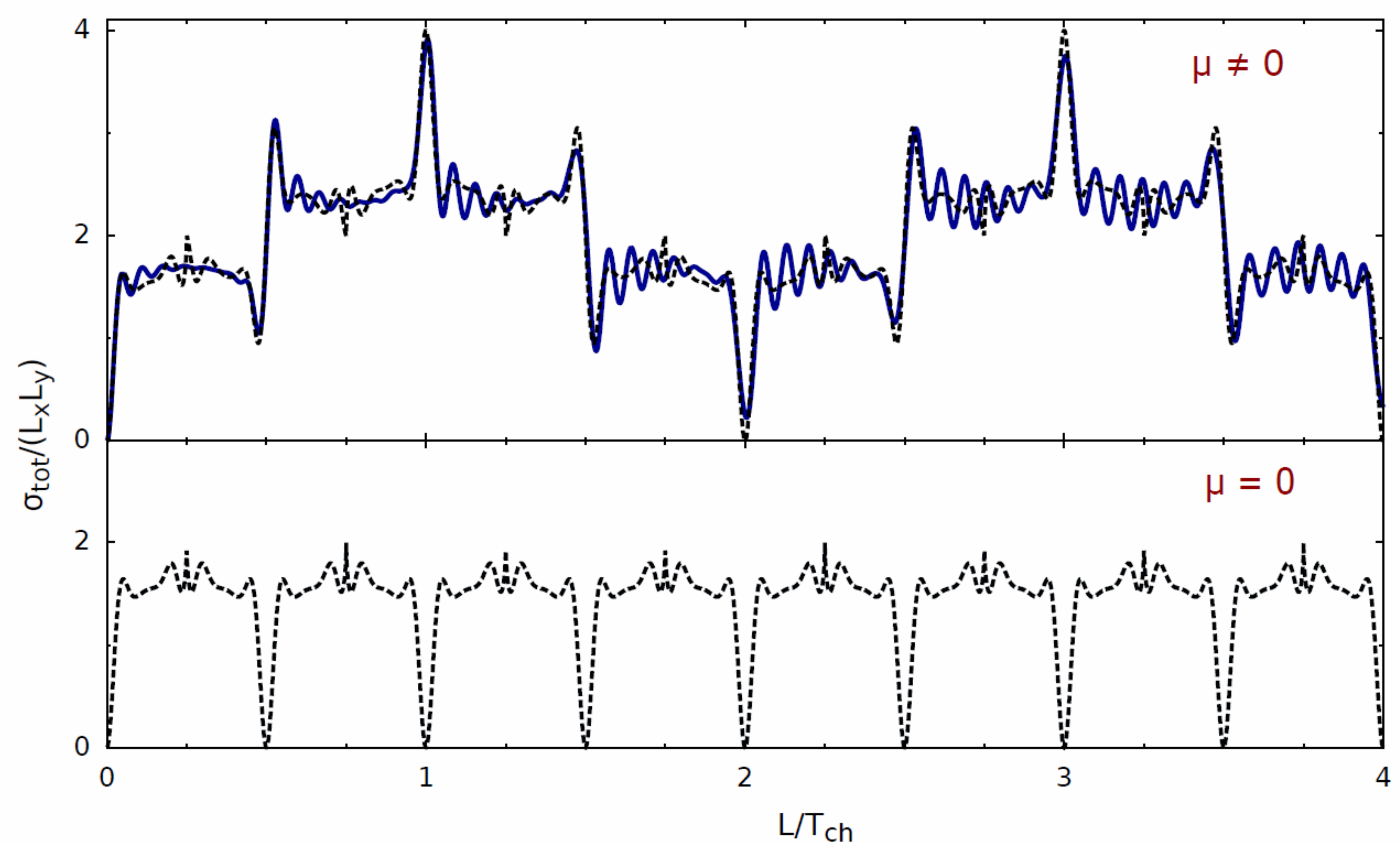
atomic field	—	present
force	—	present
scattering	—	**absent**

From: N.Mott, H.Massey. The theory of atomic collisions (fig.89), Oxford, CP (1965)

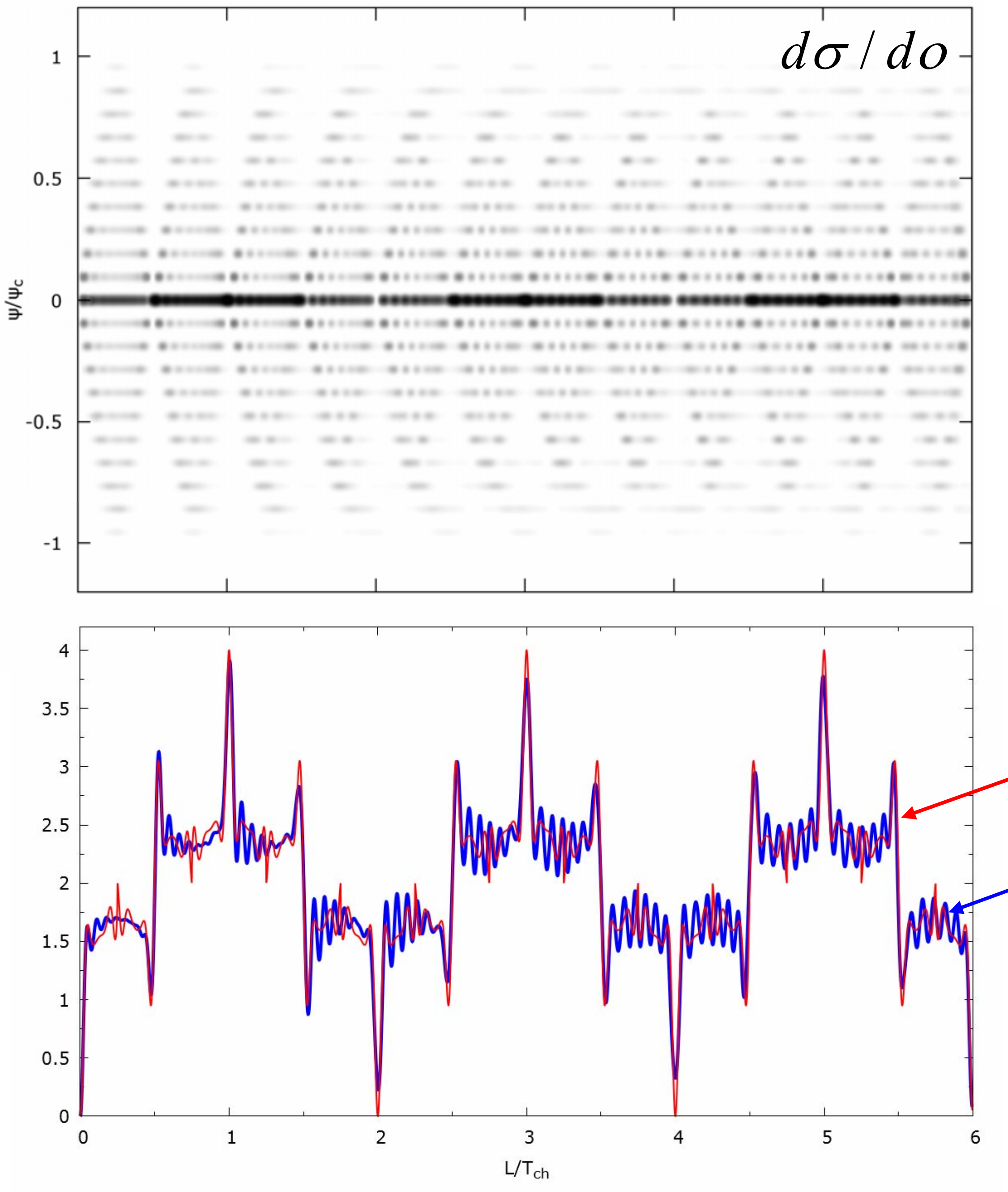
N. Bohr was delighted with this effect, because it is a quantum (interference) phenomenon

Total scattering cross-section and Maslov-Morse index μ

e^+, ε = 100 MeV



Differential and total scattering cross-sections



positrons
ε = 100 MeV

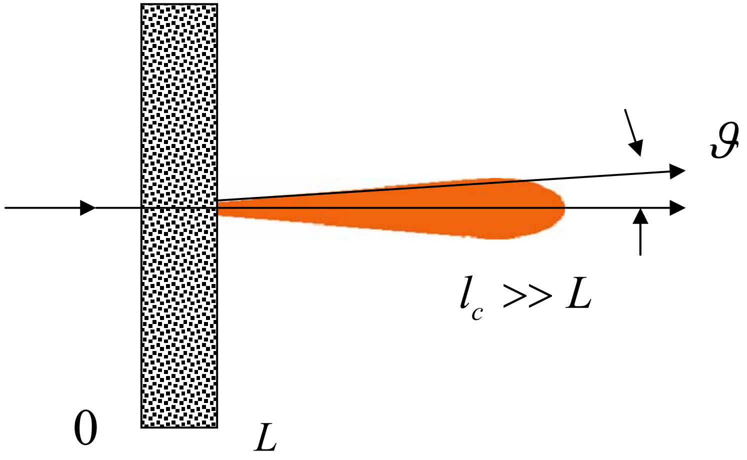
$$U_c = U_0 \left(\frac{x}{d/2} \right)^2$$

quasiclassical estimation

calculated basing upon the operator method

Radiation in thin amorphous target (TSF-effect)

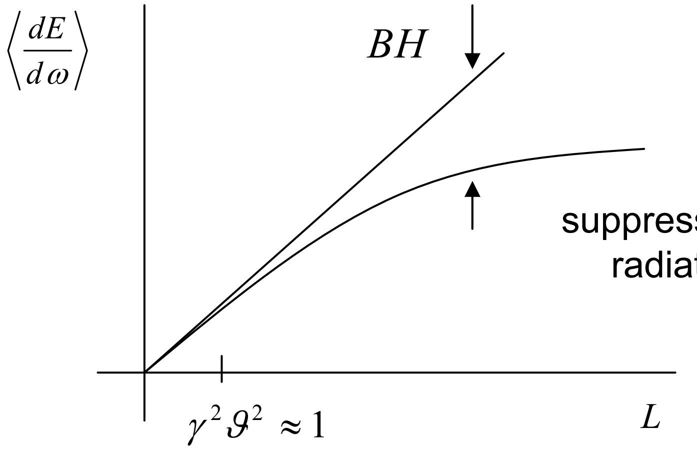
F. Ternovskii, JETF 1960, N. Shul'ga, S. Fomin JETP Lett. 1978, 1996



$$l_c = \frac{2\gamma^2}{\omega} >> L$$

$$\left\langle \frac{dE}{d\omega} \right\rangle = \frac{2e^2}{\pi} \left\langle \left[\frac{2\xi^2+1}{\xi\sqrt{\xi^2+1}} \ln\left(\xi + \sqrt{\xi^2+1}\right) - 1 \right] \right\rangle \approx$$

$$\approx \frac{2e^2}{3\pi} \begin{cases} \gamma^2 \overline{\vartheta^2} \\ 3 \ln \gamma^2 \overline{\vartheta^2} \end{cases} \approx \begin{cases} E'_{BH} \\ < E'_{BH} \end{cases}$$

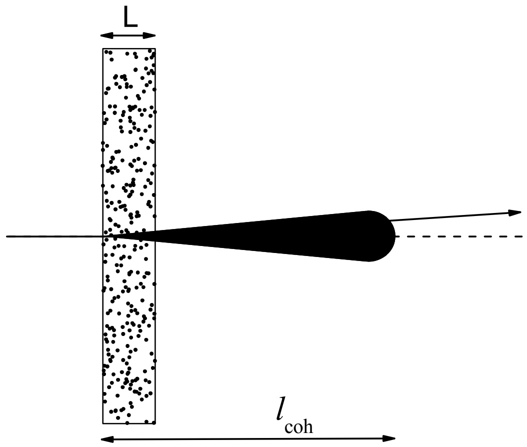


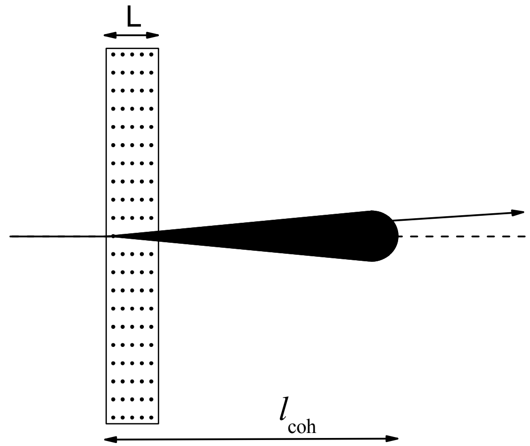
$$\xi = \frac{\gamma\vartheta}{2}$$

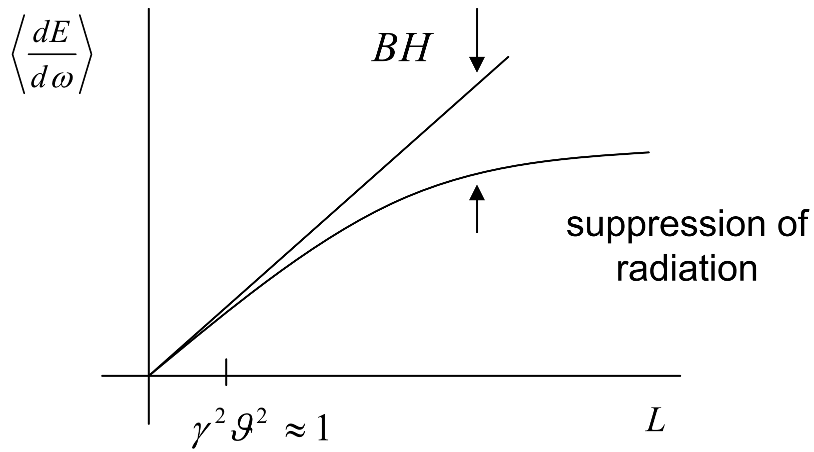
Experimental confirmation at CERN:
H.D.Thomsen et al., Physical Review D 81 (2010) 052003.

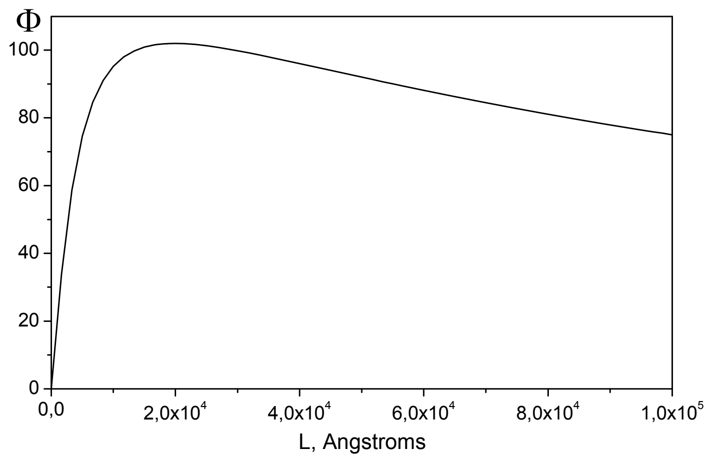
Coherent Radiation in ultrathin crystal (eikonal approx.)

N.Shul'ga, S.Shul'ga, Phys. Lett. A378 (2014) 3074



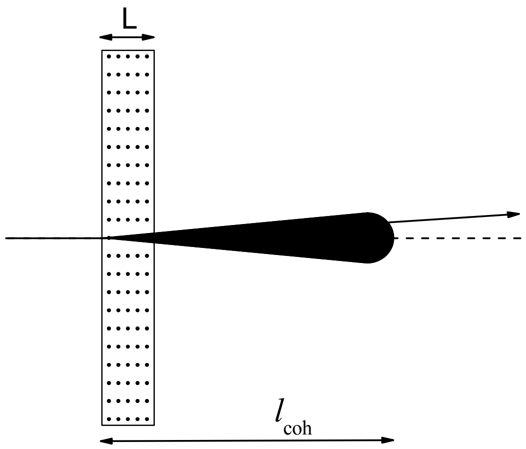


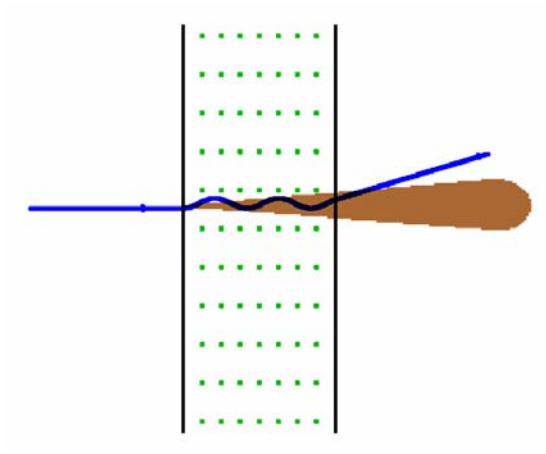


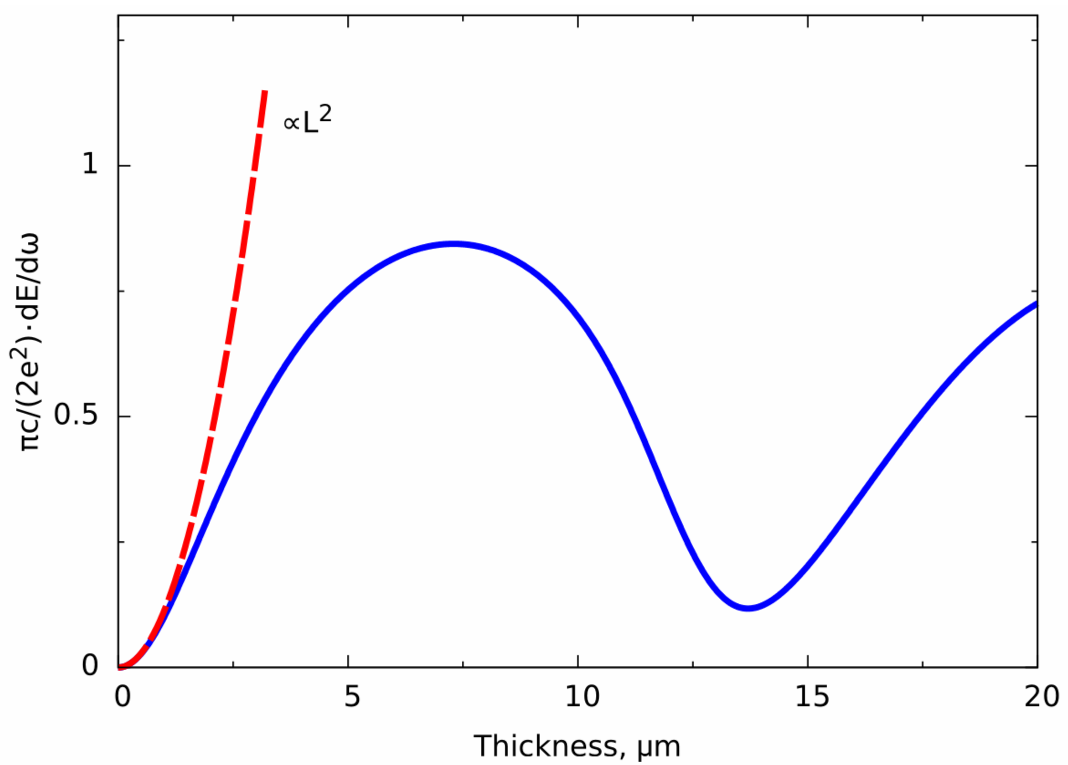


$$\Phi = \frac{d\sigma_c / d\omega}{d\sigma_{BH} / d\omega}$$

Coherent radiation and channeling



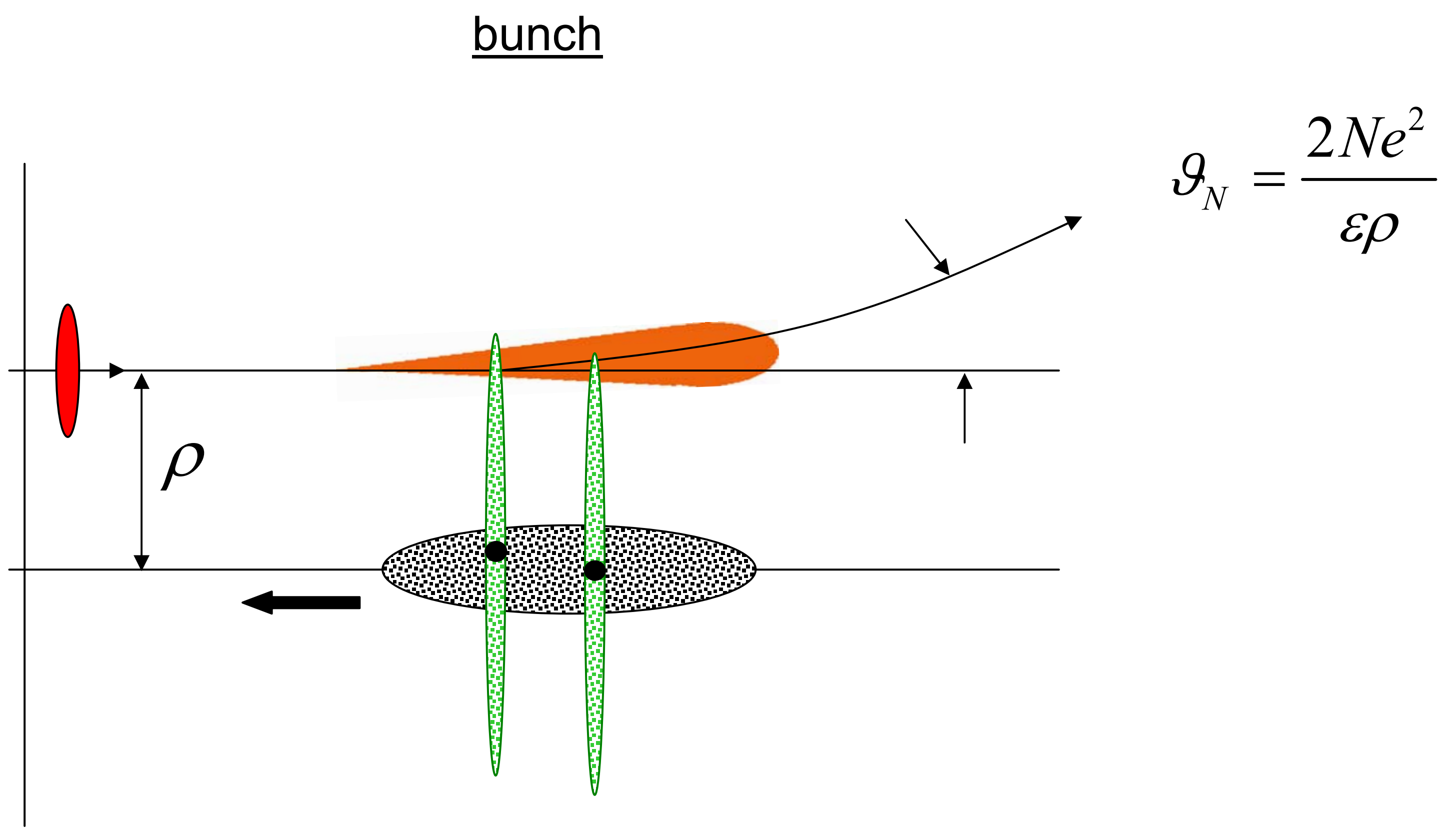




positrons
E=100GeV
Si (110)

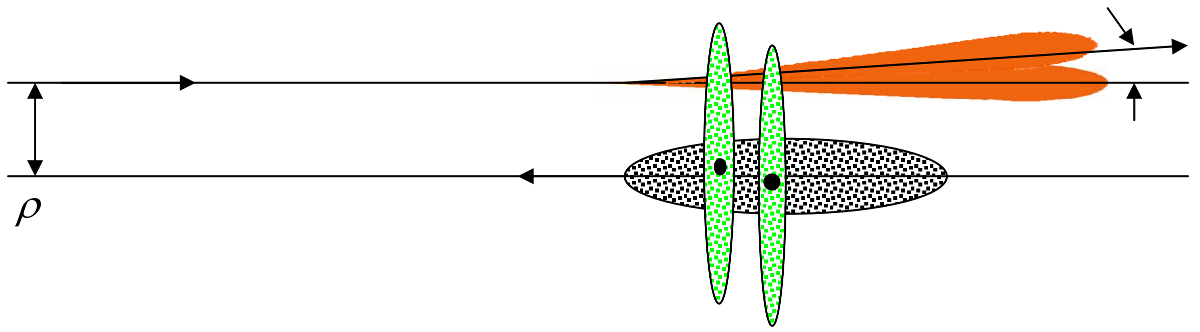
see report by S. Shulga at Channeling-2018

Coherent radiation at electron collision with a short bunch



N. Shul'ga, D. Tyutyunnik, JEPT Lett. 78(2003)700., Proc. of SPIE, v. 5974(2005)60.

Suppression of coherent radiation (analog of TSF-effect)



$$\vartheta_N = \frac{2Ne^2}{\varepsilon\rho}$$

$$\frac{dE_N}{d\omega} = \frac{2e^2}{\pi}\left[\frac{2\xi^2+1}{\xi\sqrt{\xi^2+1}}\ln\left(\xi+\sqrt{\xi^2+1}\right)-1\right], \qquad \xi = \frac{\gamma\vartheta_N}{2}$$

$$\frac{dE_N}{d\omega} \approx \begin{cases} N^2\dfrac{e^6}{m^2\rho^2} & \gamma\vartheta_N \ll 1 \\ 4e^2\ln\left(\dfrac{Ne^2}{m\rho}\right) & \gamma\vartheta_N \gg 1 \end{cases}$$

ε=5 Gev, L=0.1 cm, ρ=0.01 cm, N=10^{10}, $\omega_c = 4\gamma^2/L \approx 50 kev$, $\gamma\vartheta_N \approx 1$

N. Shul'ga, D. Tyutyunnik. JETP Lett. 78 (2003) 700.
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Conclusions

• Quantum and classical theories of scattering

• Transitional region from ultrathin to thick crystals (from channeling absence to channeling presence)

• Quantum and classical effects at scattering (coherence, interference, rainbow, …)

• Possibility of experimental observation of quantum effects

• Radiation in transitional region of thickness

• Beam-beam coherent radiation

•------------------------------- FUTURE ---

• How do quantum levels and zones appear at regular motion and dynam. chaos?

• Coherent and incoherent scattering

• Bremsstrahlung and e^+-e^- pair production in the geometrical optics approx.

• Coherent Bremsstrahlung in bent and periodically bent crystals

• Electromagnetic showers

• Multiple photon showers

•...

THANK YOU FOR YOUR ATTENTION!