# Testing general relativity with gravitational waves: results and prospects

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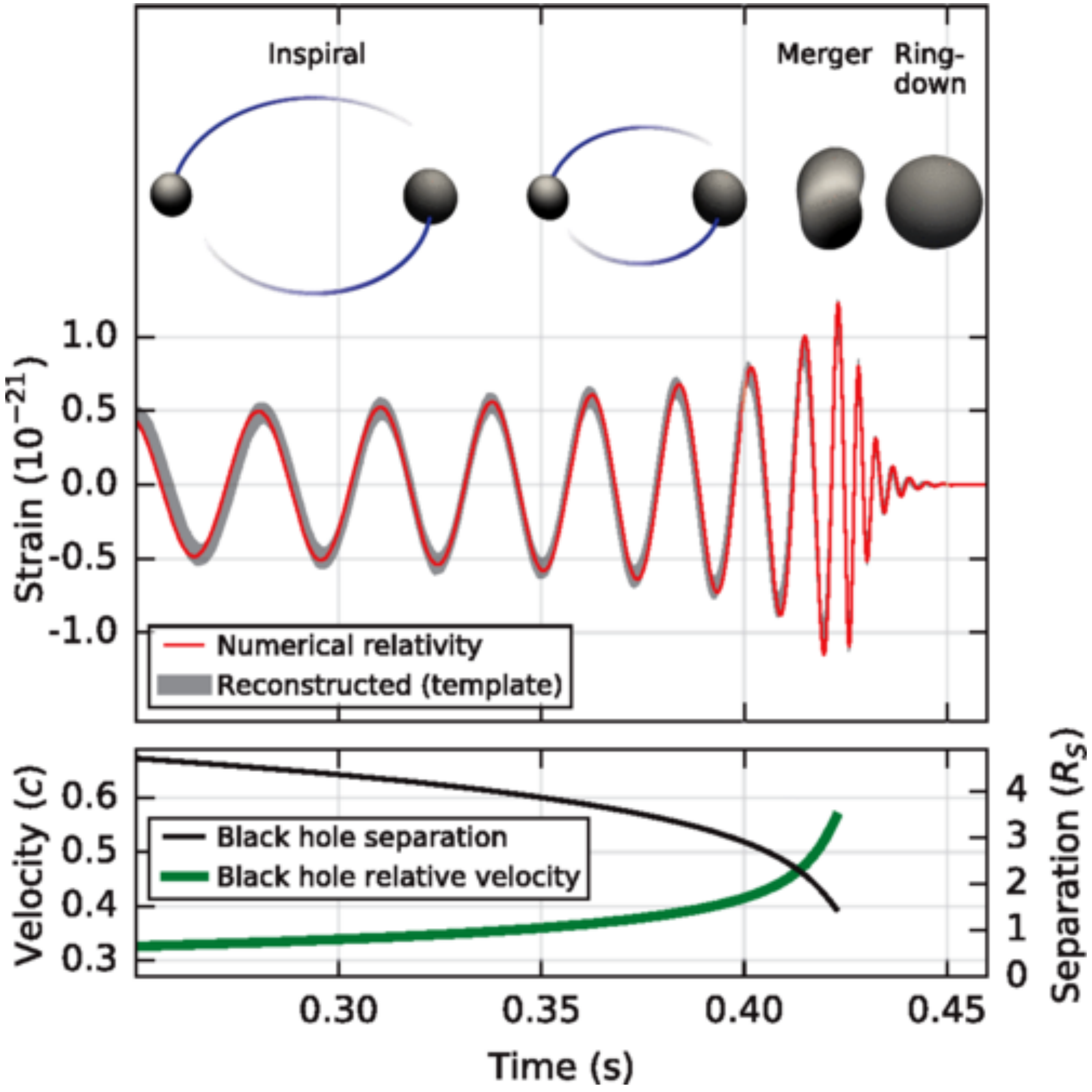
# Outline

- Fundamental aspects of gravitational wave physics
  - Dynamics of space-time
  - Nature of binary black hole binaries
  - Gravitational wave polarisation states
- Summary

# Fundamental aspects of GW physics

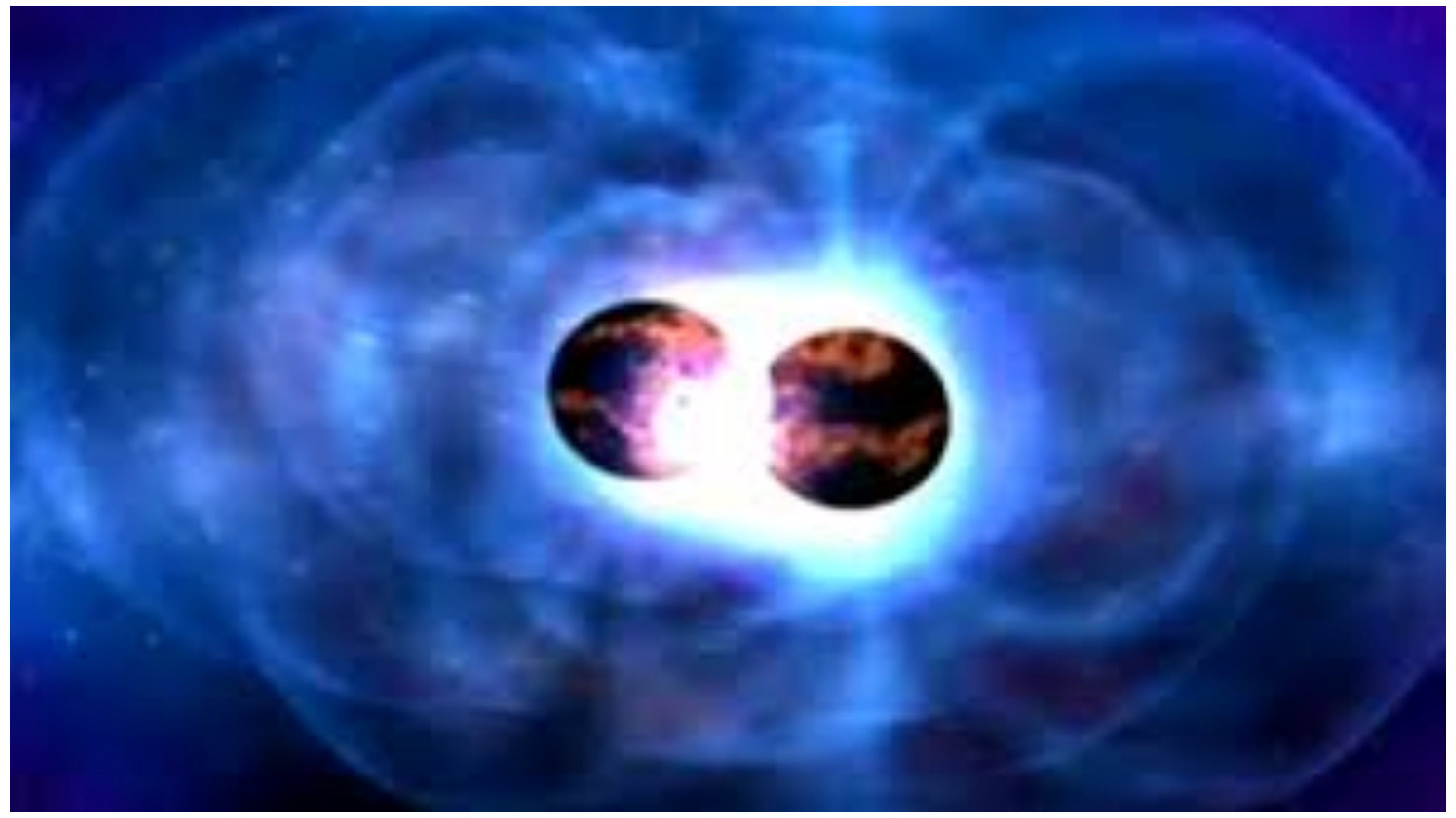
- In GR, gravitational waves (GW) are wave solutions to Einstein's equations generated from time varying mass quadrupoles and propagating at the speed of light
- Shape of GW signal carries information about
  - binary dynamics and component nature
  - non-linear dynamics of space-time
  - final object nature




LVC, arXiv:1602.03837

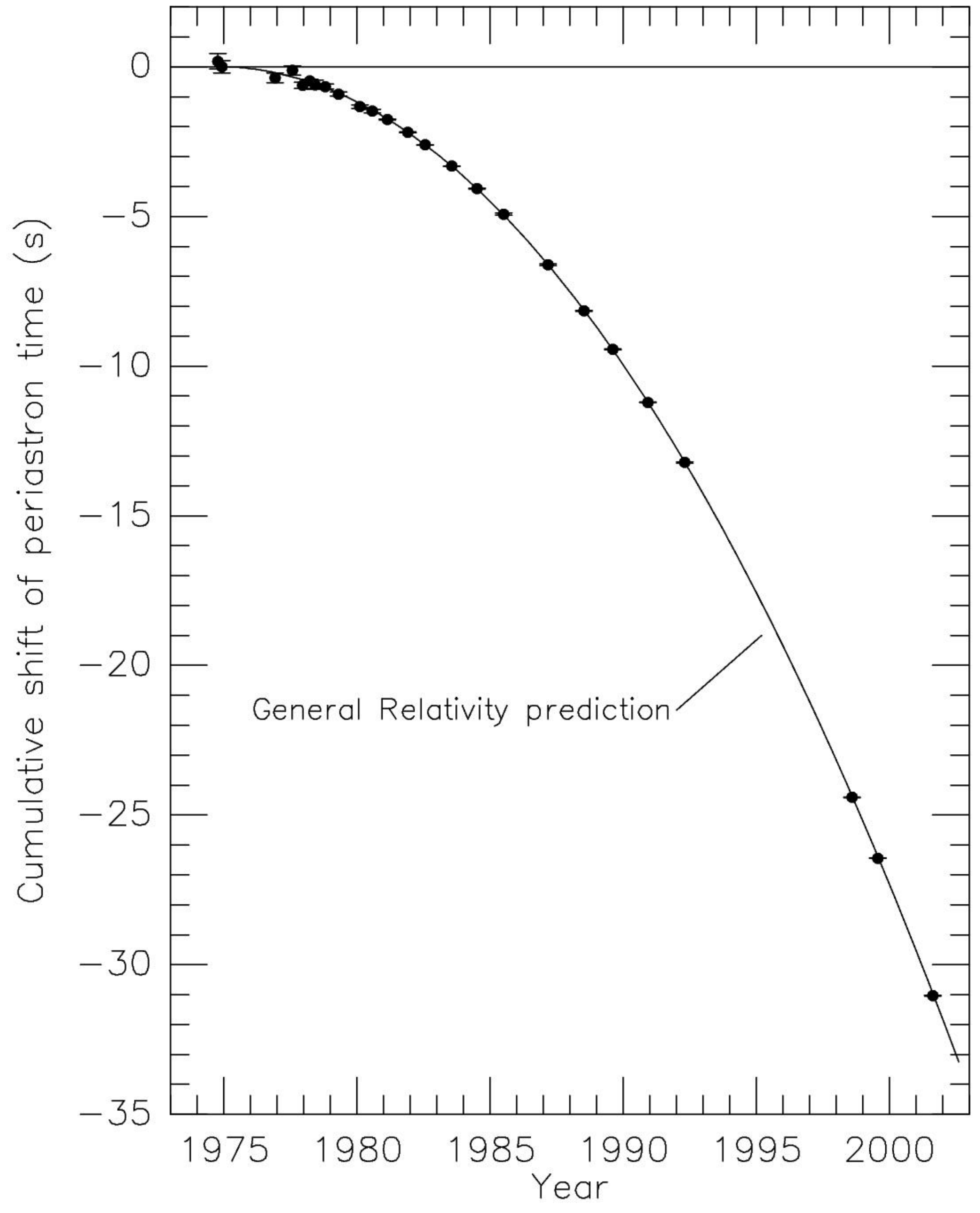
# What physics can be probed

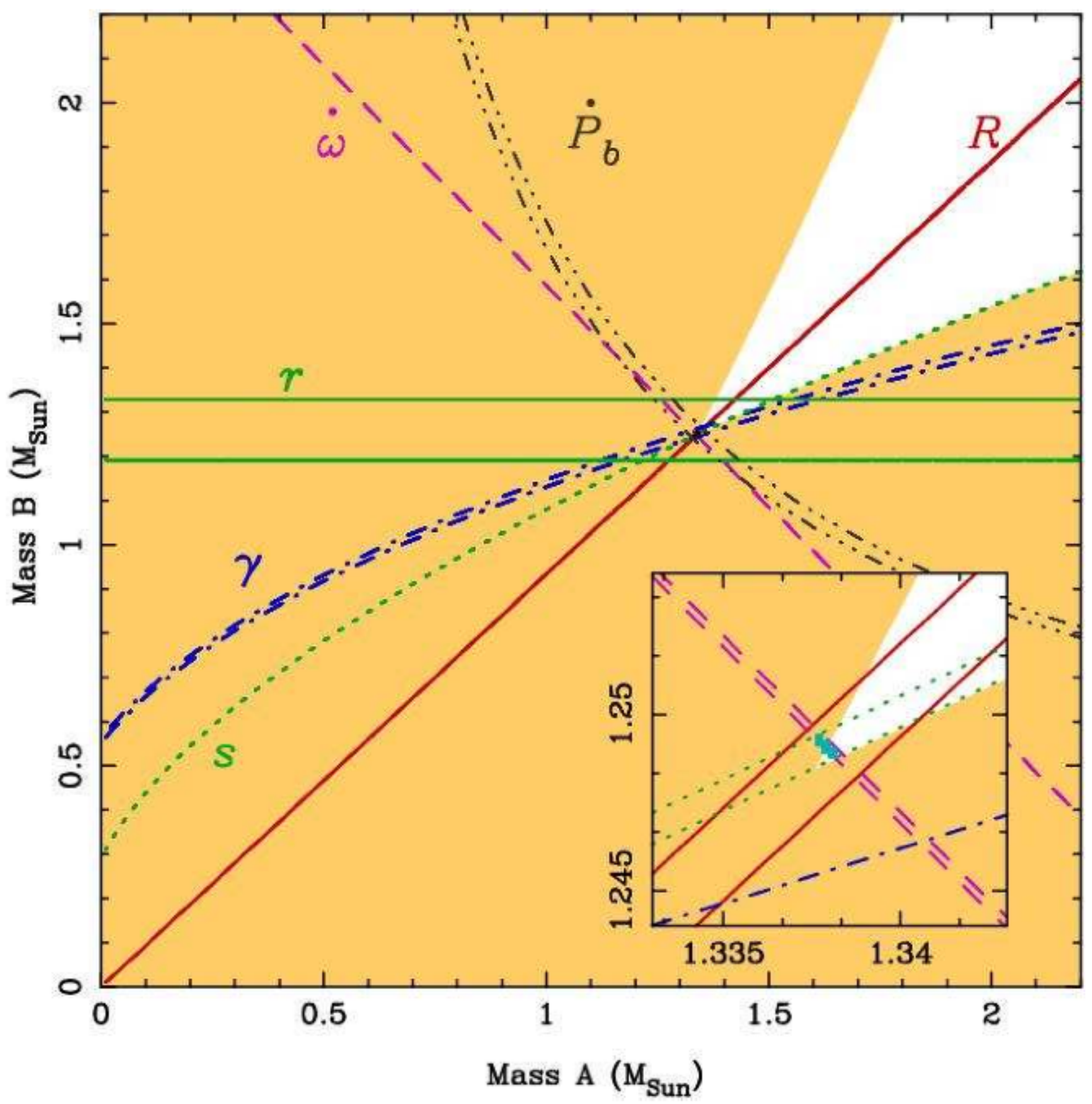
- Matching observed data with a solution to Einstein’s equations allows to probe
  - Laws of space-time dynamics
  - Nature of black holes
  - Equation of state of neutron stars
  - Cosmology



# Dynamics of space-time

- GR is non renormalisable
  - higher order terms in the action
- Dark matter & dark energy
  - signature of modified gravity?
- GR is extremely well tested in between these regimes (Will, arXiv:1403.7377, Psaltis, arXiv: 0806.1531)



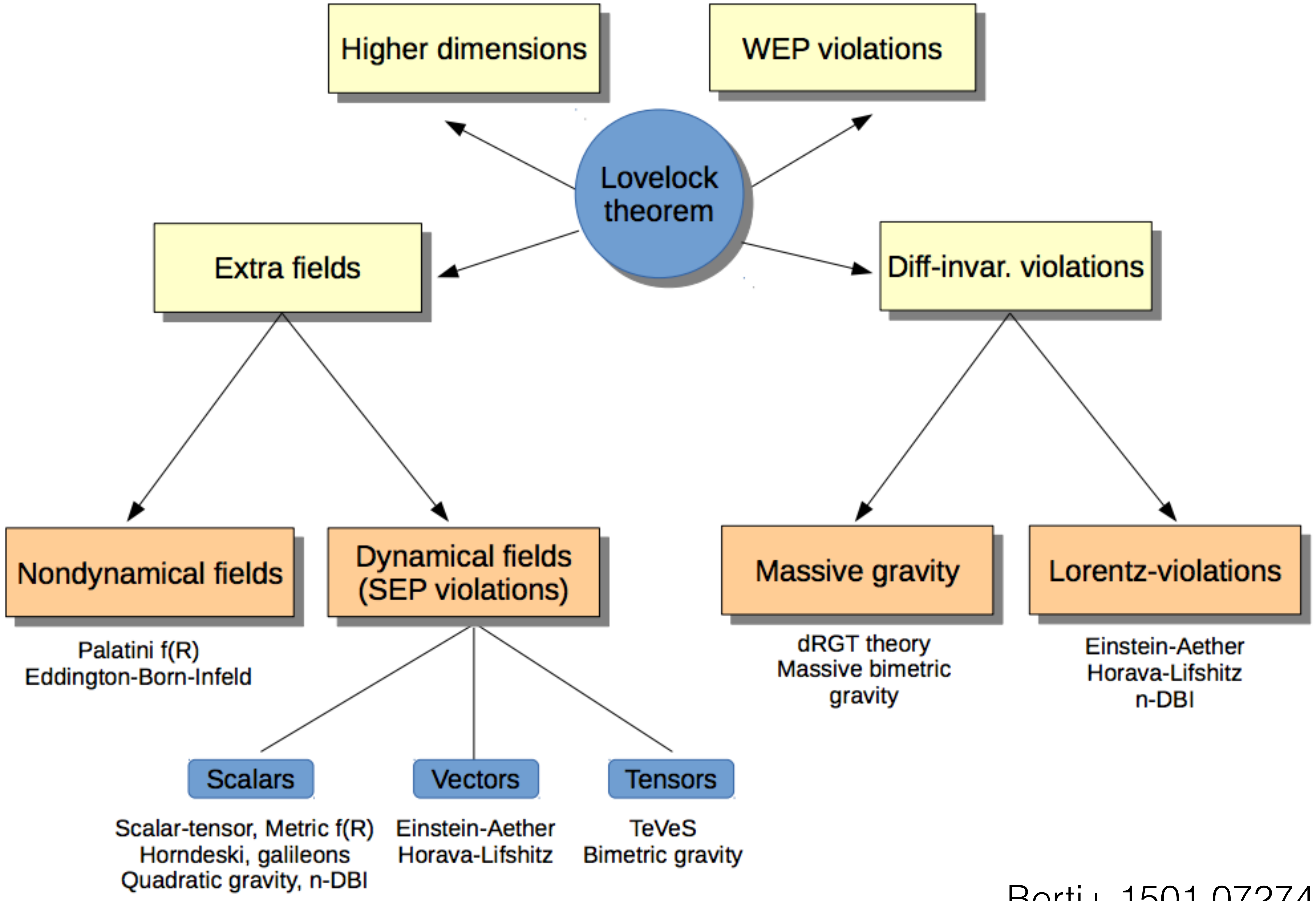




Weisberg & Taylor, arXiv:0407149
Kramer+,arXiv:0609417

# Extensions of GR

- Alternative theories
  - Introduce extra degrees of freedom:
    - additional fields
    - higher-curvature terms
  - Challenge GR assumptions:
    - Lorentz invariance
    - Equivalence principle
- Need tests in the strong-field

**Lovelock theorem:** In 4D, the only divergence free symmetric rank-2 tensor constructed only by the metric and its derivatives up to 2nd order and preserving diffeomorphism invariance is the Einstein tensor plus a constant.




Berti+,1501.07274

# Gravitational strong-field

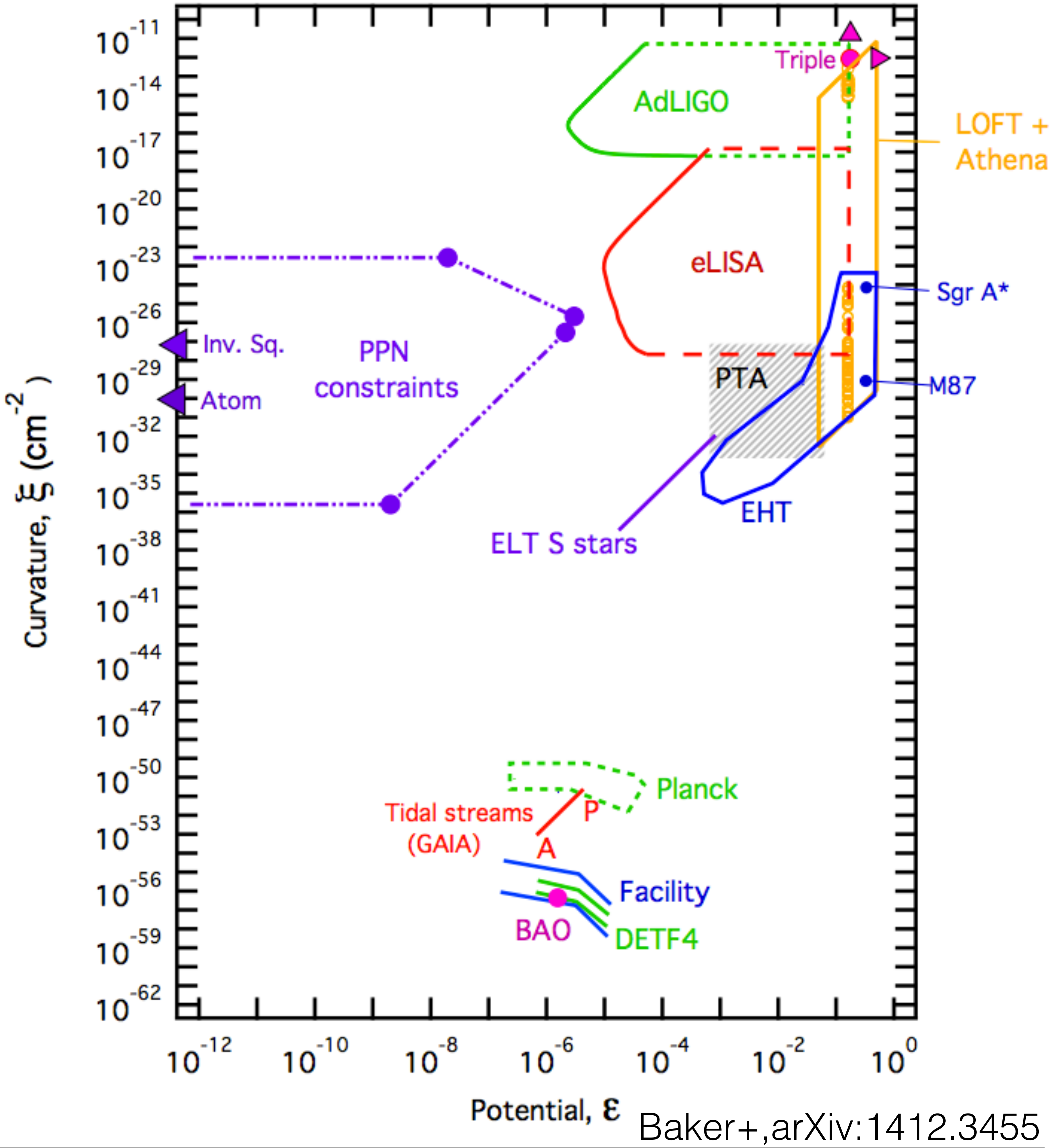
- Field strength

$$\epsilon = \frac{GM}{c^2 R}$$

- Curvature (Kretschmann scalar)

$$\xi = (R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta})^{1/2}$$

- Gravitational waves from binary black holes are the optimal probes




Baker+,arXiv:1412.3455

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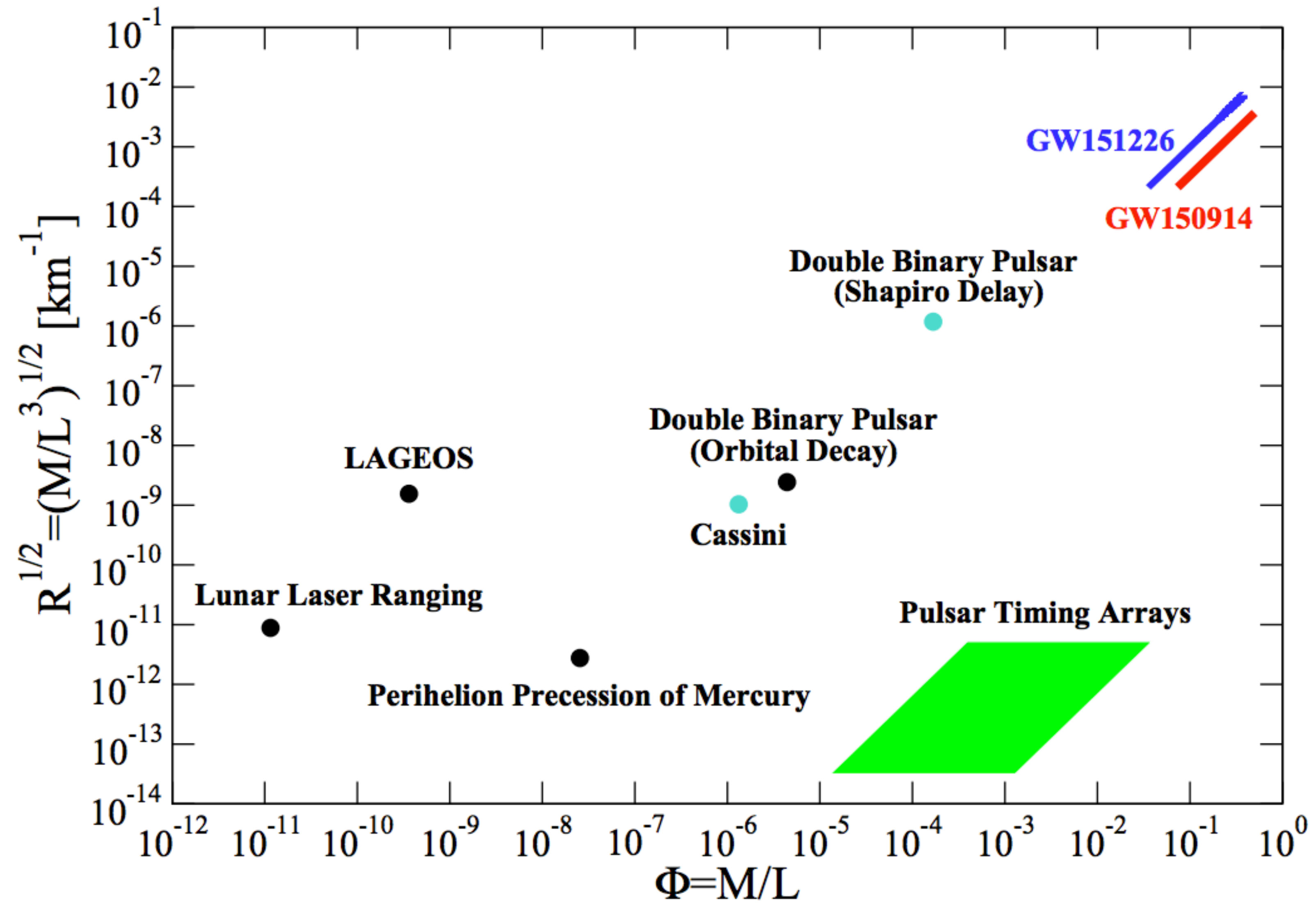
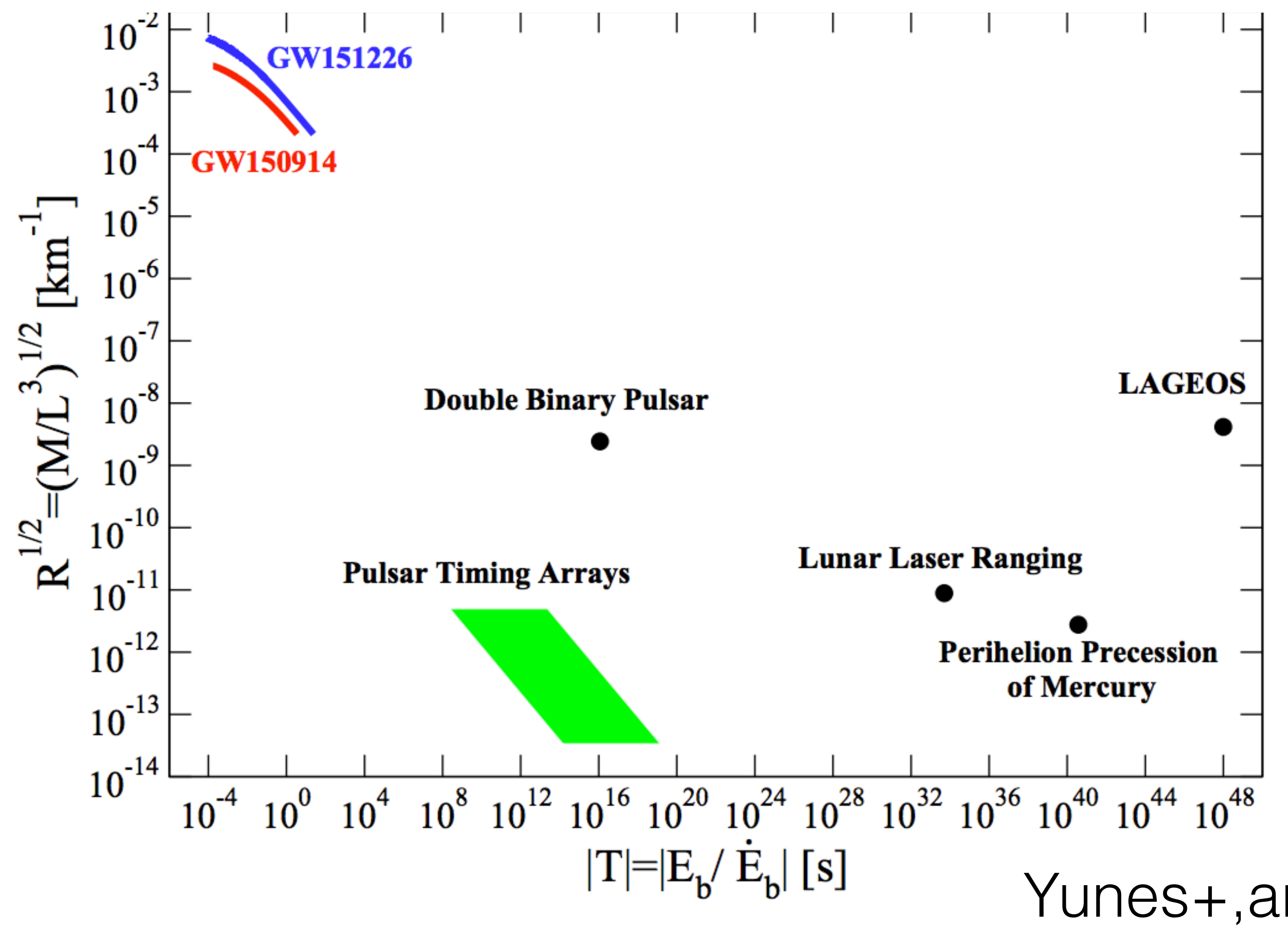
- Space-time is *dynamic*

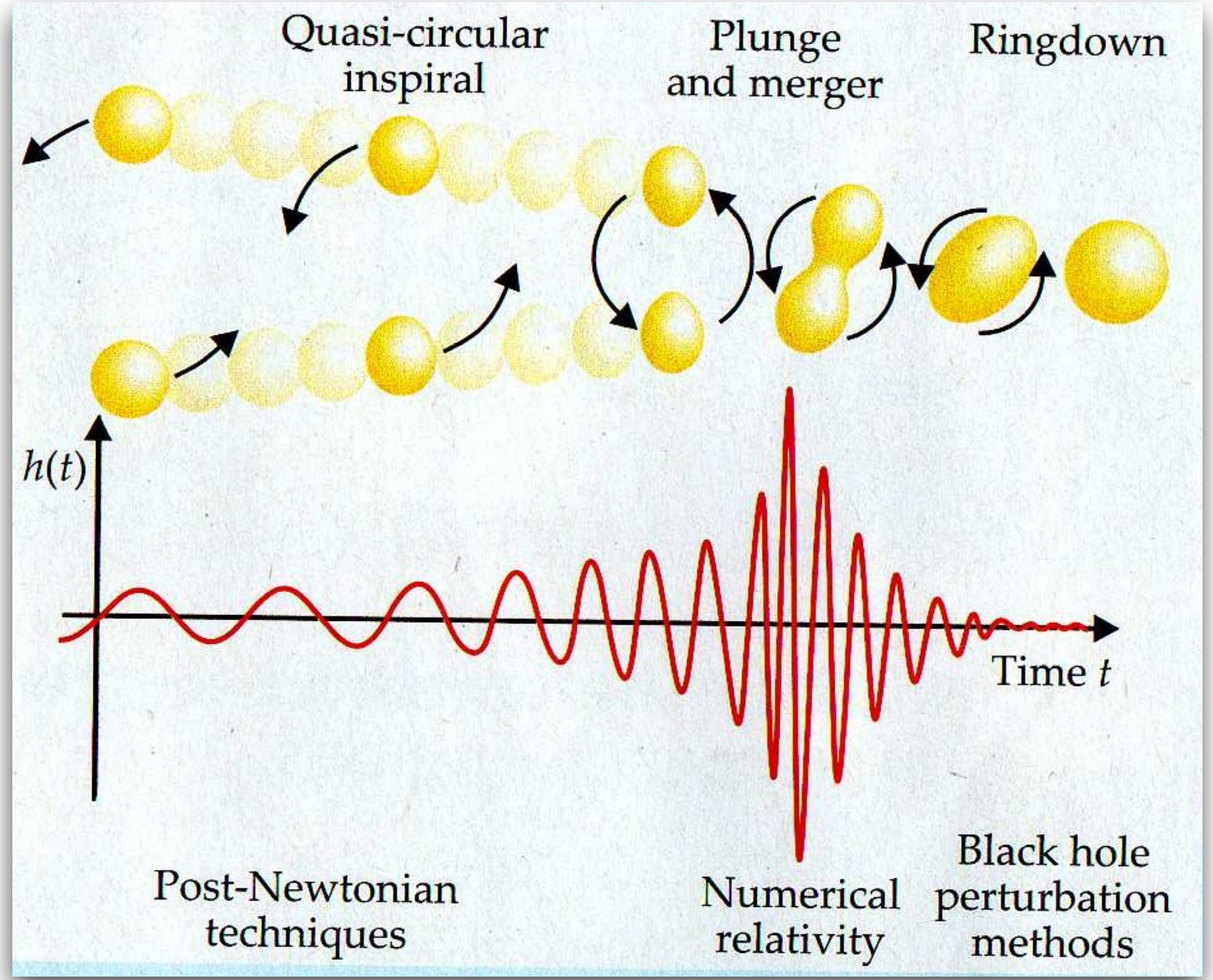



Yunes+,arXiv:1603.08955

# Gravitational wave solutions in GR

- Binary black holes solutions are constructed combining:
  - post-Newtonian theory in the weakly non-linear inspiral regime
  - direct numerical solution in the highly non-linear merger regime
  - perturbation theory in the ringdown regime

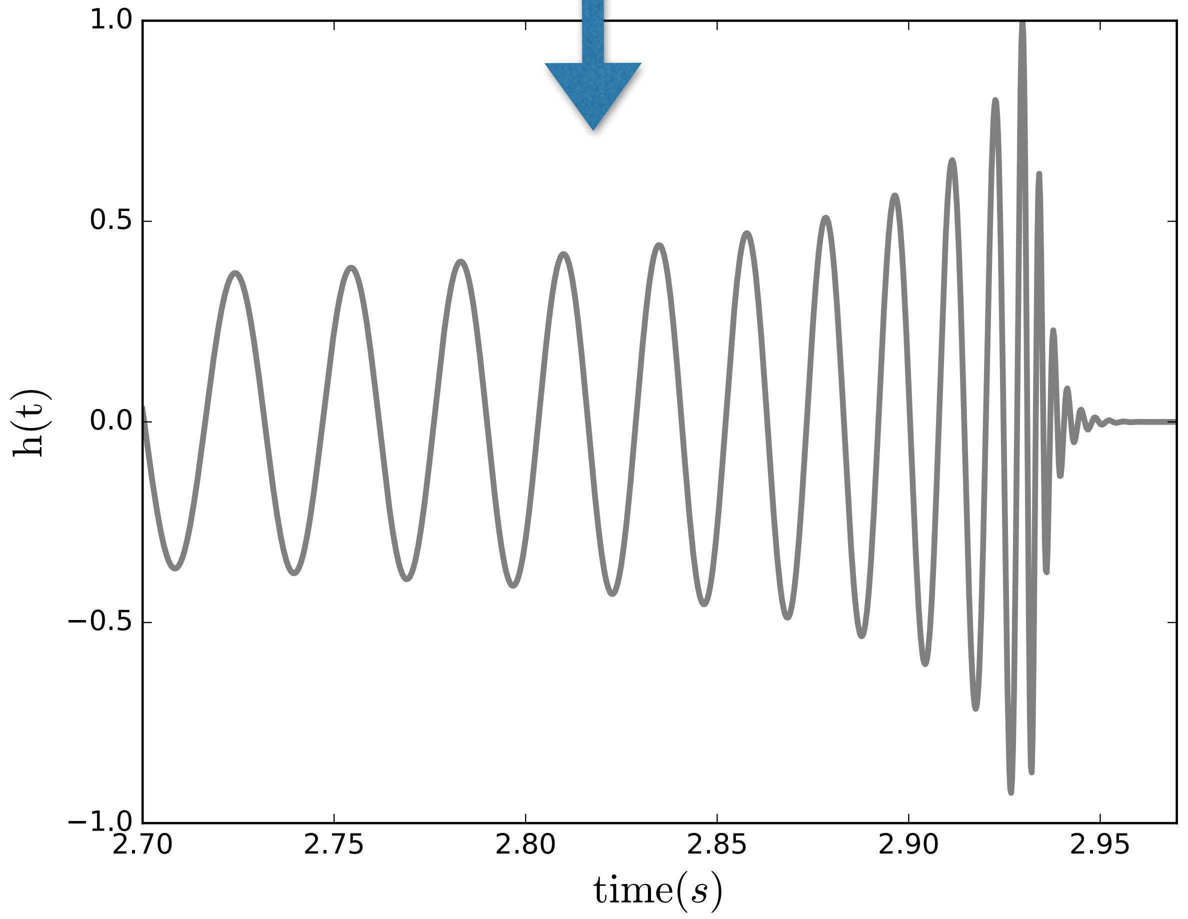


# Strong-field GR solutions

- Accurate solutions obtained by direct integration
- Formulation and implementation highly non-trivial
- Computationally challenging
- Numerical solution used to inform and complement analytical formulations:
    - Effective one body (Buonanno & Damour, arXiv: 9811091, Bohe+,arXiv:1611.03703)
    - Phenomenological (e.g. Khan+,arXiv: 1508.07253)

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

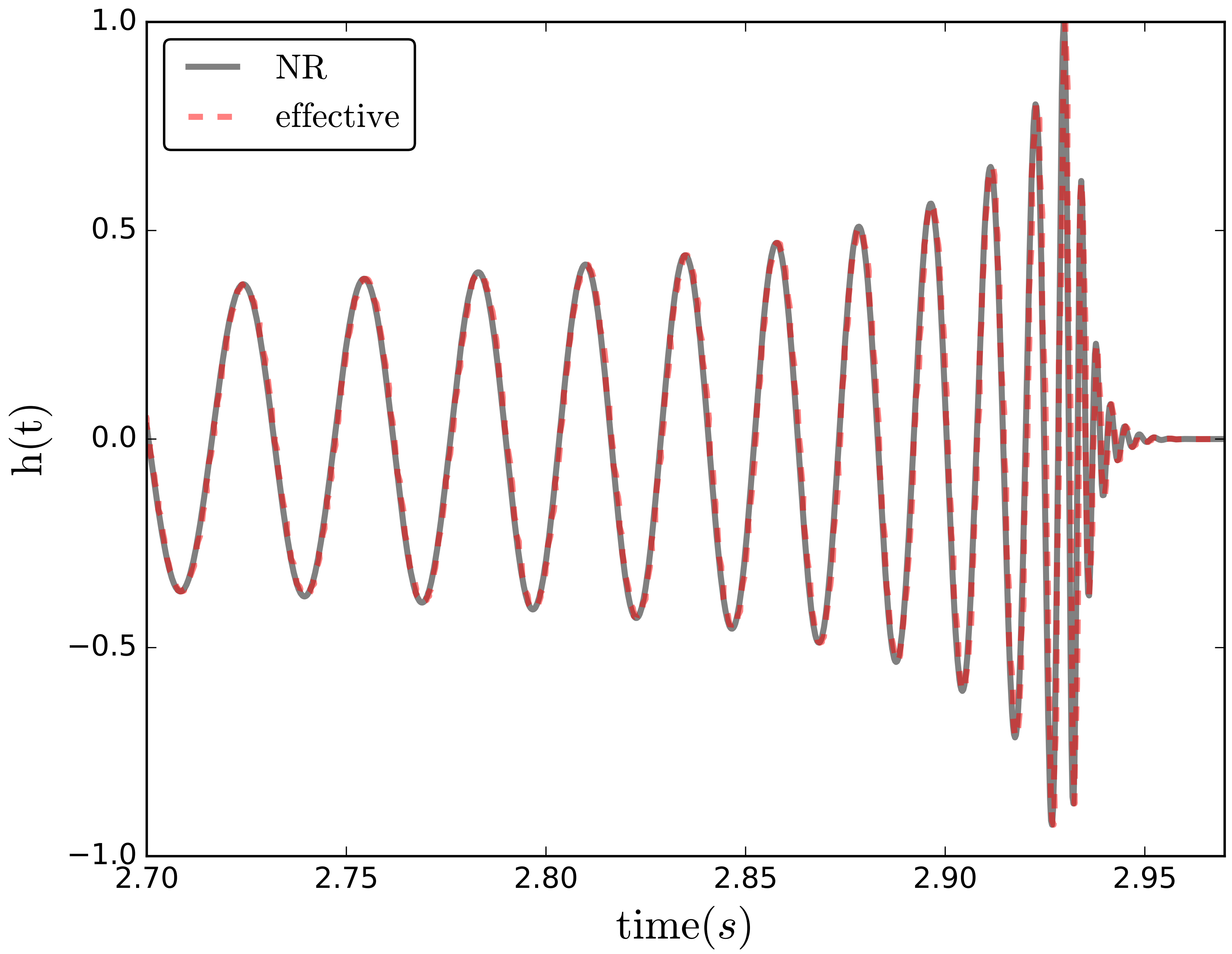
few weeks later



# GW templates in GR

- Analytical, parametric description of GW solution in GR

$$h(f;\theta) = A(f;\theta)e^{i\Phi(f;\theta)}$$
$$\Phi(f;\theta) \equiv \Phi(f;m_1,m_2,\vec{s}_1,\vec{s}_2)$$




- Suitable for detection, parameter estimation and parametric tests of general relativity

# GW in alternative gravity

- Alternative to GR can introduce extra-fields, curvature terms, challenge GR pillars, …
- Almost no full solution in non-GR known (but see Okounkova et al, arXiv:1705.07924)
- GW phase is modified:
  - non-GR action (extra fields, higher curvature, …): no full non-linear description, only post-Newtonian
  - Propagation (Lorentz violations, graviton mass, …): GR-like BBH dynamics, but modified GW propagation
  - non-GR BHs (extra-fields, exotic objects):
    - tidal deformability
    - ringdown spectrum
    - Echoes

# Parametrised tests of GR

- GW waveforms are expressed in terms of effective series, for the Phenom family:

$$h(f;\theta) = A(f;\theta)e^{i\Phi(f;\theta)}$$

$$\Phi(f;\theta) = \sum_{k=0}^{7}(\varphi_k + \varphi_k^{(l)})f^{(k-5)/3} + \sum_{i\neq k}\varphi_i g(f)$$

post-Newtonian series — effective series

$$\varphi_j \equiv \varphi_j(m_1, m_2, \vec{s}_1, \vec{s}_2)$$

- Modified theories of gravity change the series (e.g. PPE: Yunes & Pretorius, arXiv:0909.3328, Cornish+,arXiv:1105.2088)

- Perturb the GW phase around GR (Li+,arXiv:1110.0530, Agathos+,arXiv:1311.0420)

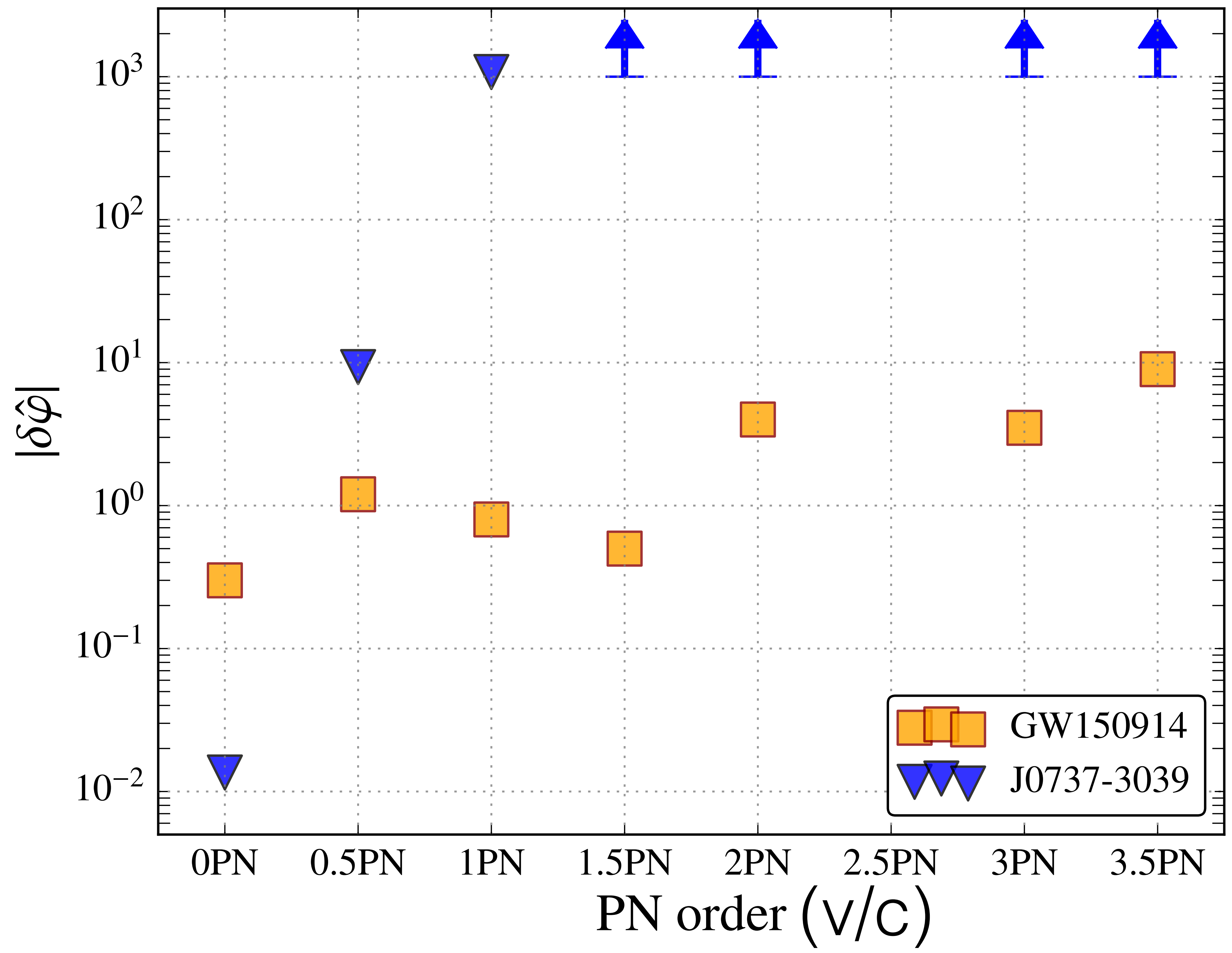
$$\hat{\varphi}_j \equiv \varphi_j^{GR}(1+\delta\hat{\varphi}_j) \qquad \delta\hat{\varphi}_j = 0 \iff \text{GR}$$

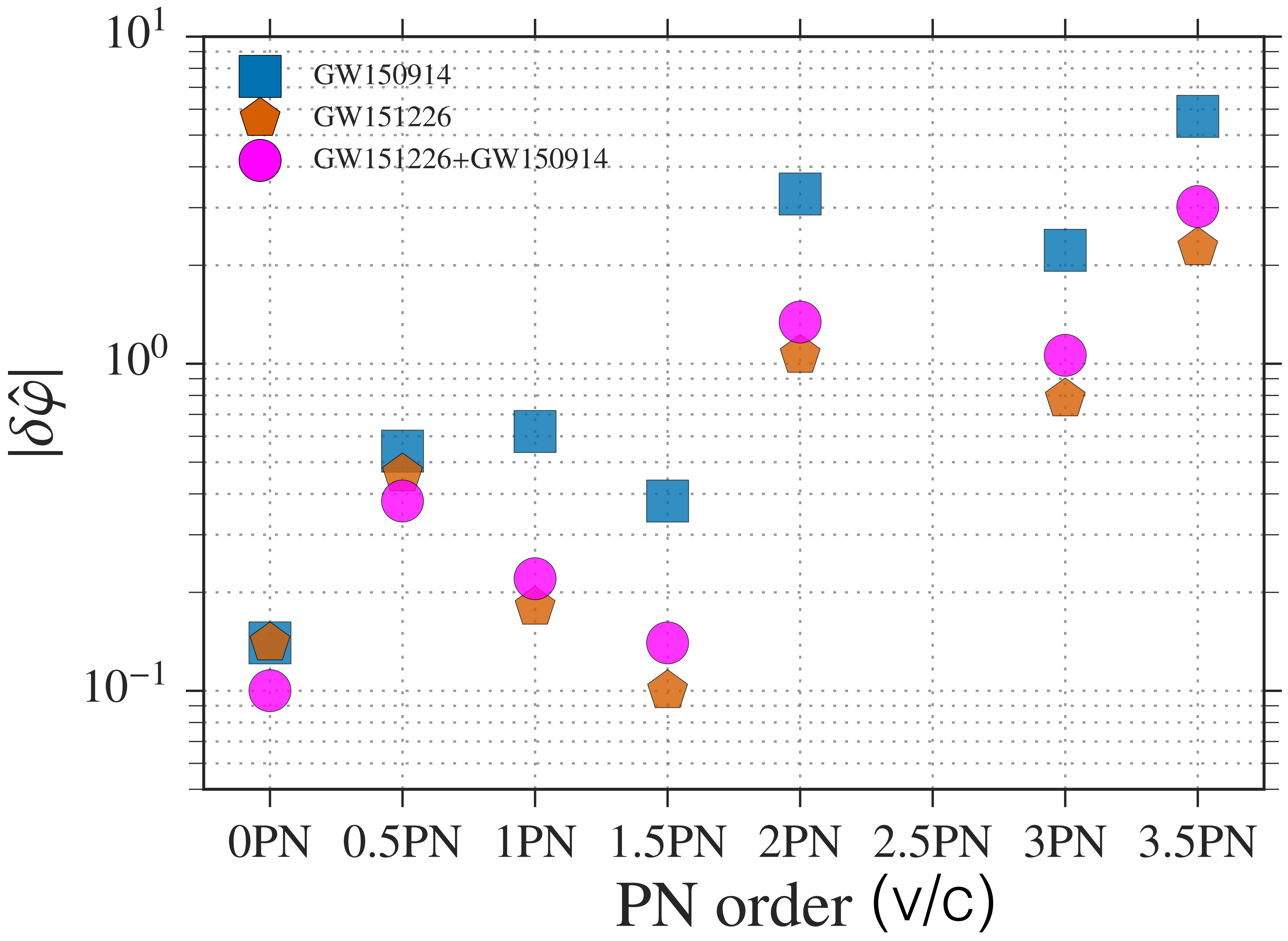
- Bound violations by computing posterior distributions for the $\delta\hat{\varphi}_j$ in concert with the physical parameters of the system

| waveform regime | parameter | $f$–dependence | |
|---|---|---|---|
| early-inspiral regime | $\delta\hat{\varphi}_0$ | $f^{-5/3}$ | post-Newtonian |
| | $\delta\hat{\varphi}_1$ | $f^{-4/3}$ | |
| | $\delta\hat{\varphi}_2$ | $f^{-1}$ | |
| | $\delta\hat{\varphi}_3$ | $f^{-2/3}$ | |
| | $\delta\hat{\varphi}_4$ | $f^{-1/3}$ | |
| | $\delta\hat{\varphi}_{5l}$ | $\log(f)$ | |
| | $\delta\hat{\varphi}_6$ | $f^{1/3}$ | |
| | $\delta\hat{\varphi}_{6l}$ | $f^{1/3}\log(f)$ | |
| | $\delta\hat{\varphi}_7$ | $f^{2/3}$ | |
| intermediate regime | $\delta\hat{\beta}_2$ | $\log f$ | effective |
| | $\delta\hat{\beta}_3$ | $f^{-3}$ | |
| merger–ringdown regime | $\delta\hat{\alpha}_2$ | $f^{-1}$ | |
| | $\delta\hat{\alpha}_3$ | $f^{3/4}$ | |
| | $\delta\hat{\alpha}_4$ | $\tan^{-1}(af+b)$ | |

LVC, arXiv:1602.03841

# Post-Newtonian constraints



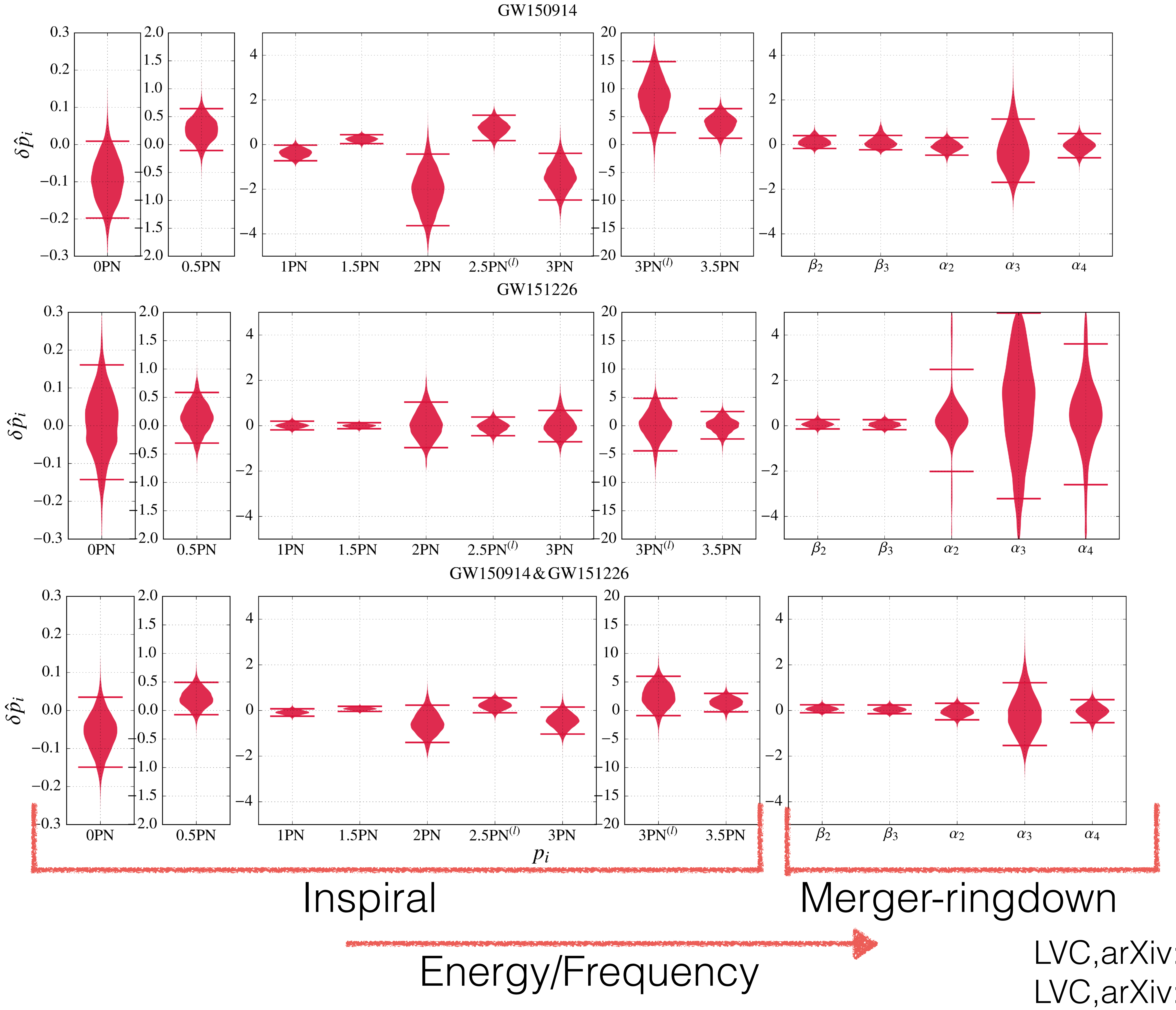




- Constraints not achievable by any other means
- Can be mapped to the space of specific theories (e.g. Yunes+,arXiv: 1603.08955)

LVC,arXiv:1602.03841
LVC,arXiv:1606.04855

# Constraints on space-time dynamics

- Only constraints on space-time dynamics
- Posterior distributions for $\delta\hat{\varphi}_j$ show no evidence for violations of GR




LVC,arXiv:1602.03841
LVC,arXiv:1606.04856

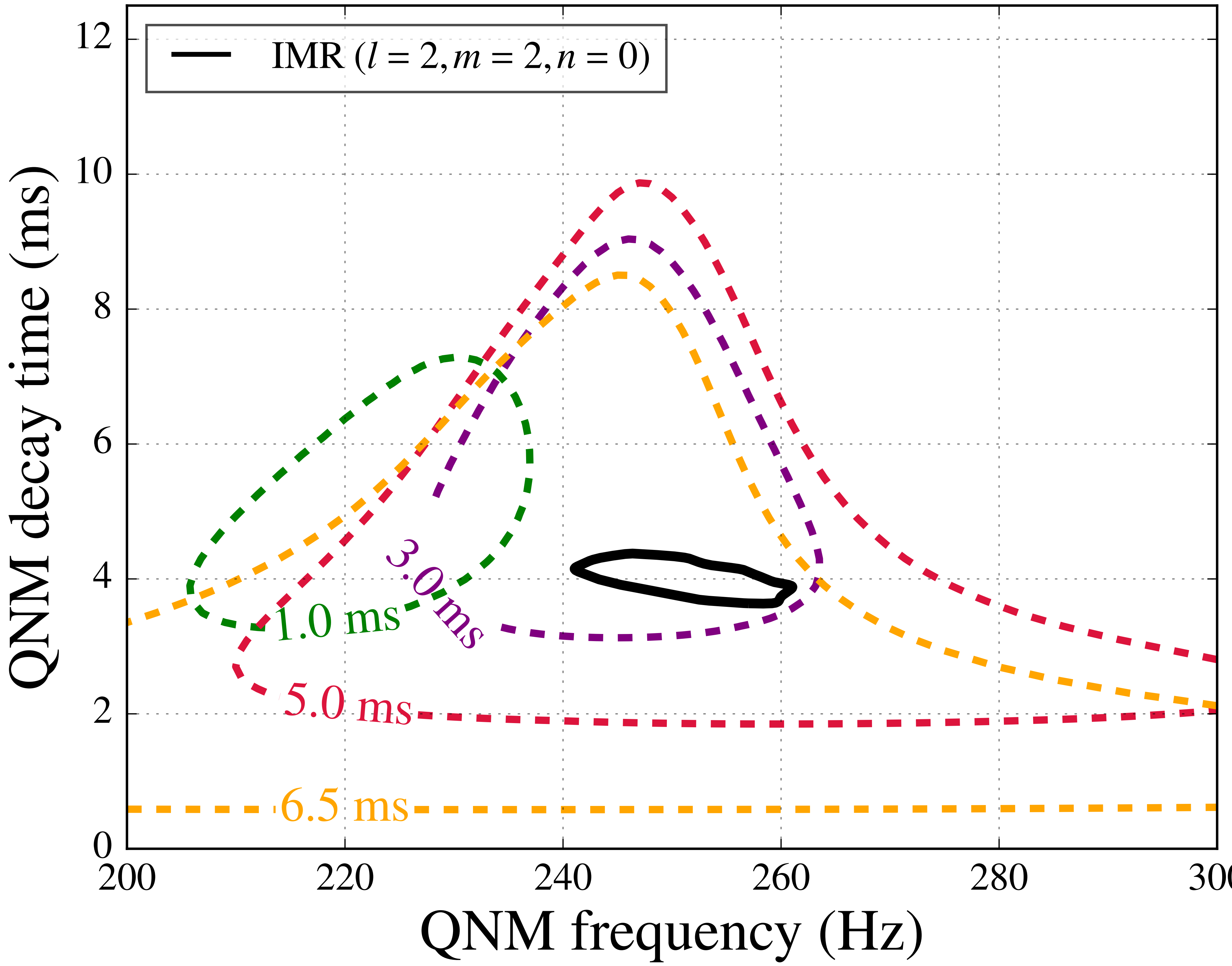
# The nature of the final object

- Ringdown signal for GR BHs is well understood

$$h(t) = \sum_{nlm} A_{nlm} e^{-\frac{t-t_0}{\tau_{nlm}}} \cos(\omega_{nlm}(t-t_0) + \varphi_{nlm})$$

- Central frequencies $\omega_{nlm}$ and decay times $\tau_{nlm}$ are functions of BH mass and spin only (the "no-hair" theorem, Berti+, arXiv:0512160)

- Multiple modes detection allows tests of BH nature and "no-hair" theorem (e.g. Gossan+, arXiv:1111.5819, Meidam+,arXiv:1406.3201




LVC,arXiv:1602.03841

# Propagation tests: massive gravity

- Families of alternative theories modify the propagation of GW
- Massive gravity (e.g. Will, arXiv:9709011)

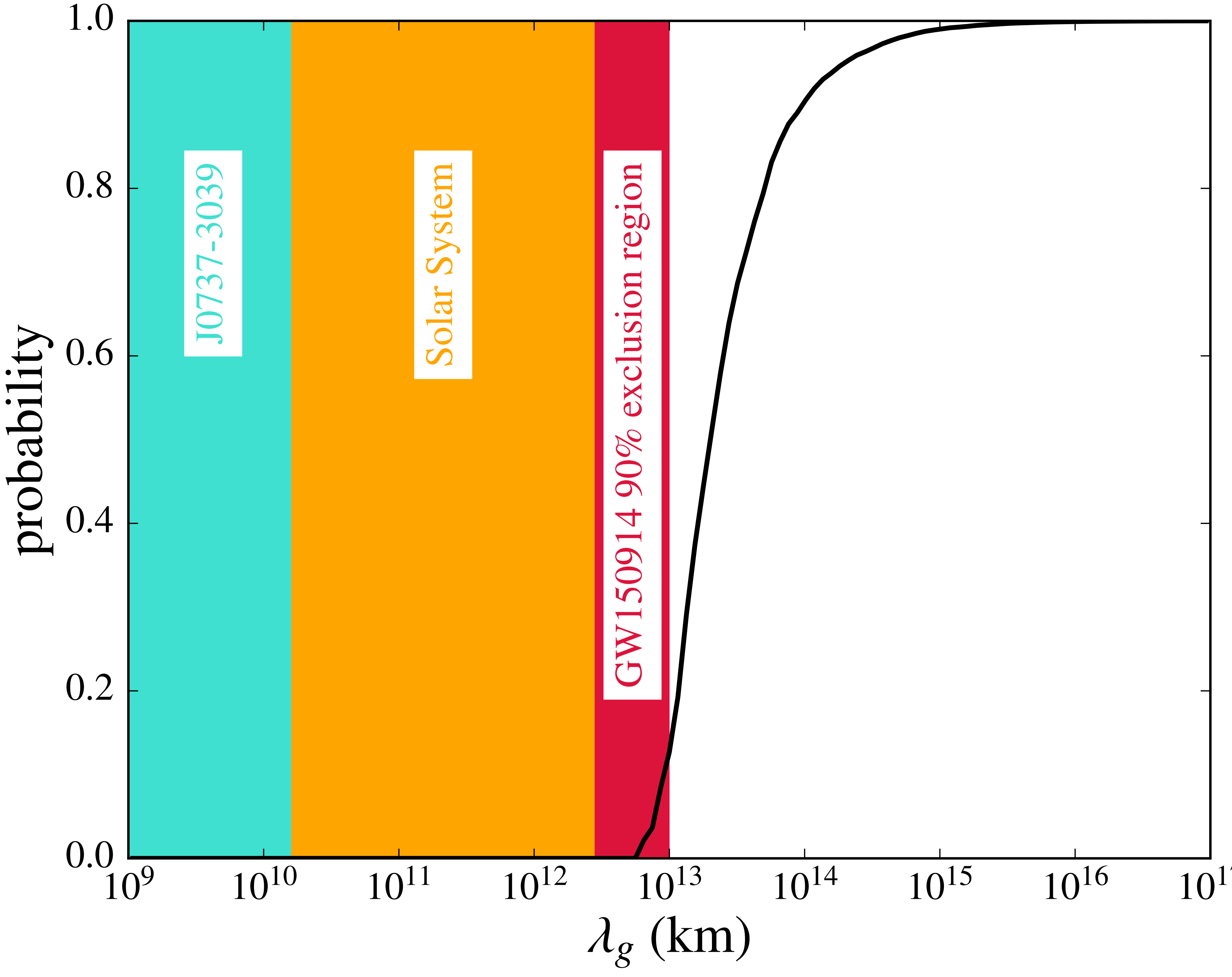
$$E^2 = p^2 v_g^2 + m_g^2 c^4$$

$$v_g^2/c^2 \simeq 1 - \frac{h^2 c^2}{\lambda_g^2 E^2} \qquad \lambda_g = \frac{h}{m_g c}$$

- GW phase affected

$$\Delta\Phi = -\frac{\pi^2 D M}{\lambda_g^2 (1+z)}$$

- GW constrains gravitons Compton wavelength




$$\lambda_g \geq 10^{13}\text{km} \ (90\%)$$

$$m_g \leq 1.2 \times 10^{-22} \text{eV/c}^2 \ (90\%)$$

LVC, arXiv:1602.03841

# Tests of Lorentz Invariance Violations

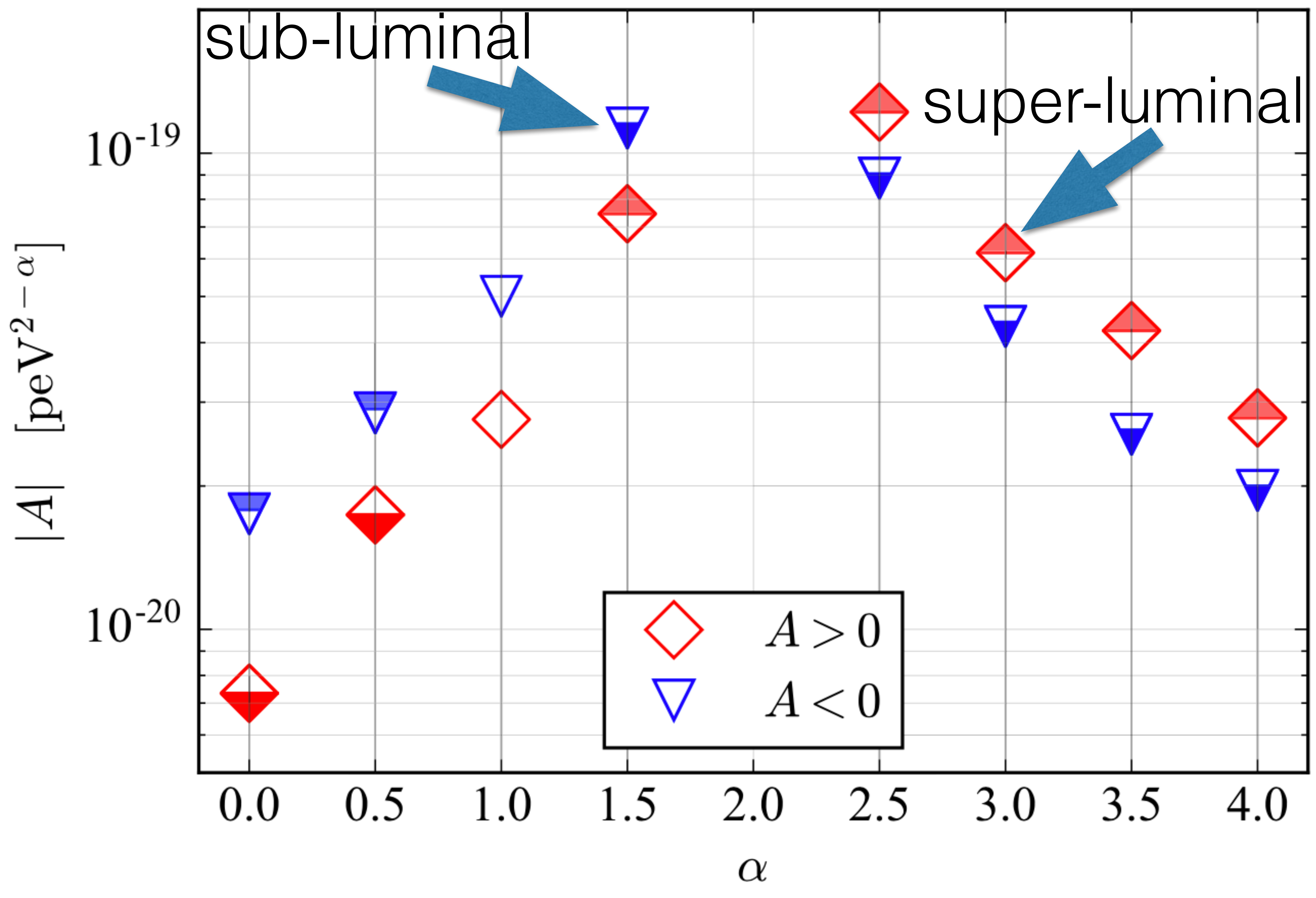
- Further generalised (e.g. Mirshekari et al, arXiv:1110.2720)

$$E^2 = p^2c^2 + Ap^\alpha c^\alpha \quad \alpha \geq 0$$

$$v_g/c = 1 + (\alpha - 1)AE^{\alpha-2}/2$$

- first bounds derived from GW

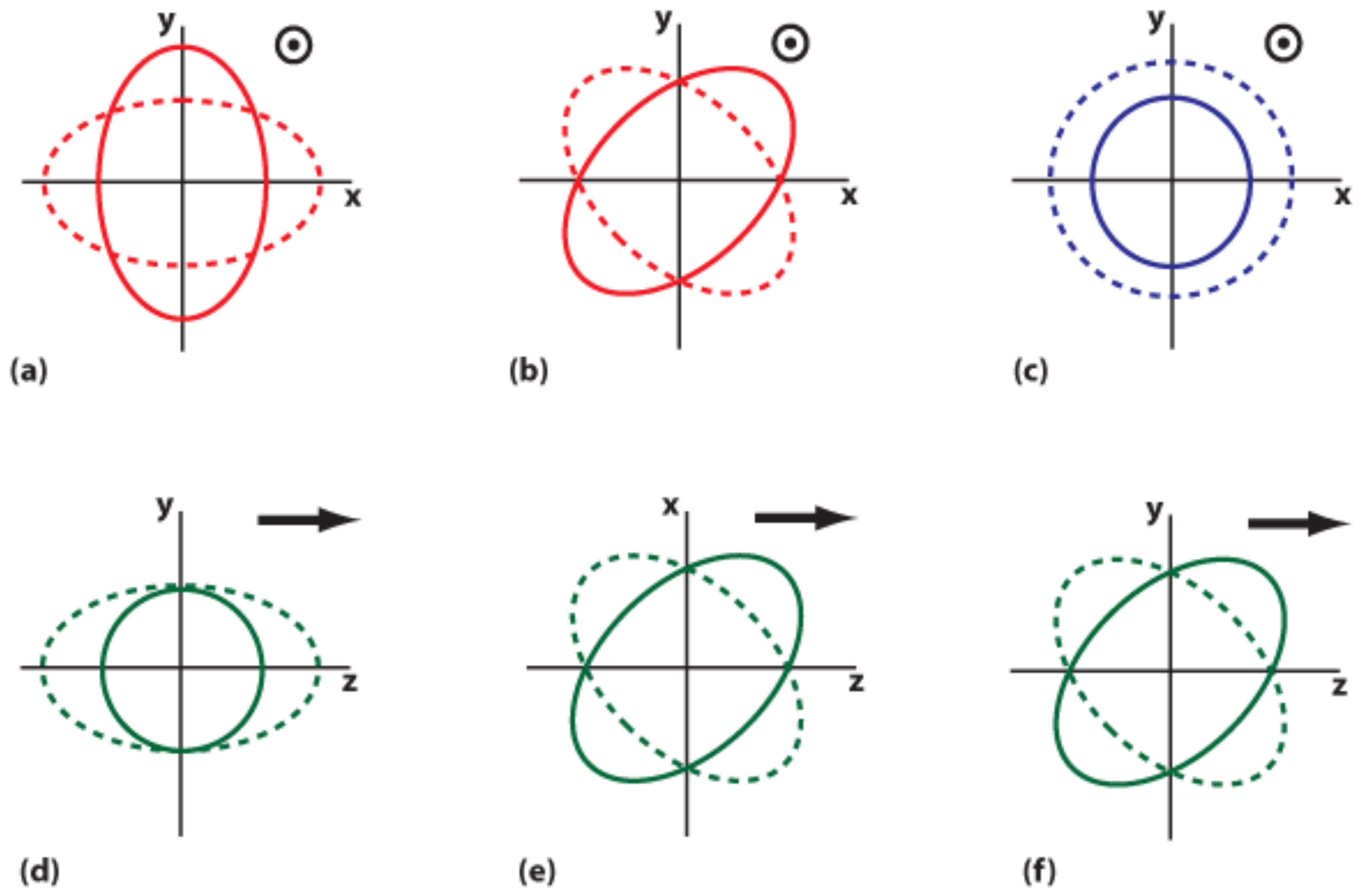
- first tests of superluminal propagation in the gravitational sector




LVC, arXiv:1706.01812

# Gravitational wave polarisation states

- Gravitational waves in general relativity are transverse, tensorial waves
- Extensions to general relativity predict up to six polarisation states
    - Two transverse tensor states
    - Two longitudinal vector states
    - Two scalar states, one longitudinal and one "breathing"




Will, arXiv:1403.7377

# Detector response to polarisation states

- Each polarisation state couples to the detector differently $h = \sum_{k=1}^{6} F_k h_k$

Antenna response functions $F_k$

Tensor

Vector

Scalar
(breathing)

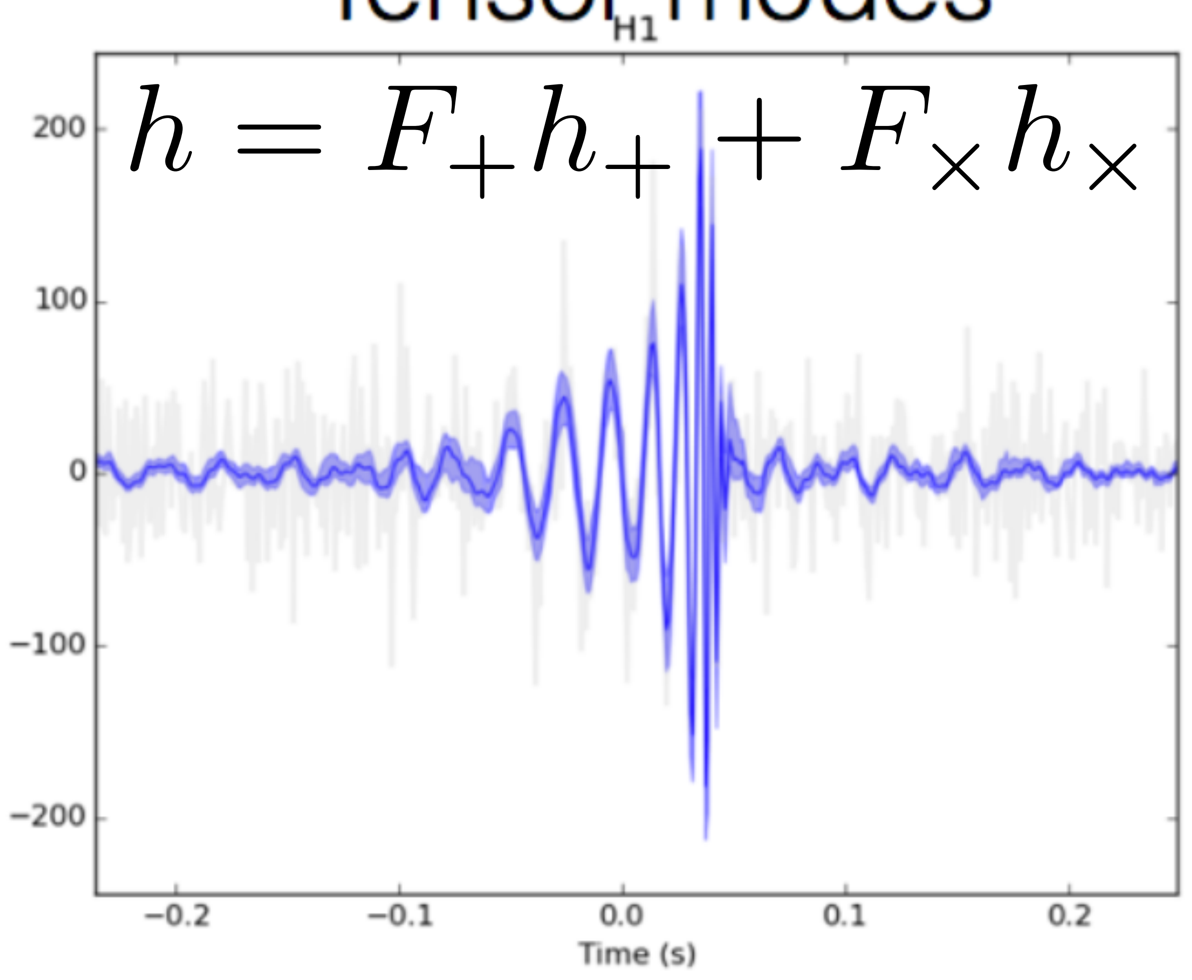
- In principle detectable with more than one detector

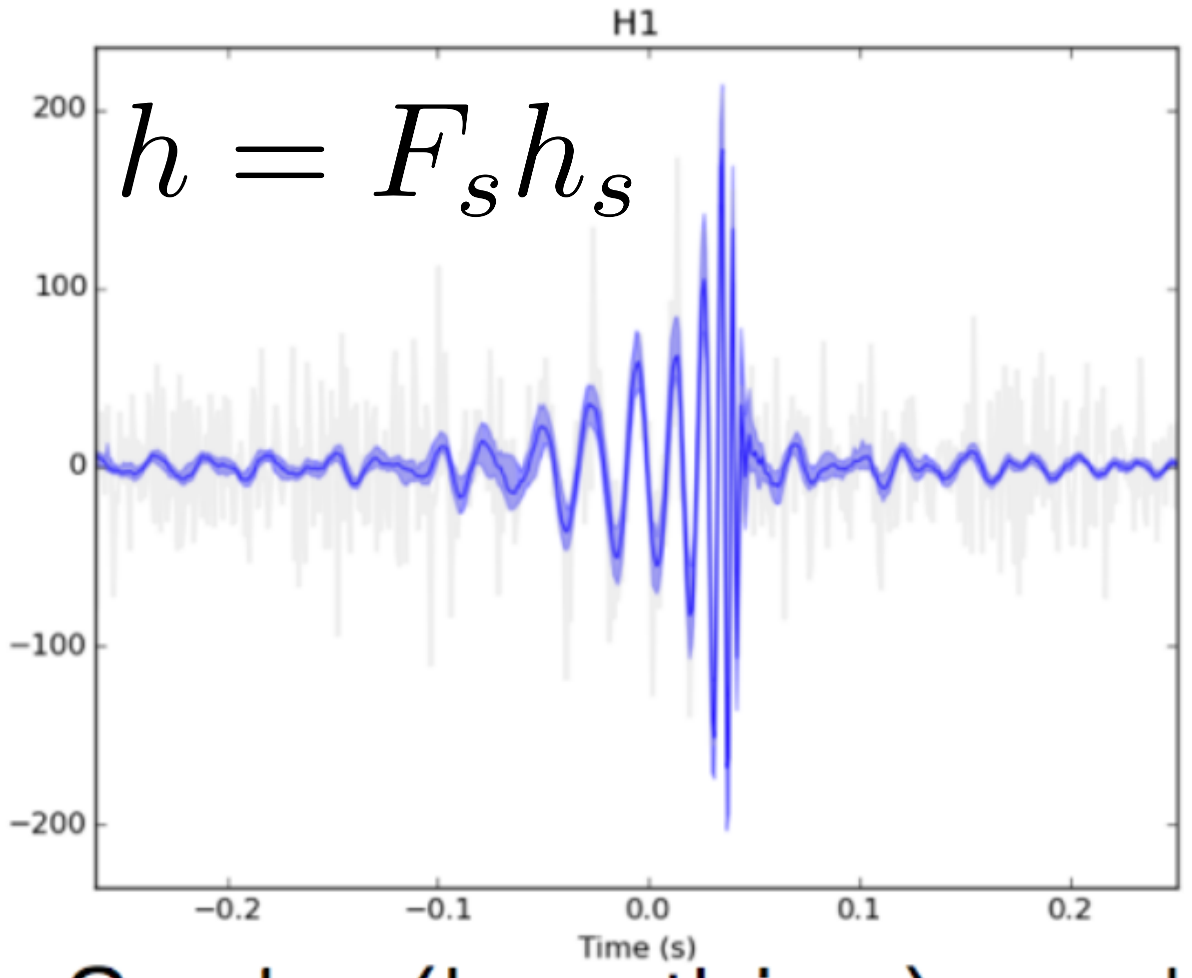
Courtesy of Max Isi

# Two detectors sensitivity to polarisation states

- The two LIGO detectors could not discriminate among different polarisation states
- Essentially aligned
  - A third detector and/or an electromagnetic counterpart would be necessary

Tensor modes

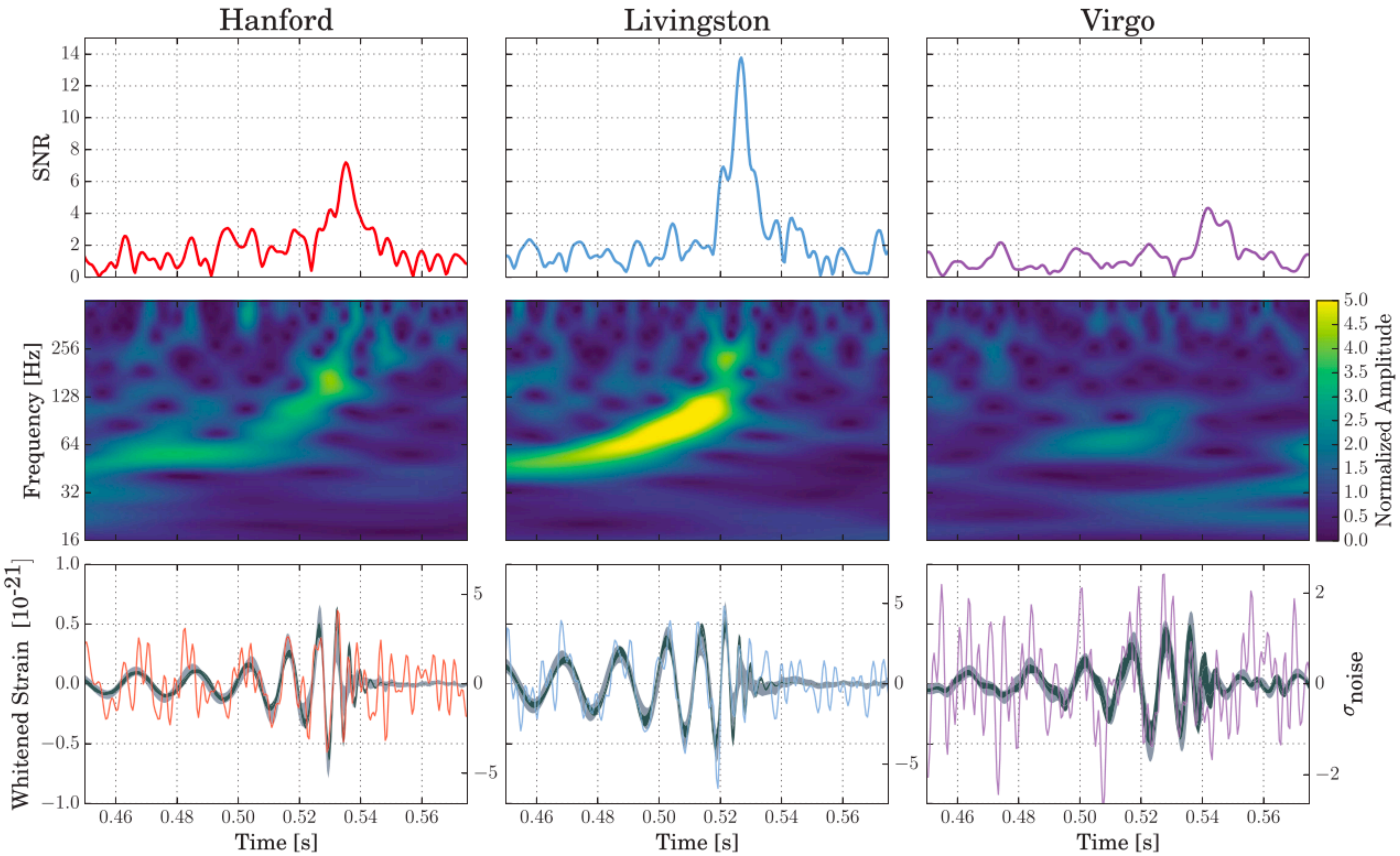



Scalar (breathing) mode

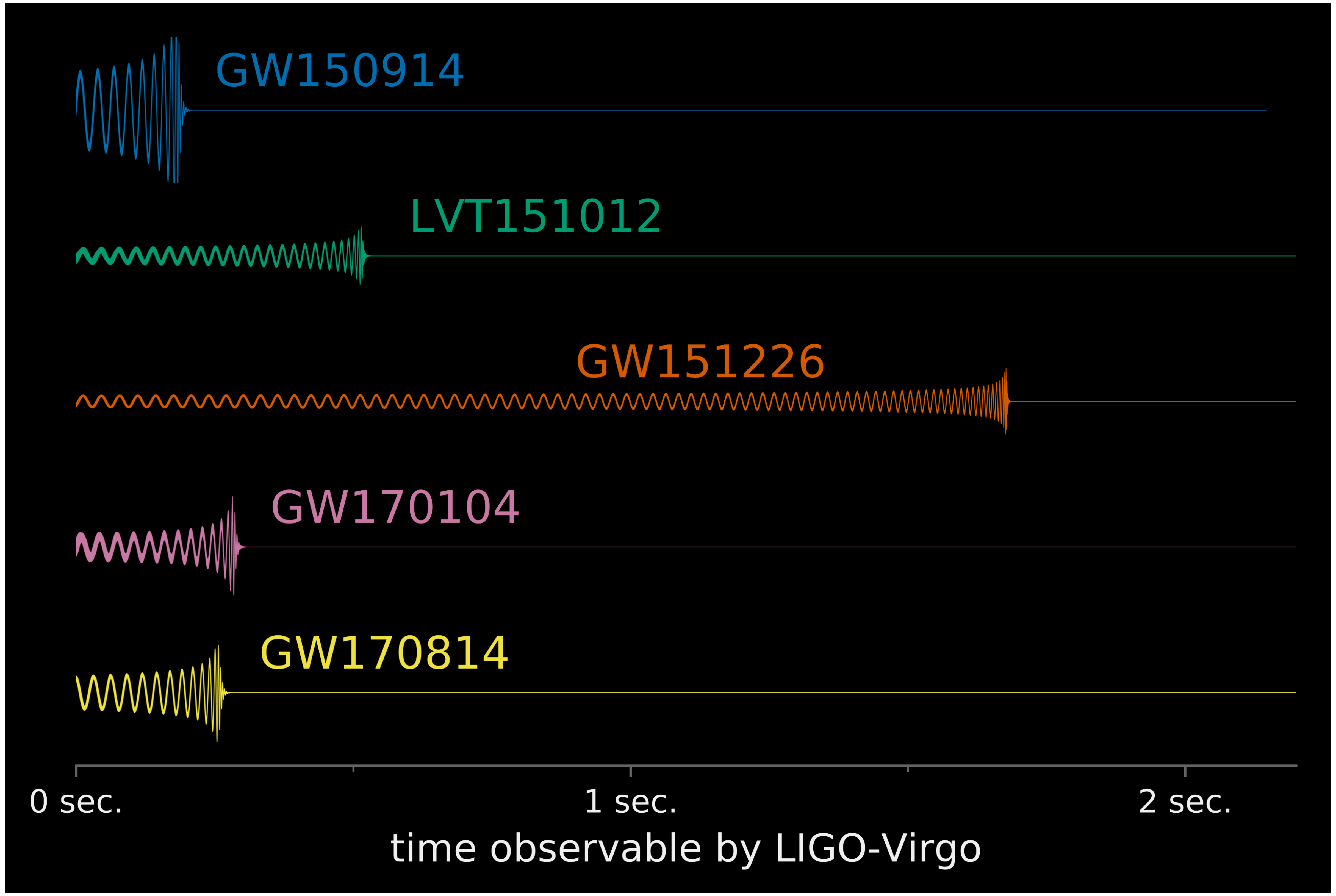
LVC,arXiv:1602.03841

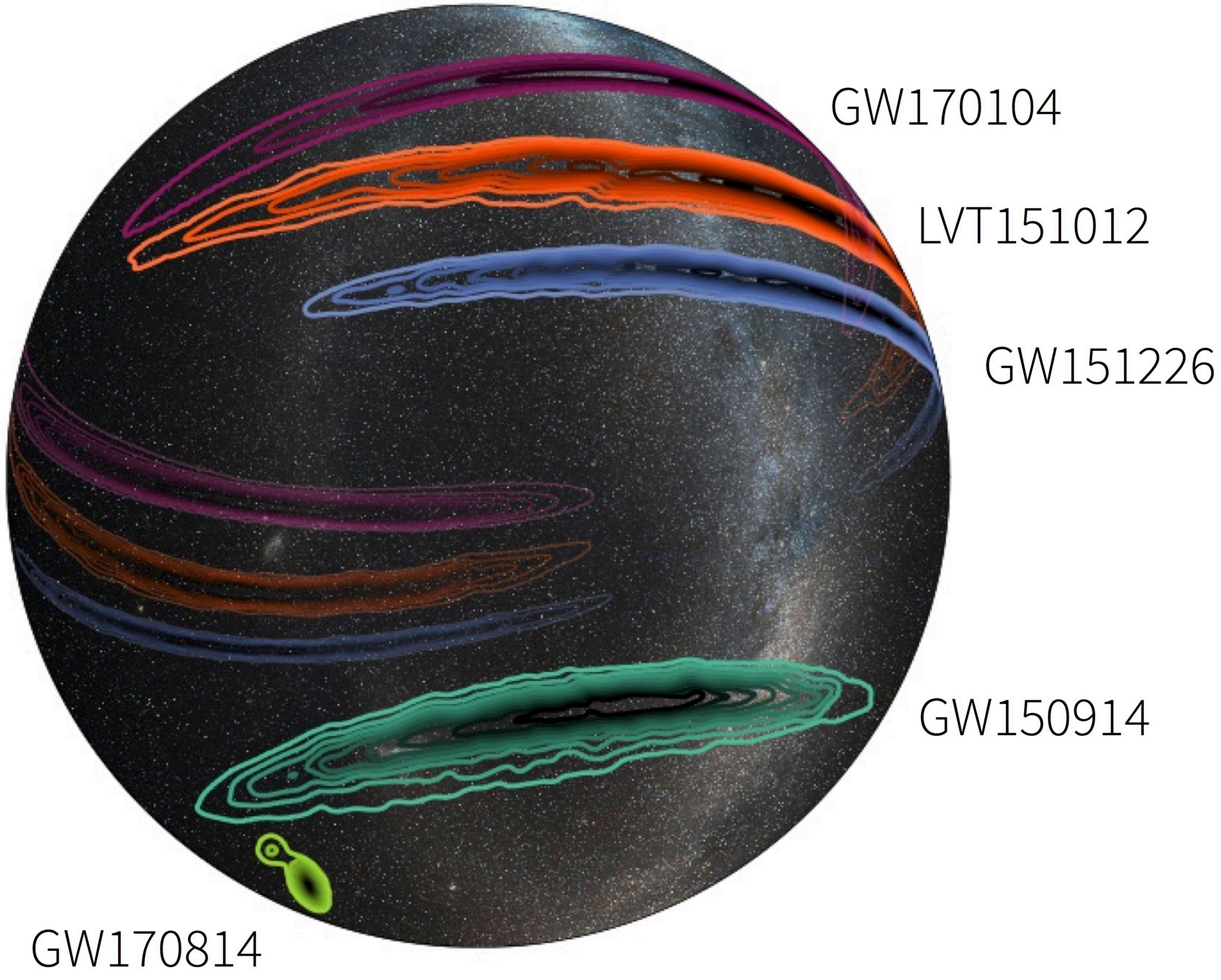
# GW170814




LVC,arXiv:1709.09660

# GW170814

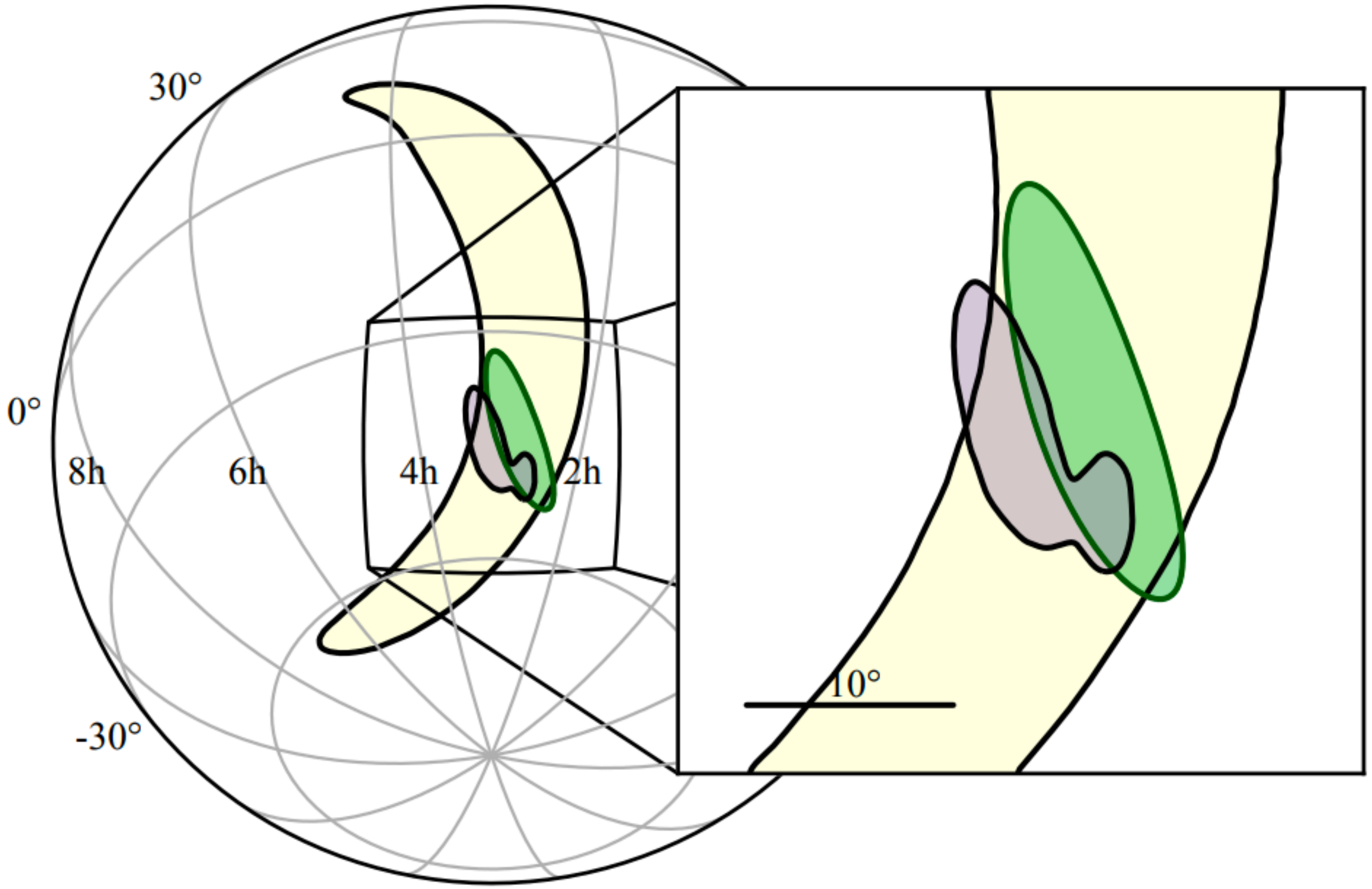



LVC,arXiv:1709.09660

# GW170814: GW polarisation

- Virgo improves dramatically the position reconstruction
- Break degeneracy with polarisation states
- Evidence for pure tensor GW against pure scalar (or pure vector)




LVC,arXiv:1709.09660

# Summary

- The era of GW astrophysics is started
- First glimpse at space-time extreme regimes:
  - **BBHs and GW behave just like GR predicts**
- Just the beginning:
  - many more detections in the future
  - improved sensitivities
  - multi-wavelength studies
- Look forward to a prolific season in gravitational physics
  - NS equation of state
  - Cosmography

# Bonus slides

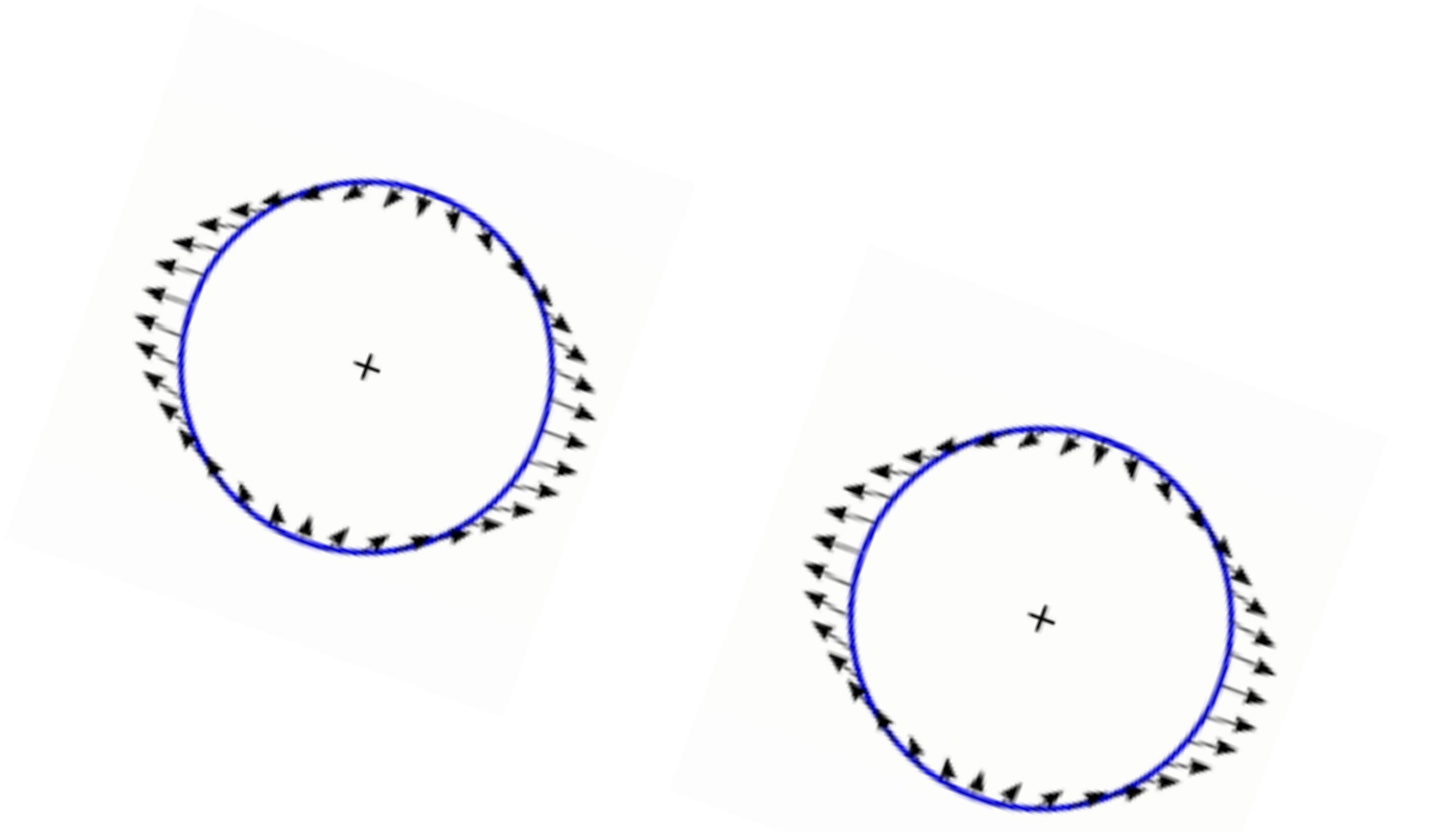
# Tidal effects in NS mergers

- Neutron stars are not point particles

- Tidal effects enter through the tidal deformability

$$Q_{ij} = -\lambda(\mathrm{EOS}; m)\tau_{ij}$$

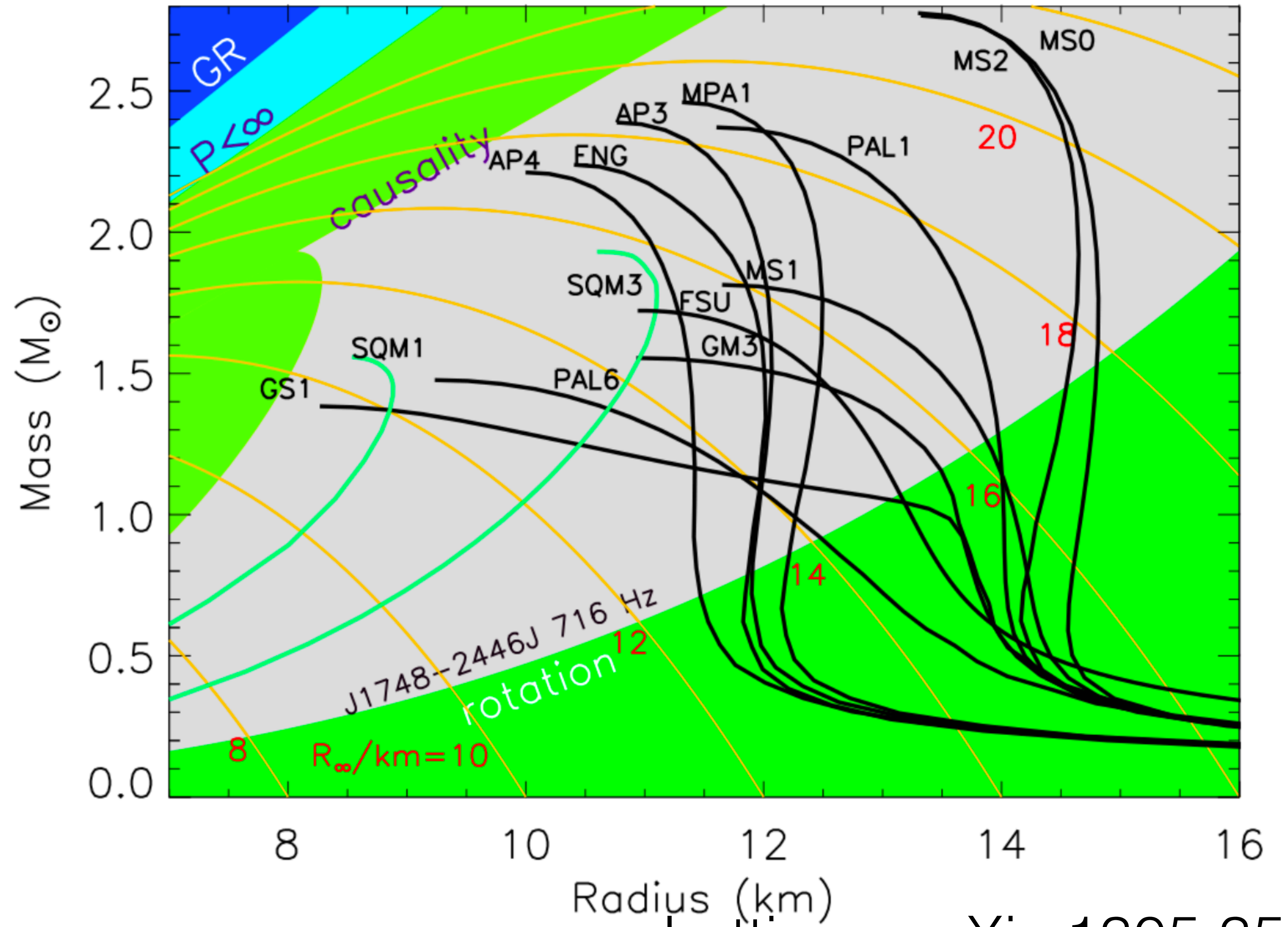
quadrupole moment — tidal field of companion star



- Tidal deformability function

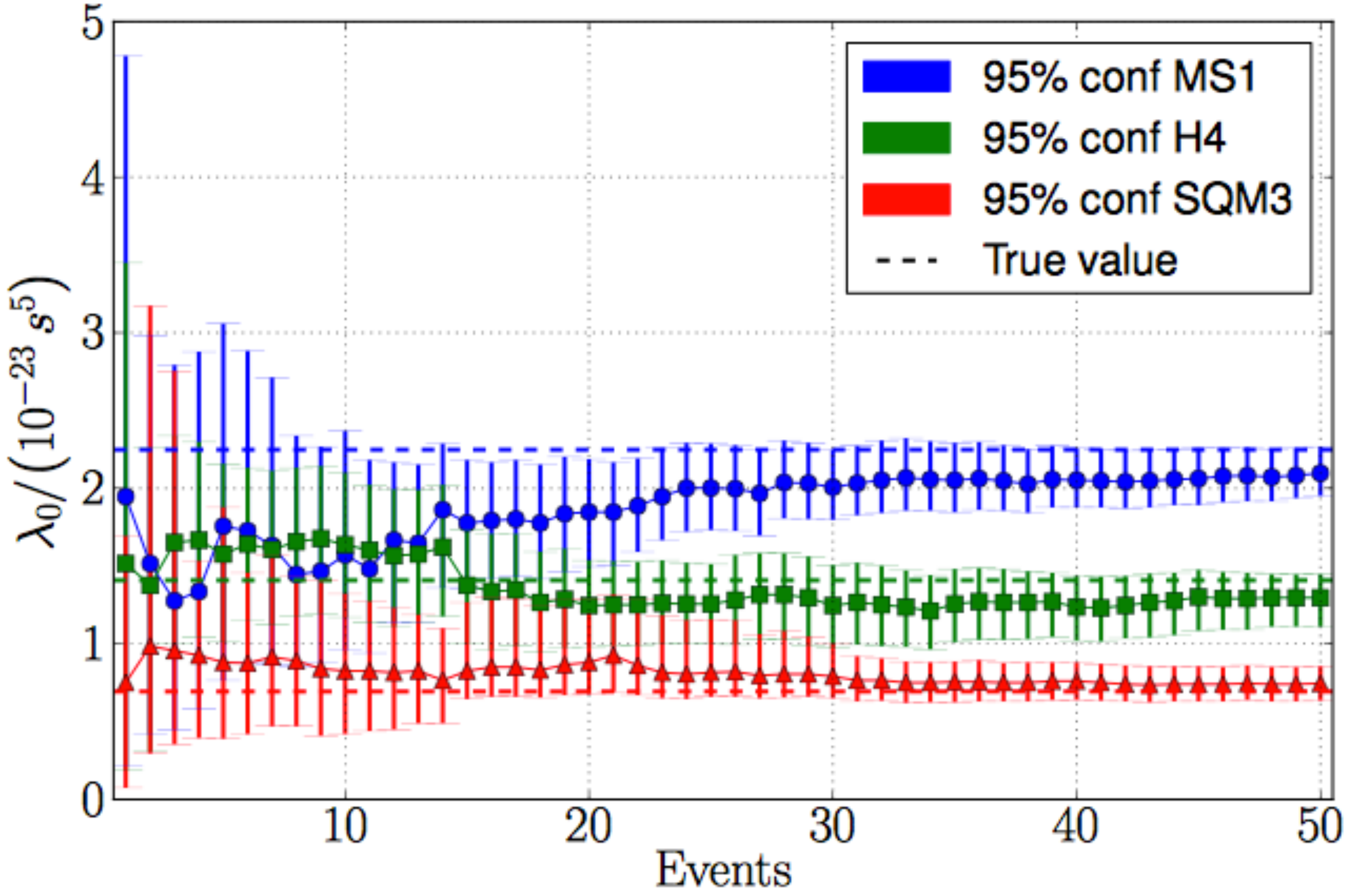
$$\lambda(m) = \frac{2}{3}k_2 R^5(m)$$

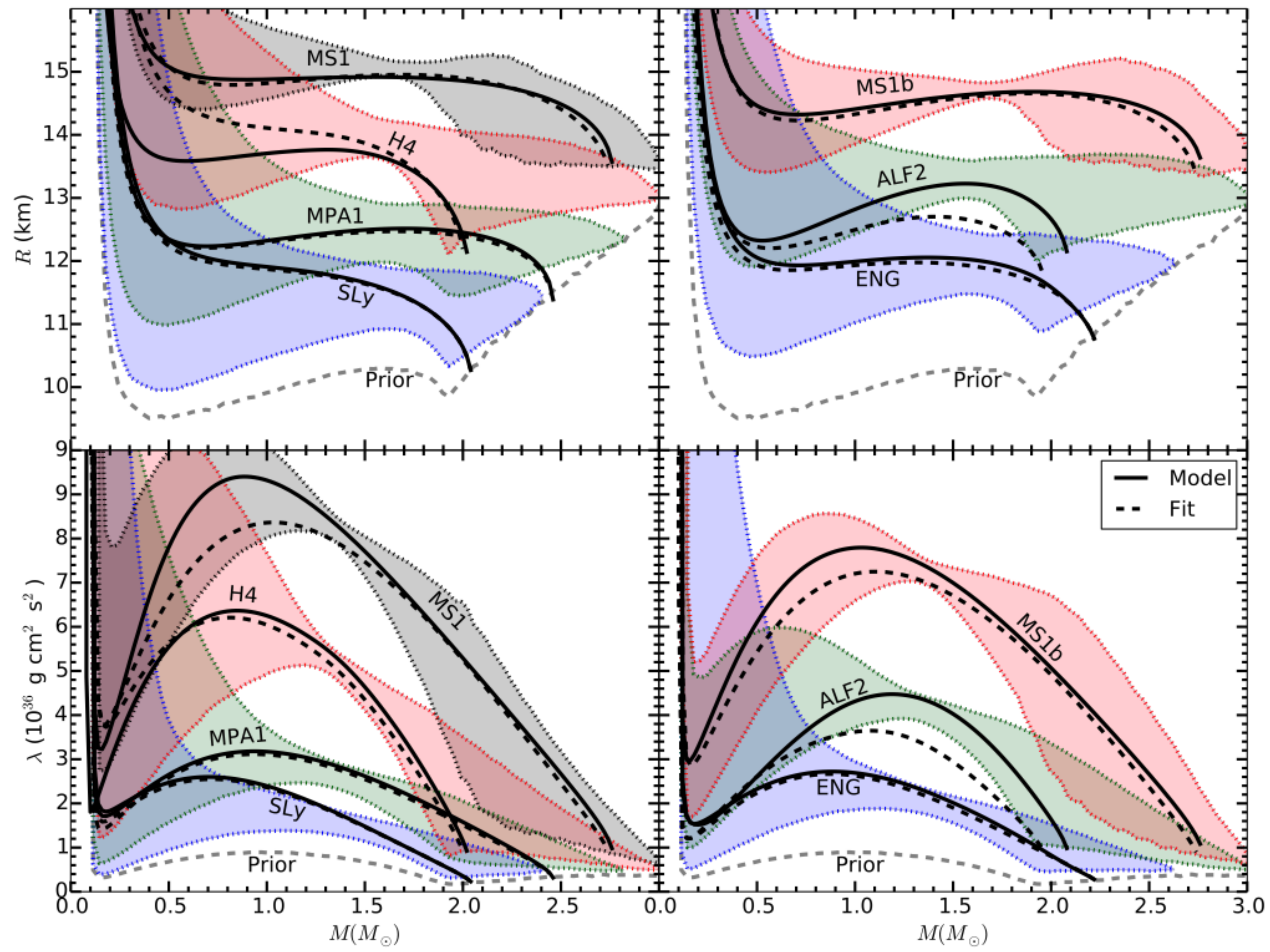
second Love number — NS radius




Lattimer, arXiv:1305.3510

# Measuring λ(m)




Del Pozzo et al, arXiv:1307.8338

Lackey & Wade, arXiv:1410.8866

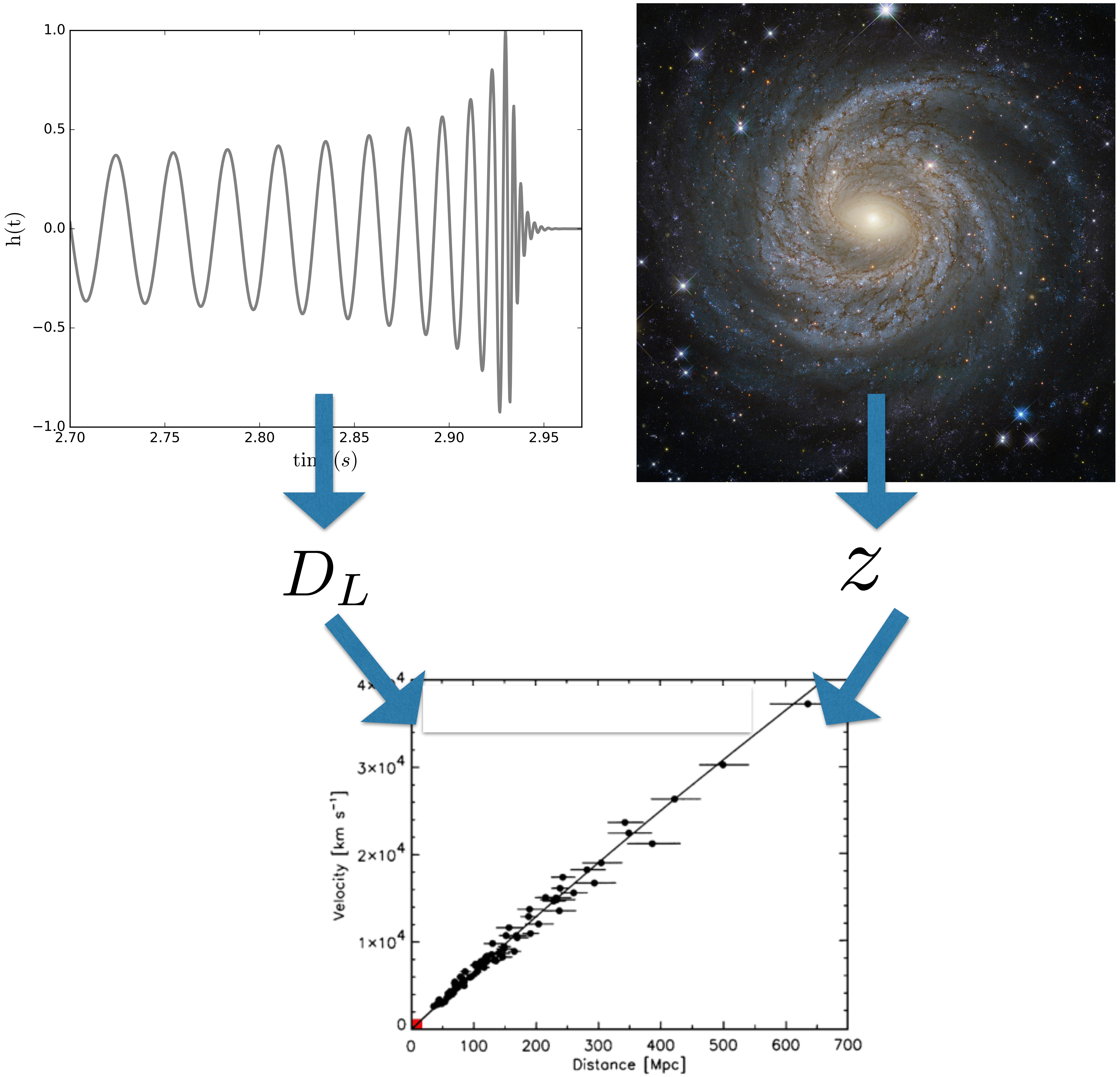
# Cosmography with GW

- GW are self-calibrating sources

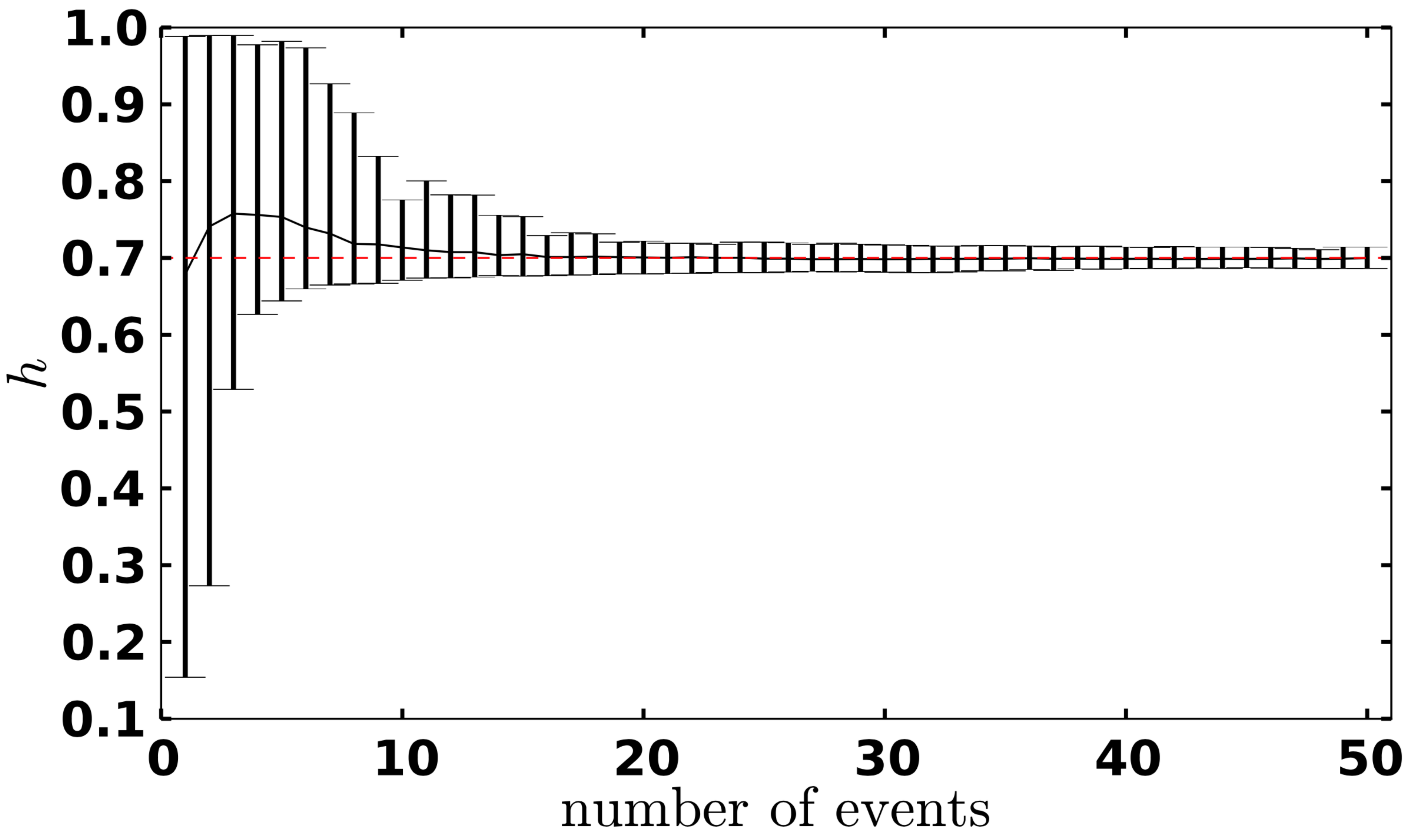
$$h \sim D_L^{-1}$$

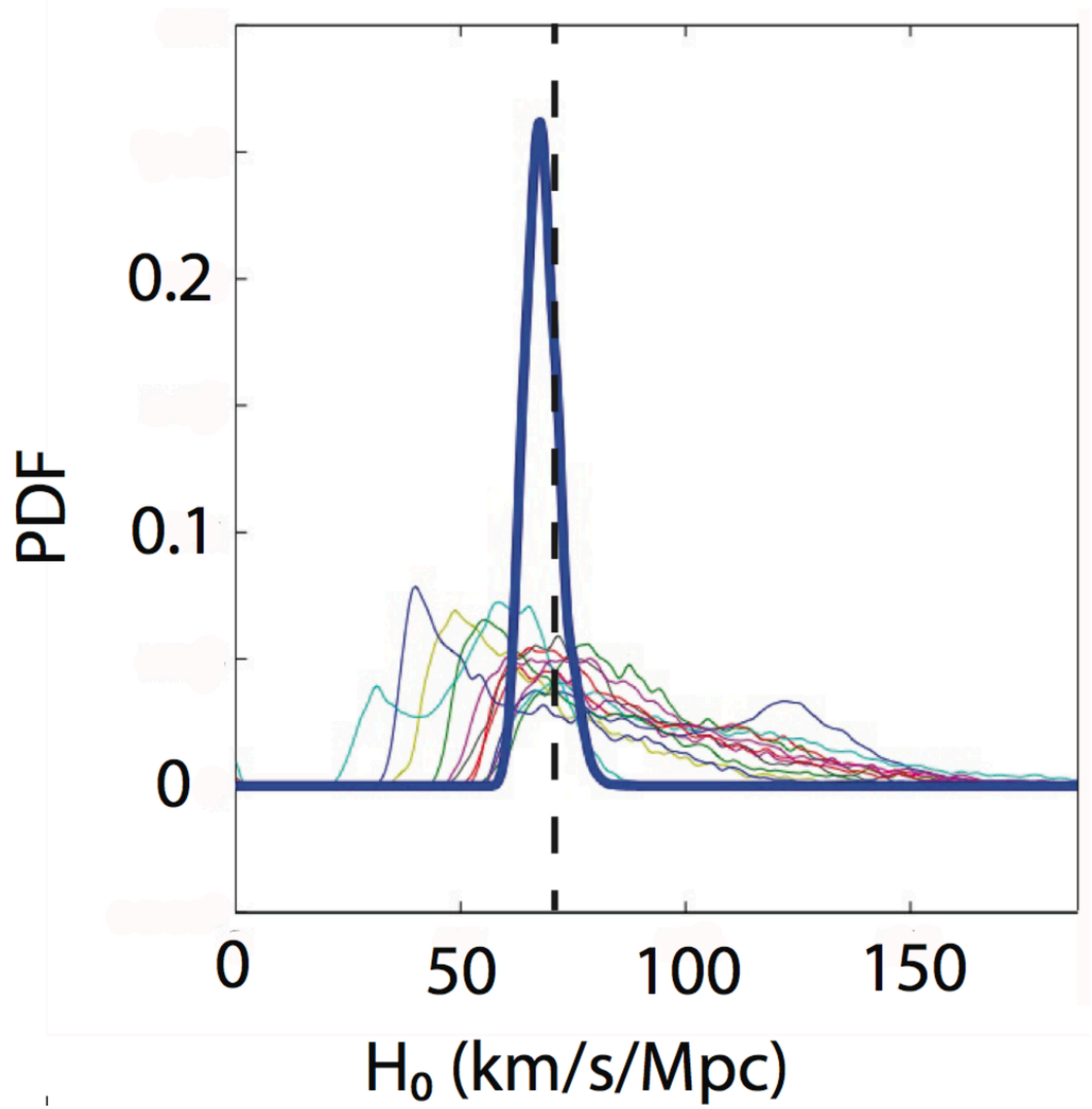
  - Direct measurement of luminosity distance

- Complemented with redshift information
  - EM counterpart
  - Host galaxy
- Determination of cosmological parameters



# Cosmography with GW




Del Pozzo, arXiv:1108.1317

Nissanke et al, arXiv:1307.2638