# Gravitational wave as probe of the equation of state of nuclear matter

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Jan 16th, 2018

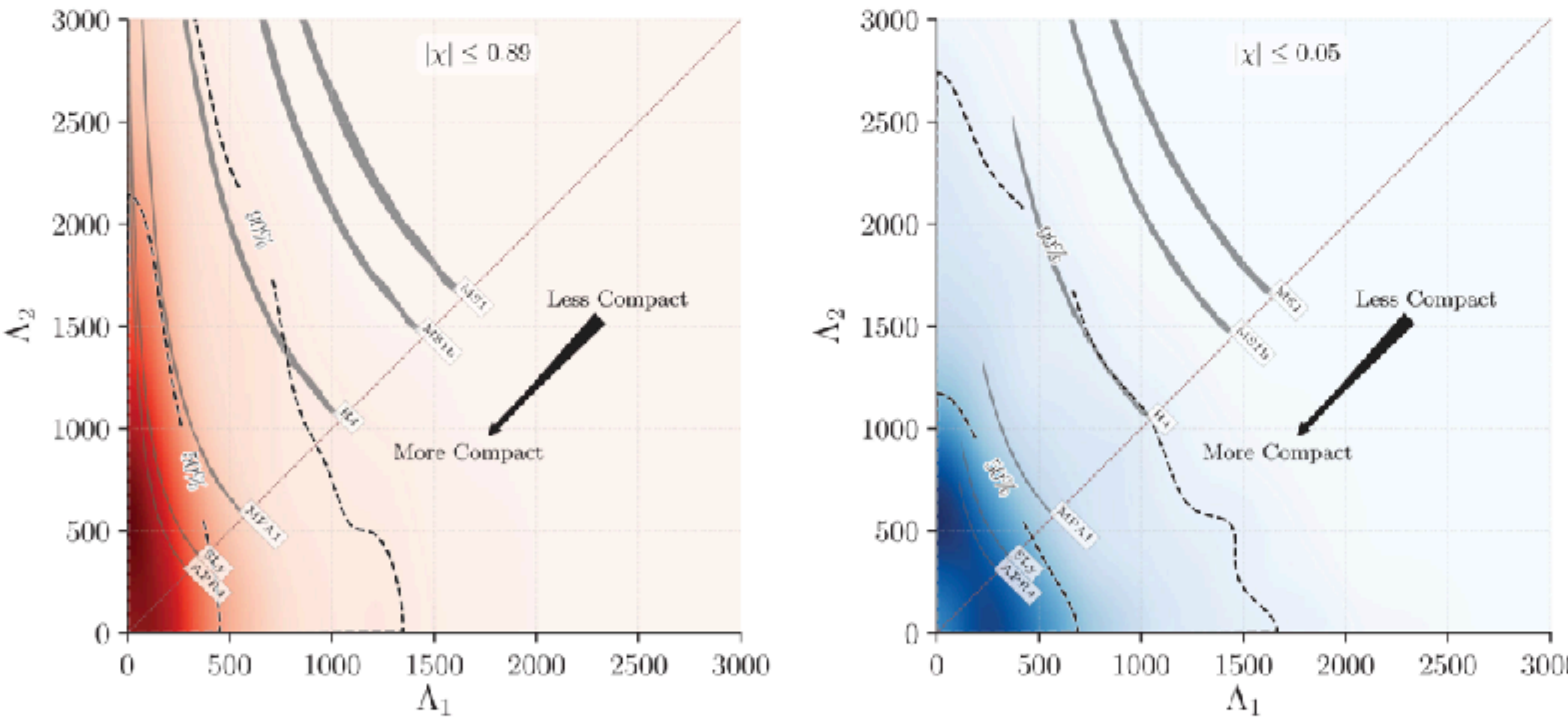
# Main goal

PRL 119, 161101 (2017) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

**GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral**

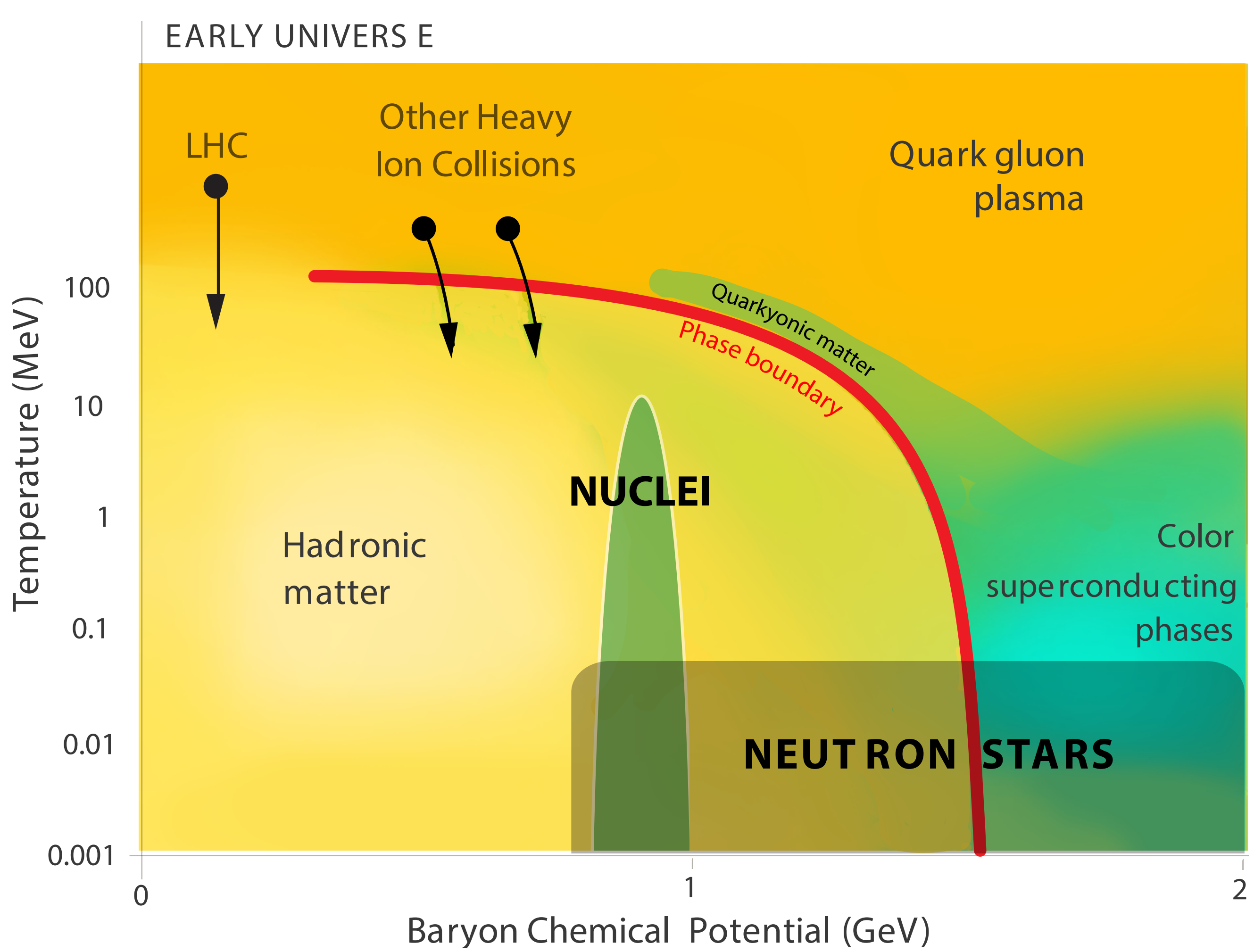
B. P. Abbott *et al.*[*]
(LIGO Scientific Collaboration and Virgo Collaboration)

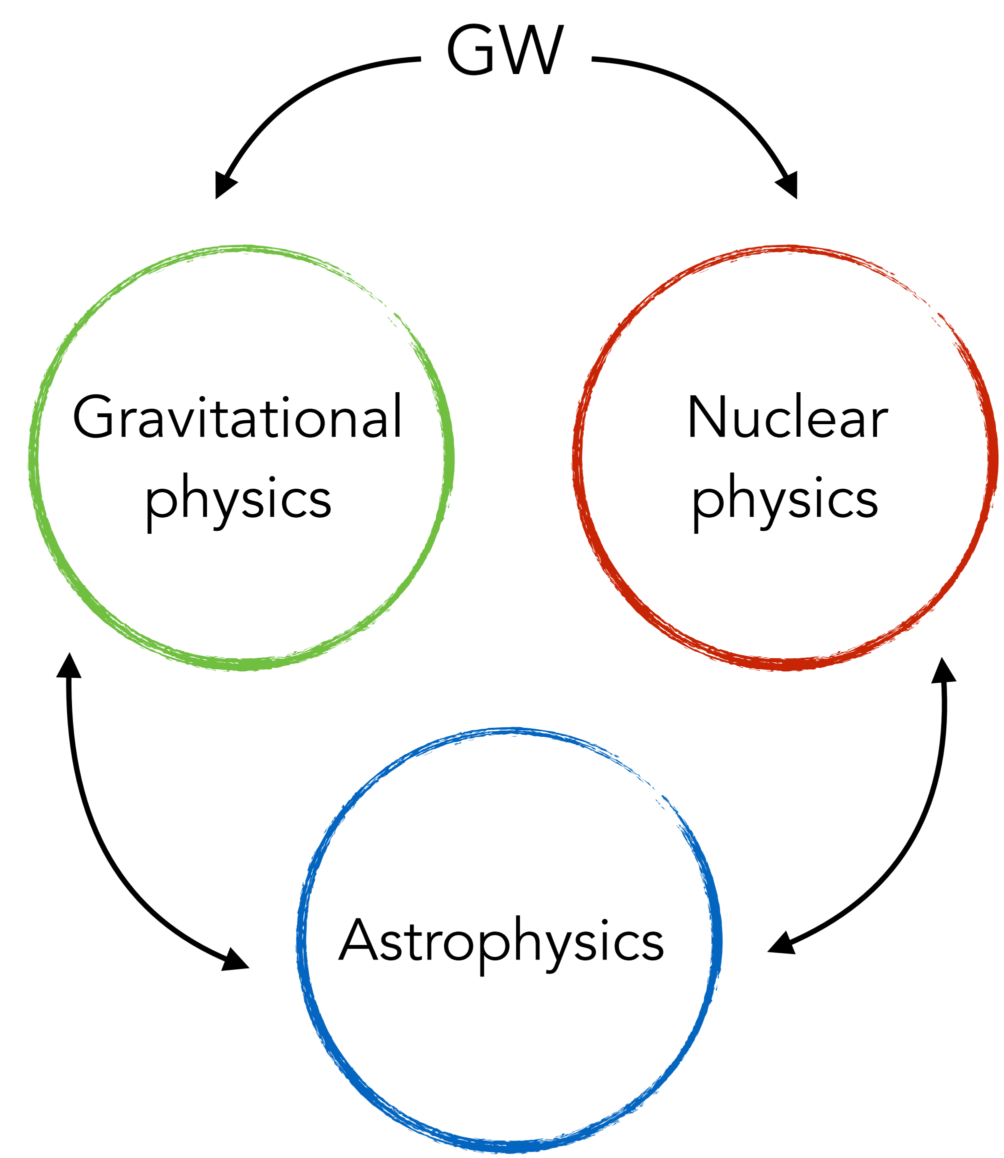
(Received 26 September 2017; revised manuscript received 2 October 2017; published 16 October 2017)



# Fundamental physics with NS

✔ How nuclear matter behaves under extreme conditions?



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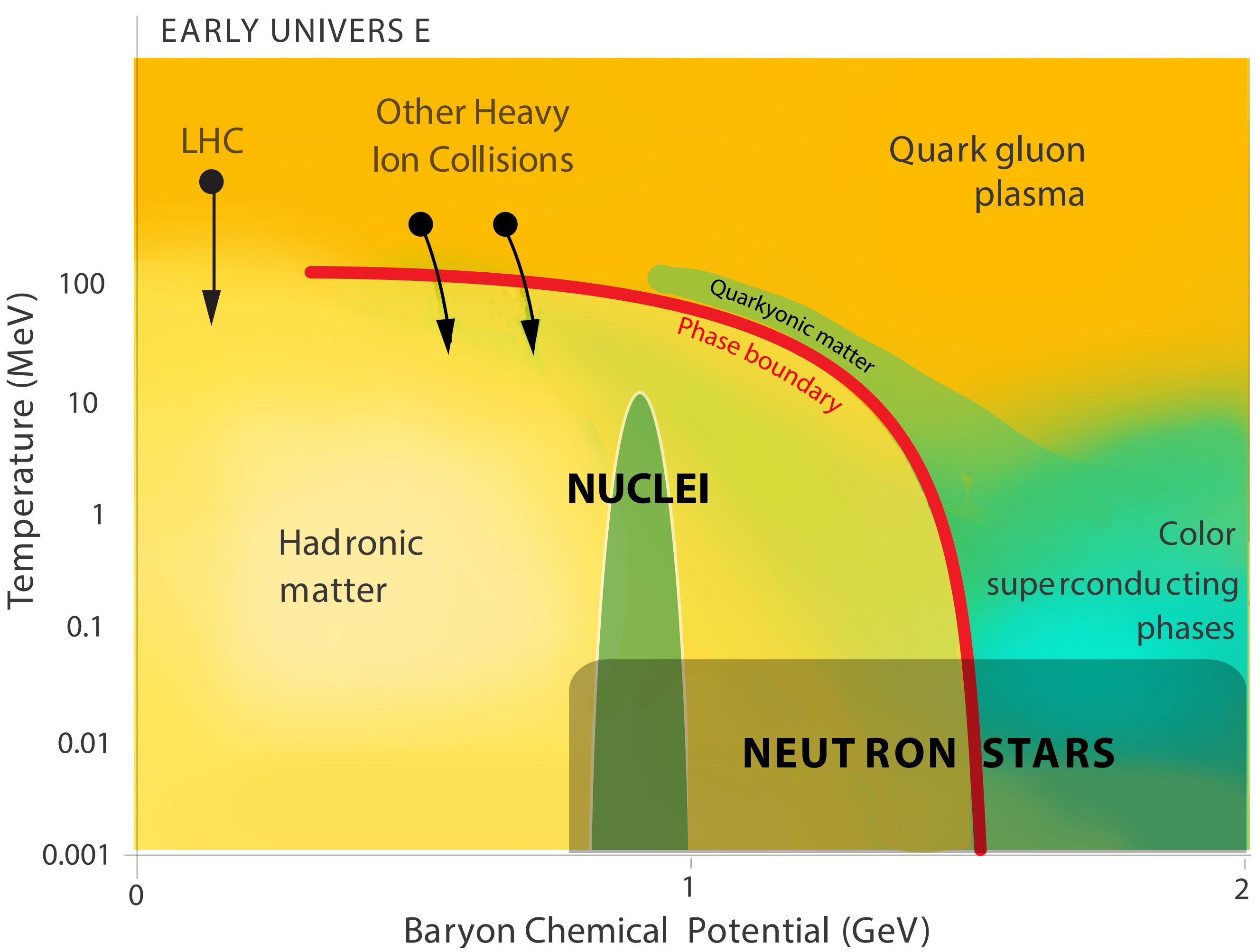
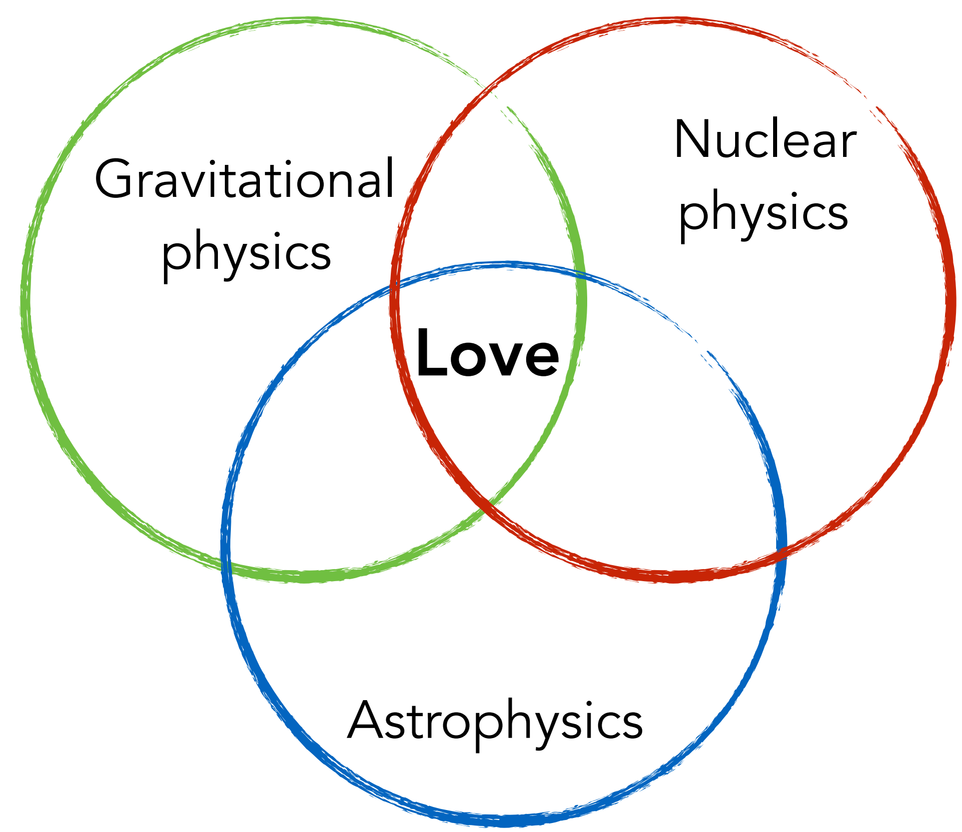
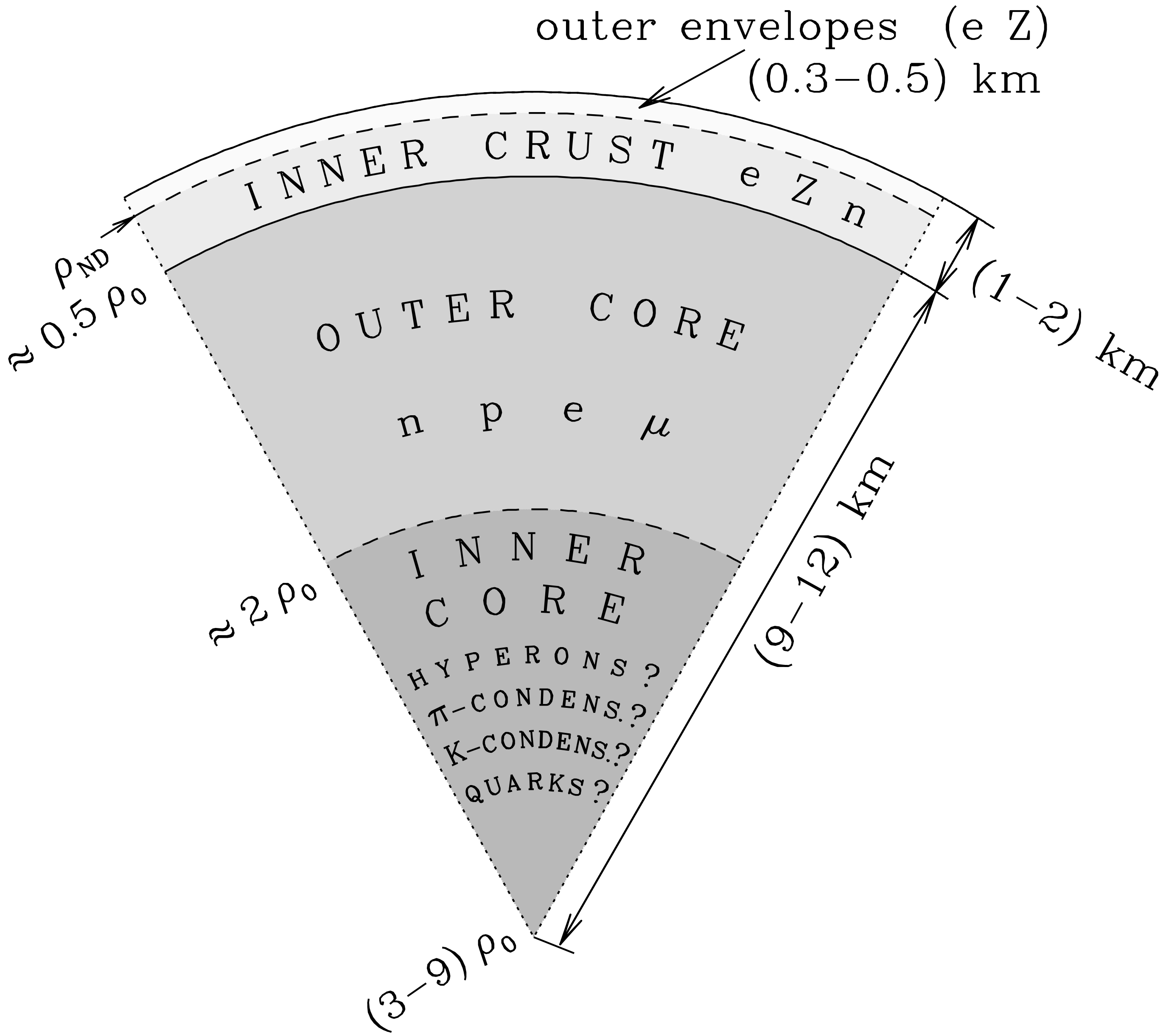


# Inside a NS




$$\rho_0 \simeq 2.67 \times 10^{14} \text{g cm}^{-3}$$

*Lattimer & Prakash, 2007*
*Caplan & Horowitz 2017*

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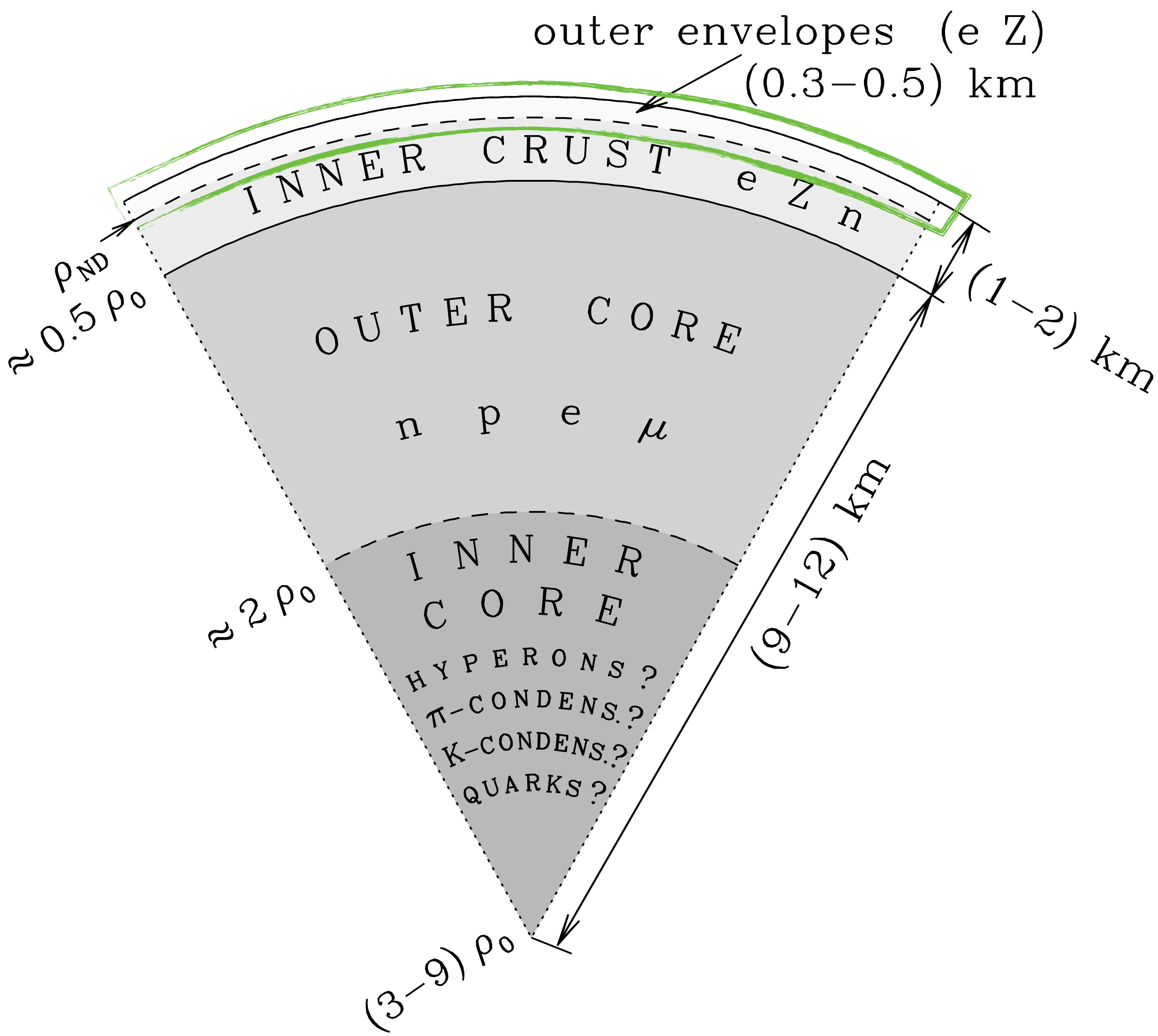


Nuclei lattice within e⁻ gas

$$\rho_0 \simeq 2.67 \times 10^{14} \mathrm{g\ cm^{-3}}$$

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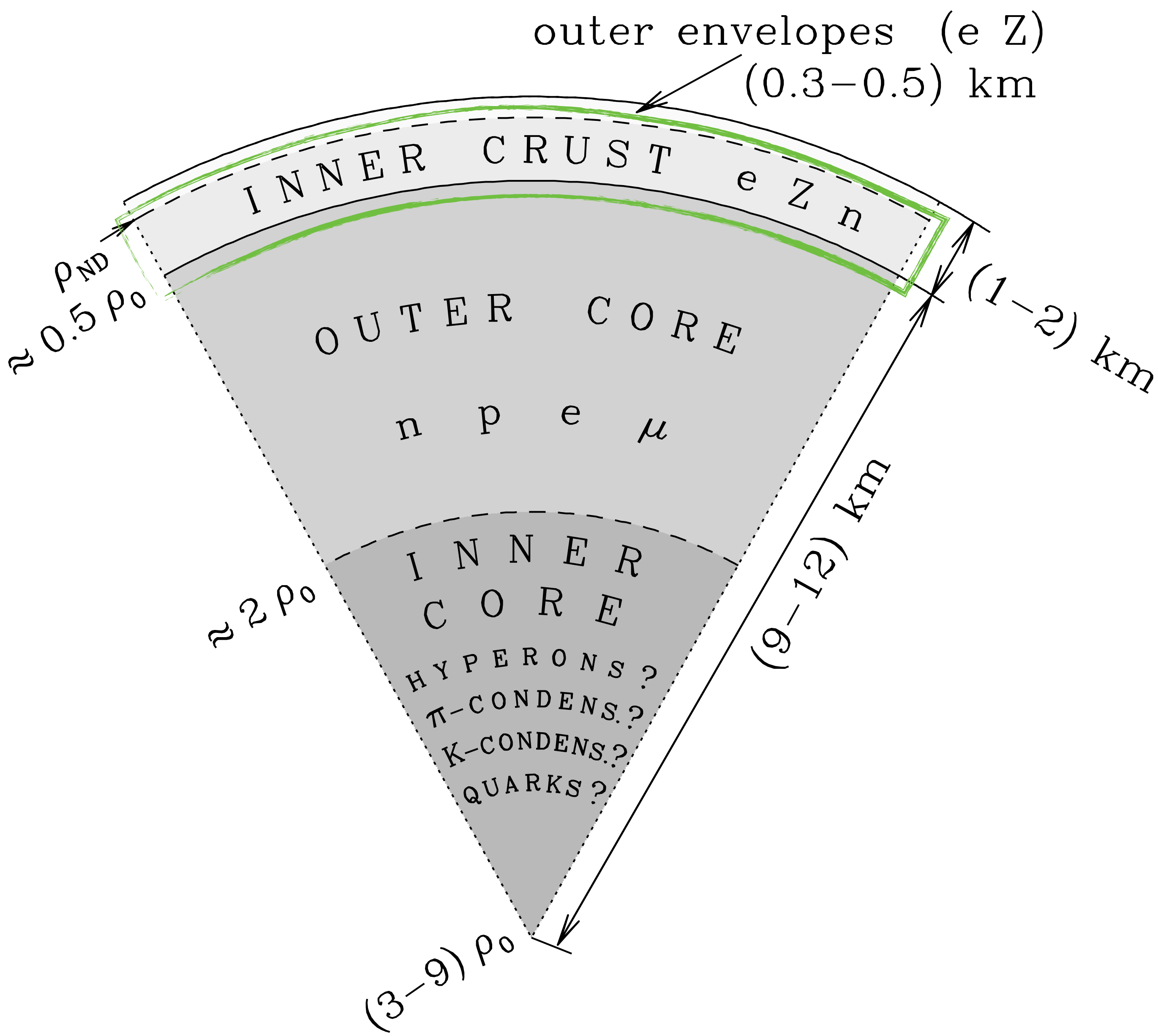
# Inside a NS

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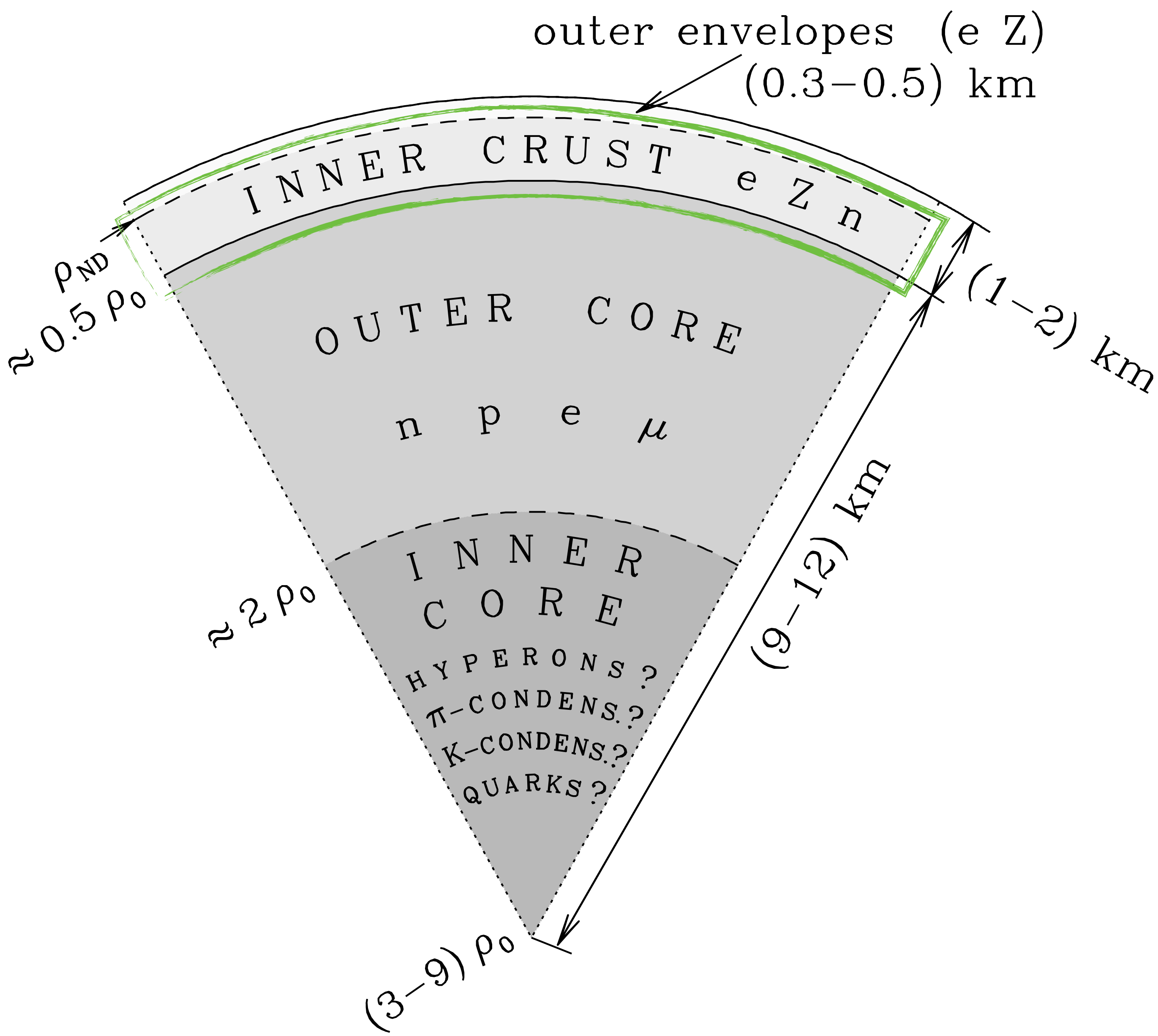
Pasta phases

*Lattimer & Prakash, 2007*
*Caplan & Horowitz 2017*

# Inside a NS

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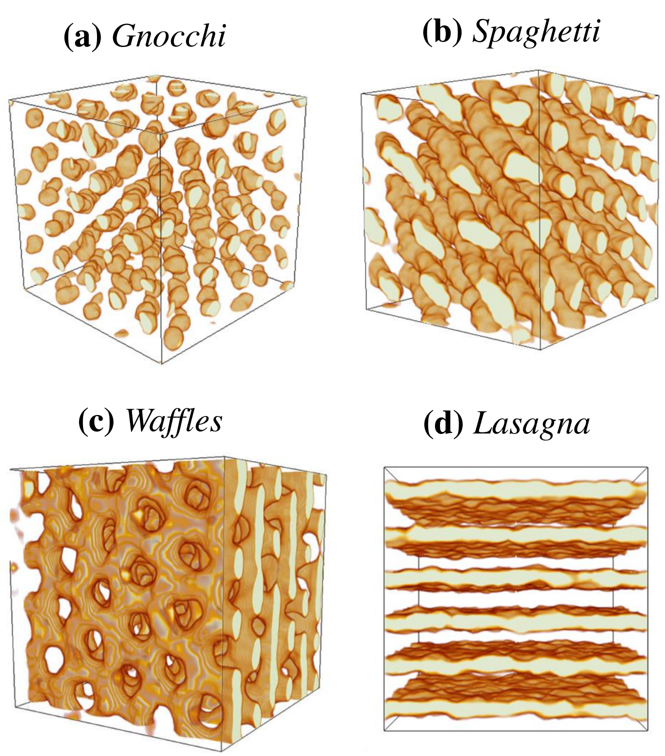
## Pasta phases




*Lattimer & Prakash, 2007*
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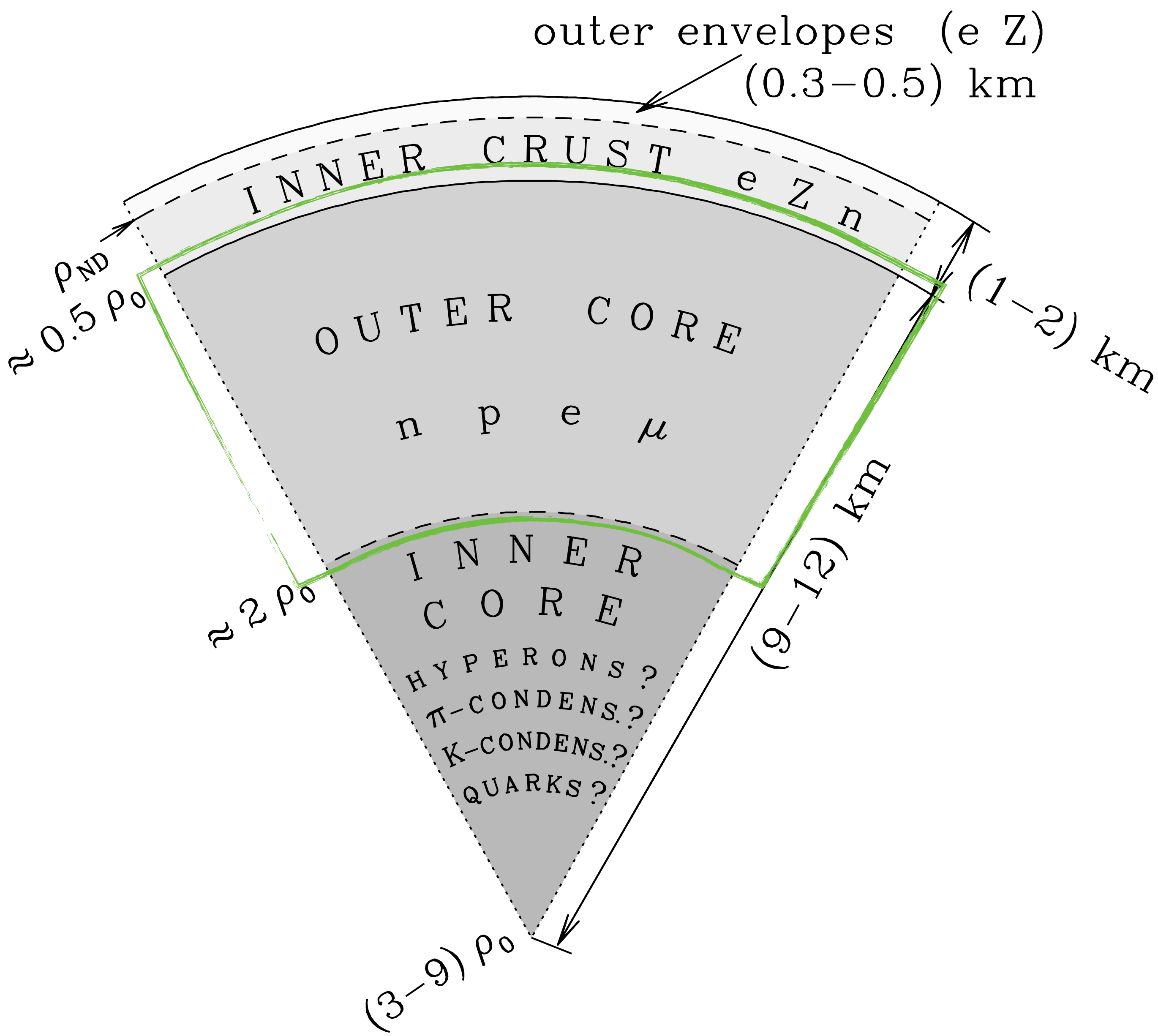
# Inside a NS

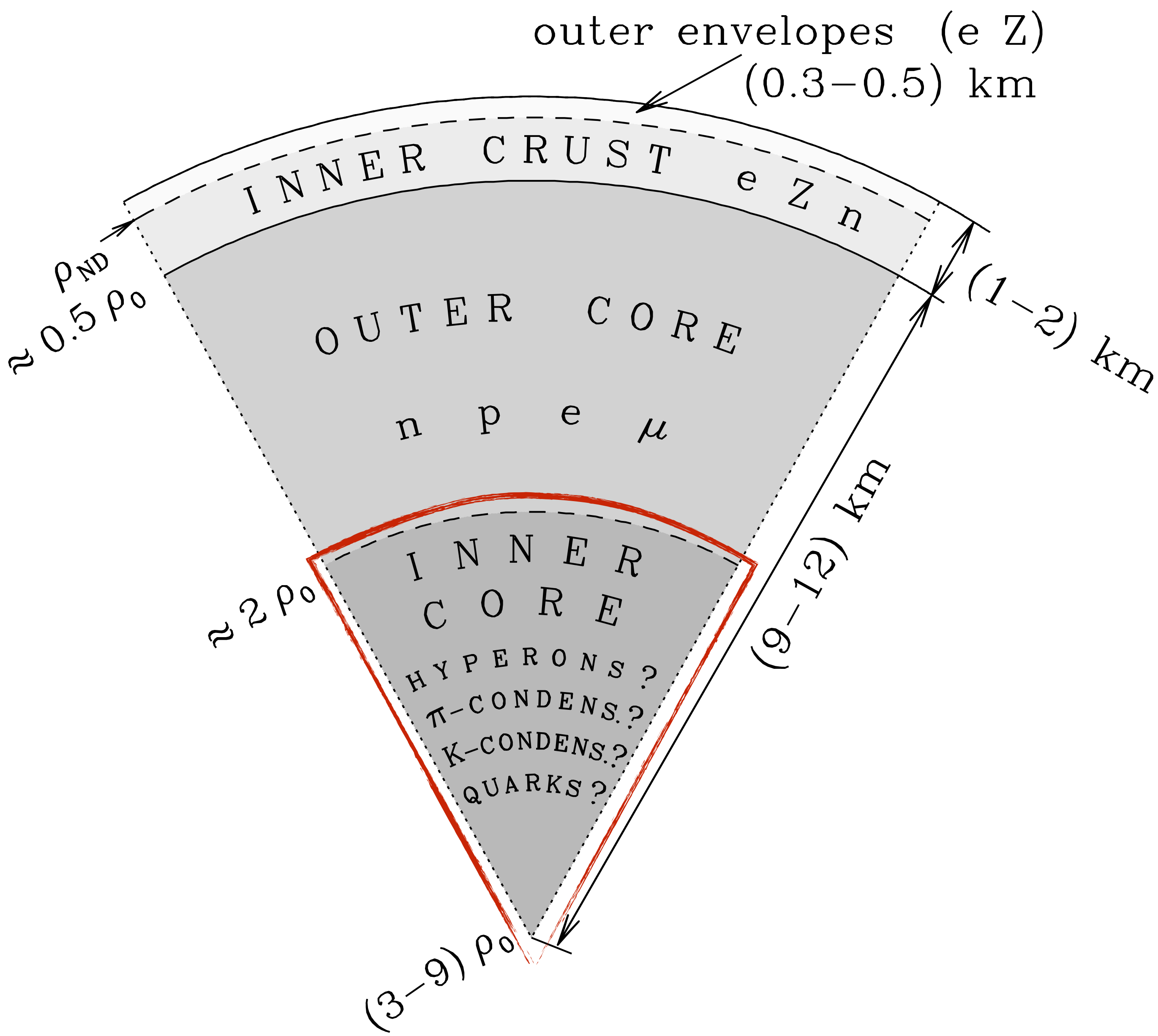



$\rho_0 \simeq 2.67 \times 10^{14} \text{g cm}^{-3}$

Nucleonic matter in $\beta-$ equilibrium

*Lattimer & Prakash, 2007*
*Caplan & Horowitz 2017*

# Inside a NS

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Phase transitions, quark-gluon plasma ...

*Lattimer & Prakash, 2007*
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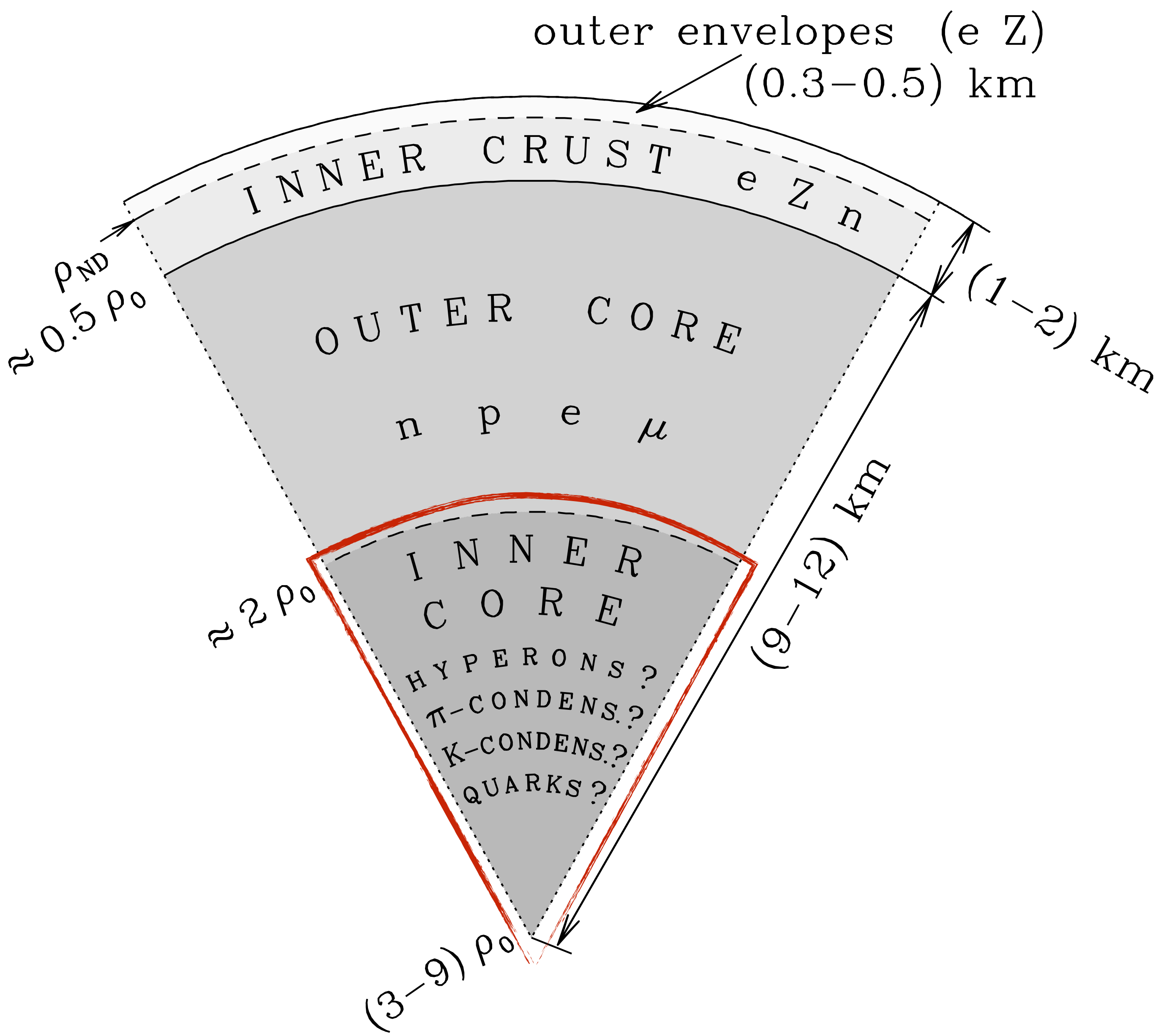
# Inside a NS

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Phase transitions, quark-gluon plasma ...

Constraints at super-nuclear density

*Lattimer & Prakash, 2007*
*Caplan & Horowitz 2017*

# Inside a NS

Different models to be tested by observations

- ✓ Non-Relativistic Many Body theory

$$\mathcal{H} = \sum_i \frac{p_i^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>i>j} V_{ijk}$$

$v_{ij}$ → 2-body; $V_{ijk}$ → 3-body

- ✓ Relativistic Mean field theory

$$\mathcal{L} = \mathcal{L}_{\rm N} + \mathcal{L}_{\rm B} + \mathcal{L}_{\rm int}$$

  - ✓ Can be generalised to include strange baryons [interactions?]

- ✓ *MIT Bag model* to describe unconfined quarks

# Inside a NS

Different models to be tested by observations

✔ Non-Relativistic Many Body theory

$$\mathcal{H} = \sum_i \frac{p_i^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>i>j} V_{ijk}$$

3-body

2-body

ask O. Benhar

✔ Relativistic Mean field theory

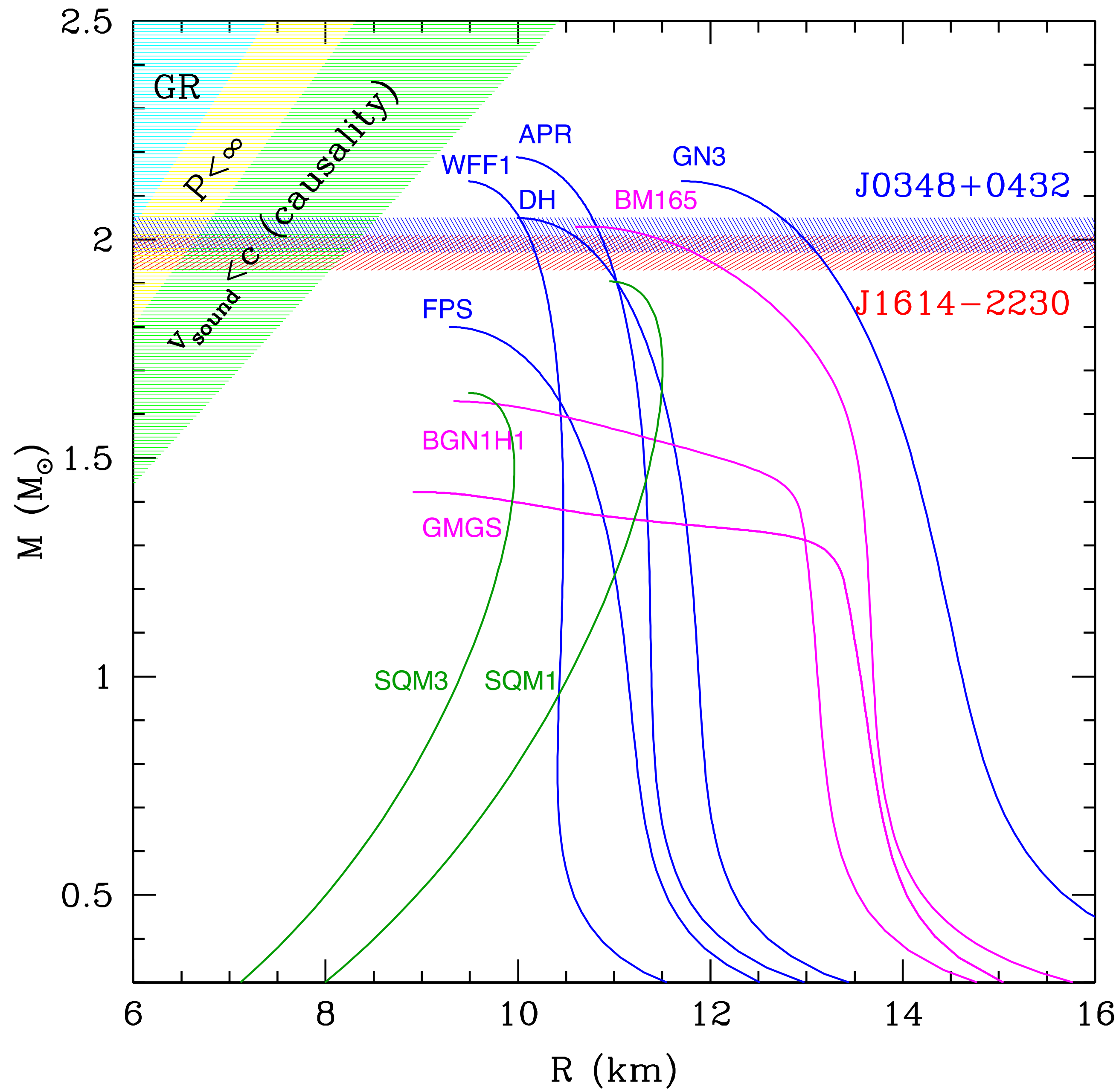
$$\mathcal{L} = \mathcal{L}_{\rm N} + \mathcal{L}_{\rm B} + \mathcal{L}_{\rm int}$$

✔ Can be generalised to include strange baryons [interactions?]

✔ *MIT Bag model* to describe unconfined quarks

# From micro to macro

## The Equation of State $p = p(\epsilon)$



# From micro to macro

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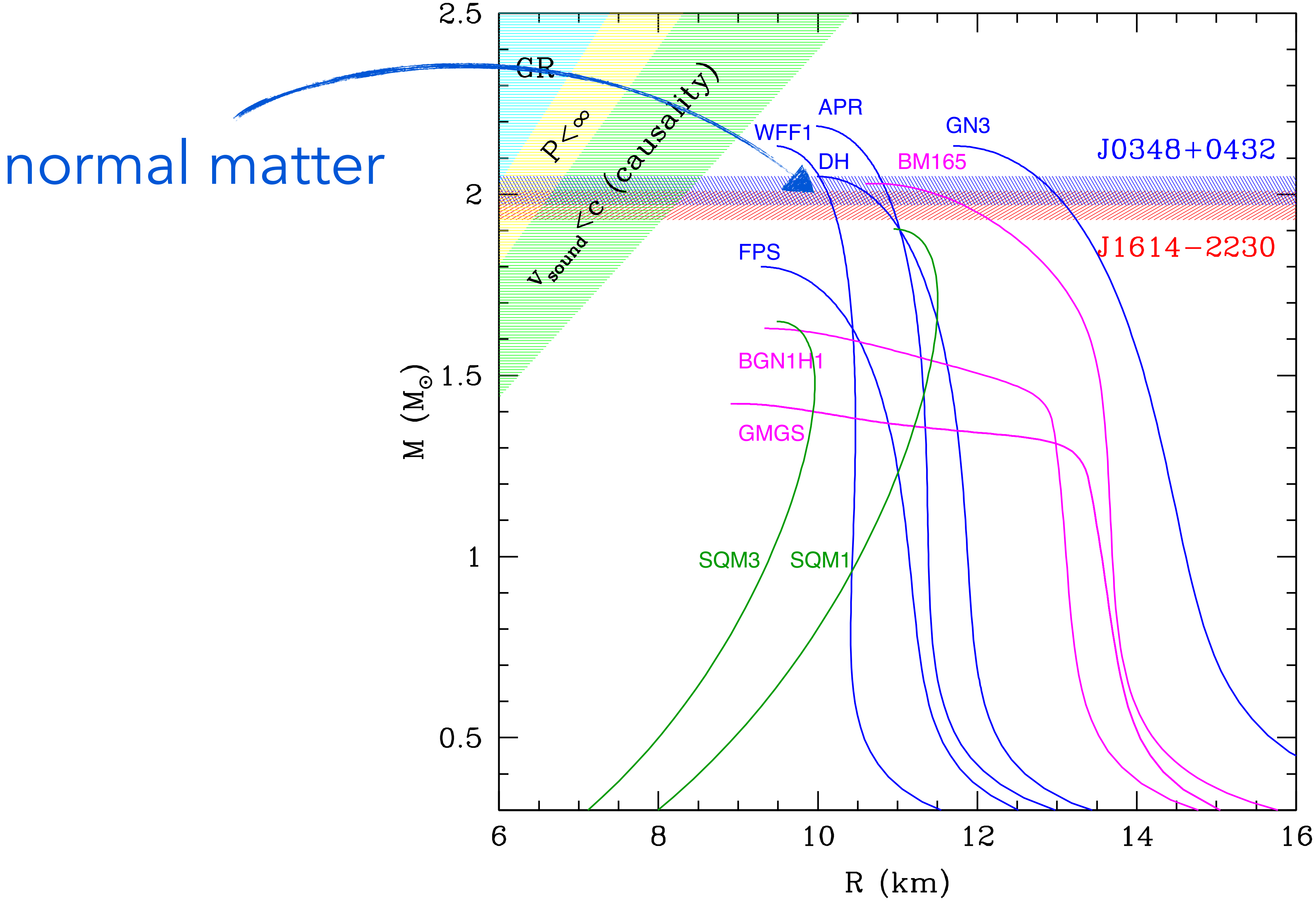
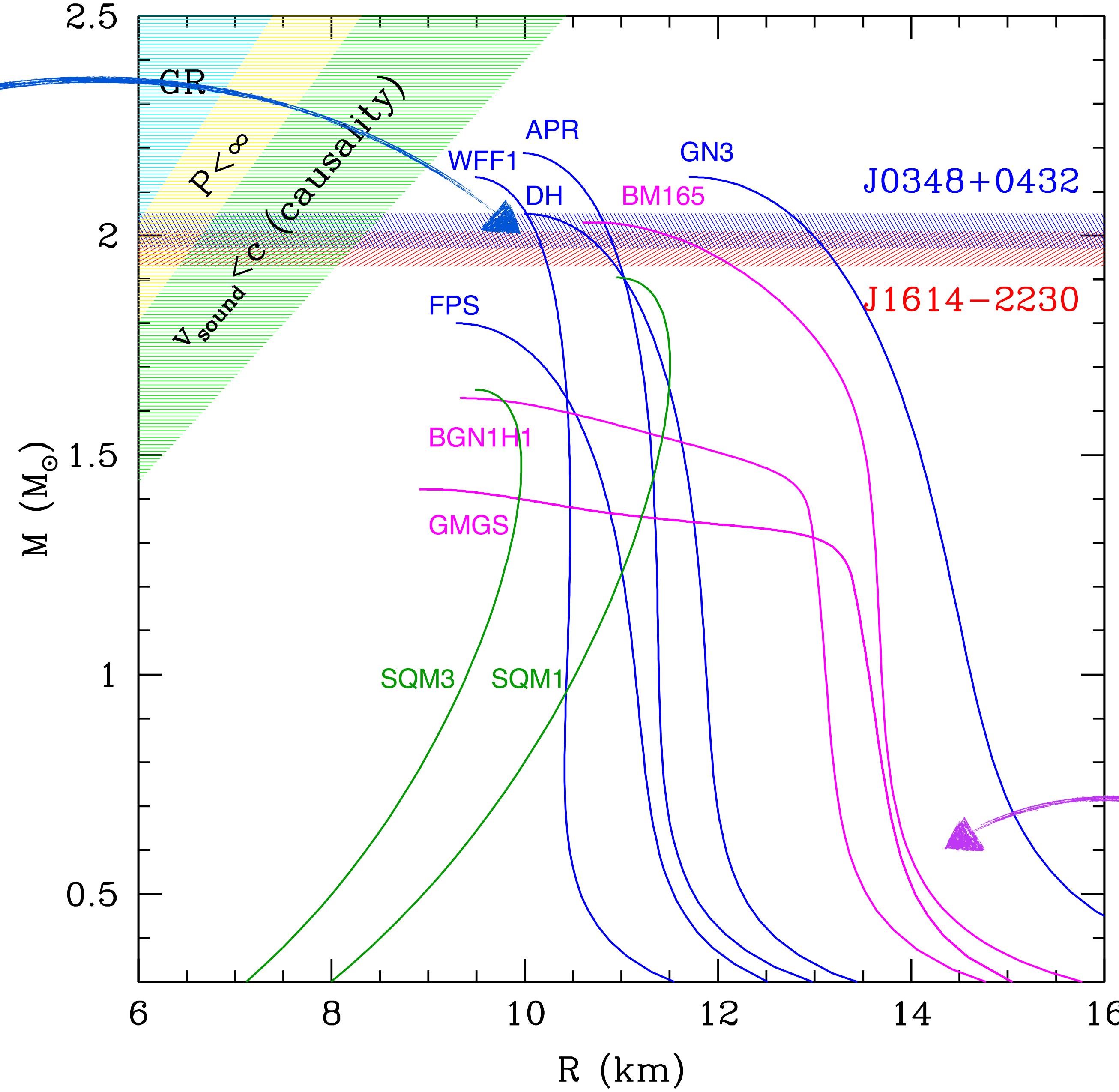


# From micro to macro

## The Equation of State $p = p(\epsilon)$

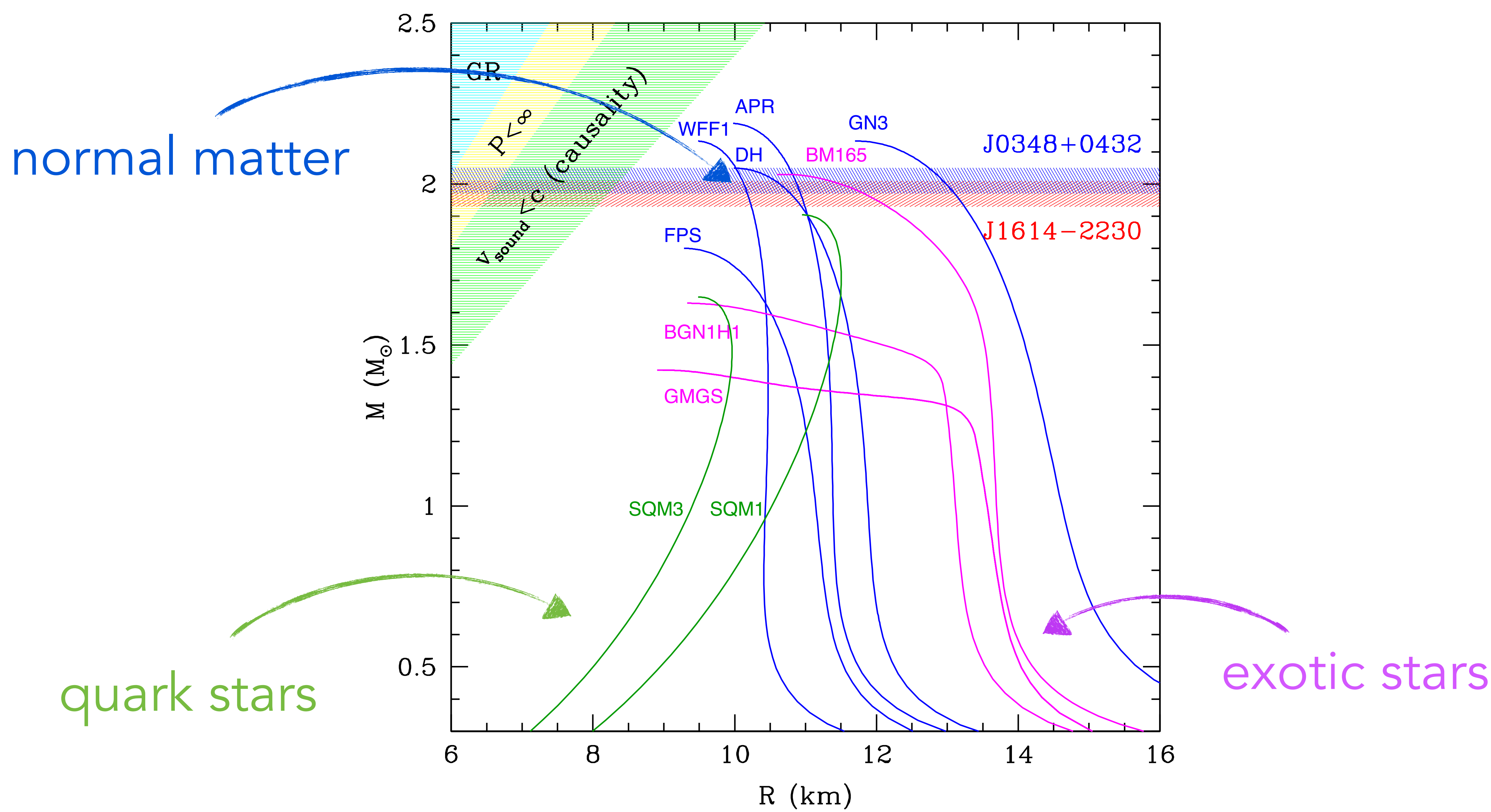
normal matter




exotic stars

# From micro to macro

The Equation of State $p = p(\epsilon)$



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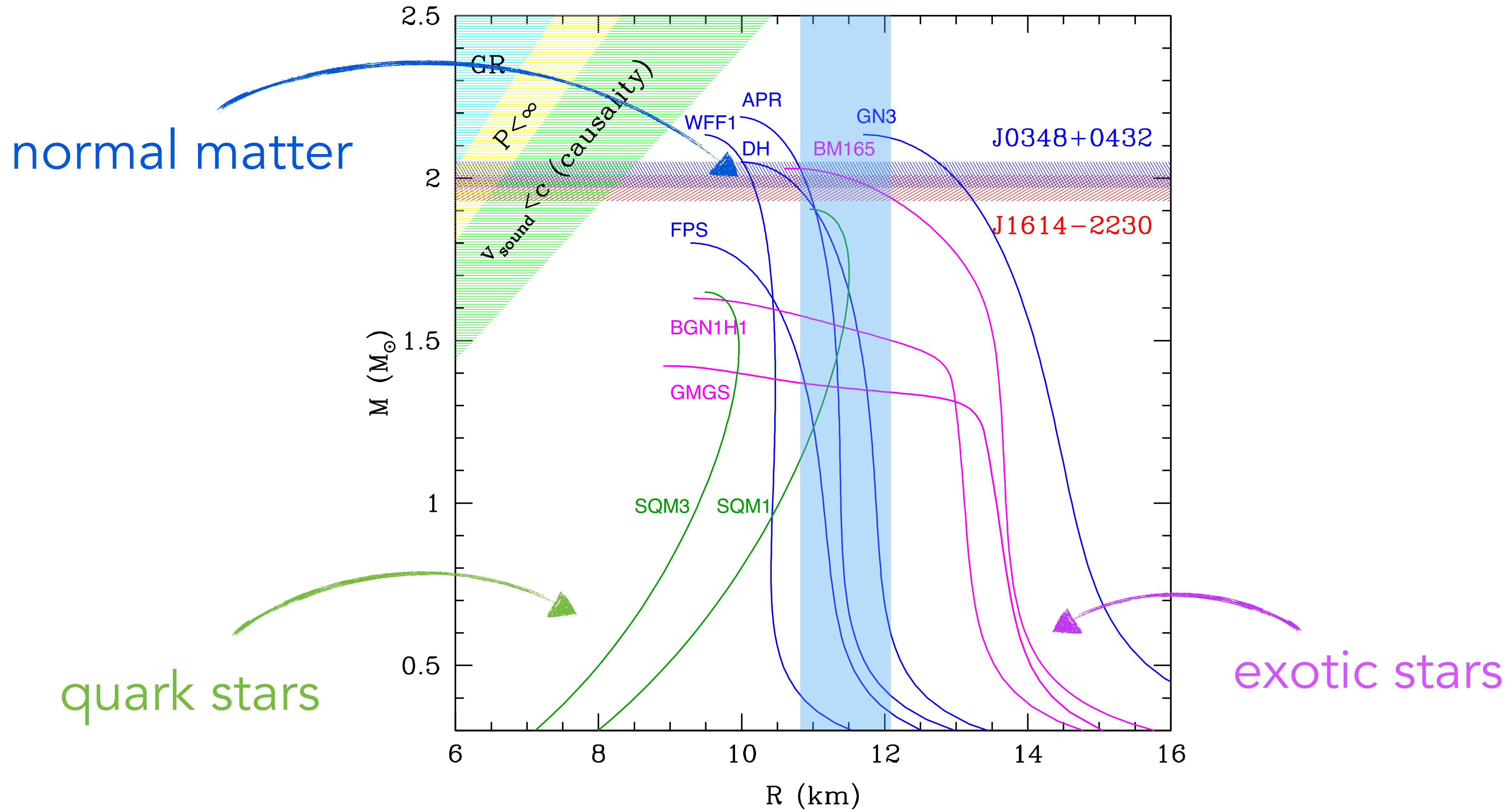
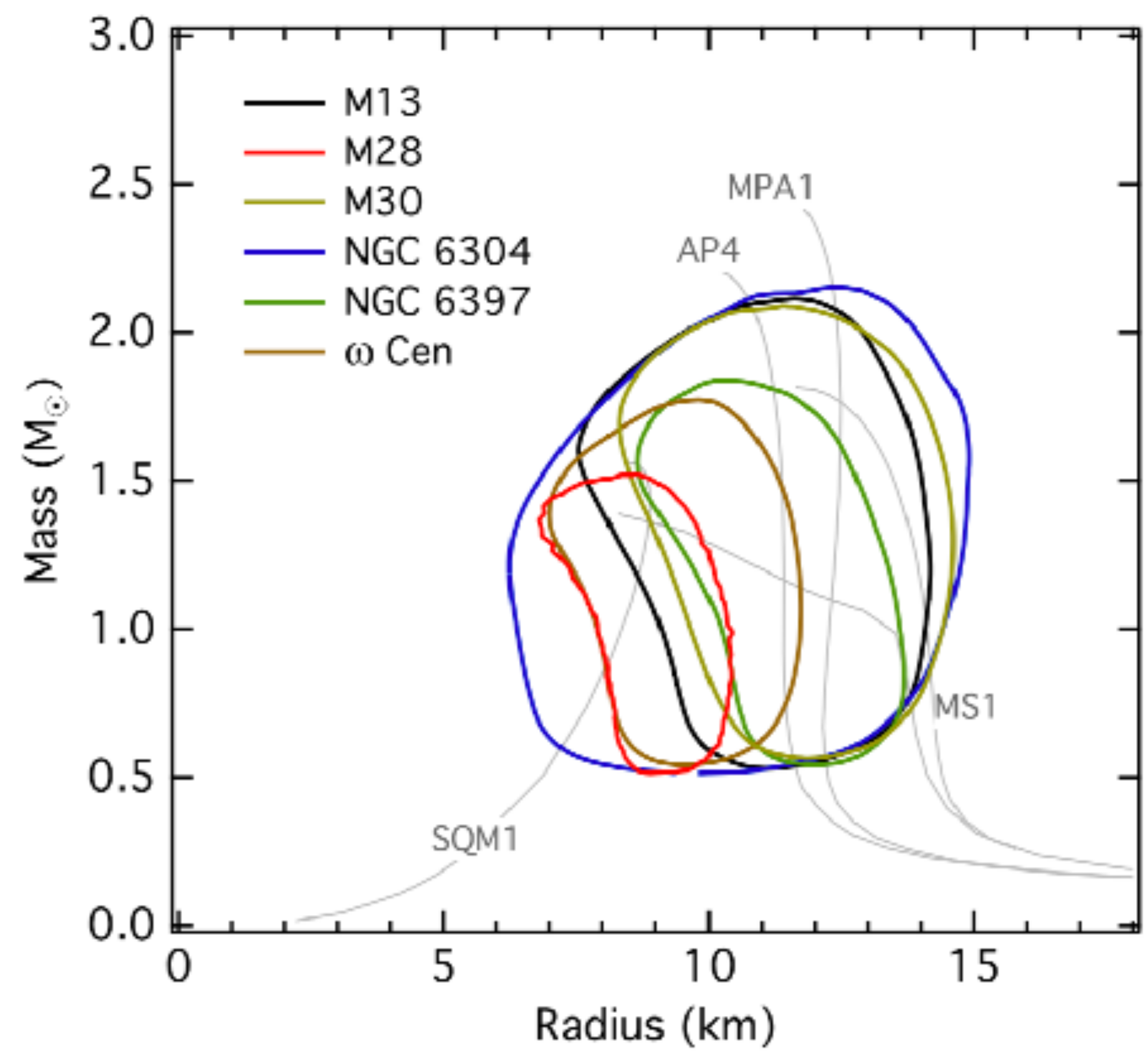


# The radius so far

Observed masses $[1 \div 2] M_{\odot}$

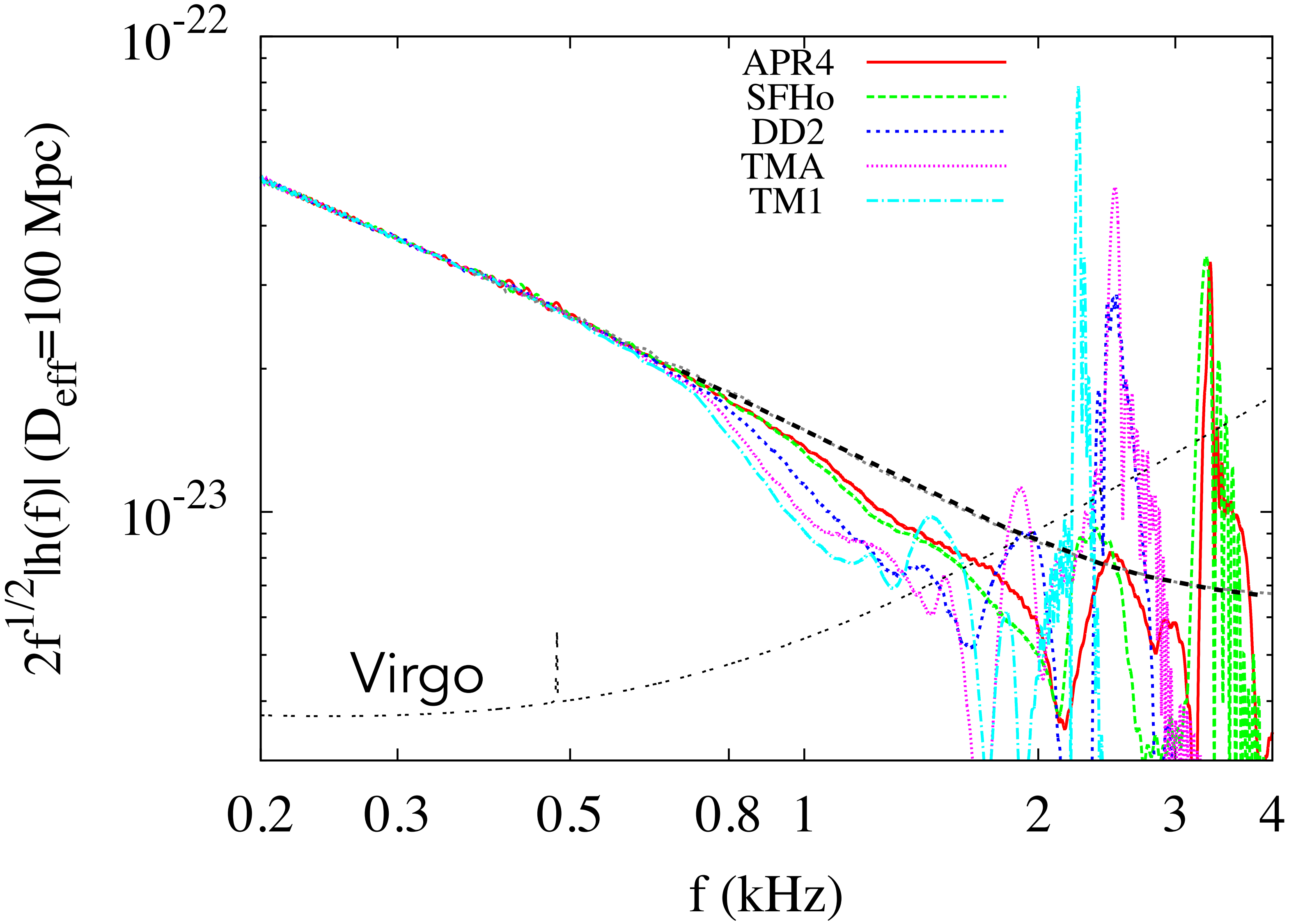
*Ozel et al., 2016*

- ✔ Radius measured with [13-15]% of accuracy
- ✔ Difficult to model emission processes (magnetic fields, atmosphere…)
- ✔ We could look for a cleaner framework



# What do we look for?

*Hotokezaka, Kyutoku, Sekiguchi, Shibata, 2016*



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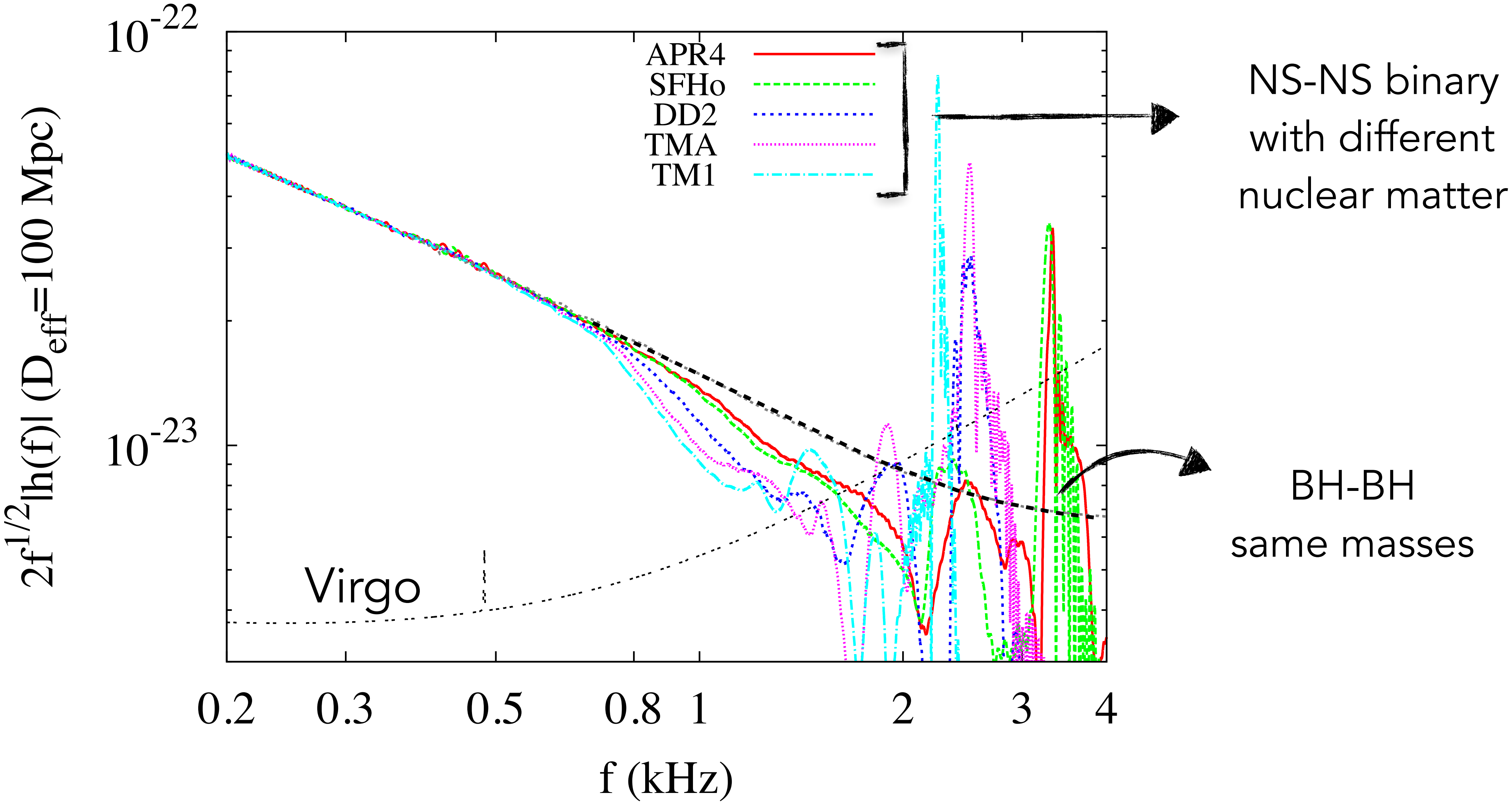

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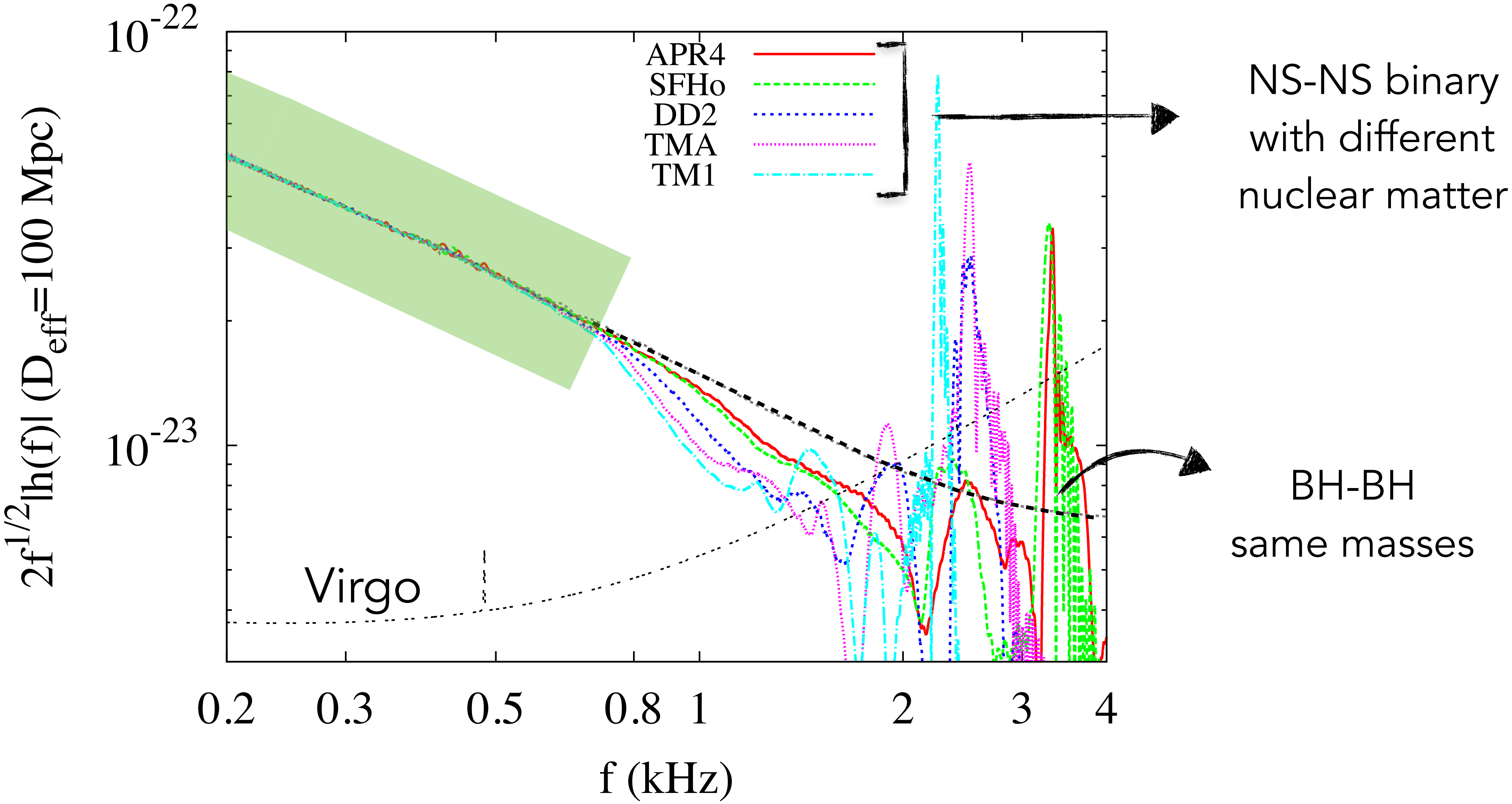


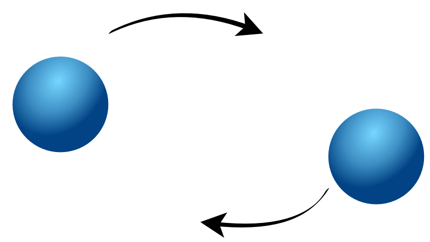

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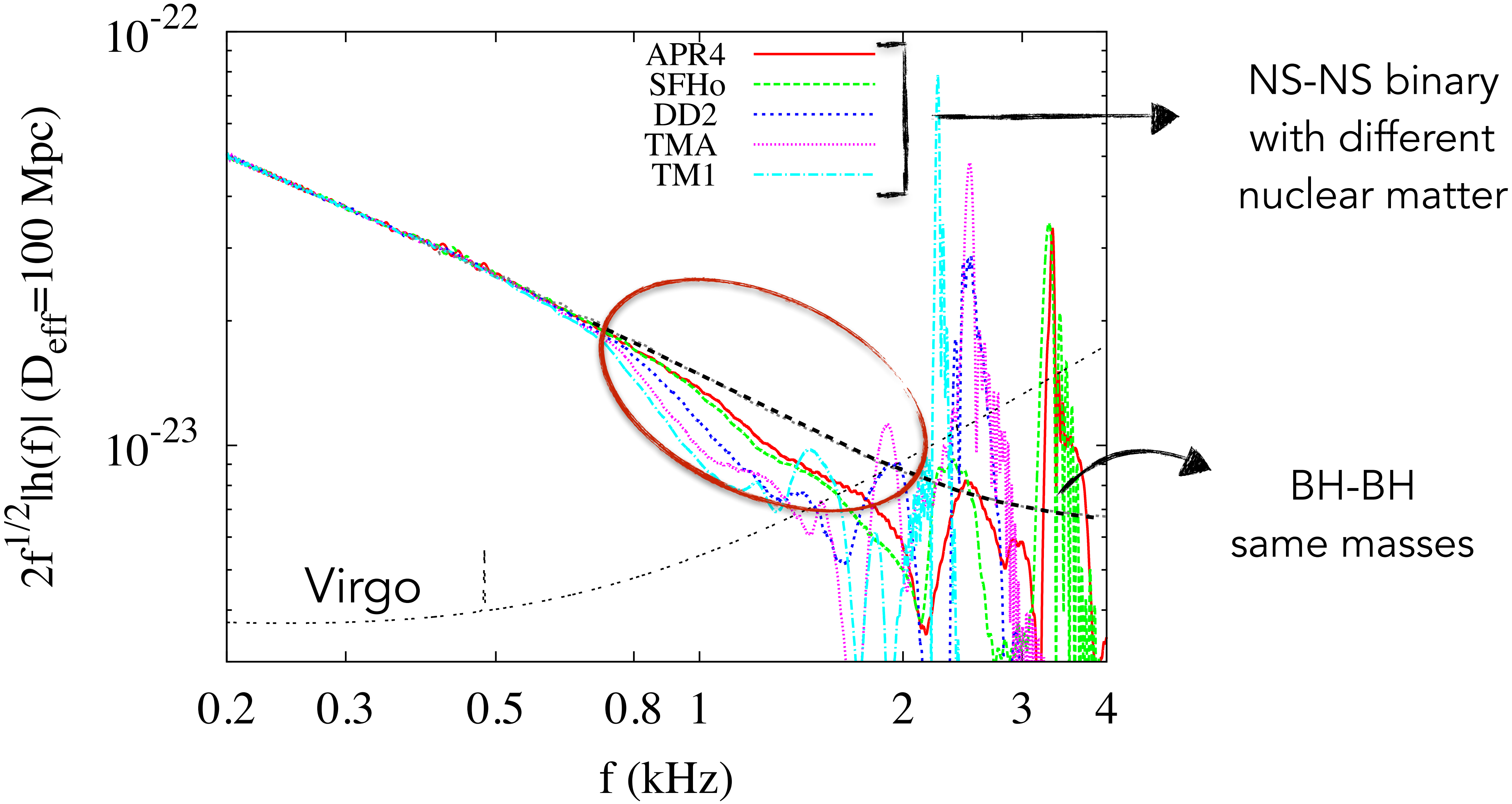


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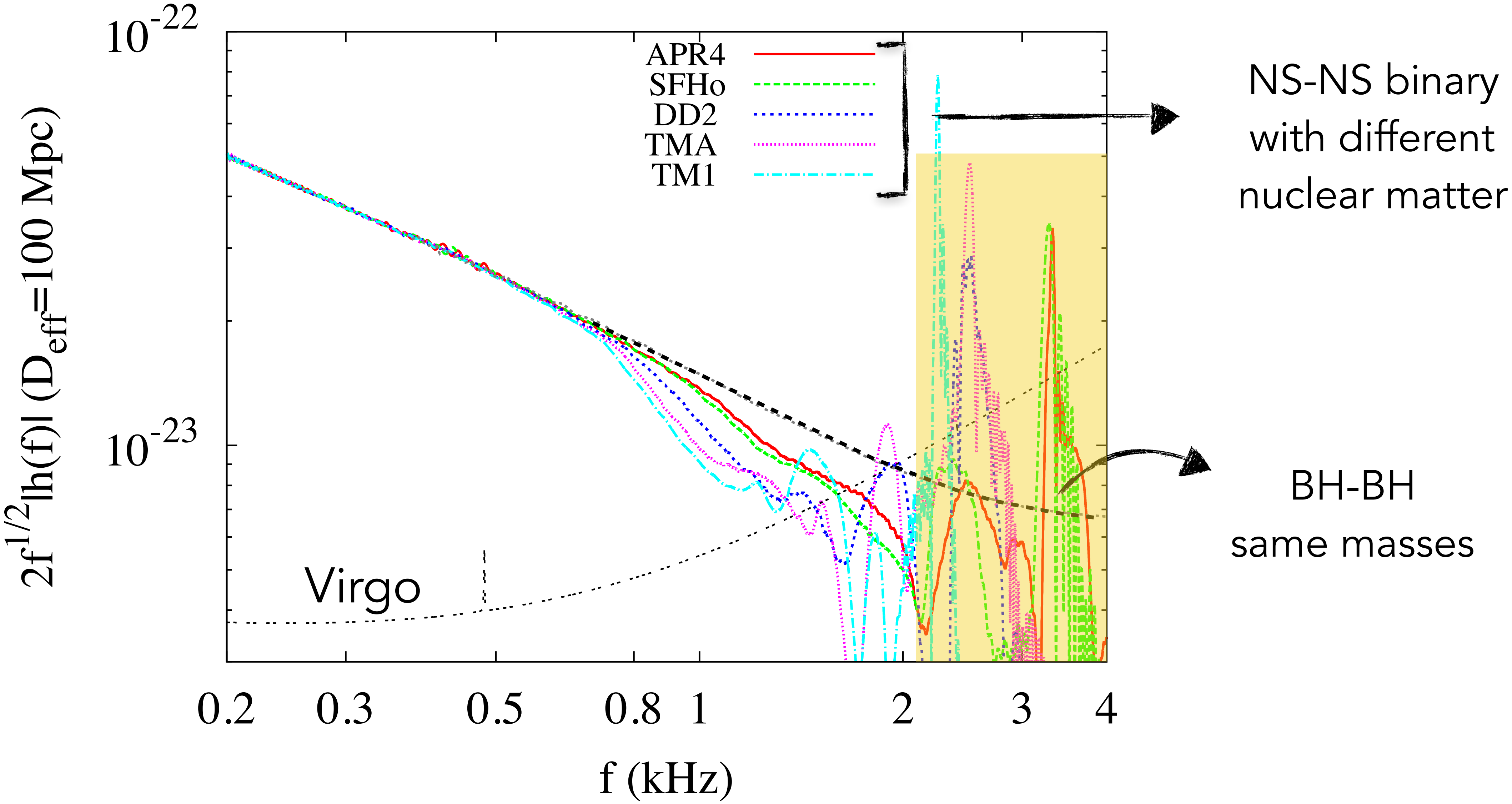
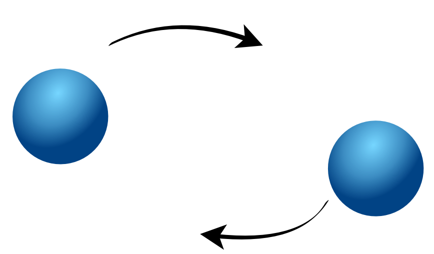
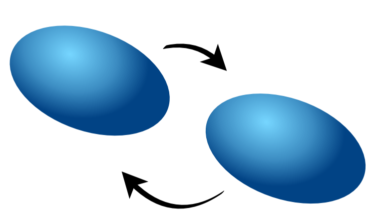
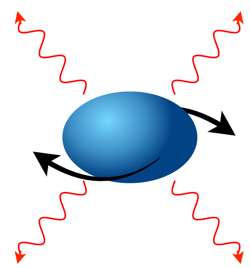


point masses



tidal interactions



merger/post-merger

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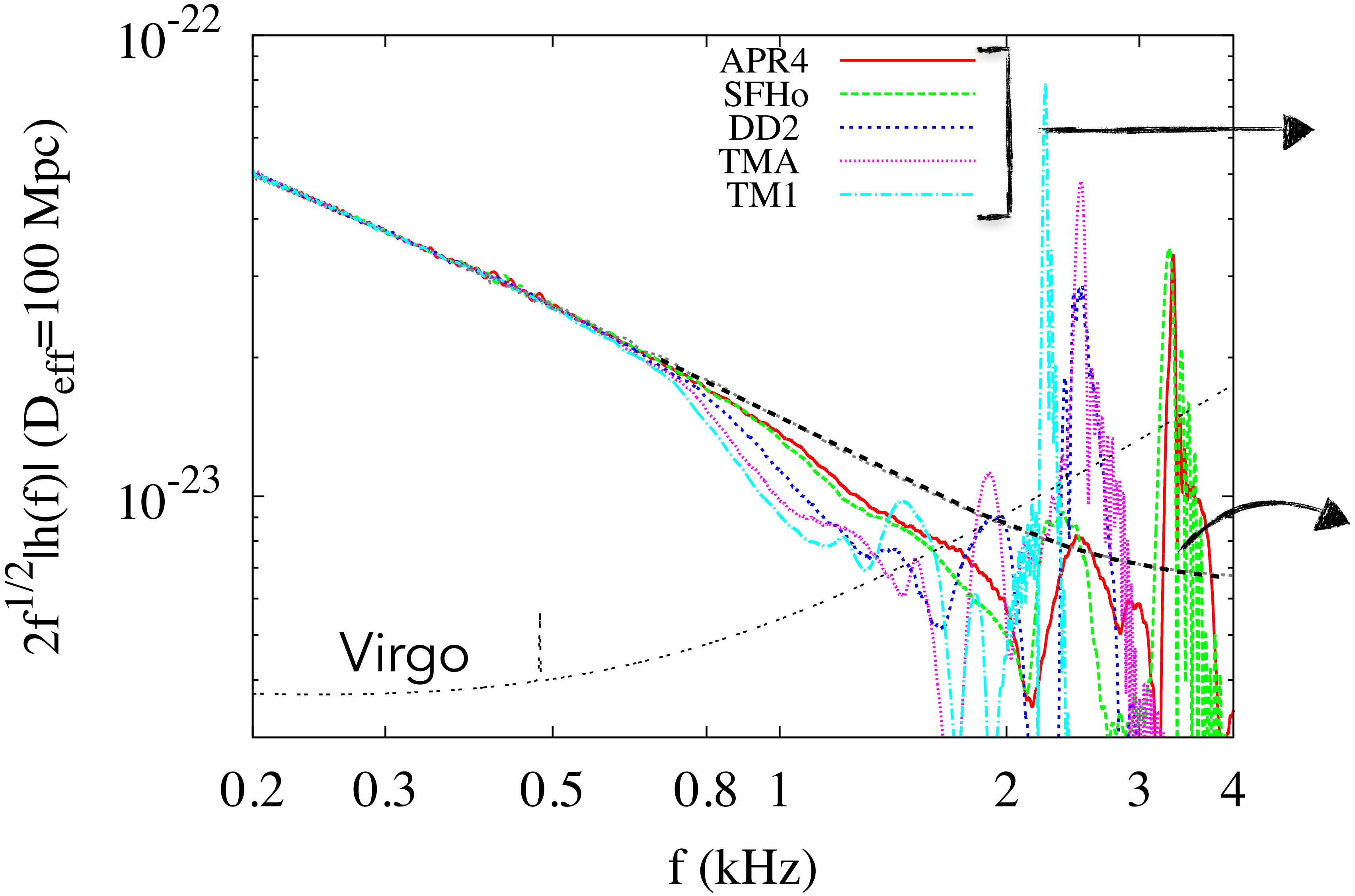
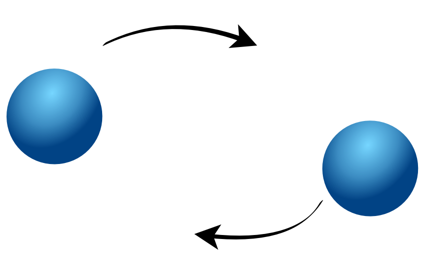



NS-NS binary with different nuclear matter

BH-BH same masses



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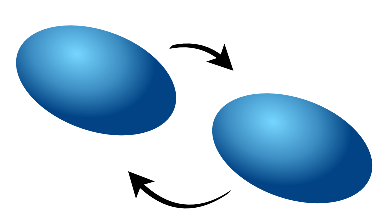
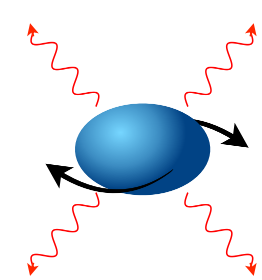


tidal interactions



merger/post-merger

# The Love number

Tidal interactions leave the footprint of the NS structure on the signal

*Hinderer, '08; Binnington & Poisson '09*
*Damour & Nagar '10*

✓ Deformation properties encoded within the Love numbers

$$\mathcal{Q}_{ij} = \frac{2}{3} k_2 R^5 \mathcal{E}_{ij} = \lambda \mathcal{E}_{ij}$$

✓ $\lambda$ depends on the EoS only, for a given compactness $M_{\rm NS}/R_{\rm NS}$

✓ $\lambda$ enters within the gravitational waveform

# The Love number

Tidal interactions leave the footprint of the NS structure on the signal

*Hinderer, '08; Binnington & Poisson '09*
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✓ Deformation properties encoded within the Love numbers

star's quadrupole

$$Q_{ij} = \frac{2}{3} k_2 R^5 \mathcal{E}_{ij} = \lambda \mathcal{E}_{ij}$$

external tidal field

✓ $\lambda$ depends on the EoS only, for a given compactness $M_{\rm NS}/R_{\rm NS}$

✓ $\lambda$ enters within the gravitational waveform

# The recipe of Love

supplementary material

## Polar-electric-type perturbation of background metric

*Regge & Wheeler '57*

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$$

$$\begin{pmatrix} -e^{\nu(r)} & 0 & 0 & 0 \\ 0 & e^{\lambda(r)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix} + \begin{pmatrix} -e^{\nu(r)} H_0(r) & 0 & 0 & 0 \\ 0 & e^{\lambda(r)} H_2(r) & 0 & 0 \\ 0 & 0 & r^2 K(r) & 0 \\ 0 & 0 & 0 & r^2 K(r) \sin^2\theta \end{pmatrix} Y_{lm}(\theta,\phi)$$

✔ Cook everything within Einstein's equations $G_{\mu\nu} = kT_{\mu\nu}$

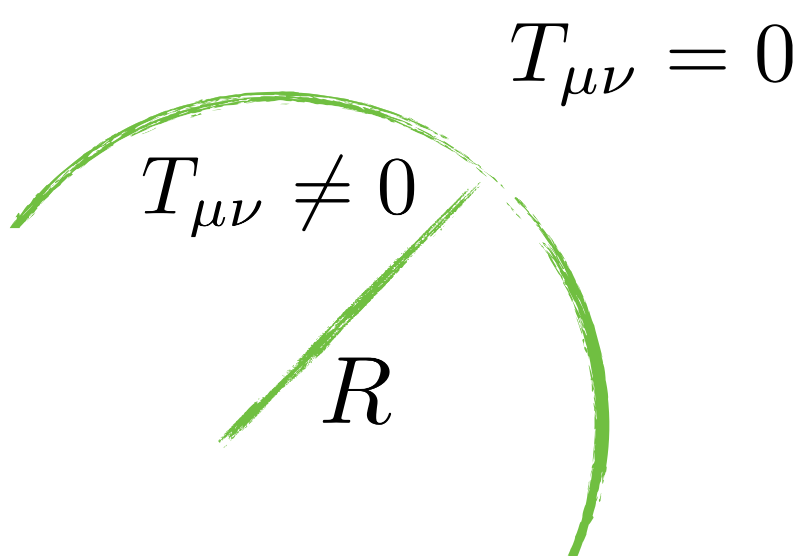
✔ Solve at linear oder in the perturbations $H_0, H_2, K$

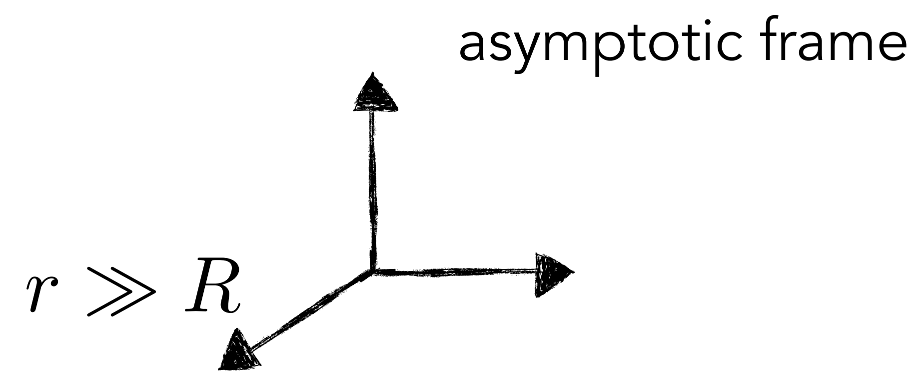
→ set of ODEs sourced by star's density perturbations

# The recipe of Love

supplementary material

- ✓ Numerically integrate from center to outside with appropriate boundary conditions



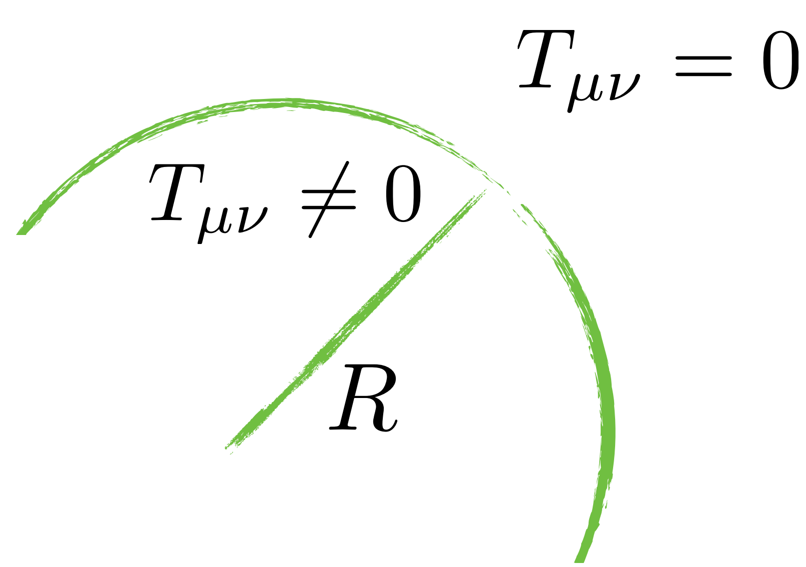

- ✓ Matching interior & exterior solution

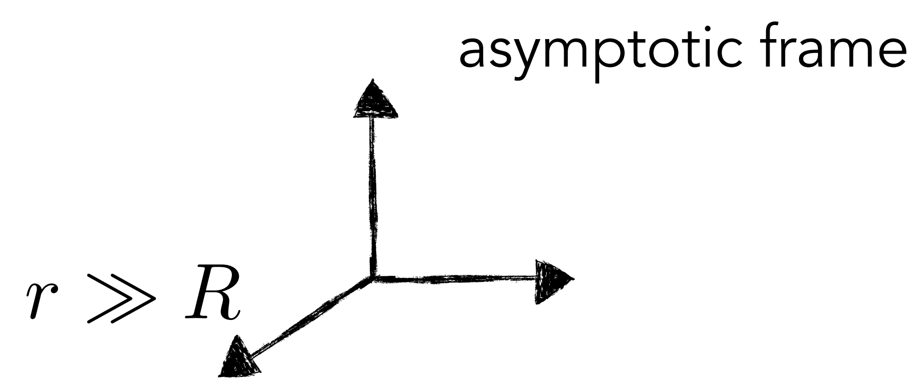
$$g_{\rm tt}^{(\rm int)} = g_{\rm tt}^{(\rm ext)} = -1 + \frac{2M}{r} + \frac{3Q_{ij}\hat{n}^i\hat{n}^j}{r^3} - \mathcal{E}_{ij}x^i x^j$$

# The recipe of Love

supplementary material

✓ Numerically integrate from center to outside with appropriate boundary conditions




read off multipole moments

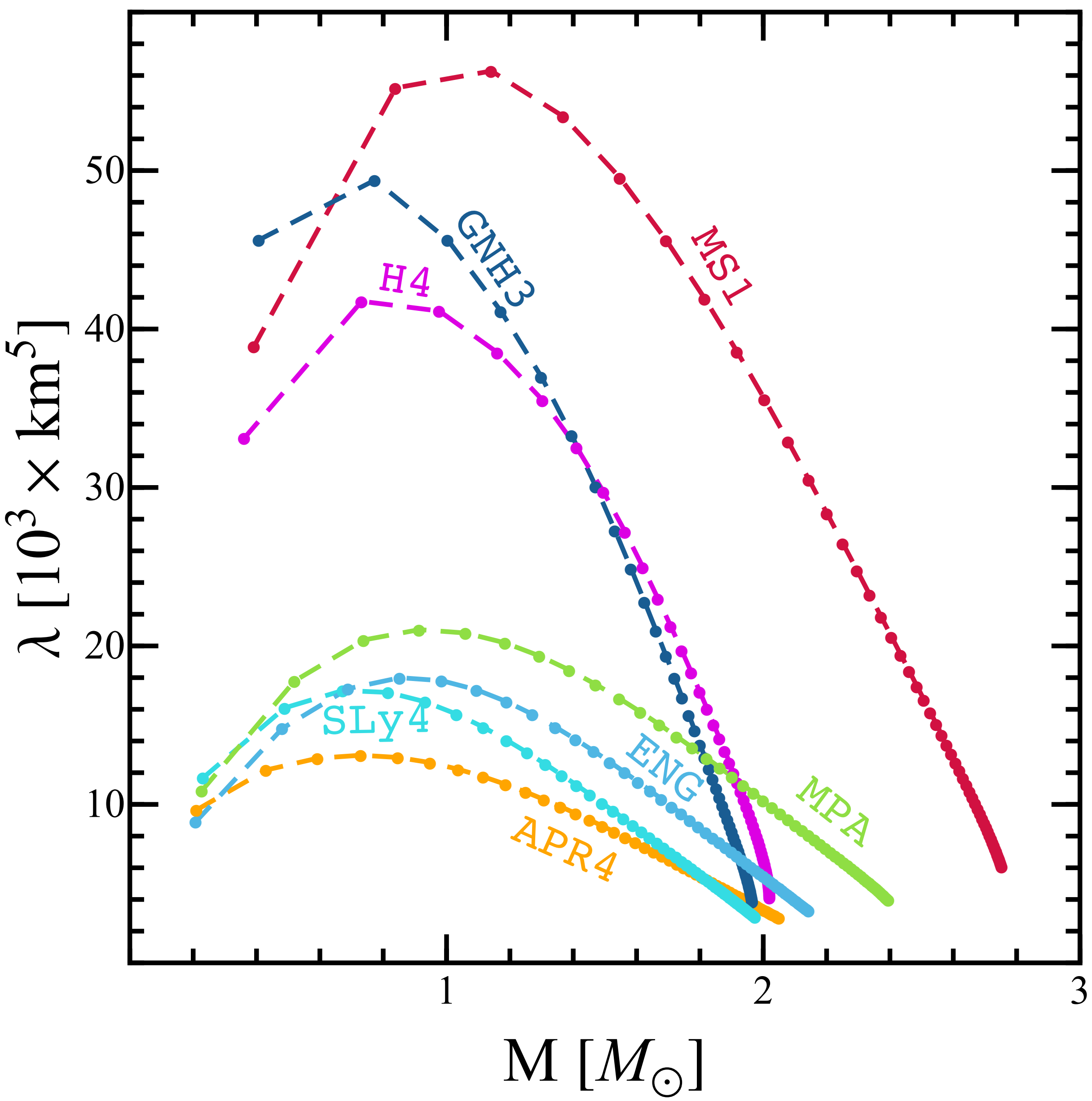
$$Q_{ij} = \lambda \mathcal{E}_{ij}$$

✓ Matching interior & exterior solution

$$g_{\rm tt}^{\rm (int)} = g_{\rm tt}^{\rm (ext)} = -1 + \frac{2M}{r} + \frac{3Q_{ij}\hat{n}^i\hat{n}^j}{r^3} - \mathcal{E}_{ij}x^i x^j$$

# The recipe of Love

✔ Build $\lambda - M$ profiles




- Neutron stars

$$M/R_{\mathrm{NS}} \in [0.1 \div 0.2]$$

↓

$$\lambda \neq 0$$

- Black holes

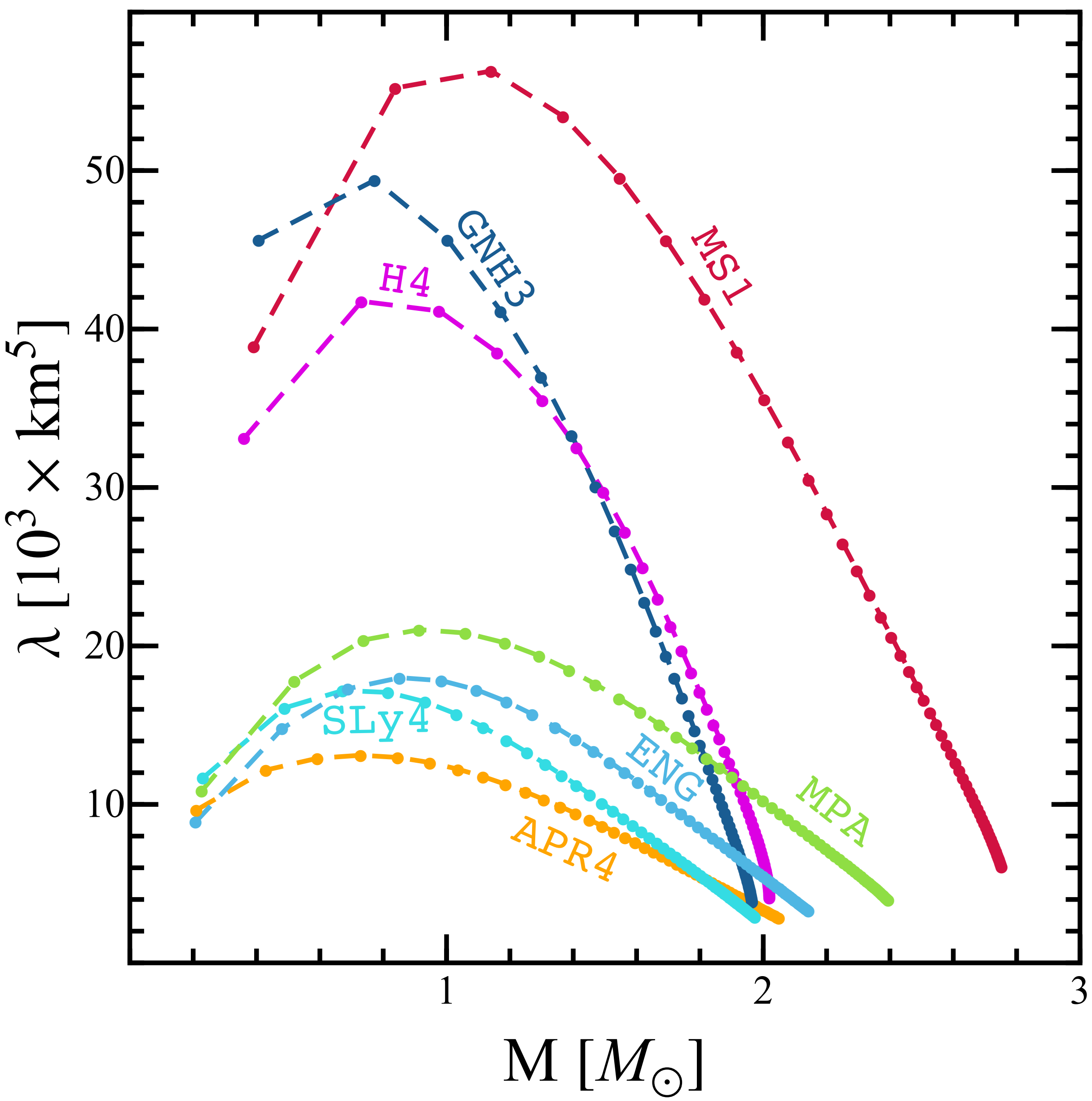
$$M/R_{\mathrm{BH}} = 0.5$$

↓

$$\lambda = 0$$

# The recipe of Love

✔ Build $\lambda - M$ profiles

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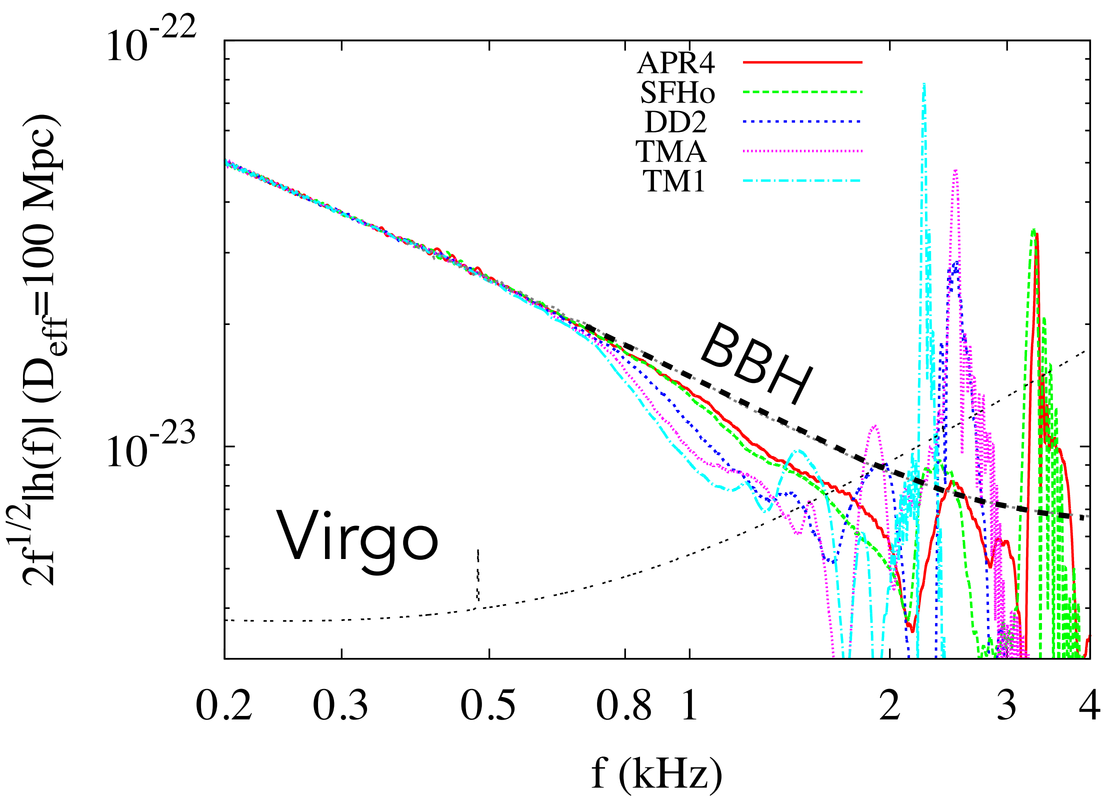
*see Paolo's talk!*

# How much Love?

We classify EoS for as stiff v.s. soft




| EoS | stiffness | $R_{NS}$ | $10^3 \times \lambda$ |
|---|---|---|---|
| APR4 | very soft | 11.09 | 10 |
| SFHo | soft | 11.91 | 13 |
| DD2 | medium soft | 13.20 | 27 |
| TMA | stiff | 13.85 | 37 |
| TM1 | very stiff | 14.48 | 45 |

## Soft EoS

- ✓ larger densities
- ✓ smaller Love

## Stiff EoS

- ✓ smaller densities
- ✓ more Love
- ✓ larger effect in the signal

# Where do we look for Love?

Tidal effects add linearly $$h(f) = \mathcal{A} e^{i[\psi_{\rm PP}(f) + \psi_{\rm T}(f)]}$$

$$\psi_{\rm PP} \propto \left[1 - \left(\frac{743}{336} + \frac{11}{4}\eta\right)\frac{(m\pi f)^{2/3}}{c^2} + \frac{\ldots}{c^3} + \ \ldots \ + \frac{\ldots}{c^7}\right]$$

Newt 1 PN 3.5 PN

$m_1$ $m_2$

$$\psi_{\rm T} \propto \frac{1}{26}\left[\left(1 + 12\frac{m_2}{m_1}\right)\lambda_1 + \left(1 + 12\frac{m_1}{m_2}\right)\lambda_2\right]\frac{(m\pi f)^{10/3}}{c^{10}} + \frac{\ldots}{c^{12}}$$

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5 PN: small term!

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Newt

1 PN

3.5 PN

$\approx$

$m_1$ $m_2$

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$\mathcal{O}(10^4)$

5 PN: small term!

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Newt — 1 PN — 3.5 PN

$\approx$

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$\Lambda$ $\mathcal{O}(10^4)$

average Love

# Status of the art

## Theoretical modelling

- ✓ We know the full form of the $\frac{1}{c^{10}}$ and $\frac{1}{c^{12}}$ contribution to the GW phase

*Vines et al, 2011*

- ✓ Rotational corrections to the Love number are under investigation

*Pani et al., 2015*
*Ask Tiziano*

- ✓ First attempts to model time-dependent corrections to the Love number

*Maselli et al., 2013*
*Hinderer et al., 2016*

- ✓ Excitations of neutron star oscillatory modes due to absorption of the tidal energy

*Weinberg, 2015*

# Before GW170817

Detectability of $\lambda$ strongly depends on the NS mass & stiffness

| Soft EoS | Stiff EoS |
|---|---|
| ✓ smaller deformations | ✓ larger deformations |
| ✓ deep inspiral to look for | ✓ affects a wider frequency range |
| ✓ weaker bounds | ✓ tighter constraints |

| | | EoS | $R_{\rm NS}^{1.4}$ | $\sigma_\lambda/\lambda$ |
|---|---|---|---|---|
| | very soft | APR4 | **11.30** | ~60% |
| LIGO/Virgo ⟼ | medium | H4 | **13.99** | ~20% |
| | very stiff | MS1 | **14.95** | ~10% |

[100 Mpc]

# Before GW170817

Massive efforts to determine real chances to measure $\lambda$

*Flanagan et al, 2008; Hinderer et al., 2010; Damour et al, 2012; Maselli et al, 2013; Read et al 2013; Favata 2014; Hotokezaka, 2016*

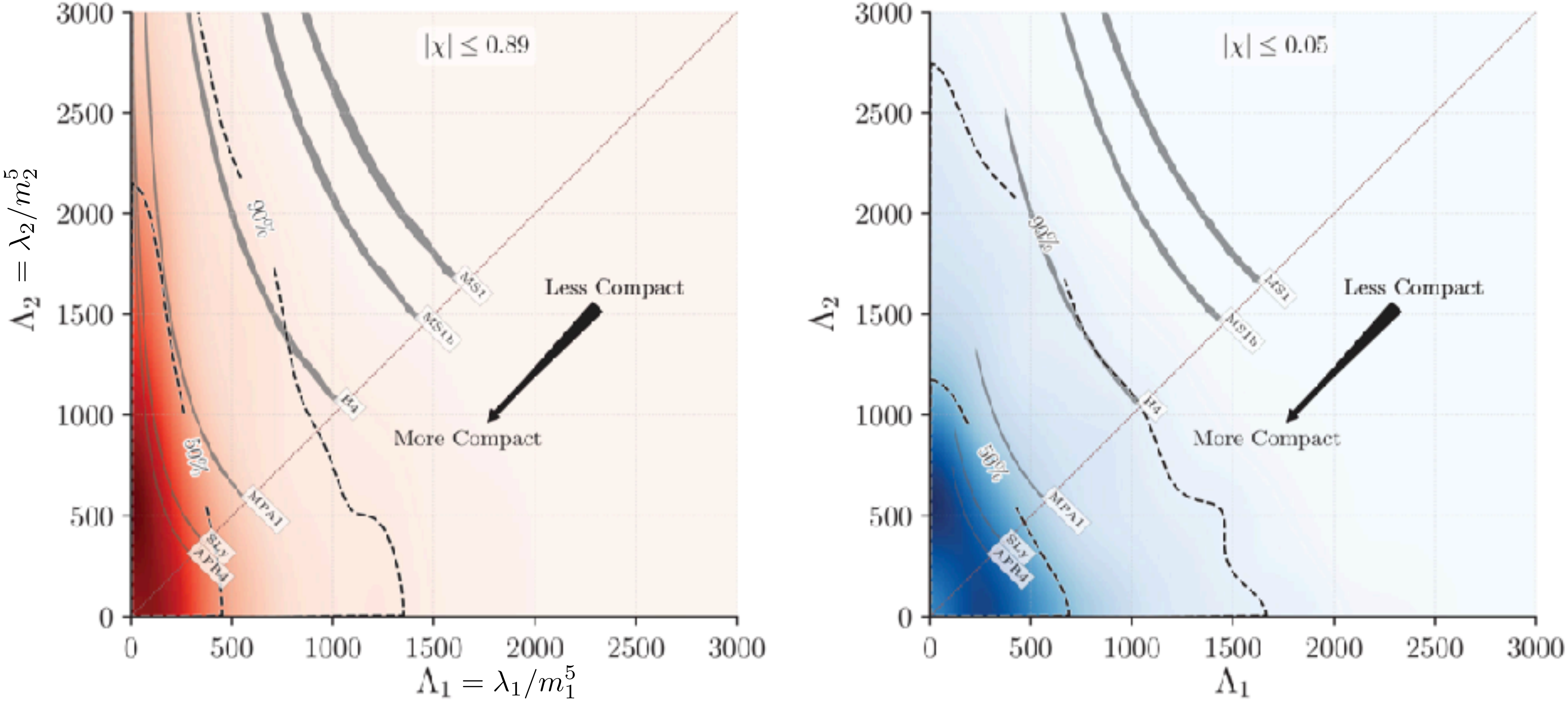
- ✓ For favourable EoS LIGO/Virgo can constrain the Love number and then the NS radius with **1 km** up 200 Mpc

- ✓ A third generation detector, like the Einstein Telescope would improve of an order of magnitude, $R_{\rm NS} \sim$ 100 m

- ✓ For softer EoS more likely to distinguish from a stiff one, than to actually determine the Love number

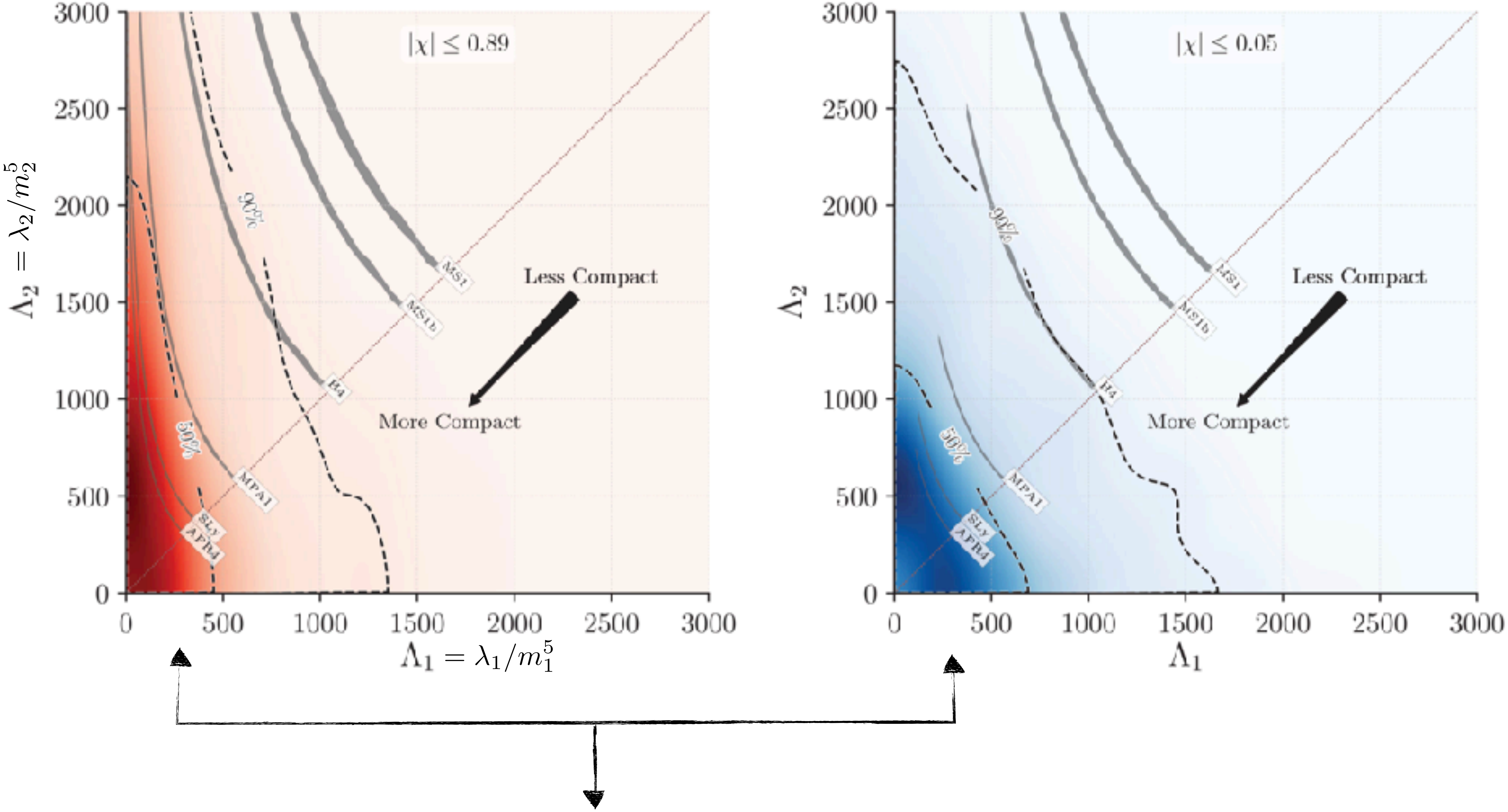
# GW170817

*Abbott et al., 2017*



# GW170817

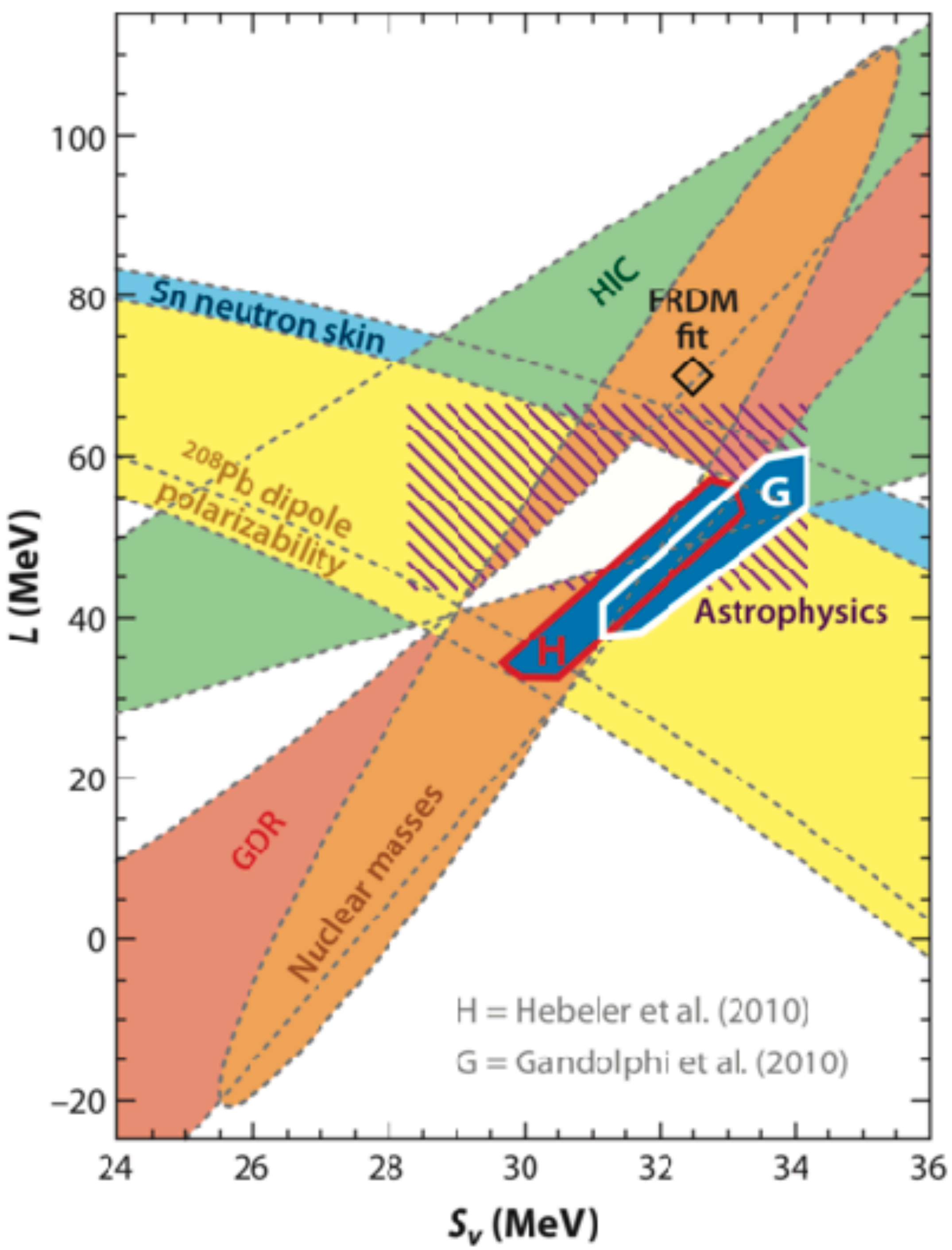
*Abbott et al., 2017*




First detection seems to favour soft nuclear matter

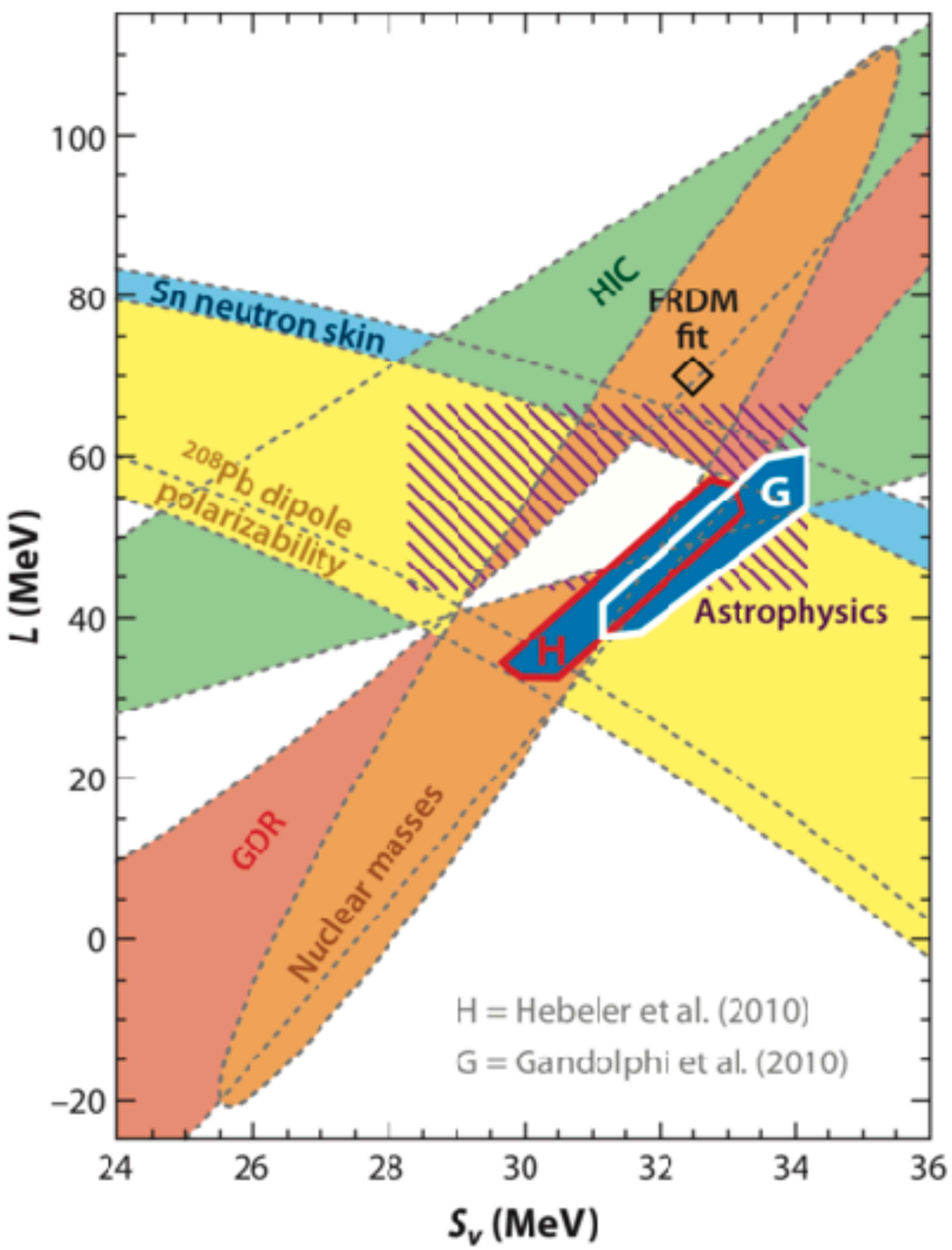
# From macro to micro

So far for electromagnetic signals



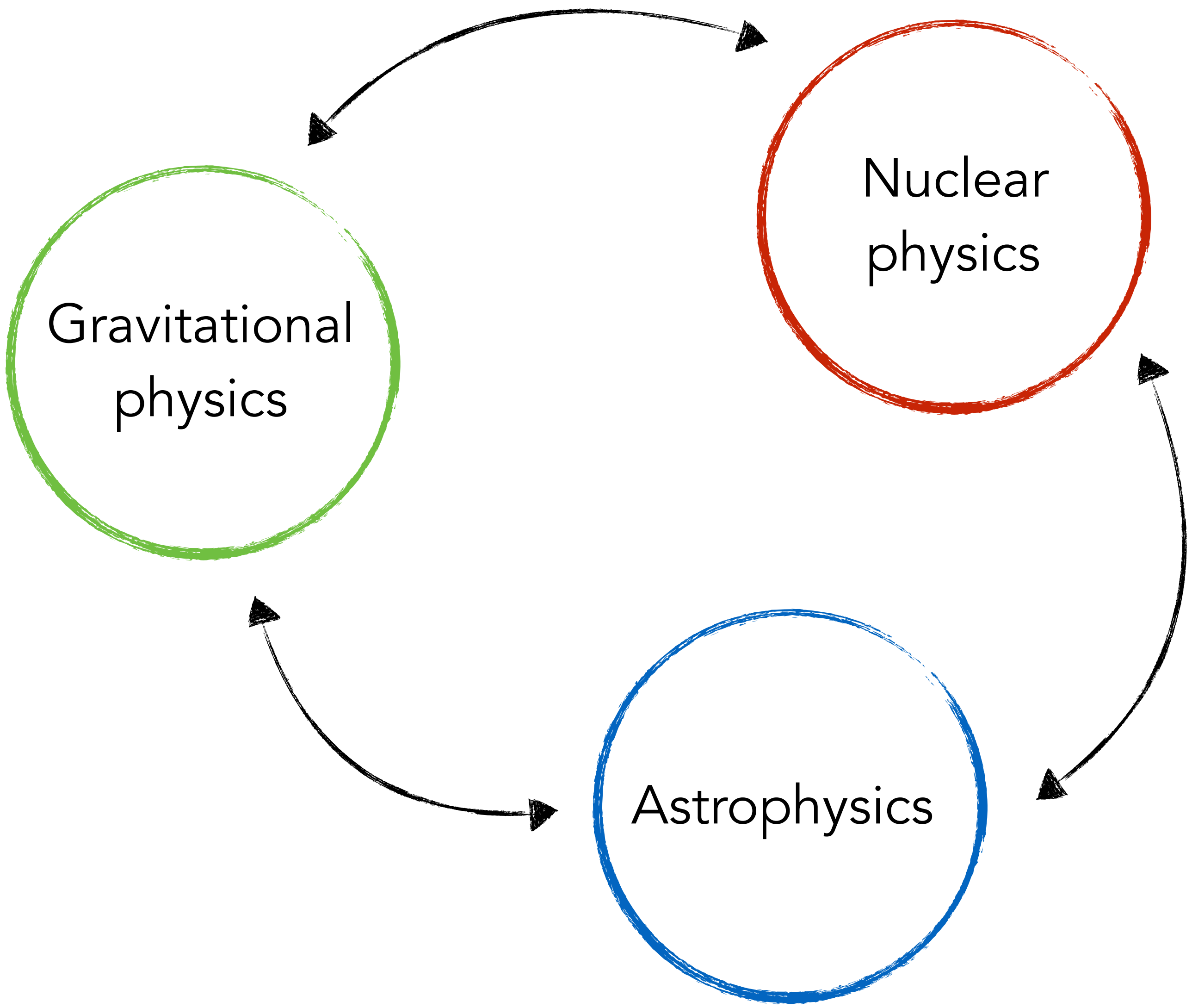
# From macro to micro

So far for electromagnetic signals

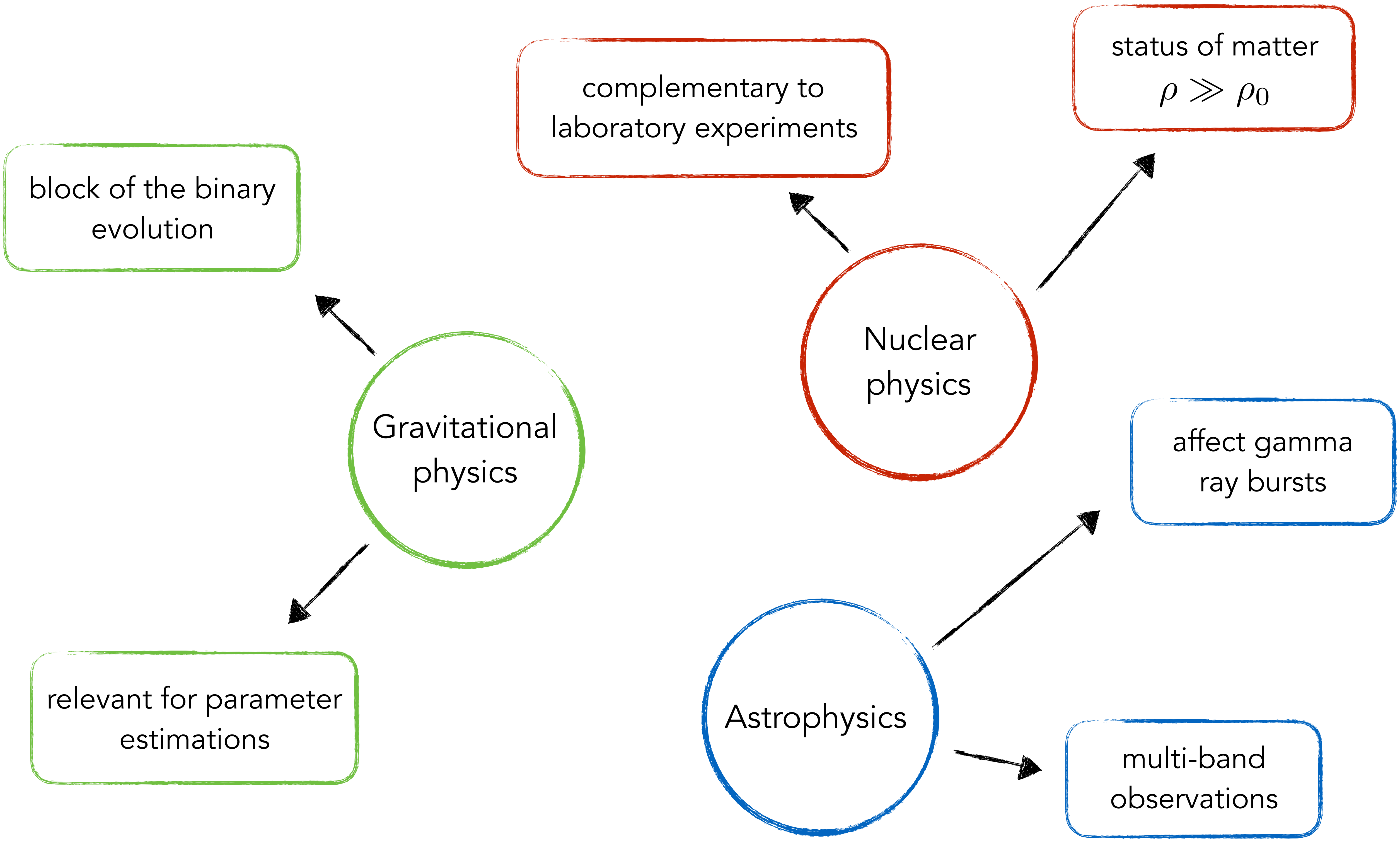



New constraints coming from **GW** and the **Love numbers**

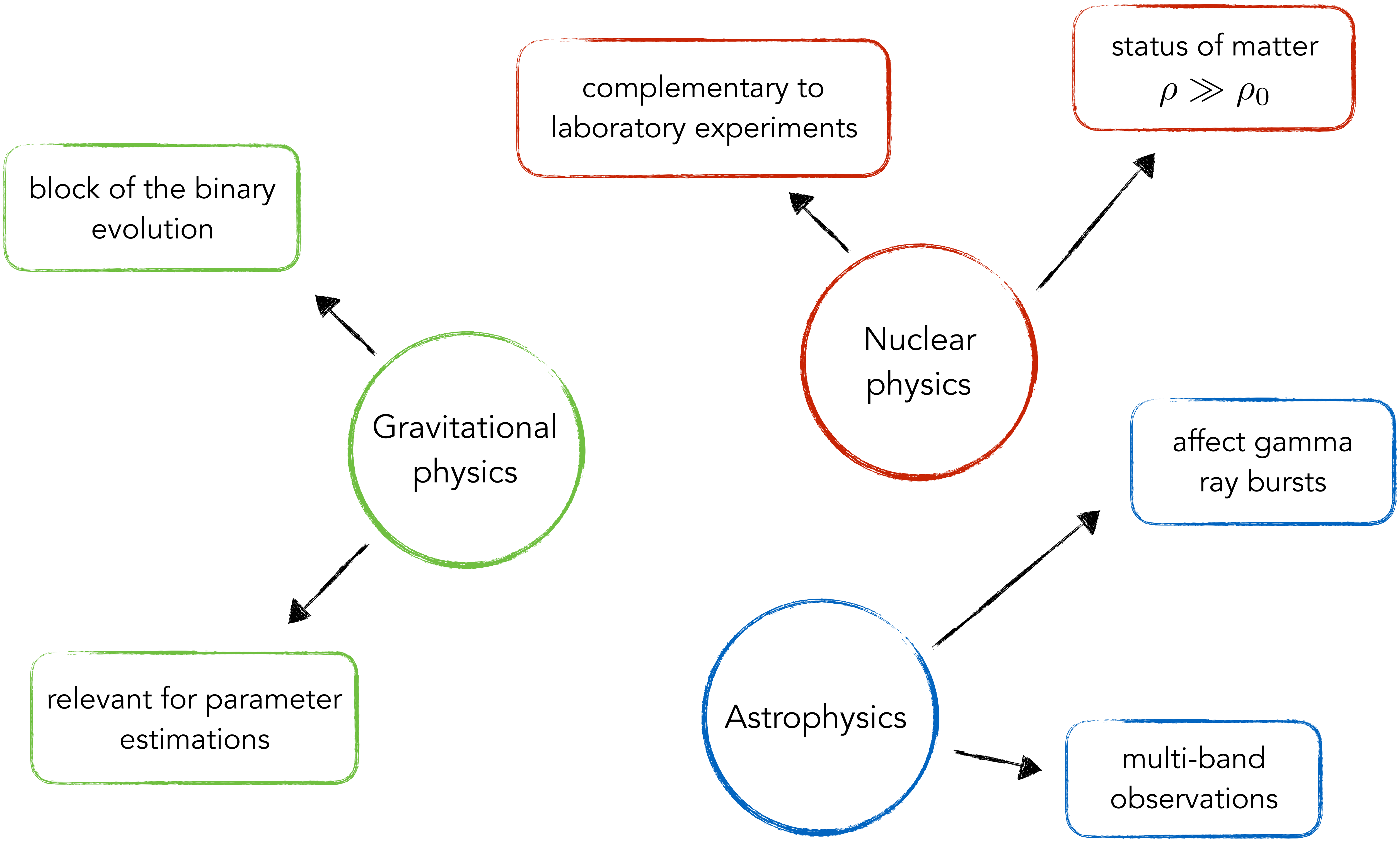
# Summary



# Summary


block of the binary evolution
Gravitational physics
relevant for parameter estimations
complementary to laboratory experiments
status of matter $\rho \gg \rho_0$
Nuclear physics
affect gamma ray bursts
Astrophysics
multi-band observations

# Summary: take care of Love



*Backup*

# Observed NS masses

*Ozel & Freire, 2017*