# Testing General Relativity with Gravitational Waves

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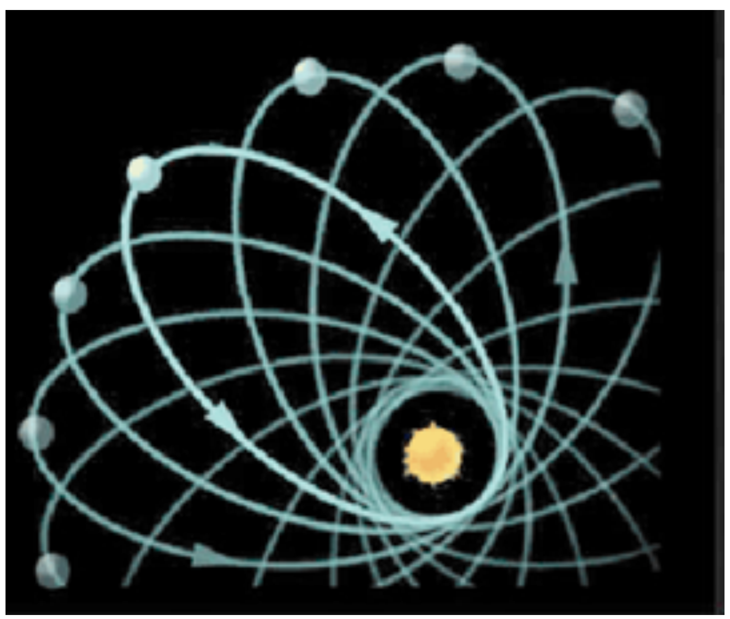
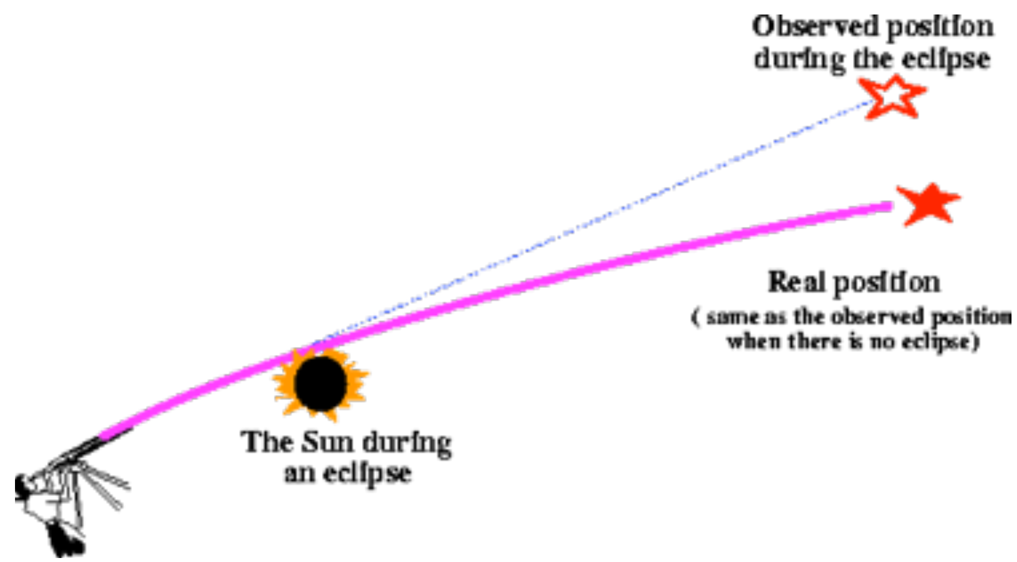
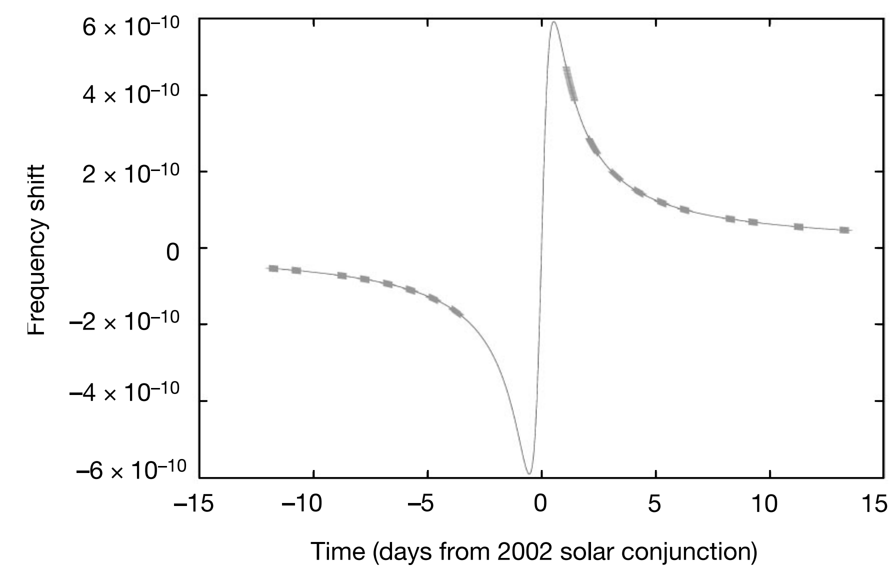
General Relativity (GR), which recenyly celebrated its centenary,
has passed several tests with flying colours:

## Solar system tests

Started when GR was first formulated, one century ago (perihelion precession, light deflection, gravitational redshift), solar system tests became more and more accurate, up to the measurement of Shapiro delay from Cassini spacecraft in 2002 with an accuracy $\sim 10^5$






Bertotti et al., Nature ’03

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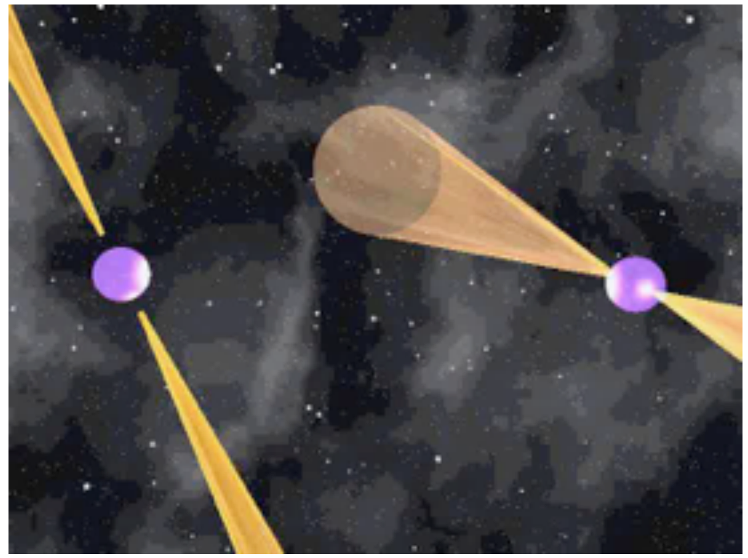
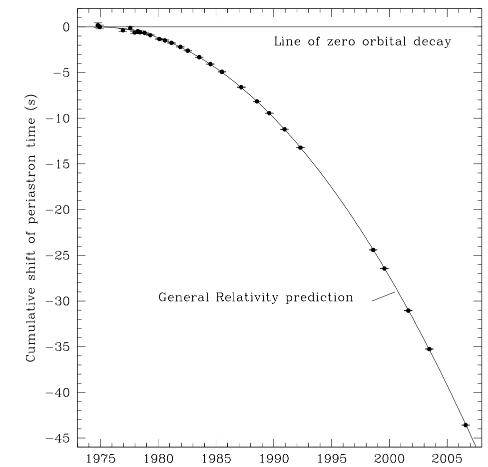
Solar system tests

Binary pulsar tests

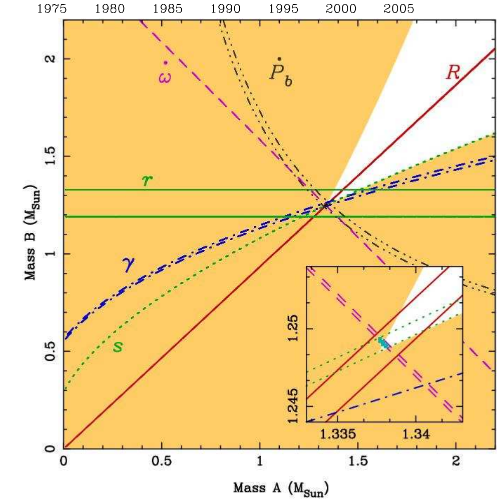


- PSR 1913+16:
  inspiral, and decrease of orbital period,
  due to energy loss through GW emission
  (first indirect proof of the existence of GWs)



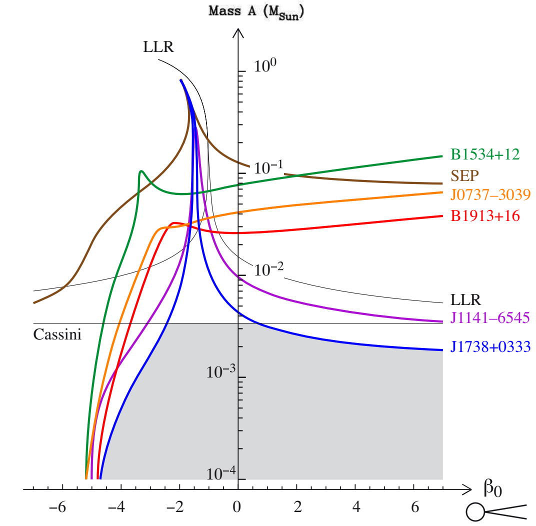

Weisberg *et al.*, '10

- PSR J0737-3039:
  double pulsar, "the most relativistic" system,
  provides strong tests of GR




Kramer *et al.*, '06

- PSR J1738+0333 (and J0348+0432):
  NS-WD systems, best to constrain parameter space
  of scalar-tensor gravity




Freire *et al.*, '12

General Relativity (GR), which recenly celebrated its centenary,
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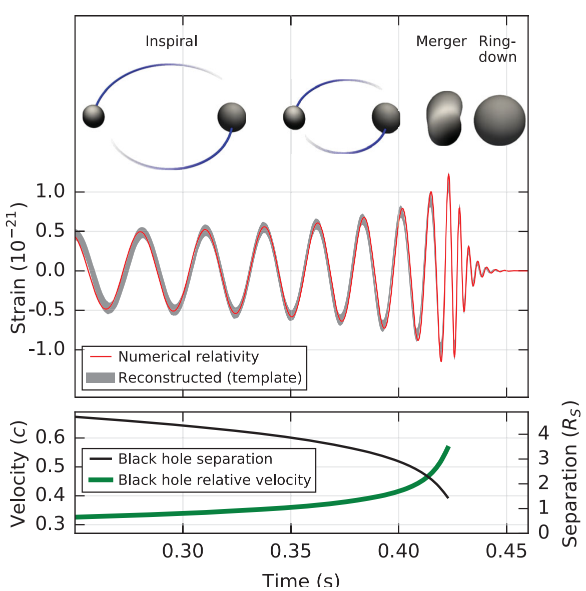
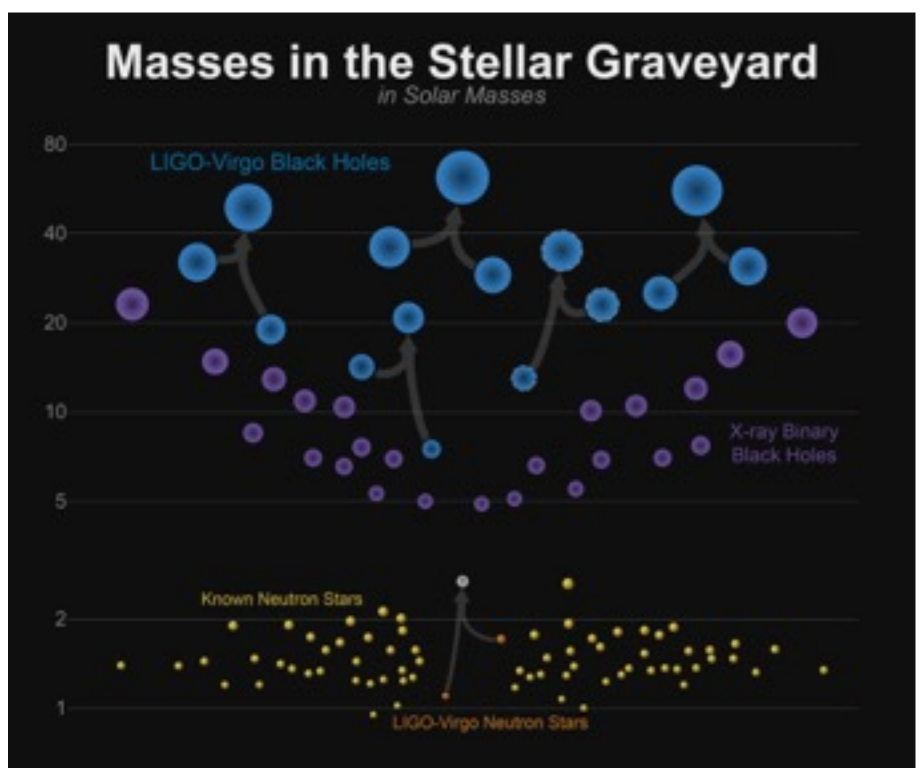
- Solar system tests
- Binary pulsar tests
- Gravitational wave tests

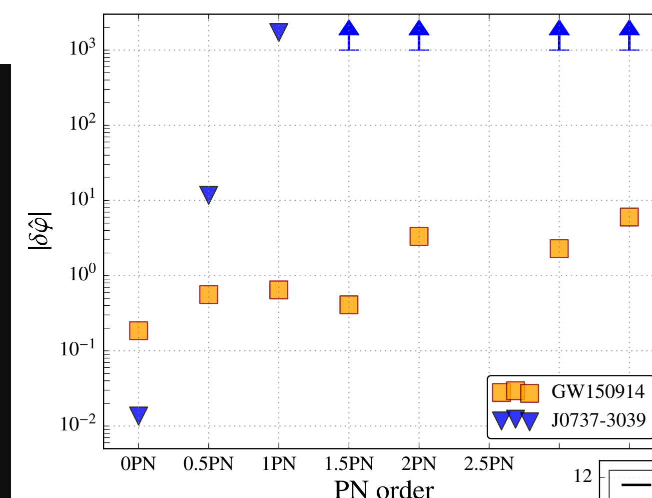
After decads of intensive theoretical and experimental effort, finally we detected GWs!

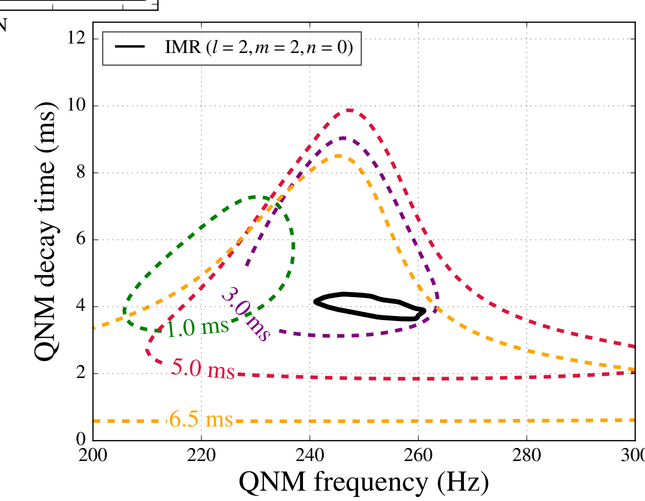



*GW150914*




Even these first signals allow for unprecedented tests of GR!



# Before GW150914, we only had tested the weak field / small curvature regime of gravity!


Solar system tests


Binary pulsar tests


Gravitational wave tests

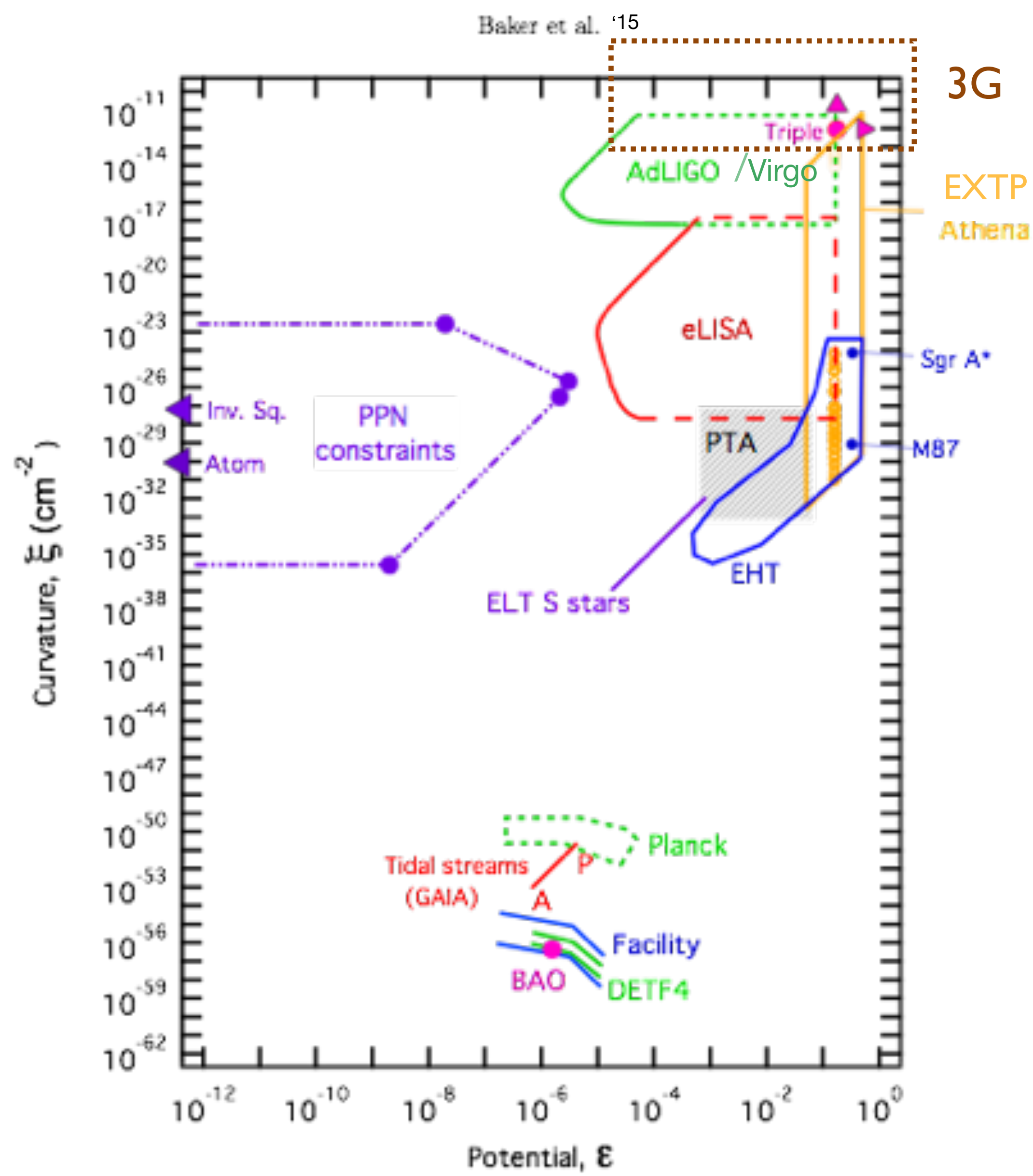
$$\frac{GM_\odot}{R_\odot c^2} \sim 10^{-6} \qquad \frac{GM_\odot}{R_\odot^3 c^2} \sim 10^{-28}\, cm^{-2}$$

$$M_{NS} \sim M_\odot \qquad R_\odot \sim d_{binary}$$

We can now start exploring the strong field/ large curvature regime of gravity.

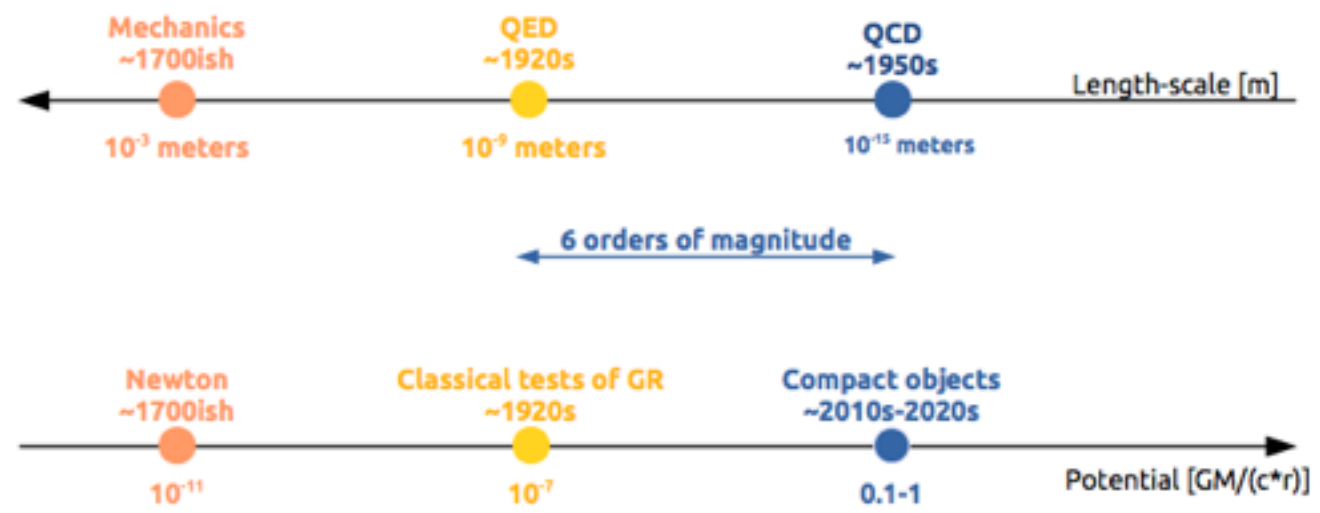
GWs are the perfect probe of this regime:

- only generated in strong-field processes
- sensitive to generation and propagation, which give complementary information
- do not interact when travelling



# Do we need to test GR?

- There is no *fundamental reason* to believe that GR works well in the strong-field regime




*Credits: Pani, '15*

- Theoretical issues
  (unification with the quantum world, singularities and other weird features of GR)

- Observational issues (dark matter & energy)

There is an open, fundamental question to answer:
**how does gravity behave in the strong-field regime**
when gravitational redshift and curvature are large
(for instance, near the surface of a neutron star, or near a black hole horizon)**?**

# Testing GR with GWs and astrophysical observations

(see e.g. *Berti et al., CQG 2015 arXiv:1501.07274)*

One can follow either a bottom-up or a top-down approach.

*Bottom-up approach*:

- choose the phenomenology to be studied, and the quantities most appropriate to describe it
- devise a parametrization of these quantities
- typically, each parameter is associated to the violation/modification of some GR property
- compute observables in terms of the parameters
- perform observations/experiments, setting bounds to the parameters

- PPN (parametrized post-Newtonian) expansion (*Eddington '22; Nordtvedt '68; Will '71)*
  PN expansion of the spacetime metric of e.g. 2-body system is extended including free parameters.
  e.g.:
  $$ds^2 = -\left(1 - 2\frac{M}{r} + 2\beta\frac{M^2}{r^2} + \dots\right)dt^2 + \left(1 + 2\gamma\frac{M}{r} + \dots\right)(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2)$$
  In GR β=0, γ=1. Appropriate framework for solar system tests

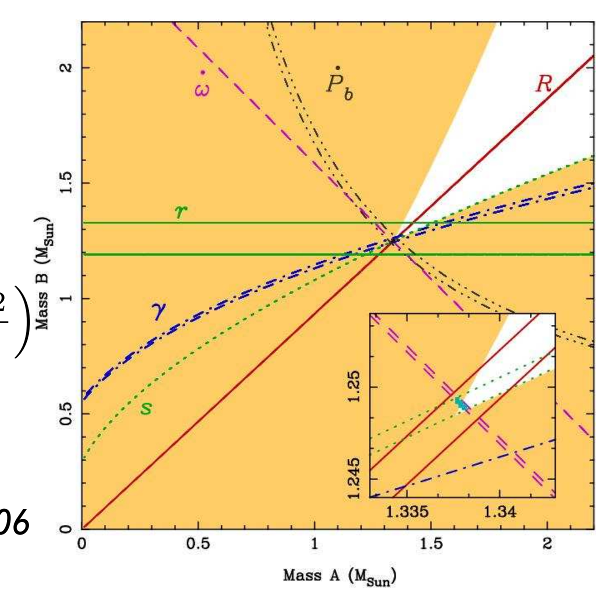
- PPK (parametrized post-Keplerian) expansion (*Damour & Taylor '92* )
  motion of compact binary characterized by Keplerian and post-Keplerian parameters. Appropriate framework for binary pulsar tests:

perhielion precession: $\langle\dot\omega\rangle = 6\pi f_{\rm b}(2\pi m f_{\rm b})^{2/3}(1-e^2)^{-1},$

time dilation: $\gamma' = e(2\pi f_{\rm b})^{-1}(2\pi m f_{\rm b})^{2/3}\frac{m_2}{m}\left(1+\frac{m_2}{m}\right)$

period decrease (inspiralling): $\dot P_{\rm b} = -\frac{192\pi}{5}(2\pi\mathcal{M} f_{\rm b})^{5/3}F(e),$

Shapiro delay: $r = m_2,$ $s = \sin i,$




*Kramer et al., '06*

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- PPE (parametrized post-Einstenian) expansion (*Yunes & Pretorius '09* )
  GW compact binary waveform is directly parametrized:

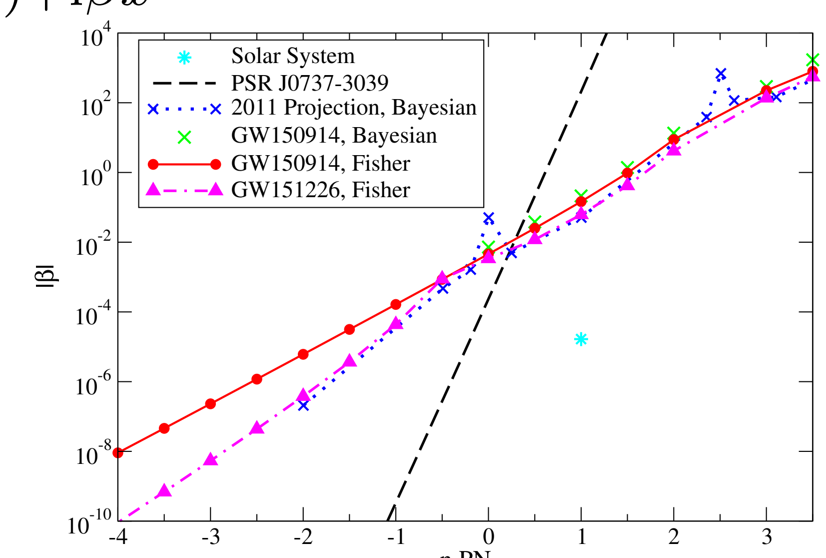
$$h(f) = A_{GR}(f)(1+\alpha x^a)e^{\mathrm{i}\Psi_{GR}(f)+\mathrm{i}\beta x^b}$$

ppE paramters:

$\alpha,\beta$ (=0 inGR): amplitude of modification;

a,b: PN order

mapping: $(\alpha,\beta,a,b)$ <=> specific theories



(*Yunes et al., PRD '16)*

- PPF (parametrized post-Friedmannian) expansion ( *Hu & Sawitcki '07* )
  cosmological quantities & equations are parametrized

## *Top-down approach*:

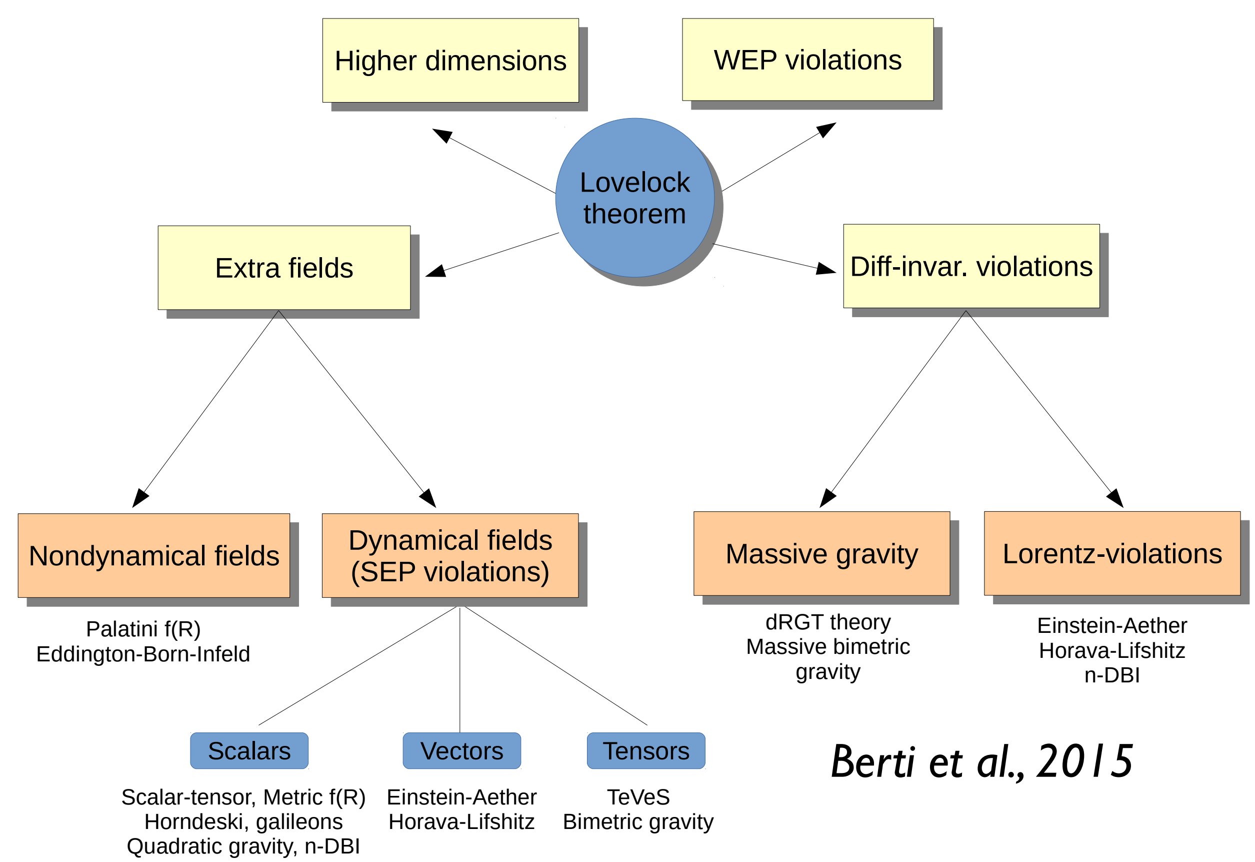
- consider GR modifications, possibly inspired by fundamental physics considerations
- work out observational consequences of these modifications

  (they typically depend on parameters describing the amplitude of the modification)
- compare with observations, setting bounds on the parameters

Remarks:

- in most cases we are looking to tiny modifications (parameters small due to existing data)
- often difficult to disentangle a truly from poorly known "standard" physics effects (BHs better than NSs)
- best (when possible) would be to find *new effects* (smoking-guns),

There are several ways to modify GR…




*Berti et al., 2015*

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**Table 1.** Catalog of several theories of gravity and their relation with the assumptions of Lovelock's theorem. Each theory violates at least one assumption (see also figure 1), and can be seen as a proxy for testing a specific principle underlying GR.

| Theory | Field content | Strong EP | Massless graviton | Lorentz symmetry | Linear $T_{\mu\nu}$ | Weak EP | Well- posed? | Weak-field constraints |
|---|---|---|---|---|---|---|---|---|
| Extra scalar field | | | | | | | | |
| Scalar–tensor | S | × | ✓ | ✓ | ✓ | ✓ | ✓[34] | [35–37] |
| Multiscalar | S | × | ✓ | ✓ | ✓ | ✓ | ✓[38] | [39] |
| Metric *f*(*R*) | S | × | ✓ | ✓ | ✓ | ✓ | ✓[40, 41] | [42] |
| Quadratic gravity | | | | | | | | |
| Gauss–Bonnet | S | × | ✓ | ✓ | ✓ | ✓ | ✓? | [43] |
| Chern–Simons | P | × | ✓ | ✓ | ✓ | ✓ | ×✓? [44] | [45] |
| Generic | S/P | × | ✓ | ✓ | ✓ | ✓ | ? | |
| Horndeski | S | × | ✓ | ✓ | ✓ | ✓ | ✓? | |
| Lorentz-violating | | | | | | | | |
| Æ-gravity | SV | × | ✓ | × | ✓ | ✓ | ✓? | [46–49] |
| Khronometric/ Hořava–Lifshitz | S | × | ✓ | × | ✓ | ✓ | ✓? | [48–51] |
| *n*-DBI | S | × | ✓ | × | ✓ | ✓ | ? | none ([52]) |
| Massive gravity | | | | | | | | |
| dRGT/Bimetric | SVT | × | × | ✓ | ✓ | ✓ | ? | [17] |
| Galileon | S | × | ✓ | ✓ | ✓ | ✓ | ✓? | [17, 53] |
| Nondynamical fields | | | | | | | | |
| Palatini *f*(*R*) | — | ✓ | ✓ | ✓ | × | ✓ | ✓ | none |
| Eddington–Born–Infeld | — | ✓ | ✓ | ✓ | × | ✓ | ? | none |
| Others, not covered here | | | | | | | | |
| TeVeS | SVT | × | ✓ | ✓ | ✓ | ✓ | ? | [37] |
| $f(R)\mathcal{L}_m$ | ? | × | ✓ | ✓ | ✓ | × | ? | |
| *f*(*T*) | ? | × | ✓ | × | ✓ | ✓ | ? | [54] |

**Table 2.** Catalog of BH properties in several theories of gravity. The column 'solutions' refers to asymptotically-flat, regular solutions. Legend: ST—'scalar–tensor'; ≡ GR—'same solutions as in GR'; ⊃ GR—'GR solutions are also solutions of the theory'; NR—'non rotating'; SR—'slowly rotating'; FR—'fast rotating/generic rotation'; ?—unknown or uncertain.

| Theory | Solutions | Stability | Geodesics | Quadrupole |
|---|---|---|---|---|
| Extra scalar field | | | | |
| Scalar–tensor | ≡GR [55–60] | [61–67] | — | — |
| Multiscalar/Complex scalar | ⊃GR [56, 68, 69] | ? | ? | [68, 69] |
| Metric *f*(*R*) | ⊃GR [58, 59] | [70, 71] | ? | ? |
| Quadratic gravity | | | | |
| Gauss–Bonnet | NR [72–74]; SR [75, 76]; FR [77] | [78, 79] | SR [75, 80, 81]; FR [77] | [76, 82] |
| Chern–Simons | SR [83–85]; FR [86] | NR [87–90]; SR [79] | [74, 91] | [85] |
| Generic | SR [80] | ? | [80] | equation (3.12) |
| Horndeski | [92–94] | ? [95, 96] | ? | ? |
| Lorentz-violating | | | | |
| Æ-gravity | NR [97–99] | ? | [98, 99] | ? |
| Khronometric/ Hořava–Lifshitz | NR, SR [98–101] | ? [102] | [98, 99] | ? |
| *n*-DBI | NR[103, 104] | ? | ? | ? |
| Massive gravity | | | | |
| dRGT/Bimetric | ⊃ GR, NR [105–108] | [109–112] | ? | ? |
| Galileon | [113] | ? | ? | ? |
| Nondynamical fields | | | | |
| Palatini *f*(*R*) | ≡GR | — | — | — |
| Eddington–Born–Infeld | ≡GR | — | — | — |

*Berti et al., 2015*

# Scalar-tensor gravity

Include a scalar field in the gravitational action, non-minimally coupled
(extension of Branse-Dicke gravity proposed in the '60s)

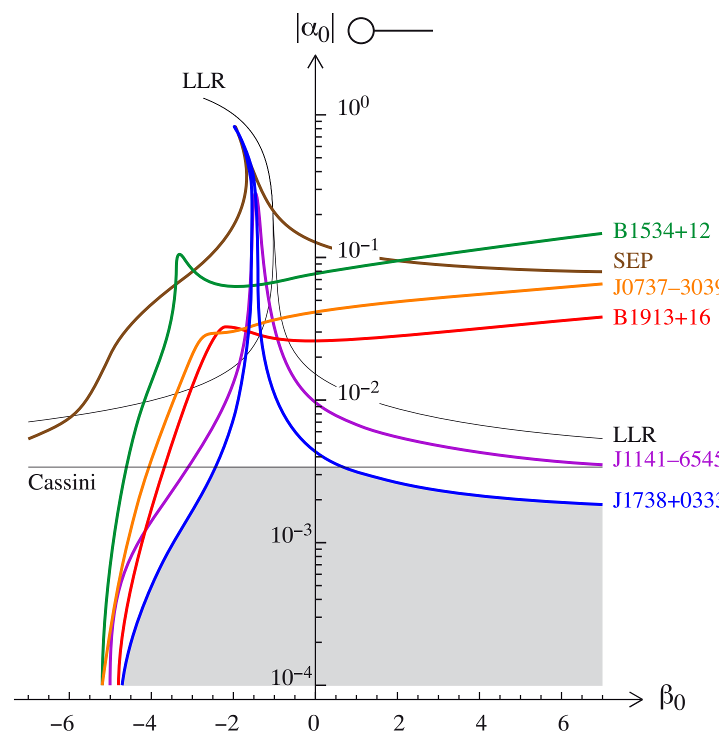
$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[F(\phi)R - 8\pi G Z(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)]$$

- Non minimal coupling $F(\phi)R$ (e.g. $\phi R$ or $e^{\phi}R$) as in String/M theory (dilaton)
- Violation of the strong equivalence principle
- It has been shown that f(R) gravity theories ($S=\int f(R)d^4x$) are equivalent to ST gravity
- BH solutions are the same as in GR, NSs are not (in some cases large deviations in NSs)
- Compact binaries with NSs emit dipole radiation, affecting the inspiral
- This is not really a strong-field effect; indeed, stringent bounds come from binary pulsars:

Massless scalar field:
coupling described by two parameters, $\alpha_0,\beta_0$
bound on $\alpha_0$ from solar system tests
bound on $\beta_0$ from binary pulsars observations

for larger $|\beta_0|$, large non-linear effects (*spontaneous scalarization*) but would affect binary pulsar motion => ruled out

Different formulations of the theory (e.g., massive scalar field) predict deviations only in the merging => not ruled out, can be tested with GWs




Freire *et al.*, '12

## Quadratic gravity: scalar field coupled to quadratic curvature terms:

$$S = \frac{1}{16\pi}\int \sqrt{-g}d^4x\Big[R - 2\nabla_a\phi\nabla^a\phi - V(\phi) + f_1(\phi)R^2 + f_2(\phi)R_{\mu\nu}R^{\mu\nu} + f_3(\phi)R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + f_4(\phi)^*RR\Big] + S_{\rm mat}\left[\Psi,\, \gamma(\phi)g_{\mu\nu}\right]\,,$$

It can be shown that, as long as BH solutions in vacuum are concerned, quadratic gravity can be reduced to two cases *(e.g. Pani et al., PRD '11)*:

Einstein-dilaton-Gauss-Bonnet (EdGB) gravity

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{\alpha e^\Phi}{4}\mathcal{R}^2_{\rm GB}\right],$$

where $\mathcal{R}^2_{\rm GB} = R_{\alpha\beta\delta\gamma}R^{\alpha\beta\delta\gamma} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2$

Dynamical Chern-Simons (DCS) gravity

$$S = \frac{1}{16\pi}\int \sqrt{-g}d^4x\left[R - 2\nabla_a\phi\nabla^a\phi - V(\phi) + \alpha_{\rm CS}\,\phi\,^*RR\right],$$

- They are probably the simplest way to modify strong-field/high-curvature regime of gravity
- Stationary BH solutions with non-trivial scalar field configurations do exist
- Large-curvature modification only show up in the final stages of coalescence => not ruled out by binary pulsar observations
- Naturally emerge in low-energy realizations of string theory (scalar field is dilaton)
- Effective field theory interpretation: can be seen as first terms in an expansion in all curvature invariants (could make theory renormalizable)
- One of them (EdGB) also exist as an *exact* theory: no instability
- Implicit assumption: there is a new scale >> Planck scale: $\alpha^{1/2} \sim$ km

## Lorentz-violating gravity:

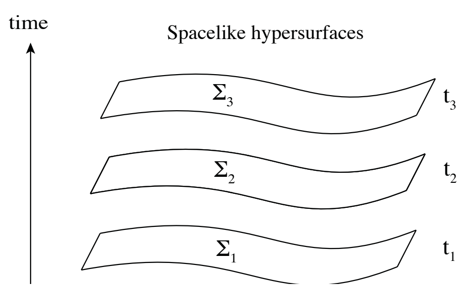
several theories in this class; the most interesting (and well-motivated) is Horava gravity *(Horava, '09)*:

Giving up Lorentz invariance in gravity, it is possible to construct a renormalizable QFT with only spatial diffeomorphism invariance

$$S_{\mathrm{H}} = \frac{1}{16\pi G_{\mathrm{H}}} \int \mathrm{d}T\, \mathrm{d}^3x\, N\sqrt{h} \left( L_2 + \frac{\hbar^2}{M_\star^2} L_4 + \frac{\hbar^4}{M_\star^4} L_6 \right),$$

Dynamical 3+1 foliation (see Valeria's talk)

$$L_2 = K_{ij}K^{ij} - \frac{1+\lambda}{1-\beta} K^2 + \frac{1}{1-\beta}\, {}^{(3)}R + \frac{\alpha}{1-\beta}\, a_i a^i$$




Prediction (*smoking gun*): modified dispersion relation, affecting GW propagation

$$\omega^2 \propto k^2 + \alpha_4 \left( \frac{\hbar}{M_\star} \right)^2 k^4 + \alpha_6 \left( \frac{\hbar}{M_\star} \right)^4 k^6 + \cdots$$

## Massive graviton theories:

most interesting is *de Rahm-Gabadadze-Tolley* (dRGT) theory *(de Rahm et al., '10)*
result of a large effort to avoid ghosts in massive graviton theories.
Gravity is described by a local, Lorentz-invariant, massive, self-interacting spin-2 field.

$$S_{\mathrm{dRGT}} = \int \mathrm{d}^4x \sqrt{-g} \left[ \frac{M_{\mathrm{Pl}}^2}{2} R + \frac{M_{\mathrm{Pl}}^2 m_{\mathrm{g}}^2}{4} \sum_{n=0}^{4} \alpha_n \mathcal{L}_n(\mathcal{K}) \right]$$

Action depends (through a tensor $\mathscr{K}$) on a fixed auxiliary metric (or dynamical, e.g. *bigravity*)

In the decoupling limit ($m_g \to 0$ with $m_g^2 M_{pl}$ finite) reduces to *Galileon theory:* scalar field action in flat space with derivative interactions

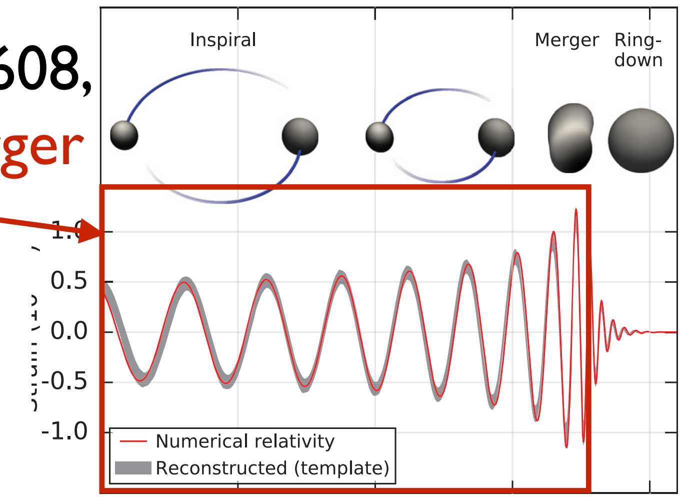
$$S_{\mathrm{gal}}[\pi] = \int \mathrm{d}^4x \left\{ -\frac{3}{4}(\partial\pi)^2 + \sum_{n=3}^{5} c_n \mathcal{L}_n^{(g)} \left[ \frac{1}{\Lambda_3^3} \partial_\mu \partial_\nu \pi \right] + \frac{g_1}{M_{\mathrm{Pl}}} \pi T + \frac{g_2}{M_{\mathrm{Pl}} \Lambda_3^3} \partial_\mu \pi \partial_\nu \pi T^{\mu\nu} \right\}$$

**Gravity with:** large extra dimensions, non-commutative geometry, time-dependent G, non-dynamical fields, etc. etc.

# GW signal from compact binary: a powerful probe for strong gravity

## 1) Inspiral and merger

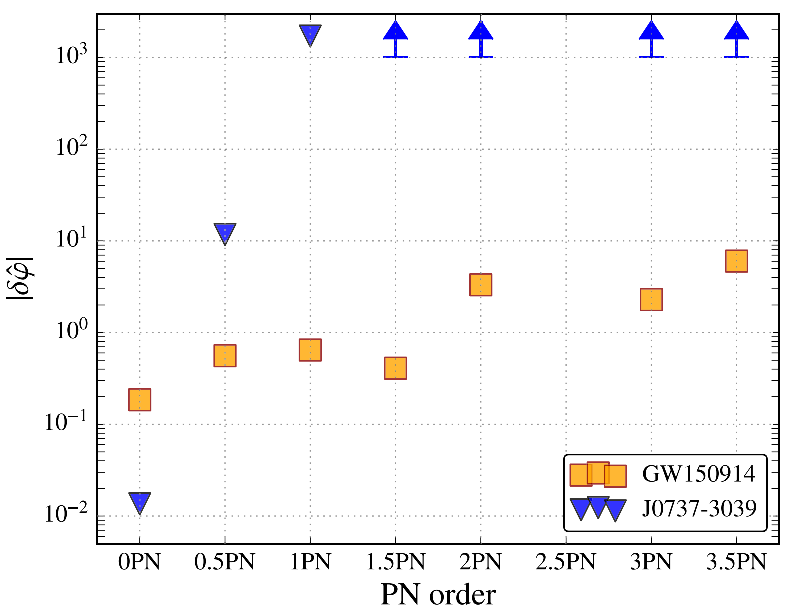
Most of the information of GW150914, GW151226, GW170104, GW170608, GW170814, GW170817 has been extracted from the inspiral and the merger




Current tests:

- Test of the PN coefficients:

comparing the signal with extended PN model of inspiral and late-inspiral, we set strict upper bounds on the PN coefficients. This is a test of several possible deviations.




*(Abbot et al., Testing of GR with GW150914, PRL '16)*

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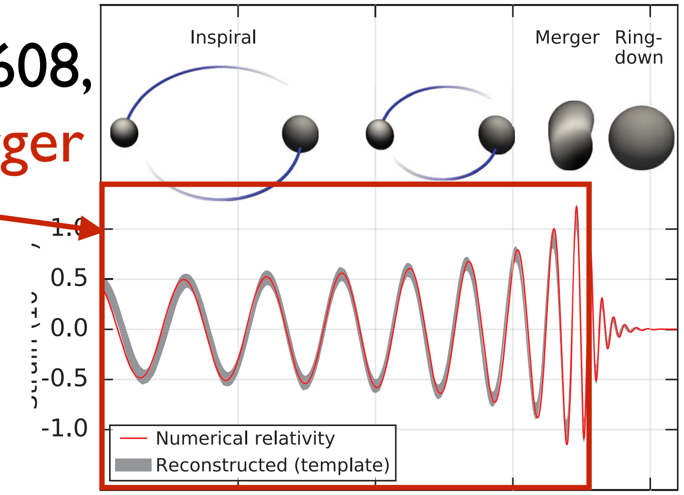


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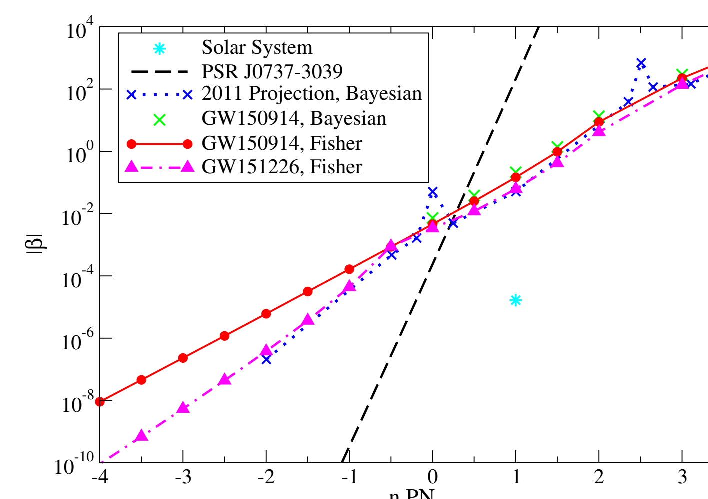
As mentioned, it has been extended through the ppE approach.

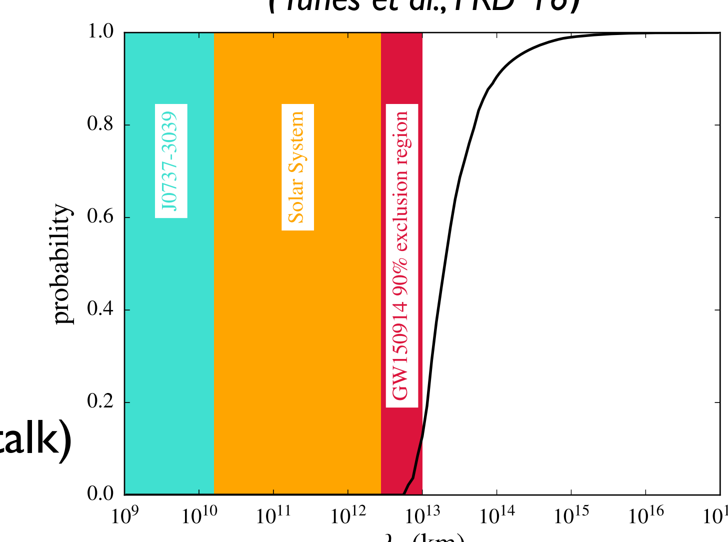



*(Yunes et al., PRD '16)*

- Test of dispersion relation (and thus of graviton mass):

if $m_g \neq 0$, $$E^2 = p^2c^2 + m_g^2 c^4\,, \qquad \lambda_g = \frac{h}{mc}\,, \qquad \frac{v_g^2}{c^2} = 1 - \frac{h^2c^2}{\lambda_g^2 E^2}$$

We get $\lambda_g > 10^{13}$km and then $m_g < 10^{-22}$eV/$c^2$




*(Abbot et al., Testing GR with GW150914, PRL '16)*

- Test of polarization: GW170814 detected by LIGO & Virgo, purely tensor polarization favored wrt scalar/vector ones (Cristiano's talk)

*(Abbot et al., PRL '17)*

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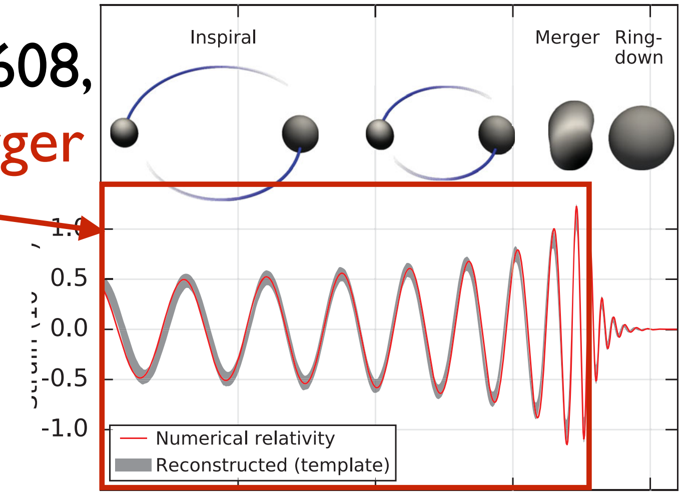
## 1) Inspiral and merger

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Future tests: *(e.g. Barausse et al., PRL '16; Yunes et al., PRD '16):*

- Bounds on dipole emission, predicted by several GR modifications (can be activated in late inspiral, thus escaping binary pulsar bounds)

  (-1)-PN effect! relevant in early inspiral, best with LISA $\rightarrow$ $\dot{E}_{\rm GW} = \dot{E}_{\rm GR}\left[1 + B\left(\frac{Gm}{r_{12}c^2}\right)^{-1}\right]$

- Bounds on more general deviations of radiated flux (due to extra dimensions, violations of Lorentz invariance, time-varying G due to extra fields, etc.)
- Bounds on modification of GW propagation (graviton mass, dispersion relation, etc.)
- Bounds on violations of the strong equivalence principle

Late inspiral and merger probe a regime currently unconstrained by binary pulsars!

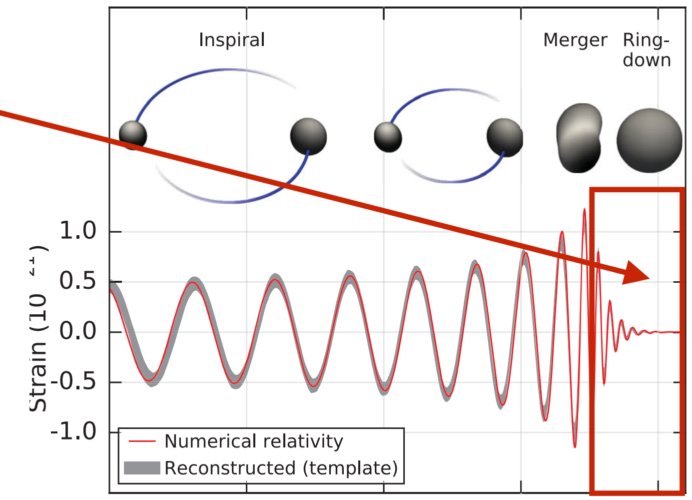
We need numerical relativity simulations in modified gravity theories!

- to test GR against other theories looking at merger (the most violent process ever observed)
- even models of late inspiral require calibration of phenomenological parameters with NR waveforms
- problem solved only for scalar-tensor theories, otherwise theoretically challenging

# GW signal from compact binary: a powerful probe for strong gravity

## 2) Ringdown:

signal emitted by the final BH,
strongly excited from the violent merger process,
which rapidly settles down to a stationary configuration,
oscillating and emitting GW at its proper (damped) oscillation frequencies:
the quasi-normal modes of the BH.



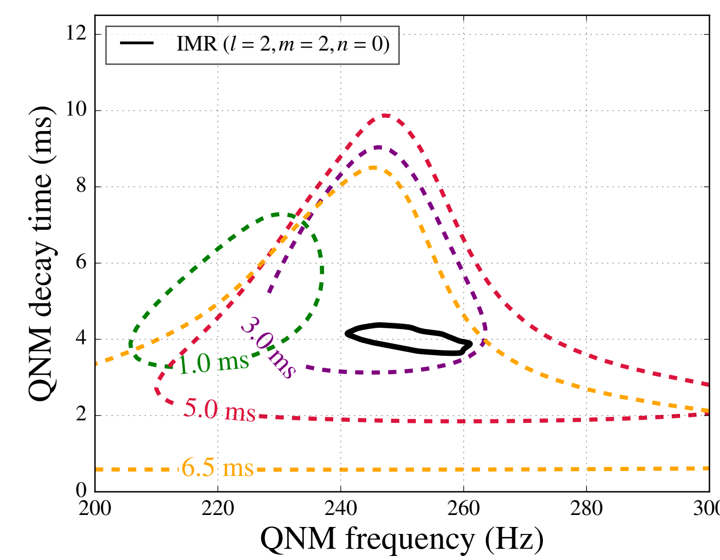

| | $M\omega_0 + iM\omega_i$ | | $M\omega_0 + iM\omega_i$ |
|---|---|---|---|
| $\ell = 2$ | 0.3737+i0.0890 | $\ell = 3$ | 0.5994+i0.0927 |
| | 0.3467+i0.2739 | | 0.5826+i0.2813 |
| | 0.3011+i0.4783 | | 0.5517+i0.4791 |
| | 0.2515+i0.7051 | | 0.5120+i0.6903 |

Caution: only *late* ringdown really containts the QNMs,
at the beginning it is determined by background (see Paolo's talk)

Current tests:

GW150914 had SNR~25 in the *entire* signal
but only SNR~7 in the ringdown,
so only weak test has been possible:

- final M obtained from inspiral+merger, matching NR
- computed the corresponding QNM frequency (*f*~251Hz)
- consistency check between this value and the signal
  (they are indeed consistent with more than 90% confidence)

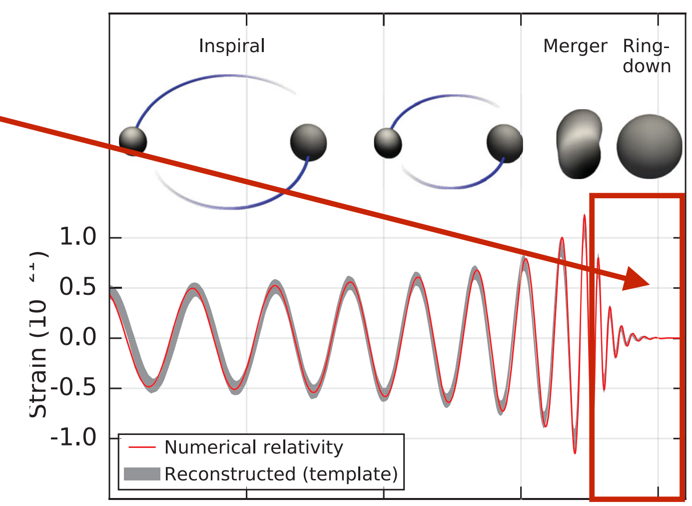



*(Abbot et al., Testing GR with GW150914, PRL '16)*

# GW signal from compact binary: a powerful probe for strong gravity

## 2) Ringdown

QNMs are a great probe of strong gravity:
BH spectroscopy!




- sensible to strong-curvature corrections
- sensible to the most dynamical content of the theory
- carry the imprint of the underlying gravity theory

(caution: only *late* ringdown contains actual QNMs, see Paolo's talk)

Problems for future tests:

- We do not expect do detect a strong enough ringdown signal soon
  - interesting proposal: stacking several detections
  - better with 3G detector or LISA

- We still know very few about BH QNMs in modified gravity theories.

  It should be important to:
  - derive QNMs of stationary BHs in different modified gravity theories
  - find how to extract information from data
  - possibly, find a parametrization of the mode shifts

## GW signal from compact binary: a powerful probe for strong gravity

### 2) Ringdown

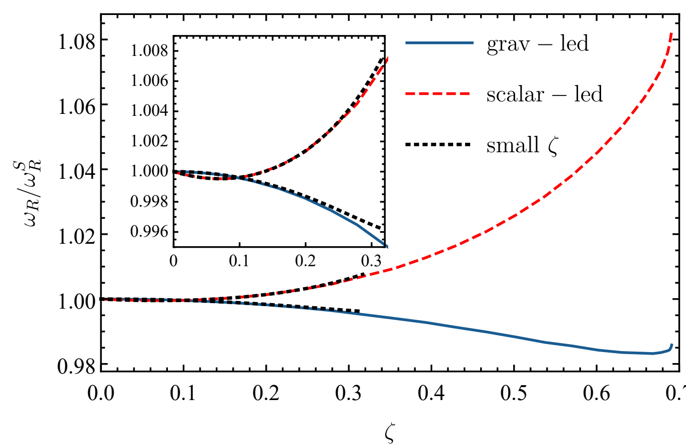
In recent years QNM of static BHs have been determined in a large class of modified gravity theories

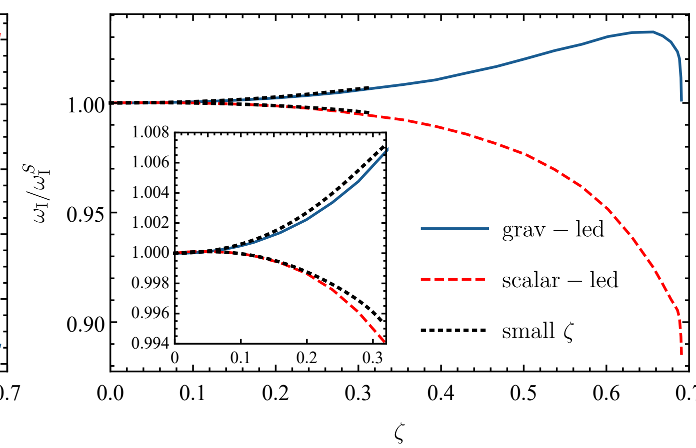
General pattern: (*Cardoso & Gualtieri PRD '09; Molina et al., PRD'10, 16, Salcedo et al., PRD. '16*):

- new classes of modes in the GW spectrum, due to coupling to extra fields
- a (small) shift in the modes predicted by GR

- The new classes of modes is likely to be poorly excited in BH coalescences.
- The shift in the "old" modes could be detectable, if SNR is large enough.

At leading order, the mode shift is O(ζ)
where ζ is the coupling parameter
of the theory




First, preliminary results:

bound on the coupling parameter of the order of $\alpha^{1/2} \lesssim 11 \left(\frac{50}{\rho}\right)^{1/4} \left(\frac{M}{10M_\odot}\right) \mathrm{km}\,,$

In order to measure this shift, we would need at least an SNR ρ~100, which can only be obtained with 3G detectors.

# Conclusions

- With GW detection, the new, uncharted territory of strong-field, large-curvature gravity is open for investigations

- General relativity should be tested against possible modifications

- Different possible approaches (bottom-up, top-down). Connections with other fields of physics (HEP, cosmology)

- We already started making non-trivial tests, but a thorough study of the strong-field regime requires next generation of detectors (3G, LISA)

- Every time a new window opened (in particle physics and in astrophyics), new physics was found. GWs could teach us something unexpected on the behaviour of the gravitational interaction!