VORTEX PRECESSION IN TRAPPED SUPERFLUIDS

AN EFT APPROACH

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Talk mostly based on:

A.E., R. Krichevsky, A. Nicolis — PRA 96 (2017); arXiv:1704.08267

OUTLINE

- Introduction
- EFT for vortex lines in superfluids
    1. An EFT for superfluids
    2. An EFT for vortices
- Vortex precession
    1. Introducing spatial confinement
    2. The vortex effective action
    3. Precession
- Conclusions and outlook

INTRODUCTION

A relativistic approach to non-relativistic matter

- The regimes of validity of **high energy theory** and that of existence of **condensed matter systems** could not seem more separated. However:

Poincarè invariance of fundamental interactions is a symmetry of the whole Universe

↓

Condensed matter systems must break Poincarè only **spontaneously**

- We can then describe CM systems using relativistic EFTs
- Advantages:
  1. Classify CM states only in terms of symmetry breaking pattern ⟶ ignore microscopic intricacies
  2. Relativistic theories are easily organized in a perturbative fashion
  3. Borrow decades of particle physics expertise to compute e.g. scattering processes, effective actions, etc.
- Disadvantages:
  1. Parameters of the theory must be extracted from experiment
  2. EFT breaks down at distances comparable to the microscopic ones.

`[for a nice discussion see e.g. Nicolis, Penco, Piazza, Rattazzi JHEP 1506; arXiv:1501.03845]`

INTRODUCTION

The broader take-home message

- Let me state right away what **broader lesson** I think should be learned from the present results. *DISCLAIMER: this is the speaker's very personal opinion*:

1. The EFT approach to condensed matter is a **powerful tool** that might allow to perform calculations that would be otherwise untreatable / very hard.

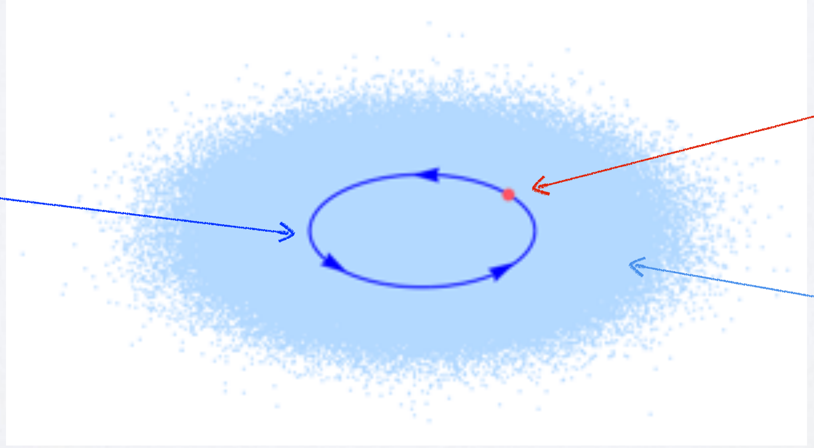
2. The formalism is now mature enough to become more than just a theoretical exercise. It is ready to be used to describe **real experimental data**, as well as expand our understanding of **unexplored phenomena.**

3. The **interaction** between the high energy community and the condensed matter one can lead to new, interesting ideas and directions for the future.

INTRODUCTION

Vortex precession in trapped superfluids

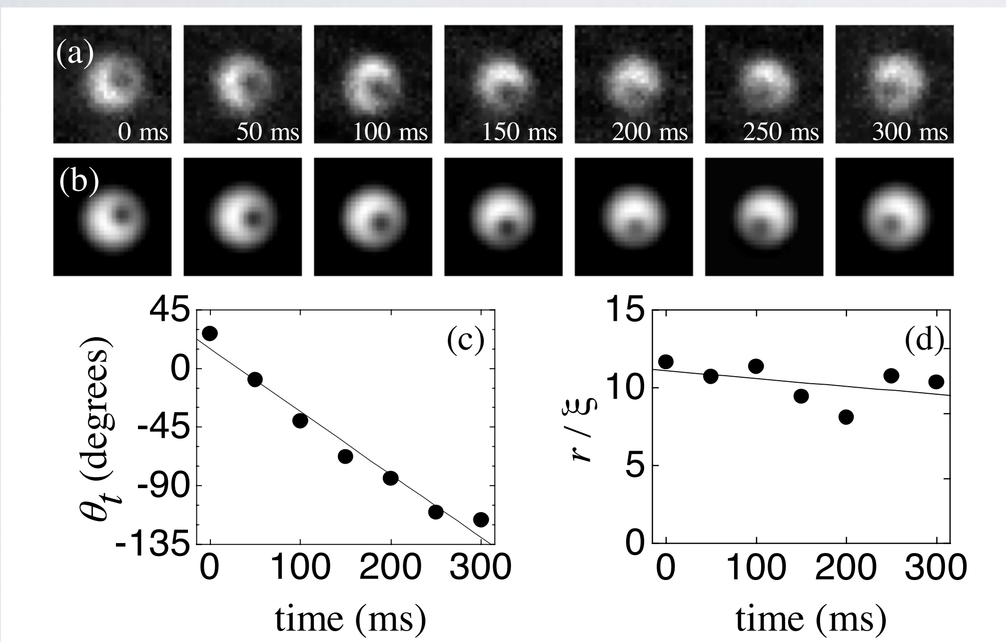
- Features of **vortex lines in superfluids**:
  1. String-like objects where the superfluid vorticity can be non-zero: $\vec{\nabla} \times \vec{v} = \hat{z}\dfrac{2\pi\ell}{m}\delta^{(2)}(\vec{r})$
  2. The velocity field away from them is irrotational but non-trivial
  3. Their circulation is quantized: $\Gamma = \oint \vec{v}\cdot d\vec{l} = \dfrac{2\pi}{m}\boxed{\ell}$ integer quantum number
- They can now be easily produced in lab
- If the vortex is created away from the center of the superfluid cloud it start **precessing** around it
- The vortex orbit is an ellipse with the same aspect ratio as for the cloud

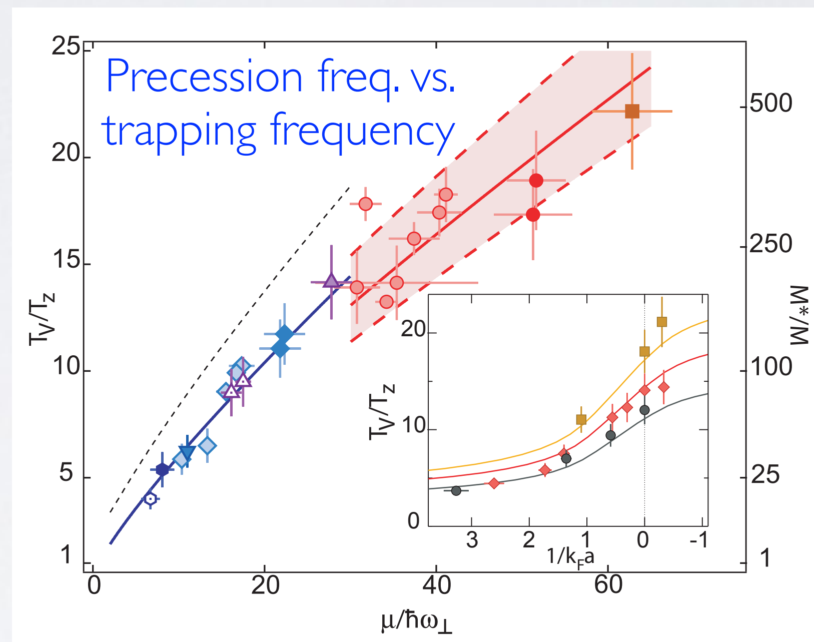


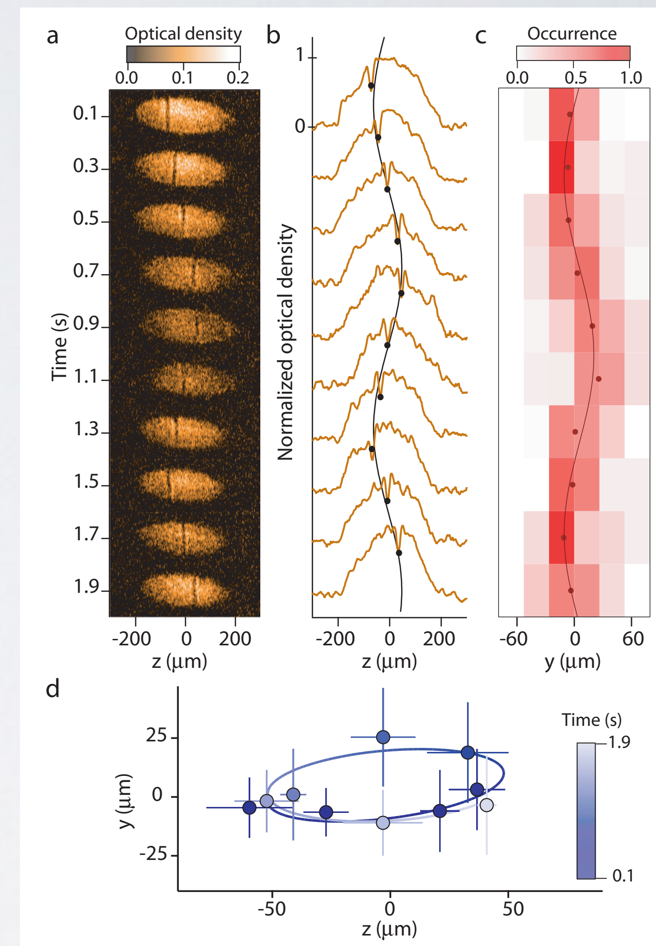
INTRODUCTION

Vortex precession in trapped superfluids

- The precession of vortices in superfluids has already been observed in several experiments







[Cornell et al. PRL 85 (2000) arXiv:cond-mat/0005368]

[Zwierlein et al. PRL 113 (2014) arXiv:1402.7052]

- Different traditional methods give different predictions for the precession frequency
- The correct one seems to be:

$$\boxed{\omega_p = \frac{3}{4}\frac{\omega_x \omega_y}{\mu}\log\left(\frac{R_\perp}{\xi}\right)}$$

[see e.g. Jackson et al. PRA61 (1999); Svidzinsky and Fetter PRL84 (2000)]

EFT FOR VORTEX LINES IN SUPERFLUIDS

An EFT for superfluids

- In field theory terms an ***s*-wave superfluid at zero temperature** is a system that carries a *U(1)* charge in a state that:
    1. has finite density for the *U(1)* charge
    2. spontaneously breaks the corresponding *U(1)* symmetry

- The simplest implementation is in terms of a single real scalar $\phi(x)$ such that

$$\phi \xrightarrow{U(1)} \phi + \boxed{a} \qquad \langle \phi \rangle = \boxed{\mu} t \qquad \phi = \mu t + \boxed{\pi}$$

Shift under U(1) — Chemical potential — Goldstone = Phonon

[see e.g. Son arXiv:hep-ph/0204199; Nicolis arXiv:1108.2513]

- An alternative description involves a 2-form $\mathcal{A}_{\mu\nu}(x)$ such that

$$\langle \mathcal{A}_{ij} \rangle = -\tfrac{1}{3}\boxed{\bar{n}}\epsilon_{ijk}x^k \qquad \mathcal{A}_{\mu\nu} \to \mathcal{A}_{\mu\nu} + \boxed{\partial_{[\mu}\xi_{\nu]}} \qquad \mathcal{A}_{0i} = \bar{n}\boxed{A_i(x)}/c \qquad \mathcal{A}_{ij} = \bar{n}\epsilon_{ijk}(-\tfrac{1}{3}x^k + \boxed{B^k(x)})$$

number density — local gauge symm. — hydrophoton — phonon

- This description is better suited to introduce vortices

EFT FOR VORTEX LINES IN SUPERFLUIDS

An EFT for vortices

- At distances much larger than its core a vortex is a string-like object
- Now we have two object to describe the superfluid and its vortices:
  1. the superfluid bulk modes $\longrightarrow$ two-form $\mathcal{A}_{\mu\nu}(x)$
  2. superfluid vortex $\longrightarrow$ string embedding $X^\mu(\tau, \sigma)$
- The symmetries of our system are now Poincarè invariance, gauge invariance for $\mathcal{A}_{\mu\nu}$ and reparametrization invariance
- The most general action is schematically

$$S = \boxed{S_{\text{bulk}}} + \boxed{S_{\text{KR}}} + \boxed{S_{\text{NG}'}} + \text{gauge fixing}$$

Superfluid modes only

Interaction between superfluid modes & string

Generalized Nambu-Goto (string only)

- We will perform an expansion in small perturbations around the background and small derivatives (low energy)

[Horn, Nicolis, Penco – JHEP 1510 (2015) arXiv:1507.05635]

EFT FOR VORTEX LINES IN SUPERFLUIDS

An EFT for vortices

- Given the 2-form the following quantity is gauge and Lorentz invariant:

$$F^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\partial_\nu A_{\alpha\beta} \qquad \Longrightarrow \qquad Y = -F_\mu F^\mu$$

- The **most general action for the superfluid modes** is then $S_{\text{bulk}} = \int d^4x\, G(Y)$

- These objects can be related to measurable thermodynamical quantities:

Number density

$$\boxed{n} = \sqrt{Y} \qquad \boxed{\rho} = -G(Y)$$

Energy density

Pressure

$$\boxed{p} = G(Y) - 2YG'(Y) \qquad \boxed{u_\mu} = -\frac{F_\mu}{\sqrt{Y}}$$

Superfluid velocity

- Disclaimer: this dictionary is useful for clear experimental reasons. It is by no means necessary from the theory side though!

EFT FOR VORTEX LINES IN SUPERFLUIDS

An EFT for vortices

- Action for vortex line, bulk fluctuations and their couplings:

Bkg enthalpy density

Sound speed

$$S \to \frac{\boxed{\bar{w}}}{c^2}\int d^3x dt \left[\frac{1}{2}(\vec{\nabla}\times\vec{A})^2 + \frac{1}{2}(\dot{\vec{B}}^2 - \boxed{c_s^2}(\vec{\nabla}\cdot\vec{B})^2) + \frac{1}{2}\left(1-\frac{c_s^2}{c^2}\right)\vec{\nabla}\cdot\vec{B}(\dot{\vec{B}}-\vec{\nabla}\times\vec{A})^2\right]$$

phonon & hydrophoton kinetic terms and interactions

$$-\int dt d\sigma \left[\frac{1}{3}\bar{n}\lambda\epsilon_{ijk}X^k\partial_t X^i\partial_\sigma X^j + \boxed{T_{(00)}}|\partial_\sigma\vec{X}|\right]$$

free string

$$+\int dt d\sigma \left[\boxed{\bar{n}\lambda}(A_i\partial_\sigma X^i + \epsilon_{ijk}B^k\partial_t X^i\partial_\sigma X^j) + |\partial_\sigma\vec{X}|(2\boxed{T_{(01)}}\vec{\nabla}\cdot\vec{B} + 2\boxed{T_{(10)}}(\dot{\vec{B}}-\vec{\nabla}\times\vec{A})\cdot\frac{\vec{v}_\perp}{c^2})\right]$$

String / superfluid interaction

Kalb-Ramond coupling

Generalized string tension

- No need to read all the details!

- The **hydrophoton** and **phonon** propagators are:

$$G_A^{ij}(k) = \frac{c^2}{\bar{w}}\frac{i(\delta^{ij}-\hat{k}^i\hat{k}^j)}{k^2} =$$ 

(Hydrophoton: non-dynamical degree of freedom)

$$G_B^{ij}(k) = \frac{c^2}{\bar{w}}\frac{i\hat{k}^i\hat{k}^j}{\omega^2 - c_s^2 k^2} =$$ 

(Phonon: gapless, propagates at the speed of sound)

[Horn, Nicolis, Penco — JHEP 1510 (2015) arXiv:1507.05635]

VORTEX PRECESSION

Trapping the superfluid

- From now on we work in the non-relativistic limit ($c \to \infty$)
- We need to introduce the spatial confinement of the superfluid. Forget about the vortex for now and write the most general trapping term compatible with our symmetries:

$$S_{\rm tr} = -\int d^3x dt\, \mathcal{E}(\boxed{\sqrt{Y}}, \vec{x}) \longrightarrow \int d^3x dt\, \bar{n} \boxed{V(\vec{x})}\, \vec{\nabla}\cdot\vec{B} + \text{cubic terms}$$

$Y = -F_\mu F^\mu$

$V(\vec{x}) \equiv \partial\mathcal{E}/\partial\sqrt{Y}$

- Therefore $\vec{\nabla}V$ provides an external source for the phonon field. To lowest order in the trapping potential this modifies the superfluid density:

$$n(\vec{x}) = \sqrt{Y} = \bar{n}\left(1 - \frac{\bar{n}c^2}{\bar{w}c_s^2}V(\vec{x})\right) \xrightarrow{c\to\infty} \boxed{\bar{n}\left(1 - \frac{V(\vec{x})}{mc_s^2}\right)}$$

- This is exactly the result obtained with other standard techniques.
- ***Can we improve it?***

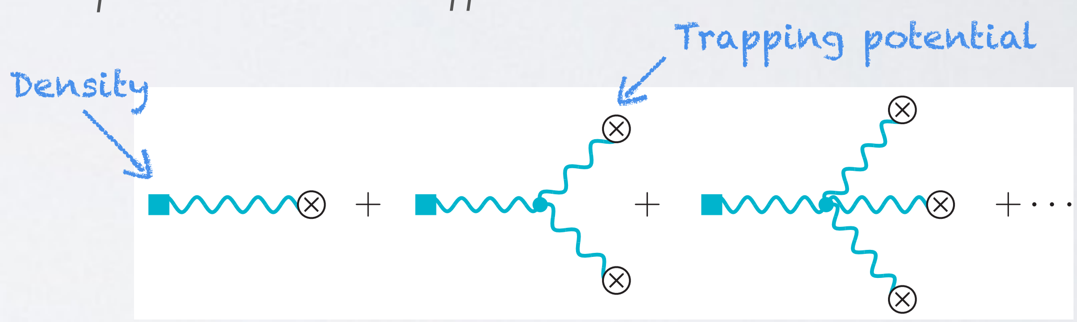
VORTEX PRECESSION

Trapping the superfluid

- First order in the trap $\longrightarrow$ the result can only be trusted for small $V(\vec{x})$ (close to center)
- The point around which we are expanding is arbitrary $\longrightarrow$ truly an expansion in small $\vec{\nabla}V(\vec{x})$
- Standard RG logic allows to *rewrite the previous equation as a differential one*

$$\boxed{c_s^2(n)\frac{dn}{n} = -\frac{dV}{m}}$$

Resum



- If the equation of state $c_s(n)$ is known, this equation allows to **find the fully non-linear relation between the density and the potential**
- Standard approach (Gross-Pitaevskii) assumes the naive $c_s^2(n) \propto n$
- To the best of our knowledge this result has never appeared in the literature

VORTEX PRECESSION

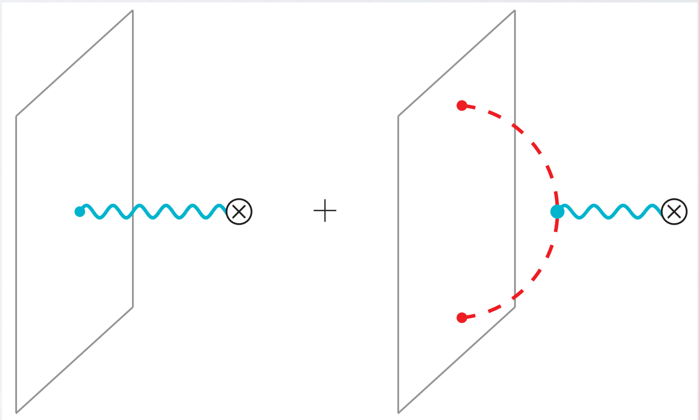
The vortex effective action

- We can now reintroduce the vortex $\longrightarrow$ consider a straight one $\vec{X}(t,z) = (X(t), Y(t), z)$
- The vortex provides another external source for the phonon and hydrophoton fields

$$\vec{J}_A(x) = \bar{n}\lambda\delta^2(\vec{x}_\perp - \vec{X})\hat{z}; \qquad \vec{J}_B(x) = \left[(\bar{n}\lambda\epsilon_{ab}\dot{X}^b - 2T_{(01)}\partial_a)\delta^2(\vec{x}_\perp - \vec{X}) - \bar{n}\partial_a V(\vec{x}_\perp)\right]\hat{x}^a_\perp$$

- **Integrate out phonon and hydrophoton at tree level** to get an effective action for the vortex:

$$S_{\text{eff}}[\vec{X}] = \int \frac{d^3k\,d\omega}{(2\pi)^4}\left[J^i_A(-k)iG^{ij}_A(k)J^j_A(k) + J^i_B(-k)iG^{ij}_B(k)J^j_B(k)\right] \longleftrightarrow$$



- To lowest order in the trapping we obtain the non-relativistic vortex action:

$$\boxed{S^{(\text{NR})}_{\text{eff}}[\vec{X}] = \int dt dz\left[\frac{\bar{n}\lambda}{3}\epsilon_{ab}X^a\dot{X}^b + \frac{2T_{(01)}}{mc_s^2}V(\vec{X}) + \frac{\bar{n}\lambda^2}{8\pi^2m^2c_s^2}\int d^2x_\perp \frac{V(\vec{x}_\perp + \vec{X})}{x^2_\perp}\right]}$$

- *The motion of the vortex is now just a point particle problem!*

VORTEX PRECESSION

Precession

- The eqs. of motion for a vortex close to the center of the cloud is easily found:

$$\frac{2\bar{n}\lambda}{3}\epsilon_{ab}\dot{X}^b + X^b\left[\frac{2T_{(01)}}{mc_s^2}\partial_a\partial_b V(0) + \frac{\bar{n}\lambda^2}{8\pi^2 m^2 c_s^2}\int d^2x_\perp \frac{\partial_a\partial_b V(\vec{x}_\perp)}{x_\perp^2}\right]$$

- Note that:

Eq. of motion for the vortex is only **first order in time derivatives**

$\Downarrow$

Only need to specify the initial position to completely determine the motion!

- There are two qualitatively different scenarios:

  1. Trapping potential such that $\partial_a\partial_b V(0) \neq 0$ (***harmonic***)
  2. Trapping potential such that $\partial_a\partial_b V(0) = 0$ (***non-harmonic***)

- Let's analyze them separately

VORTEX PRECESSION

Harmonic trap

- For harmonic traps the previous integral is log divergent

UV cutoff

Microscopic length to be determined from exp.

$$\int d^2x_\perp \frac{\partial_a \partial_b V(\vec{x}_\perp)}{x_\perp^2} = \partial_a\partial_b V(0)\, 2\pi \, \log(\boxed{R_\perp}/\boxed{a}) + \cdots$$

Typical size of the cloud (IR cutoff)

- We hide the log divergence in a running of the coupling constant: $T_{(01)}(q) = -\frac{\bar{n}\lambda^2}{8\pi m}\log(q\boxed{\ell})$

- This is a running at tree level!

- Experiments typically provide this harmonic trapping potential: $V(\vec{x}_\perp) = \frac{m}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2\right) + \mathcal{O}(x_\perp^4)$

- The eqs. of motion for the vortex become:

$$\boxed{\dot{X}(t) = \omega_p \frac{\omega_y}{\omega_x} Y(t); \qquad \dot{Y}(t) = -\omega_p \frac{\omega_x}{\omega_t} X(t) \qquad \text{with} \qquad \omega_p = \frac{3\Gamma}{8\pi c_s^2}\omega_x\omega_y \log(R_\perp/\ell)}$$

- This is exactly the elliptic motion with the right frequency to fit the experimental data!

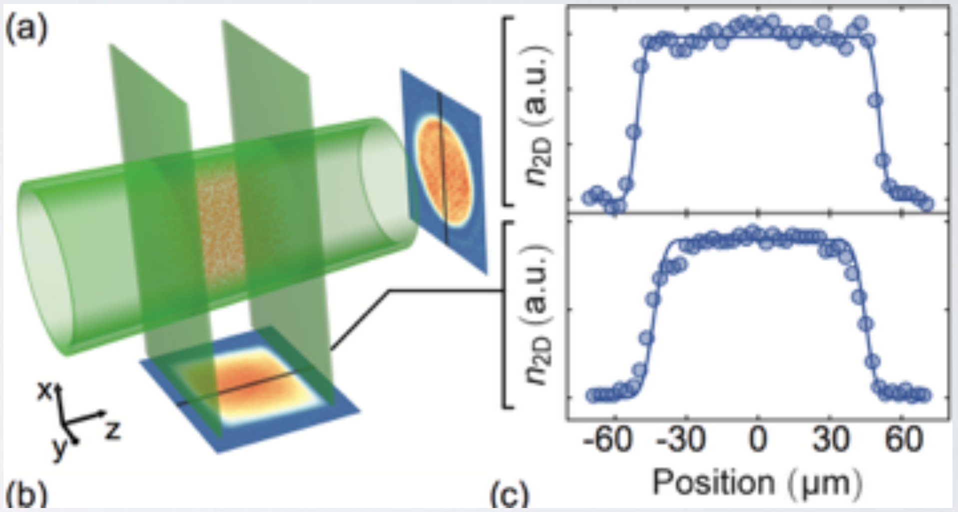
- The precession frequency has a dramatic **log enhancement**, which is found in data as well.
Measurable effect of the tree level running!

VORTEX PRECESSION

Flatter traps

- Recent experiments managed to produce trapping potentials very close to a perfect box
- Can our formalism deal with them?
- Just consider a trap for which $\partial_a \partial_b V(0) = 0$
- One can write quite generally:

$$\int d^2 x_\perp \frac{\partial_a \partial_b V(\vec{x}_\perp)}{x_\perp^2} = \frac{mc_s^2}{R_\perp} f_{ab}$$



[see e.g. Mukherjee et al. – PRL118 (2017)]

- Choose the axes as the eigenvectors of f_{ab} and find the eqs. of motion for the vortex

$$\boxed{\dot{X}(t) = \omega_p \sqrt{\frac{f_{yy}}{f_{xx}}} Y(t); \qquad \dot{Y}(t) = -\omega_p \sqrt{\frac{f_{xx}}{f_{yy}}} X(t) \qquad \text{with} \qquad \omega_p = \frac{3}{16\pi^2} \frac{\Gamma}{R_\perp^2} \sqrt{f_{xx} f_{yy}}}$$

- Still elliptical motion but the log enhancement is gone!
- This result has also been found recently with more traditional methods

[Kevrekidis, Wang, Carretero-Gonzales, Frantzekakis, Xie – PRA96 (2017), arXiv:1706.07137]

CONCLUSIONS

The strengths of our approach

- What have we achieved applying an EFT viewpoint to superfluids?

1. EFT is only based on symmetry arguments ⟶ **true for every superfluid**, even the strongly coupled ones (beyond Gross-Pitaevskii)

2. We can use simple renormalization group arguments ⟶ easily allowed to find the **fully non-linear relation between density and trap** + easily allowed to **recover the log enhancement observed experimentally**

3. We were able to compute the vortex effective action for **every regular potential**, without the need to rely to different physical effects (Bernoulli effects vs. inhomogeneity)

   [see e.g. discussion in Groszek et al. — arXiv:1708.09202]

4. The formalism is completely relativistic ⟶ we **computed the relativistic corrections to the vortex action**

   [see A.E., Nicolis, Krichevsky — PRA 96 (2017), arXiv:1704.08267]

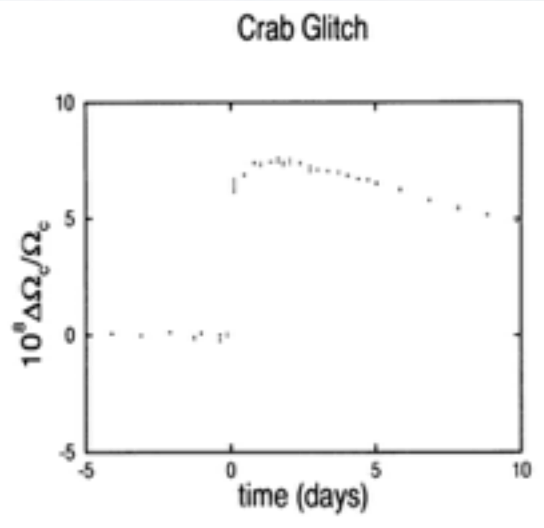
CONCLUSIONS

Possible future directions

- The EFT approach to condensed matter is powerful. **What can we do with it?**
- Some half-baked ideas:

1. Vortex-phonon interactions at finite temperature are experimentally relevant for the vortex lifetime ⟶ hard to treat with traditional models, easy with the EFT description
2. Neutron stars present ``glitches'' in the observed rotational frequency. They might be related to the liberation of pinned vortices ⟶ relativistic corrections are necessary

   [see e.g. Anderson, Itoh – Nature 256 (1975)]



3. Recent works propose to detect sub-GeV dark matter using superfluid He. Interaction between DM and the superfluid collective modes is complicated in a non-relativistic formalism ⟶ the EFT approach can make it considerably easier

   [see e.g. Schutz, Zurek – PRL 117 (2016), arXiv:1604.08206]

Thanks for your attention!

BACK UP

MOST GENERAL ACTION

- Symmetries of the superfluid alone:
  1. Gauge invariance of the two form: $F^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\partial_\nu \mathcal{A}_{\alpha\beta}$ $\longrightarrow$ $\boxed{S_{\text{bulk}} = \int d^4x\, G(Y)}$
  2. Poincare' invariance: $Y = -F_\mu F^\mu$
- Nambu-Goto action for the string alone: $S_{\text{NG}} \propto \int d\tau d\sigma \sqrt{-\det\left(G_{\mu\nu}(X)\partial_\alpha X^\mu \partial_\beta X^\nu\right)}$

  Any bulk metric
- Two possible bulk ``metrics'': $\eta_{\mu\nu}$ $\quad u_\mu u_\nu$
- From bi-gravity theory, the most general diff-invariant action is:

$$\boxed{S_{\text{NG}'} = -\int d\tau d\sigma \sqrt{-\det g}\,\mathcal{T}\left(g^{\alpha\beta}h_{\alpha\beta}, Y\right) \quad \text{with} \quad g_{\alpha\beta} = \eta_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu \quad h_{\alpha\beta} = u_\mu u_\nu \partial_\alpha X^\mu \partial_\beta X^\nu}$$

- For the interaction between vortex and superfluid we have Kalb-Ramond:

$$\boxed{S_{\text{KR}} = \lambda \int d\tau d\sigma \mathcal{A}_{\mu\nu}\partial_\tau X^\mu \partial_\sigma X^\nu}$$

PRECESSION FREQUENCY AND UV SCALE

- The result obtained for the precession frequency in harmonic potential is

$$\omega_p = \frac{3\Gamma}{8\pi c_s^2}\omega_x\omega_y \log(R_\perp/\ell)$$

- Microscopic theories tell us that $\ell \sim$ size of the vortex core. However, for us it is an unknown parameter to be measured from experiment.

- We can nevertheless define $\chi \equiv \dfrac{\omega_p}{\omega_x\omega_y}$

- The quantity $\chi(R_{\perp,1}) - \chi(R_{\perp,2})$ is then independent of any UV scale and it is hence a completely predictive result of our theory

RELATIVISTIC CORRECTIONS

- To include relativistic corrections we need to add some more ingredients:

  1. The trapping potential could now depend on the superfluid velocity (coupled via Doppler effect):

$$S_{\rm tr} = -\int dt d^3x\, \mathcal{E}\left(\sqrt{Y}, \boxed{\vec{u}}, \vec{x}\right)$$

$$\vec{u} = -\frac{F_i}{\sqrt{Y}} = \frac{\dot{\vec{B}} - \vec{\nabla}\times\vec{A}}{1-\vec{\nabla}\cdot\vec{B}}$$

  2. Keep the speed of light finite

- The new interactions between trapping potential and the fields are

$$S_{\rm tr} \to \int dt\, d^3x\, \bar{n}\left\{V(\vec{x})\left[\vec{\nabla}\cdot\vec{B} + \frac{1}{2c^2}(\dot{\vec{B}} - \vec{\nabla}\times\vec{A})^2\right] - \frac{1}{2c^2}\boxed{V_{ij}(\vec{x})}(\dot{\vec{B}} - \vec{\nabla}\times\vec{A})^i(\dot{\vec{B}} - \vec{\nabla}\times\vec{A})^j\right\}$$

$$V_{ij} \equiv \frac{c^2}{\bar{n}}\frac{\partial^2\mathcal{E}}{\partial u^i\partial u^j}$$

- We then need to compute one more diagram. The final action is

$$\boxed{\begin{aligned} S_{\rm eff}[\vec{X}] = \int dt d\sigma \Bigg[&\frac{\bar{n}\lambda}{3}\epsilon_{ab}X^a\dot{X}^b + \frac{2T_{(01)}\bar{n}c^2}{\bar{w}c_s^2}V(\vec{X}) + \frac{\bar{n}^3\lambda^2c^4}{8\pi^2\bar{w}^2c_s^2}\int d^2x_\perp \frac{V(\vec{x}_\perp+\vec{X})}{x_\perp^2} \\ &-\frac{\bar{n}^3\lambda^2c^2}{8\pi^2\bar{w}^2}\int d^2x_\perp\, \epsilon^{ab}\epsilon^{cd}V_{ac}(\vec{x}_\perp+\vec{X})\frac{x_\perp^b x_\perp^d}{x_\perp^4}\Bigg]\end{aligned}}$$