Accidental Peccei-Quinn symmetry in a model of flavour

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Based on work in
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+
work in progress

Strong CP problem

$$\mathcal{L} \supset \bar{\theta} \frac{g^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

Physical angle

$$\bar{\theta} = \theta_0 + \arg \det M^q \lesssim 10^{-10}$$

from neutron EDM
[Pendlebury et al '15]

Solutions

1. Massless up quark $m_u = 0$
   Disfavoured
   [PDG '16, Aoki et al '16]
2. Calculable models
   [Nelson '84, Barr '84]
   Ex: [FB, de Anda, de Medeiros Varzielas, King '15]
3. Peccei-Quinn mechanism
   [Peccei, Quinn '77, Weinberg '78, Wilczek '78]

Ingredients in a PQ solution

- $U(1)_{PQ}$ symmetry with chiral anomaly
- Complex scalar field φ
- Spontaneous symmetry breaking as $\varphi \to \langle\varphi\rangle$

Archetypal "invsibible axion" models

KSVZ

$$\mathcal{L} \supset \lambda\varphi\overline{Q}Q$$

- Heavy quarks Q integrated out below scale v_{PQ}
- Chiral anomaly $N_{DW} = 1$
- No tree-level couplings to matter

DFSZ

$$\mathcal{L} \supset \lambda\varphi^2 H_u H_d$$

- Chiral anomaly $N_{DW} = 6$
- Tree-level couplings to matter (since Higgs carries PQ charge)

Accidental PQ symmetry

- From $\mathbb{Z}_N$ with large N

  [Babu, Gogoladze, Wang '03, Dias, Pleitez, Tonasse '02, '04]
- In SUSY

  [Chun, Lukas '92]
- From gauge symmetry

  [Di Luzio, Nardi, Ubaldi '16]

Recent developments

- Flaxion [Ema, Hamaguchi, Moroi, Nakayama '16]
- Axiflavon [Calibbi, Goertz, Redigolo, Ziegler, Zupan '16]

"A to Z" model of flavour [King '14, King, Di Bari '15]

Key ingredients

- Pati-Salam gauge group
  Vertical unification
- A_4 family symmetry
  Horizontal unification (of LH fermions)
- $\mathbb{Z}_N$ (family) symmetries
  shapes superpotential, forbids dangerous terms
- CSD(4) vacuum alignment
  explain large neutrino mixing, Cabibbo angle
- Supersymmetry
  resolve hierarchy problem, gauge coupling unification

Pati-Salam $[SU(4)_C \times SU(2)_L \times SU(2)_R]$

- Left-handed fermions in

$$F_i \sim (4,2,1)_i = \begin{pmatrix} u_r & u_g & u_b & \nu \\ d_r & d_g & d_b & e \end{pmatrix}_i$$

- Right-handed fermions in

$$F_i^c \sim (\bar{4},1,2)_i = \begin{pmatrix} u_r^c & u_g^c & u_b^c & N^c \\ d_r^c & d_g^c & d_b^c & e^c \end{pmatrix}_i$$

$\mathbf{A}_4$

- Left-handed fermions in triplet

$$F \sim 3 = (F_1, F_2, F_3)$$

- Right-handed fermions in singlets

$$F_1^c,\ F_2^c,\ F_3^c \sim 1$$

Constrained sequential dominance (CSD) [King '99, '00, '02]

- SD originally devised for neutrinos:
  1) $N_{\rm atm} \to$ atmospheric mass m_{ν_3} and mixing $\theta_{23} \sim 45^\circ$
  2) $N_{\rm sol} \to$ solar mass m_{ν_2} and solar+reactor mixing θ_{12}, θ_{13}
  3) $N_{\rm dec}$, if present, nearly decoupled from theory $\to m_{\nu_1} \ll m_{\nu_{2,3}}$

CSD(n) with two neutrinos:

$$Y^\nu = \begin{pmatrix} 0 & b & * \\ a & nb & * \\ a & (n-2)b & * \end{pmatrix}, \qquad M_R \sim \mathrm{diag}(M_{\rm atm}, M_{\rm sol}, M_{\rm dec})$$

$$\begin{aligned} m^\nu &= v^2 Y^\nu M_R^{-1} (Y^\nu)^T \\ &= m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b \begin{pmatrix} 1 & n & n-2 \\ n & n^2 & n(n-2) \\ n-2 & n(n-2) & (n-2)^2 \end{pmatrix} + m_c \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix} \end{aligned}$$

- In unified scenario, CSD is extended to the quarks!
- Consider $n = 4$ [King '13]. With Y^d diagonal,

$$Y^u = Y^\nu = \begin{pmatrix} 0 & b & * \\ a & 4b & * \\ a & 2b & * \end{pmatrix}$$

- To first approximation, Cabibbo angle

$$\theta_{12}^q \approx \frac{Y_{12}^u}{Y_{22}^u} \approx \frac{1}{4}$$

- This is compellingly close to the true value $\theta_{12}^q \approx 0.227$.

- CSD(4) achieved by A_4 triplet flavons ϕ
- Flavons acquire VEVs with particular alignments:

$$\langle\phi_1^u\rangle = v_{\phi_1^u}(0,1,1), \qquad \langle\phi_1^d\rangle = v_{\phi_1^d}(1,0,0)$$
$$\langle\phi_2^u\rangle = v_{\phi_2^u}(1,4,2), \qquad \langle\phi_2^d\rangle = v_{\phi_2^d}(0,1,0)$$

- Example: first-generation up-type quarks

$$W \supset \frac{(F\cdot\phi_1^u)h_u F_1^c}{M} \to v_u \frac{v_{\phi_1^u}}{M}\left(F_1 F_2 F_3\right)\begin{pmatrix}0\\1\\1\end{pmatrix} F_1^c$$

- Alignments can be fixed by A_4 and orthogonality arguments, implemented by a superpotential

Full Yukawa/mass superpotential

$$W_F^{\text{eff}} = (F \cdot h_3)F_3^c + \frac{(F \cdot \phi_1^u)h_u F_1^c}{\langle \Sigma_u \rangle} + \frac{(F \cdot \phi_2^u)h_u F_2^c}{\langle \Sigma_u \rangle} + \frac{(F \cdot \phi_1^d)h_d F_1^c}{\langle \Sigma_{15}^d \rangle} + \frac{(F \cdot \phi_2^d)h_{15}^d F_2^c}{\langle \Sigma_d \rangle} + \frac{(F \cdot \phi_1^u)h_d F_1^c}{\langle \Sigma_d \rangle}$$

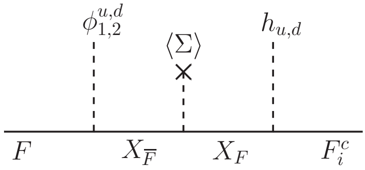
$$W_{\text{Maj}}^{\text{eff}} = \frac{\overline{H^c}\,\overline{H^c}}{\Lambda} \left(\frac{\xi^2}{\Lambda^2} F_1^c F_1^c + \frac{\xi}{\Lambda} F_2^c F_2^c + F_3^c F_3^c + \frac{\xi}{\Lambda} F_1^c F_3^c \right)$$

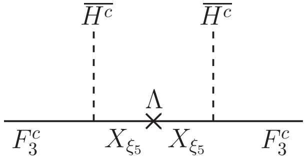
Notes

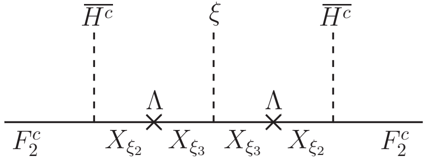
- $\overline{H^c} \sim (4,1,2)$ breaks $SU(4)_C \to SU(3)_C$, generates RHν masses
- $\Sigma \sim (1/15,1,1) \to \langle \Sigma \rangle \lesssim M_{\text{GUT}}$
- $\xi \sim (1,1,1) \to \langle \xi \rangle / \Lambda \sim 10^{-5}$

● 3rd family ● Up quarks ● Down quarks/charged leptons

Sample diagrams







Field	G_{PS}	A_4	$\mathbb{Z}_5$	$\mathbb{Z}_3$	$\mathbb{Z}_5'$	R	$U(1)_{PQ}$
F	$(4,2,1)$	3	1	1	1	1	0
$F^c_{1,2,3}$	$(\bar{4},1,2)$	1	$\alpha,\alpha^3,1$	$\beta,\beta^2,1$	$\gamma^3,\gamma^4,1$	1	$-2,-1,0$
$\overline{H^c}$	$(4,1,2)$	1	1	1	1	0	0
H^c	$(\bar{4},1,2)$	1	1	1	1	0	0
$\phi^u_{1,2}$	$(1,1,1)$	3	α^4,α^2	β^2,β	γ^2,γ	0	$2,1$
$\phi^d_{1,2}$	$(1,1,1)$	3	α^3,α	β^2,β	γ^2,γ	0	$2,1$
h_3	$(1,2,2)$	3	1	1	1	0	0
h_u	$(1,2,2)$	$1''$	α	1	1	0	0
h^u_{15}	$(15,2,2)$	1	α	1	1	0	0
h_d	$(1,2,2)$	$1'$	α^3	1	1	0	0
h^d_{15}	$(15,2,2)$	$1'$	α^4	1	1	0	0
Σ_u	$(1,1,1)$	$1''$	α	1	1	0	0
Σ_d	$(1,1,1)$	$1'$	α^3	1	1	0	0
Σ^d_{15}	$(15,1,1)$	$1'$	α^2	1	1	0	0
ξ	$(1,1,1)$	1	α^4	β^2	γ^2	0	2

Discrete $\mathbb{Z}_N$ symmetries

- $\mathbb{Z}_5$
  - Shaping symmetry of original A to Z model
  - Ensures CSD(4)
- $\mathbb{Z}_3$
  - Ensures PQ symmetry at renormalisable level
  - Forbids most off-diagonal terms in $Y^{d,e}$ (new!)
- $\mathbb{Z}_5'$
  - Protects PQ symmetry to sufficient order

Yukawa and mass matrices

$$Y^u = Y^\nu = \begin{pmatrix} 0 & b & \epsilon_{13} c \\ a & 4b & \epsilon_{23} c \\ a & 2b & c \end{pmatrix} \qquad Y^d = \begin{pmatrix} y_d^0 & 0 & 0 \\ B y_d^0 & y_s^0 & 0 \\ B y_d^0 & 0 & y_b^0 \end{pmatrix}$$

$$Y^e = \begin{pmatrix} -(y_d^0/3) & 0 & 0 \\ B y_d^0 & x y_s^0 & 0 \\ B y_d^0 & 0 & y_b^0 \end{pmatrix} \qquad M_R = \begin{pmatrix} M_1 & 0 & M_{13} \\ 0 & M_2 & 0 \\ M_{13} & 0 & M_3 \end{pmatrix}$$

Neutrino matrix after seesaw,

$$m^\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & 4 & 2 \\ 4 & 16 & 8 \\ 2 & 8 & 4 \end{pmatrix} + m_c e^{i\xi} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

PQ charges

$$
\begin{aligned}
W_F^{\text{eff}} \sim\ & (\underset{0}{F} \cdot \underset{0}{h_3})\underset{0}{F_3^c} + (\underset{0}{F} \cdot \underset{2}{\phi_1^u})\underset{0}{h_u}\underset{-2}{F_1^c} + (\underset{0}{F} \cdot \underset{1}{\phi_2^u})\underset{0}{h_u}\underset{-1}{F_2^c} \\
& + (\underset{0}{F} \cdot \underset{2}{\phi_1^d})\underset{0}{h_d}\underset{-2}{F_1^c} + (\underset{0}{F} \cdot \underset{1}{\phi_2^d})\underset{0}{h_{15}^d}\underset{-1}{F_2^c} + (\underset{0}{F} \cdot \underset{2}{\phi_1^u})\underset{0}{h_d}\underset{-2}{F_1^c}
\end{aligned}
$$

$$
W_{\text{Maj}}^{\text{eff}} \sim \underset{0}{\overline{H^c}}\,\underset{0}{\overline{H^c}}\,(\underset{2}{\xi}\ \underset{2}{\xi}\ \underset{-2}{F_1^c}\underset{-2}{F_1^c} + \underset{2}{\xi}\ \underset{-1}{F_2^c}\underset{-1}{F_2^c} + \underset{0}{F_3^c}\underset{0}{F_3^c} + \underset{2}{\xi}\ \underset{-2}{F_1^c}\underset{0}{F_3^c})
$$

Notes

- PQ symmetry realised also at renormalisable level
- Higgs sector completely neutral $\rightarrow$ no GUT-scale PQ breaking
- $U(1)_{PQ}$ assignments unique
- Third family is neutral

Breaking $U(1)_{PQ}$

- $\phi_i^f \to \langle \phi_i^f \rangle \sim v_{\phi_1^f}$ breaks all discrete symmetries *and* $U(1)_{PQ}$
- PQ-breaking scale

$$v_{PQ}^2 = (N_a f_a)^2 = \sum_{\phi} x_{\phi}^2 v_{\phi}^2$$

- Dominated by largest VEV: $\langle \phi_2^u \rangle$ (related to charm mass)

Axion

$$a = \frac{1}{v_{PQ}} \sum_{\varphi} x_{\varphi} v_{\varphi} a_{\varphi}$$

Domain wall number

$$N_a \equiv \left| 6x_F + 2\sum_i x_{F_i^c} \right| = \left| 6(0) + 2(-2 + -1 + 0) \right| = 6$$

Protecting the PQ symmetry

Consider terms like

$$\frac{\{\phi\}^n}{M_P^n} W$$

These generate a PQ-breaking axion mass

$$m_*^2 \sim m_{3/2}^2 \frac{v_{PQ}^{n-2}}{M_P^{n-2}}$$

[Holman et al '92]

[Kamionkowski, March-Russell '92]

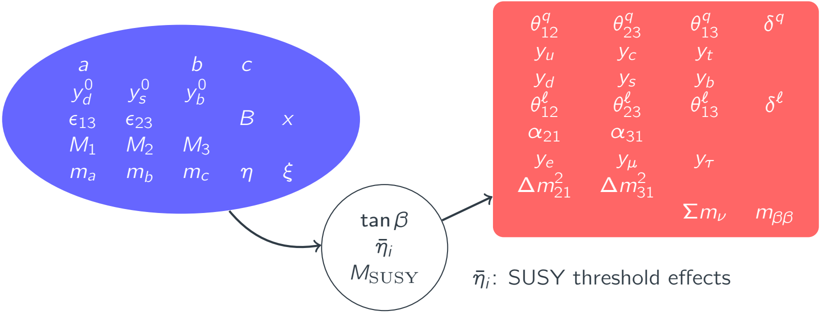
[Barr, Seckel '92]

We require $m_*^2/m_a^2 < 10^{-10}$, where

$$m_a^2 \approx m_\pi^2 \frac{f_\pi^2}{f_a^2}$$

To protect our solution, we forbid all PQ-violating terms like $\{\phi\}^n$ up to $n = 7$ (or $dim = 10$)!

Fitting to quark and lepton mixing data



$\bar{\eta}_i$: SUSY threshold effects

Simple MCMC

- Minimise χ^2 to find best fit

$$\chi^2 = \sum_i \left(\frac{P(x_i) - \mu_i}{\sigma_i} \right)^2$$

- Calculate 95% credible intervals (hpd)

Measured values run up to M_{GUT} (assuming MSSM) [Antusch, Maurer '13]

Leptons

Observable	Data		Model	
	Central value	1σ range	Best fit	Interval
θ^ℓ_{12} /°	33.57	$32.81 \to 34.32$	32.88	$32.72 \to 34.23$
θ^ℓ_{13} /°	8.460	$8.310 \to 8.610$	8.611	$8.326 \to 8.882$
θ^ℓ_{23} /°	41.75	$40.40 \to 43.10$	39.27	$37.35 \to 40.11$
δ^ℓ /°	261.0	$202.0 \to 312.0$	242.6	$231.4 \to 249.9$
y_e $/10^{-5}$	1.004	$0.998 \to 1.010$	1.006	$0.911 \to 1.015$
y_μ $/10^{-3}$	2.119	$2.106 \to 2.132$	2.116	$2.093 \to 2.144$
y_τ $/10^{-2}$	3.606	$3.588 \to 3.625$	3.607	$3.569 \to 3.643$
Δm^2_{21} $/10^{-5}\,\mathrm{eV}^2$	7.510	$7.330 \to 7.690$	7.413	$7.049 \to 7.762$
Δm^2_{31} $/10^{-3}\,\mathrm{eV}^2$	2.524	$2.484 \to 2.564$	2.540	$2.459 \to 2.616$
m_1 /meV			0.187	$0.022 \to 0.234$
m_2 /meV			8.612	$8.400 \to 8.815$
m_3 /meV			50.40	$49.59 \to 51.14$
$\sum m_i$ /meV		< 230	59.20	$58.82 \to 60.19$
α_{21}			10.4	$-38.0 \to 70.1$
α_{31}			272.1	$218.2 \to 334.0$
$m_{\beta\beta}$ /meV			1.940	$1.892 \to 1.998$

We set $\tan\beta = 5$, $M_{\mathrm{SUSY}} = 1$ TeV and $\bar{\eta}_b = -0.24$

Quarks

Observable	Data		Model	
	Central value	1σ range	Best fit	Interval
θ^q_{12} /°	13.03	12.99 → 13.07	13.04	12.94 → 13.11
θ^q_{13} /°	0.1471	0.1418 → 0.1524	0.1463	0.1368 → 0.1577
θ^q_{23} /°	1.700	1.673 → 1.727	1.689	1.645 → 1.753
δ^q /°	69.22	66.12 → 72.31	68.85	63.00 → 75.24
y_u /10^{-6}	2.982	2.057 → 3.906	3.038	1.098 → 4.957
y_c /10^{-3}	1.459	1.408 → 1.510	1.432	1.354 → 1.560
y_t	0.544	0.537 → 0.551	0.545	0.530 → 0.558
y_d /10^{-5}	2.453	2.183 → 2.722	2.296	2.181 → 2.966
y_s /10^{-4}	4.856	4.594 → 5.118	4.733	4.273 → 5.379
y_b	3.616	3.500 → 3.731	3.607	3.569 → 3.643

We set $\tan\beta = 5$, $M_{\mathrm{SUSY}} = 1$ TeV and $\bar{\eta}_b = -0.24$

Input parameters

Parameter	Value
$a\,/10^{-5}$	$1.246\,e^{4.047i}$
$b\,/10^{-3}$	$3.438\,e^{2.080i}$
c	-0.545
$y_d^0\,/10^{-5}$	$3.053\,e^{4.816i}$
$y_s^0\,/10^{-4}$	$3.560\,e^{2.097i}$
$y_b^0\,/10^{-2}$	3.607
$\epsilon_{13}\,/10^{-3}$	$6.215\,e^{2.434i}$
$\epsilon_{23}\,/10^{-2}$	$2.888\,e^{3.867i}$
B	$10.20\,e^{2.777i}$
x	5.880

Parameter	Value
m_a /meV	3.646
m_b /meV	1.935
m_c /meV	1.151
η	2.592
ξ	2.039

Recall

$$v_{PQ}^2 = (N_a f_a)^2 = \sum_\phi x_\phi^2 v_\phi^2$$

This is dominated by $v_{\phi_2^u}$, giving

$$f_a \approx \frac{v_{\phi_2^u}}{N_a} = \frac{|b| \langle \Sigma_u \rangle}{\lambda_{2u} N_a}$$

- b is known from fit
- $\langle \Sigma \rangle \sim M_{\mathrm{GUT}}$ ($\simeq 2 \times 10^{16}$ GeV)
- $\lambda_{2u} = \mathcal{O}(1)$ coupling

Ex 1: $\langle \Sigma \rangle = M_{\mathrm{GUT}}$, $\lambda_{2u} = 1$

$$f_a \approx 1.1 \times 10^{13}\ \mathrm{GeV}$$

Ex 2: $\langle \Sigma \rangle = 0.1 M_{\mathrm{GUT}}$, $\lambda_{2u} = 2$

$$f_a \approx 5 \times 10^{11}\ \mathrm{GeV}$$

f_a is close to cosmological upper bound!

Axion couplings to matter

- In "SUSY" basis, $(Y^f_{a,0})_{ij} = x_j (Y^f)_{ij}$, e.g. $Y^d_{a,0} = \begin{pmatrix} 2y^0_d & 0 & 0 \\ 2By^0_d & y^0_s & 0 \\ 2By^0_d & 0 & 0 \end{pmatrix}$
- Rotate to mass basis. To leading order, we get

$$Y^u_a = \begin{pmatrix} -\frac{1}{2}a & -\frac{1}{4\sqrt{17}}a & 0 \\ \frac{\sqrt{17}}{4}a & 4b & 0 \\ -\frac{3}{2}a & -\frac{8}{\sqrt{17}}b & 0 \end{pmatrix} \qquad Y^d_a = \begin{pmatrix} 2y^0_d & 0 & 0 \\ -By^0_d & y^0_s & 0 \\ -2By^0_d & 0 & 0 \end{pmatrix}$$

$$Y^e_a = \begin{pmatrix} -2y^0_d/3 & 0 & 0 \\ -By^0_d & xy^0_s & 0 \\ -2By^0_d & 0 & 0 \end{pmatrix}$$

$\boxed{\text{Off-diagonal elements lead to flavour violation!}}$

Kaon decays: $K^+ \to \pi^+ a$

$$\mathcal{L}_{asd} = i\frac{a}{N_a f_a}\left[\mathrm{Re}(m_{21}^d)\bar{s}\gamma_5 d + \mathrm{Im}(m_{21}^d)\bar{s}d\right]$$

Decay rate [Ema, Hamaguchi, Moroi, Nakayama '16]

$$\Gamma(K^+ \to \pi^+ a) = \frac{m_K^3}{32\pi v_{PQ}^2}\left(1 - \frac{m_\pi^2}{m_K^2}\right)^3 \left|\frac{m_{21}^d}{m_s - m_d}\right|^2$$

Experimental limit on branching fraction [Adler et al '08]

$$\mathrm{Br}(K^+ \to \pi^+ a) \lesssim 7.3 \times 10^{-11} \Rightarrow N_a f_a \gtrsim 2.3 \times 10^{10}\ \mathrm{GeV}$$

Future sensitivity from NA62 experiment should improve limit by approx. an order of magnitude.

Other meson channels

- B decays to
  pions (allowed) and
  kaons (forbidden?)
- D decays

$$Y_a^d = \begin{pmatrix} 2y_d^0 & 0 & 0 \\ -By_d^0 & y_s^0 & 0 \\ -2By_d^0 & 0 & 0 \end{pmatrix}$$

Charged lepton flavour violation

- $\mu \to e\gamma$
- $\mu \to e$ conversion
- $\mu \to 3e$

Axion mass [di Cortona, Hardy, Pardo Vega, Villadoro '16]

$$m_a = 5.70(6)(4)\left(\frac{10^{12}\ \mathrm{GeV}}{f_a}\right)\mu\mathrm{eV}$$

Axionic dark matter density

$$\Omega_a \approx \left(\frac{6\ \mu\mathrm{eV}}{m_a}\right)^{7/6}$$

The A to Z model naturally predicts a very light axion ($f_a \sim 10^{12}$ GeV), and significant contribution to dark matter

May be in range of ADMX

Summary

- A to Z: a rather complete model of quarks and leptons
  Now with axions!
  - Renormalisable, breaking to MSSM
  - Sheds light on the flavour puzzle
  - All quark/lepton masses and mixings fitted

- Strong *CP* problem solved
- Axion and flavour scales are linked!
- Potentially rich axion phenomenology
  - Flavour violating decays
  - Dark matter

Backup slides

$$W_{\mathrm{driving}} = P^{u,d}_{1,2}\left(\bar{\phi}^{u,d}_{1,2}\phi^{u,d}_{1,2} - M^2\right) + P_\xi\left(\bar{\xi}\xi - M^2\right),$$

Field	G_{PS}	A_4	$\mathbb{Z}_5$	$\mathbb{Z}_3$	$\mathbb{Z}'_5$	R	$U(1)_{PQ}$
$\phi^u_{1,2}$	$(1,1,1)$	3	α^4,α^2	β^2,β	γ^2,γ	0	$2,1$
$\phi^d_{1,2}$	$(1,1,1)$	3	α^3,α	β^2,β	γ^2,γ	0	$2,1$
ξ	$(1,1,1)$	1	α^4	β^2	γ^2	0	2
$\bar{\phi}^u_{1,2}$	$(1,1,1)$	3	α,α^3	β,β^2	γ^3,γ^4	0	$-2,-1$
$\bar{\phi}^d_{1,2}$	$(1,1,1)$	3	α^2,α^4	β,β^2	γ^3,γ^4	0	$-2,-1$
$\bar{\xi}$	$(1,1,1)$	1	α	β	γ^3	0	-2

Yukawa matrices can be diagonalised by bi-unitary matrices $V_{L,R}^{u,d}$, $U_{L,R}^{e}$

$$Y^{u,\mathrm{diag}} = V_L^u Y^u (V_R^u)^\dagger,$$
$$Y^{d,\mathrm{diag}} = V_L^d Y^d (V_R^d)^\dagger,$$
$$Y^{e,\mathrm{diag}} = U_L^e Y^e (U_R^e)^\dagger.$$

We transform the fields by

$$Q \to (V_L^u)^\dagger Q,$$
$$d^c \to (V_R^d)^\dagger d^c,$$
$$u^c \to (V_R^u)^\dagger u^c.$$

Then $Y^u \to Y^{u,\mathrm{diag}}$, $Y^d \to V_{\mathrm{CKM}} Y^{d,\mathrm{diag}}$, where $V_{\mathrm{CKM}} = V_L^u (V_L^d)^\dagger$.