Conventional and exotic states in the DSE/BSE framework

Christian S. Fischer

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GGI 2018

Reviews: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]; Sanchis-Alepuz, Williams, [1710.04903]

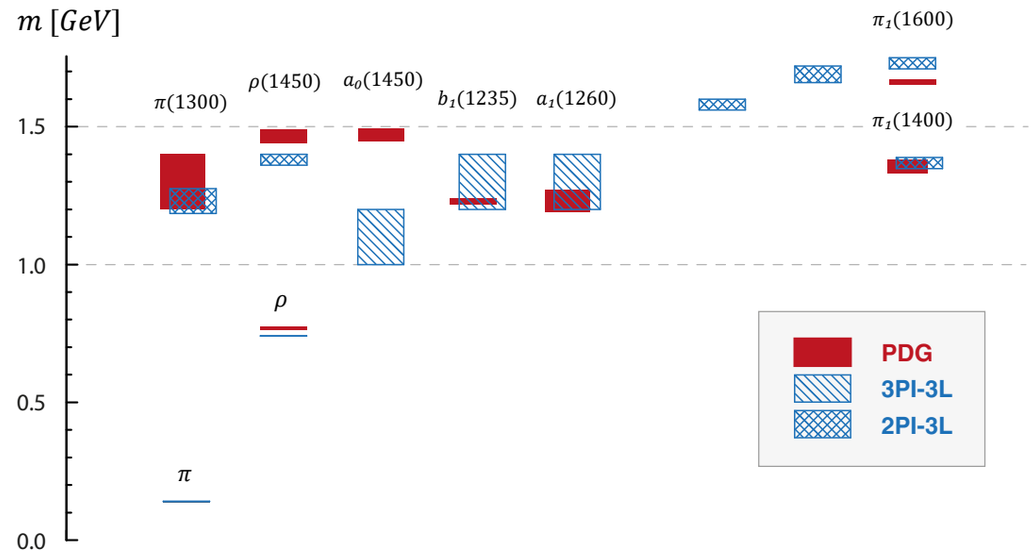
Bundesministerium für Bildung und Forschung

HIC for FAIR
Helmholtz International Center

Overview - Take home messages

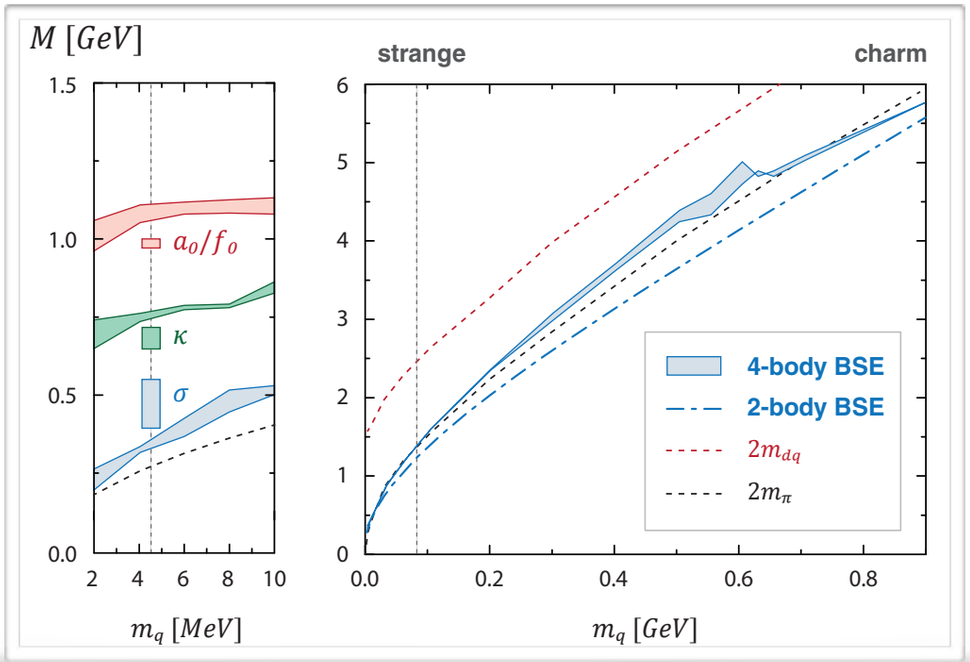
- Glueballs: $M(0^{++}) = 1.64\,\mathrm{GeV}$

- Light and heavy meson spectrum:



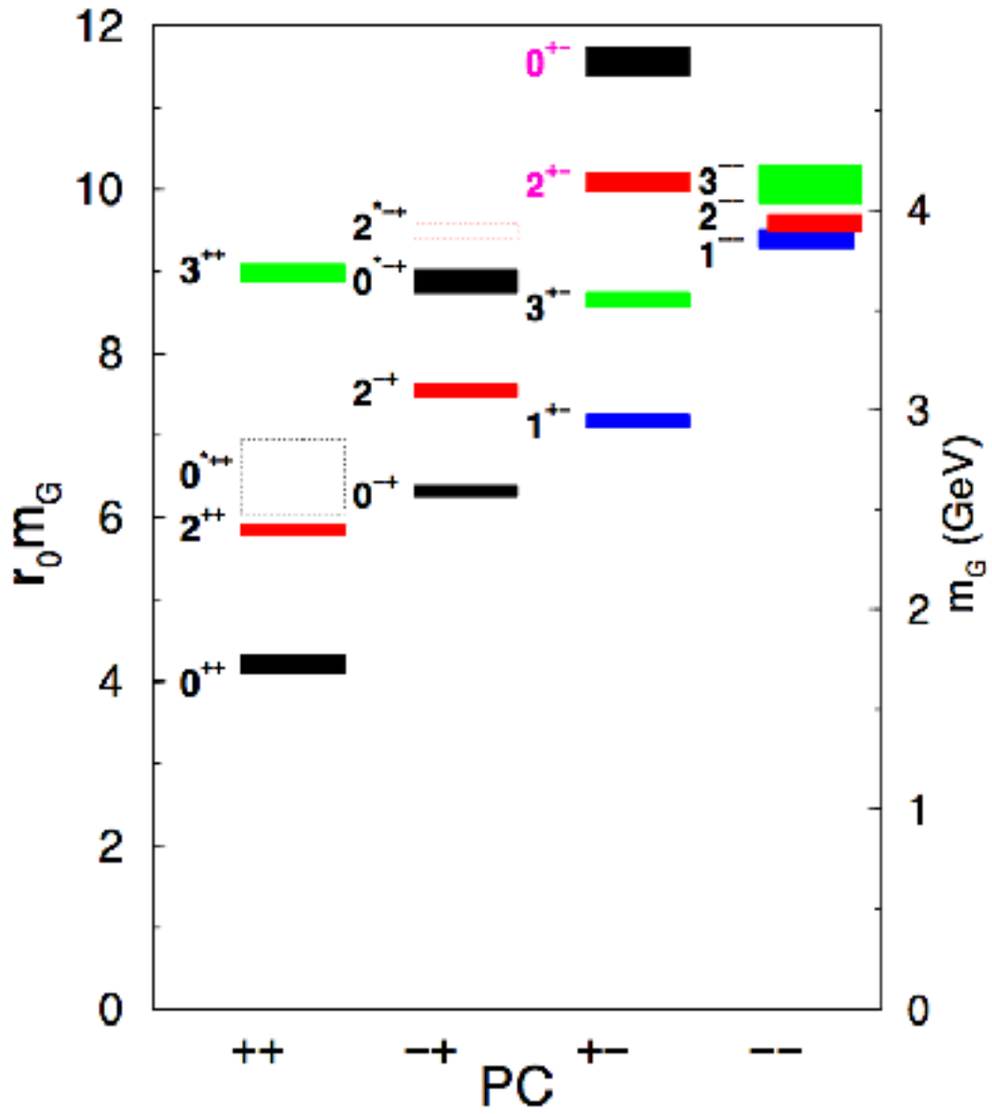
Williams, CF, Heupel, PRD93 (2016) 034026

- Light tetraquarks:



Eichman, CF, Heupel, PLB 753 (2016) 282-287

Glueballs



Morningstar and Peardon, PRD 60 (1999) 034509
Y.~Chen et al., PRD 73 (2006) 014516

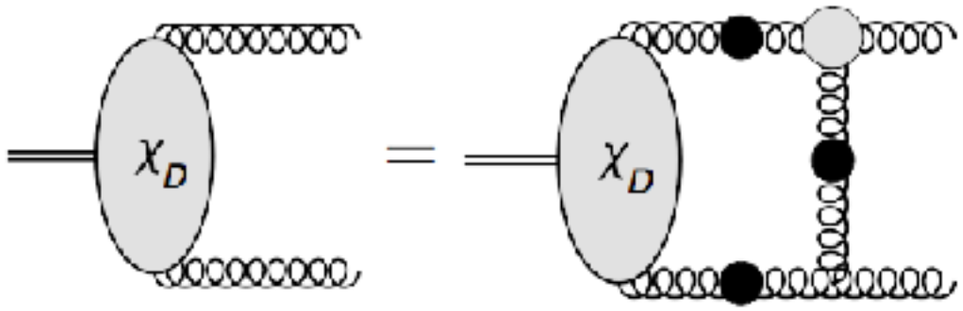
Lattice:

- States in the light and heavy quark energy regions
- Most calculations quenched
- Unquenched calculations very involved

Gregory et al., JHEP 1210 (2012) 170

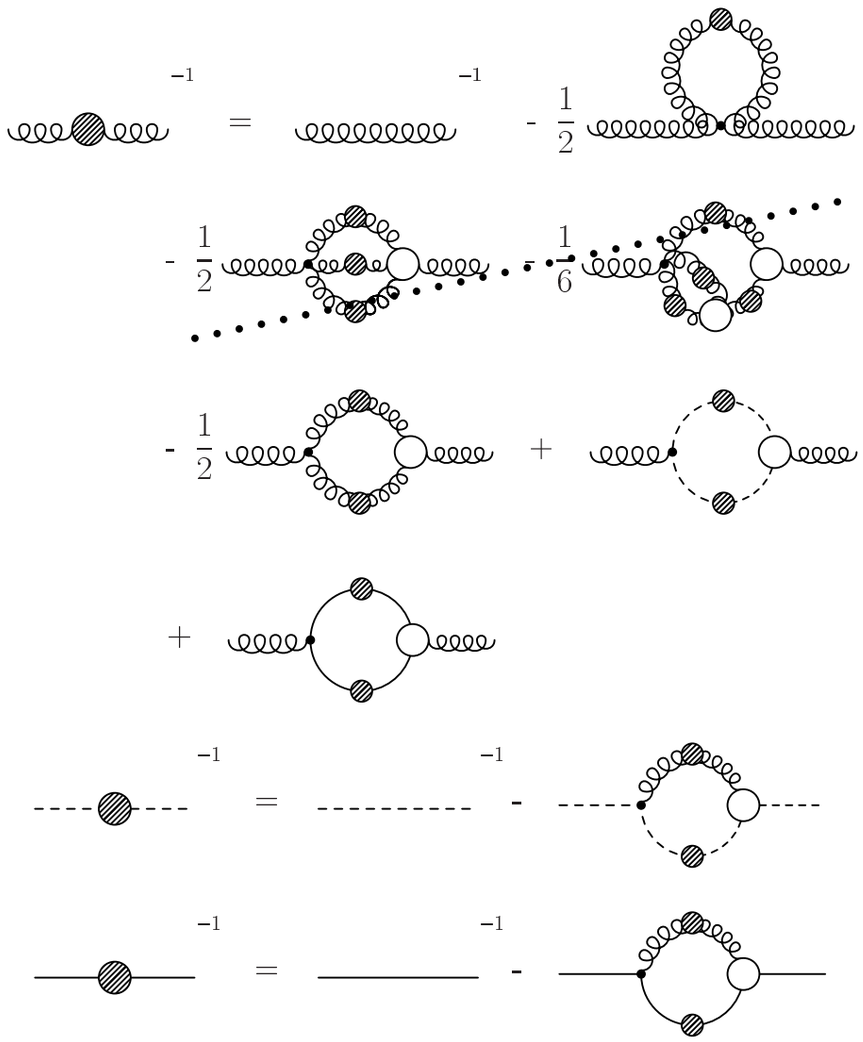
DSE:

- structural information

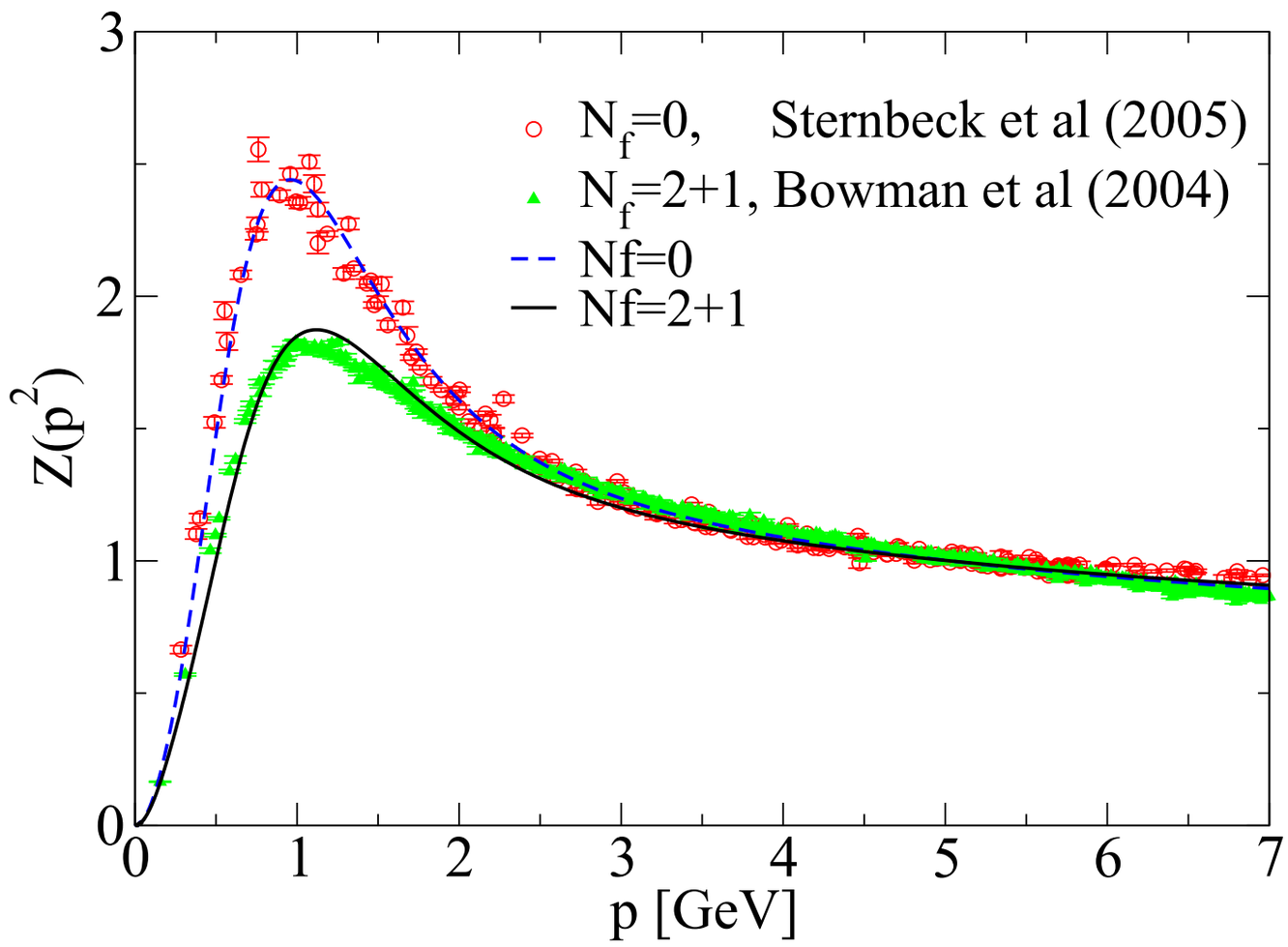


Meyers, Swanson, PRD 87 (2013) 3, 036009
Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

Landau gauge gluon propagator



$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p^2)}{p^2}$$

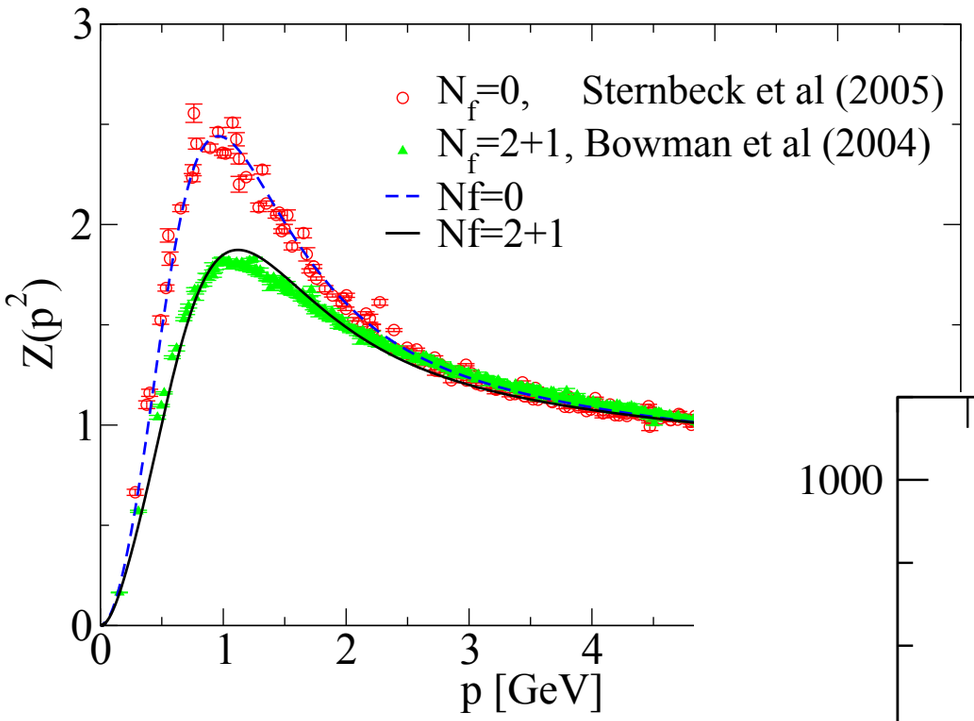


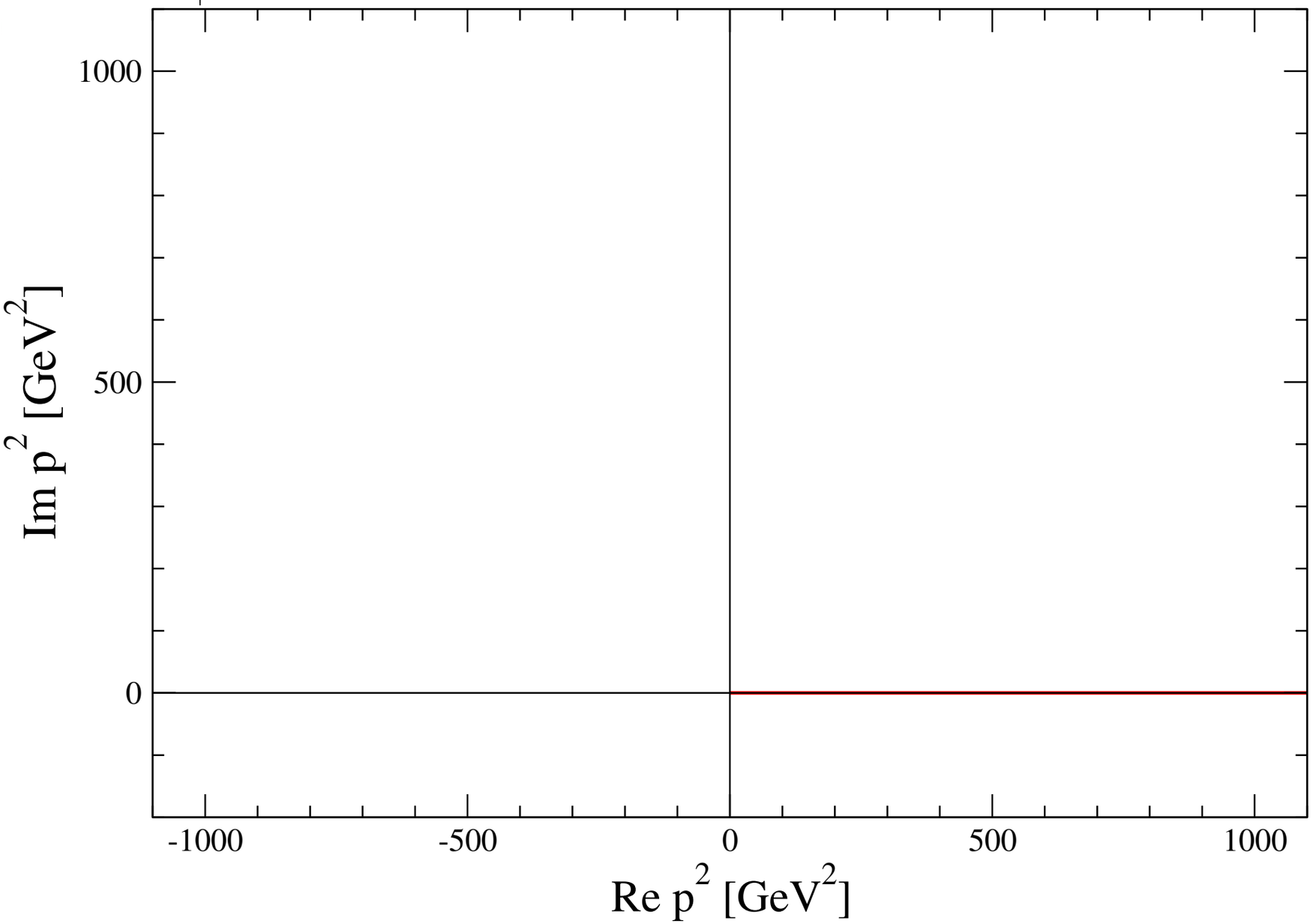
- spacelike momenta:
  good agreement with lattice
- fully dressed gluon appears massive

Cornwall PRD 26 (1982);
Cucchieri, Mendes PoS Lat2007 297
Aguilar, Binosi, Papavassiliou, PRD 78, 025010 (2008);
Boucaud et al. JHEP 0806 (2008) 099;
CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408

Huber and von Smekal, JHEP 1304 (2013)149
Hopfer, CF and Alkofer, JHEP 1411 (2014) 035
Huber, EPJC 77 (2017)

Analytic structure I





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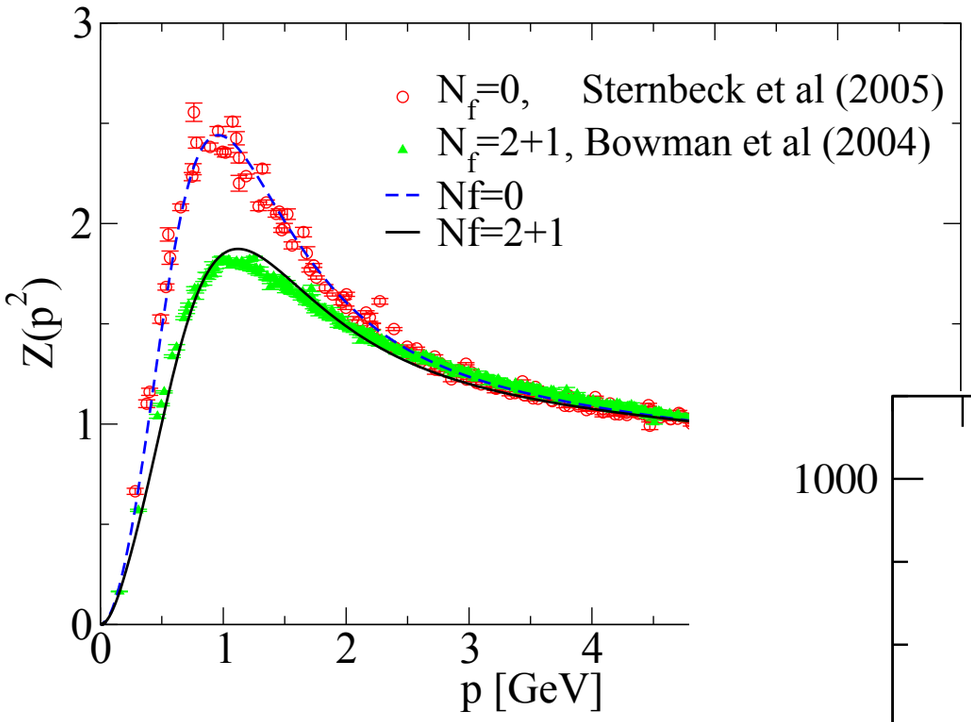

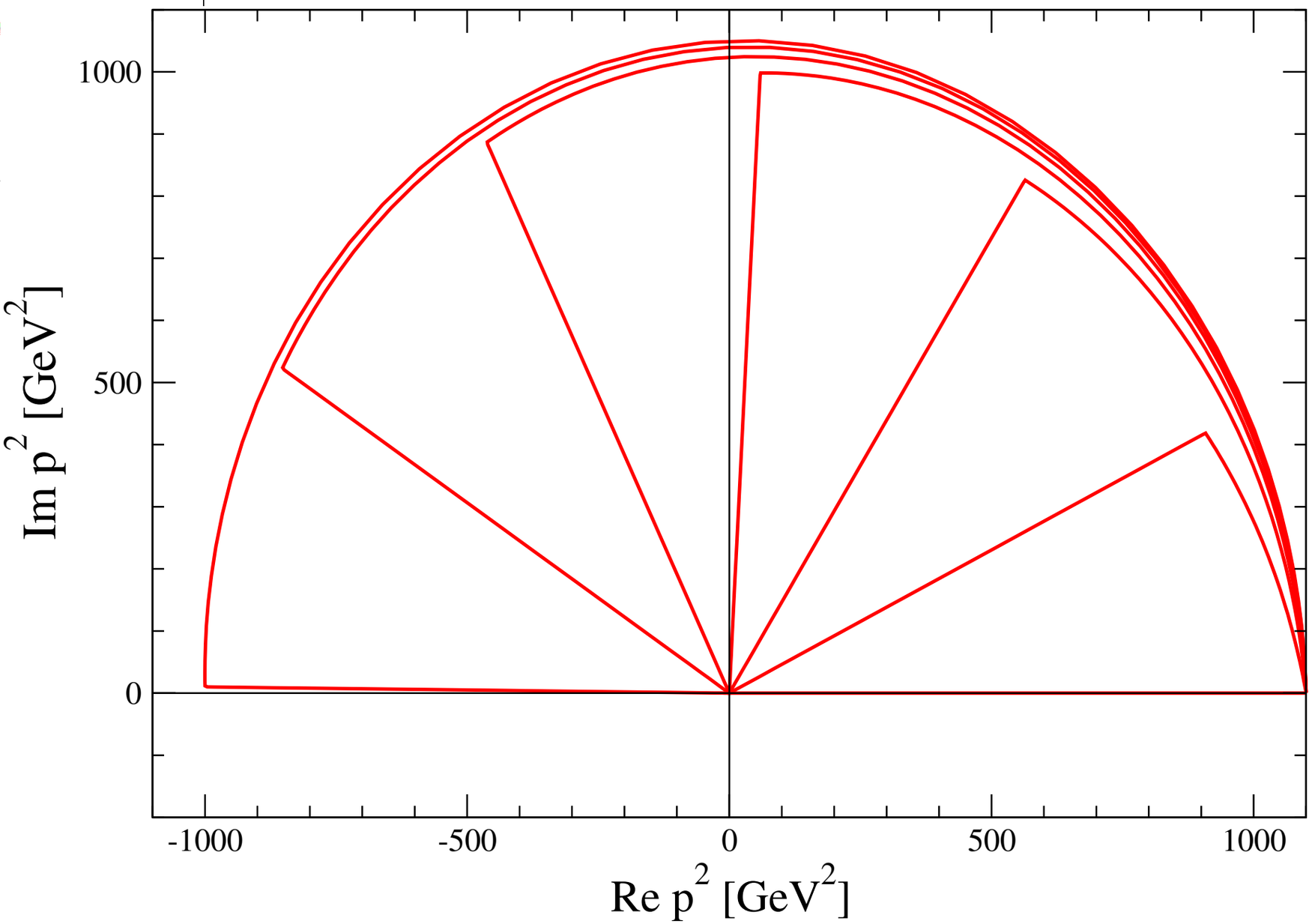

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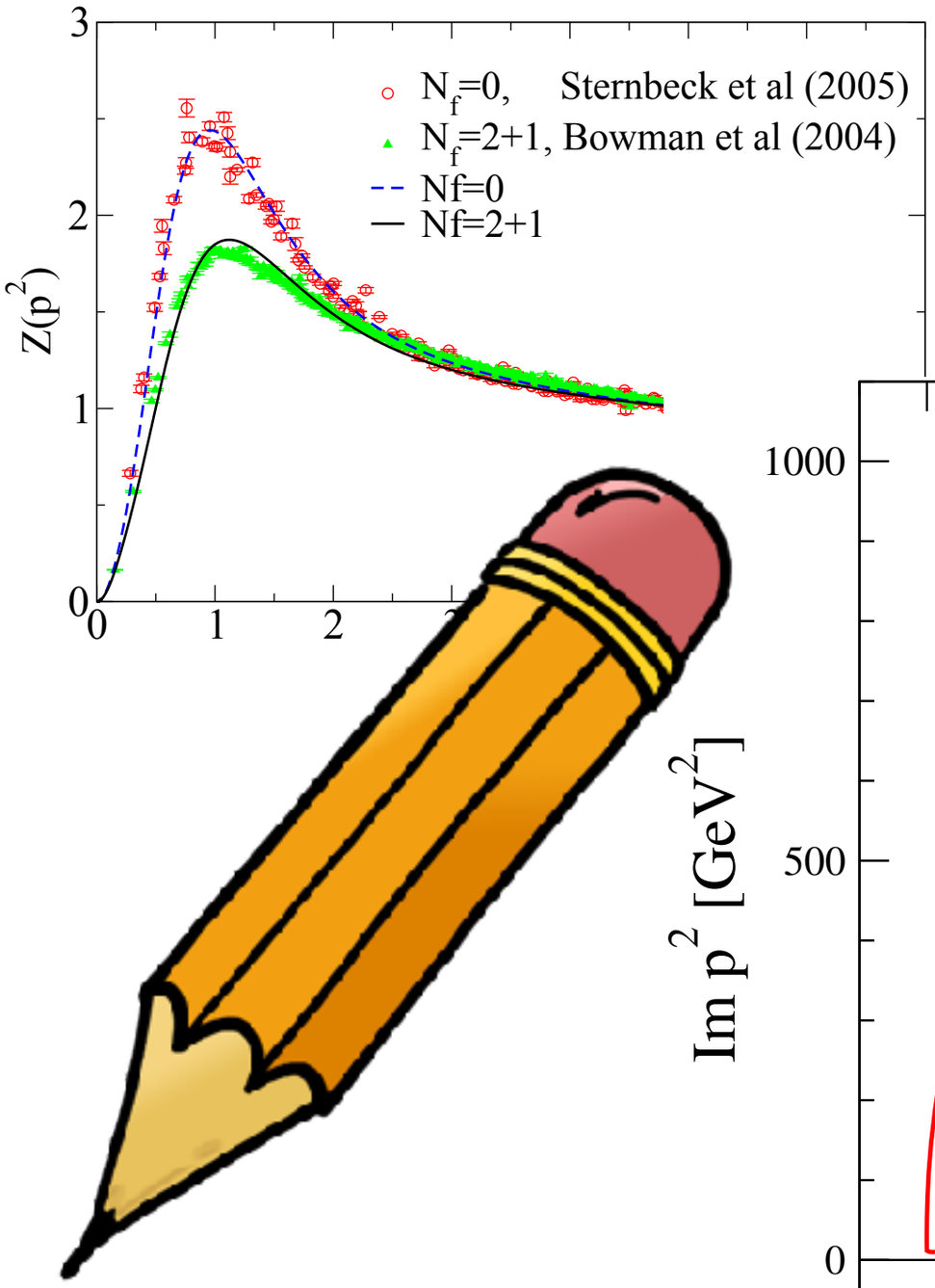

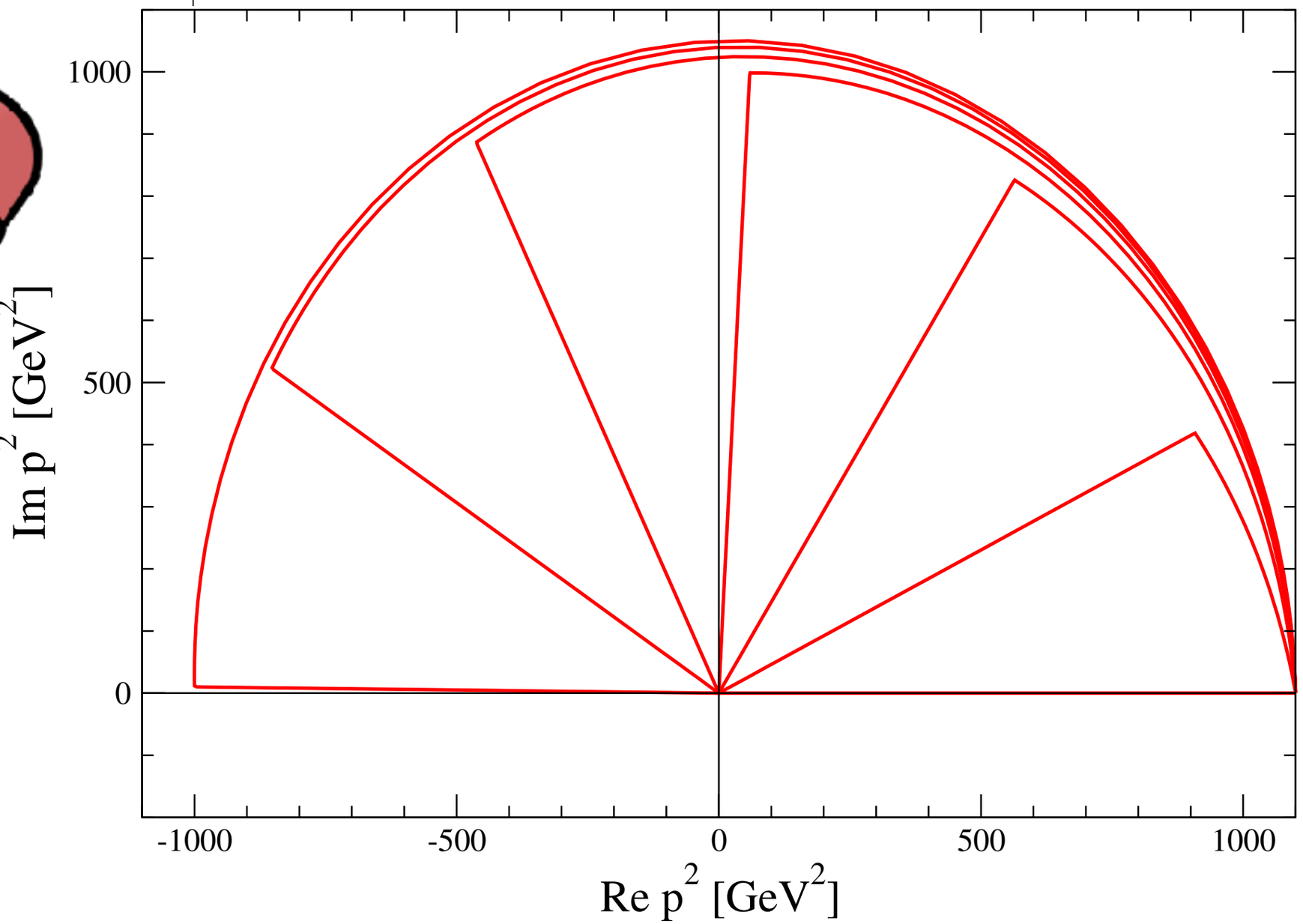

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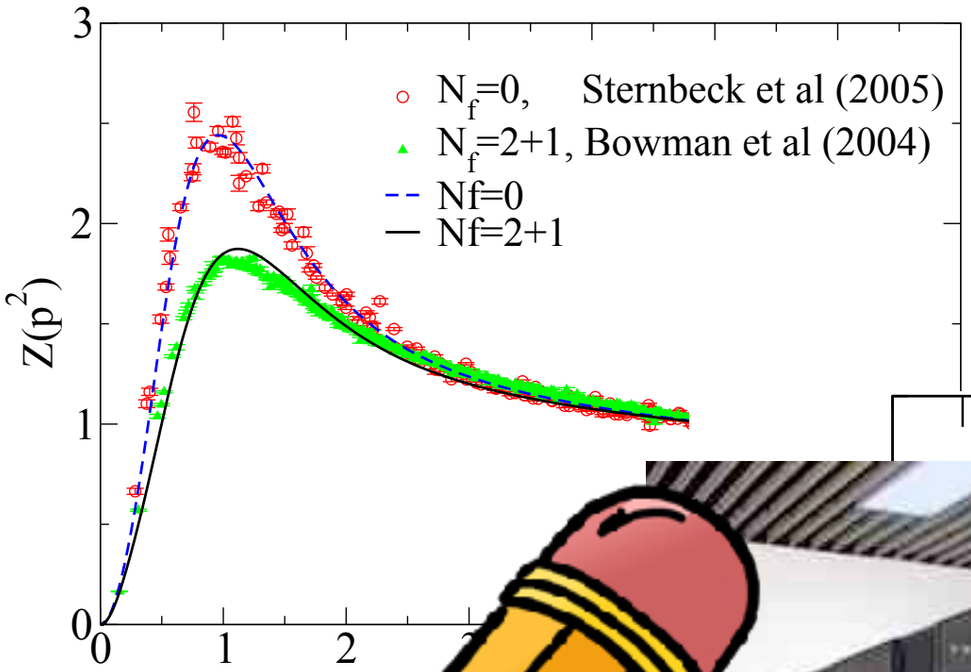
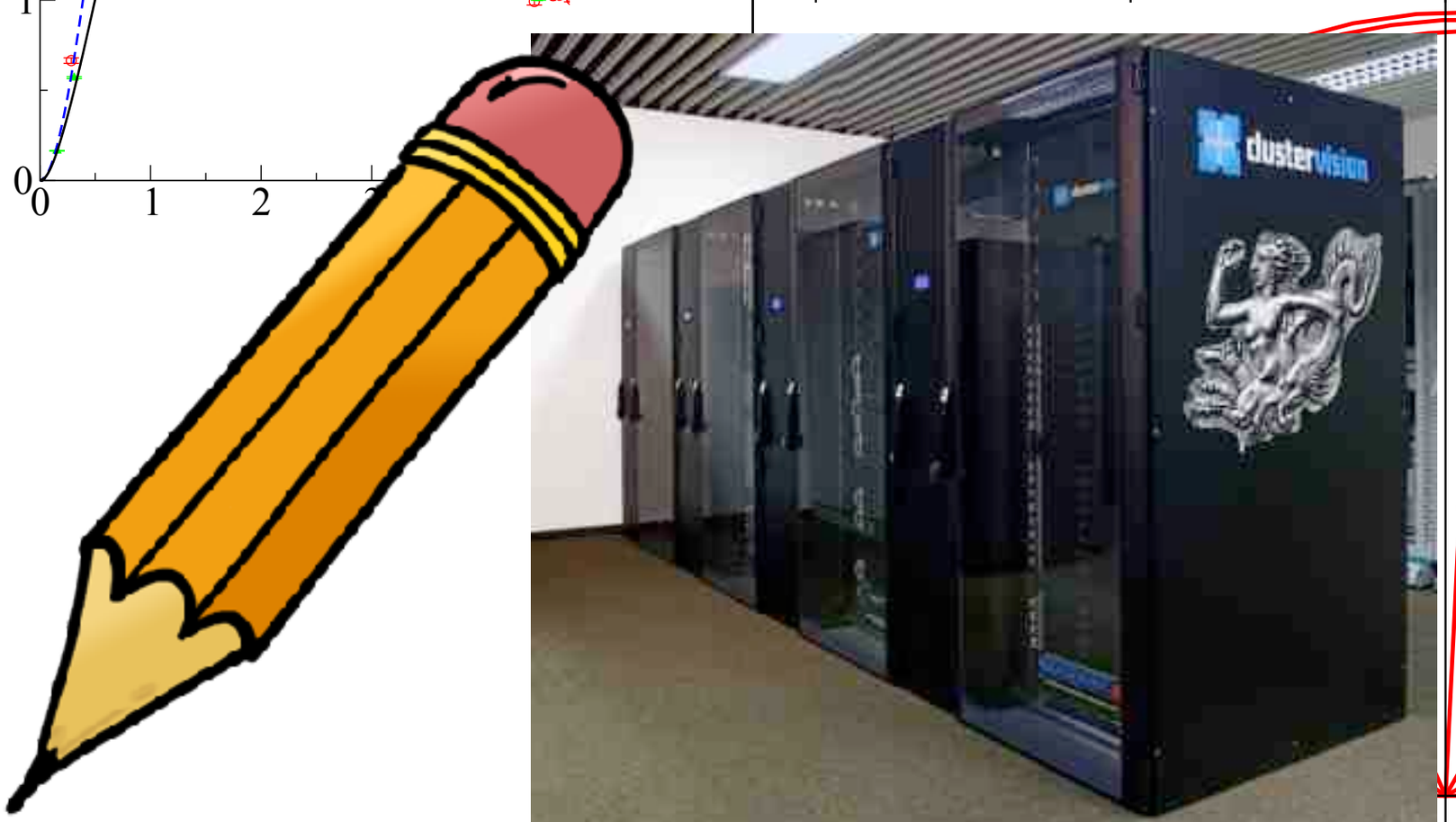
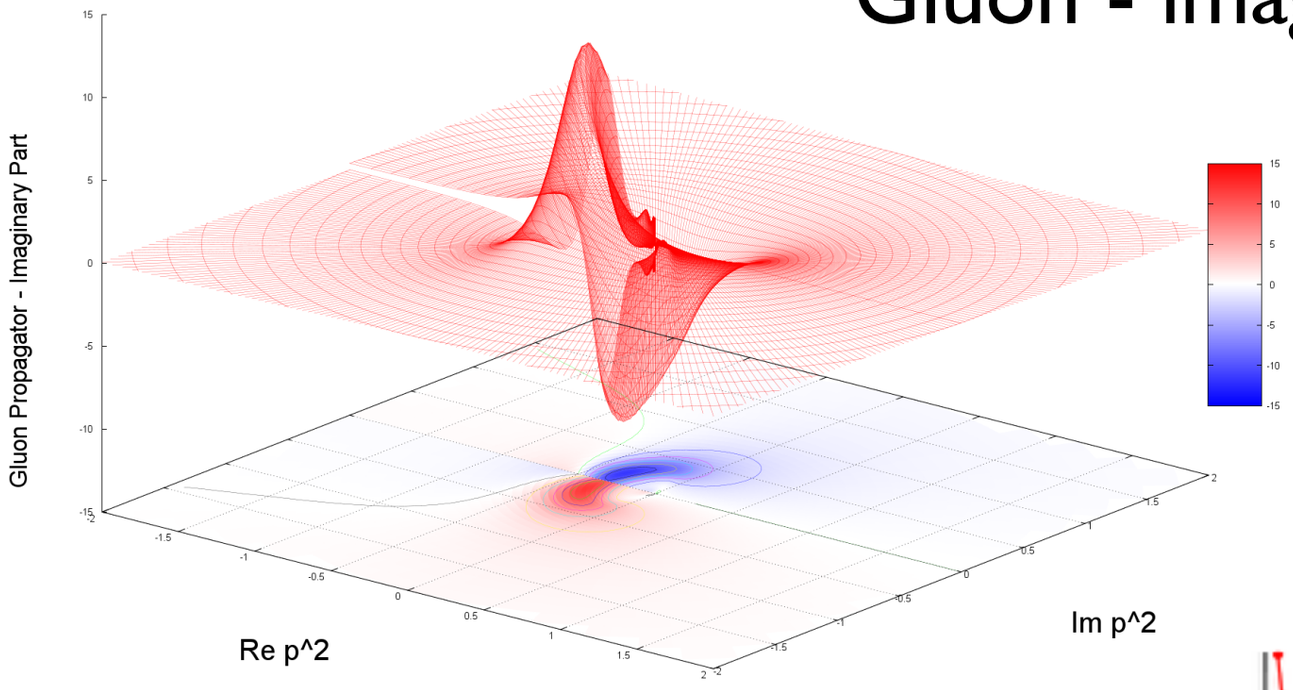
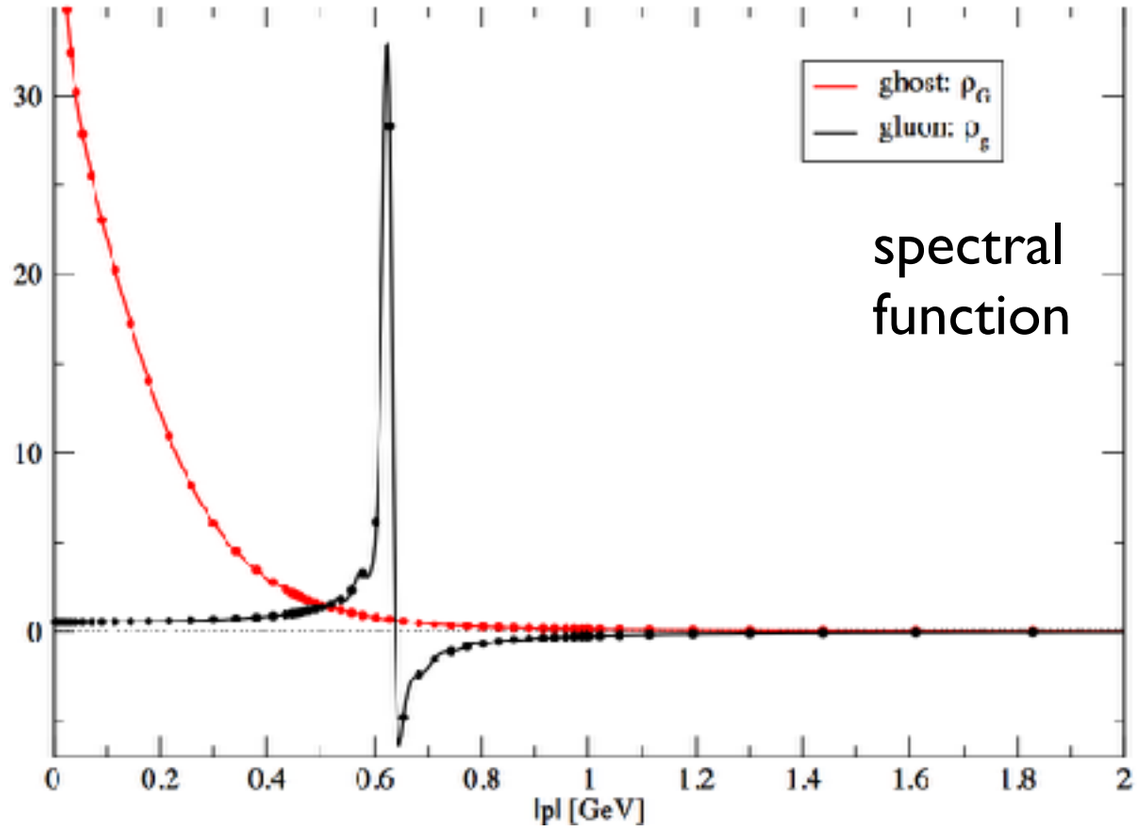




Analytic structure II

Gluon - imaginary part





spectral function

- spectral function: positivity violations

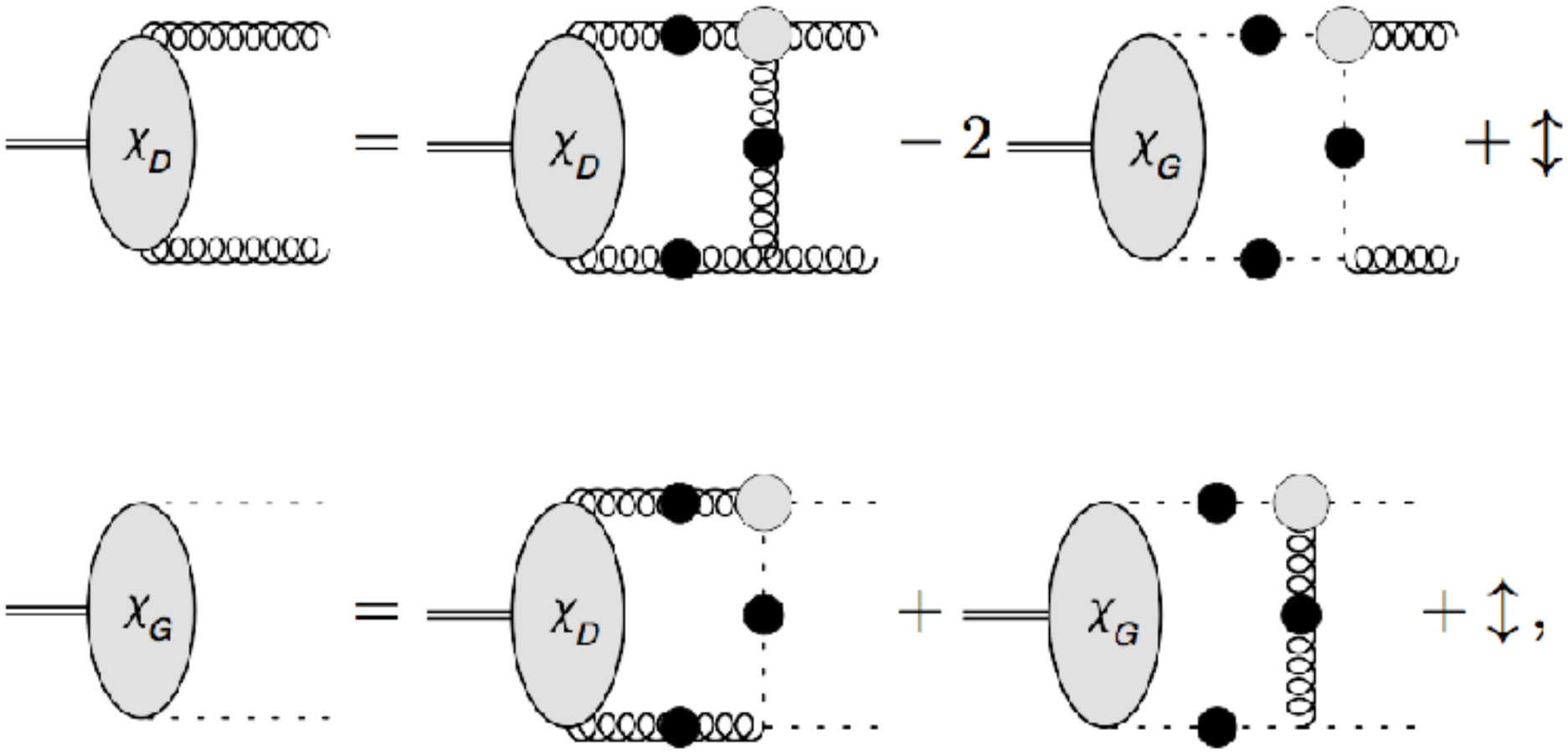
$$600\,\mathrm{MeV} < m_g < 700\,\mathrm{MeV}$$

Cornwall PRD 26 (1982); Cucchieri, Mendes PoS Lat2007 297
Aguilar, Binosi, Papavassiliou, PRD 78, 025010 (2008);
Boucaud et al. JHEP 0806 (2008) 099

Gluon cannot appear in detector!

Strauss, CF, Kellermann, Phys. Rev. Lett. 109, (2012) 252001

Glueballs from DSE/BSEs



- Mixing of two-gluon amplitudes with ghost-antighost
- Probes analytical structure of gluons and ghosts

Results: $M(0^{++}) = 1.64\,\mathrm{GeV}$

$M(0^{-+}) = 4.53\,\mathrm{GeV}$ ← ghost do not contribute !

Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

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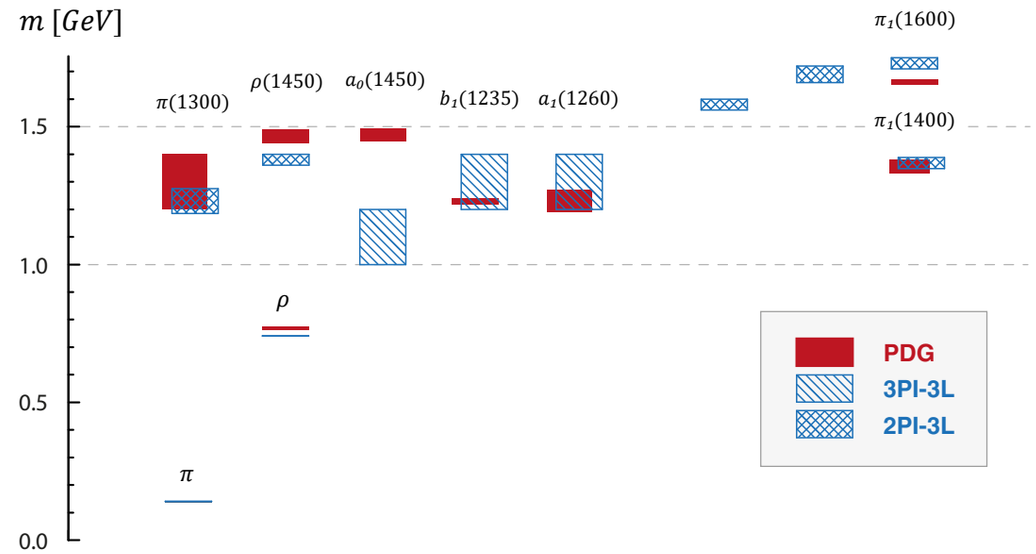
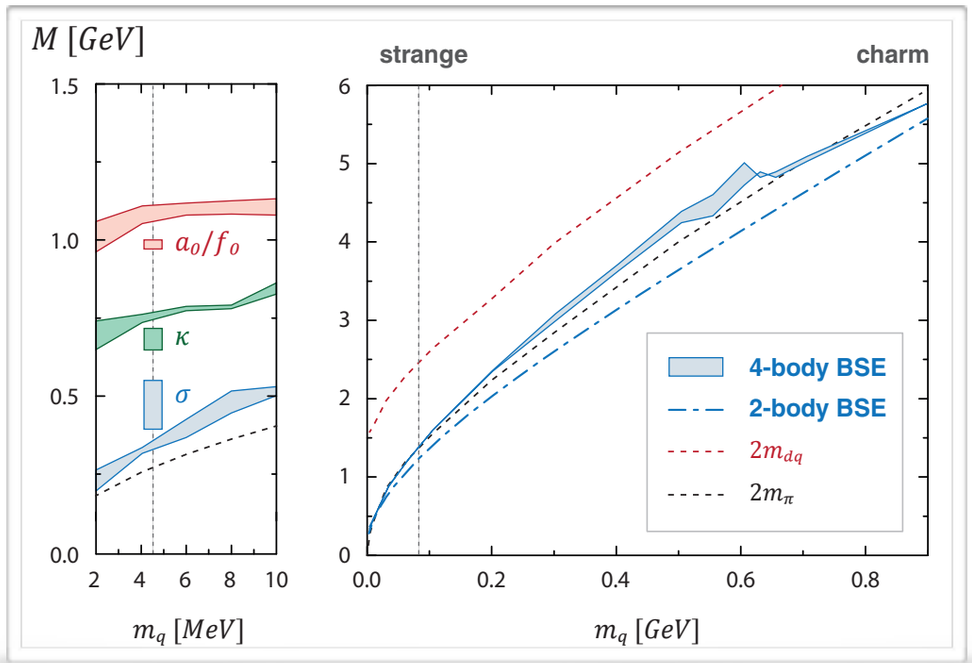
- **Light and heavy meson spectrum:**



Williams, CF, Heupel, PRD93 (2016) 034026

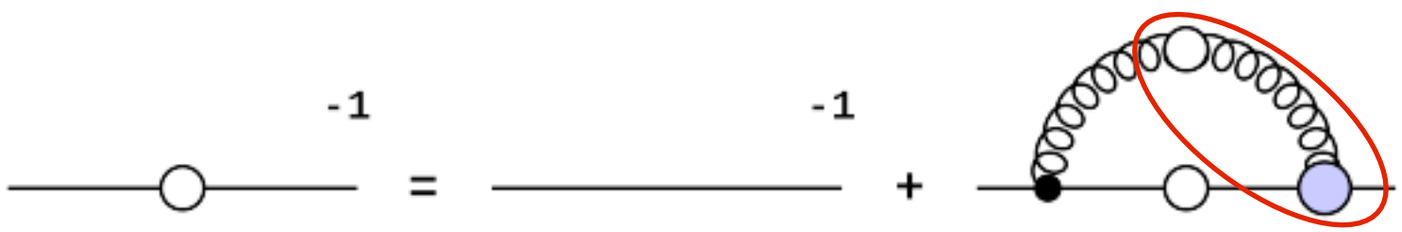
- **Light tetraquarks:**

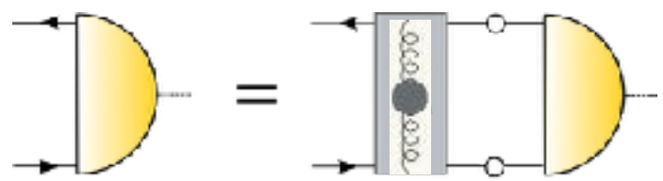


Eichman, CF, Heupel, PLB 753 (2016) 282-287

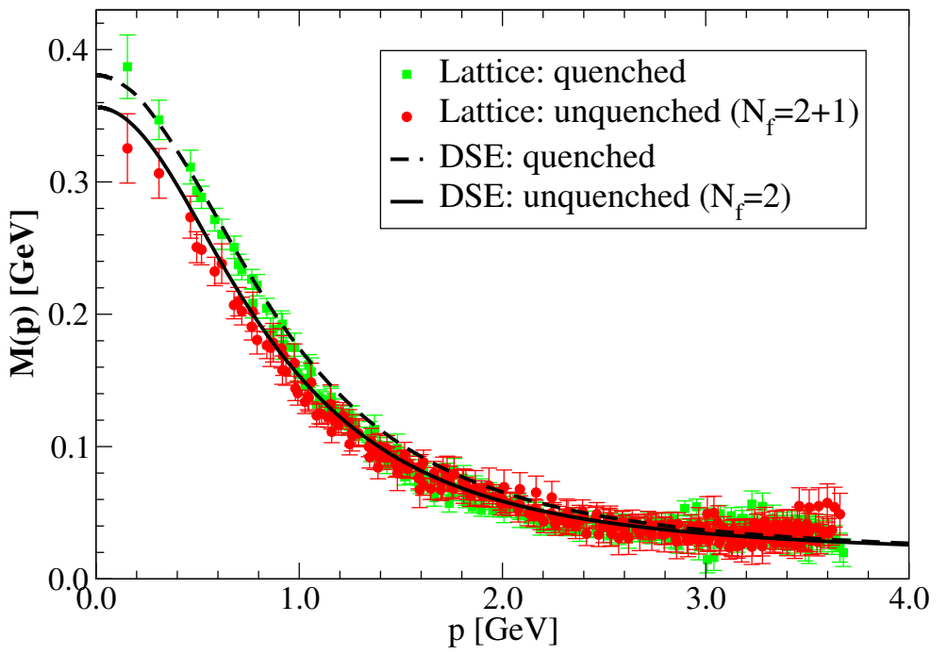
Quark mass

Rainbow-Ladder (RL) vs beyond the rainbow (BRL)





$$[S(p)]^{-1} = [-i\not{p} + M(p^2)]/Z_f(p^2)$$



CF, Nickel, Williams, EPJ C 60 (2009) 47

- $M(p^2)$: momentum dependent!
- Dynamical mass: $M_{strong} \approx 350$ MeV
- Flavour dependence because of m_{weak}
- Chiral condensate: $\langle \bar{\Psi}\Psi \rangle \approx (250\,\mathrm{MeV})^3$

Quark mass

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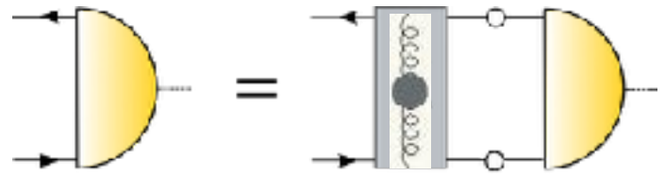
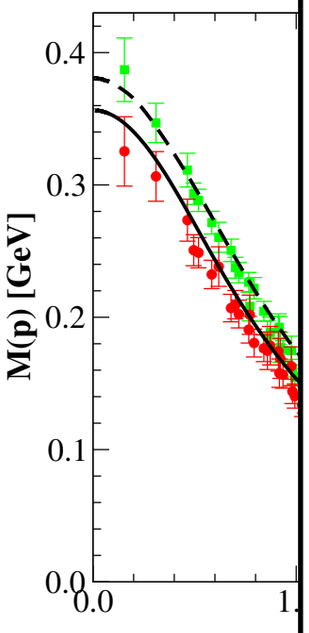


$$[S(p)]^{-1} = \Gamma^\mu(p,k) = \sum_{i=1,12} \tau_i(p,k) T_i^\mu$$

$$\sim \gamma^\mu \tau(k^2)$$ "approximation" !

$$D^{\mu\nu}(k) = \left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) \frac{Z(k^2)}{k^2}$$

$$\frac{g^2}{4\pi}\tau(k^2)Z(k^2) \sim \alpha(k^2)$$



veak

$V)^3$

CF, Nickel, Williams, EPJ C 60 (2009) 47

Quark mass

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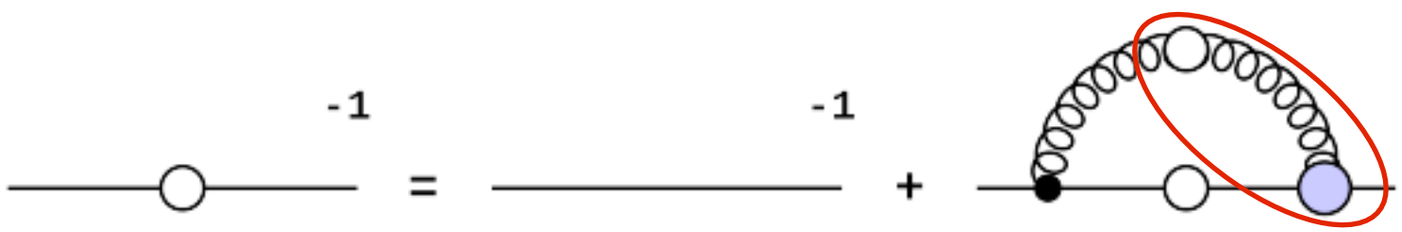

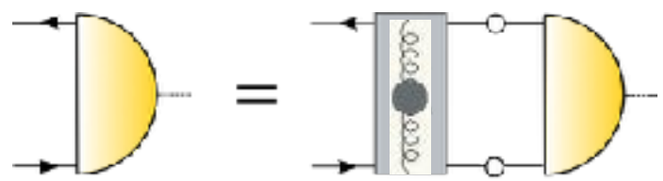

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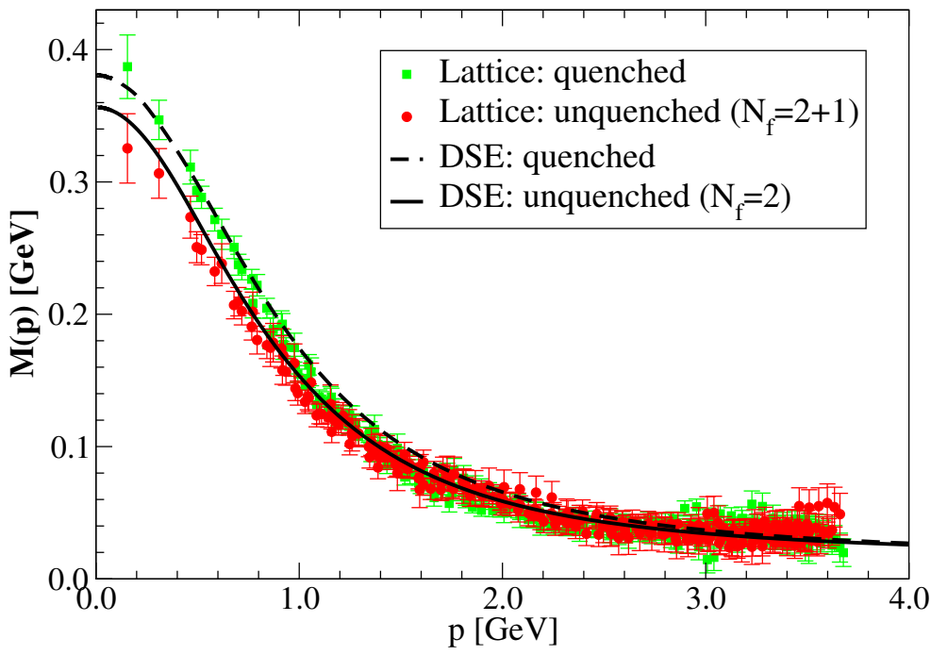

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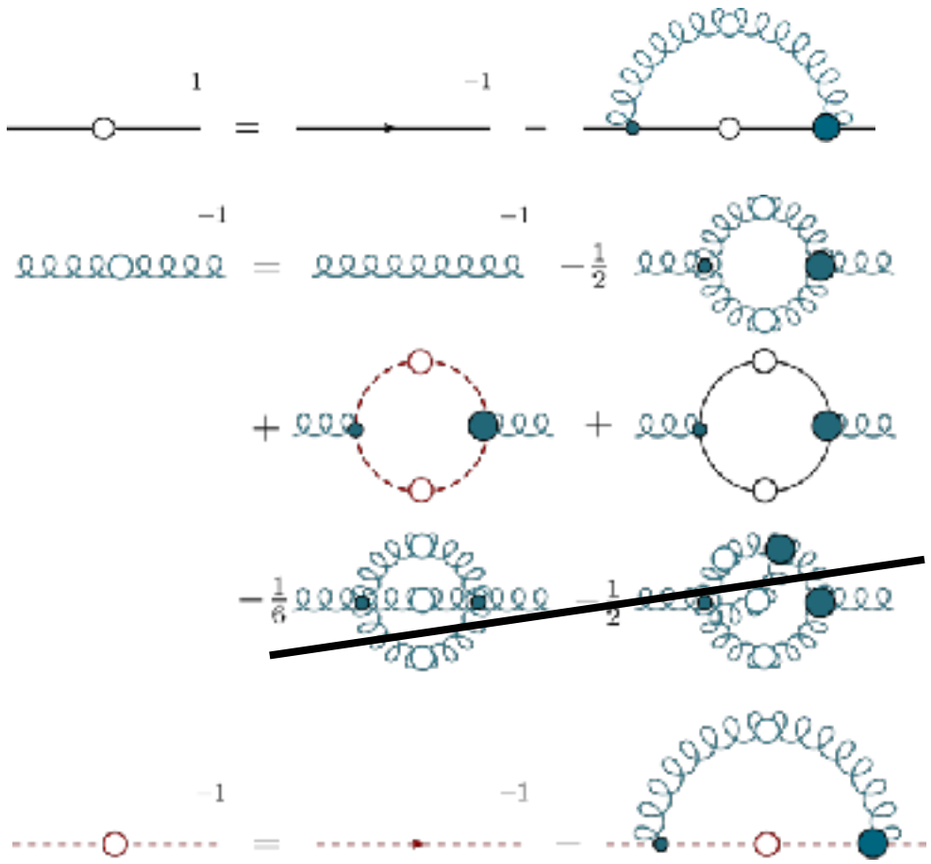
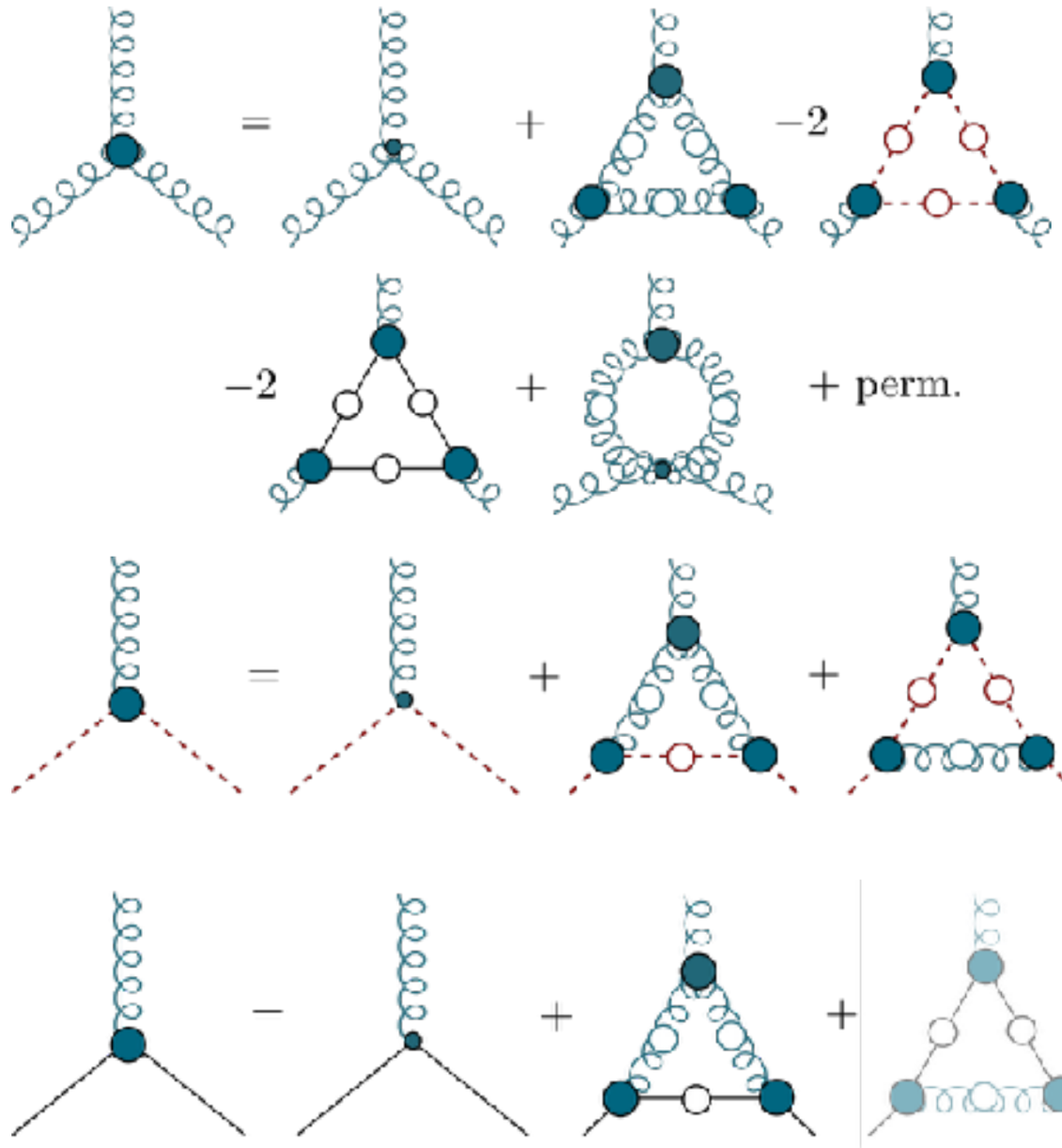
3PI-truncation

propagators



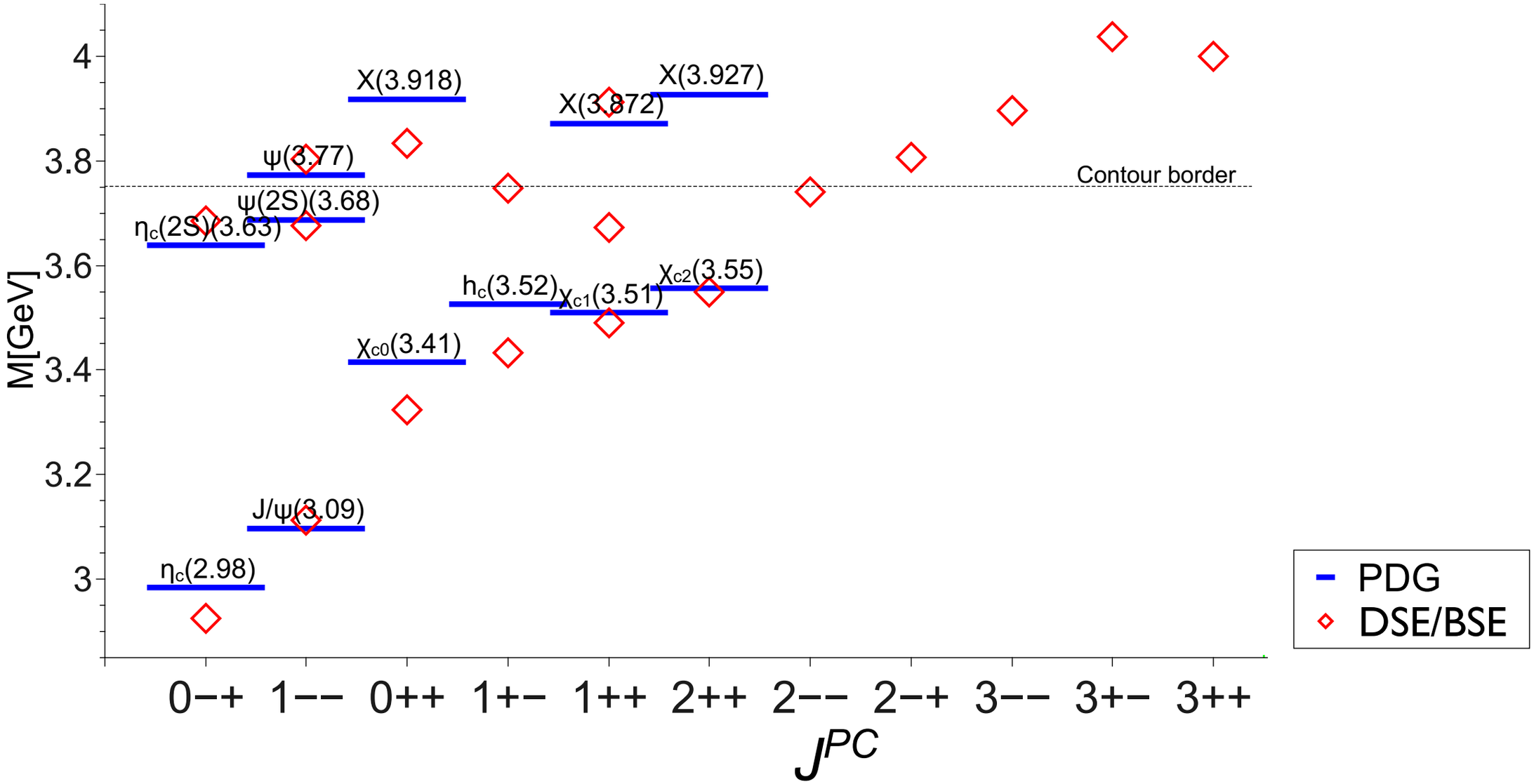
for different BRL approaches see work of
Aguilar, Alkofer, Binosi, Blum, Chang, Cyrol, Eichmann, Fister, Huber, Maas, Mitter, Papavassiliou, Pawlowski, Roberts, Smekal, Strodthoff, Vujinovic, Watson, Williams…

vertices



Williams, CF, Heupel, PRD 93 (2016) 034026
CF, Williams, PRL 103 (2009) 122001

Charmonium spectrum



- good channels: $1^{--}, 2^{++}, 3^{--}, \ldots$
- acceptable channels: 0^{-+}
- clear deficiencies in other channels: missing spin-structure
- excited states fine ! (in good channels)

CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

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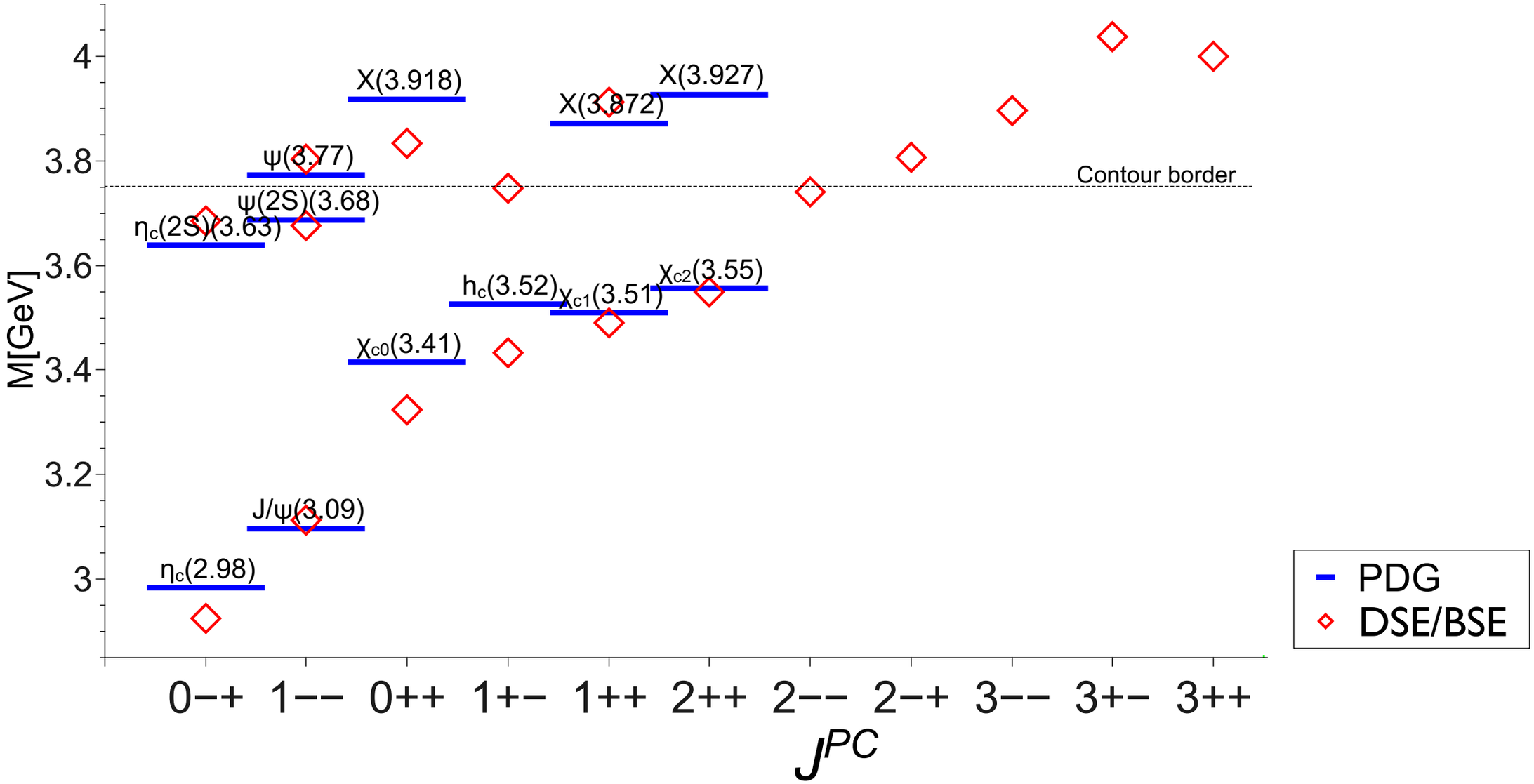

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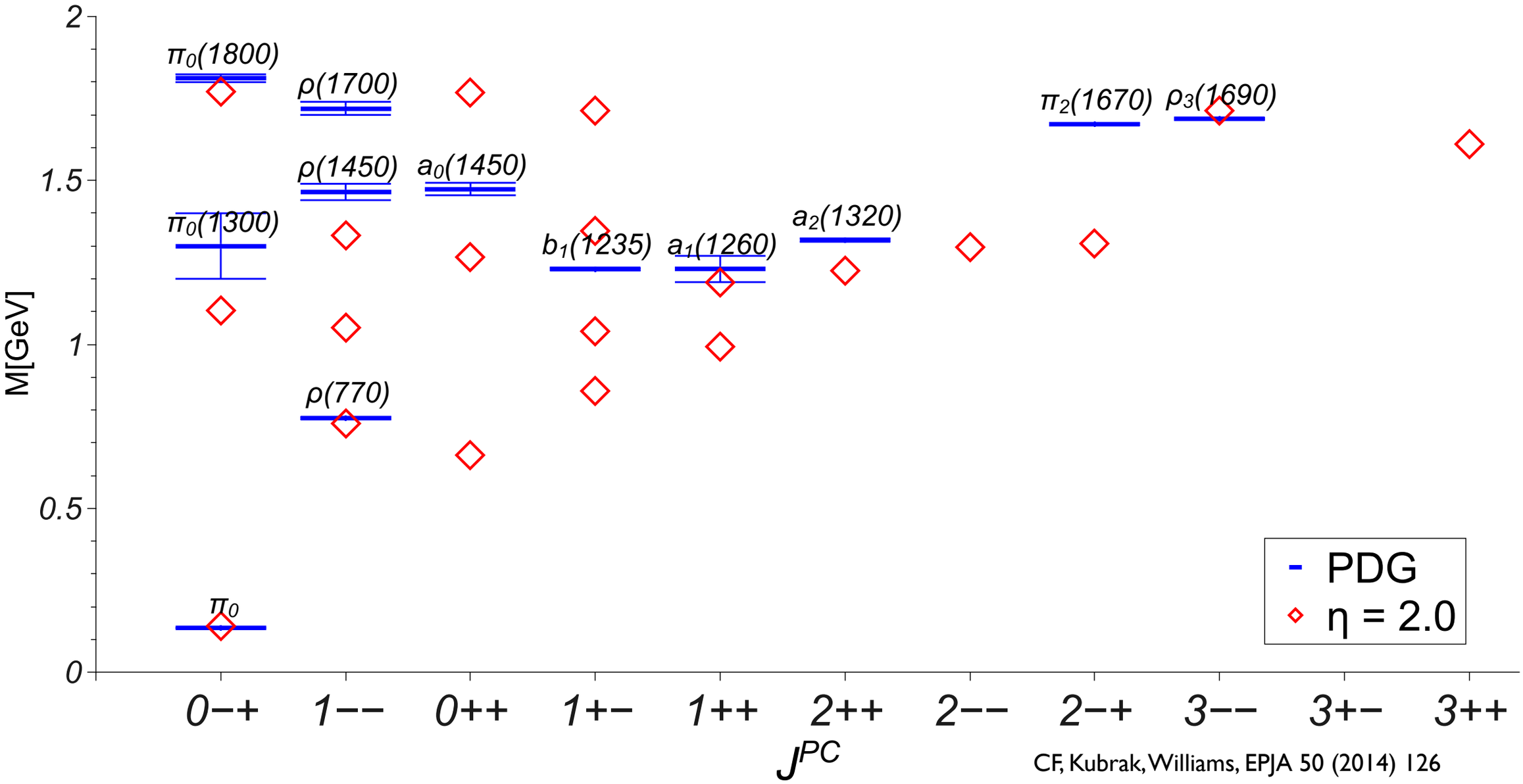

CF, Kubrak, Williams, EPJA 50 (2014) 126

Williams, CF, Heupel, PRD93 (2016) 034026

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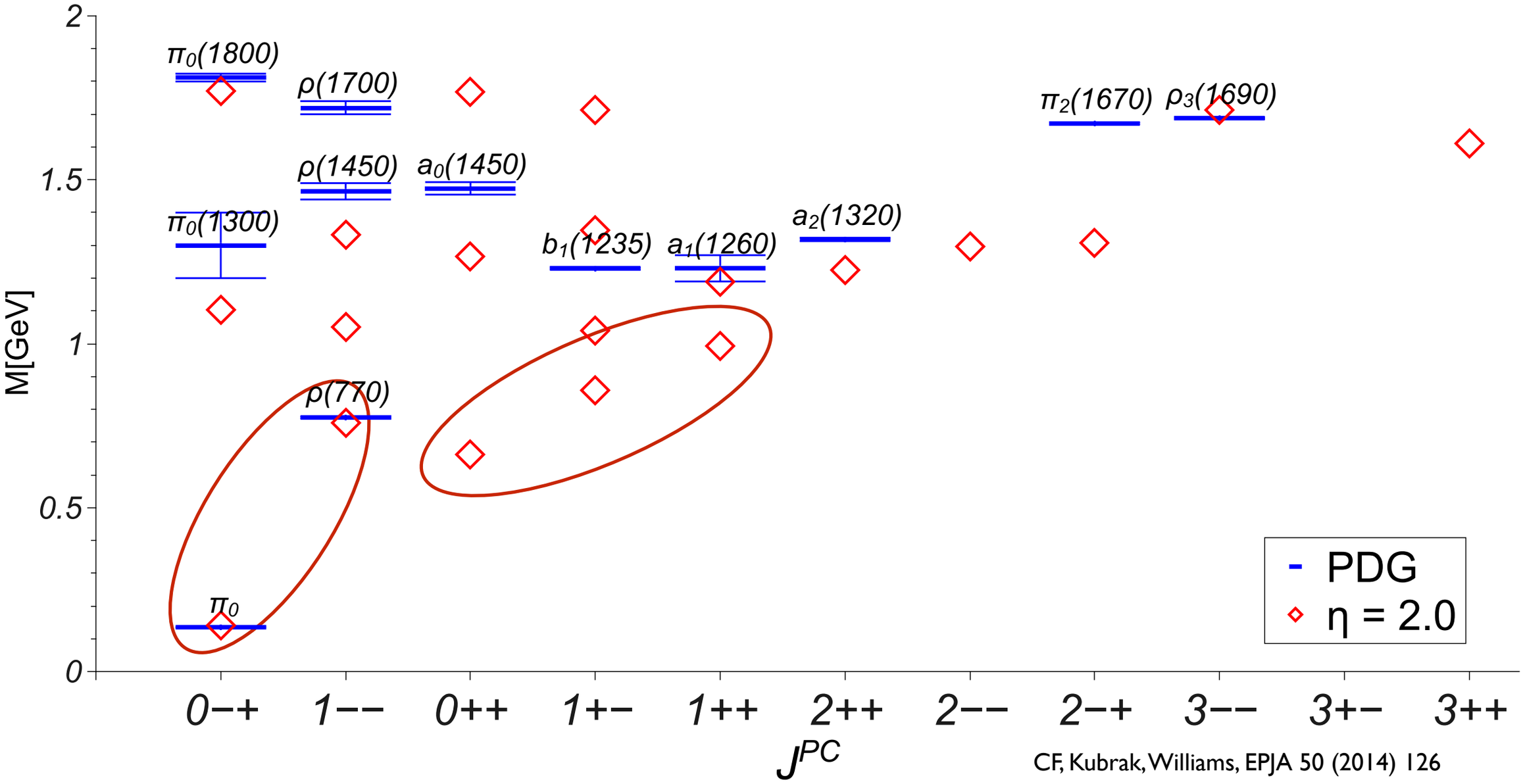

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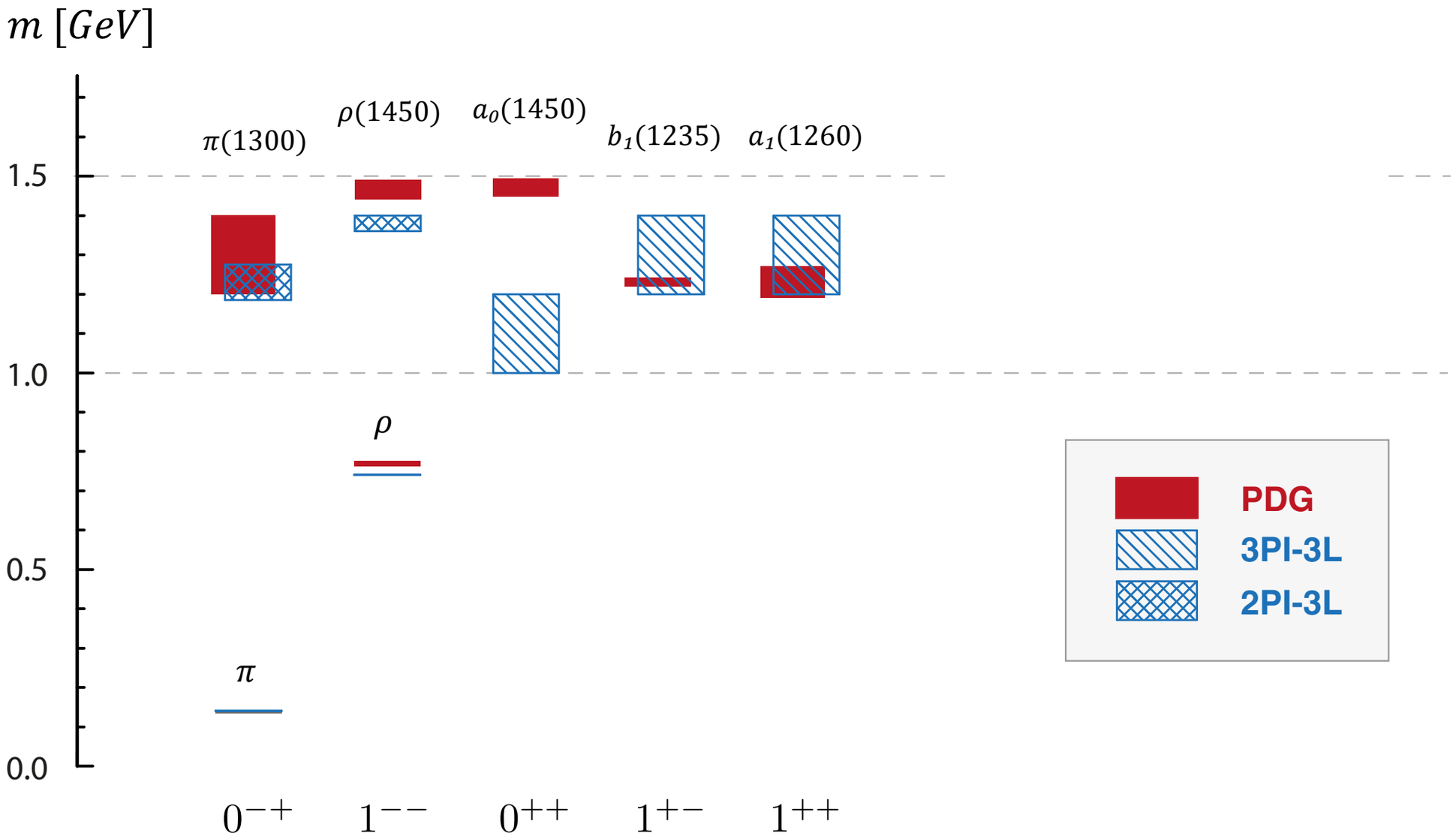

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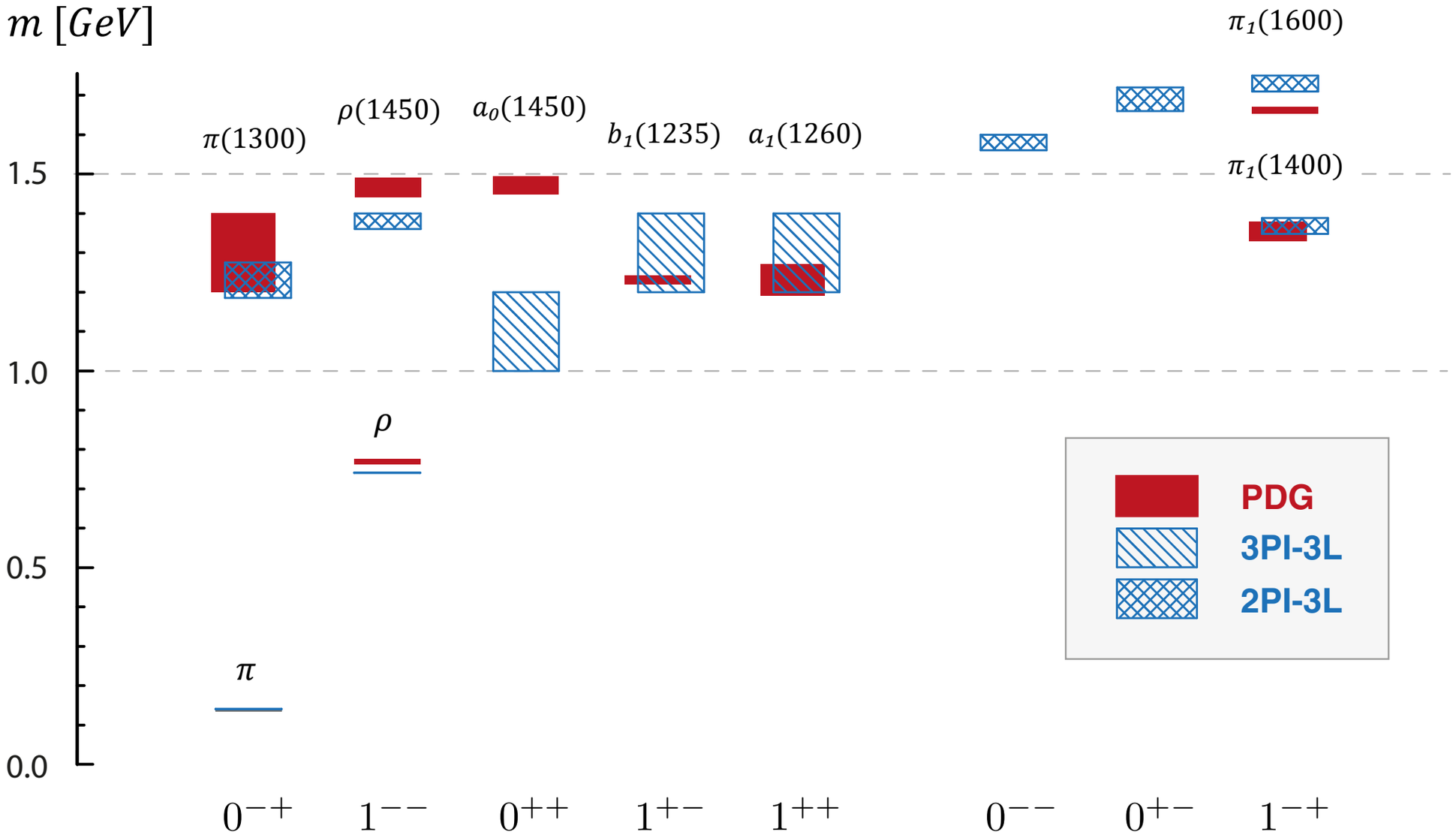

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Quantum numbers: non-relativistic vs relativistic

non-relativistic $q\bar{q}$

$$S : 1/2 \otimes 1/2 \to 0 \oplus 1$$

$$P : (-1)^{L+1}$$

S	L	J^{PC}	
0	0	0^{-+}	
1	0	1^{--}	
0	1	1^{+-}	$^1P_1$
1	1	0^{++}	$^3P_0$
		1^{++}	$^3P_1$
		2^{++}	$^3P_2$

J^{PC} or $^{2S+1}L_J$

relativistic $q\bar{q}$

$$\Gamma_\pi(P,p) = \gamma_5 \left[F_1(P,p) \right. \qquad \text{s-wave}$$
$$+ F_2(P,p) i\not{P}$$
$$+ F_3(P,p) pP i\not{p} \qquad \text{p-wave}$$
$$\left. + F_4(P,p) [\not{p}, \not{P}] \right]$$

(rest frame of π)

~~$P : (-1)^{L+1}$~~

Bethe,Salpeter, Llewelyn-Smith 1950ies

- conventional states more complicated
- 'exotic' quantum numbers possible !

$$0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \ldots$$

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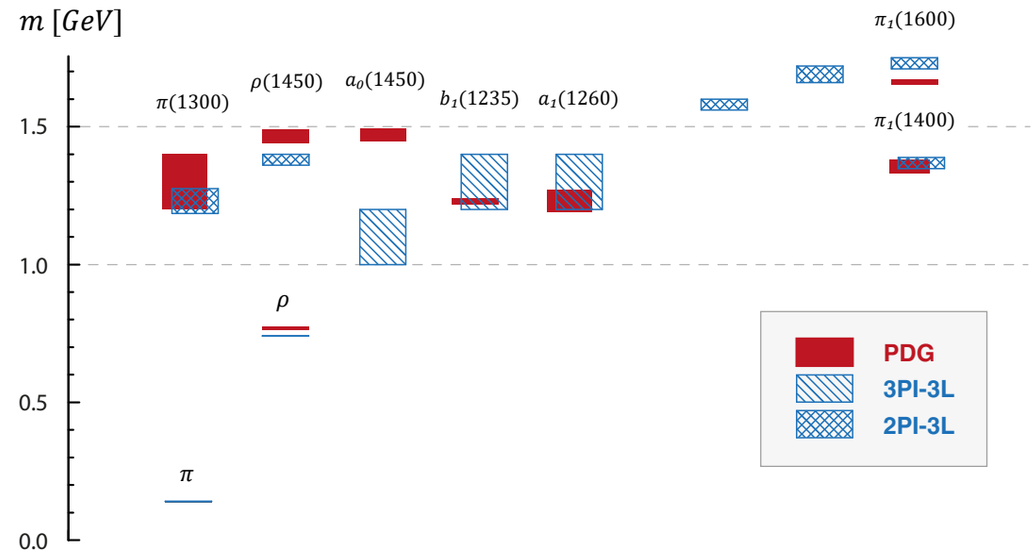

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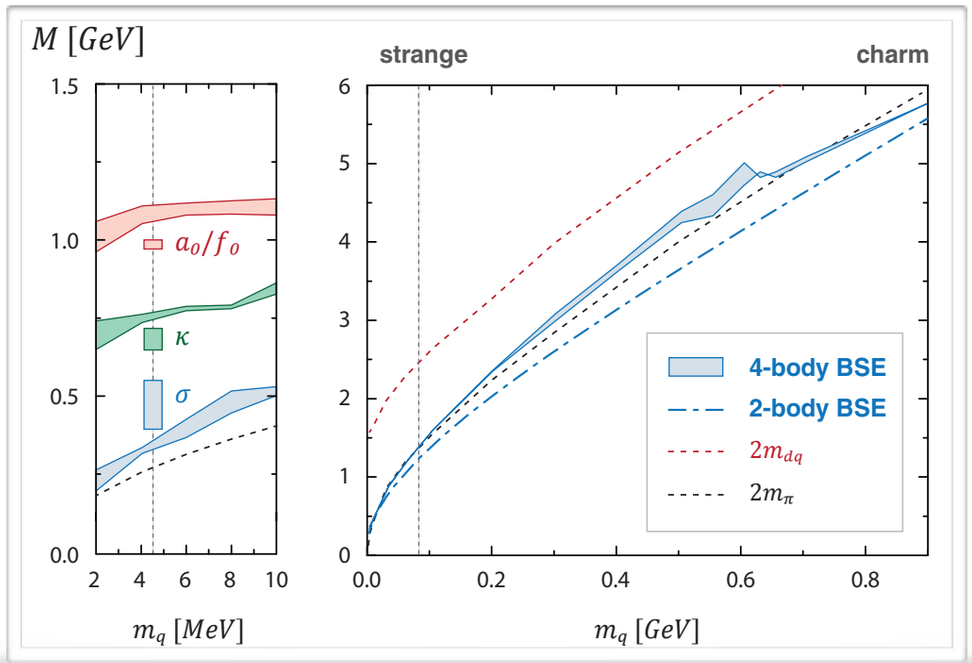
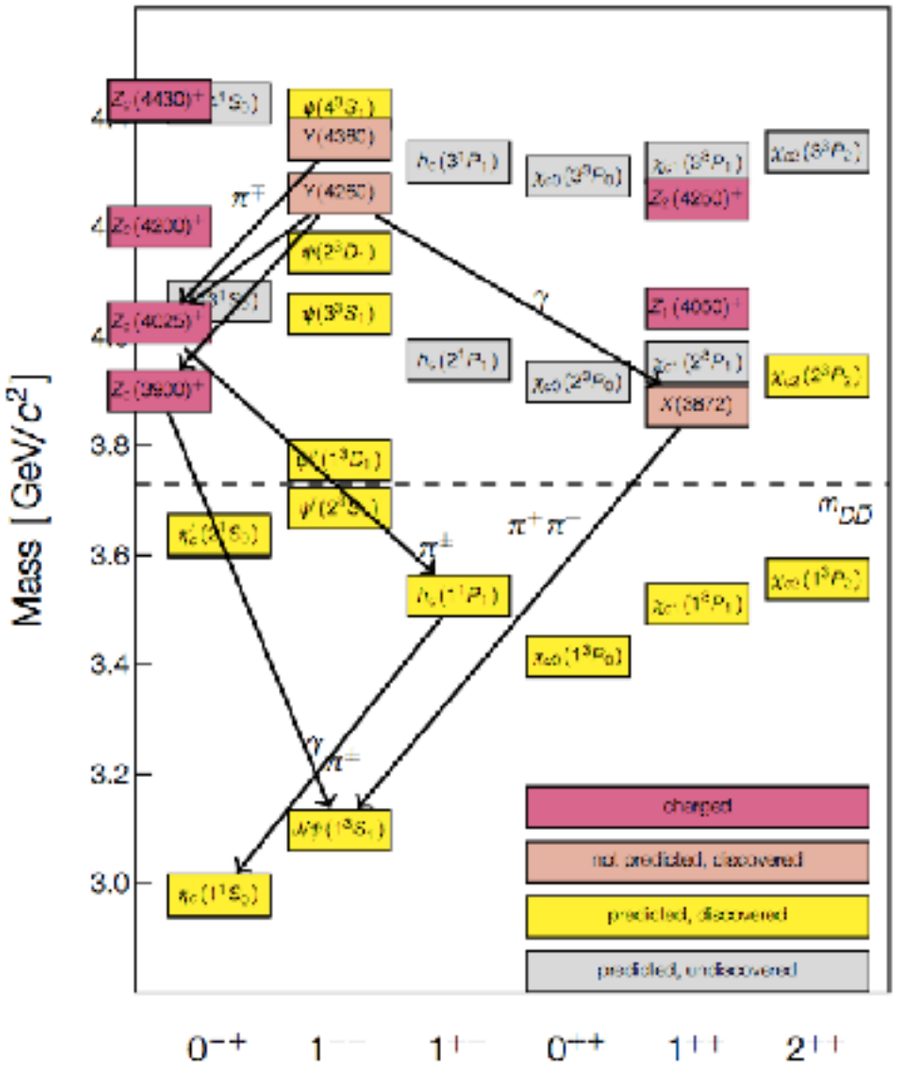


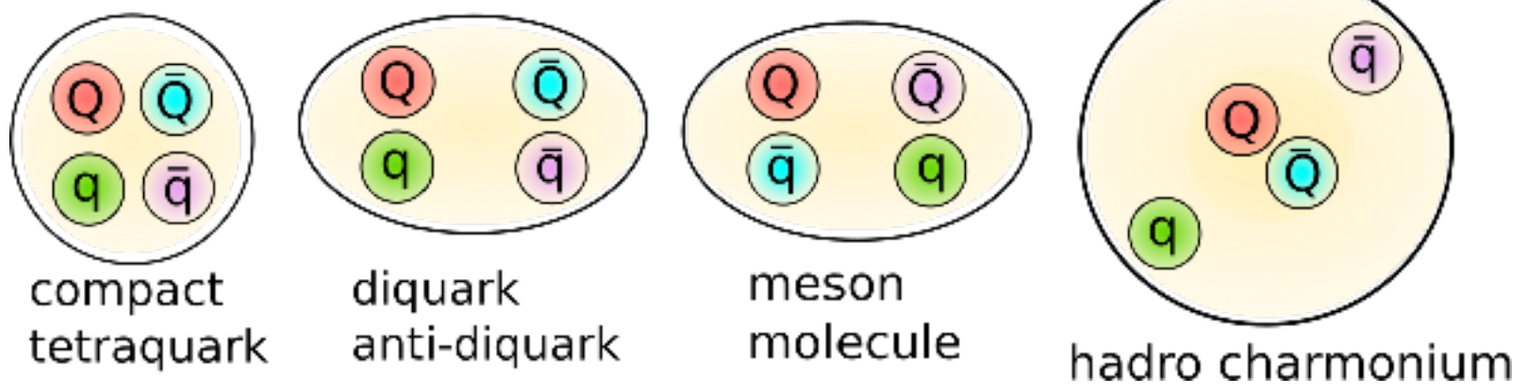
Eichman, CF, Heupel, PLB 753 (2016) 282-287

Heavy and light tetraquark



Wolfgang Gradl, BESIII, St Goar 2015

Internal structure ??



compact tetraquark

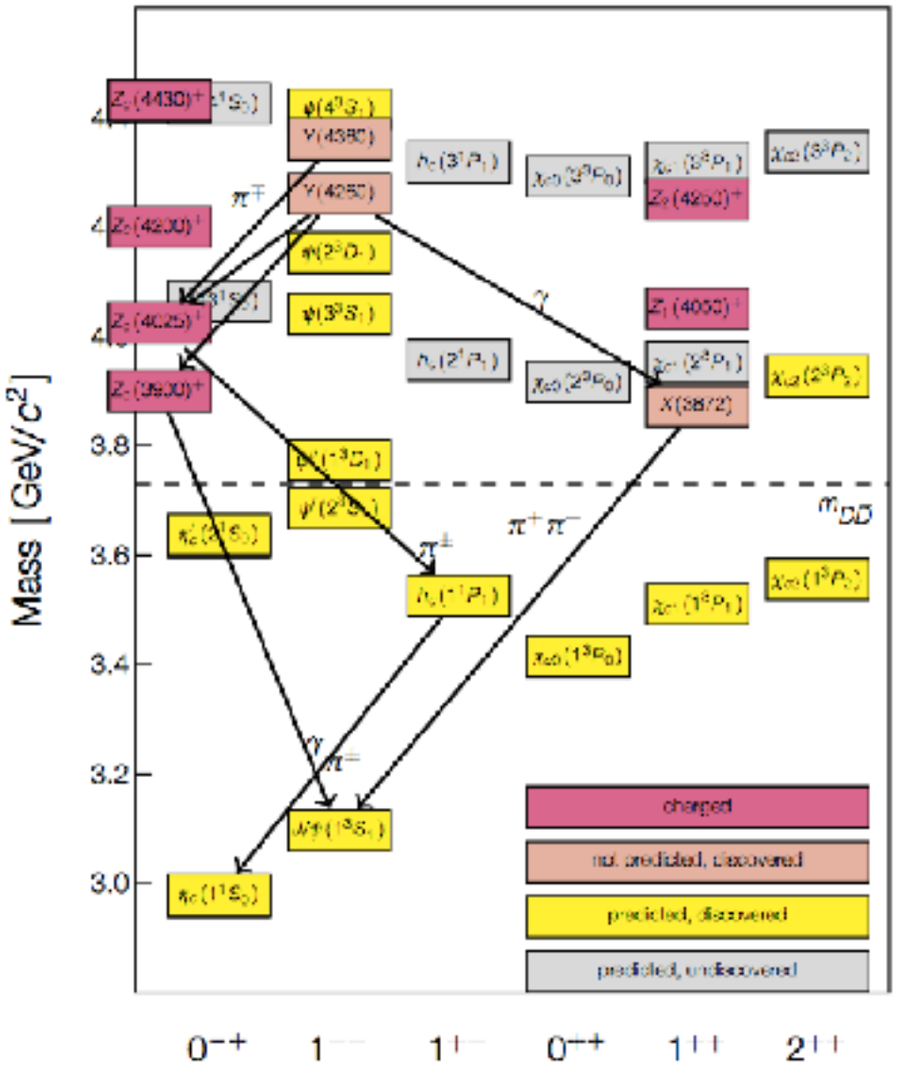
diquark anti-diquark

meson molecule

hadro charmonium

Related to details of underlying QCD forces between quarks

Heavy and light tetraquark



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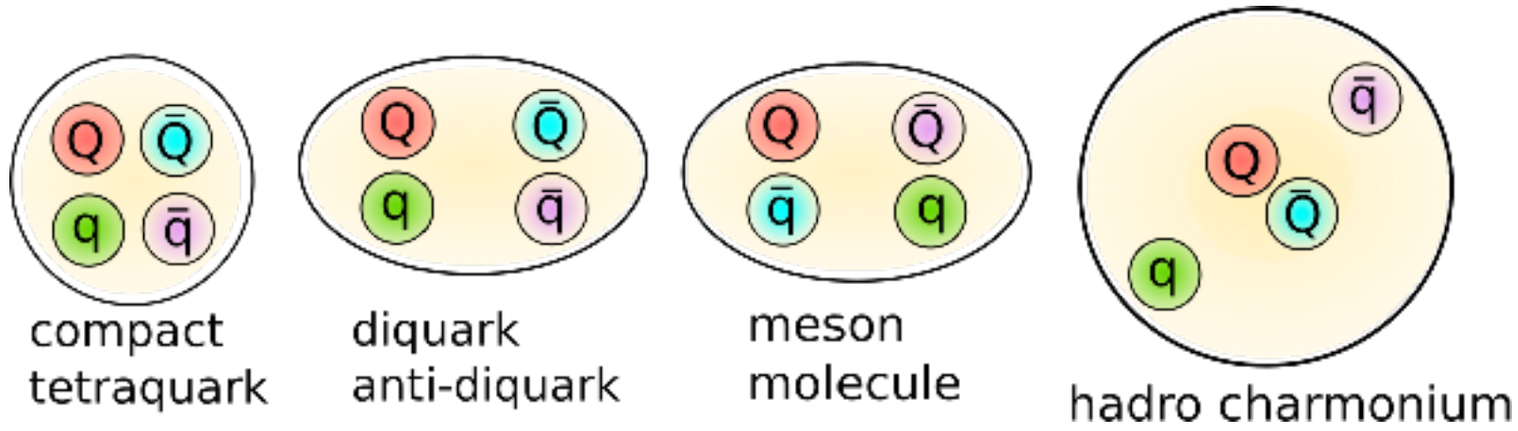
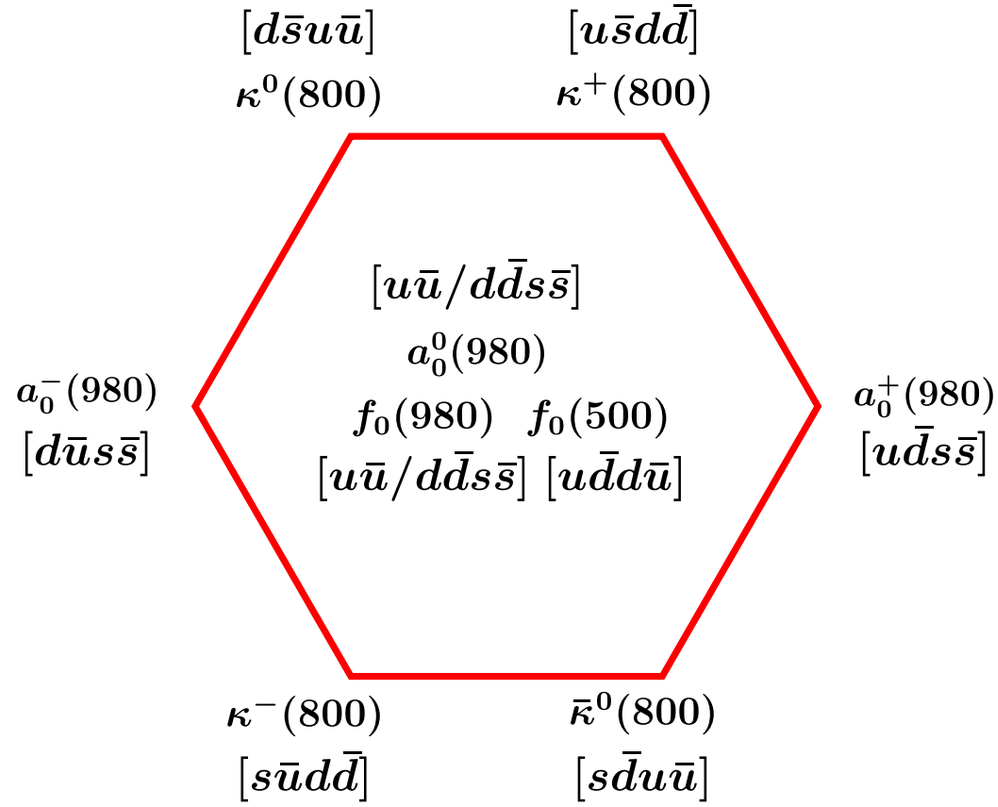
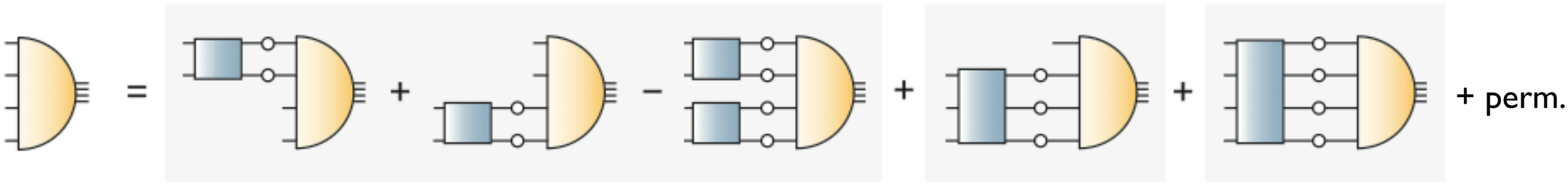




Related to details of underlying QCD forces between quarks

Tetraquarks from the four-body equation

Exact equation:



Two-body interactions

Three- and four-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

Heupel, Eichmann, CF, PLB 718 (2012) 545-549

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Basic idea:
  solve four-body equation without any assumption on internal clustering
- Key elements: quark propagator and interaction kernels

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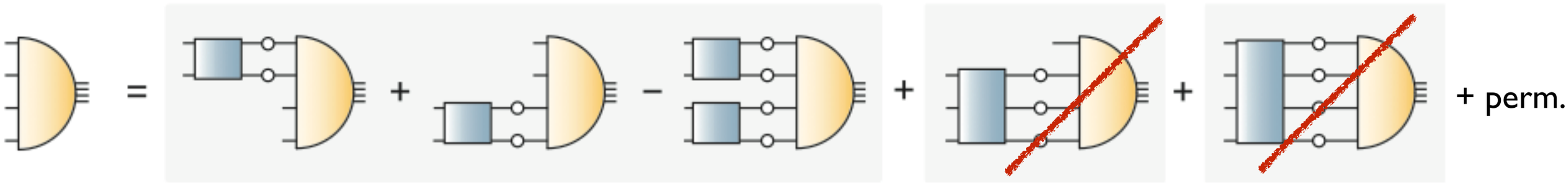

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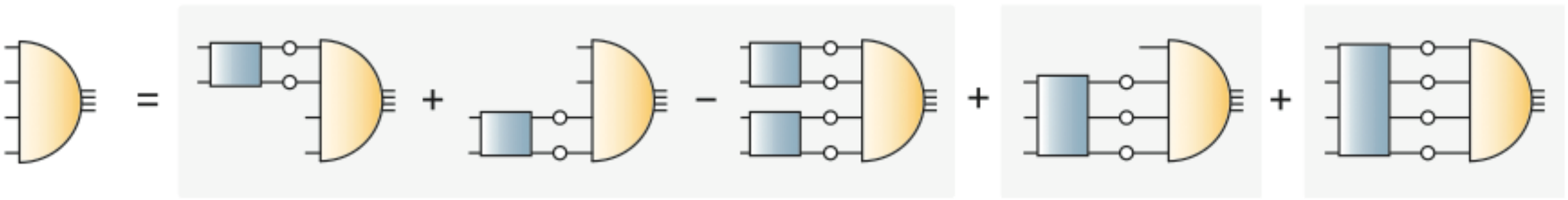
Two-body approximation



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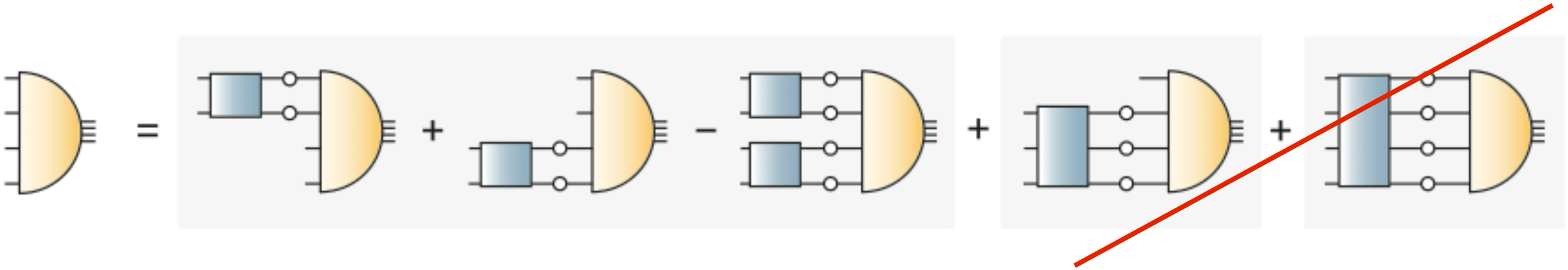
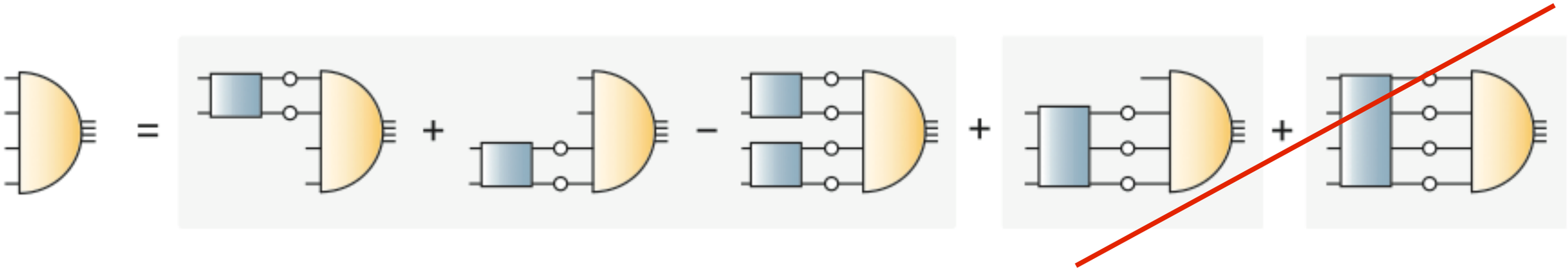
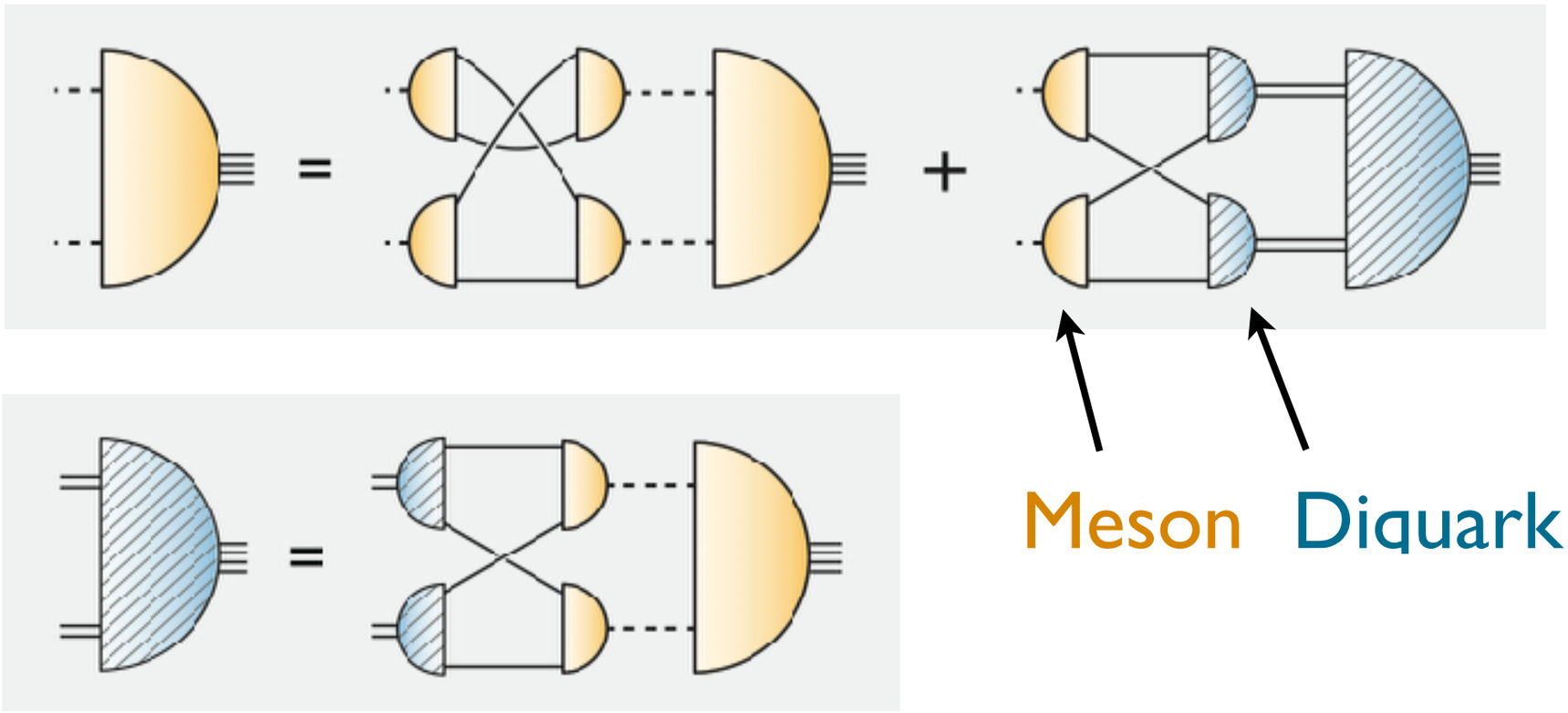


Two-body approximation

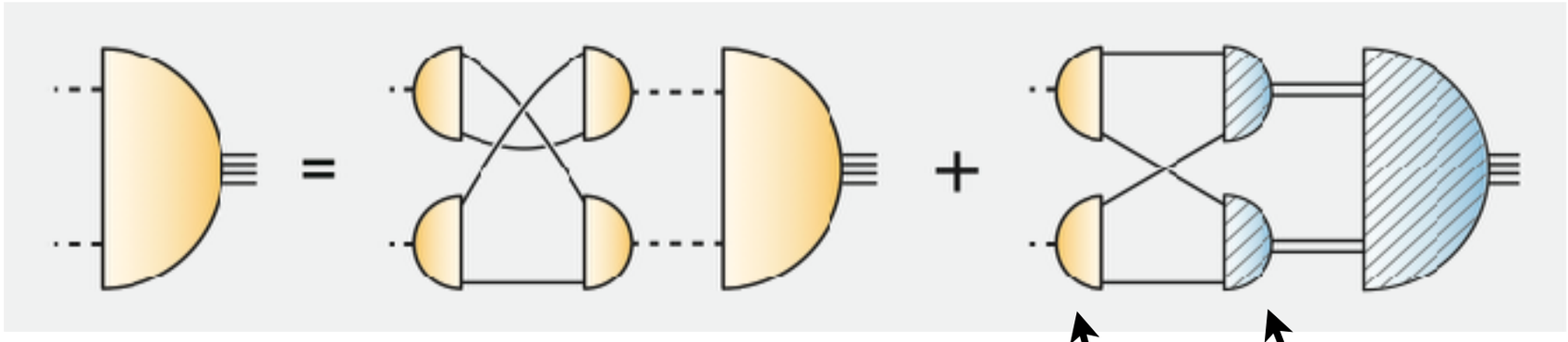


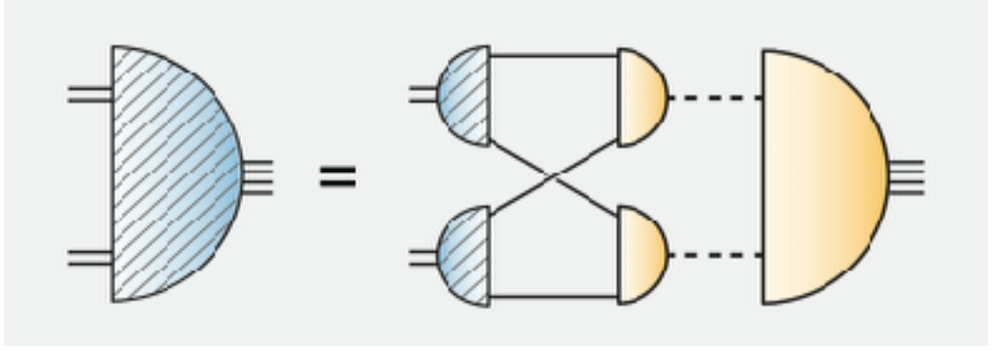
approximation: separable ansatz for interaction kernel

Heupel, Eichman, CF, PLB 718 (2012) 545-549



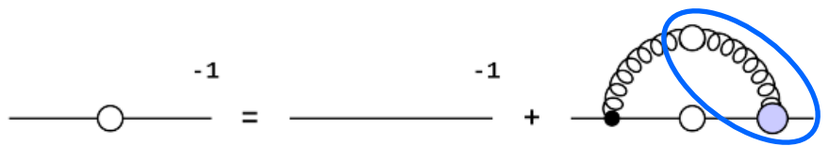
Tetraquark-BSEs - two-body equations





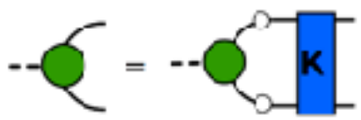
Meson Diquark

- Input: Covariant Quark-Gluon interaction - Maris-Tandy model



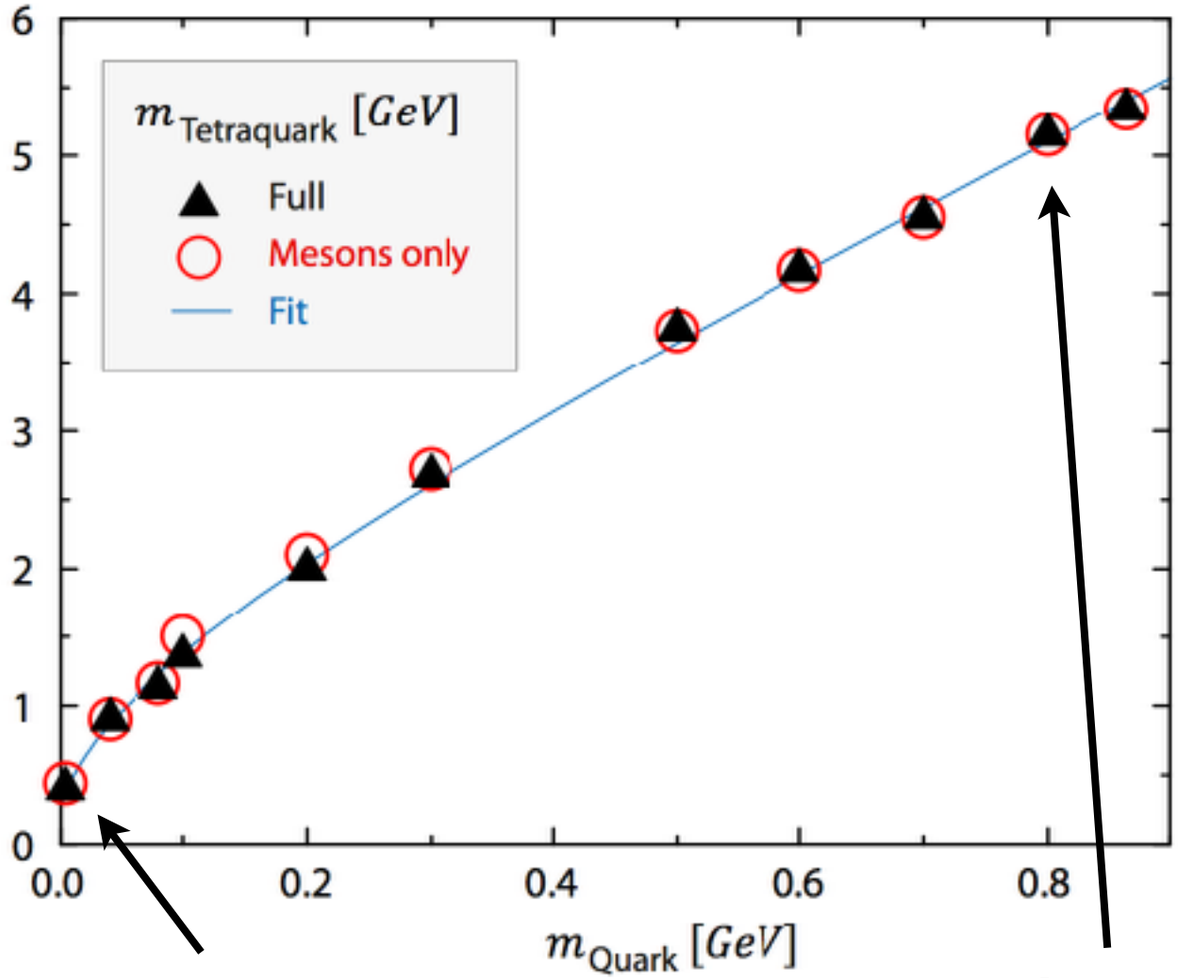
$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

- Mesons and Diquarks via Bethe-Salpeter equation



Dynamical decision between Meson- and Diquark-configurations

Results: scalar tetraquarks



Heupel, Eichman, CF, PLB 718 (2012) 545-549

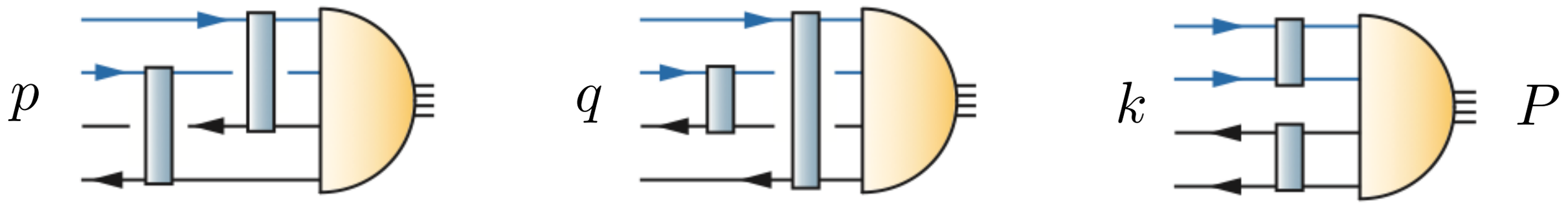
- Pion-Pion-contribution dominates ! } $f_0(500)$
- $m(0^{++}) = 403$ MeV

see also Caprini, Colangelo and Leutwyler, PRL. 96 (2006) 132001
Parganlija, Kovacs, Wolf, Giacosa and Rischke, PRD 87 (2013) 014011

- Narrow scalar $cc\overline{cc}$: $m(0^{++}) = 5.3 \pm (0.5)$ GeV

Structure of the amplitude

Scalar tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

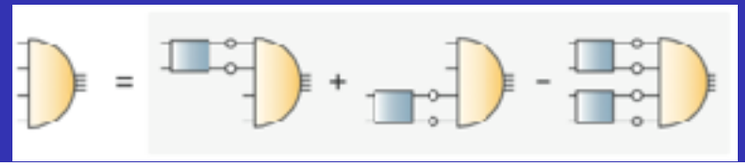
9 Lorentz scalars (built from P,p,q,k)

256 tensor structures (scalar tetra)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

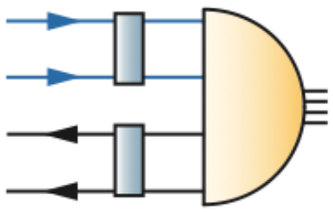
- reasonable approximation: keep s-waves only;
  → 16 tensor structures

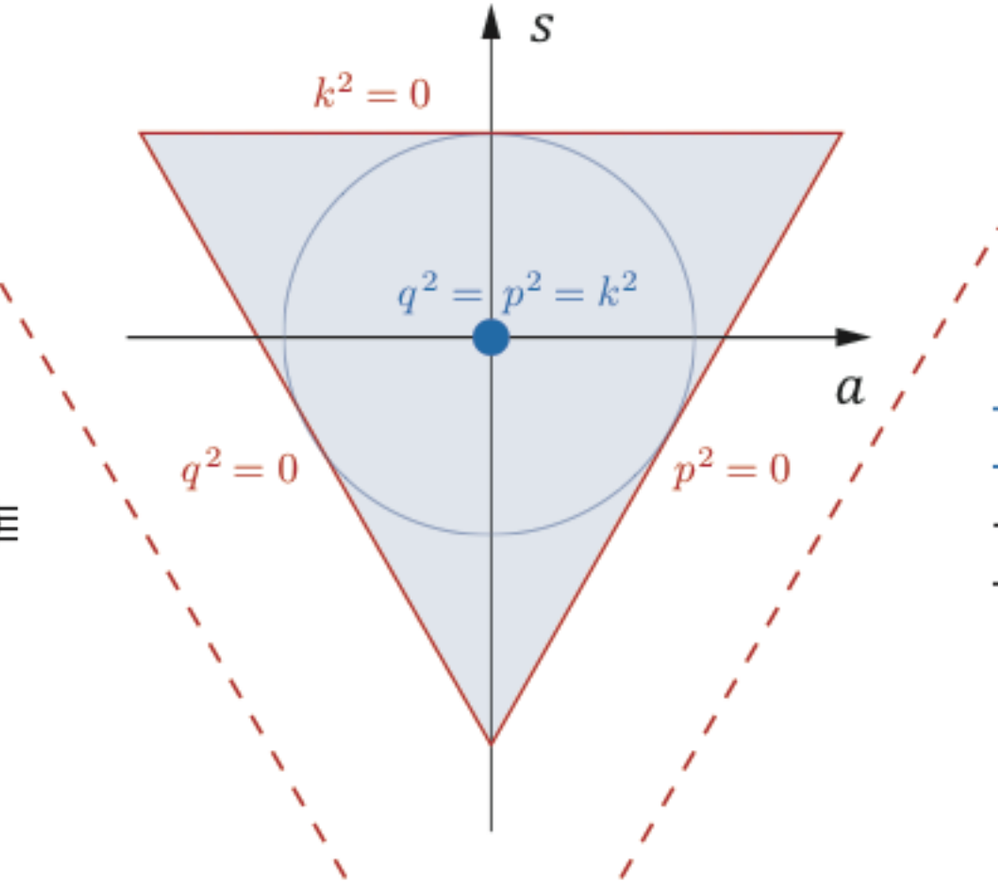
Four-body equation:



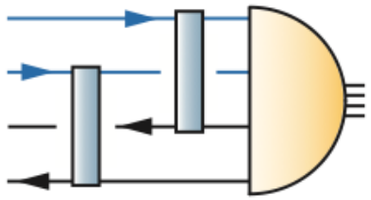
Organise Dirac-Lorentz-tensors into multiplets of **S4**

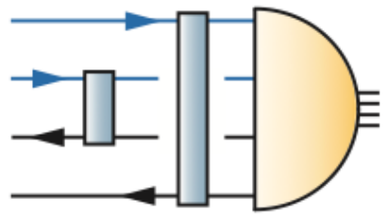
- Singlet, carries overall scale
- Doublet





diquark pole



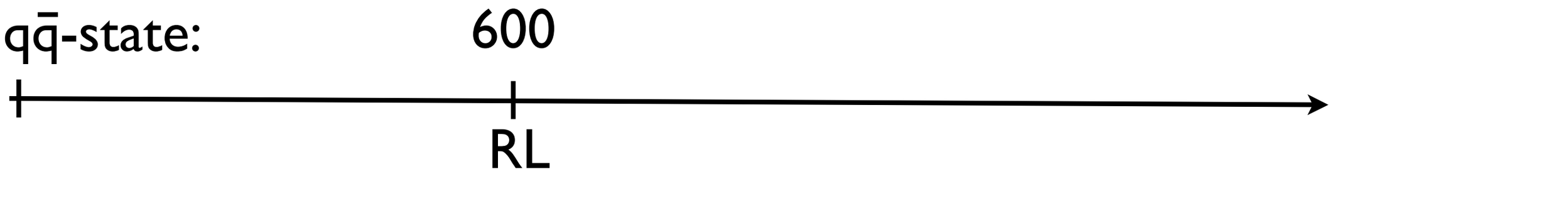


meson poles

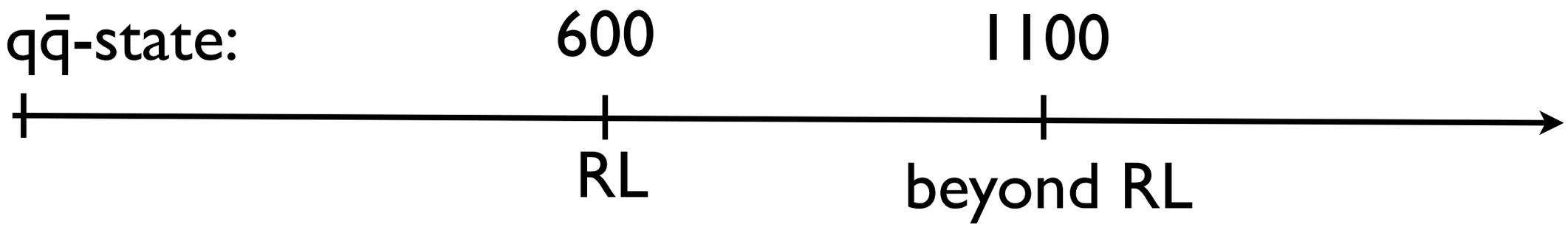
- Two triplets

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Bound state vs resonance: light scalars



Bound state vs resonance: light scalars



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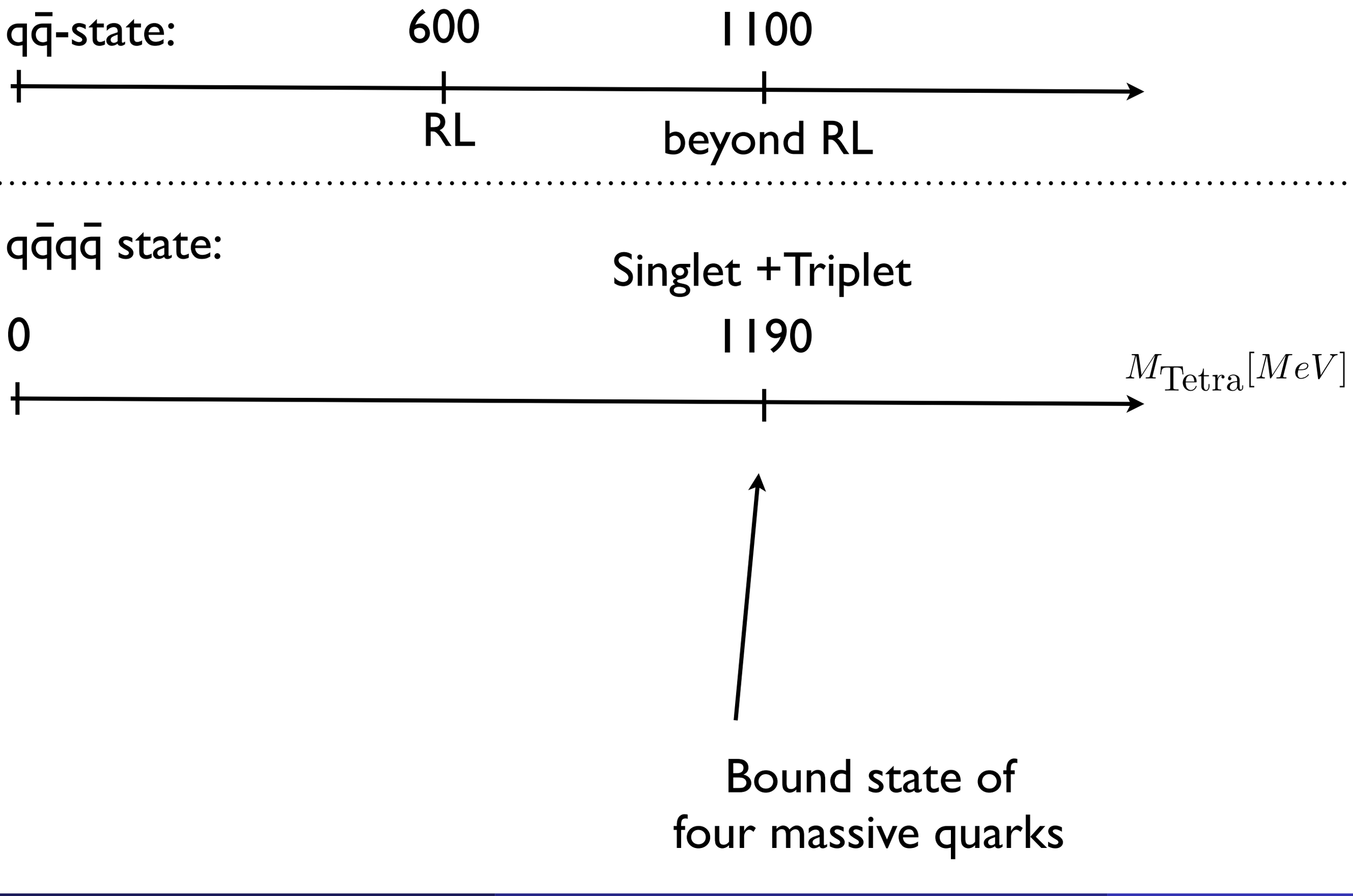


Bound state vs resonance: light scalars

qq̄-state:

600

1100

RL

beyond RL

qq̄qq̄ state:

Singlet +Triplet

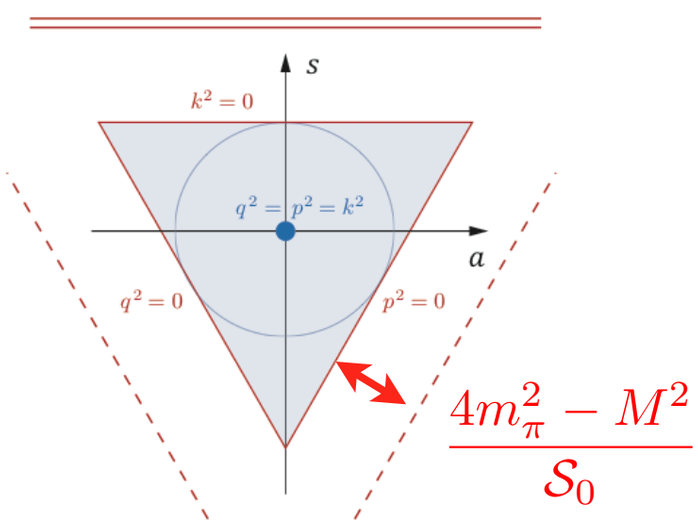
0

350

1190

$M_{\rm Tetra}[MeV]$

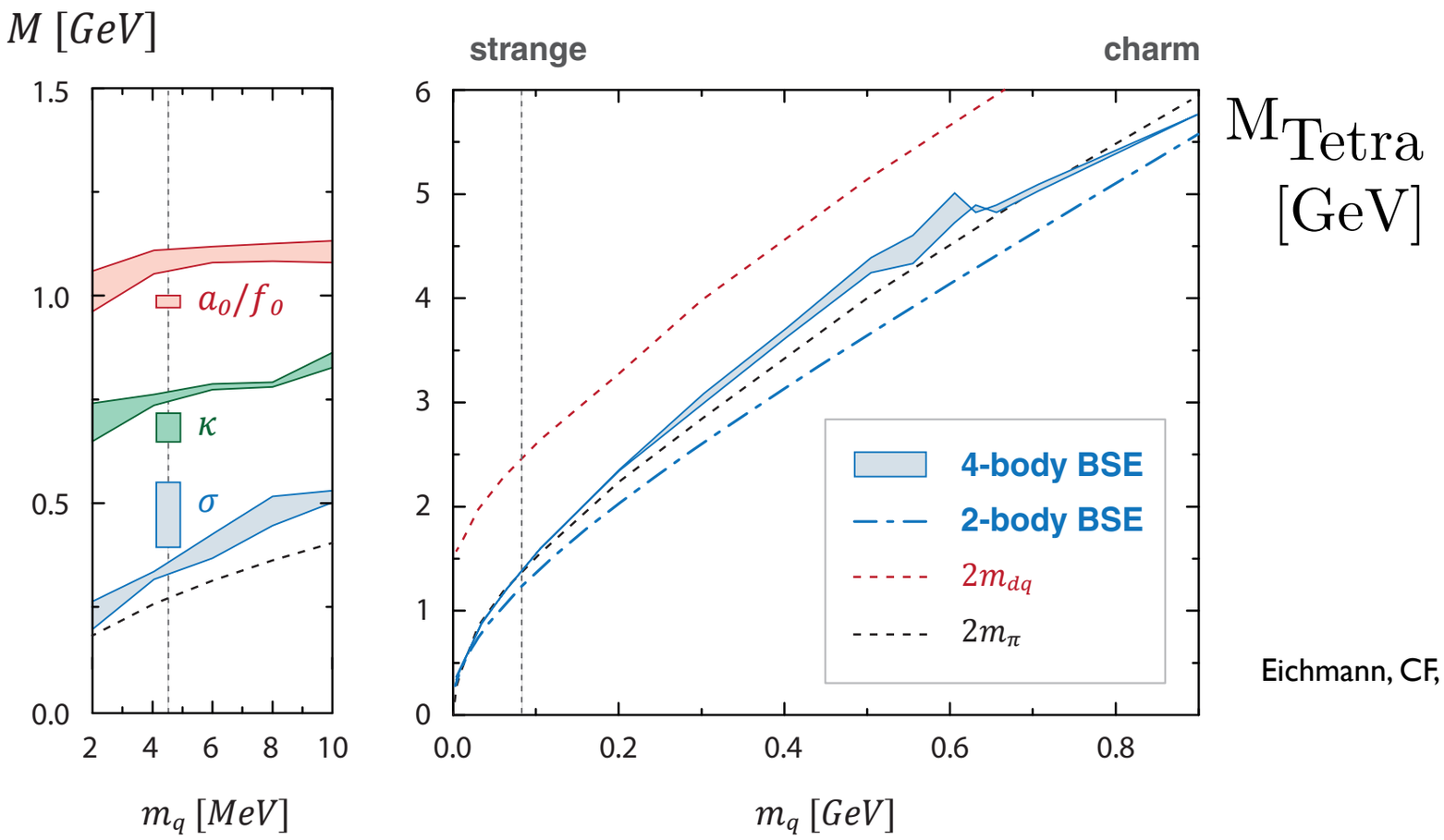
Singlet + Doublet



Two-pion resonance

Bound state of
four massive quarks

Mass evolution of tetraquark



Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Resonance becomes bound state for large m_q
- Dynamical decision: meson clusters, not diquarks
- Results:

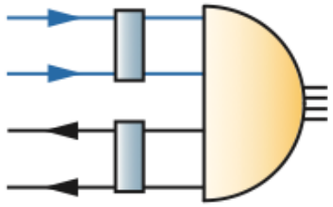
$$m_\sigma \sim 350\,\mathrm{MeV}$$
$$m_\kappa \sim 750\,\mathrm{MeV} \qquad m_{ss\bar{s}\bar{s}} \sim 1.5\,\mathrm{GeV}$$
$$m_{a_0,f_0} \sim 1080\,\mathrm{MeV} \qquad m_{cc\bar{c}\bar{c}} \sim 5.7\,\mathrm{GeV}$$

qualitatively similar to two-body framework

Heupel, Eichmann, CF, PLB 718 (2012) 545-549

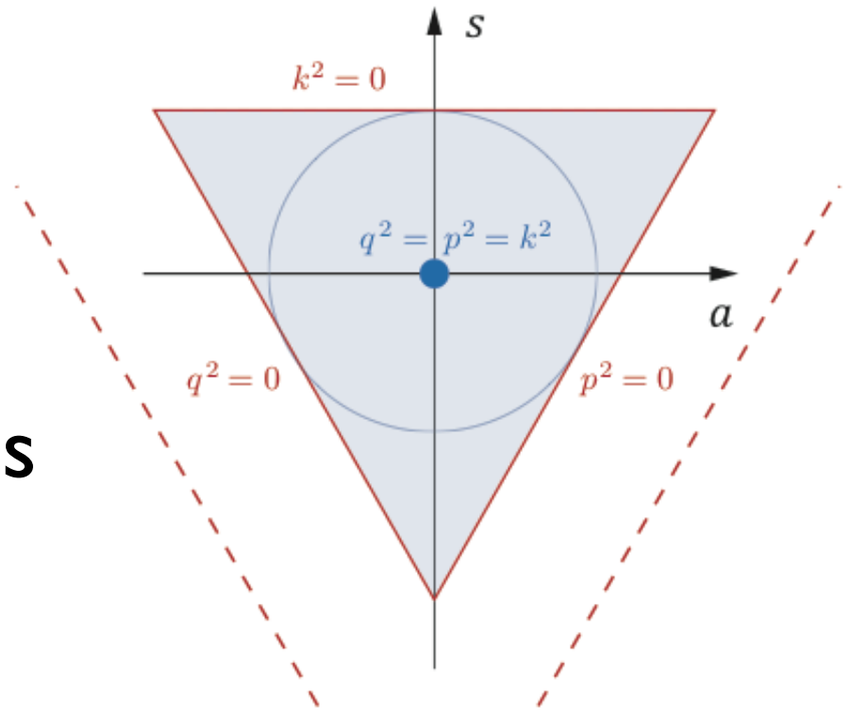
Outlook: heavy-light systems

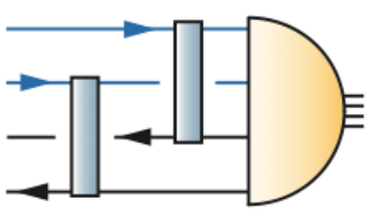
Dynamical situation in **S4**-doublet:



heavy-light diquarks

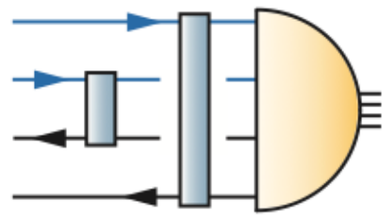
$$cq - \bar{c}\bar{q}$$





meson-molecules

$$c\bar{q} - \bar{c}q$$



hadro-charmonium

$$c\bar{c} - q\bar{q}$$

Dynamical decision of most important clustering!

Summary and outlook

One approach to find them all…

Baryons: Gernot Eichmann
Helios Sanchis-Alepuz

Summary

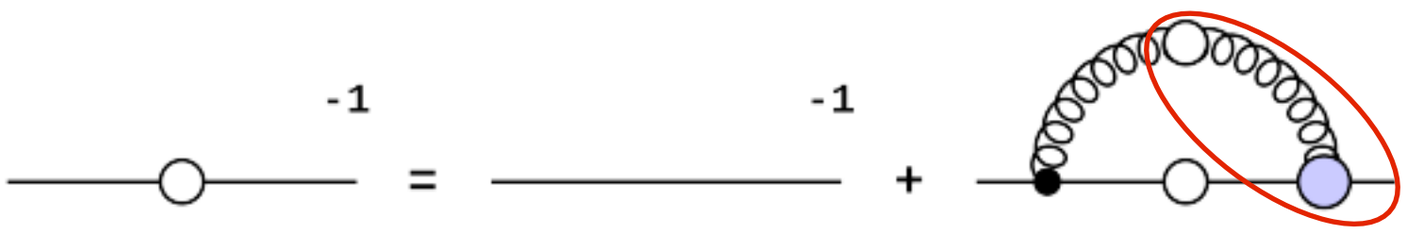
- Glueballs: $M(0^{++}) = 1.64\ \mathrm{GeV}$
- Hybrids in $q\bar{q}$-BRL
- Four-quarks states dominated by meson-meson configurations
- Dynamical description of σ as π-π resonance

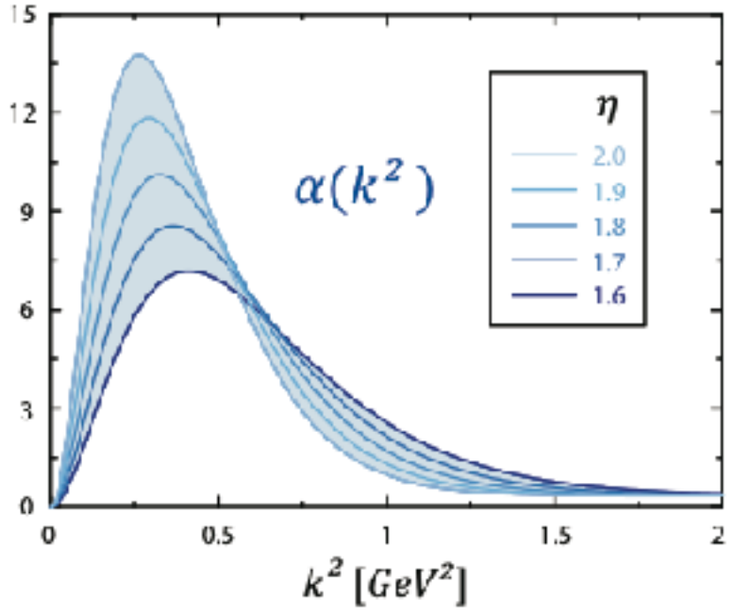
Outlook

- Glueballs: refine, explore other quantum numbers, unquench
- Hybrids: $q\bar{q}g$ ⟶ work in progress
- Tetraquarks: explore heavy-light systems

Backup

Rainbow-ladder model for quark-gluon interaction





Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

Combine gluon with quark-gluon vertex:

effective coupling

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- scale Λ from f_π, masses m_u=m_d, m_s from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : band of results

Binosi, Chang, Papavassiliou and Roberts, PLB 742 (2015) 183

Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

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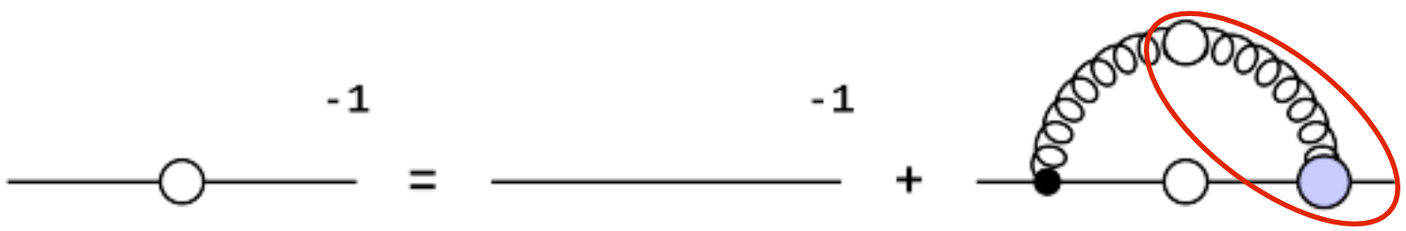

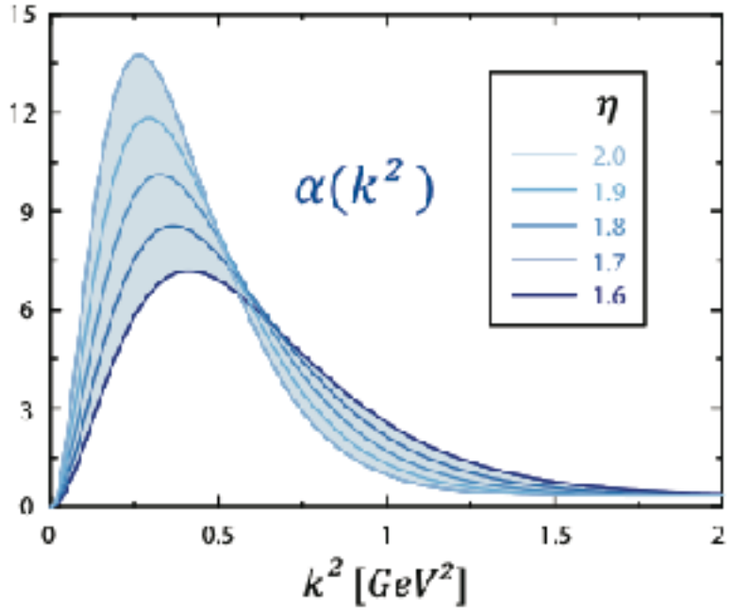

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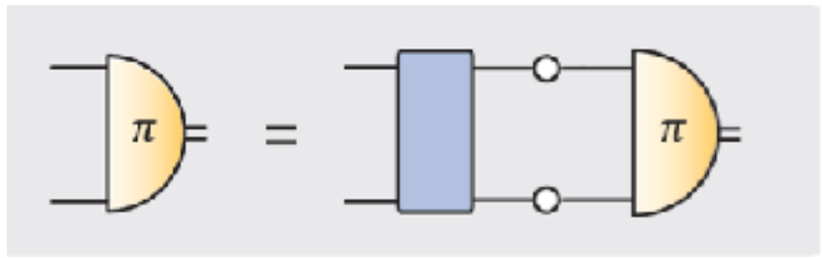

Binosi, Chang, Papavassiliou and Roberts, PLB 742 (2015) 183

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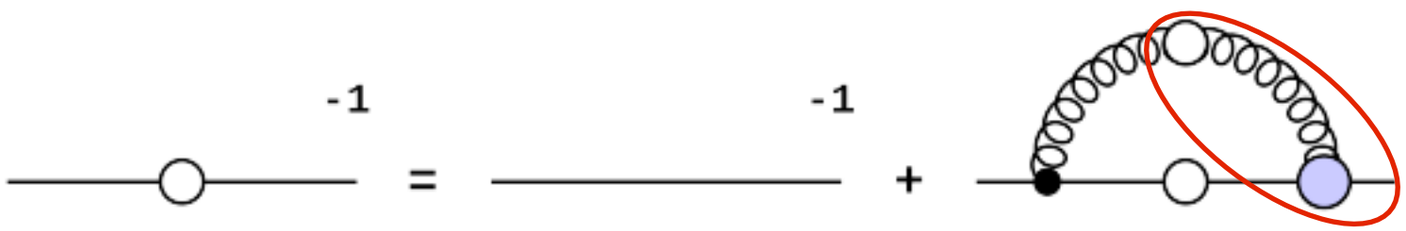

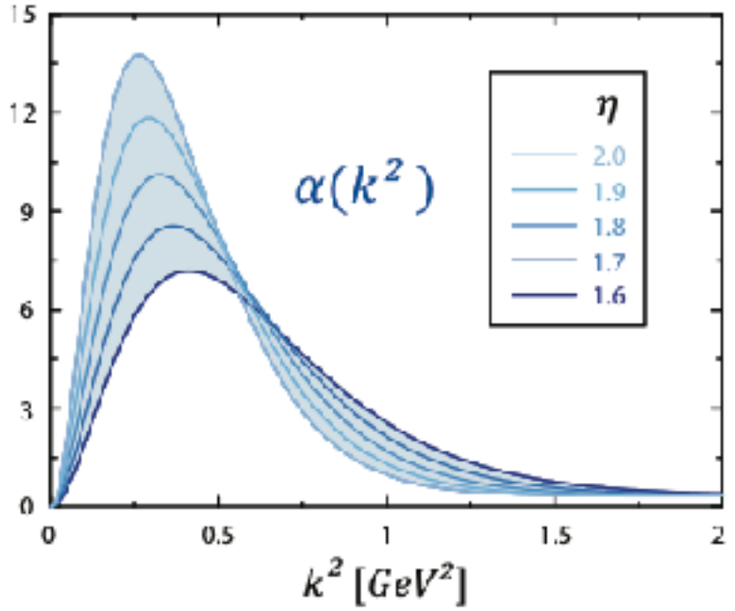

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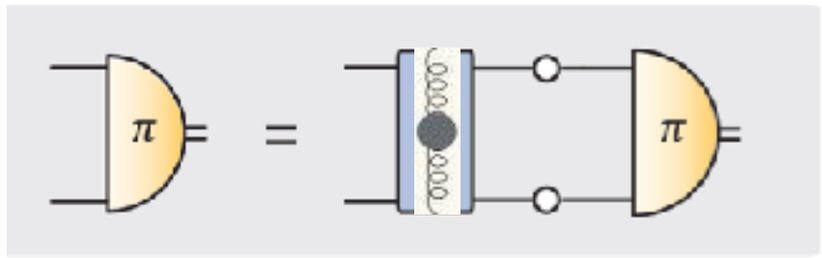

Binosi, Chang, Papavassiliou and Roberts, PLB 742 (2015) 183

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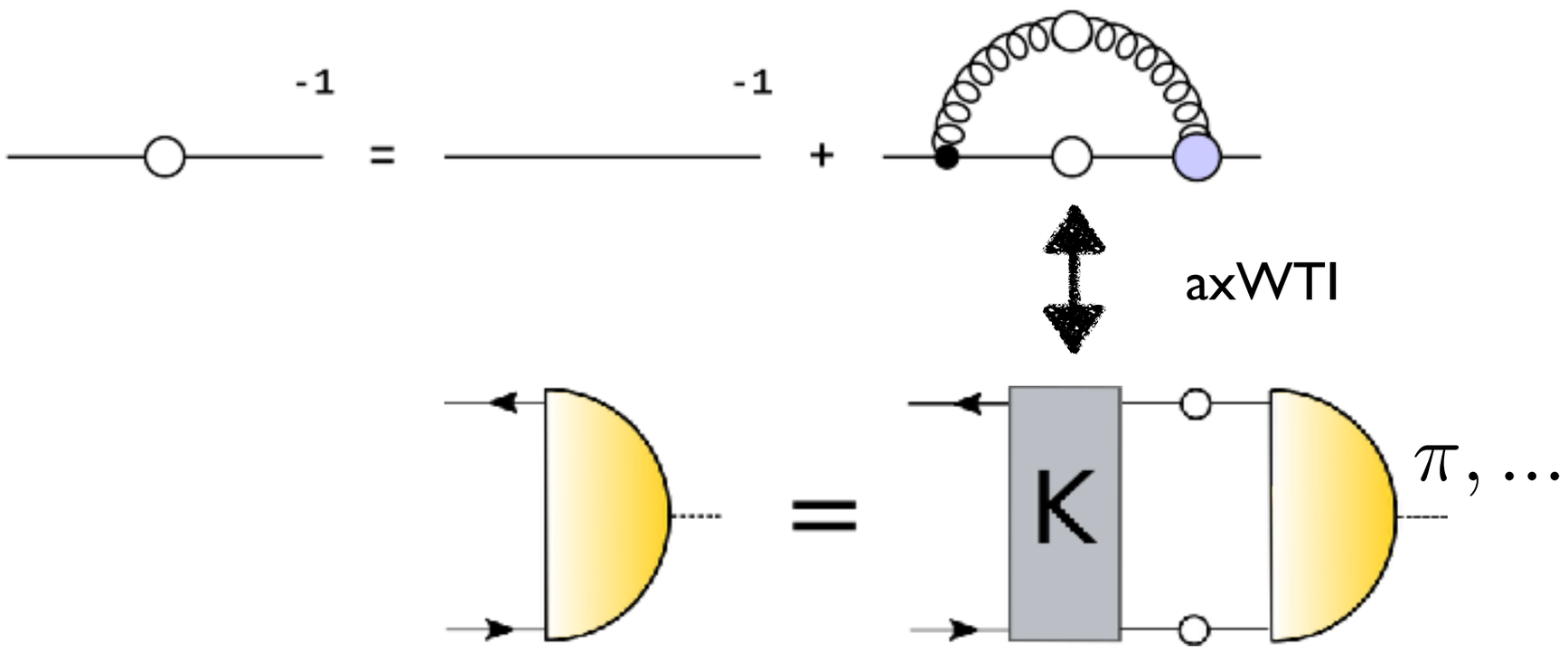
DSEs and Bethe-Salpeter equation



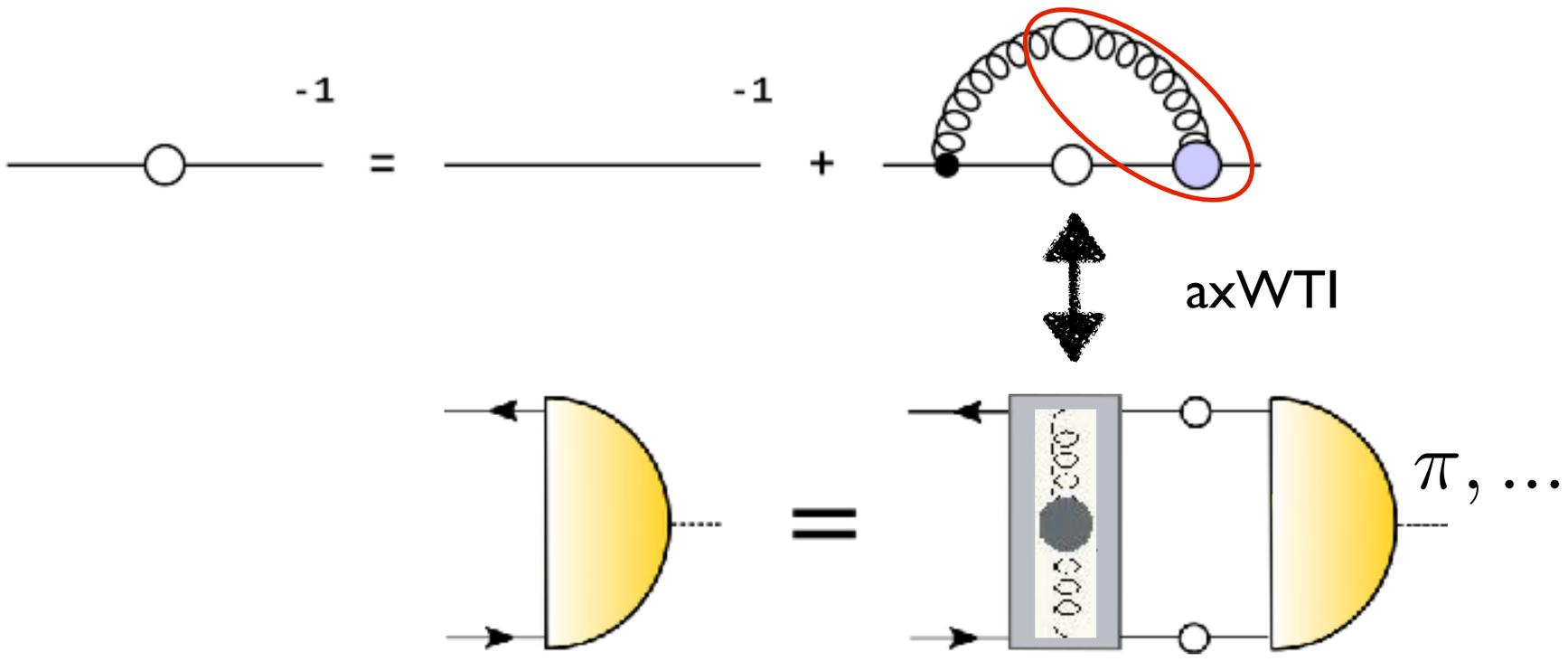
Kernel K uniquely related to quark-DSE via axialvector Ward-Takahashi-Identity (axWTI):

$$-i\int(K\gamma_5 S_- + K S_+\gamma_5) = \int \gamma_\mu S_+ D_{\mu\nu}\Gamma_\nu\gamma_5 + \int \gamma_5\gamma_\mu S_- D_{\mu\nu}\Gamma_\nu$$

→Pion is bound state **and** Goldstone boson

Maris, Roberts, Tandy, PLB 420 (1998) 267

DSEs and Bethe-Salpeter equation



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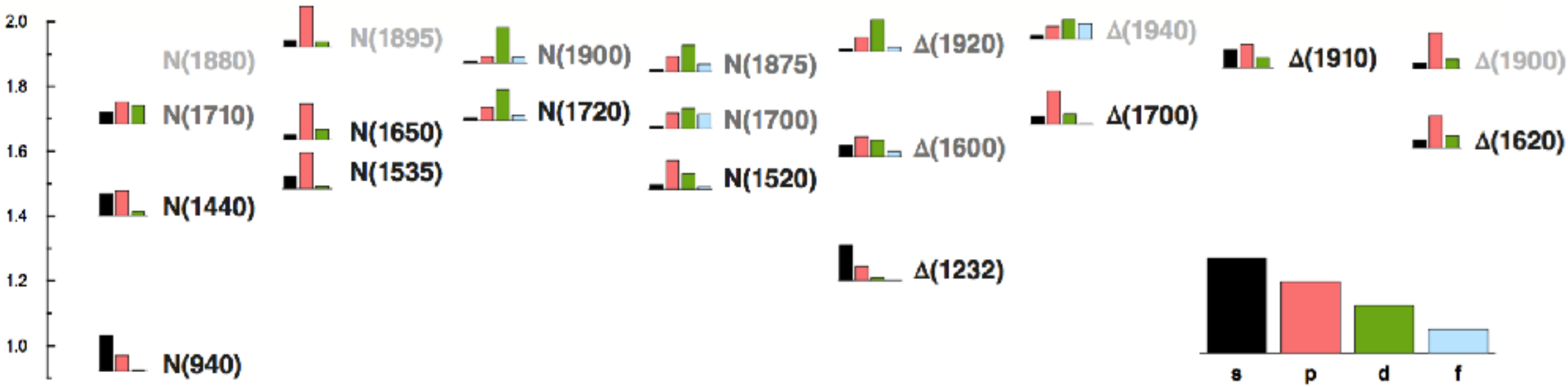
Maris, Roberts, Tandy, PLB 420 (1998) 267

Angular momentum



$$J^P = \frac{1}{2}^+ \quad \frac{1}{2}^- \quad \frac{3}{2}^+ \quad \frac{3}{2}^- \quad \frac{3}{2}^+ \quad \frac{3}{2}^- \quad \frac{1}{2}^+ \quad \frac{1}{2}^-$$

- non-relativistic quark model: restriction to certain ang. mom.
- here: quark-model forbidden contributions always present

Diquarks beyond rainbow-ladder