Energy-momentum tensor for unpolarized proton target

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GGI, 16th Mar, 2018

The goal

The quantity of our interest is the energy momentum tensor (EMT) on unpolarized proton state,

$$\langle T^{\mu\nu}\rangle = \frac{1}{2}\sum_{s=\uparrow,\downarrow}\frac{\langle p', s| T^{\mu\nu}(0)|p, s\rangle}{\sqrt{2p_0'\,2p_0}}\ ,$$

which Fourier transformation leads to the EMT in the position space,

$$\widetilde{\langle T^{\mu\nu}\rangle}(x) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3}\, e^{i\Delta\cdot x}\ \langle T^{\mu\nu}\rangle\ ,$$

where $\Delta = p' - p$, $P = \dfrac{1}{2}(p' + p)$ and $t = \Delta^2$.

M. V. Polyakov, Phys. Lett. B555, 57 (2003)
C. Lorcé, L. Mantovani, B. Pasquini, Phys. Lett. B776, 38 (2018)

The matrix element of the general local asymmetric energy–momentum tensor for a spin-1/2 target reads

$$
\begin{aligned}
&\left\langle p', s' \middle| T^{\mu\nu}(0) \middle| p, s \right\rangle = \\
&\qquad = \bar{u}(p', s')\Bigg\{ \frac{P^\mu P^\nu}{M} A(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C(t) + M\eta^{\mu\nu}\bar{C}(t) \\
&\qquad\qquad + \frac{P^\mu i\sigma^{\nu\lambda}\Delta_\lambda}{4M}\left[A(t) + B(t) + D(t)\right] \\
&\qquad\qquad + \frac{P^\nu i\sigma^{\mu\lambda}\Delta_\lambda}{4M}\left[A(t) + B(t) - D(t)\right]\Bigg\} u(p, s)\,,
\end{aligned}
$$

where $A(t) = A_q(t) + A_g(t)$ and similar others.

X.-D. Ji, Phys. Rev. Lett., 78, 610 (1997)
C. Lorcé, L. Mantovani, B. Pasquini, Phys. Lett. B776, 38 (2018)

Motivation

The study of the EMT became especially important after obtaining by Ji a relation between the EMT and DVCS process.

- DVCS gives a way to experimentally measure $T^{\mu\nu}$, *e.g.* JLab.
- $T^{\mu\nu}$ is a more fundamental quantity, which allows to access for example a spin decomposition.
- Because $\vec{\Delta}^{\,2}$ is related to $\vec{r}^{\,2}$, one has a clear interpretation as spatial densities.
- EMT form factors and GPDs constrains each other.
- Studding EMT form factors we have an access to the limit $t = \Delta^2 \to 0$, which is excluded experimentally.

X.-D. Ji, Phys. Rev. Lett., 78, 610 (1997)
X.-D. Ji, Phys. Rev. D55, 7114 (1997)

The origin of the definition

This definition of $\langle T^{\mu\nu}\rangle$ originate from derivation based on the Wigner distribution

$$\widetilde{\langle T^{\mu\nu}\rangle}(x) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3}\, e^{i\Delta\cdot x}\ \langle T^{\mu\nu}\rangle$$

$$= \mathrm{Tr}[T^{\mu\nu}(x)\, \rho(\vec{0}\,, \vec{P}\,)]\,,$$

where $\rho(\vec{X}\,, \vec{P}\,)$ is the Wigner distribution of the proton state of momentum $\vec{P}$ at space point $\vec{X}$ and reads,

$$\rho(\vec{X}\,, \vec{P}\,) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} \frac{e^{-i\vec{\Delta}\cdot\vec{X}}}{\sqrt{2p_0'\,2p_0}} \left| P - \frac{\Delta}{2} \right\rangle \left\langle P + \frac{\Delta}{2} \right| .$$

E. P. Wigner, Phys. Rev. 40, 749 (1932)

Outline

To better understand these formal definitions we write the EMT in a position space $x = (x_0, \vec{x})$ as

$$\widetilde{\langle T^{\mu\nu} \rangle}(x) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{+i\Delta_0 x_0 - i\vec{\Delta}\cdot\vec{x}} \langle T^{\mu\nu} \rangle ,$$

where $\Delta_0 = \dfrac{\vec{P} \cdot \vec{\Delta}}{P_0}$ and we consider some special cases:

(a) forward limit (FL), $\vec{\Delta} = \vec{0}$;

(b) Breit frame (BF), $\vec{P} = \vec{0}$;

(c) elastic frame (EF), $\vec{P} \cdot \vec{\Delta} = 0$;

(d) infinite momentum frame (IMF), $\vec{P} \cdot \vec{\Delta} = 0$ and $|\vec{P}| \to \infty$.

Since in all these frames energy transfer $\Delta^0 = 0$, they all lead to computing the static (average of time) EMT.

$$\widetilde{\langle T^{\mu\nu} \rangle}(\vec{x}) = \int dx_0 \widetilde{\langle T^{\mu\nu} \rangle}(x)$$

$$= \int \frac{d^3\vec{\Delta}}{(2\pi)^3} \left(\int dx_0\, e^{i\Delta_0 x_0} \right) e^{-i\vec{\Delta}\cdot\vec{x}} \langle T^{\mu\nu} \rangle$$

$$= \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{x}} \langle T^{\mu\nu} \rangle \Big|_{\Delta_0=0}$$

(a) Forward limit

The EMT in the FL, $\vec{\Delta} = \vec{0}$, reads

$$\langle T^{\mu\nu}\rangle\Big|_{\text{FL}} = \int d^3\vec{r}\,\widetilde{\langle T^{\mu\nu}\rangle}(\vec{r}) = A(0)\frac{P^\mu P^\nu}{P_0} + \bar{C}(0)\frac{M^2}{P_0}\eta^{\mu\nu}\,,$$

where $P_0\big|_{FL} = \sqrt{M^2 + \vec{P}^{\,2}}$.
This expression has the same structure as the perfect fluid density

$$\theta^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - p\,\eta^{\mu\nu}\,,$$

where ϵ is the energy density, p is the isotropic pressure, and $u^\mu = P^\mu/M$ is the four-velocity.

C. Lorcé, Eur. Phys. J. C78, 120 (2018)
C. Eckart, Phys. Rev. 58, 919 (1940)

Interpretation of $A_{q/g}(0)$ and $\bar{C}_{q/g}(0)$

The physical velocity $\vec{v}$ is related to the proper velocity, $\vec{u}$, by $\frac{M}{P_0}$,

$$\vec{v} = \frac{M}{P_0}\vec{u} = \frac{M}{\sqrt{M^2 + M^2\vec{u}^{\,2}}}\,\vec{u} = \frac{\vec{u}}{\sqrt{1+\vec{u}^{\,2}}}\,,$$

similar, the volume $d^3\vec{r}$ is related to the proper volume $d^3\mathcal{V}$,

$$d^3\vec{r} = \frac{M}{P_0}d^3\mathcal{V} = \frac{d^3\mathcal{V}}{\sqrt{1+\vec{u}^{\,2}}}\,.$$

Since $\bar{C}(t) = C_q(t) + C_g(t) = 0$ and $A(0) = A_g(0) + A_q(0) = 1$,

$$\int d^3\mathcal{V}\,(p_q + p_g) = 0\,,\quad \text{where}\quad \int d^3\mathcal{V}\,p_{q/g} = -\bar{C}_{q/g}(0)M\,,$$

$$\int d^3\mathcal{V}\,(\epsilon_q + \epsilon_g) = M\,,\quad \text{where}\quad \int d^3\mathcal{V}\,\epsilon_{q/g} = \Big[A_{q/g}(0) + \bar{C}_{q/g}(0)\Big]M\,.$$

(b) Breit frame

The EMT in the BF, $\vec{P} = \vec{0}$, reads

$$\langle T^{\mu\nu}\rangle\Big|_{\rm BF} = \Big[MA(t) - \frac{\vec{\Delta}^{\,2}}{4M}B(t)\Big]\eta^{\mu 0}\eta^{\nu 0} + \Big[M\bar{C}(t) + \frac{\vec{\Delta}^2}{M}C(t)\Big]\eta^{\mu\nu} + \frac{\Delta^\mu\Delta^\nu}{M}C(t)\,,$$

where $P_0\big|_{BF} = \sqrt{M^2 + \frac{\vec{\Delta}^2}{4}}$. This expression has the same structure as the anisotropic fluid density,

$$\Theta^{\mu\nu} = (\epsilon + p_t)u^\mu u^\nu - p_t\eta^{\mu\nu} + (p_r - p_t)\chi^\mu\chi^\nu\,,$$

for $u^\mu = \eta^{\mu 0}$ and $\chi^\mu = (0, \vec{r}/|\vec{r}|)$, where $(p_r - p_t)$ is a pressure anisotropy.

S.S. Bayin, Astrophys. J. 303, 101 (1986)

The Fourier transforms of the EMT leads to

$$p_r(r) = -M\tilde{\bar{C}}(r) + \frac{1}{M}\frac{2}{r}\frac{d\tilde{C}(r)}{dr}\,,$$

$$p_t(r) = -M\tilde{\bar{C}}(r) + \frac{1}{M}\left(\frac{1}{r}\frac{d\tilde{C}(r)}{dr} + \frac{d^2\tilde{C}(r)}{dr^2}\right),$$

$$\begin{aligned}\epsilon(r) = M\tilde{A}(r) &+ \frac{1}{4M}\left(\frac{2}{r}\frac{d\tilde{B}(r)}{dr} + \frac{d^2\tilde{B}(r)}{dr^2}\right)\\ &+ M\tilde{\bar{C}}(r) - \frac{1}{M}\left(\frac{2}{r}\frac{d\tilde{C}(r)}{dr} + \frac{d^2\tilde{C}(r)}{dr^2}\right).\end{aligned}$$

(c) Elastic frame

The EMT in the EF, $\vec{P} \cdot \vec{\Delta} = \vec{0}$, reads

$$\langle T^{\mu\nu} \rangle \Big|_{\text{EF}} = \left\{ \left[1 - \frac{\vec{P}^{\,2}}{N^2} \right] A(t) - \frac{\vec{\Delta}_\perp^{\,2}}{4N^2} \left[A(t) + B(t) \right] \right\} \frac{P^\mu P^\nu}{M}$$
$$+ \left[1 - \frac{\vec{P}^{\,2}}{N^2} \right] \left\{ \left[M\bar{C}(t) + \frac{\vec{\Delta}_\perp^{\,2}}{M} C(t) \right] \eta^{\mu\nu} + \frac{\Delta^\mu \Delta^\nu}{M} C(t) \right\}$$
$$- \frac{\vec{\Delta}_\perp^{\,2}}{8N^2} \left[A(t) + B(t) \right] \left[\eta^{0\mu} P^\nu + \eta^{0\nu} P^\mu \right]$$
$$+ \frac{\vec{\Delta}_\perp^{\,2}}{8N^2} D(t) \left[\eta^{0\mu} P^\nu - \eta^{0\nu} P^\mu \right],$$

where $N^2 = P_0(M + P_0)$ and $P_0\big|_{\text{EF}} = \sqrt{M^2 + \vec{P}^{\,2} + \frac{\vec{\Delta}_\perp^{\,2}}{4}}$.

C. Lorcé, L. Mantovani, B. Pasquini, Phys. Lett. B776, 38 (2018)

The EMT in the EF is not symmetric,

$$\left\langle T^{[\mu\nu]} \right\rangle \Big|_{\mathrm{EF}} = \frac{\Delta_\perp^2}{4N^2} D(t)\, \eta^{0\mu} P^\nu = -i\Delta_\lambda \left\langle S^{\lambda\mu\nu} \right\rangle ,$$

where spin tensor reads

$$\left\langle S^{\lambda\mu\nu} \right\rangle = -i\, \frac{g_A(t)}{4N^2} \epsilon^{\lambda\mu\nu\sigma} \epsilon_{\sigma 0\alpha\beta} P^\alpha \Delta^\beta .$$

Thus $D(t) = -g_A(t)$ and

$$\vec{S}\,(\vec{r}_\perp) = -i \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{r}_\perp} \frac{D(t)}{4N^2} \left(\vec{P} \times \vec{\Delta}_\perp\right)\Big|_{\mathrm{EF}} .$$

$$= \left(\vec{P} \times \vec{\partial}_\perp\right) \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp \cdot \vec{r}_\perp} \frac{D(t)}{4N^2}$$

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(d) Infinite momentum frame

In the IMF, where $\vec{P} \cdot \vec{\Delta} = \vec{0}$ and $|\vec{P}| \to \infty$, thus

$$P^\mu \simeq |\vec{P}|(\eta^{\mu 0} + \eta^{\mu 3}) + \frac{M^2 + \frac{\vec{\Delta}_\perp^2}{4}}{2|\vec{P}|}\eta^{\mu 0}$$

$$N^2 = P_0^2 + P_0\, M \simeq |\vec{P}|^2 + M\, |\vec{P}|\, ,$$

where $\simeq$ means that terms $\mathcal{O}(\vec{P}^{-2})$ have been suppressed.

We also introduce a notation, that for each vector/tensor v^μ,

$$v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^3)\, .$$

S. Weinberg, Phys. Rev. 150, 1313 (1966)

The EMT in the IMF reads,

$$\left\langle T^{++}\right\rangle \simeq 2|\vec{P}\,|A + \dots\,,$$

$$\left\langle T^{+-}\right\rangle \simeq \frac{1}{2|\vec{P}\,|}\left[\left(M^2 + \frac{\vec{\Delta}_\perp^{\,2}}{4}\right)A + 2\left(M^2\bar{C} + \vec{\Delta}_\perp^{\,2}C\right) - \frac{\vec{\Delta}_\perp^{\,2}}{4}(A+B+D)\right],$$

$$\left\langle T^{ij}\right\rangle \simeq \frac{1}{|\vec{P}\,|}\left[\Delta_\perp^i\Delta_\perp^j C + \left(M^2\bar{C} + \vec{\Delta}_\perp^{\,2}C\right)\eta^{ij}\right] \quad \text{for } i,j \in \{1,2\}\,.$$

It is easy to make Fourier transform.

(d') Drell-Yan frame

The EMT in the DYF, $\Delta^+ = 0$, reads,

$$\langle T^{++} \rangle = P^+ A \,,$$

$$\langle T^{+-} \rangle = \frac{1}{2P^+} \left[\left(M^2 + \frac{\vec{\Delta}_\perp^{\,2}}{4} \right) A + 2 \left(M^2 \bar{C} + \vec{\Delta}_\perp^{\,2} C \right) - \frac{\vec{\Delta}_\perp^{\,2}}{4} (A + B + D) \right] ,$$

$$\langle T^{ij} \rangle = \frac{1}{P^+} \left[\Delta_\perp^i \Delta_\perp^j C + \left(M^2 \bar{C} + \vec{\Delta}_\perp^{\,2} C \right) \eta^{ij} \right] \quad \text{for } i, j \in \{1, 2\} \,.$$

P.A.M. Dirac, Rev. Mod. Phys. 21, 392 (1949)

Conclusion

Study of the $\langle T^{\mu\nu} \rangle$ in different frames of reference gives us clear interpretation of the EMT form factors

- $A(t)$, $B(t)$ are related to the energy + pressure density,
- $C(t)$, $\bar{C}(t)$ are related to the isotropy pressure,
- $C(t)$ is related to the anisotropy pressure,
- $D(t)$ is related to the spin density.

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