ECT* European Centre for Theoretical Studies in Nuclear Physics and Related Areas

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Colored bound states and dynamical gluon mass generation

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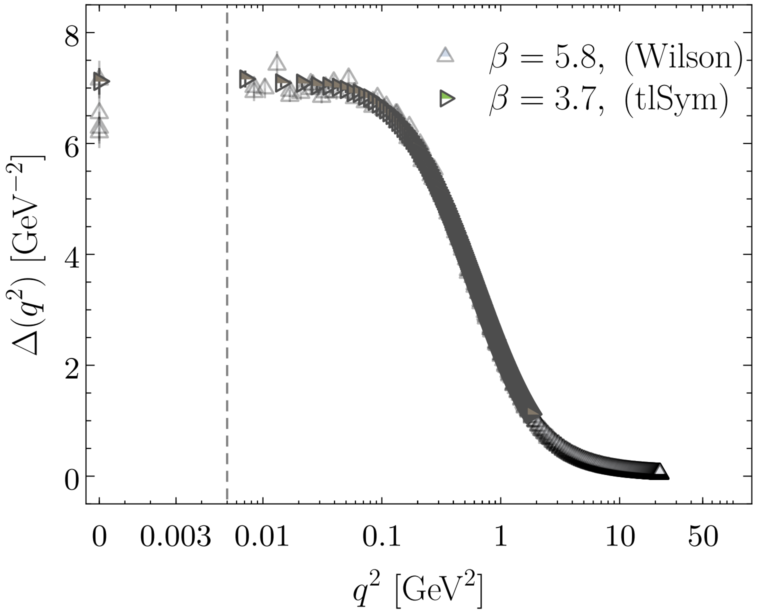
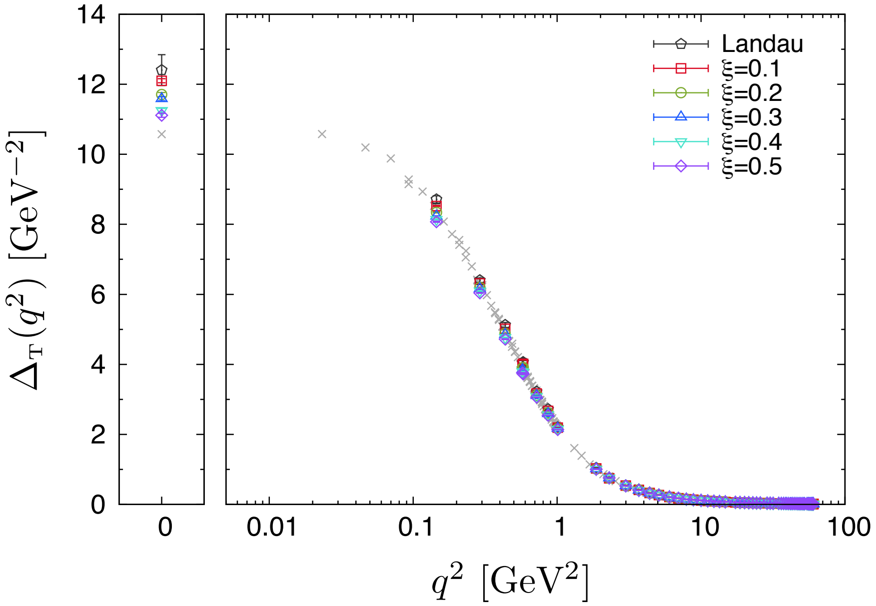
Bound states in strongly coupled systems
GGI, Firenze, Italy
March 16, 2018

Gluon propagator saturation

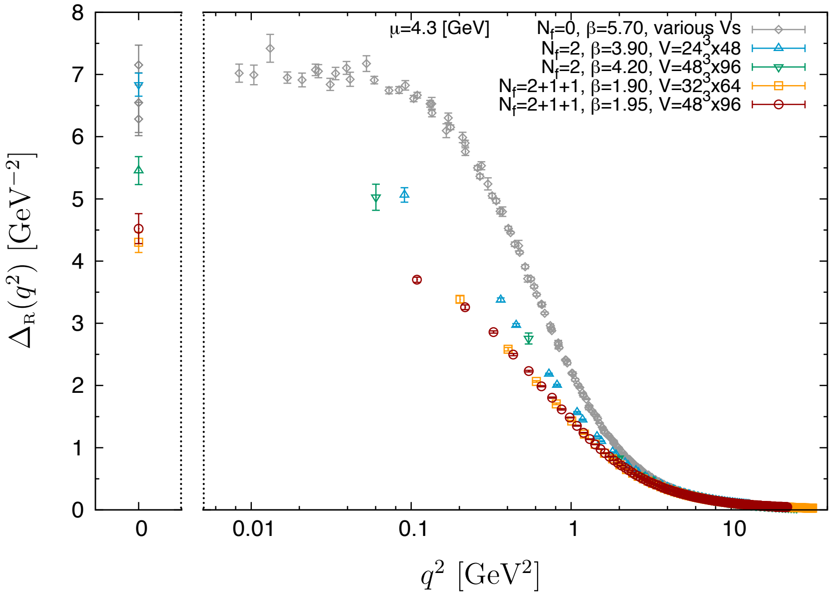
- **Landau gauge**
  (quenched)
  Bogolubsky, Ilgenfritz, Muller-Preussker, Sternbeck, PLB 676 (2009)



- **Linear gauges**
  (quenched)
  Bicudo, DB, Cardoso, Oliveira, Silva, PRD 92 (2015)



- **Landau gauge**
  (unquenched)
  Ayala, Bashir, DB, Cristoforetti, Rodriguez-Quintero, PRD 86 (2012)



- **Can be explained through "dynamical gluon mass generation"**
  generates running mass protecting (most of) perturbative logs
  Cornwall, PRD 26 (1982)

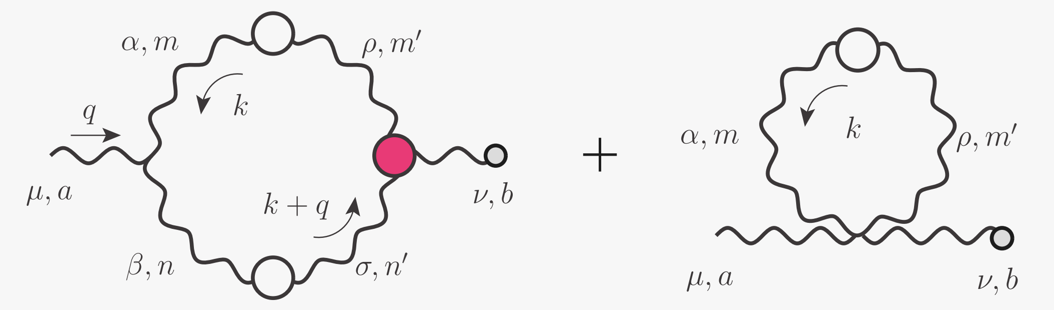
- **Gluon propagator saturation: emergent phenomenon**
  low level rules producing high level phenomena with enormous apparent complexity

PT-BFM framework

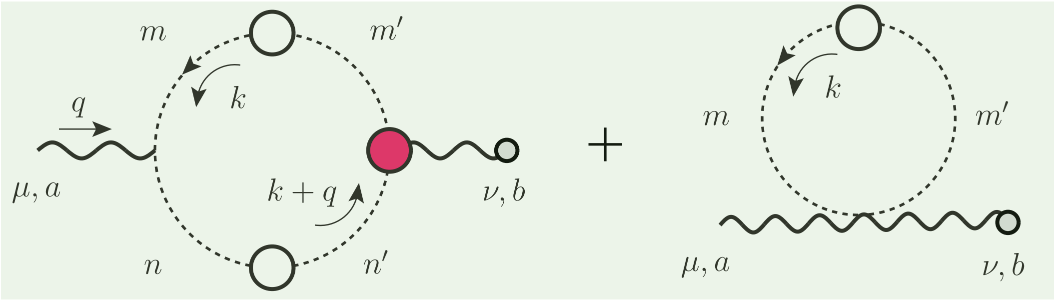
- **Resummed gluon SDE**
  expressed in terms of QB self-energy

$$\Delta^{-1}(q^2)P_{\mu\nu}(q) = \frac{q^2 P_{\mu\nu}(q) + i\widetilde{\Pi}_{\mu\nu}(q)}{1+G(q^2)}$$

- **Stronger version of transversality**
  allows gauge invariant truncation

$q^\nu \times$


$= 0$

$q^\nu \times$


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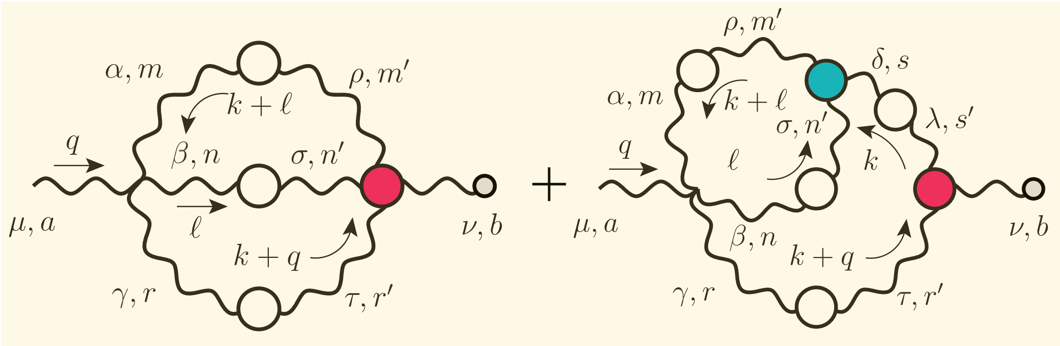
$q^\nu \times$


$= 0$

- **G function known**
  constrained by antiBRST symmetry
  DB, Quadri, PRD 88 (2013)

$$1 + G(0) = F^{-1}(0)$$

- **Divergence of B legs**
  gives rise to Abelian STIs

$$q^\nu \widetilde{\Gamma}_{\nu\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p)$$

$$q^\nu \widetilde{\Gamma}_{\nu}(q,r,p) = iD^{-1}(r^2) - iD^{-1}(p^2)$$

$$\begin{aligned} q^\nu \widetilde{\Gamma}^{mnrs}_{\nu\alpha\beta\gamma}(q,r,p,t) &= f^{mse}f^{ern}\Gamma_{\alpha\beta\gamma}(r,p,q+t) \\ &+ f^{mne}f^{esr}\Gamma_{\beta\gamma\alpha}(p,t,q+r) \\ &+ f^{mre}f^{ens}\Gamma_{\gamma\alpha\beta}(t,r,q+p) \end{aligned}$$

$\Delta(0)$ in the absence of poles

- **Abelian STI for BQ^2 and BQ^3 vertices:**

$$q^\mu \widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p)$$

$$\begin{aligned} q^\nu \widetilde{\Gamma}^{mnrs}_{\nu\alpha\beta\gamma}(q,r,p,t) &= f^{mse}f^{ern}\Gamma_{\alpha\beta\gamma}(r,p,q+t) \\ &+ f^{mne}f^{esr}\Gamma_{\beta\gamma\alpha}(p,t,q+r) \\ &+ f^{mre}f^{ens}\Gamma_{\gamma\alpha\beta}(t,r,q+p) \end{aligned}$$

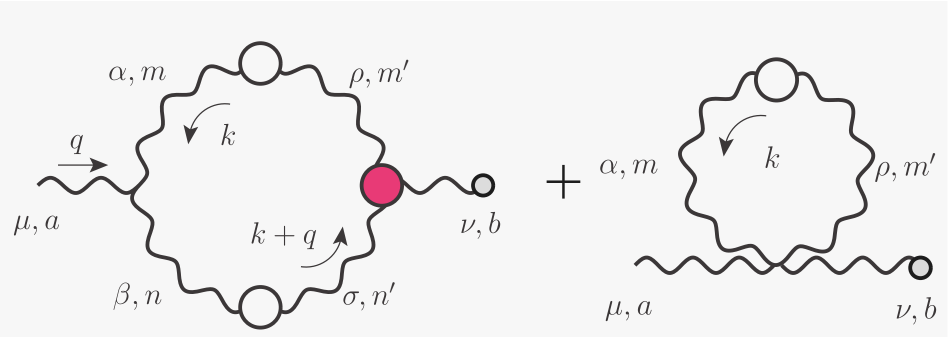
- **Taylor expand around q=0** assuming no $1/q^2$ poles are present

$$\widetilde{\Gamma}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial}{\partial r^\mu}\Delta^{-1}_{\alpha\beta}(r)$$

$$\begin{aligned} \widetilde{\Gamma}^{mnrs}_{\mu\alpha\beta\gamma}(0,-r,-p,r+p) &= -f^{mne}f^{esr}\frac{\partial}{\partial r^\mu}\Gamma_{\alpha\beta\gamma}(-r,-p,r+p) \\ &+ f^{mre}f^{ens}\frac{\partial}{\partial p^\mu}\Gamma_{\alpha\beta\gamma}(-r,-p,r+p) \end{aligned}$$

- **Plug into BQ gluon self-energy:**

$$\Delta^{-1}(0) = \lim_{q\to 0}\mathrm{Tr}$$



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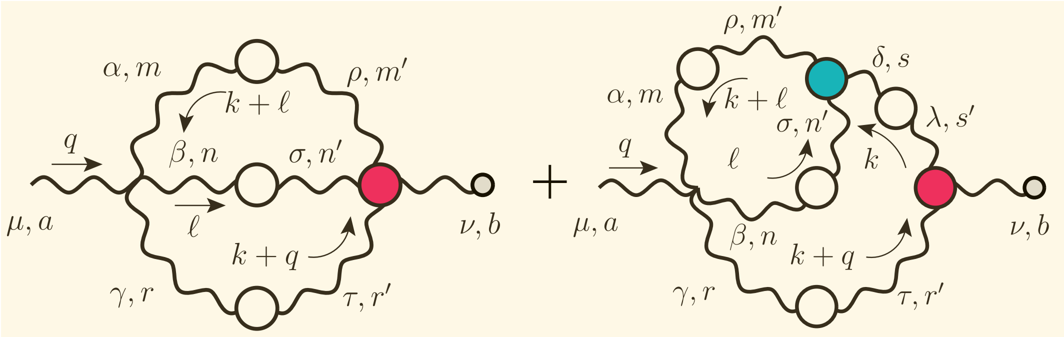


$$\sim \int_k \frac{\partial}{\partial_\mu}\left[k_\mu \Delta(k^2)\right] + \int_k \frac{\partial}{\partial k_\mu}\left[k_\mu \Delta(k^2)Y(k^2)\right]$$

- **Seagull identity $\Delta^{-1}(0) = 0$** valid independently for each diagrams set

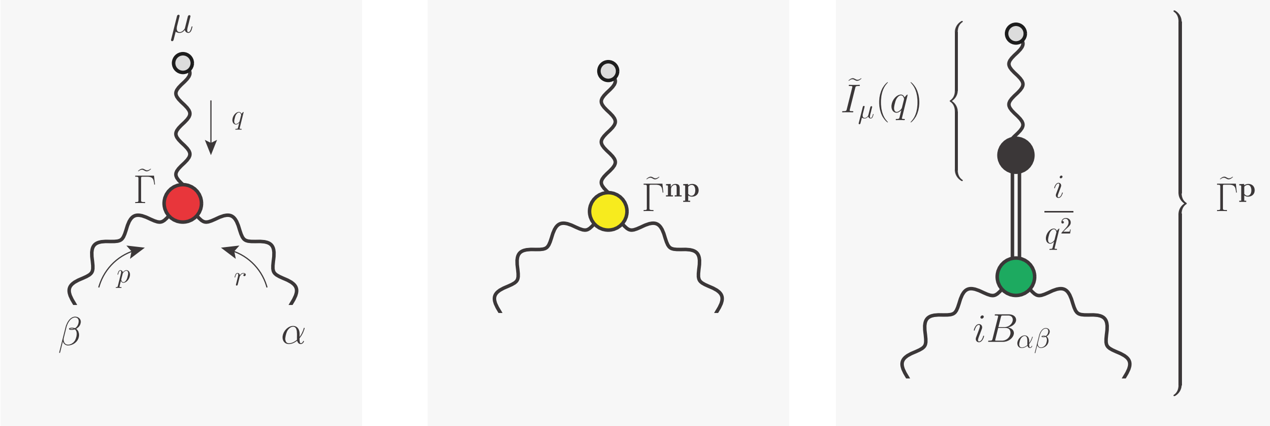
  Aguilar, DB, Figueiredo, Papavassiliou, PRD 94 (2016)

- **No gluon mass scale generation** need to relax one fo the underlying assumptions…

Schwinger mechanism

- **Derivation of the q=0 Abelian STI**
  hinges on the absence of massless poles
- **Assume now that such structures are dynamically generated in the vertex**
  Schwinger, PR 125 (1962); PR 128 (1962)
  Jackiv, Johnson, PRD 8 (1973)
  Eichten, Feinberg, PRD 10 (1973)

$$\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p)$$



- **Colored composite**
  bound state excitations

- **Poles are longitudinally coupled**
  do not participate in the seagull cancellation

$$\widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p) = \frac{q_\mu}{q^2}\widetilde{C}_{\alpha\beta}(q,r,p)$$

- **5 possible form factors**
  in Landau gauge only $\widetilde{C}_1(q,r,p)g_{\alpha\beta}$ survives
- **Bose symmetry**
  Implies $\widetilde{C}_{\alpha\beta}(0,r,-r) = 0$

Evading the seagull identity

- **Vertex satisfies the same Abelian STI**

$$q^{\mu}\Gamma^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{C}_{\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p)$$

- **Expand around q=0**

  match orders in q

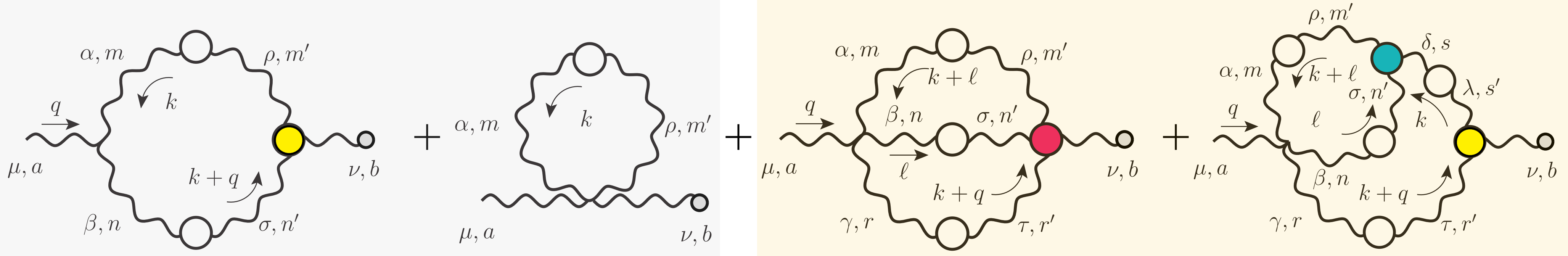
$$\Gamma^{\mathbf{np}}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial}{\partial r^{\mu}}\Delta^{-1}_{\alpha\beta}(r) - \left\{\frac{\partial}{\partial q^{\mu}}\widetilde{C}_{\alpha\beta}(q,r,-r-q)\right\}_{q=0}$$

- **Plug into BQ gluon self-energy again**

  only no-pole part participates in the seagull identity

  Aguilar, DB, Figueiredo, Papavassiliou, PRD 94 (2016)

$$\Delta^{-1}(0) = \lim_{q\to 0}\mathrm{Tr}$$



$$\sim \quad \int_k k^2\Delta^2(k^2)\widetilde{C}_1'(k^2) \quad - \quad \int_k k^2\Delta^2(k^2)Y(k^2)\widetilde{C}_1'(k^2)$$

- **If $\widetilde{C}_1' \neq 0$ a gluon mass scale can be generated**

Gluon mass scale

- **Physically motivated parametrization**

$$\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)$$

- **Kinetic term**
  associated with np part of the STI

- **Running mass**
  associated with massless poles

$$\widetilde{C}_{\alpha\beta}(q,r,p) = m^2(p^2)P_{\alpha\beta}(p) - m^2(r^2)P_{\alpha\beta}(r)$$

- **Focus on the metric part**
  take limit as $q \to 0$

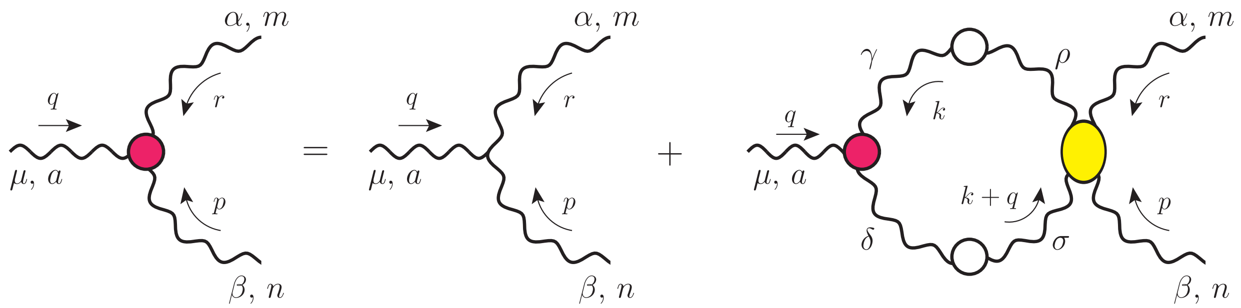
$$\widetilde{C}_1'(r^2) = \frac{\mathrm{d}m^2(r^2)}{\mathrm{d}r^2} \quad \Rightarrow \quad m^2(x) = \Delta^{-1}(0) + \int_0^x \mathrm{d}y\, \widetilde{C}_1'(y)$$

- **Not yet a running mass**
  must be monotonically decreasing and

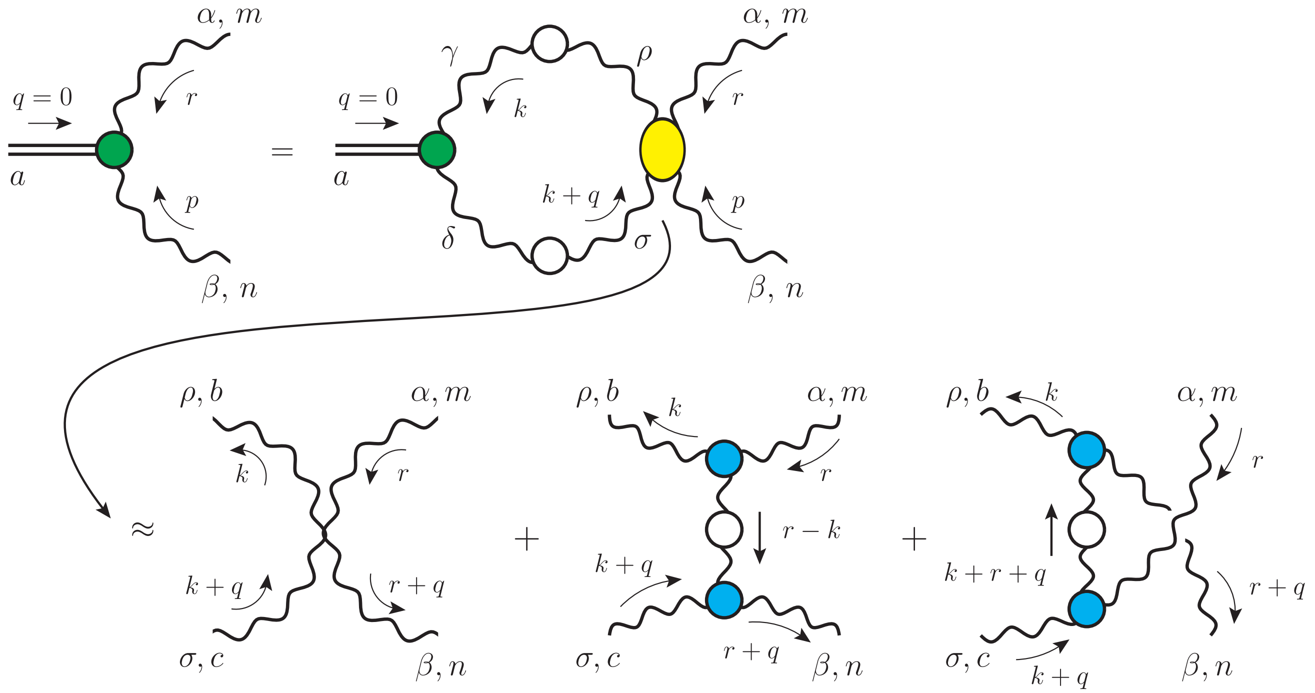
$$m^2(\infty) = 0 \quad \Rightarrow \quad m^2(x) = -\int_x^\infty \mathrm{d}y\, \widetilde{C}_1'(y)$$

BSE for massless poles

- **Consider BSE for the full vertex**



- **Replace vertex:** $\Gamma \to \Gamma^{\mathbf{np}} + \Gamma^{\mathbf{p}}$
  expand and equate terms linear in q
  DB, Papavassiliou, PRD (2018)



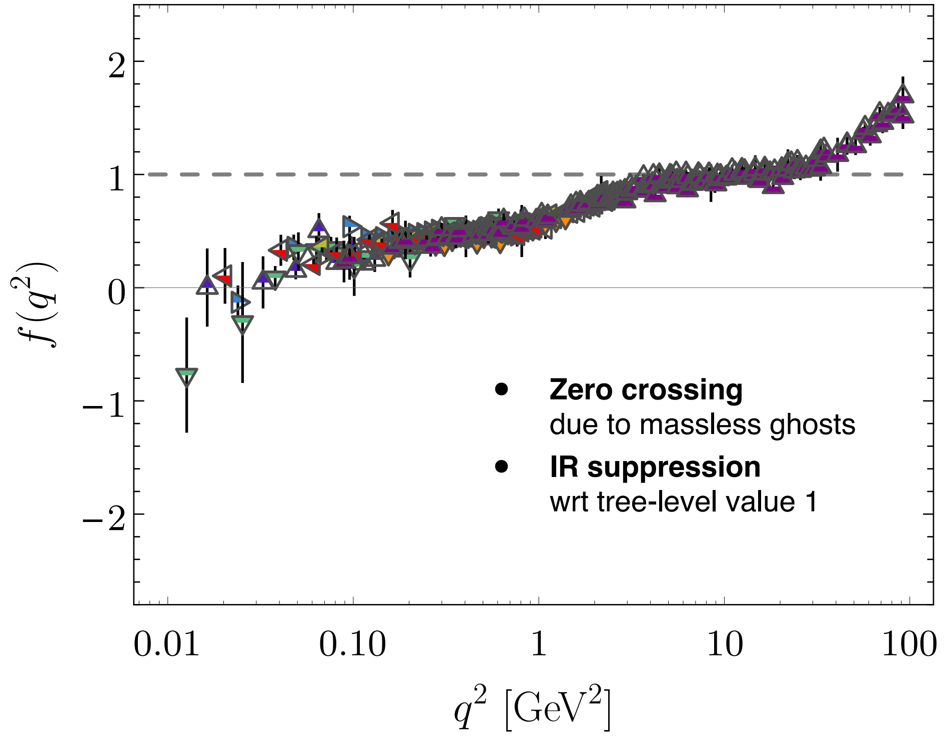
- **Homogeneous BSE**
  eigenvalue proportional to the coupling

$$\widetilde{C}_1'(x) = \alpha_s \int_0^{\pi} \mathrm{d}\theta \int_0^{\infty} \mathrm{d}y\, \mathcal{K}(x,y,\theta)\widetilde{C}_1'(y)$$

- **Four gluon kernel:**
  tractable in one-loop dressed approximation
- **Three gluon vertex**

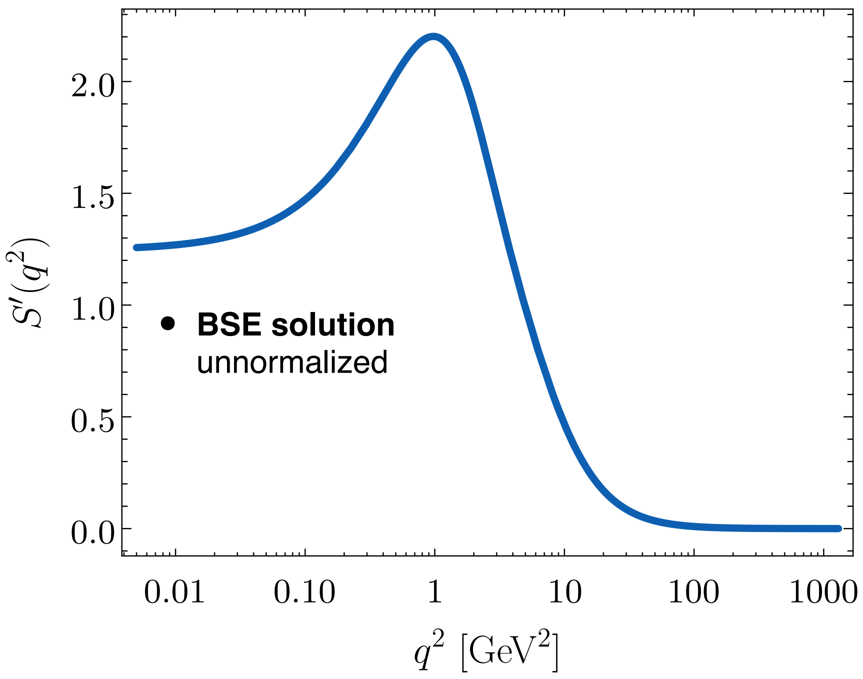
$$\Gamma_{\mu\alpha\beta}(k_1,k_2,k_3) = f(k_2)\Gamma^{(0)}_{\mu\alpha\beta}(k_1,k_2,k_3)$$

- **Vertex form factor**
  motivated by continuous/lattice studies
  Cucchieri, Maas, Mendes, PRD 74 (2006)
  Athenodorou, DB, Boucaud, De Soto, Papavassiliou, Rodriguez-Quintero, Zafeiropoulos, PLB 761 (2016)
  Aguilar, DB, Ibañez, Papavassiliou, PRD 89 (2014)

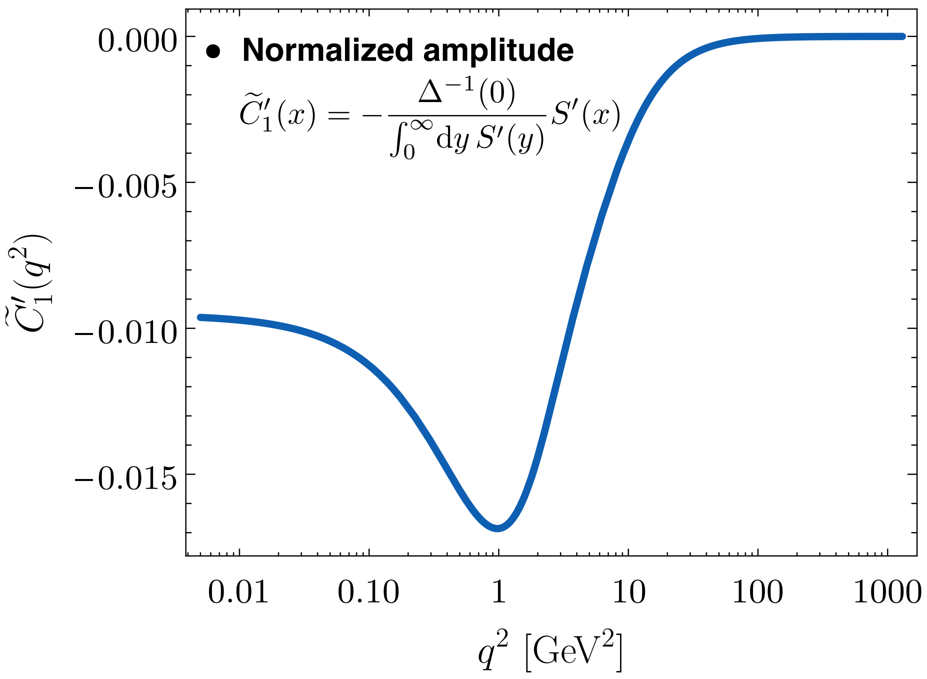


- **Zero crossing**
  due to massless ghosts
- **IR suppression**
  wrt tree-level value 1

Poles BS amplitude

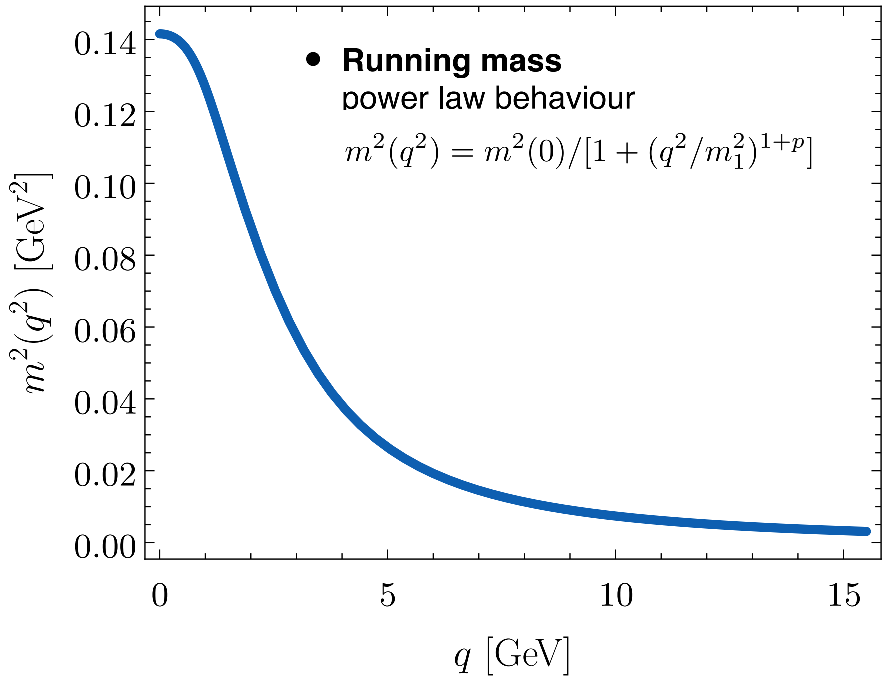


- **BSE solution**
  unnormalized



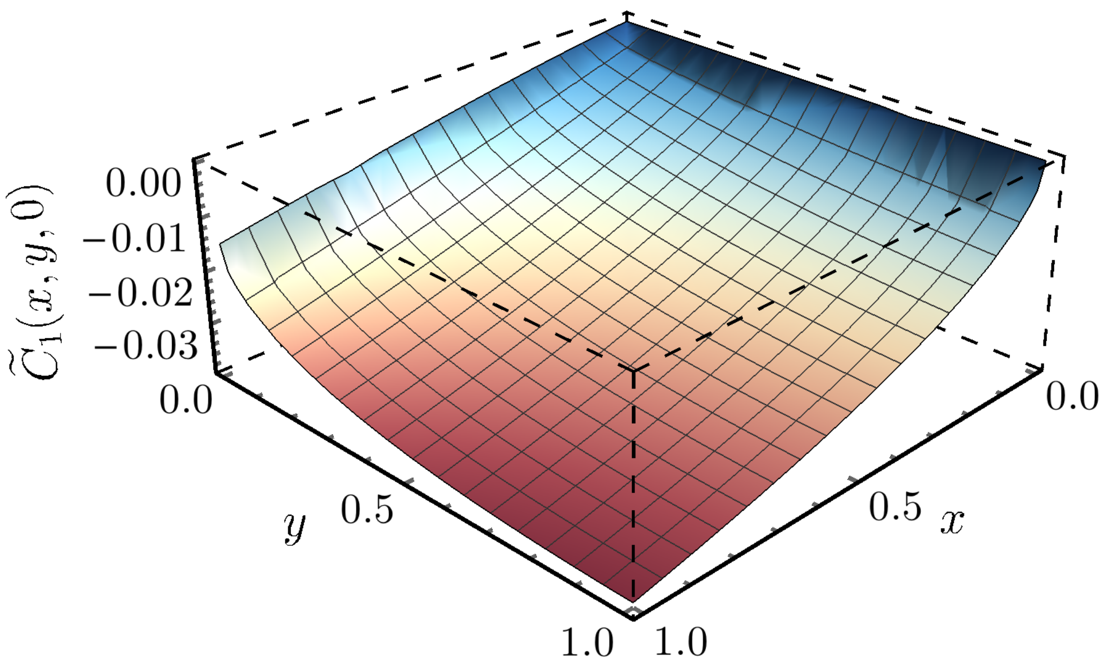
- **Normalized amplitude**

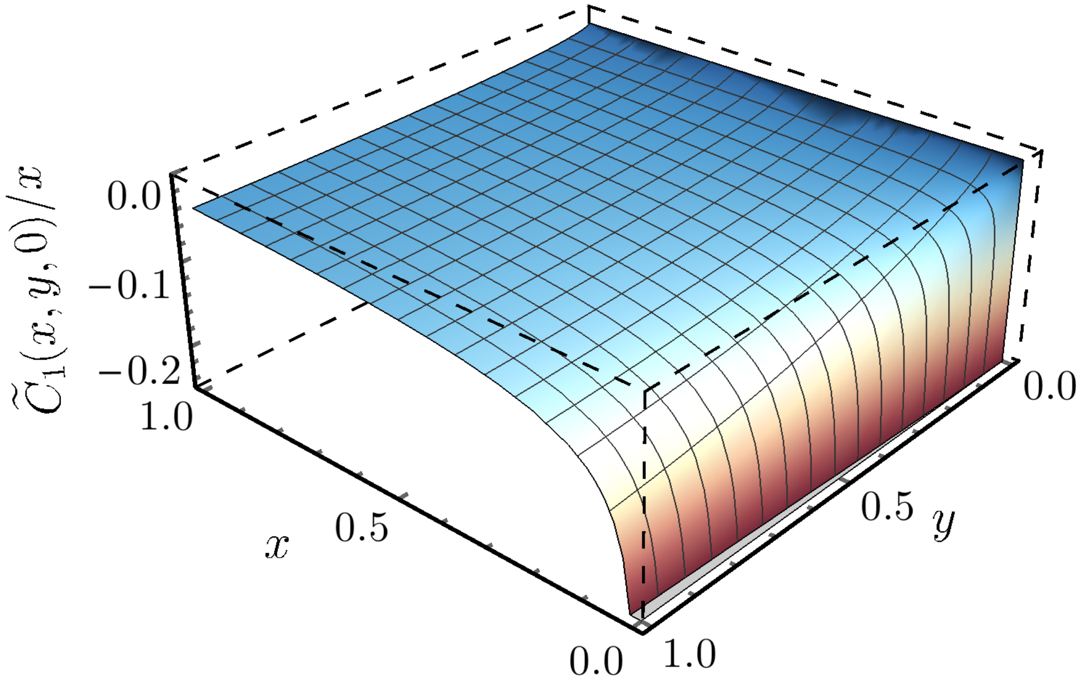
$$\widetilde{C}'_1(x) = -\frac{\Delta^{-1}(0)}{\int_0^\infty \mathrm{d}y\, S'(y)} S'(x)$$



- **Running mass**
  power law behaviour

$$m^2(q^2) = m^2(0)/[1 + (q^2/m_1^2)^{1+p}]$$

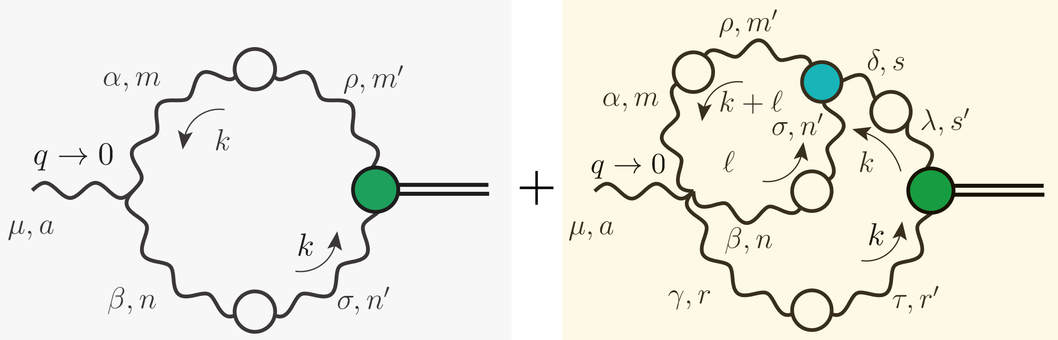




Coupling the gluon SDE

- **Gluon SDE at q=0**
  yields quadratic equation in the coupling
  DB, Papavassiliou, PRD (2018)

$m^2(0) =$



$$-C = B\alpha_s + A\alpha_s^2$$

- **Consistency condition**
  for a given MOM subtraction point μ

$$\alpha_s^{\rm SDE} = \frac{-B + \sqrt{B^2 - 4AC}}{2A} = \alpha_s^{\rm BSE}$$

- **Vertex form factor crucial**
  f=1 implies $\alpha_s^{\rm SDE} = 0.42$ and $\alpha_s^{\rm BSE} = 0.27$

- **Lattice data fit**

$$f \sim \lambda \left[1 + \underbrace{b \ln \frac{q^2 + \mathcal{M}^2}{\mu^2 + \mathcal{M}^2}}_{\text{gluon loops}} + \underbrace{c \ln \frac{q^2}{\mu^2}}_{\text{ghost loops}} + \cdots \right]$$

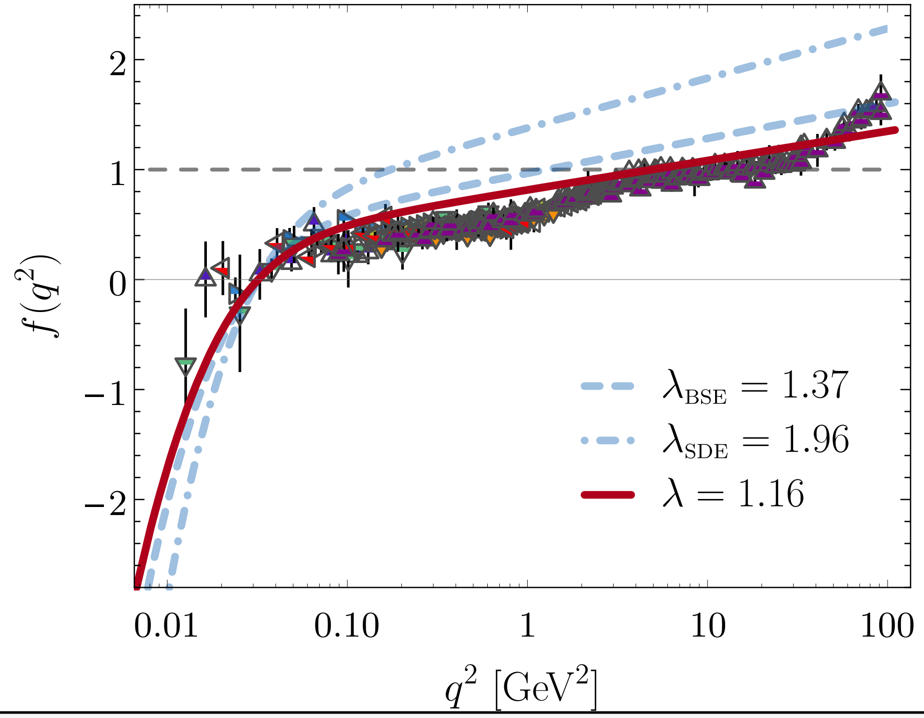
- **Two-loop dressed diagrams fundamental**
  one-loop dressed alone requires negative coupling

$$A = \frac{3C_A^2}{32\pi^3} F(0) \int_0^\infty \mathrm{d}y\, y^2 \Delta^2(y) Y(y) S'(y)$$

$$B = -\frac{3C_A}{8\pi} F(0) \int_0^\infty \mathrm{d}y\, y^2 \Delta^2(y) S'(y)$$

$$C = -\int_0^\infty \mathrm{d}y\, S'(y)$$

- **Start with** $\lambda_0 = 1$
  evaluate $A_0,\ B_0,\ C_0$ and α_0
- **Rescale** f
  track rescaling through BSE/SDE
  $C_0\lambda^3 + \alpha_0 B_0 \lambda + \alpha_0^2 A_0 = 0$
- **Solved @** $\mu = 4.3$ GeV **by**
  $\lambda = 1.16 \quad \alpha_s^{\rm BSE} = \alpha_s^{\rm SDE} = 0.45$
- **Expected value:** $\alpha_s = 0.32$
  obtained if
  $\lambda_{\rm BSE} = 1.36,\ \lambda_{\rm SDE} = 1.97$



Renormalization point dependence

- **Consistency condition**
  depends on the MOM subtraction point
- **Changes in the subtraction point**
  amounts to finite renormalizations

$$\Delta(q^2,\overline{\mu}^2) = z_A(\overline{\mu}^2,\mu^2)\Delta(q^2,\mu^2)$$

$$F(q^2,\overline{\mu}^2) = z_c(\overline{\mu}^2,\mu^2)F(q^2,\mu^2)$$

$$f(q^2,\overline{\mu}^2) = z_3(\overline{\mu}^2,\mu^2)f(q^2,\mu^2)$$

- **BSE formally RG invariant:**

$$\alpha^{\mathrm{BSE}}(\overline{\mu}^2) = z_A^{-3}z_3^{-2}\alpha_s$$

- **Consistency condition:**

$$Cz_3^3 + z_3\left(z_A^{-1}z_c\right)\alpha_s B + \left(z_A^{-2}z_c\right)\alpha_s^2 A = 0$$

- **Gluon and ghost renormalization**
  can be obtained from lattice data

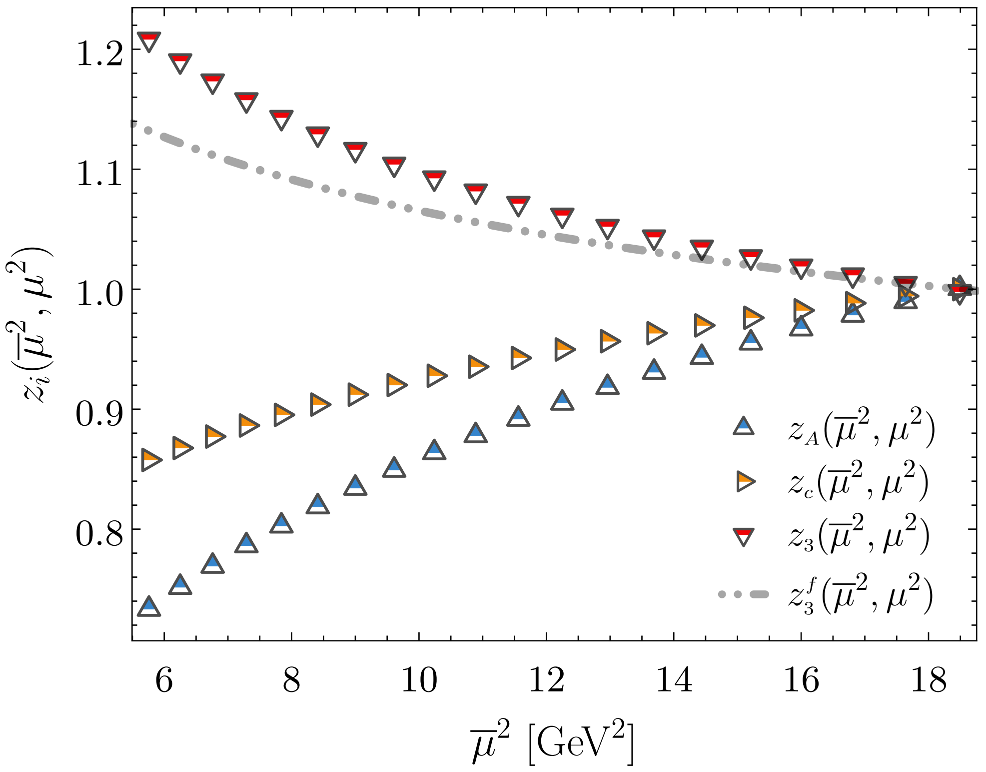
$$z_A(\overline{\mu}^2,\mu^2) = \frac{1}{\overline{\mu}^2\Delta(\overline{\mu}^2,\mu^2)}$$

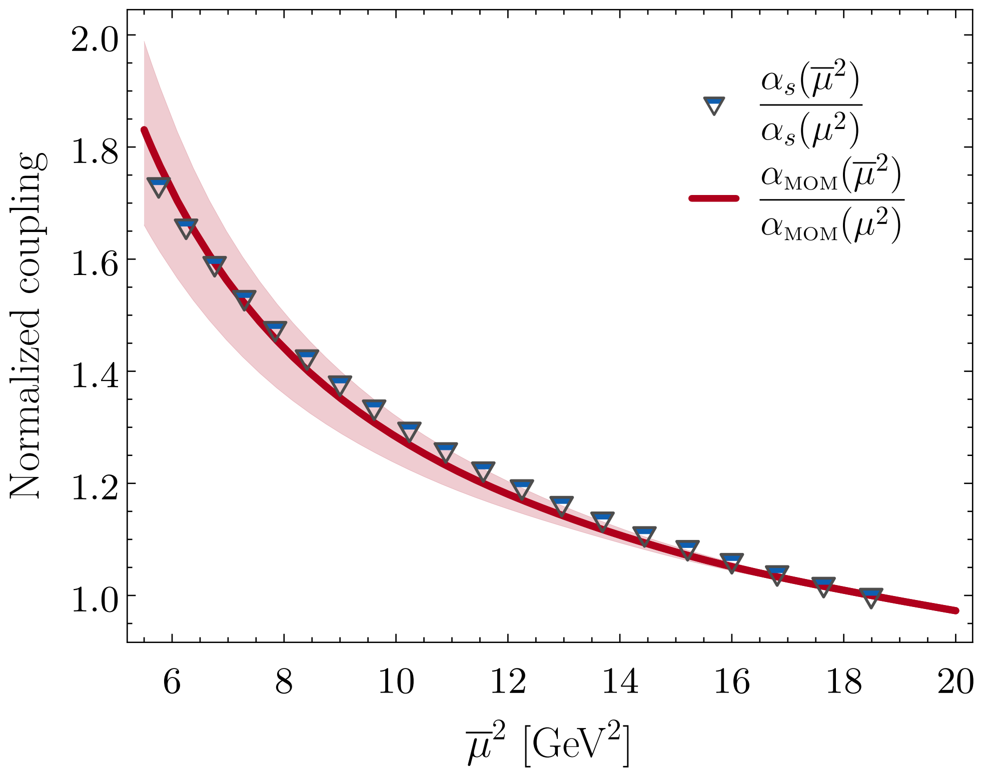
$$z_c(\overline{\mu}^2,\mu^2) = \frac{1}{F(\overline{\mu}^2,\mu^2)}$$

- **Solve cubic equation for z_3**
  values found will enforce identity

$$\alpha_s^{\mathrm{BSE}}(\overline{\mu}^2) = \alpha_s^{\mathrm{SDE}}(\overline{\mu}^2)$$

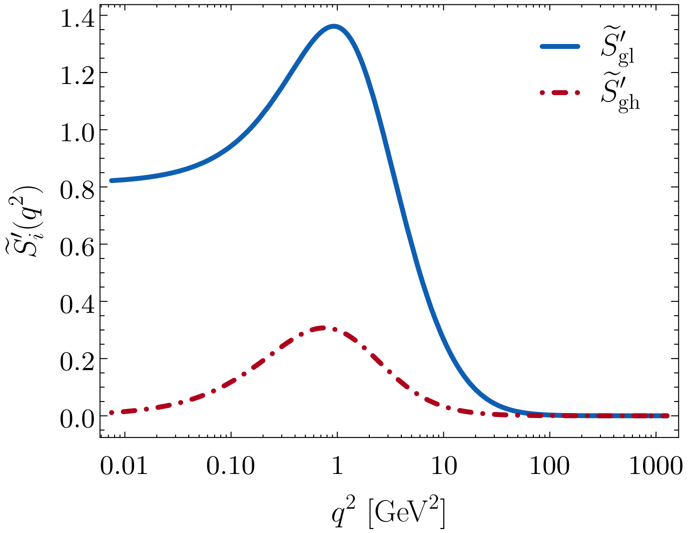
- **Results compare favourably**
  with expected MOM results
  Boucaud, de Soto, Leroy, Le Yaouanc, Micheli, Moutarde, Pene, Rodriguez-Quintero PRD 74 (2006)

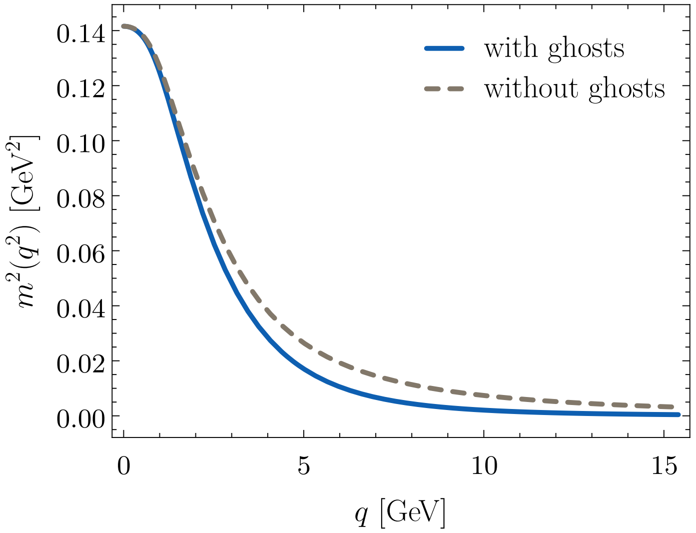




Summary

- **Schwinger mechanism**
  successfully generates gluon mass scale
  - **Massless poles: indispensable ingredients**
    yield running mass with expected features
  - **Only poles survive within a BSE/SDE system**
    stronger/milder divergences lead to vanishing/divergent propagator at zero
- **Two-loop dressed terms crucial**
  reverse one-loop dressed ‘wrong’ sign
  - **4-gluon vertex double role**
    enforces seagull cancellation; enables mass dynamical generation
  - **Tree-level 4-gluon suffices**
    non-perturbative 4-gluon effects suppressed
- **3-gluon vertex needs to be dressed**
  required for consistency between BSE/SDE
  - **Zero crossing / suppression**
    important in glueballs/hybrids?
- **Ghost terms**
  almost irrelevant at the quantitative level
  Aguilar, DB, Figueiredo, Papavassiliou, EPJC 78 (2018)





QCD Effective charge

- **Remarkable feature of QCD:**
  $\widehat{d}(k^2)$ saturates in the IR
  DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)

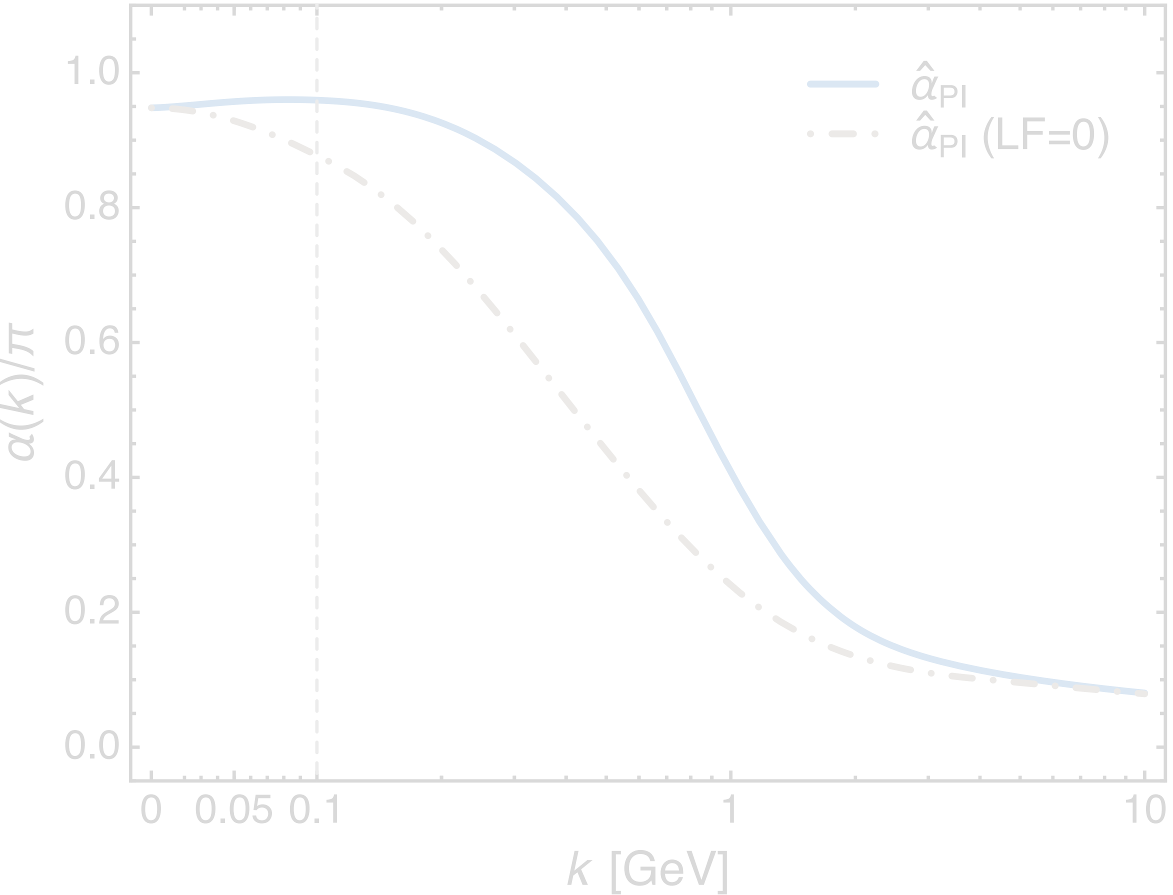
$$\widehat{d}(0) = \frac{\alpha_0}{m_0^2} \approx \frac{0.9\pi}{(m_P/2)^2}$$

- **Define the RG invariant function**

$$\mathcal{D}(k^2) = \frac{\Delta(k^2;\mu^2)}{\Delta(0;\mu^2)m_0^2}$$

- **Extract (process independent) coupling**
  using the quark gap equation
  DB, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero, 1612.04835

$$\widehat{\alpha}(k^2) = \frac{\widehat{d}(k^2)}{\mathcal{D}(k^2)} \xrightarrow[k^2 \gg m_0^2]{} \mathcal{I}(k^2)$$



- **Parameter free**
  completely determined from 2-point sector
- **No Landau pole**
  physical coupling showing an IR fixed point
- **Smoothly connects IR and UV domains**
  no need for matching procedures
- **Essentially non-perturbative result**
  continuum/lattice results plus setting of single mass cale
- **Ghost gluon dynamics critical**
  produces enhancement at intermediate momenta

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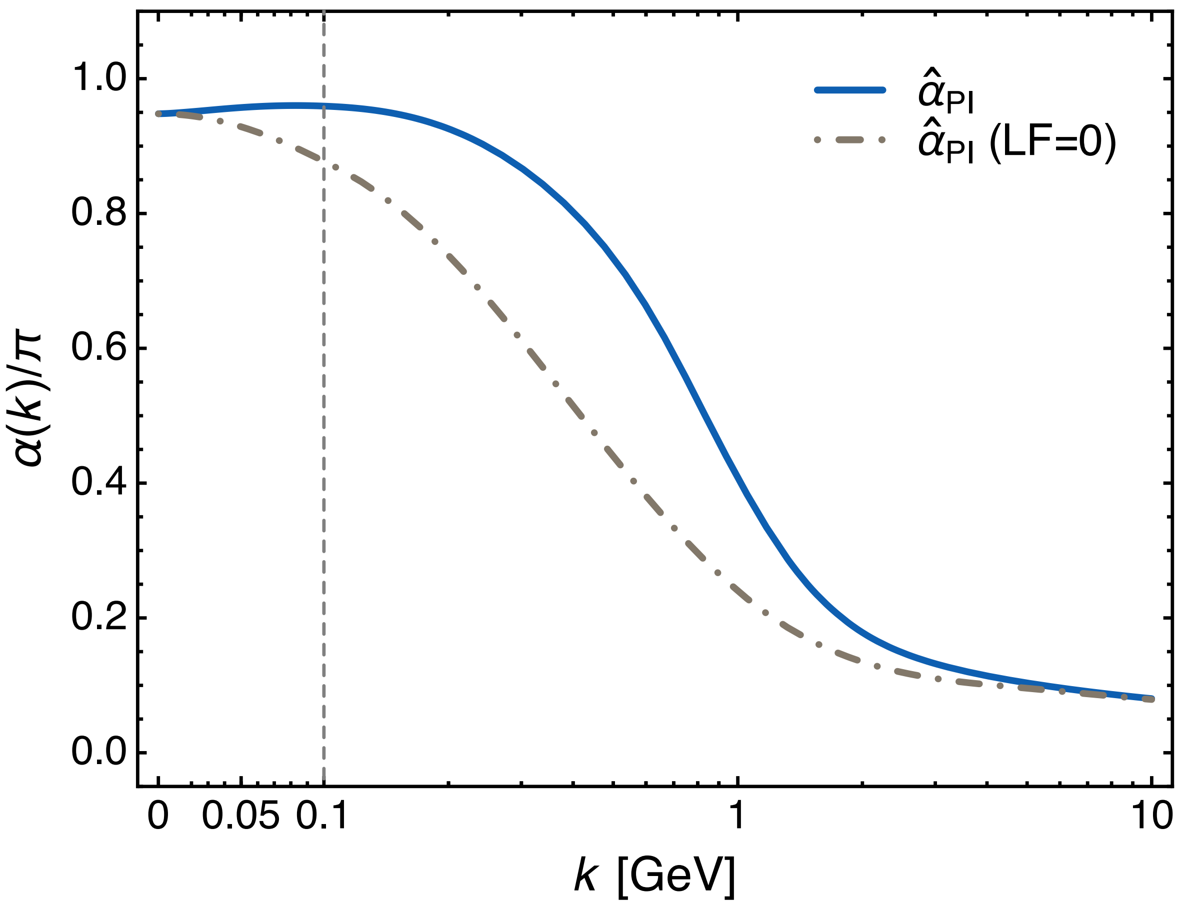

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QCD Effective charge

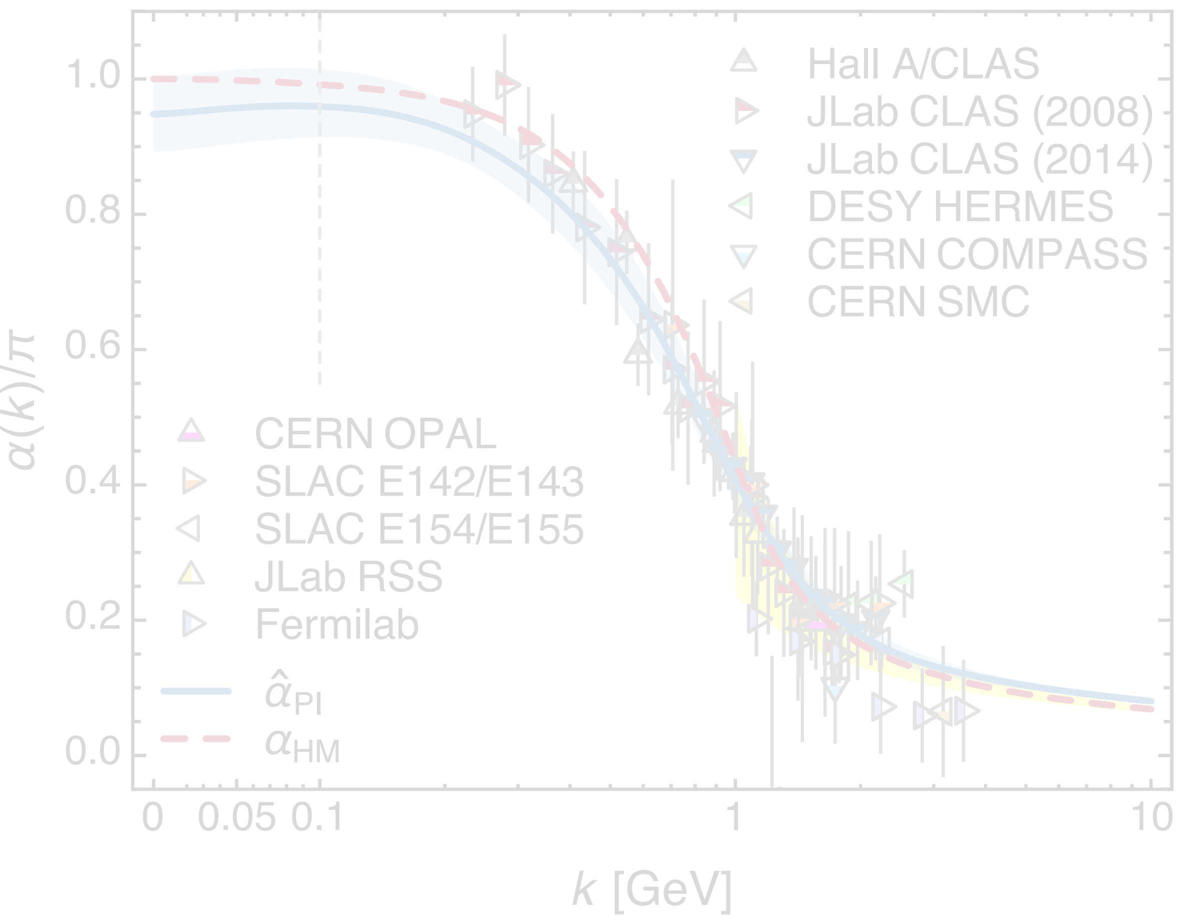
- **Process dependent effective charges**
  fixed by the leading-order term in the expansion of a given observable
  Grunberg, PRD 29 (1984)

- **Bjorken sum rule**
  defines such a charge
  Bjorken, PR 148 (1966); PRD 1 (1970)

$$\int_0^1 \mathrm{d}x\,[g_1^p(x,k^2) - g_1^n(x,k^2)] = \frac{g_A}{6}[1-\alpha_{g_1}(k^2)/\pi]$$

  - $g_1^{p,n}$ **spin dependent p/n structure functions**
    extracted from measurements using unpolarized targets
  - g^A **nucleon flavour-singlet axial charge**

- **Many merits**
  - **Existence of data**
    for a wide momentum range
  - **Tight sum rules constraints on the integral**
    at IR and UV extremes
  - **Isospin non-singlet**
    suppress contributions from hard-to-compute processes



- **Equivalence in the perturbative domain**
  reasonable definitions of the charge

$$\alpha_{g_1}(k^2) = \alpha_{\overline{\mathrm{MS}}}(k^2)[1+1.14\alpha_{\overline{\mathrm{MS}}}(k^2)+\cdots]$$
$$\widehat{\alpha}_{PI}(k^2) = \alpha_{\overline{\mathrm{MS}}}(k^2)[1+1.09\alpha_{\overline{\mathrm{MS}}}(k^2)+\cdots]$$

- **Equivalence in the non-perturbative domain**
  highly non-trivial (ghost-gluon interactions)

- **Agreement with light-front holography**
  model for α_{g_1}
  Deur, Brodsky, de Teramond, PPNP 90 (2016)

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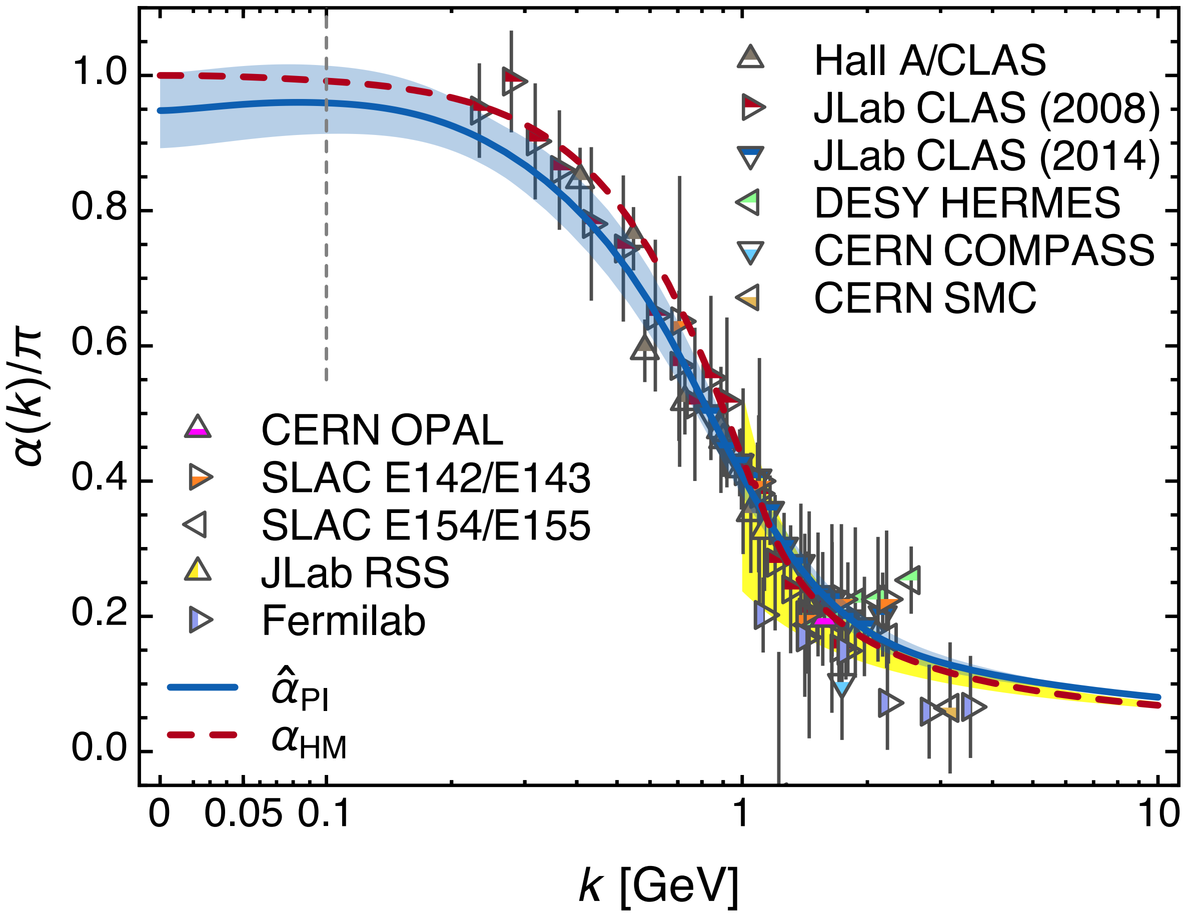
- **Bjorken sum rule**
  defines such a charge
  Bjorken, PR 148 (1966); PRD 1 (1970)

$$\int_0^1 \mathrm{d}x\,[g_1^p(x,k^2) - g_1^n(x,k^2)] = \frac{g_A}{6}[1 - \alpha_{g_1}(k^2)/\pi]$$

  - $g_1^{p,n}$ **spin dependent p/n structure functions** extracted from measurements using unpolarized targets
  - g^A **nucleon flavour-singlet axial charge**

- **Many merits**
  - **Existence of data**
    for a wide momentum range
  - **Tight sum rules constraints on the integral**
    at IR and UV extremes
  - **Isospin non-singlet**
    suppress contributions from hard-to-compute processes



- **Equivalence in the perturbative domain**
  reasonable definitions of the charge

$$\alpha_{g_1}(k^2) = \alpha_{\overline{\mathrm{MS}}}(k^2)[1 + 1.14\alpha_{\overline{\mathrm{MS}}}(k^2) + \cdots]$$
$$\widehat{\alpha}_{PI}(k^2) = \alpha_{\overline{\mathrm{MS}}}(k^2)[1 + 1.09\alpha_{\overline{\mathrm{MS}}}(k^2) + \cdots]$$

- **Equivalence in the non-perturbative domain**
  highly non-trivial (ghost-gluon interactions)

- **Agreement with light-front holography**
  model for α_{g_1}
  Deur, Brodsky, de Teramond, PPNP 90 (2016)