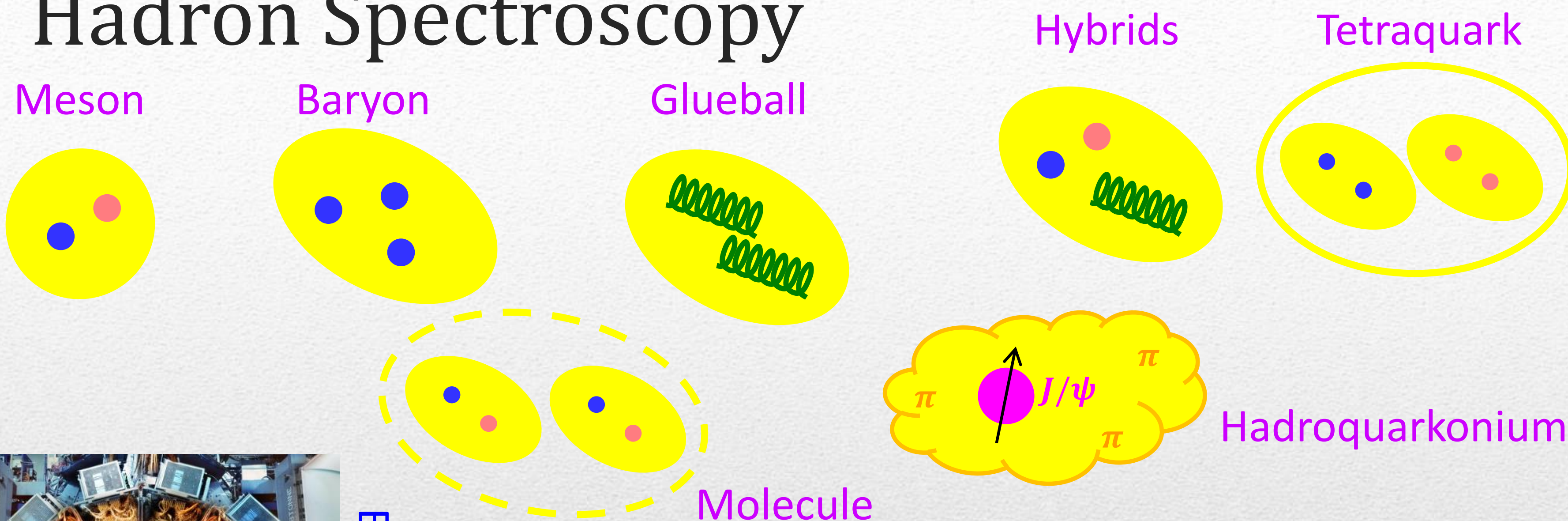
Amplitude analysis for exotic states

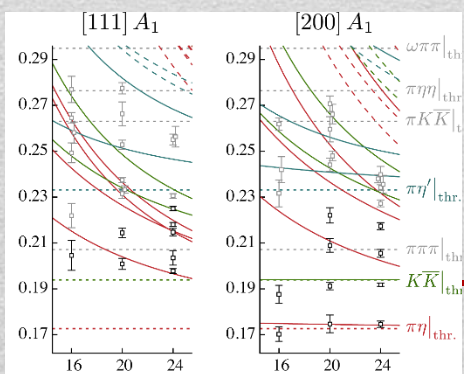
Alessandro Pilloni

GGI, Florence, March 15th, 2018

Hadron Spectroscopy



Experiment



Lattice QCD

Data → ? → Fundamental properties, Model building

Interpretations on the spectrum leads to understanding fundamental laws of nature

Hadron Spectroscopy

$\rho(770)$ $I^G(J^{PC}) = 1^+(1^{--})$

Review: The $\rho(770)$

$\rho(770)$ **MASS**

NEUTRAL ONLY, e^+e^-	775.26 ± 0.25 MeV
CHARGED ONLY, τ DECAYS and e^+e^-	775.11 ± 0.34 MeV
MIXED CHARGES, OTHER REACTIONS	763.0 ± 1.2 MeV
Mass m	
CHARGED ONLY, HADROPRODUCED	766.5 ± 1.1 MeV
NEUTRAL ONLY, PHOTOPRODUCED	769.0 ± 1.0 MeV
NEUTRAL ONLY, OTHER REACTIONS	769.0 ± 0.9 MeV (S = 1.4)
$m_{\rho(770)^0} - m_{\rho(770)^\pm}$	-0.7 ± 0.8 MeV (S = 1.5)
$m_{\rho(770)^+} - m_{\rho(770)^-}$	
$\rho(770)$ RANGE PARAMETER	$5.3^{+0.9}_{-0.7}$ GeV^{-1}

$\rho(770)$ **WIDTH**

NEUTRAL ONLY, e^+e^-	147.8 ± 0.9 MeV (S = 2.0)
CHARGED ONLY, τ DECAYS and e^+e^-	149.1 ± 0.8 MeV
MIXED CHARGES, OTHER REACTIONS	149.5 ± 1.3 MeV
CHARGED ONLY, HADROPRODUCED	150.2 ± 2.4 MeV
NEUTRAL ONLY, PHOTOPRODUCED	151.7 ± 2.6 MeV
NEUTRAL ONLY, OTHER REACTIONS	150.9 ± 1.7 MeV (S = 1.1)
$\Gamma_{\rho(770)^0} - \Gamma_{\rho(770)^\pm}$	0.3 ± 1.3 (S = 1.4)
$\Gamma_{\rho(770)^+} - \Gamma_{\rho(770)^-}$	1.8 ± 2.1

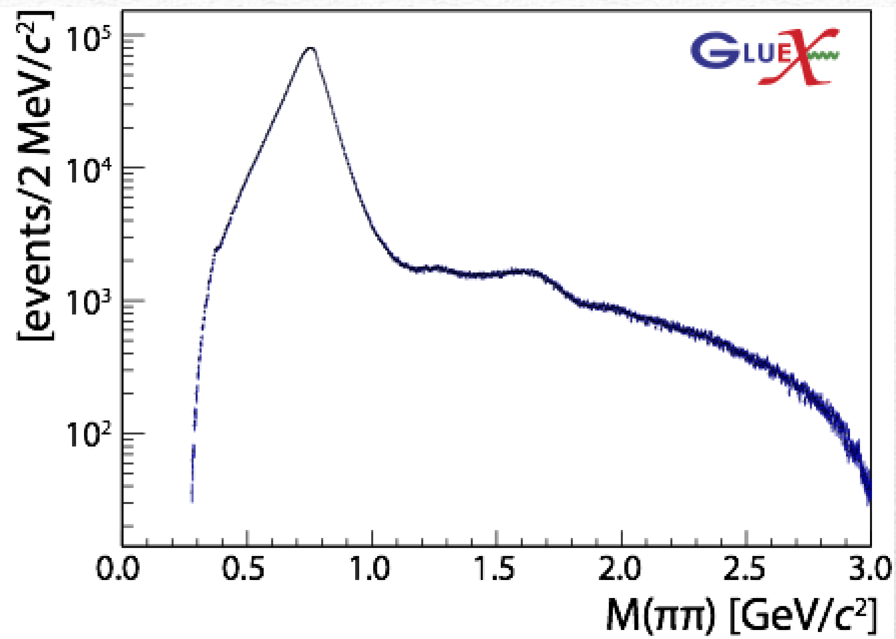
Hadron Spectroscopy

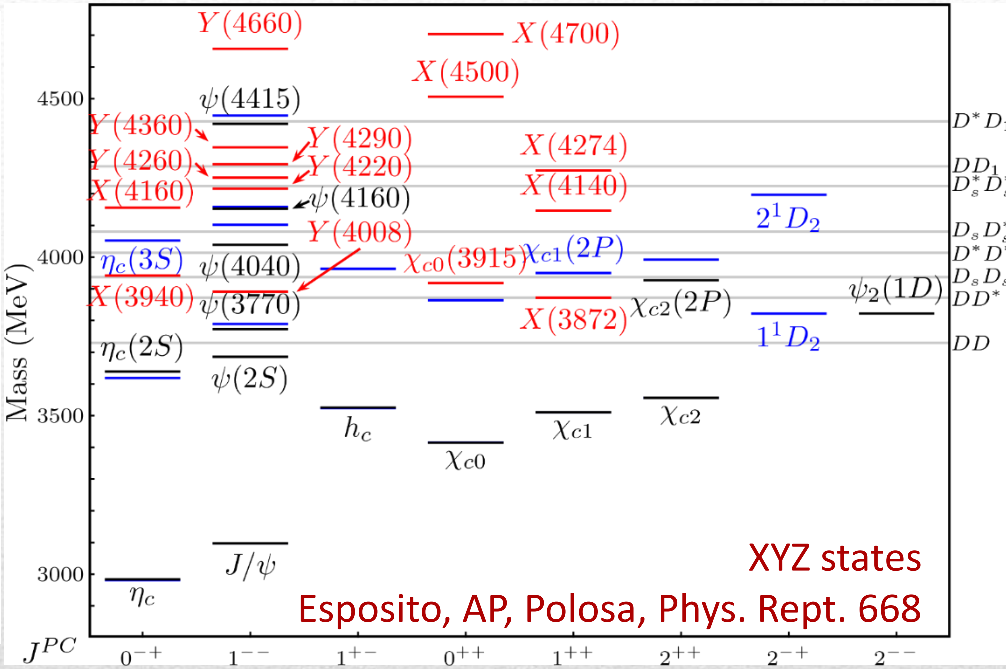
$a_1(1260)$ WIDTH

INSPIRE search

VALUE (MeV)	EVTS		DOCUMENT ID		TECN	COMMENT
250 to 600	**OUR ESTIMATE**					
$367 \pm 9\,^{+28}_{-25}$	420k		ALEKSEEV	2010	COMP	$190\ \pi^- \to \pi^-\pi^-\pi^+ Pb'$
• • • We do not use the following data for averages, fits, limits, etc. • • •						
$410 \pm 31 \pm 30$		1	AUBERT	2007AU	BABR	$10.6\ e^+ e^- \to \rho^0 \rho^\pm \pi^\mp \gamma$
520 - 680	6360	2	LINK	2007A	FOCS	$D^0 \to \pi^-\pi^+\pi^-\pi^+$
480 ± 20		3	GOMEZ-DUMM	2004	RVUE	$\tau^+ \to \pi^+\pi^+\pi^-\nu_\tau$
580 ± 41	90k		SALVINI	2004	OBLX	$\bar{p}\,p \to 2\pi^+ 2\pi^-$
460 ± 85	205	4	DRUTSKOY	2002	BELL	$B^{(*)} K^- K^{*0}$
$814 \pm 36 \pm 13$	37k	5	ASNER	2000	CLE2	$10.6\ e^+ e^- \to \tau^+\tau^-,\ \tau^- \to \pi^-\pi^0\pi^0\nu_\tau$

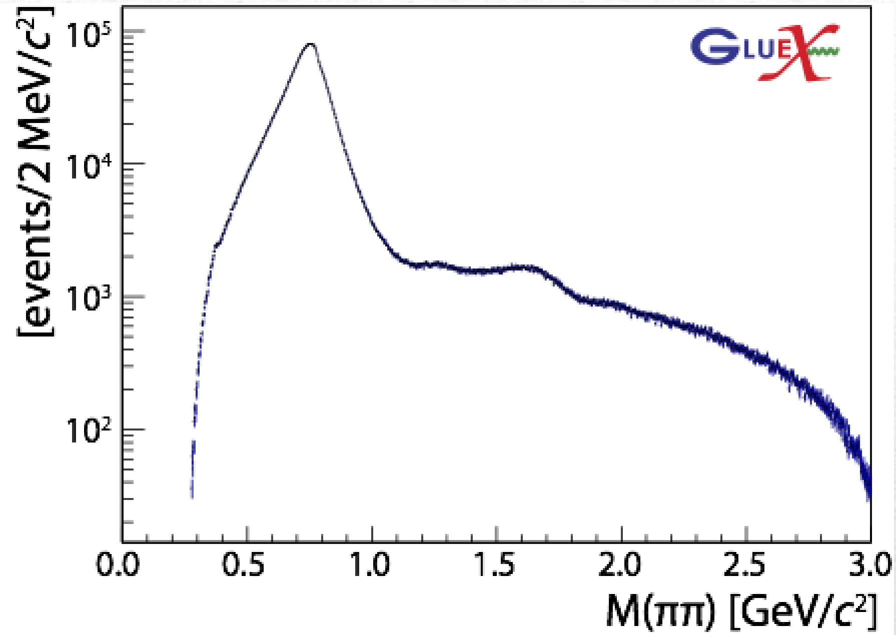
Hadron Spectroscopy

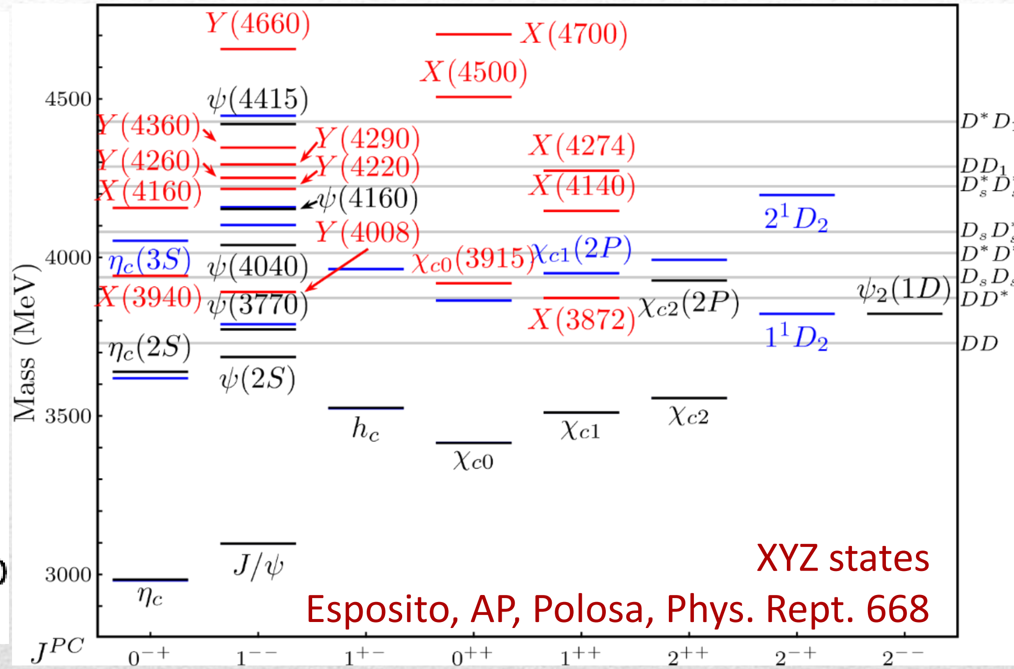




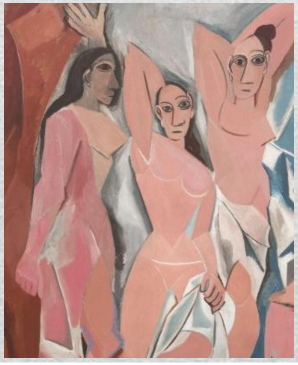
Data → Amplitude analysis → Fundamental properties, Model building

Hadron Spectroscopy





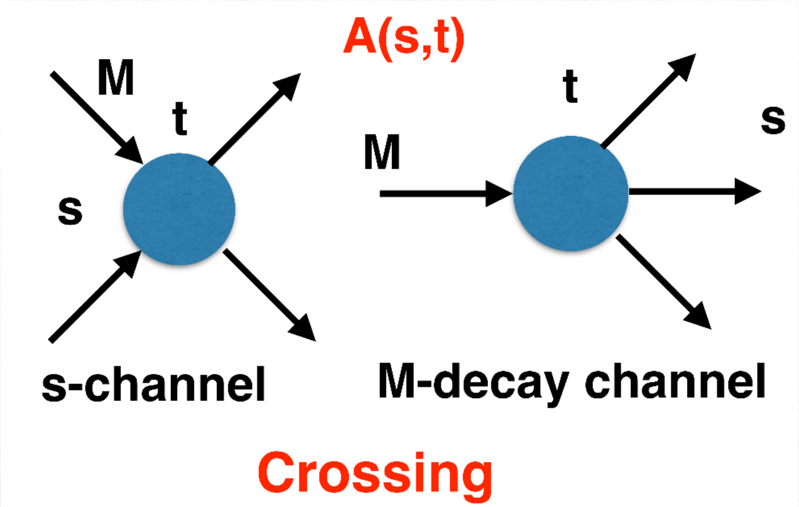
Data

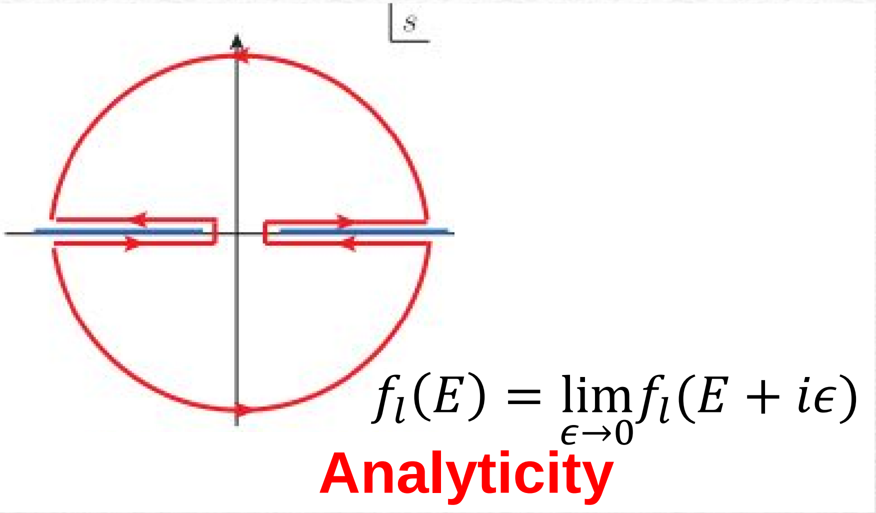


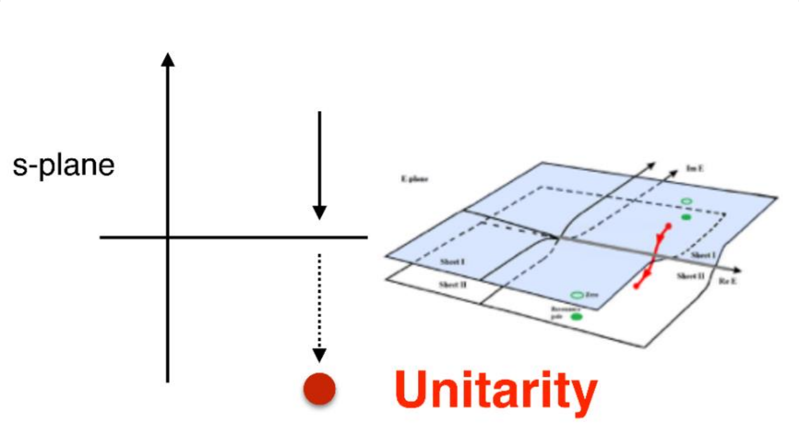
Fundamental properties, Model building

Improvement needed! With great statistics comes great responsibility!

S-Matrix principles





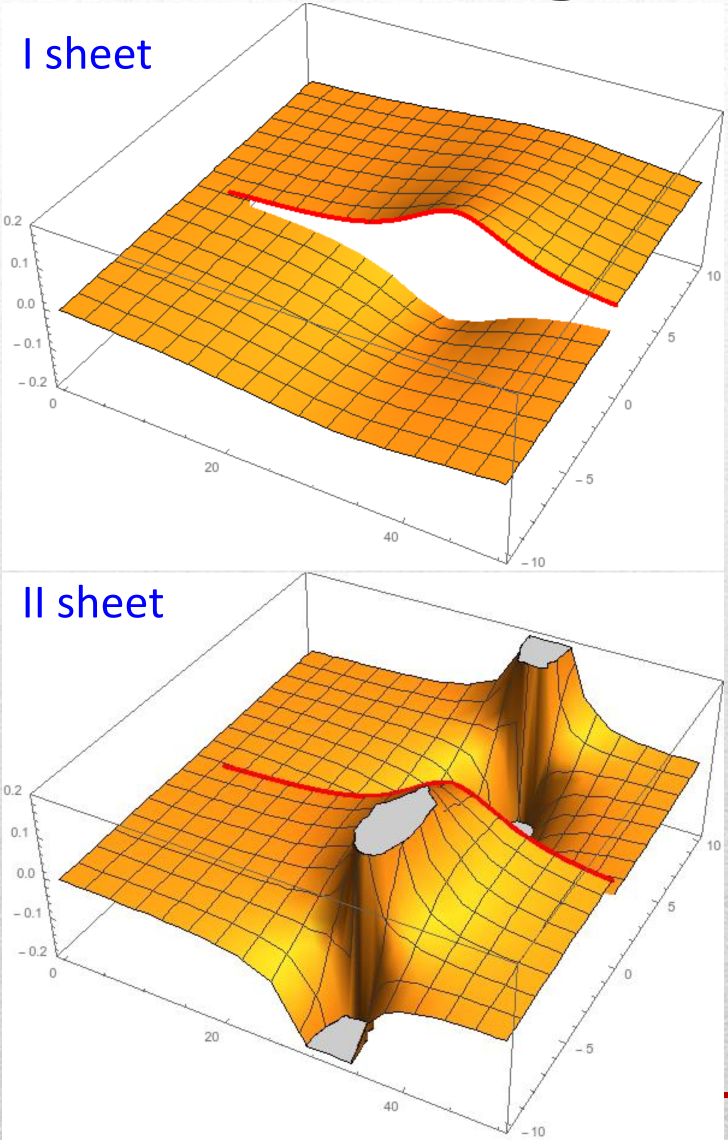


+ Lorentz, discrete & global symmetries

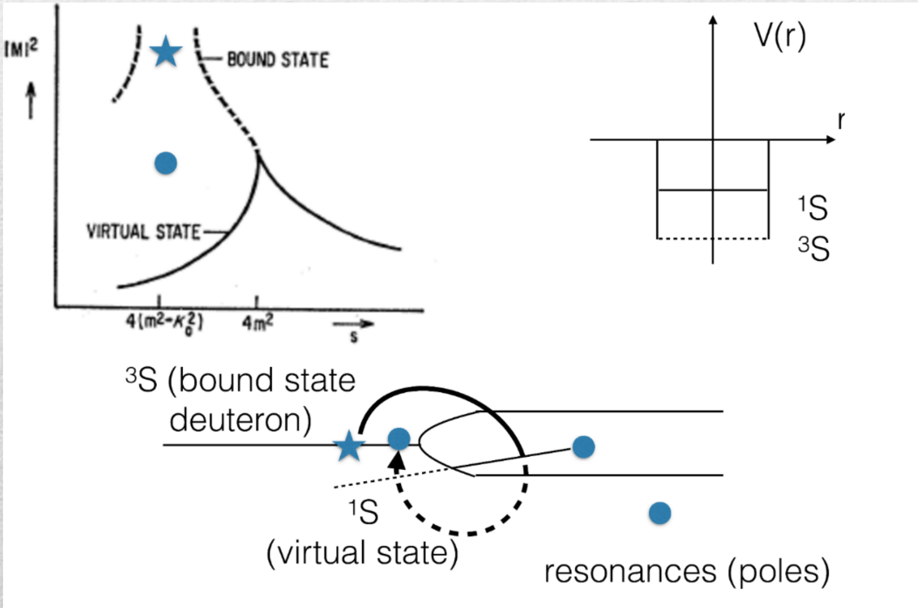
These are constraints the amplitudes have to satisfy, but do not fix the dynamics

Resonances (QCD states) are poles in the unphysical Riemann sheets

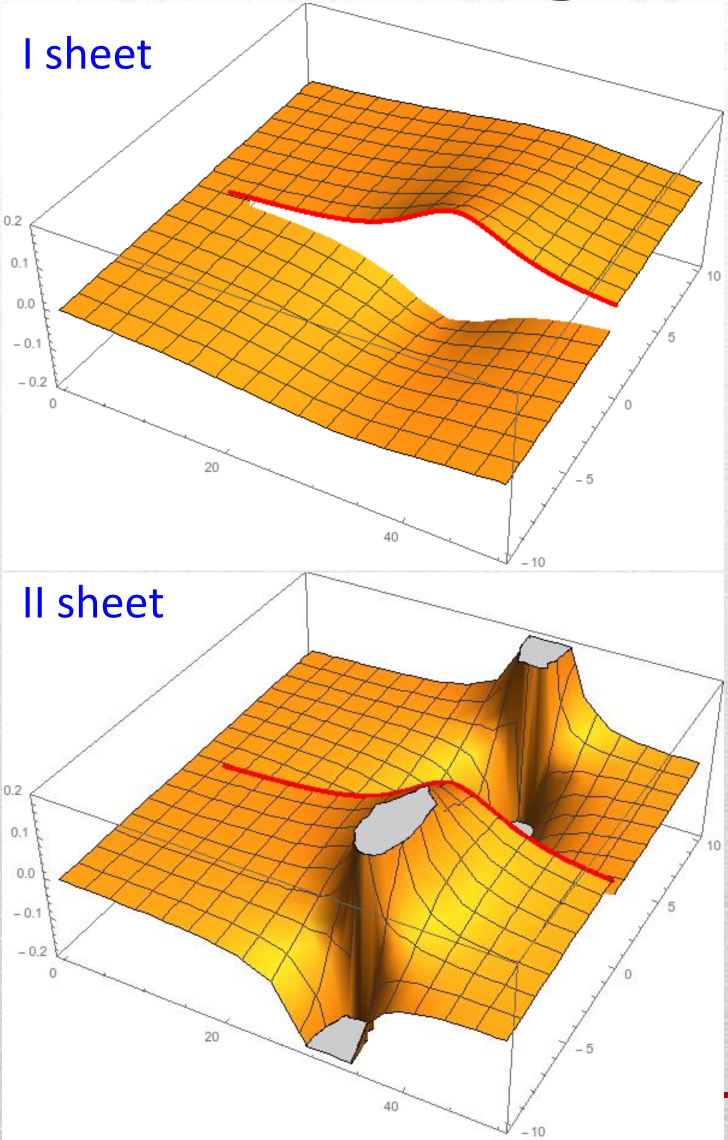
Pole hunting



Bound states on the real axis 1st sheet
Not-so-bound (virtual) states on the real axis 2nd sheet

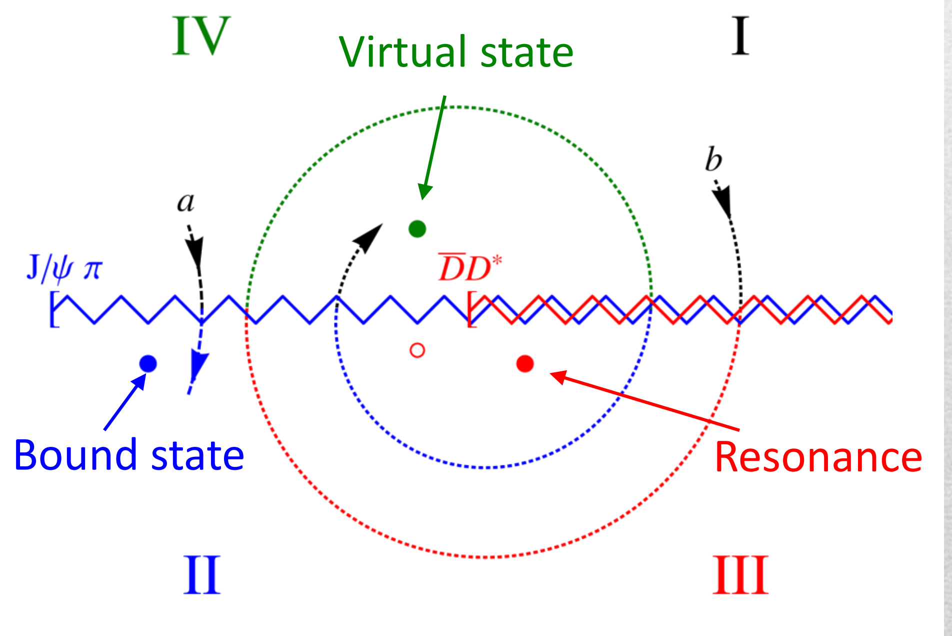


Pole hunting



More complicated structure when more thresholds arise:
two sheets for each new threshold

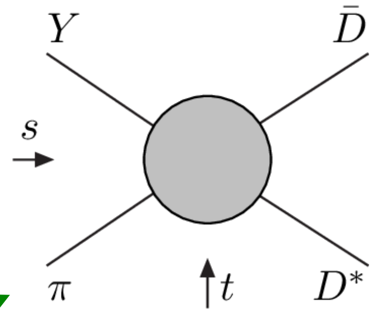
III sheet: usual resonances
IV sheet: cusps (virtual states)



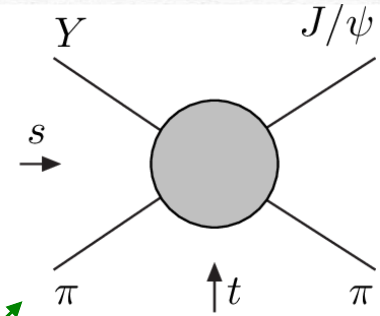
Amplitude analysis for $Z_c(3900)$

One can test different parametrizations of the amplitude, which correspond to
different singularities → different natures

AP *et al.* (JPAC), PLB772, 200



$Z_c(3900)$?



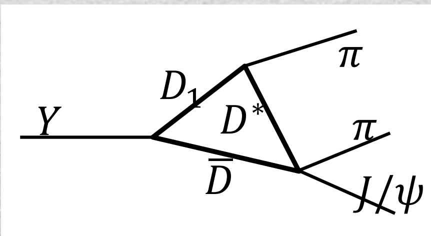
u: $D_0(2400)$

$D_1(2420)$

u: $Z_c(3900)$?

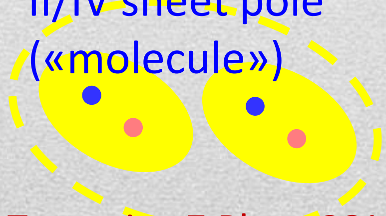
"$\sigma, f_0(980)$"

Triangle rescattering,
logarithmic branching point



Szczepaniak, PLB747, 410

(anti)bound state,
II/IV sheet pole
(«molecule»)



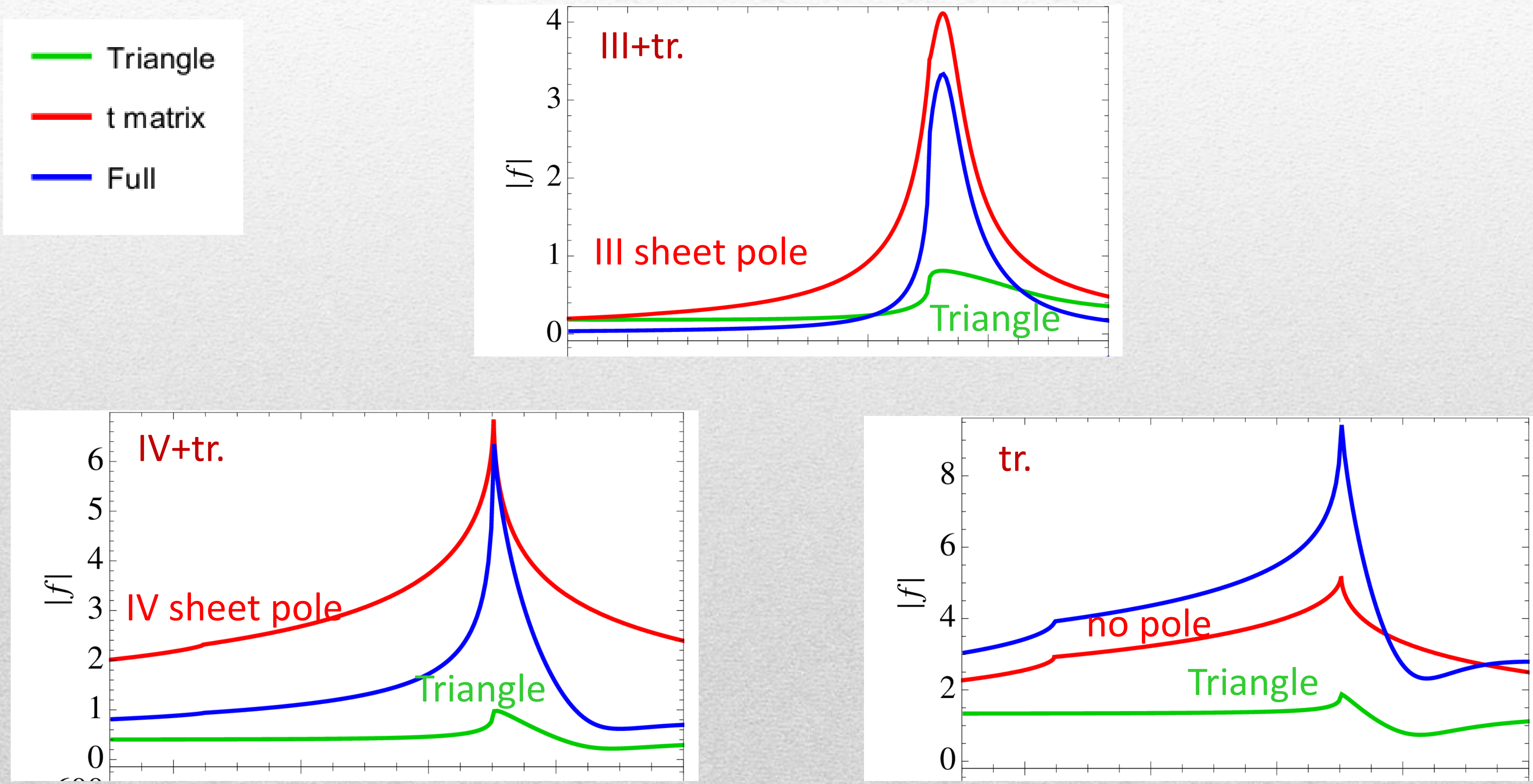
Tornqvist, Z.Phys. C61, 525
Swanson, Phys.Rept. 429
Hanhart *et al.* PRL111, 132003

Resonance,
III sheet pole
(«compact state»)

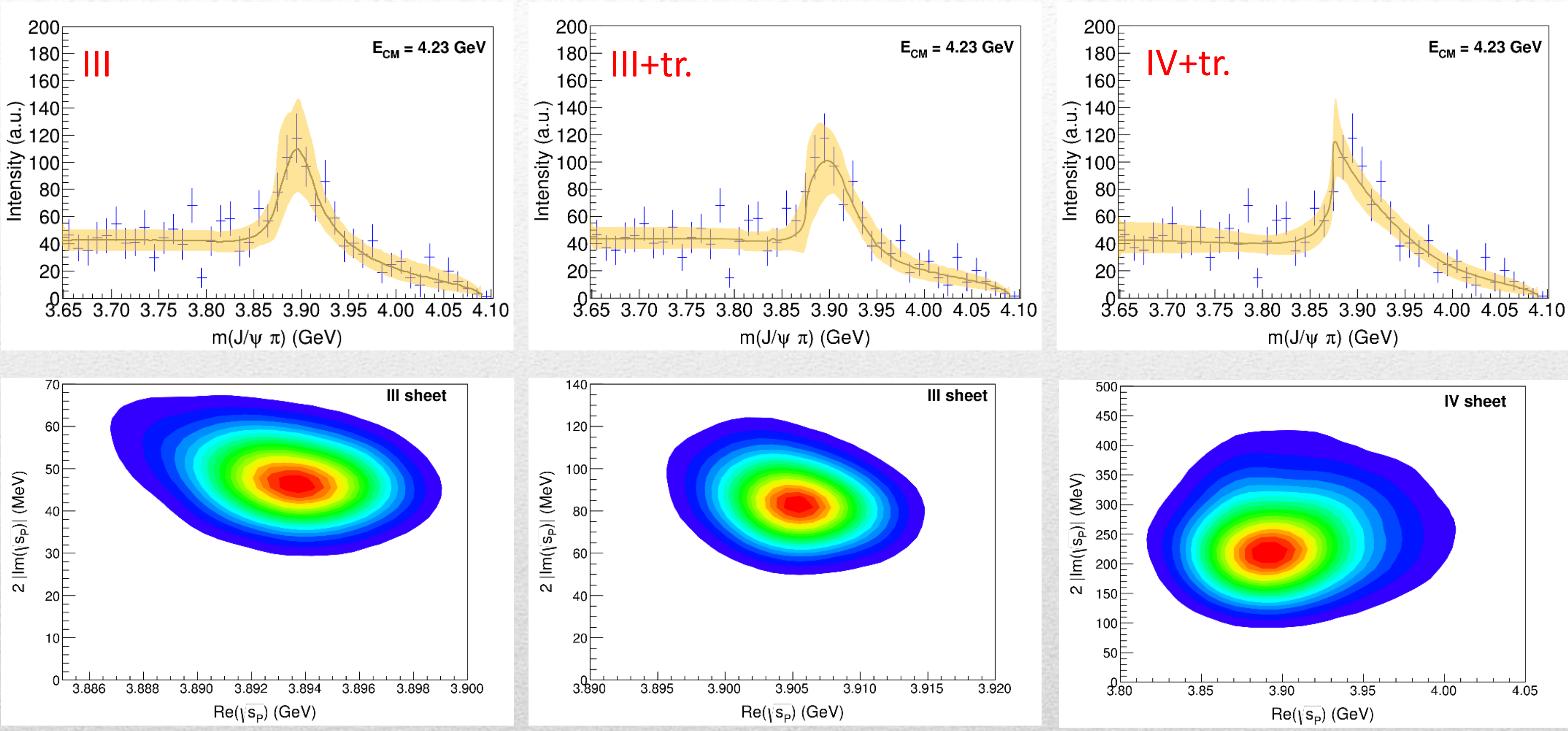
Maiani *et al.*, PRD71, 014028
Faccini *et al.*, PRD87, 111102
Esposito *et al.*, Phys.Rept. 668

Singularities and lineshapes

Different lineshapes according to different singularities



Pole extraction



Scenario	III+tr.	IV+tr.	tr.
III	1.5σ (1.5σ)	1.5σ (2.7σ)	"2.4σ" ("1.4σ")
III+tr.	–	1.5σ (3.1σ)	"2.6σ" ("1.3σ")
IV+tr.	–	–	"2.1σ" ("0.9σ")

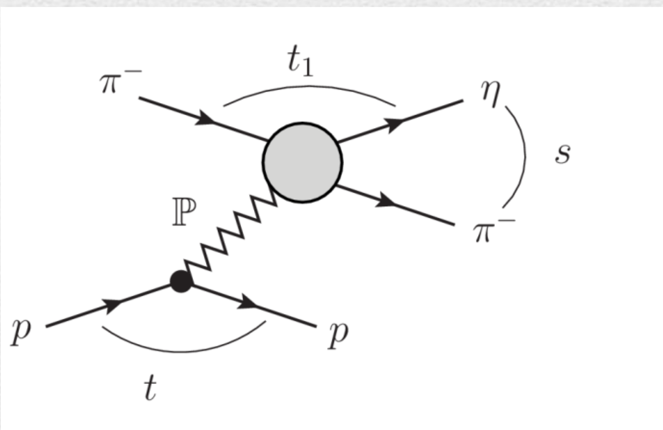
	III	III+tr.	IV+tr.
M (MeV)	$3893.2^{+5.5}_{-7.7}$	3905^{+11}_{-9}	3900^{+140}_{-90}
Γ (MeV)	48^{+19}_{-14}	85^{+45}_{-26}	240^{+230}_{-130}

Not conclusive at this stage

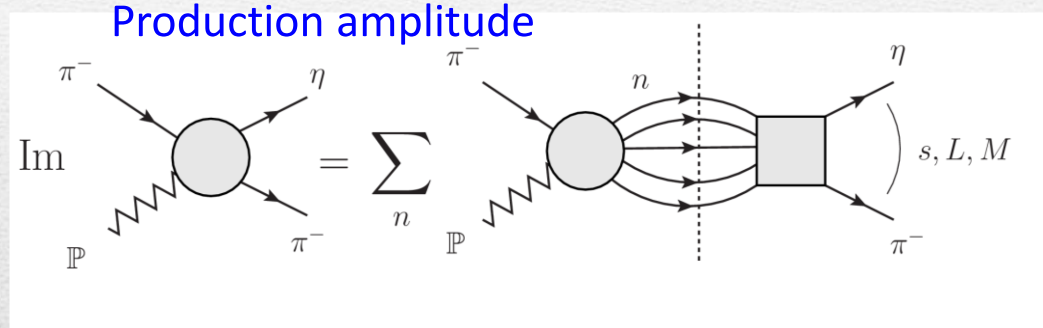
Searching for resonances in $\eta\pi$

- The $\eta\pi$ system is one of the golden modes for hunting hybrid mesons
- We build the partial wave amplitudes according to the N/D method
- We test against the D-wave data, where the a_2 and the a_2' show up

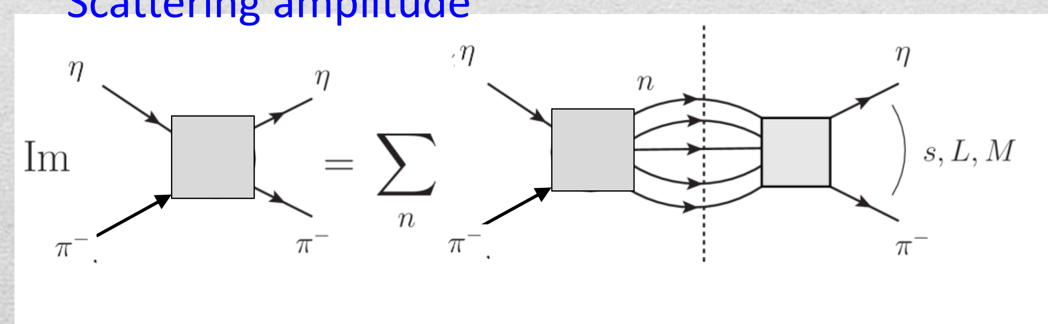
A. Jackura, M. Mikhasenko, AP *et al.* (JPAC & COMPASS), PLB779, 464-472



Production amplitude



Scattering amplitude



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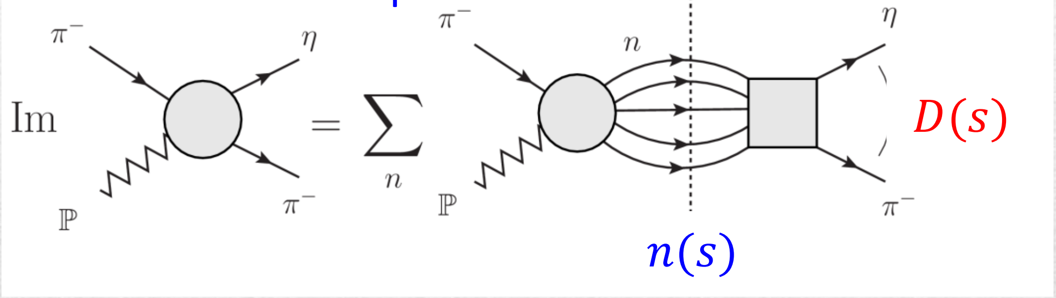
A. Jackura, M. Mikhasenko, AP *et al.* (JPAC & COMPASS), PLB779, 464-472

$$t(s) = \frac{N(s)}{D(s)}$$

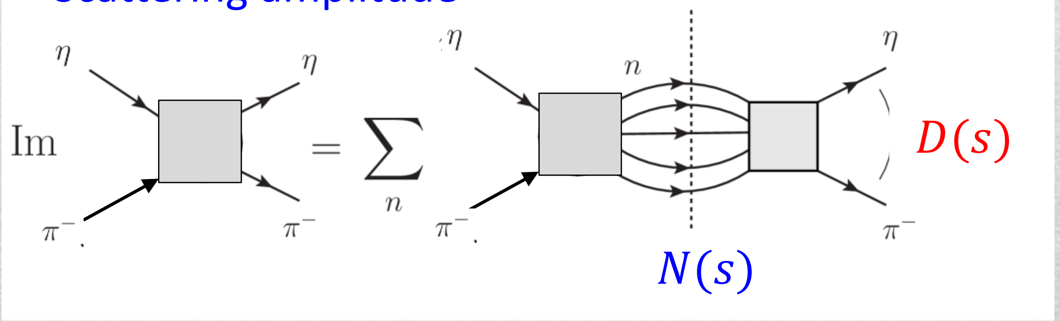
The $D(s)$ has only right hand cuts;
it contains all the Final State Interactions constrained by unitarity → universal

$$\mathrm{Im}\, D(s) = -\rho\, N(s)$$

Production amplitude



Scattering amplitude



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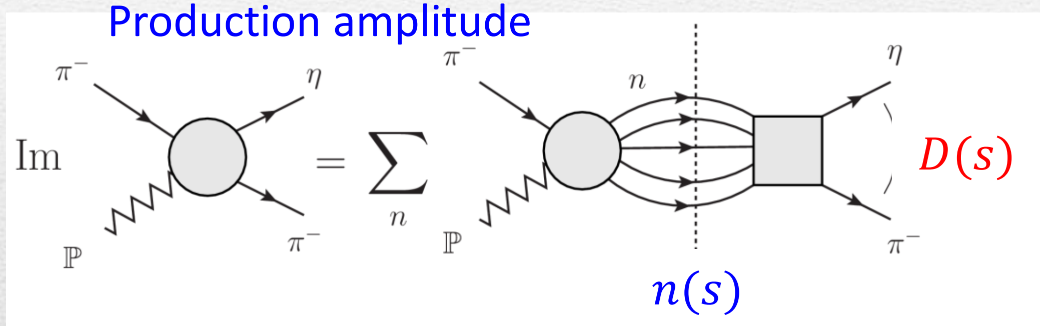

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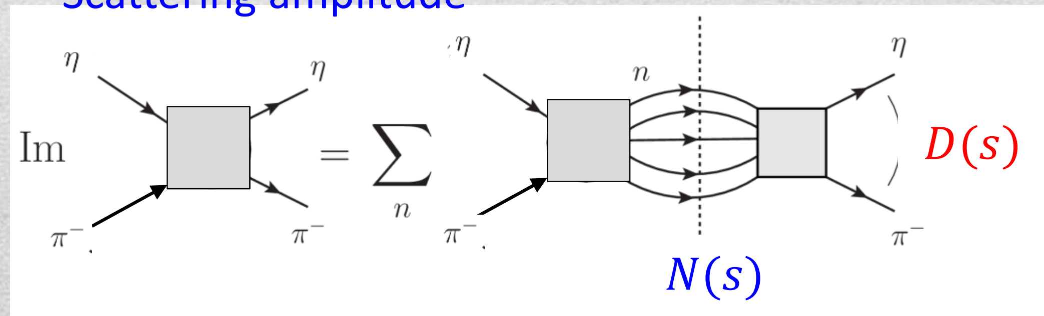


The $n(s), N(s)$ have left hand cuts only, they depend on the exchanges → process-dependent, smooth

Searching for resonances in $\eta\pi$

The denominator $D(s)$ contains all the FSI constrained by unitarity → universal

$$D(s)_{ij} = (K^{-1})_{ij}(s) - \frac{s}{\pi}\int_{s_i}^{\infty} \frac{\rho_i(s')N_{ij}(s')}{s'\left(s'-s\right)}ds'$$

$$K_{ij}(s) = \sum_R \frac{g_i^R g_j^R}{M_R^2 - s} \quad \textbf{OR} \quad K_{ij}^{-1}(s) = c_0 - c_1 s + \sum_i \frac{c_i}{M_i^2 - s}$$

$$\rho_i(s)N_{ij}(s) = g\frac{\lambda^{(2l+1)/2}\left(s, m_\pi^2, m_\eta^2\right)}{(s+\Lambda)^7}$$

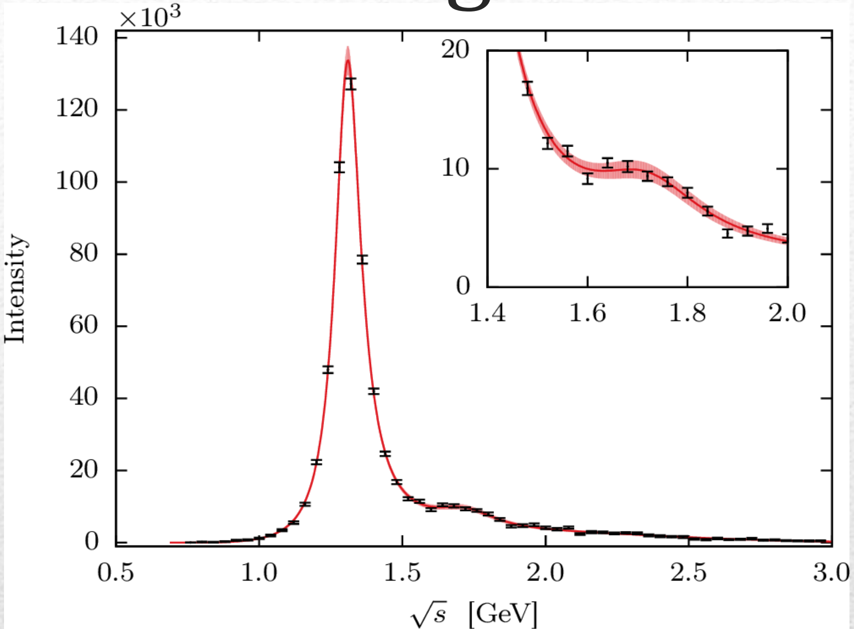
Searching for resonances in $\eta\pi$

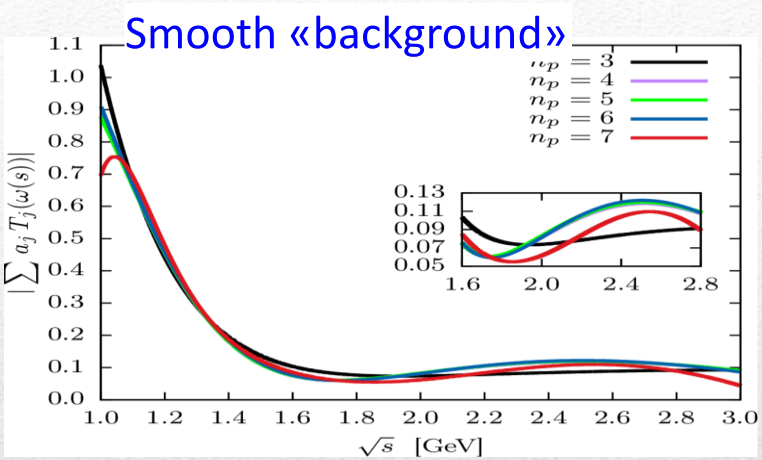
The $n(s)$ is process-dependent, smooth

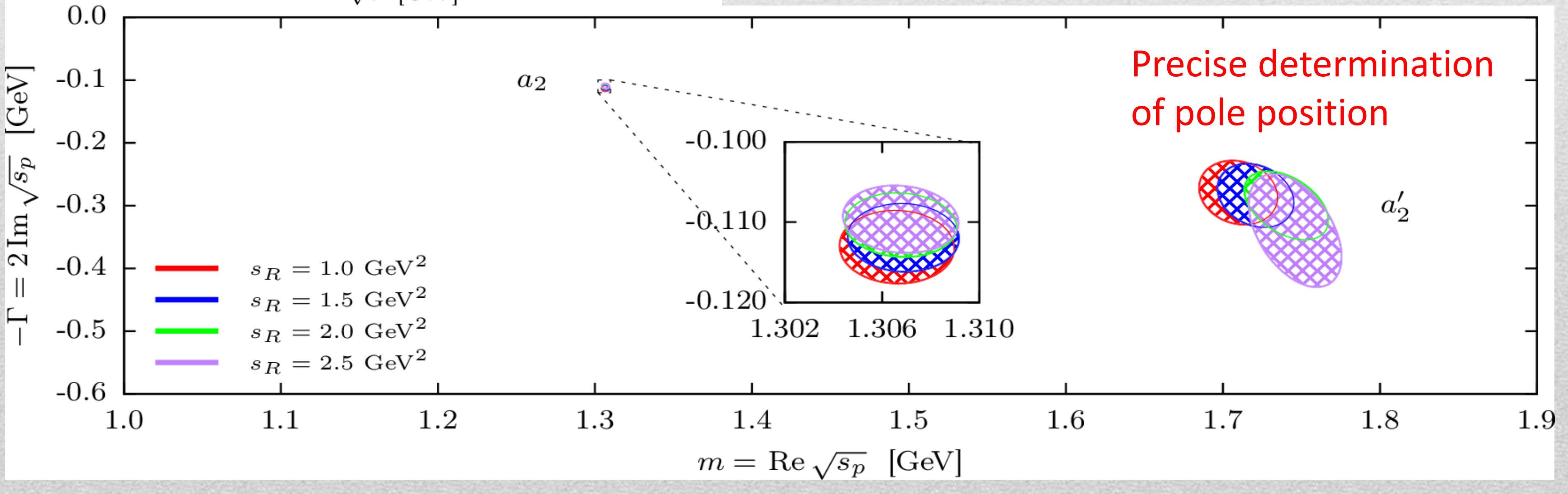
$$n(s) = \sum_j a_j \omega^j(s)$$

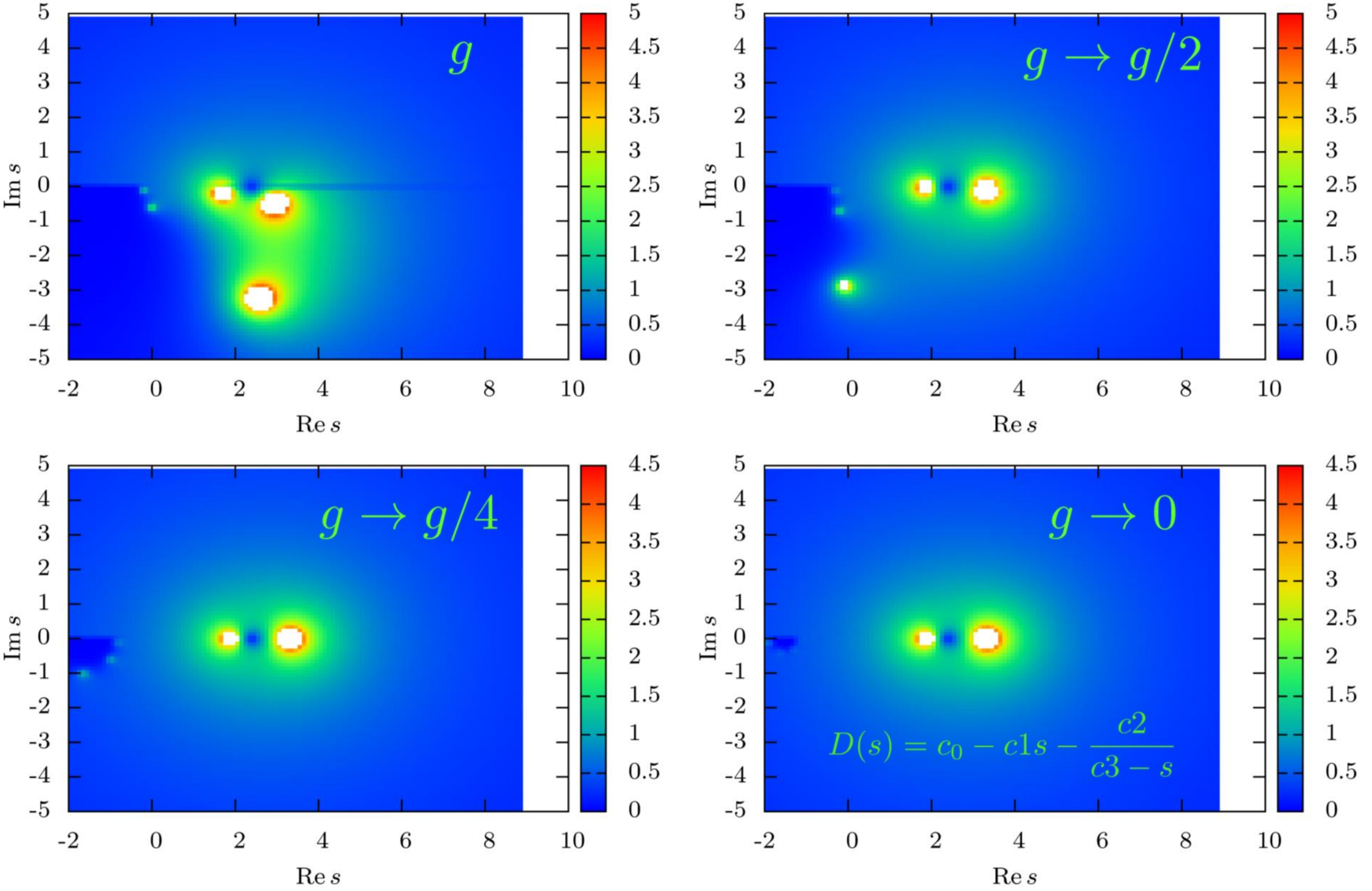
$$\omega(s) = \frac{s}{s + s_0}$$

Searching for resonances in $\eta\pi$





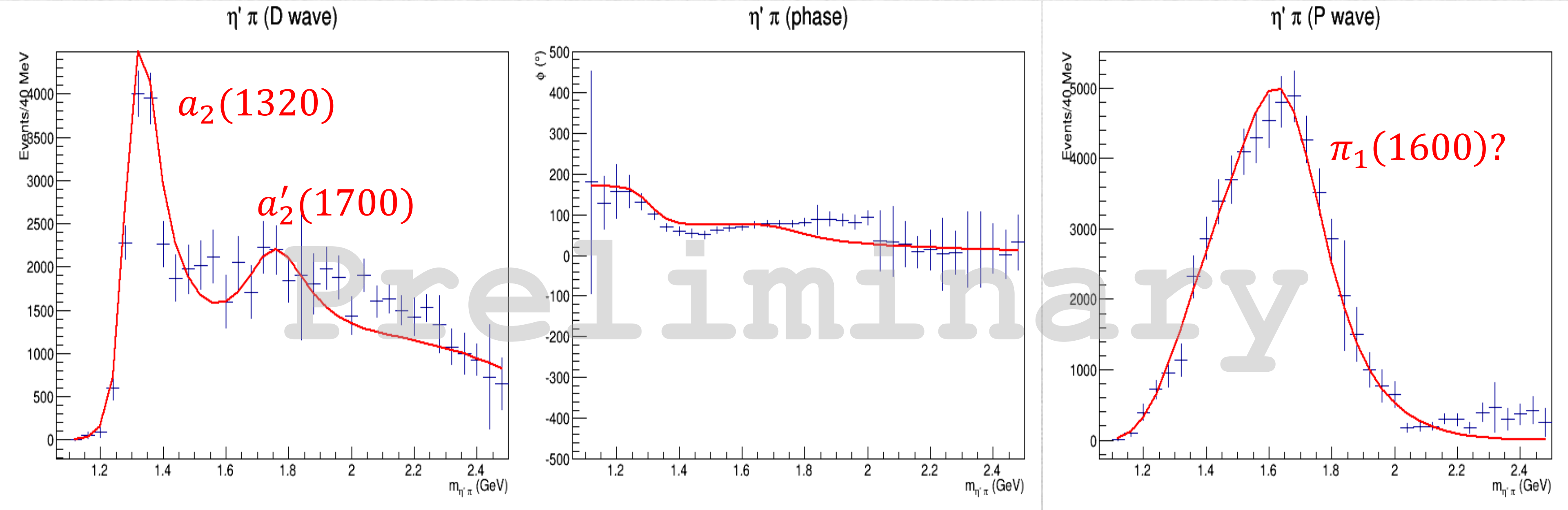



g
g → g/2
g → g/4
g → 0
$D(s) = c_0 - c1s - \frac{c2}{c3 - s}$
Im s
Re s

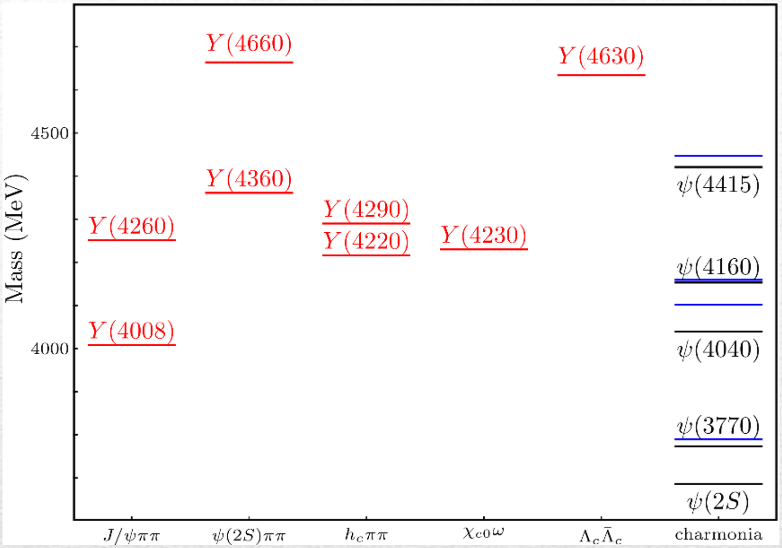
Searching for resonances in $\eta\pi$

$$m(a_2) = (1307 \pm 1 \pm 6)\ \mathrm{MeV} \qquad m(a_2') = (1720 \pm 10 \pm 60)\ \mathrm{MeV}$$
$$\Gamma(a_2) = (112 \pm 1 \pm 8)\ \mathrm{MeV} \qquad \Gamma(a_2') = (280 \pm 10 \pm 70)\ \mathrm{MeV}$$

- The coupled channel analysis involving the exotic P-wave is ongoing, as well as the extention to the GlueX production mechanism and kinematics

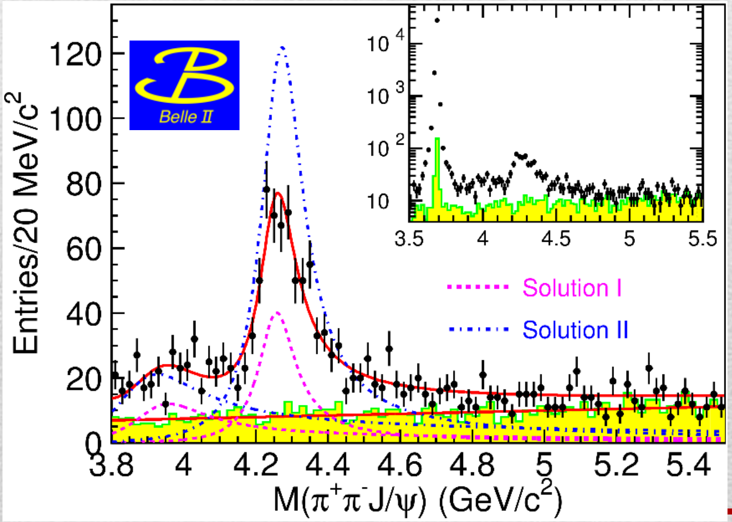


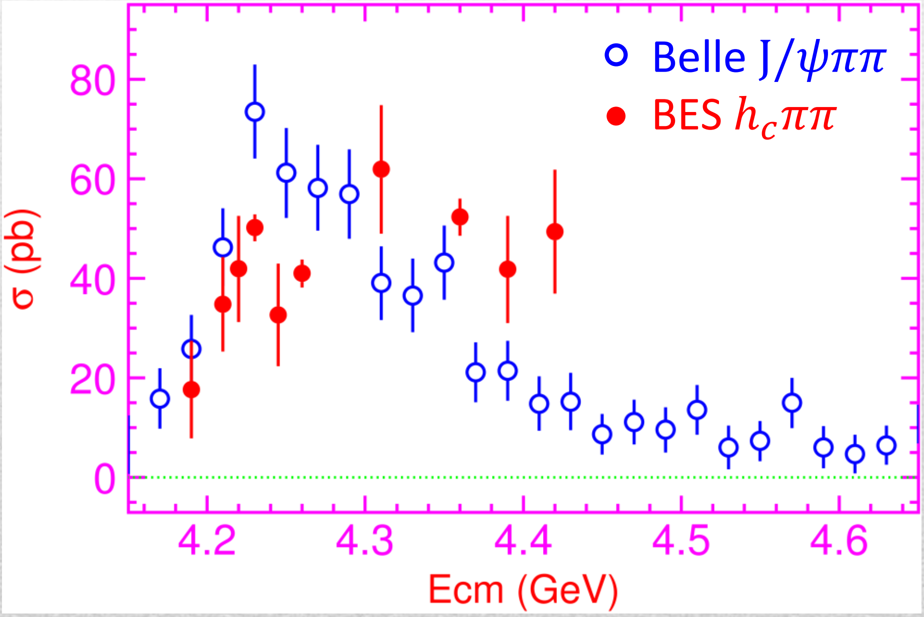
Vector Y states



Lots of unexpected $J^{PC} = 1^{--}$ states
found in ISR/direct production (and nowhere else!)
Seen in few final states,
mostly $J/\psi\,\pi\pi$ and $\psi(2S)\,\pi\pi$

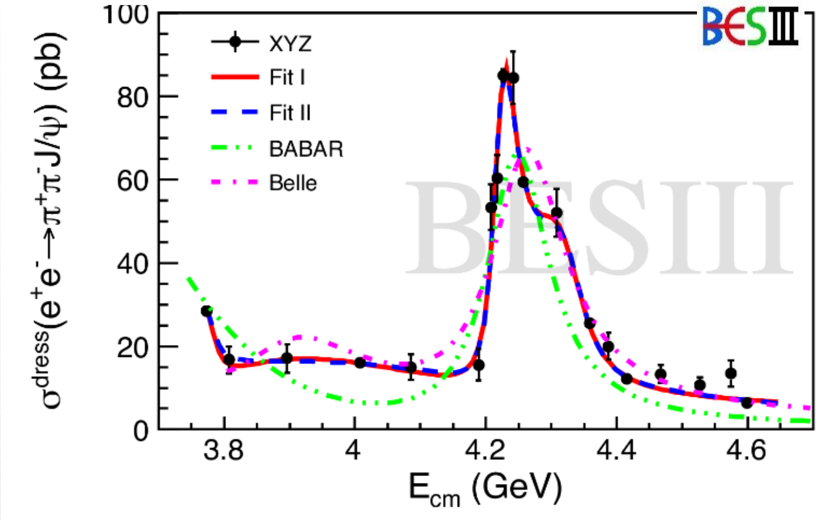
Not seen decaying into open charm pairs
Large HQSS violation





Vector Y states in BESIII

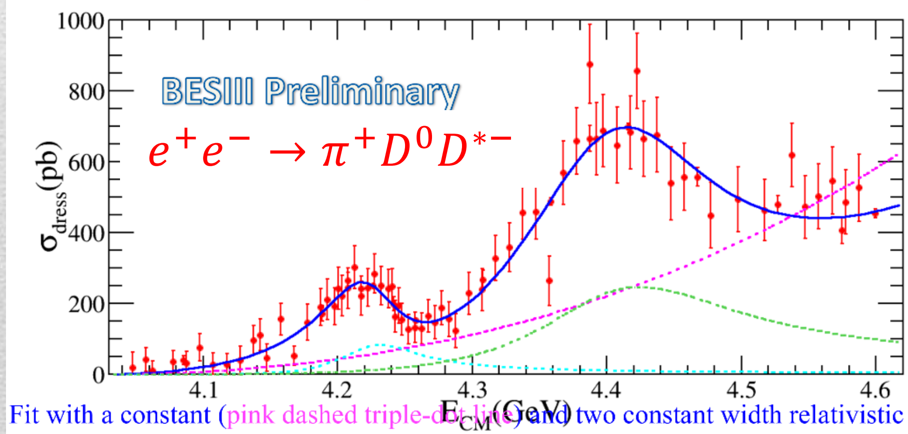
BESIII, PRL118, 092001 (2017) $e^+e^- \to J/\psi\,\pi\pi$



BESIII, PRL118, 092002 (2017)

Parameters	Solution I	Solution II
$\Gamma_{e^+e^-}\mathcal{B}[\psi(3770)\to\pi^+\pi^- J/\psi]$		0.5 ± 0.1 (0
$\Gamma_{e^+e^-}\mathcal{B}(R_1\to\pi^+\pi^- J/\psi)$	$8.8^{+1.5}_{-2.2}\,(\cdots)$	$6.8^{+1.1}_{-1.5}\,(\cdots)$
$\Gamma_{e^+e^-}\mathcal{B}(R_2\to\pi^+\pi^- J/\psi)$	$13.3 \pm 1.4\,(12.0 \pm 1.0)$	$9.2 \pm 0.7\,(8.9 \pm 0.6)$
$\Gamma_{e^+e^-}\mathcal{B}(R_3\to\pi^+\pi^- J/\psi)$	$21.1 \pm 3.9\,(17.9 \pm 3.3)$	$1.7^{+0.8}_{-0.6}\,(1.1^{+0.5}_{-0.4})$
ϕ_1	$-58 \pm 11\,(-33 \pm 8)$	$-116^{+9}_{-10}\,(-81^{+7}_{-8})$
ϕ_2	$-156 \pm 5\,(-132 \pm 3)$	$68 \pm 24\,(107 \pm 20)$

New BESIII data show a peculiar lineshape for the $Y(4260)$
The state appear lighter and narrower, compatible with the ones in $h_c\pi\pi$ and $\chi_{c0}\omega$
A broader old-fashioned $Y(4260)$ is appearing in $\bar{D}D^*\pi$, maybe indicating a $\bar{D}D_1$ dominance



Fit with a constant (pink dashed triple-dot line) and two constant width relativistic BW functions (green dashed double-dot line and aqua dashed line).

$M(Y(4220)) = (4224.8\pm5.6\pm4.0)$ MeV/c^2, $\Gamma(Y(4220)) = (72.3\pm9.1\pm0.9)$ MeV.
$M(Y(4390)) = (4400.1\pm9.3\pm2.1)$ MeV/c^2, $\Gamma(Y(4220)) = (181.7\pm16.9\pm7.4)$ MeV.

The $Y(4260)$

A. Amor, C. Fernandez-Ramirez, AP, U. Tamponi, in preparation

$$f(s) = \frac{N(s)}{K^{-1}(s) - \frac{i}{2}\rho_3(s)},$$

Same game,
we start analyzing the single channel $e^+e^- \to J/\psi\,\pi\pi$ data

We consider the amplitude in the elastic, quasi two-body approximation

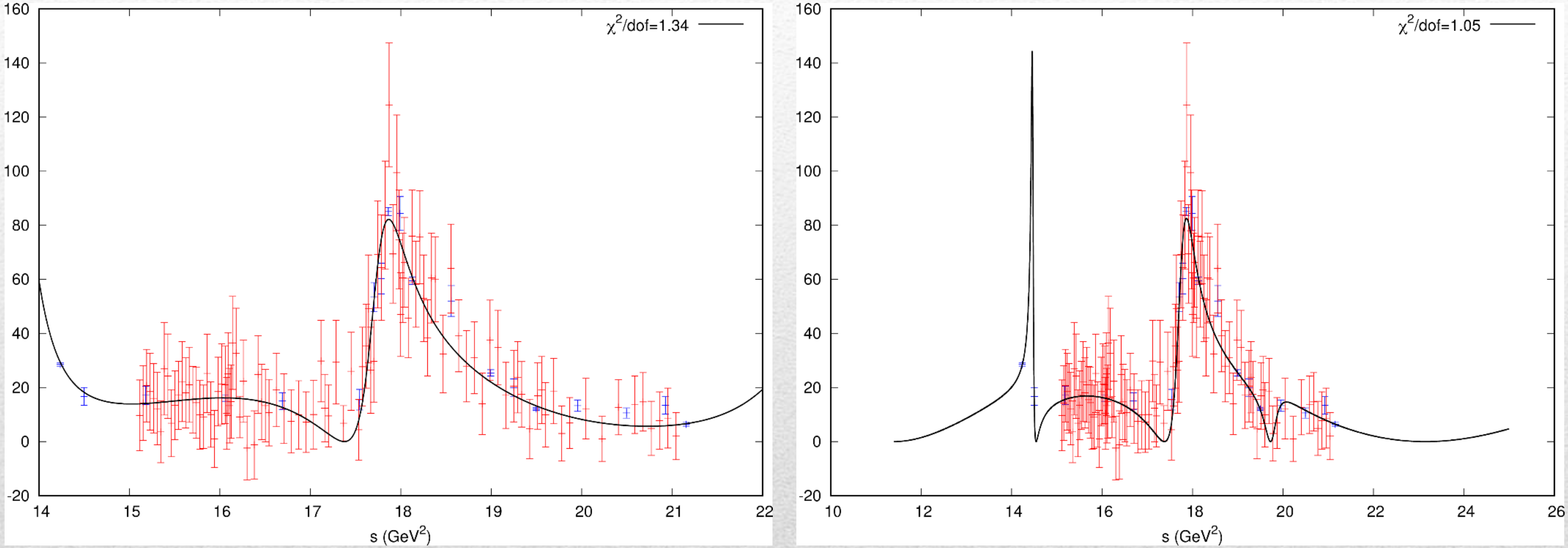
$$i\rho_3(s) = \sum_{k=0}^{n-1} a_k\,(s-s_0)^k + \frac{(s-s_0)^n}{\pi}\int_{(2m_\pi+M_\psi)^2}^{\infty} \frac{\rho_2(s')}{(s'-s)}\frac{ds'}{(s'-s_0)^n}$$

$$\rho_2(s') = \int_{4m_\pi^2}^{\left(\sqrt{s'}-M_\psi\right)^2} \frac{ds_{\pi\pi}}{2\pi}\,\frac{\lambda^{1/2}(s',s_{\pi\pi},m_{J/\psi}^2)}{8\pi s'}\,\frac{\lambda^{1/2}(s_{\pi\pi},m_\pi^2,m_\pi^2)}{8\pi s_{\pi\pi}}\,\left|t_{2\to2}(s_{\pi\pi})\right|^2$$

Need model for the Dalitz distribution

The $Y(4260)$

A. Amor, C. Fernandez-Ramirez, AP, U. Tamponi, in preparation



LASSO method (linear penalization in the χ^2)
is helpful in constraining the number of resonances
and parameters in the numerator

Conclusions & prospects

- We aim at developing new theoretical tools, to get insight on QCD using first principles of QFT (unitarity, analyticity, crossing symmetry, low and high energy constraints,…) to extract the physics out of the data
- Many other ongoing projects (both for meson and baryon spectroscopy, and for high energy observables), with a particular attention to producing complete reaction models for the golden channels in exotic meson searches

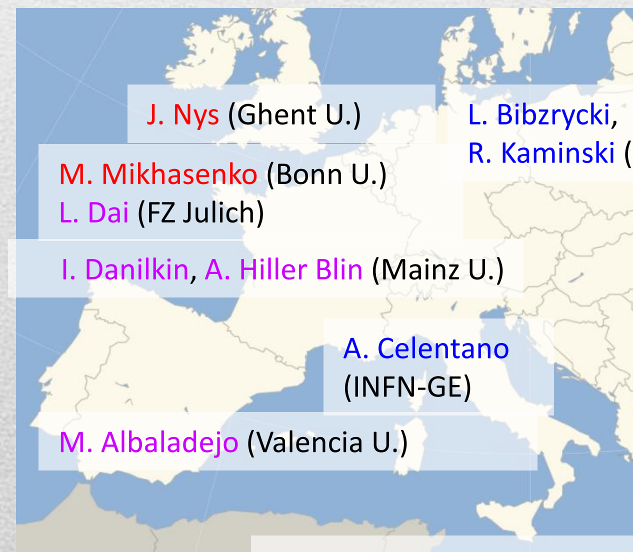
A. Jackura, N. Sherrill, G. Fox, T. Londergan (IU), E. Passemar, A. Szczepaniak (IU/JLab)

R. Workman (GWU), M. Döring (GWU/JLab)

V. Mathieu, A. Pilloni, V. Mokeev (JLab)

P. Guo (Cal. State U.)

J. Castro, C. Fernandez-Ramirez (UNAM)



J. Nys (Ghent U.)

L. Bibzrycki, R. Kaminski (Krakow)

M. Mikhasenko (Bonn U.)
L. Dai (FZ Julich)

I. Danilkin, A. Hiller Blin (Mainz U.)

A. Celentano (INFN-GE)

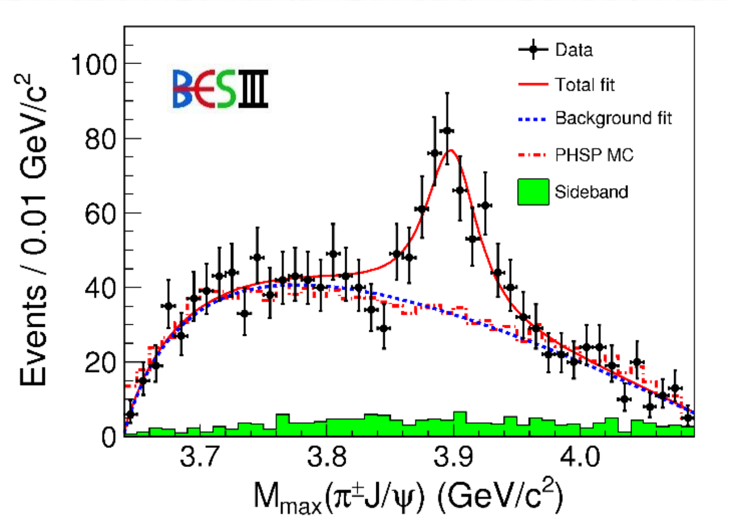
M. Albaladejo (Valencia U.)

Students, Postdocs, Faculties

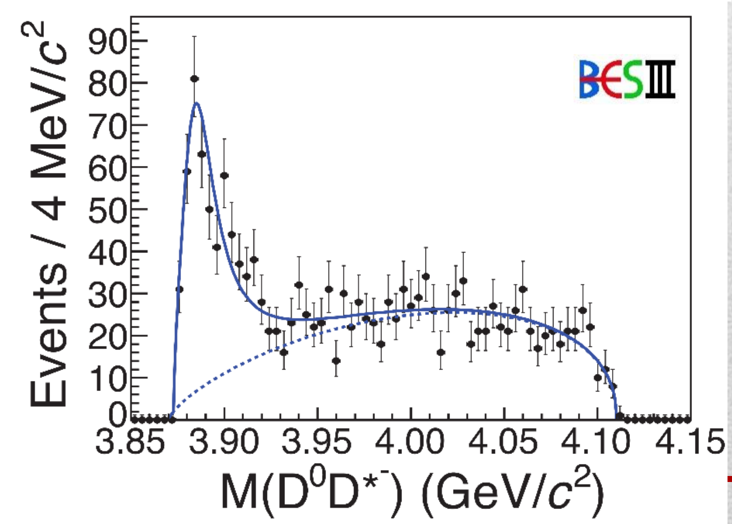
BACKUP

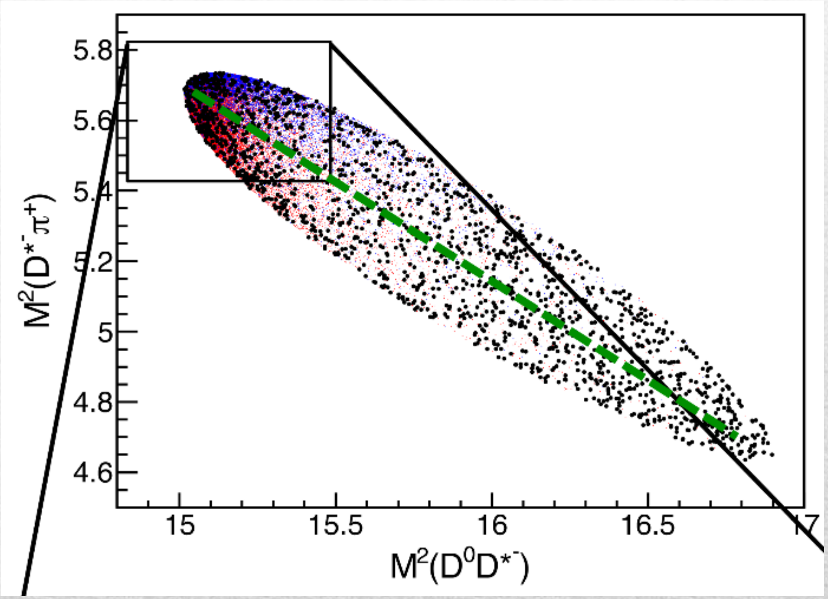
Example: The charged $Z_c(3900)$

A charged charmonium-like resonance has been claimed by BESIII in 2013.



$$e^+e^- \to Z_c(3900)^+\pi^- \to J/\psi\,\pi^+\pi^- \text{ and } \to (DD^*)^+\pi^-$$
$$M = 3888.7 \pm 3.4 \text{ MeV}, \Gamma = 35 \pm 7 \text{ MeV}$$





Such a state would require a minimal 4q content and would be manifestly exotic

Testing scenarios

- We approximate all the particles to be scalar – this affects the value of couplings, which are not normalized anyway – but not the position of singularities.
  This also limits the number of free parameters

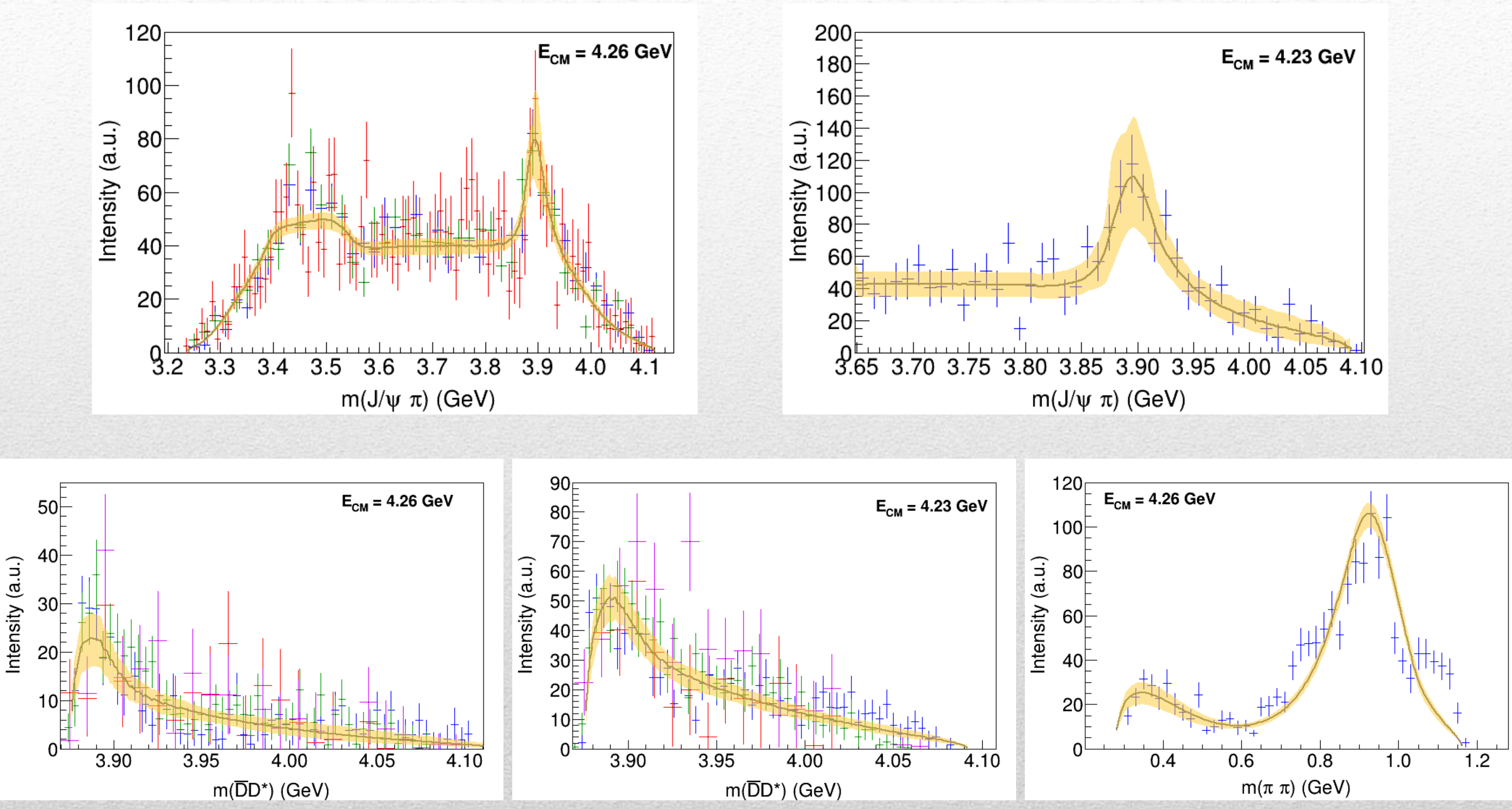
$$f_i(s,t,u) = 16\pi \left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left(c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s'(s'-s)} \right) \right],$$

The scattering matrix is parametrized as $(t^{-1})_{ij} = K_{ij} - i\,\rho_i\,\delta_{ij}$

Four different scenarios considered:

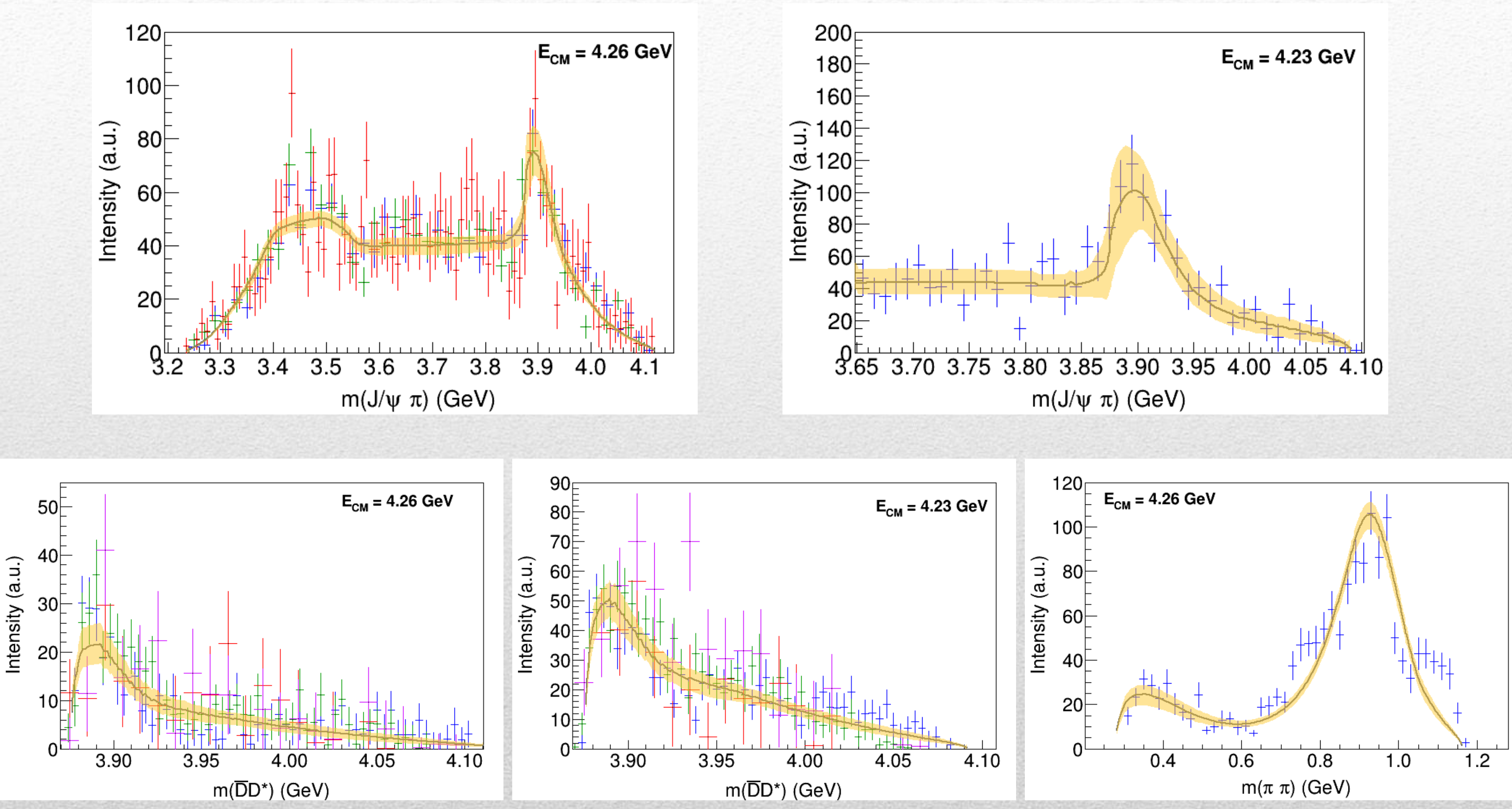
- «III»: the K matrix is $\frac{g_i\,g_j}{M^2-s}$, this generates a pole in the closest unphysical sheet the rescattering integral is set to zero
- «III+tr.»: same, but with the correct value of the rescattering integral
- «IV+tr.»: the K matrix is constant, this generates a pole in the IV sheet
- «tr.»: same, but the pole is pushed far away by adding a penalty in the χ^2

Fit: III



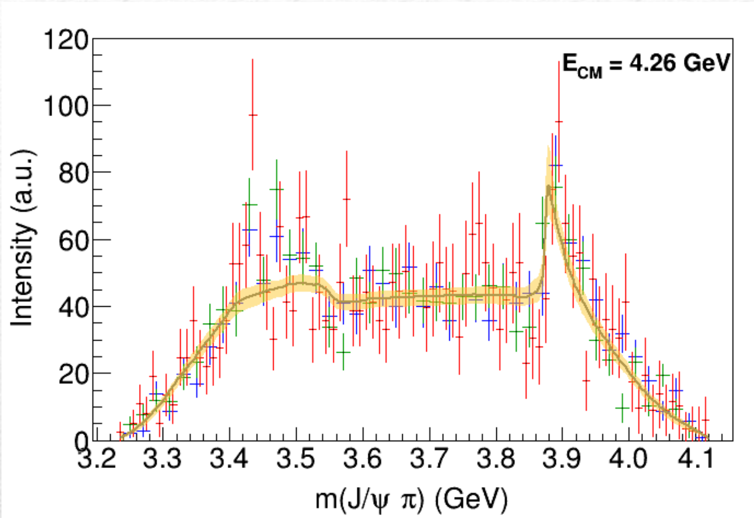
E_CM = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
E_CM = 4.23 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
E_CM = 4.26 GeV
Intensity (a.u.)
m(D̄D*) (GeV)
E_CM = 4.23 GeV
Intensity (a.u.)
m(D̄D*) (GeV)
E_CM = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

Fit: III+tr.

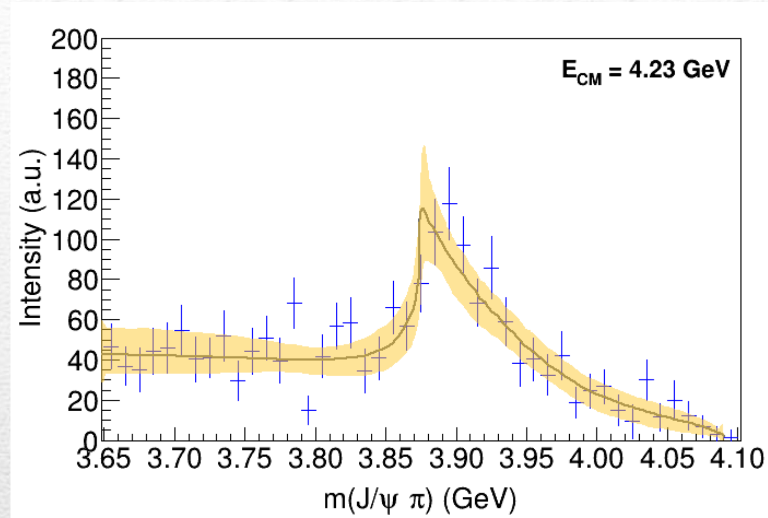


E_CM = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
E_CM = 4.23 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
E_CM = 4.26 GeV
Intensity (a.u.)
m(D̄D*) (GeV)
E_CM = 4.23 GeV
Intensity (a.u.)
m(D̄D*) (GeV)
E_CM = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

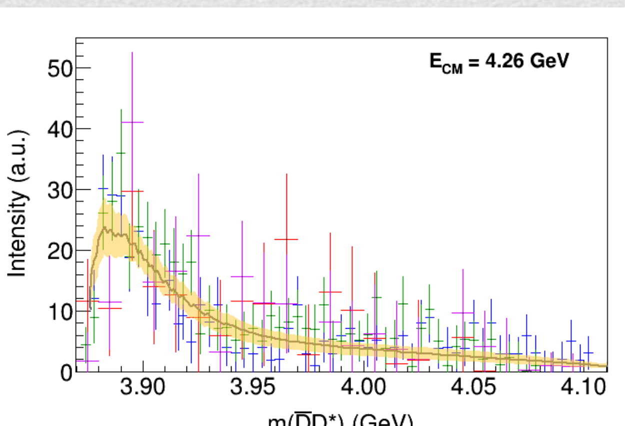
Fit: IV+tr.



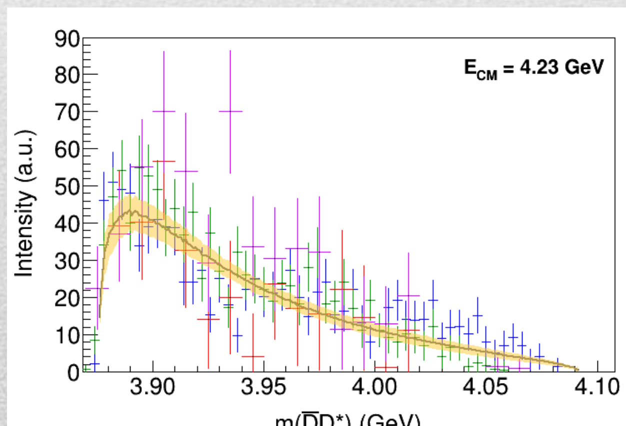
E_{CM} = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)



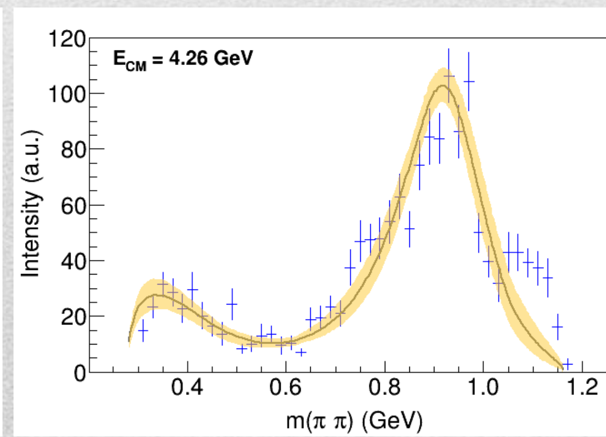
E_{CM} = 4.23 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)



E_{CM} = 4.26 GeV
Intensity (a.u.)
m($\bar{D}D^*$) (GeV)

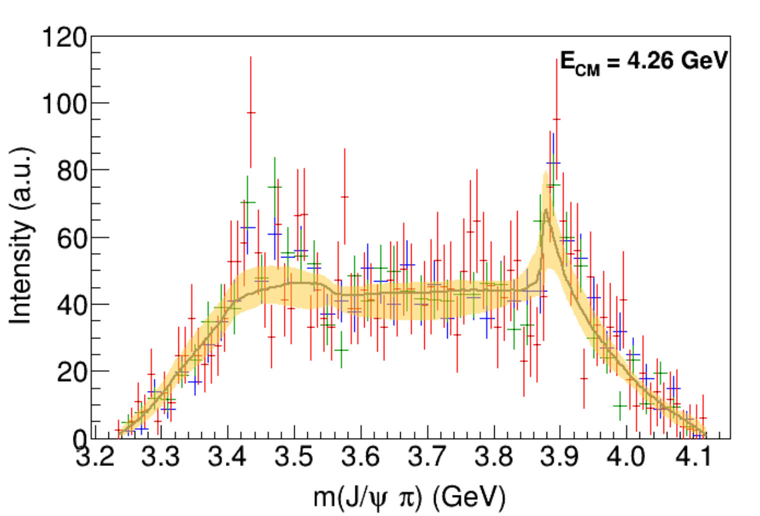


E_{CM} = 4.23 GeV
Intensity (a.u.)
m($\bar{D}D^*$) (GeV)

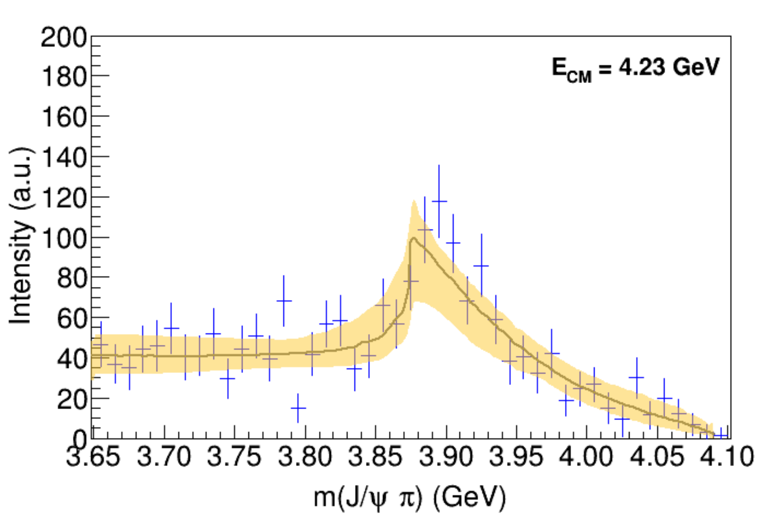


E_{CM} = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

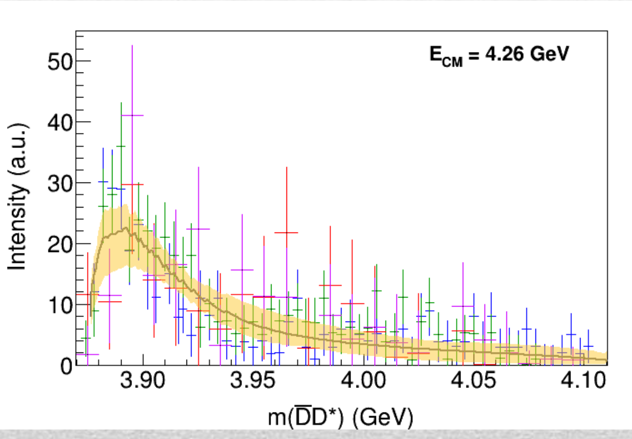
Fit: tr.



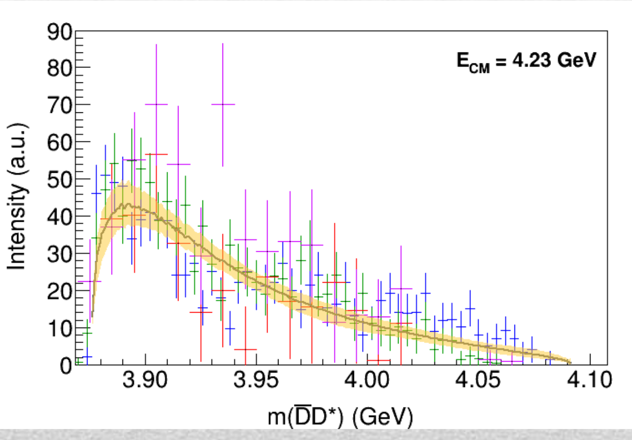
E_{CM} = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)



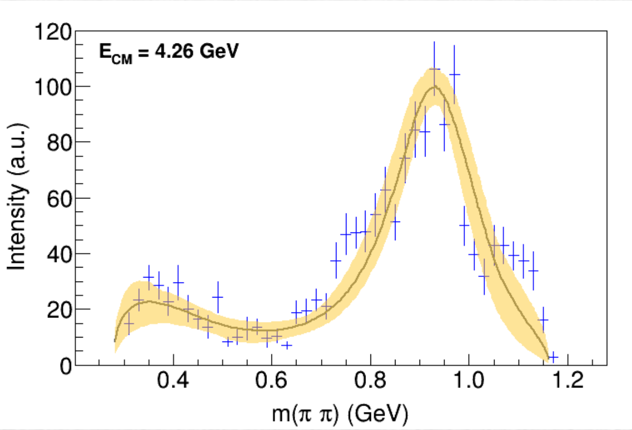
E_{CM} = 4.23 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)



E_{CM} = 4.26 GeV
Intensity (a.u.)
m(D̄D*) (GeV)

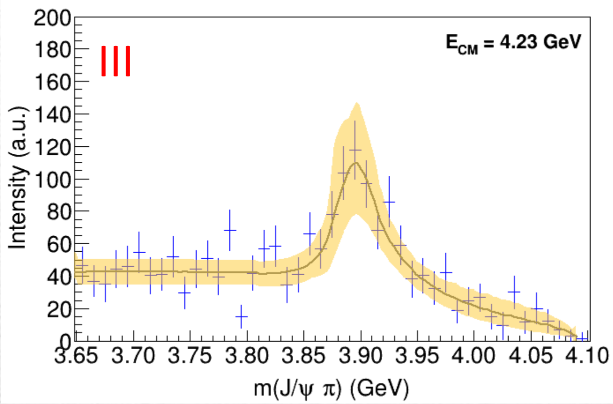


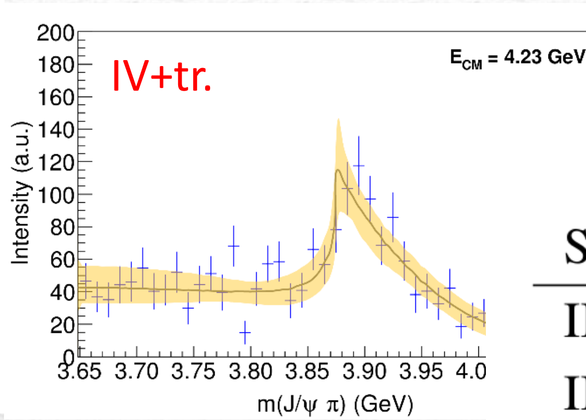
E_{CM} = 4.23 GeV
Intensity (a.u.)
m(D̄D*) (GeV)



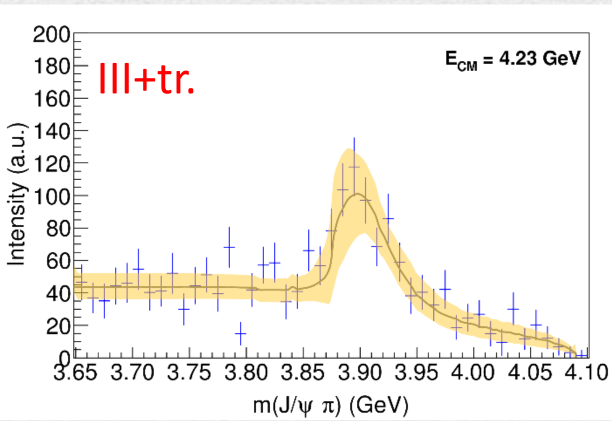
E_{CM} = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

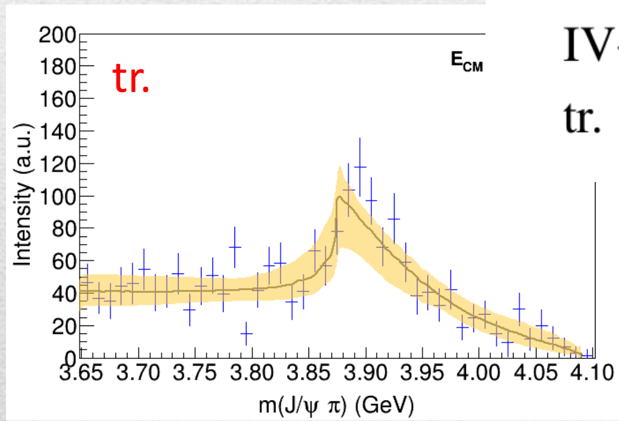
Fit summary





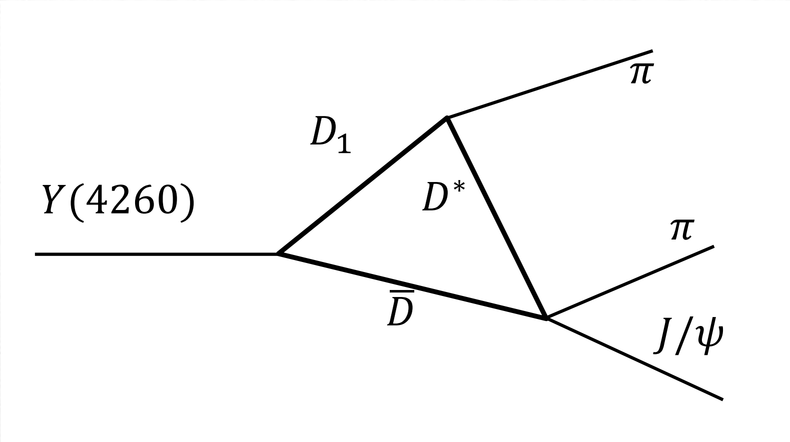
Scenario	χ^2	DOF	χ^2/DOF
III	644	532	1.21
III+tr.	642	532	1.21
IV+tr.	666	532	1.25
tr.	695	532	1.31

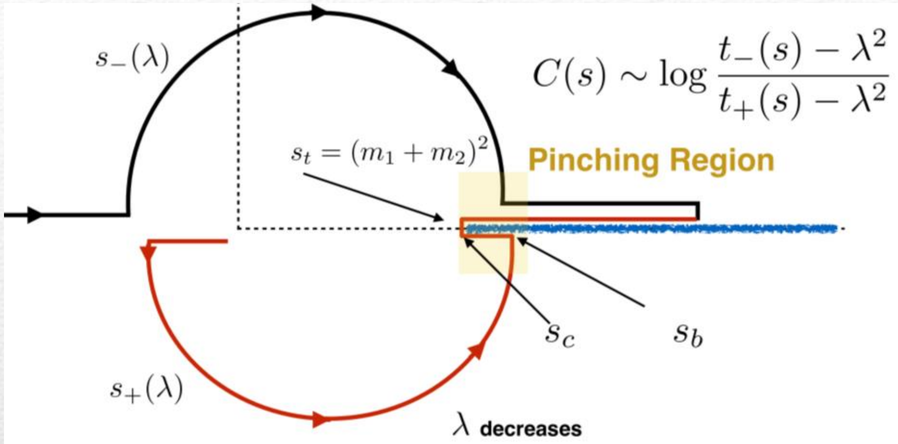




Naive loglikelihood ratio test give a $\sim 4\sigma$ significance of the scenario III+tr. over IV+tr., looking at plots it looks too much – better using some more solid test

Triangle singularity





- Logarithmic branch points due to exchanges in the cross channels can simulate a resonant behavior, only in very special kinematical conditions (Coleman and Norton, Nuovo Cim. 38, 438)
- However, this effects cancels in Dalitz projections, no peaks (Schmid, Phys.Rev. 154, 1363)
- But the cancellation can be spread in different channels, you might still see peaks in other channels!

Production

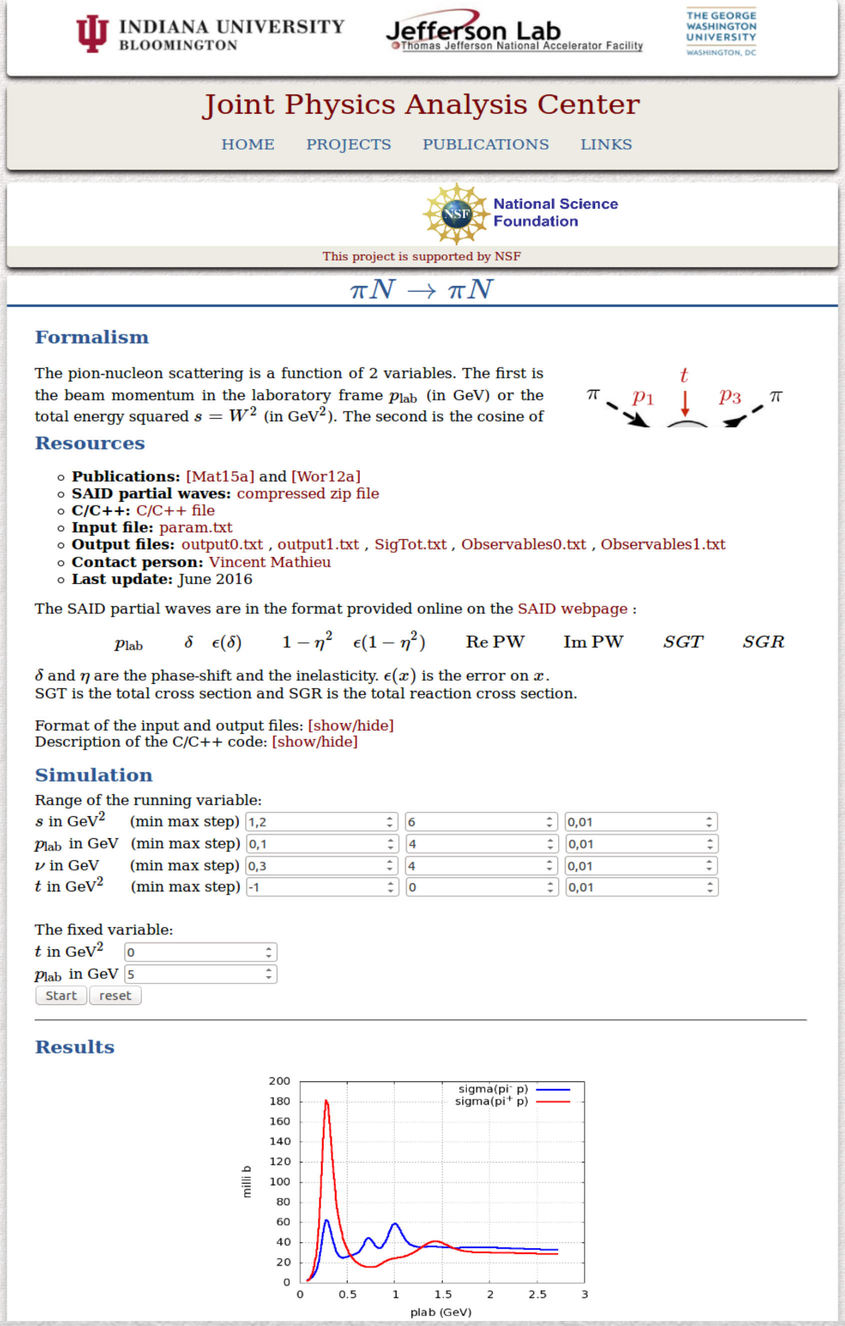
- $>$ 40 Research Papers (Phys.Rev., Phys.Lett, Eur.J. Phys.)
- $\sim$120 Invited Talks and Seminars
- $O(10)$ ongoing analyses
- Summer Schools on Reaction Theory (IU, 2015 and 2017)
- Workshop "Future Directions in Hadron Spectroscopy" (JLab, 2014 and UNAM 2017)

FESR	V. Mathieu *et al.*,	arXiv:1708.07779
$\pi N \to \eta \pi N$	A. Jackura *et al.*,	arXiv:1707.02848
$\gamma N \to \eta N$ vs. $\to \eta' N$	V. Mathieu *et al.*,	arXiv:1704.07684
$Z_c(3900)$	A. Pilloni *et al.*,	PLB772, 200
$\gamma N \to \eta N$	J. Nys *et al.*,	PRD95, 034014
$\gamma p \to J/\psi p$	A. Blin *et al.*,	PRD94, 034002
$K N \to K N$	C. Fernandez-Ramirez *et al.*,	PRD93, 034029; PRD93, 074015
$\gamma p \to \pi^0 p$	V. Mathieu *et al.*,	PRD92, 074013
$\pi N \to \pi N$	V. Mathieu *et al.*,	PRD92, 074004
$\eta \to \pi^+ \pi^- \pi^0$	P. Guo *et al.*,	PRD92, 054016; PLB771, 497
$\omega, \phi \to \pi^+ \pi^- \pi^0$	I. Danilkin *et al.*,	PRD91, 094029
$\gamma p \to K^+ K^- p$	M. Shi *et al.*,	PRD91, 034007

Interactive tools

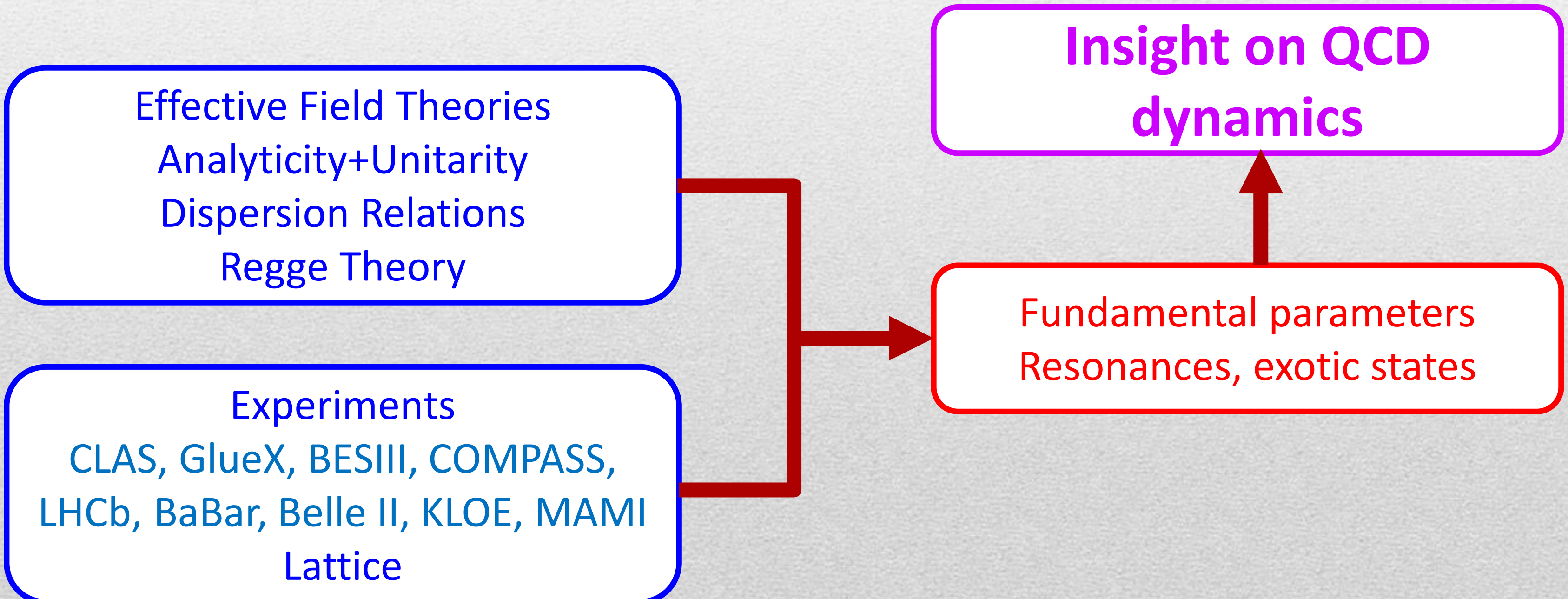
- Completed projects are fully documented on interactive portals
- These include description on physics, conventions, formalism, etc.
- The web pages contain source codes with detailed explanation how to use them. Users can run codes online, change parameters, display results.

http://www.indiana.edu/~jpac/



Joint Physics Analysis Center

- Joint effort between theorists and experimentalists to work together to make the best use of the next generation of very precise data taken at JLab and in the world
- Created in 2013 by JLab & IU agreement
- It is engaged in education of further generations of hadron physics practitioners



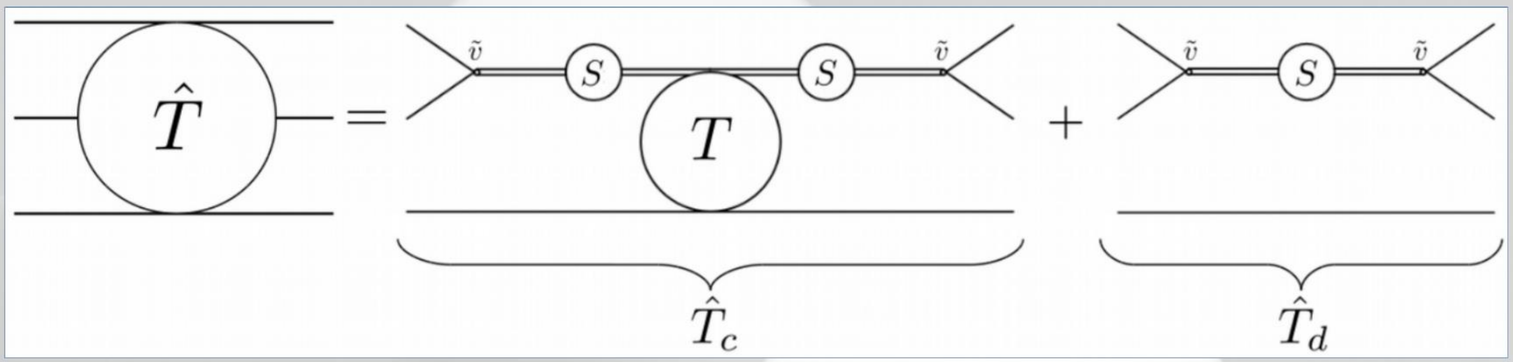
Three-Body Unitarity

Mai, Hu, Doring, AP, Szczepaniak, EPJA53, 9, 177

Original study by Amado/Aaron/Young **AAY(1968)**

- 3-dimensional integral equation from unitarity constraint & BSE ansatz
- valid below break-up energies ($E < 3m$)
- analyticity constraints unclear

One has to begin with asymptotic states



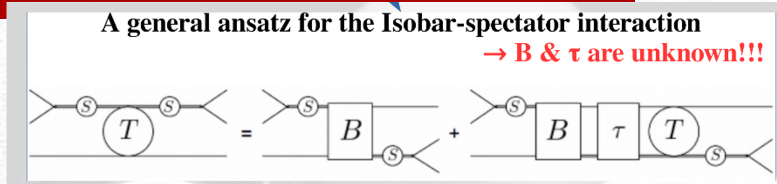
- v a general but cut-free (in the phys. region) function
- two-body interaction is parametrized by an *"isobar"*

= has definite QN and correct r.h.-singularities w.r.t invariant mass

- ***S*** **and** ***T*** **are yet unknown functions**

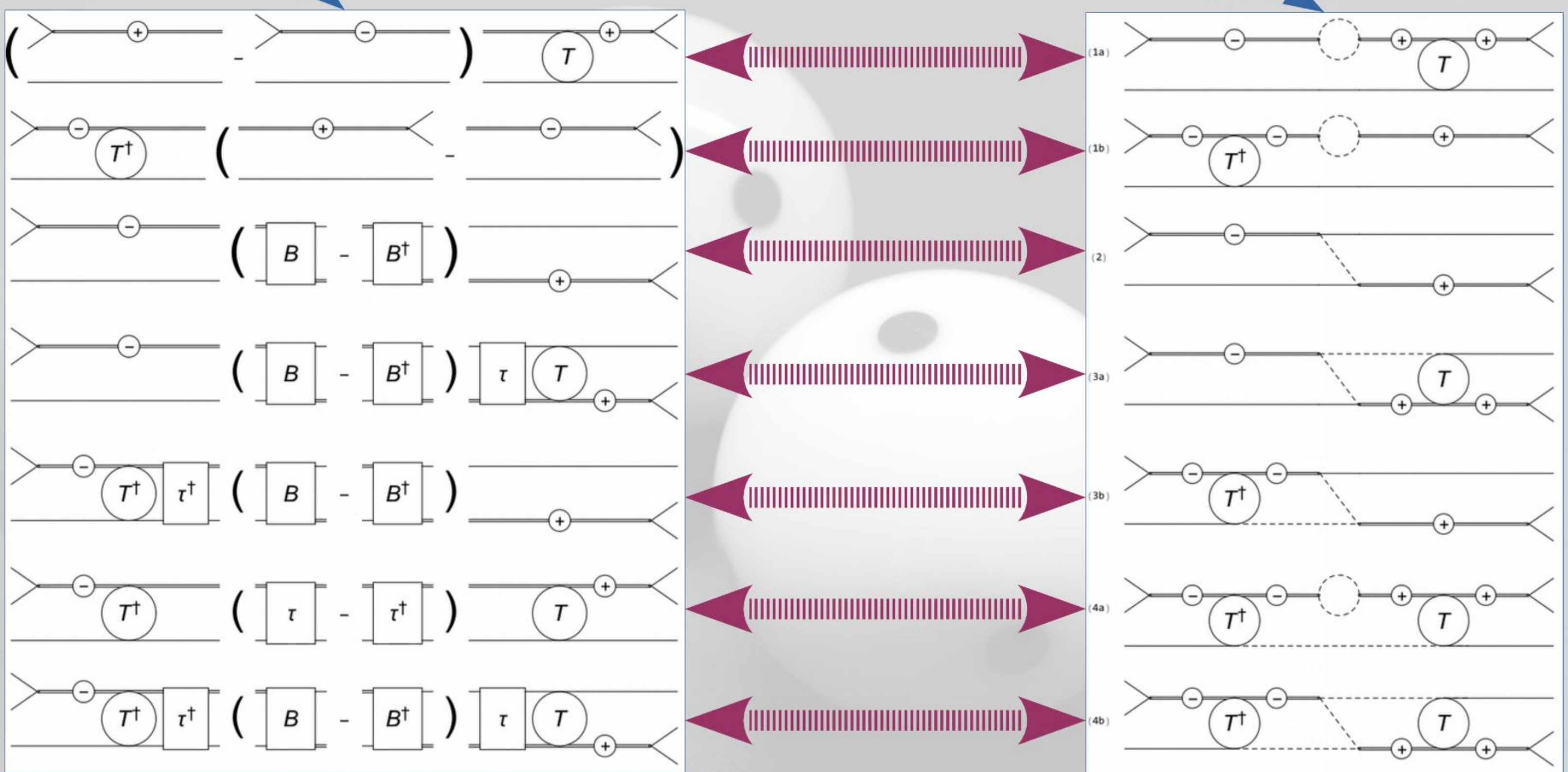
M. Mai

Three-Body Unitarity



3-body Unitarity (normalization condition ↔ phase space integral)

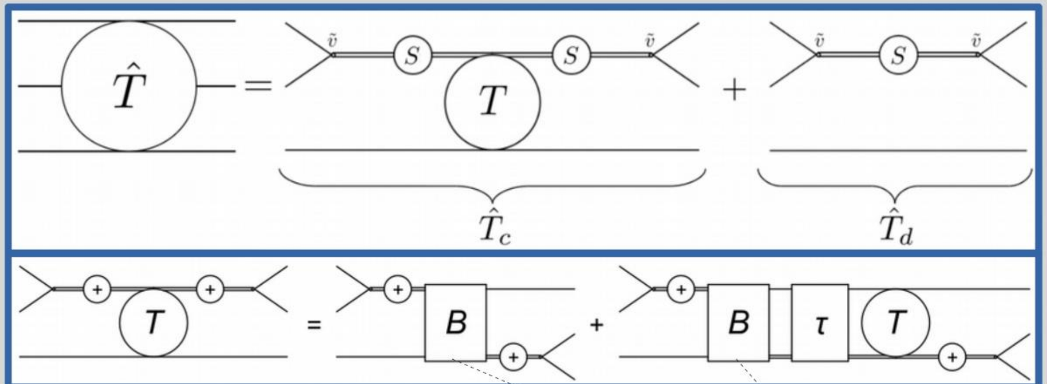
$$\langle q_1,q_2,q_3|(\hat{T}-\hat{T}^\dagger)|p_1,p_2,p_3\rangle \;=\; i\int_P \langle q_1,q_2,q_3|\hat{T}^\dagger|k_1,k_2,k_3\rangle\langle k_1,k_2,k_3|\hat{T}|p_1,p_2,p_3\rangle$$



M. Mai

Three-Body Unitarity

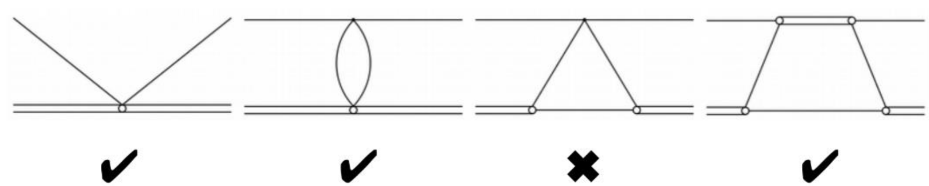
$3 \to 3$ scattering amplitude is a 3-dimensional integral equation



– Imaginary parts (***B, τ, S***) are fixed by **unitarity/matching**
For simplicity ***v=λ*** (full relations available)

$$\langle q|B(s)|p\rangle = -\frac{\lambda^2}{2\sqrt{m^2+\mathbf{Q}^2}\left(E_Q-\sqrt{m^2+\mathbf{Q}^2}+i\epsilon\right)}$$

- un-subtracted dispersion relation
- one-π exchange in TOPT
- real contributions can be added to B



$$\tau(\sigma(k)) = (2\pi)\delta^+(k^2-m^2)S(\sigma(k))$$

$$-\frac{1}{S(P^2)} = \sigma(k) - M_0^2 - \frac{1}{(2\pi)^3}\int d^3\boldsymbol{\ell}\,\frac{\lambda^2}{2E_\ell(\sigma(k)-4E_\ell^2+i\epsilon)}$$

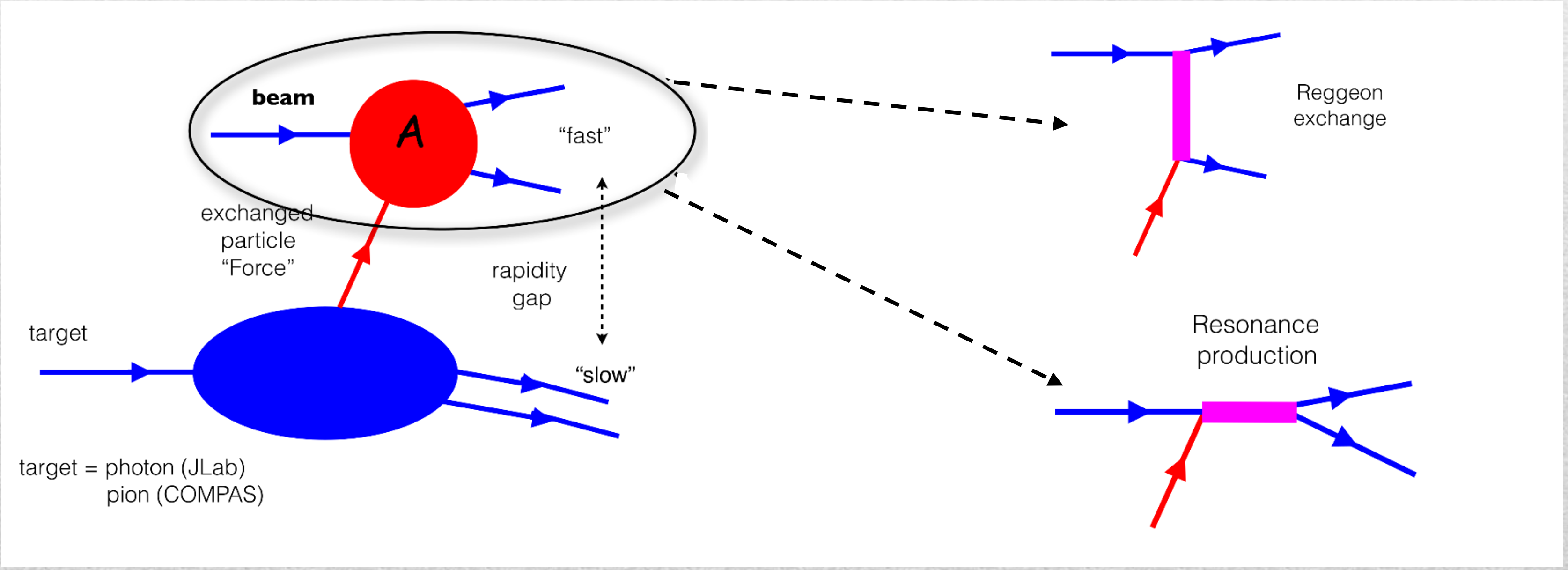
Higher energies: Regge exchange

Resonances are poles in s for fixed l
dominate low energy region

$$A_l \sim \frac{g_1 g_2}{s_p - s}$$

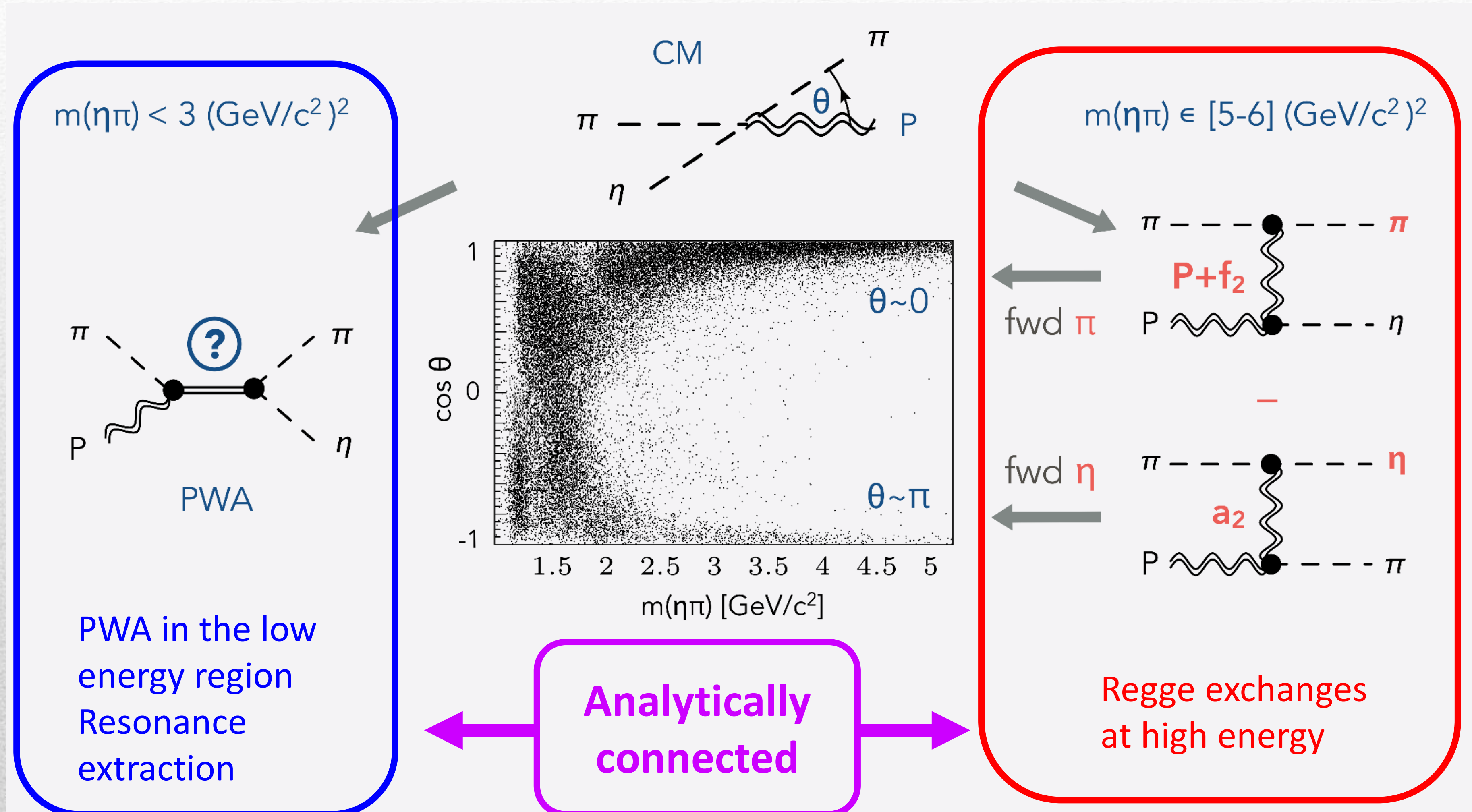
Reggeons are poles in l for fixed s
dominate high energy region

$$A \sim \sum s^l \sim \beta(t)\, s^{\alpha(t)}$$

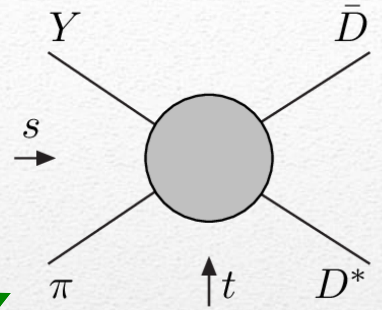


Finite energy sum rules

See J. Nys and V. Mathieu talk on friday



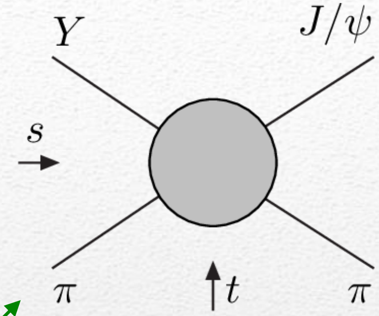
Amplitude model



u: $D_0(2400)$

$D_1(2420)$

$Z_c(3900)$?



u: $Z_c(3900)$?

"$\sigma, f_0(980)$"

$$f_i(s,t,u) = 16\pi \sum_{l=0}^{L_{\max}} (2l+1)\left(a_{l,i}^{(s)}(s)P_l(z_s) + a_{l,i}^{(t)}(t)P_l(z_t) + a_{l,i}^{(u)}(u)P_l(z_u)\right)$$

Khuri-Treiman

$$f_{0,i}(s) = \frac{1}{32\pi}\int_{-1}^{1} dz_s\, f_i\left(s, t(s,z_s), u(s,z_s)\right) = a_{0,i}^{(s)} + \frac{1}{32\pi}\int_{-1}^{1} dz_s \left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u)\right) \equiv a_{0,i}^{(s)} + b_{0,i}(s)$$

$$f_{l,i}(s) = \frac{1}{32\pi}\int_{-1}^{1} dz_s\, P_l(z_s)\left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u)\right) \equiv b_{l,i}(s) \quad \text{for } l > 0. \quad f_{0,i}(s) = b_{0,i}(s) + \sum_j t_{ij}(s)\frac{1}{\pi}\int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s'-s},$$

$$\boxed{f_i(s,t,u) = 16\pi\left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s)\left(c_j + \frac{s}{\pi}\int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s'(s'-s)}\right)\right],}$$

Strategy

AP *et al.* (JPAC), arXiv:1612.06490

- We fit the following invariant mass distributions:
  - BESIII PRL110, 252001 $J/\psi\,\pi^+, J/\psi\,\pi^-, \pi^+\pi^-$ at $E_{CM} = 4.26$ GeV
  - BESIII PRL110, 252001 $J/\psi\,\pi^0$ at $E_{CM} = 4.23, 4.26, \cancel{4.36}$ GeV
  - BESIII PRD92, 092006 $\overline{D^0}D^{*+}, \overline{D^{*0}}D^+$ (double tag) at $E_{CM} = 4.23, 4.26$ GeV
  - BESIII PRL115, 222002 $\overline{D^0}D^{*0}, \overline{D^{*0}}D^0$ at $E_{CM} = 4.23, 4.26$ GeV
  - ~~BESIII PRL112, 022001 $\overline{D^0}D^{*+}, \overline{D^{*0}}D^+$ (single tag) at $E_{CM} = 4.26$ GeV~~
  - ~~Belle PRL110, 252002 $J/\psi\,\pi^\pm$ at $E_{CM} = 4.26$ GeV~~
  - ~~CLEO-c data PLB727, 366 $J/\psi\,\pi^\pm, J/\psi\,\pi^0$ at at $E_{CM} = 4.17$ GeV~~

- Published data are not efficiency/acceptance corrected,
  → we are not able to give the absolute normalization of the amplitudes

- No given dependence on E_{CM} is assumed – the couplings at different E_{CM} are independent parameters

Strategy

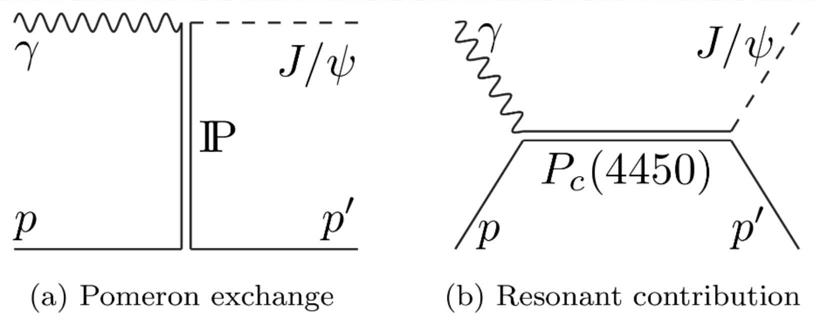
AP *et al.* (JPAC), PLB772, 200

- Reducible (incoherent) backgrounds are pretty flat and do not influence the analysis, except the peaking background in $\overline{D^0}D^{*0}$, $\overline{D^{*0}}D^0$ (subtracted)
- Some information about angular distributions has been published, but it's not constraining enough → we do not include in the fit
- Because of that, we approximate all the particles to be scalar – this affects the value of couplings, which are not normalized anyway – but not the position of singularities. This also limits the number of free parameters

Pentaquark photoproduction

To exclude any rescattering mechanism, we propose to search the $P_c(4450)$ state in photoproduction

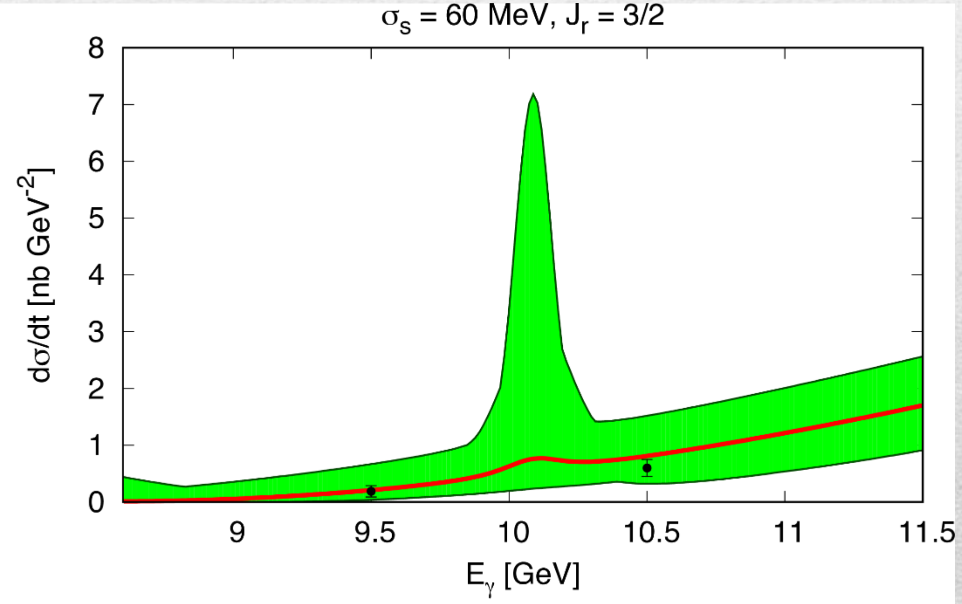
We use the (few) existing data and VMD + pomeron inspired bkg to estimate the cross section



GlueX data coming soon!

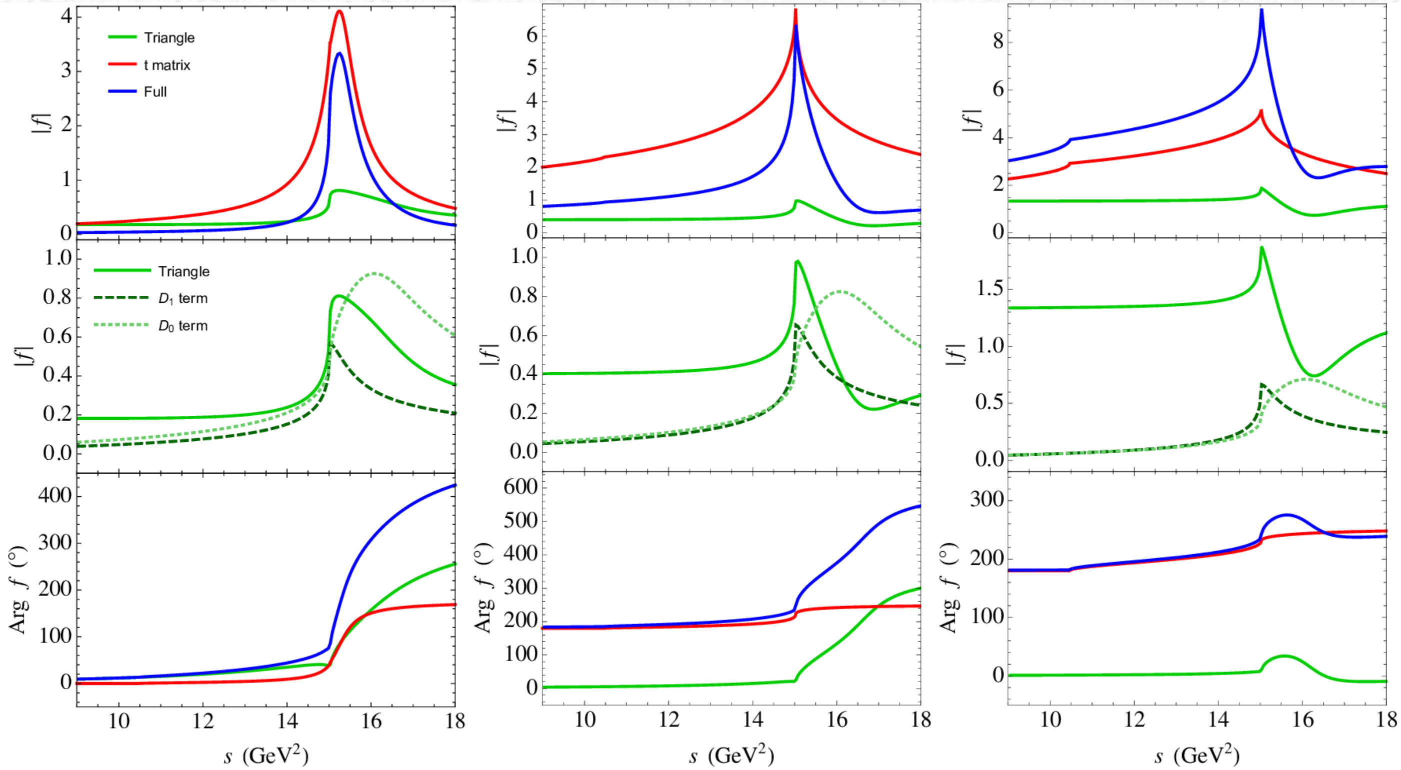
$J^P = (3/2)^-$

σ_s (MeV)	0	60	120
A	$0.156^{+0.029}_{-0.020}$	$0.157^{+0.039}_{-0.021}$	$0.157^{+0.037}_{-0.022}$
α_0	$1.151^{+0.018}_{-0.020}$	$1.150^{+0.018}_{-0.026}$	$1.150^{+0.015}_{-0.023}$
α' (GeV^{-2})	$0.112^{+0.033}_{-0.054}$	$0.111^{+0.037}_{-0.064}$	$0.111^{+0.038}_{-0.054}$
s_t (GeV^2)	$16.8^{+1.7}_{-0.9}$	$16.9^{+2.0}_{-1.6}$	$16.9^{+2.0}_{-1.1}$
b_0 (GeV^{-2})	$1.01^{+0.47}_{-0.29}$	$1.02^{+0.61}_{-0.32}$	$1.03^{+0.49}_{-0.31}$
$\mathcal{B}_{\psi p}$ (95% CL)	≤ 29 %	≤ 30 %	≤ 23 %



Hiller Blin, AP *et al.* (JPAC), PRD94, 034002

Lineshapes at 4260



(a) III+tr. (b) IV+tr. (c) tr.

Figure 7: Interplay of scattering amplitude poles and triangle singularity to reconstruct the peak. We focus on the $J/\psi\,\pi$ channel, at $E_{CM} = 4.26$ GeV. The red curve is the t_{12} scattering amplitude, the green curve is the $c_1 + H(s, D_1) + +H(s, D_0)$ term in Eq. (9), and the blue curve is the product of the two. The upper plots show the magnitudes of these terms, the lower plots the phases. The middle row shows the contributions to the unitarized term due to the D_1 (dashed) and the D_0 (dotted). Only for D_1 the singularity is close enough to the physical region to generate a large peak. (a) The pole on the III sheet generates a narrow Breit-Wigner-like peak. The contribution of the triangle is not particularly relevant. (b) The sharp cusp in the scattering amplitude is due to the IV sheet pole close by; the triangle contributes to make the peak sharper. (c) The scattering amplitude has a small cusp due to the threshold factor, and the triangle is needed to make it sharp enough to fit the data.

Lineshapes at 4230

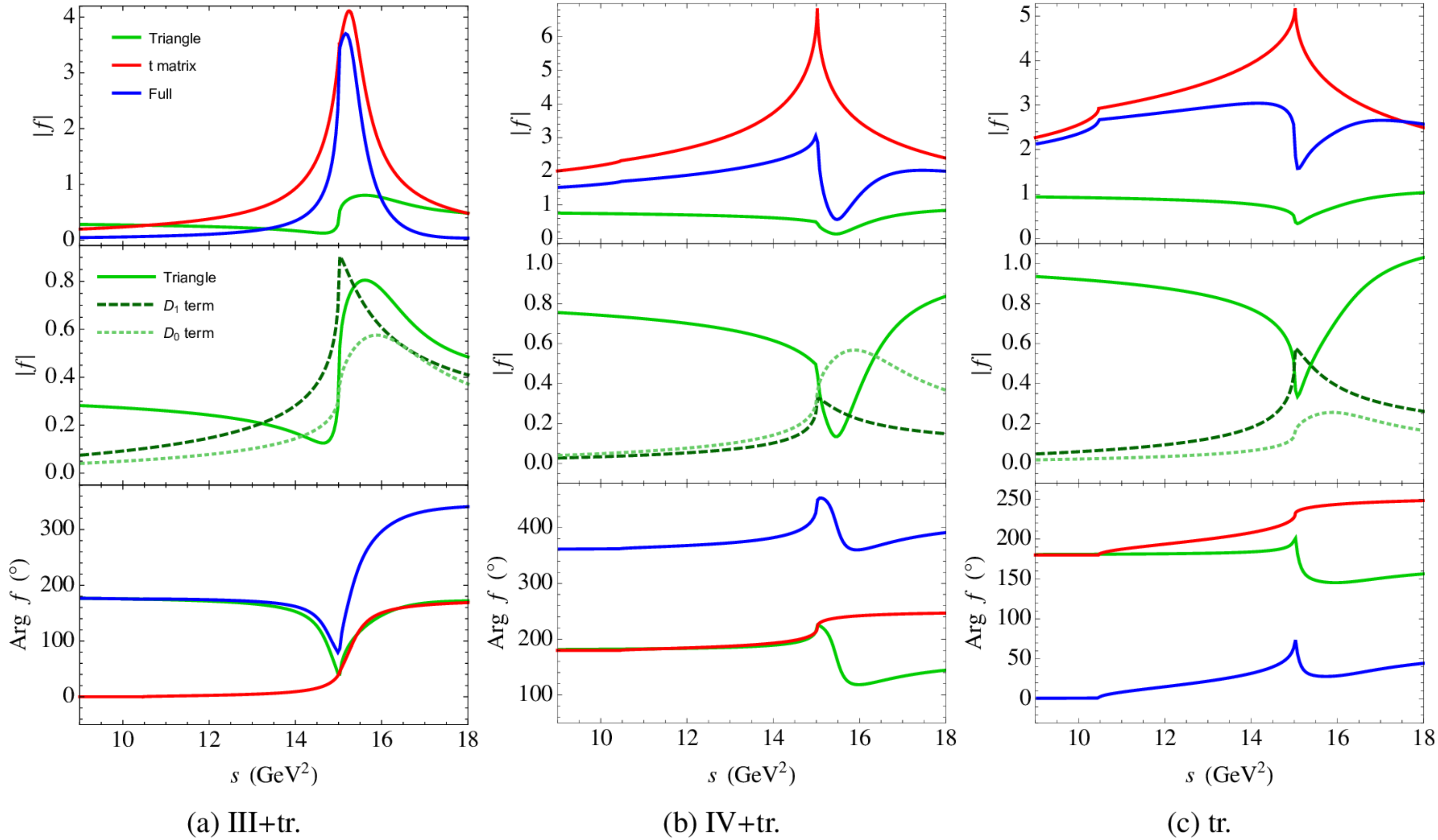
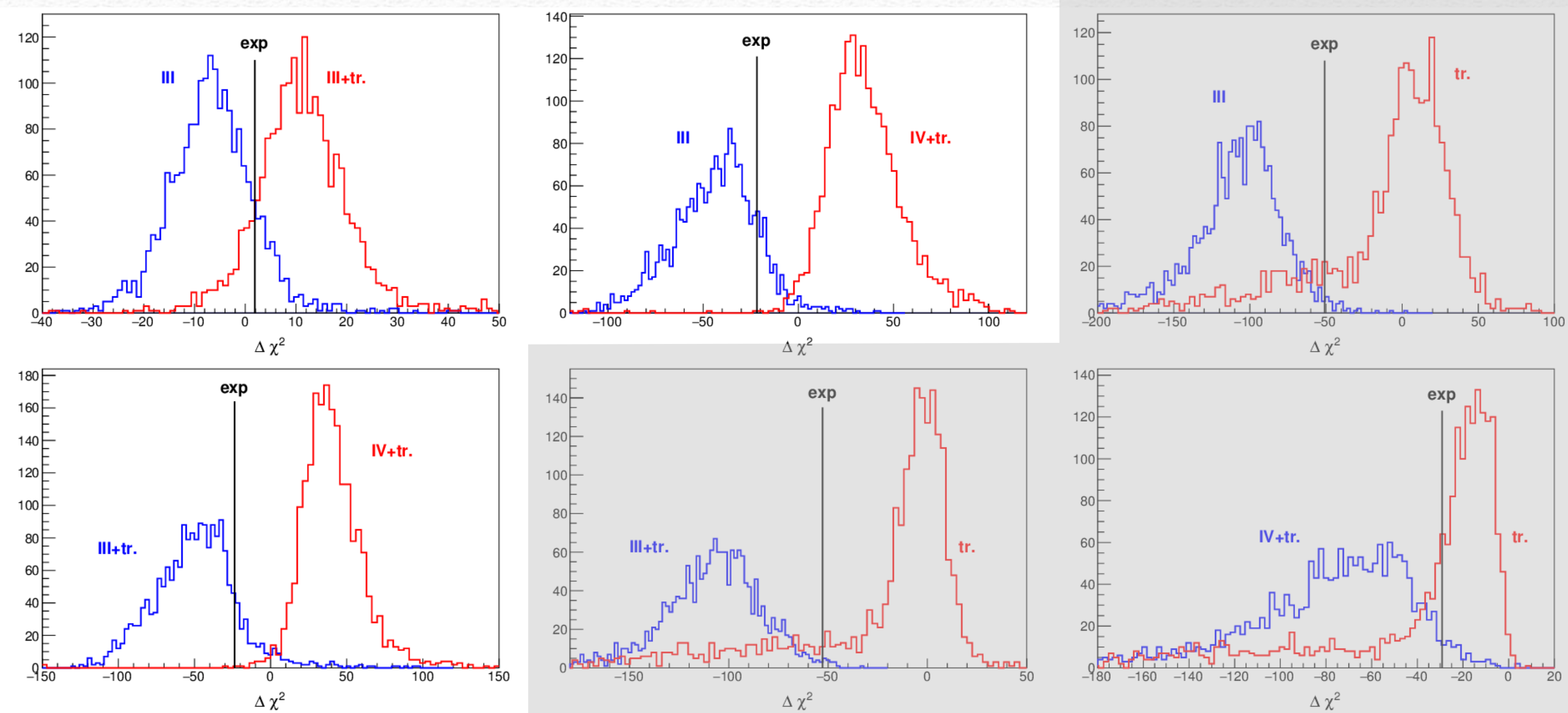


Figure 8: Same as Figure 7, but for E_{CM} = 4.23 GeV.

Statistical analysis



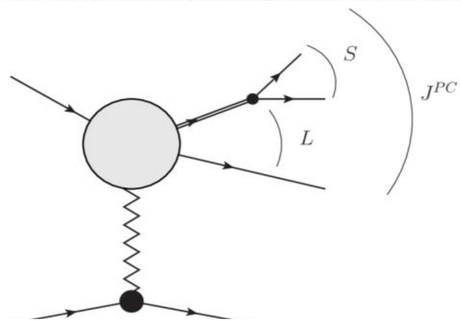
Toy experiments according to the different hypotheses, to estimate the relative rejection of various scenarios

Scenario	III+tr.	IV+tr.	tr.
III	1.5σ (1.5σ)	1.5σ (2.7σ)	"2.4σ" ("1.4σ")
III+tr.	–	1.5σ (3.1σ)	"2.6σ" ("1.3σ")
IV+tr.			"2.1σ" ("0.9σ")

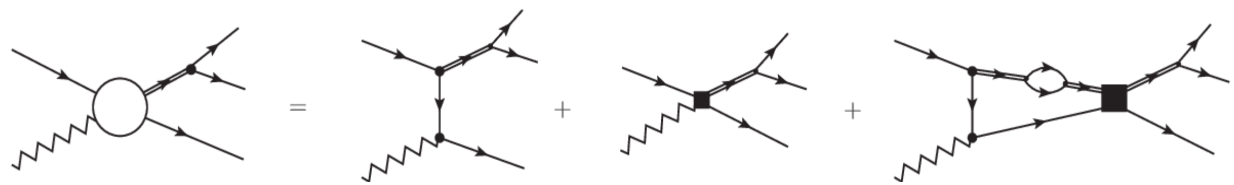
Not conclusive at this stage

PWA of 3π sytem

We start from 2^{-+}, long standing puzzle about $\pi_2(1670) - \pi_2(1880)$ interplay



$$F_{LS}(s) = b_{LS}(s) + h_L \bar{T}(s) c_{L'S'} + \frac{h_L \bar{T}(s)}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\rho(s') b_{L'S'}(s') h_L(s')}{s' - s - i0} \mathrm{d}s'$$



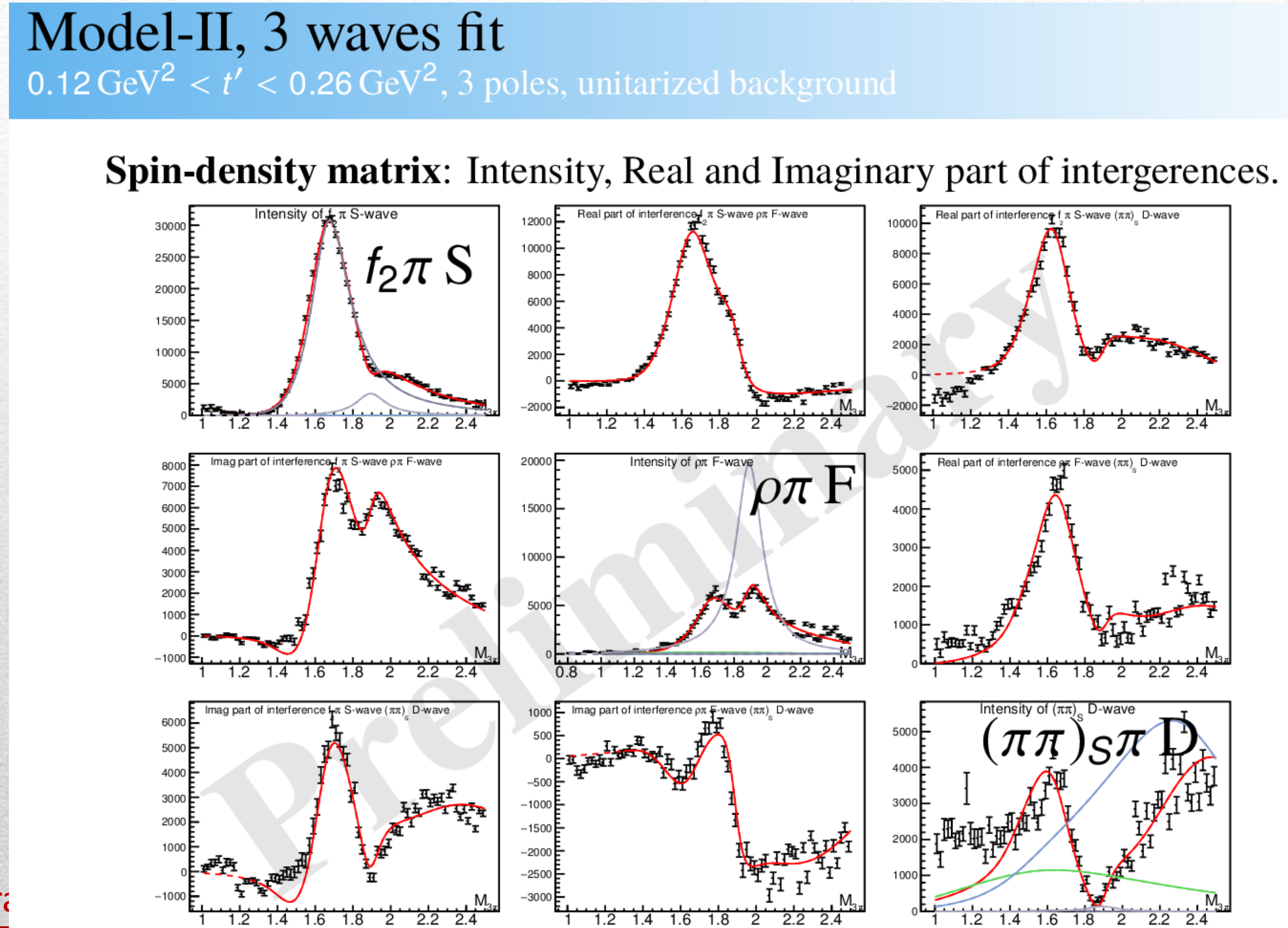
- The rescattering (Unitarisation) term has to be added to preserve usitarity.
- Shape of the background is fixed by projections of one-pion-exchange diagram
- Fit parameters are strengths of background for each channel, production constants c_{LS} and K-matrix parameters.

Details of one-pion-exchange amplitude calculations

- Pomeron trajectory $(s/s_0)^{\alpha(t)}$, $s_0 = 1\,\text{GeV}^2$, $\alpha(t) = 1$.
- Pion propagator is not "reggeized"
- Proton spin and structure is neglected
- Isobar decay amplitude is taken out, remaining isobar mass dependence is smeared out.

A. Jackura, M. Mikhasenko (JPAC), in progress

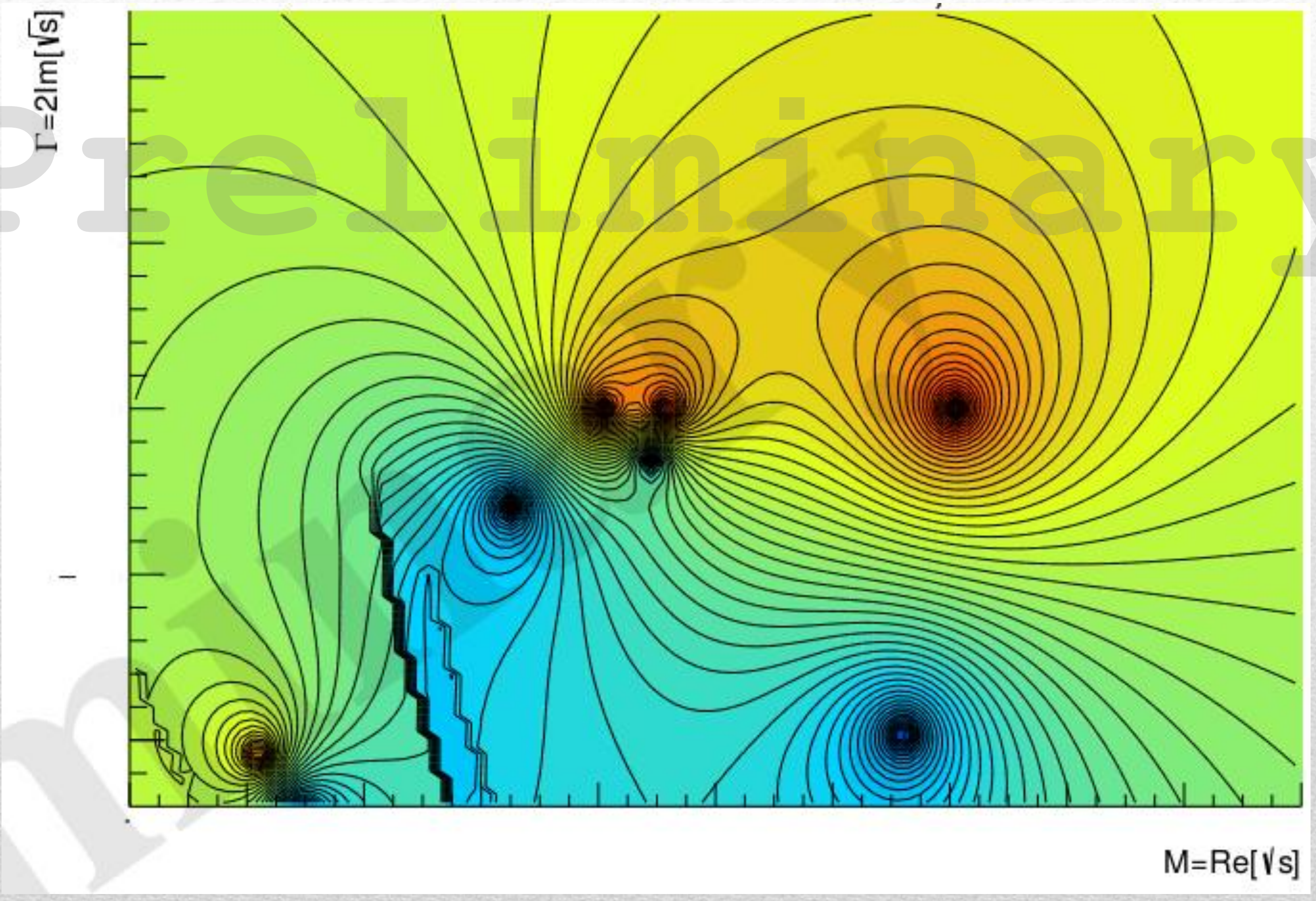
PWA of 3π sytem



A. Jackura

PWA of 3π sytem

We start from 2^{-+}, long standing puzzle about $\pi_2(1670) - \pi_2(1880)$ interplay



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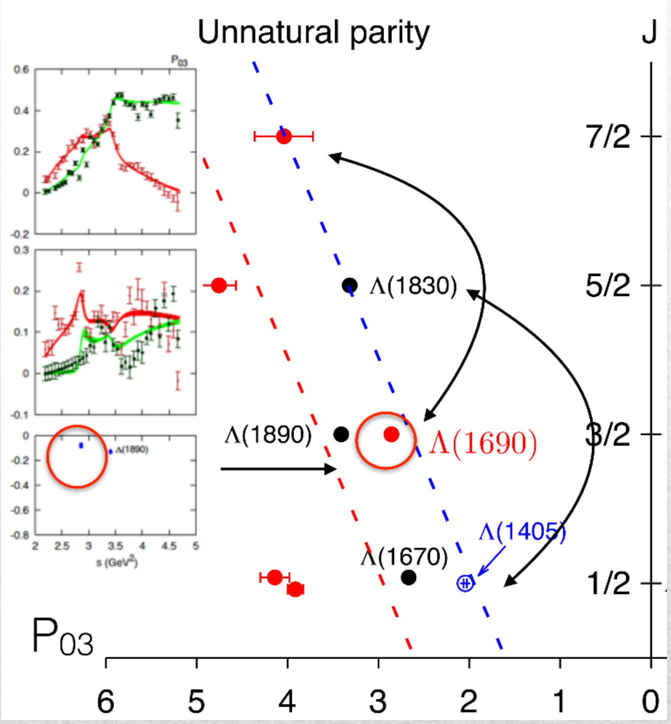
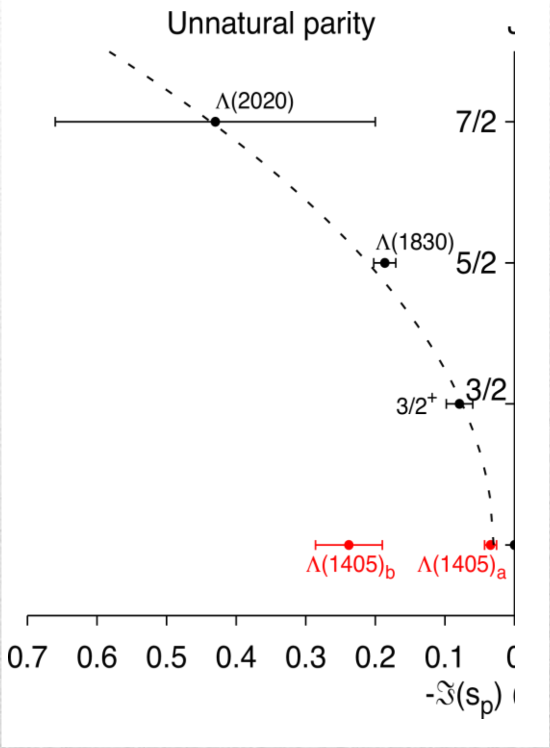
$K N$ scattering and the $\Lambda(1405)$

Coupled-channel K matrix model (up to 13 channels per partial wave), analyticity in angular momentum enforced, fit to KSU partial waves

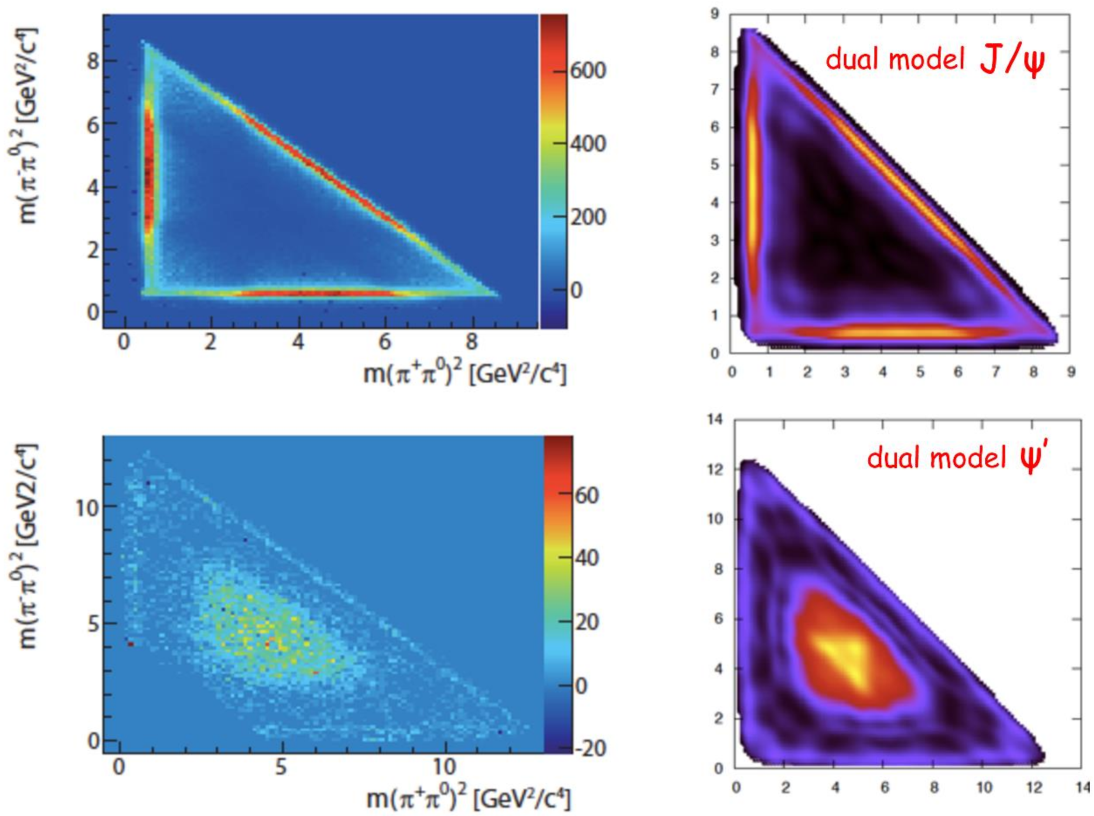




One of the $\Lambda(1405)$ poles is out of the trajectory → non 3-q state

Fernandez-Ramirez *et al*. (JPAC), PRD93, 034029
Fernandez-Ramirez *et al*. (JPAC), PRD93, 074015

$\psi^{(\prime)} \to \pi^+\pi^-\pi^0$ within dual models



$$A(s,t) = \frac{\Gamma(-J(s))\Gamma(-J(t))}{\Gamma(-J(s)-J(t))}$$

BESIII, Phys.Lett. B710 (2012) 594-599

Szczepaniak and Pennington, PLB737, 283