Nonperturbative Applications of the Functional Renormalization Group

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Bound states in strongly coupled systems, GGI, Arcetri
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Plan of the Talk

- Introduction to the functional renormalization group (FRG)
- Review of applications to the chiral Ising universality class
- A glimpse of supersymmetry
- Towards total asymptotic freedom in (beyond) the standard model
- Remarks on gauge symmetry in the FRG

Wilsonian RG

- Regularization cutoff Λ

$$e^{W} = \int^{\Lambda} \mathcal{D}\phi \, e^{-S[\phi]} \equiv \int \mathcal{D}\phi \, e^{-S^{\Lambda}[\phi]}$$

- Coarse-graining, rescaling, renormalization

$$\Lambda \to \Lambda' \,, \quad x' = x \frac{\Lambda}{\Lambda'} \,, \quad \phi' = \phi \left(\frac{\Lambda'}{\Lambda} \right)^{d_\phi} \,, \quad e^{W} = \int \mathcal{D}\phi' \, e^{-S^{\Lambda'}[\phi']}$$

- RG flow $\qquad S^{\Lambda} \twoheadrightarrow S^{\Lambda'}$

$$\Lambda \frac{\partial}{\partial \Lambda} S^{\Lambda}[\phi] = \cdots$$

Functional RG

1PI FRG (Bonini, D'Attanasio, Marchesini, Wetterich, Morris, Ellwanger '93)

- Quadratic regularization

$$e^{W_k[J]} = \int \mathcal{D}\phi \, e^{-S[\phi]-\Delta S_k[\phi]+J\phi}$$

$$\Delta S_k[\phi] = \frac{1}{2}\phi(-p)R_k(p^2)\phi(p)$$

- Effective average action Γ_k

$$\Gamma_k[\phi] = \sup_J\{J\phi - W_k[J]\}$$

- Flow equation

$$\partial_k\Gamma_k = \tfrac{1}{2}\,\mathrm{STr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1}\partial_k R_k\right]$$

Functional RG

Truncations: systematic nonperturbative approximation schemes

- Derivative expansion

$$\Gamma_k[\phi] = U_k(\phi) + \frac{1}{2} Z_k(\phi) \partial_\mu \phi \partial^\mu \phi + \cdots$$

- Vertex expansion

$$\Gamma_k[\phi] = \Gamma_k^{(0)} + \frac{1}{2} \Gamma_k^{(2)}(p_1, p_2) \phi(p_1) \phi(p_2) + \frac{1}{3!} \Gamma_k^{(3)}(p_1, p_2, p_3) \phi(p_1) \phi(p_2) \phi(p_3) + \cdots$$

- Scaling Field Expansion, BMW, Momenta expansion, . . .

Nonperturbative Dynamics: Critical Phenomena

- Gross–Neveu–Yukawa model

$$\mathcal{L}_{\mathrm{GNY}} = \frac{1}{2}\,\partial^\mu\phi\partial_\mu\phi + \frac{\bar{m}^2}{2}\phi^2 + \frac{\bar{\lambda}_2}{2}\phi^4 + \bar{\psi}\gamma^\mu \mathrm{i}\partial_\mu\psi + \mathrm{i}\,\bar{h}\phi\,\bar{\psi}\,\psi$$

- Chiral $\mathbb{Z}_2$ & flavor $\mathrm{U}(N_f)$ symmetries

- $\mathbb{Z}_2$ quantum phase transition in $d = 3$

  symmetric $\phi_0 = 0$ VS broken $\phi_0 \neq 0$

- Non-Gaussian FP, perturbative in $d = (4-\epsilon)$, $(2+\epsilon)$ or $N_f \to \infty$

- same as Gross-Neveu (or chiral Ising) universality class

Chiral Ising

(Rosa, Vitale, Wetterich '01)

$$\Gamma_k = \frac{1}{2} Z_\phi \, \partial^\mu \phi \partial_\mu \phi + \lambda_1 \rho + \frac{\lambda_2}{2} \rho^2 + \frac{\lambda_3}{3!} \rho^3 + Z_\psi \bar{\psi} \gamma^\mu \mathrm{i} \partial_\mu \psi + \mathrm{i} \bar{h} \, \phi \, \bar{\psi} \, \psi$$

where $\rho = \frac{\phi^2}{2}$

$d_\gamma N_f = 4, 6, 8, \ldots, 24$

For instance for $d_\gamma N_f = 4$:

- $\nu = 0.961 \,, \quad \eta_\phi = 0.561 \quad \eta_\psi = 0.066$

Chiral Ising

(Hofling, Nowak, Wetterich '02)

$$\Gamma_k = \frac{1}{2} Z_\phi \, \partial^\mu \phi \partial_\mu \phi + U_k(\rho) + Z_\psi \bar{\psi} \gamma^\mu \mathrm{i} \partial_\mu \psi + \mathrm{i} \bar{h} \, \phi \, \bar{\psi} \, \psi$$

where $\rho = \frac{\phi^2}{2}$

$d_\gamma N_f = 2, 4, 6, 8, \ldots, 24$

For instance for $d_\gamma N_f = 4$:

- $\nu = 0.961\,, \quad \eta_\phi = 0.561 \quad \eta_\psi = 0.066$
- $\nu = 0.927\,, \quad \eta_\phi = 0.525 \quad \eta_\psi = 0.071$

Chiral Ising

(Vacca, LZ '15)

$$\Gamma_k = \frac{1}{2} Z_\phi \, \partial^\mu \phi \partial_\mu \phi + U_k(\rho) + Z_\psi \bar{\psi} \gamma^\mu \mathrm{i} \partial_\mu \psi + \mathrm{i} H(\rho) \, \phi \, \bar{\psi} \, \psi$$

$d_\gamma N_f = 1, 2, 4, 8, \infty$

+ correction to scaling exponents
+ dimensional dependence

For instance for $d_\gamma N_f = 4$:

- $\nu = 0.961, \quad \eta_\phi = 0.561, \quad \eta_\psi = 0.066$
- $\nu = 0.927, \quad \eta_\phi = 0.525, \quad \eta_\psi = 0.071$
- $\nu = 0.929, \quad \eta_\phi = 0.602, \quad \eta_\psi = 0.069$

Chiral Ising

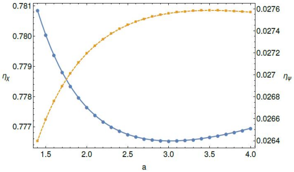
(Knorr '16)

$$\Gamma_k = \frac{1}{2} Z_\phi(\rho)\, \partial^\mu\phi\partial_\mu\phi + U_k(\rho) + Z_\psi(\rho)\bar\psi\gamma^\mu \mathrm{i}\partial_\mu\psi + \mathrm{i}H(\rho)\,\phi\,\bar\psi\,\psi$$
$$+\mathrm{i}J(\rho)\bar\psi(\partial_\mu\rho)\psi + X_1(\rho)\phi(\partial_\mu\bar\psi)(\partial^\mu\psi) + \mathrm{i}X_2(\rho)(\partial^\mu\phi)\left(\bar\psi\partial_\mu\psi - \text{p.i.}\right)$$
$$+X_3(\rho)\bar\psi(\partial^2\phi)\psi + X_4(\rho)(\partial_\mu\phi)\left(\bar\psi\Sigma^{\mu\nu}(\partial_\nu\psi) - \text{p.i.}\right) + X_5(\rho)(\partial^\mu\phi)^2\phi\bar\psi\psi$$

$d_\gamma N_f = 4, 8$ $\quad$ + optimization of regulator dependence

For instance for $d_\gamma N_f = 4$:

- $\nu = 0.961$, $\quad \eta_\phi = 0.561$, $\quad \eta_\psi = 0.066$
- $\nu = 0.927$, $\quad \eta_\phi = 0.525$, $\quad \eta_\psi = 0.071$
- $\nu = 0.929$, $\quad \eta_\phi = 0.602$, $\quad \eta_\psi = 0.069$
- $\nu = 0.930$, $\quad \eta_\phi = 0.551$, $\quad \eta_\psi = 0.065$



Chiral Ising

For instance for $d_\gamma N_f = 4$:

- $\nu = 0.930, \quad \eta_\phi = 0.551, \quad \eta_\psi = 0.065$ FRG [1]
- $\nu = 0.91, \quad \eta_\phi = 0.49, \quad \eta_\psi = 0.077(1)$ $(4-\epsilon)$, $O(\epsilon^4)$ [2]
- $\nu = 0.877, \quad \eta_\phi = 0.54(6)$ Monte Carlo [3]
- $\nu = 1.3, \quad \eta_\phi = 0.54, \quad \eta_\psi = 0.084$ conformal bootstrap [4]

[1] Knorr '16

[2] Zerf, Mihaila, Marquard, Herbut, Scherer '17

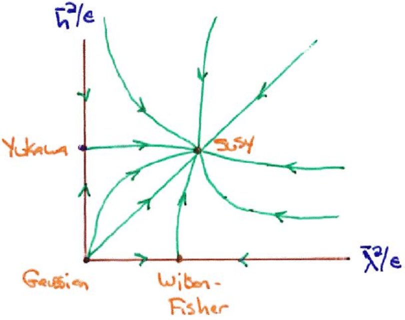
[3] Huffman, Chandrasekharan '17

[4] Iliesu, Kos, Poland, Pufu, Simmons-Duffin '17

Emergent Supersymmetry: $\mathcal{N} = 1$

For $d_\gamma N_f = 1$ emergent $\mathcal{N} = 1$ supersymmetry (Thomas '05)

- The FP enjoy SUSY: one Majorana + one real scalar, $\lambda_2 = h^2$
- There are only two relevant directions
- One is a SUSY-preserving massive deformation $m_\phi = m_\psi$
- One is a SUSY-breaking massive deformation $m_\phi \neq m_\psi$
- A one-parameter tuning results in long distance SUSY



Emergent Supersymmetry: $\mathcal{N} = 1$

On-shell SUSY is nonlinear: FRG breaks it

Off-shell SUSY is linear: FRG preserves it

$d_\gamma N_f = 1$

- $\nu = 0.693, \quad \eta_\phi = 0.154, \ \eta_\psi = 0.221$ FRG [1]
- $\nu = 0.722, \quad \eta_\phi = \eta_\psi = 0.174$ FRG [2]
- $\nu = 0.717, \quad \eta_\phi = \eta_\psi = 0.167$ FRG [2]
- $\nu = 0.707, \quad \eta_\phi = \eta_\psi = 0.171$ $(4 - \epsilon), \ O(\epsilon^4)$ [3]
- $\eta_\phi = \eta_\psi = 0.164$ conformal bootstrap [4]

[1] Vacca, LZ '15

[2] Gies, Hellwig, Wipf, Zanusso '17

[3] Zerf, Mihaila, Marquard, Herbut, Scherer '17

[4] Iliesu, Kos, Poland, Pufu, Simmons-Duffin, Yacoby '16

Emergent Supersymmetry: $\mathcal{N} = 2$

- Nambu–Jona-Lasinio–Yukawa model

$$\mathcal{L}_{\mathsf{NJLY}} = \partial_\mu \bar{\phi} \partial^\mu \phi + \frac{\bar{\lambda}_2}{2} \left(\bar{\phi} \phi \right)^2 + \bar{\psi} \gamma^\mu \mathrm{i} \partial_\mu \psi + \mathrm{i}\, \bar{h}\, \bar{\psi} \left(\phi_1 + \mathrm{i} \gamma_5 \phi_2 \right) \psi$$

- Chiral U(1) & Flavor U(N_f) symmetries

$$\psi \to e^{\mathrm{i}\alpha\gamma_5} \psi \,, \quad \phi \to e^{-2\mathrm{i}\alpha} \phi$$

For $d_\gamma N_f = 2$ emergent $\mathcal{N} = 2$ supersymmetry (Thomas '05)

- The FP enjoy SUSY: one Dirac + one complex scalar, $\lambda_2 = h^2$
- i.e. the reduction of the $d = 4$, $\mathcal{N} = 1$ Wess-Zumino model

$$\mathcal{L}_{3,\mathsf{WZ}} = Z \left(\partial_\mu \bar{\phi} \partial^\mu \phi + \bar{\psi} \gamma^\mu \mathrm{i} \partial_\mu \psi - \bar{f} f \right) - \left(W'(\phi) f - \frac{1}{2} W''(\phi) \psi^T \gamma^2 \psi + \mathsf{h.c.} \right)$$

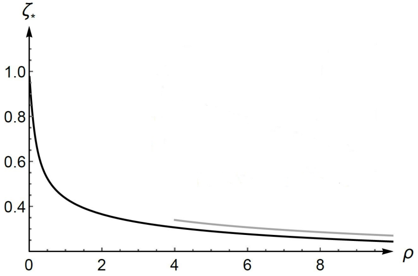
Emergent Supersymmetry: $\mathcal{N} = 2$

$$\mathcal{L}_{3,\mathrm{WZ}} = -\frac{1}{4}\int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\, K\big(\Phi, \bar{\Phi}\big) - \left\{\frac{1}{2\mathrm{i}}\int \mathrm{d}^2\theta\, W(\Phi) + \text{h.c.}\right\}$$

where $\Phi = \phi + \sqrt{2}\,\theta^T\gamma^2\phi + \theta^T\gamma^2\theta\, f$

- Nonrenormalization of the superpotential W
- Exact $\eta = \frac{1}{3}$ (Aharony, Hanany, Intriligator, Seiberg, Strassler '97)
- Kähler potential K is unprotected (metric $\zeta(\rho) = \partial\bar{\partial}K$)

(Feldmann, Wipf, LZ '17)



Emergent Supersymmetry: $\mathcal{N} = 2$

First correction-to-scaling exponent $\qquad \omega = 2 - \nu^{-1}$

- $\omega = 0.8344$ FRG [1]
- $\omega = 0.871(1)$ $(4 - \epsilon)$, $O(\epsilon^4)$ [2]
- $\omega = 0.9098(20)$ conformal bootstrap [3]

[1] Feldmann, Wipf, LZ '17
[2] Zerf, Mihaila, Marquard, Herbut, Scherer '17
[3] Bobev, El-Showk, Mazac, Paulos '15

Towards the Nonperturbative Standard Model?

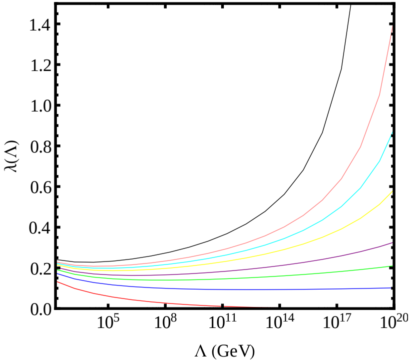
Lessons from critical phenomena:

- Nonperturbative approximations are controllable
- Regulator dependence can be optimized
- Chiral symmetry can be preserved
- Supersymmetry can be preserved
- The FRG provides good estimates of nonperturbative observables

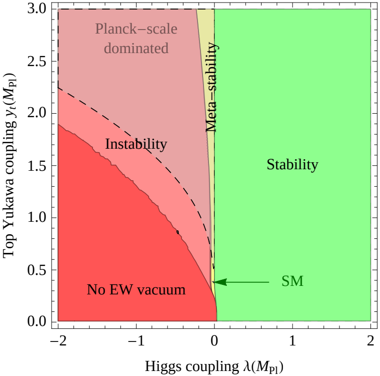
Issues in approaching the standard model (SM):

- Can we test triviality?
- Can gauge symmetry be preserved?

RG Flow of λ in the Standard Model



(Holthausen, Lim, Lindner '12)



(Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia '13)

Light Higgs = almost vanishing self-interaction to high scales

Total Asymptotic Freedom (TAF): Known Facts

Perturbatively renormalizable TAF models have been classified:

(Cheng, Eichten, Li '74) (Chang '74) (Fradkin, Kalashnikov '75)
(Chang, Perez-Mercader '78) (Bais, Weldon '78) (Callaway '88)

(Giudice, Isidori, Salvio, Strumia '15) (Holdom, Ren, Zhang '15)
(Hetzel, Stech '15) (Pelaggi, Strumia, Vignali '15)
(Pica, Ryttov, Sannino '16) (Molgaard, Sannino '16)

strong constraints on matter content and symmetries!

no TAF in the SM, guiding principle for BSM

Total Asymptotic Freedom (TAF): New Results

How close can TAF be to the SM?

NEW! TAF is possible already in the generic nonabelian Higgs model

(Gies, LZ '15 & '17)

Where is the loophole?

NEW!

Higher-dimensional operators are needed

&

TAF is realized OUT of the Deep Euclidean Limit

The vev scales like the RG scale in the UV: $v^2 \sim k^2$

Standard One-Loop Analysis

Non-Abelian Higgs model + fermions

$$\mathcal{L}_{\rm cl} = \frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + (D^\mu \phi)^\dagger (D_\mu \phi) + \frac{\bar{\lambda}}{2} \left(\phi^\dagger \phi\right)^2 + i\bar{\psi} \not{D} \psi$$

$$\beta_{g^2} = -b_0 g^4 \ , \quad \beta_\xi = g^2 (A\xi^2 + B\xi + C), \quad \xi = \frac{\lambda}{g^2}$$

UV asymptotics $\lambda \sim g^2 =$ RG FP ξ_* (Gross, Wilczek '73)

TAF condition: $\Delta = B^2 - 4AC > 0$

e.g.: holds for SU($N \gg 1$) and $b_0 \approx 0$ (many fermions)

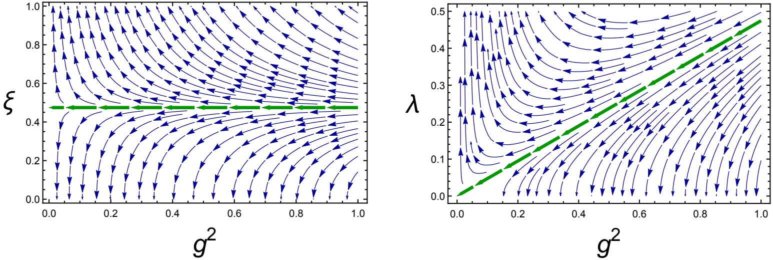
Higher-Dimensional Operators

EFT analysis = polynomial truncation of the local potential

$$U = \frac{\lambda}{2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 + \frac{\lambda_3}{6k^2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^3 + \dots$$

$$\beta_\xi = g^2\left(A\xi^2 + B\xi + C\right) - \frac{1}{g^2}\left(\frac{\lambda_3}{16\pi^2} + \frac{9\lambda_3}{64\pi^2\xi}\right) + \dots$$

$\xi_* > 0$ if $\lambda_3/g^4 = \chi$ is kept free and nonvanishing



Line of Fixed Points

$\lambda/g^2 = \xi, \quad \lambda_3/g^4 = \chi, \quad v^2/k^2 = 2\kappa$ finite and nonvanishing

Higher-dimensional operators suppressed by higher powers of g^2

$$x = (g Z_\phi/k^2)\ \phi^\dagger\phi\ , \qquad f(x) = k^{-4} U$$

Weak-g^2 expansion:

$$\beta_f = -4\, f(x) + 2x f'(x) - g \left(\frac{9x}{64\pi^2} + \frac{f'(x)}{8\pi^2} + \frac{x f''(x)}{16\pi^2} \right) + O(g^2)$$

FP condition = linear ODE

Line of Fixed Points

$\lambda/g^2 = \xi, \quad \lambda_3/g^4 = \chi, \quad v^2/k^2 = 2\kappa$ finite and nonvanishing

Higher-dimensional operators suppressed by higher powers of g^2

$$x = (g Z_\phi/k^2)\ \phi^\dagger \phi\ , \qquad f(x) = k^{-4} U$$

Weak-g^2 expansion:

$$f(x) \sim \frac{\xi}{2}\ x^2 - \frac{3(3+4\xi_*)}{128\pi^2} g\ x + O(g^2)$$

Stable potential!

One free parameter (here ξ) = boundary condition of a linear ODE

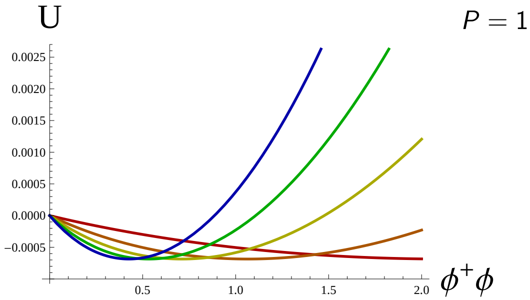
= ambiguity in resumming an infinite number of vertices

Generalization: $\lambda \sim g^{4P}$

Look for scaling solutions: $\lambda = g^{4P}\xi > 0$

$P \in (0, 1/2)$ is possible if $\lambda_3 = g^{8P}\chi$

$P \in (1/2, +\infty)$ is possible if $v^2/k^2 = 2\kappa \to +\infty$ in the UV



Conclusions

A novel understanding of nonabelian Higgs models :

- they generically allow for Total Asymptotic Freedom (TAF)
- TAF is compatible with any wanted Higgs/W mass ratio
- small number of free UV parameters (g^2, v^2, λ, P)
- infinitely many higher-dimensional operators predicted

Work in progress (Gies, Sondenhemer, Ugolotti, LZ)

Similar new TAF solutions

- have been revealed also in $\mathbb{Z}_2-$Yukawa$-$QCD models
- can be constructed in other RG schemes, e.g. $\overline{MS}$

Conclusions

A novel understanding of nonabelian Higgs models :

- they generically allow for Total Asymptotic Freedom (TAF)
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Take away points

These new TAF solutions

- require functional methods (i.e. ∞-many couplings) to be revealed
- are visible in EFT studies, if one allows for free parameters
- show singularities at vanishing fields, as well as nontrivial minima

(Coleman, Weinberg '73)

Gauge Symmetry in the FRG

A quadratic mass-like regulator breaks gauge symmetry

- One could simply not care?!
  UV fine tuning for IR restoration
- Discover novel FRG equations that preserve the symmetry
  higher derivative, Pauli−Villars, propertime
- Preserve part of the symmetry
  background field method
- Treat the breaking as a gauge fixing?
  Work in progress (Asnafi, Gies, LZ)

Mass as a Nonlinear Gauge Fixing

Consider the generating functional

$$Z(\nu) = \int \mathcal{D}A\mathcal{D}c\mathcal{D}\bar{c}\mathcal{D}\lambda\, e^{-S_{\rm YM}(A) - S_{\rm gauge}(A,c,\bar{c},\lambda,\nu)}\,,$$

where

$$S_{\rm gauge} = \lambda^a\,(F^a[A] - \nu^a) - \bar{c}^a M^{ab} c^b\,,$$

and

$$M^{ab} = \left.\frac{\delta F^a[A]}{\delta A^c_\mu}\,\frac{\delta A^{\omega\;c}_\mu}{\delta \omega^b}\right|_{\omega=0} = \frac{\delta F^a[A]}{\delta A^c_\mu}\,D^{cb}_\mu\,.$$

Here ν^a is a noise field, and λ^a a Nakanishi-Lautrup field.

Mass as a Nonlinear Gauge Fixing

Average over the noise with measure $\mu(\nu)$

$$\int \mathrm{d}\mu(\nu)\, e^{\lambda^a \nu^a} = e^{w(\lambda)}\,.$$

A Gaussian weight

$$\mu(\nu) = e^{-\frac{1}{2\xi}\nu^a \nu^a}\,,$$

corresponds to

$$w(\lambda) = -\frac{\xi}{2}\lambda^a \lambda^a\,,$$

which entails

$$S_{\mathrm{gf}} = \frac{1}{2\xi} F^a F^a\,.$$

Mass as a Nonlinear Gauge Fixing

Average over the noise with measure $\mu(\nu)$

$$\int \mathrm{d}\mu(\nu)\, e^{\lambda^a \nu^a} = e^{w(\lambda)}\,.$$

A Fourier weight

$$\mu(\nu) = e^{v^a \nu^a}\,,$$

corresponds to

$$e^{w(\lambda)} = \delta\left[\lambda^a - v^a\right]\,,$$

which entails

$$S_{\mathrm{gf}} = v^a F^a\,.$$

v^a is a fixed vector

Mass as a Nonlinear Gauge Fixing

For any $\mu(\nu)$ one has the off-shell BRST symmetry

$$sA_\mu^a = D_\mu^{ab} c^b\,, \qquad sc^a = \frac{1}{2} f^{abc} c^b c^c\,, \qquad s\lambda^a = 0\,, \qquad s\bar{c}^a = \lambda^a\,.$$

A Fourier weight gives the same on-shell BRST symmetry, with

$$\lambda^a = v^a$$

which is nihilpotent

Mass as a Nonlinear Gauge Fixing

Add sources

$$S_{\rm so} = K_\mu^a \, (D^\mu c)^a + L^a \frac{1}{2} g f^{abc} c^b c^c - J^{\mu a} A_\mu^a - \bar{\eta}^a c^a - \bar{c}^a \eta^a - \lambda^a \ell^a \, .$$

Define the effective action

$$\Gamma[A, c, \bar{c}, \lambda, K, L] = \sup_{J,\eta,\bar{\eta}} \left\{ J^{\mu a} A_\mu^a + \bar{\eta}^a c^a + \bar{c}^a \eta^a + \lambda^a \ell^a - W[J, \eta, \bar{\eta}, \ell, K, L] \right\} .$$

The master equation reads

$$\frac{\delta \Gamma}{\delta A^{\mu a}} \frac{\delta \Gamma}{\delta K_\mu^a} + \frac{\delta \Gamma}{\delta c^a} \frac{\delta \Gamma}{\delta L^a} + \lambda^a \frac{\delta \Gamma}{\delta \bar{c}^a} = 0 \, .$$

Mass as a Nonlinear Gauge Fixing

Accommodate a nonlinear gauge fixing

$$F^a = v^a \tilde{A}^b_\mu \tilde{A}^{\mu b} + f \partial^\mu A^a_\mu \,,$$

where

$$\tilde{A}^a_\mu = \ell_{\mu\nu} A^{\nu a} \,.$$

Lorentz gauge plus an additive quadratic regularization

$$\ell_{\mu\nu} = \sqrt{-\Box + R_1(-\Box)}\,\Pi_{/\!/\mu\nu} + \sqrt{R_2(-\Box)}\,\Pi_{\perp\mu\nu}$$

Then take ∂_k and go with the flow ...

Thank You!

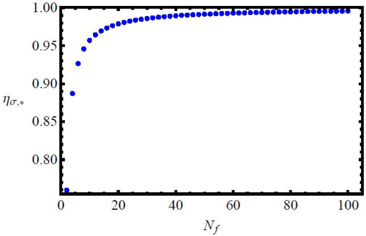
Backup

Chiral Ising

(Scherer, Braun, Gies '13)

$$\Gamma_k = \frac{1}{2} Z_\phi\, \partial^\mu \phi \partial_\mu \phi + U_k(\rho) + Z_\psi \bar{\psi} \gamma^\mu \mathrm{i} \partial_\mu \psi + \mathrm{i} \bar{h}\, \phi\, \bar{\psi}\, \psi$$

$d_\gamma N_f = 2, 4, 6, 8, \ldots, \infty \qquad (d_\gamma = 2) \qquad +$ correction to scaling exponents



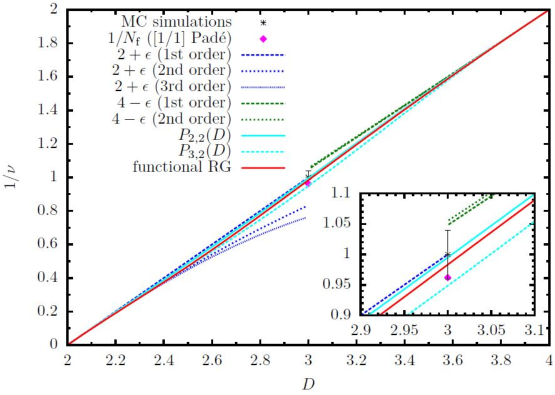
N_f	Θ^1	Θ^2	Θ^3	Θ^4	Θ^5	Θ^6
2	0.9821	−0.8722	−1.0916	−3.5135	−6.0514	−8.5820
4	0.9775	−0.9240	−1.1010	−3.3910	−5.7739	−8.2429
12	0.9903	−0.9735	−1.0506	−3.1810	−5.3665	−7.6004
50	0.9975	−0.9936	−1.0143	−3.0510	−5.1062	−7.1789
100	0.9987	−0.9968	−1.0073	−3.0263	−5.0550	−7.0934
∞	1	−1	−1	−3	−5	−7

Chiral Ising

(Janssen, Herbut '14)

$$\Gamma_k = \frac{1}{2} Z_\phi\, \partial^\mu \phi \partial_\mu \phi + U_k(\rho) + Z_\psi \bar{\psi} \gamma^\mu \mathrm{i} \partial_\mu \psi + \mathrm{i} \bar{h}\, \phi\, \bar{\psi}\, \psi$$

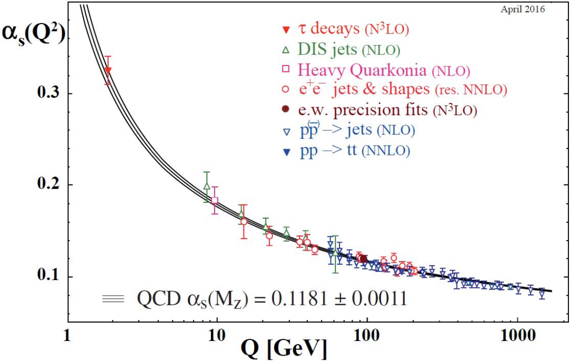
$d_\gamma N_f = 8$ + dimensional dependence



Asymptotic freedom in QCD

RG equation: $\partial_t g^2 = \beta_{g^2} = -b_0 g^4,$

$$b_0 = \frac{1}{12\pi^2}\left(\frac{11}{2}\mathrm{N} - \mathrm{N}_f\right) > 0$$

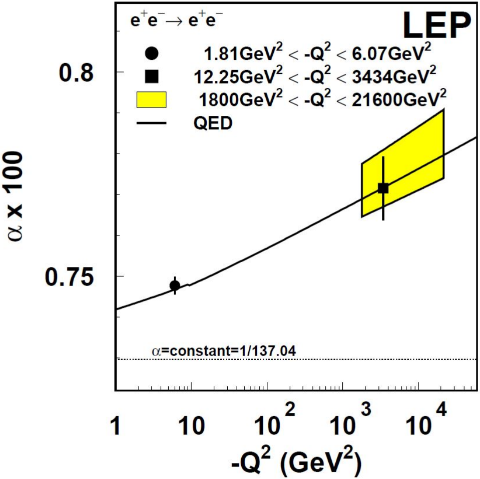


(Particle Data Group '16)

RG Flow of QED

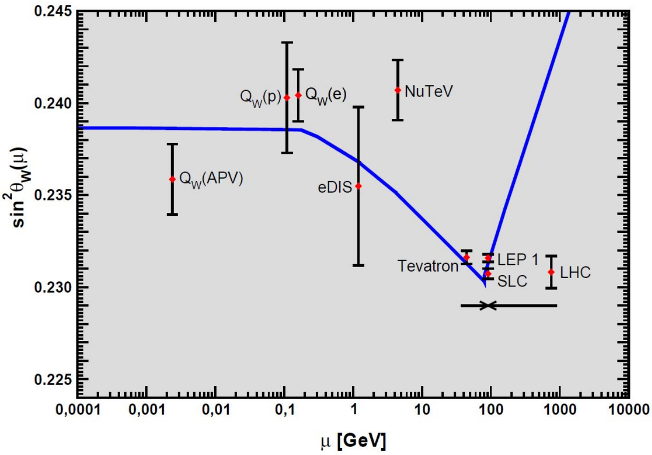
RG equation: $\partial_t e^2 = \beta_{e^2} = a_0 e^4,$

$$a_0 = \frac{1}{6\pi^2} > 0$$



(Mele '06)

RG Flow of $\sin^2\theta_W = e^2/g^2$ in the Standard Model



(Particle Data Group '16)

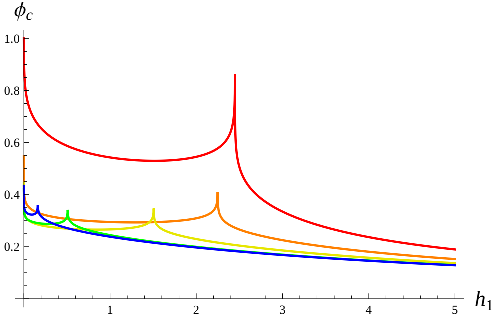
RG Flow of Simple Scalar Theories

Pure scalar theory: $\partial_t \lambda = \beta_\lambda = A\lambda^2,$

$$A = \frac{3}{16\pi^2} > 0$$

Yukawa theories:

(Vacca, LZ '15)



Motivations

Triviality of the Higgs sector:

Landau pole for the Higgs self-interaction

$$\beta_\lambda^{1\text{loop}} = a\lambda^2, \qquad \lambda_\Lambda = \frac{\lambda_R}{1 - a\lambda_R \log\frac{\Lambda}{m_R}} \qquad (a > 0)$$

$$\Lambda \to \infty \iff \lambda_R = 0 \text{ trivial QFT}$$
$$\Lambda < \infty \iff \lambda_R \neq 0 \text{ effective QFT}$$

confirmed by lattice for pure scalar theories

Generalization: $\lambda \sim g^{4P}$

Back to

$$\beta_\lambda = A\lambda^2 + B'\lambda g^2 + Cg^4 - \left(\frac{\lambda_3}{16\pi^2} + \frac{9\lambda_3 g^2}{64\pi^2\lambda}\right) + \ldots$$

look for scaling solutions: $\lambda = g^{4P}\xi$

if $0 < P < 1/2$ and $\lambda_3 = g^{8P}\chi$ then

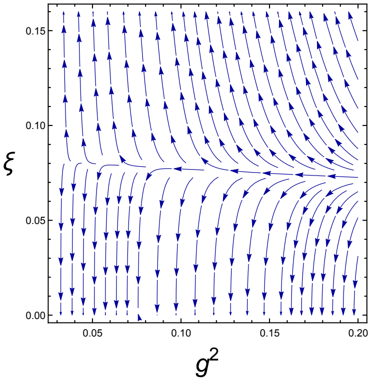
$$\beta_\xi = \left(A\xi^2 - \frac{\chi}{16\pi^2}\right) g^{4P} + O(g^2)$$

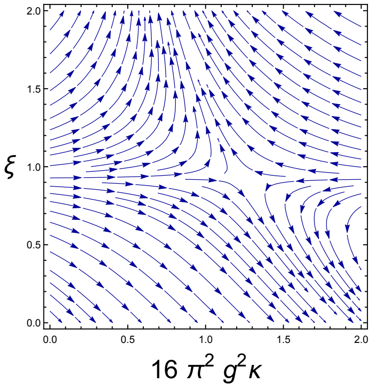
Fixed points at $\xi > 0$ if χ is kept free and nonvanishing

Relevant − Irrelevant

For all solutions: v^2 = relevant, λ = marginally irrelevant

$$P = 1$$

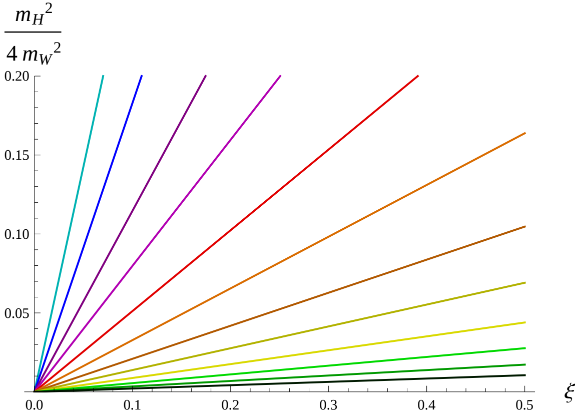




Higgs Phase and Masses

Every TAF scaling solution can be connected to the Higgs phase

- by choosing compatible scale-dependent boundary conditions
- by adding a suitable relevant component



$$S_{\rm so} = -J^\mu_a A^a_\mu - \bar{c}^a \eta_a - \bar{\eta}^a c^a$$
$$+K^\mu_a (D_\mu c^a) - M^\mu_a \bar{c}^a \tilde{D}_\mu c^a - \bar{c}^a \tilde{A}^a_\mu I^\mu_a - L^a \tfrac{g}{2} f^{abc} c^b c^c$$

Effective action

$$\Gamma_k[A, \bar{c}, c, K, \bar{I}, L, M] = (J^\mu_a A^a_\mu + \bar{c}^a \eta_a + \bar{\eta}^a c^a) - W_k[J, \eta, \bar{\eta}, K, \bar{I}, L, M]$$

The master equation

$$\frac{\delta \Gamma_k}{\delta A^a_\mu} \frac{\delta \Gamma_k}{\delta K^a_\mu} - \frac{f}{\xi} v_a \frac{\delta \Gamma_k}{\delta \bar{c}^a} - \frac{f}{\xi} M^\mu_a \ell_{\mu\nu} \frac{\delta \Gamma_k}{\delta K^a_\nu} - \frac{f}{\xi} \tilde{A}^a_\mu I^a_\mu$$
$$+ \frac{\delta \Gamma_k}{\delta M^\mu_a} I^\mu_a + \frac{\delta \Gamma_k}{\delta c^a} \frac{\delta \Gamma_k}{\delta M^a} = 0 \qquad (1)$$