Bound states and Interquark Potential in High Temperature Lattice Gauge Theories

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Summary:

1. Introduction and motivations
2. The ξ/ξ_{2nd} ratio
3. Effective String Model
4. Bound states in the deconfined phase and conformal perturbation

Introduction and motivations

- The hadronic spectrum of QCD can be studied using the Lattice regularization and Montecarlo simulations.
- The key ingredient is the evaluation of the interquark potential using Polyakov Loop correlators.
- In the limit of infinite mass quarks (i.e. approximating QCD with a pure Lattice Gauge Theory) these correlators can be studied with two powerful effective models: Conformal Perturbation and Effective String Models. Within their range of validity these effective models agree very well with high precision Montecarlo Simulations.
- Using these effective theories one can see that besides the standard hadronic states non Abelian Lattice Gauge Theories (LGTs) have a rich spectrum of non trivial excited/bound states with a highly non-trivial temperature dependence.
- This finding is confirmed by high precision montecarlo simulations in different models ($SU(N)$ LGTs both in 2+1 and 3+1 dimensions) and different regimes (both low T and high T)

Lattice determination of the interquark potential.

In pure lattice gauge theories at Finite Temperature the interquark potential is extracted from Polyakov loop correlators $< P(0)P(R)^\dagger >$

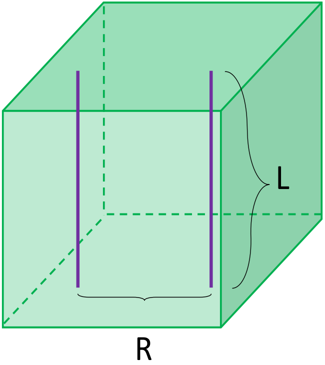
$$< P(0)P(R)^\dagger > \sim \sum_{n=0}^{\infty} c_n \; e^{-LE_n}$$

where L is the inverse temperature, i.e. the length of the lattice in the compactified imaginary time direction.
At low temperature the potential is dominated by the ground state energy E_0

$$E_0 = V(R) = -\lim_{L\to\infty} \frac{1}{L} \log < P(0)P(R)^\dagger >$$

Polyakov loop correlator.

Expectation value of two Polyakov loops at distance R and Temperature $T = 1/L$



$$V(R) = -\lim_{L\to\infty} \frac{1}{L} \log < P(0)P(R)^{\dagger} >$$

Montecarlo Simulations

Can we test if there are excited states using Montecarlo Simulations?
Which is their energy? Is there a dependence on the finite temperature T?

A direct search of subleading exponentials in the Polyakov loop correlator is difficult due to the presence of the ground state.

There are two possible ways to address the problem

- Construct a basis of lattice operators (generalizations of Polyakov loops with different shapes) and evaluate the matrix of cross-correlations. The eigenvalues of this matrix are the E_n energy states that we are looking for[1].

  This strategy is very effective but very intensive from a computational point of view and at the moment out of reach for real QCD simulations.

- A much simpler and economic way to address this issue is to study the ξ/ξ_{2nd} ratio[2]

[1] Brandt, B.B. J. High Energ. Phys. (2017) 2017: 8. arXiv:1705.03828
[2] M. Caselle, A. Nada, Phys. Rev. D 96, 074503 (2017), arXiv:1707.02164

ξ versus ξ_{2nd} in spin models

In a d-dimensional spin model the exponential correlation length ξ describes the long distance behavior of the connected two point function.

$$\frac{1}{\xi} = - \lim_{|\vec{n}| \to \infty} \frac{1}{|\vec{n}|} \log \langle s_{\vec{0}} s_{\vec{n}} \rangle_c \quad .$$

where

$$\langle s_{\vec{m}} s_{\vec{n}} \rangle_c = \langle s_{\vec{m}} s_{\vec{n}} \rangle - \langle s_{\vec{m}} \rangle^2$$

The square of the second moment correlation length ξ_{2nd} is defined as:

$$\xi^2_{2nd} = \frac{\mu_2}{2d\mu_0} \ ,$$

where

$$\mu_0 = \lim_{L \to \infty} \frac{1}{V} \sum_{\vec{m},\vec{n}} \langle s_{\vec{m}} s_{\vec{n}} \rangle_c$$

and

$$\mu_2 = \lim_{L \to \infty} \frac{1}{V} \sum_{\vec{m},\vec{n}} |\vec{m} - \vec{n}|^2 \langle s_{\vec{m}} \, s_{\vec{n}} \rangle_c \ ,$$

where $V = L^d$ is the lattice volume.

ξ versus ξ_{2nd} in spin models

ξ_{2nd} is not exactly equivalent to ξ. The difference is in general very small, but it carries important information on the spectrum of the underlying theory.

The relation between ξ and ξ_{2nd} can be understood introducing the “time slice” variables

$$S_{n_0} = \frac{1}{L^2} \sum_{n_1,n_2} s_{(n_0,n_1,n_2)}$$

and the “time-slice” correlation function

$$G(\tau) = \sum_{n_0} \left\{ \langle S_{n_0} S_{n_0+\tau} \rangle - \langle S_{n_0} \rangle^2 \right\} \ .$$

whose large distance behaviour is controlled by ξ

$$G(\tau) \sim \exp(-\tau/\xi) \ .$$

ξ versus ξ_{2nd} in spin models

Using time slice variables, μ_2 and μ_0 can be rewritten as:

$$\mu_2 \quad = \quad \frac{d}{V} \sum_{\vec{m},\vec{n}} (n_0 - m_0)^2 \ \langle s_{\vec{m}}\, s_{\vec{n}} \rangle_c \quad .$$

i.e.

$$\mu_2 = dL^2 \sum_{\tau=-\infty}^{\infty} \tau^2 \ \langle S_0 \ S_\tau \rangle_c$$

and

$$\mu_0 = L^2 \sum_{\tau=-\infty}^{\infty} \langle S_0 \ S_\tau \rangle_c \quad .$$

From which we have

$$\xi_{2nd}^2 = \frac{\sum_{\tau=-\infty}^{\infty} \tau^2 \ G(\tau)}{2\sum_{\tau=-\infty}^{\infty} \ G(\tau)} \quad .$$

ξ versus ξ_{2nd} in spin models

Assuming a multiple exponential decay for $G(\tau)$,

$$\langle S_0 \, S_\tau \rangle_c \propto \sum_i c_i \; \exp(-|\tau|/\xi_i) \quad ,$$

and replacing the summation by an integration over τ we get

$$\xi_{2nd}^2 = \frac{1}{2} \frac{\int_{\tau=0}^{\infty} \mathrm{d}\tau \; \tau^2 \sum_i c_i \; \exp(-\tau/\xi_i)}{\int_{\tau=0}^{\infty} \mathrm{d}\tau \; \sum_i c_i \exp(-\tau/\xi_i)} = \frac{\sum_i c_i \xi_i^3}{\sum_i c_i \xi_i} \; ,$$

which is equal to ξ^2 if only one state contributes. It is thus clear that we can use the ξ/ξ_{2nd} to have some insight on the spectrum of the theory and on the amplitude c_i of these states.

Example: the Ising case

	d	ξ/ξ_{2nd}	Method
High T phase	2	1.00040...	
	3	1.00016(2)	strong-coupling + ϵ-expansion [1]
	3	1.00021(3)	perturbative $d = 3$ calculation[2]
	3	1.000200(3)	strong-coupling expansion [1]
Low T phase	2	1.58188...	
	3	1.031(6)	Monte Carlo simulations[3]
	3	1.032(4)	strong-coupling expansion [1]
critical isotherm	2	1.07868...	
$(t = 0, \lvert H\rvert \neq 0)$	3	1.024(4)	strong-coupling + ϵ-expansion [1]

Table : Values of the ξ/ξ_{2nd} ratio for an Ising spin system in three different conditions: in the high-temperature symmetric phase, in the low-temperature broken symmetry phase and along the critical isotherm.

[1] M. Campostrini et al. Phys. Rev.E60 (1999) 3526-3563 cond-mat/9905078
[2] M. Campostrini et al. Phys. Rev.E57 (1998) 184-210 cond-mat/9705086
[3] M. Caselle et al. Nucl. Phys. B556 (1999) 575-600 hep-lat/9903011

These values have a natural interpretation:

- In the high-T symmetric phase, where the spectrum is composed by a single massive state, we would expect that $\xi/\xi_{2nd} = 1$: the small but not negligible difference from 1 is due to the cut above the pair production threshold at momentum p equal twice the lowest mass.
- In the low-T broken symemtry phase in $d = 3$ the spectrum is more complex, most likely it is composed by an infinite tower of bound states and one of them lies below the two particles threshold: $m_{bound} = 1.83(3)\, m_{ph}$ and in fact $\xi/\xi_{2nd} \sim 1.03$
- In the $d = 2$, $T = T_c$, $H \neq 0$ thanks to the exact solution S-matrix solution of Zamolodchikov we know that there are three particles in the spectrum below the two-particle threshold and accordingly we find $\frac{\xi}{\xi_{2nd}} = 1.07868...$
- Finally, in the $d = 2$, low-T case, the Fourier transform of the correlators starts with a cut whcih can be shown to be due to the coalescence of an infinite number of states. Accordingly we find $\frac{\xi}{\xi_{2nd}} = 1.58188...$

The $d = 3 + 1$ SU(2) LGT

To test the behaviour of ξ/ξ_{2nd} in Lattice Gauge Theories we performed a set of simulations in the $d = 3 + 1$ $SU(2)$ model [1] looking at correlators of Polyakov loops in the confined phase: the temperature $T = 1/(a(\beta)N_t)$ is varied using the inverse coupling β and the temporal extent N_t.
Here are a few info on the simulations

β	$N_s^3 \times N_t$	T/T_c	n_{conf}
2.27	$32^3 \times 6$	0.59	4.5×10^5
2.33	$32^3 \times 6$	0.71	2.25×10^5
2.3	$32^3 \times 5$	0.78	5.5×10^5
2.357	$32^3 \times 6$	0.78	2.25×10^5
2.25	$64^3 \times 4$	0.84	3×10^4
2.4	$64^3 \times 6$	0.90	2×10^4

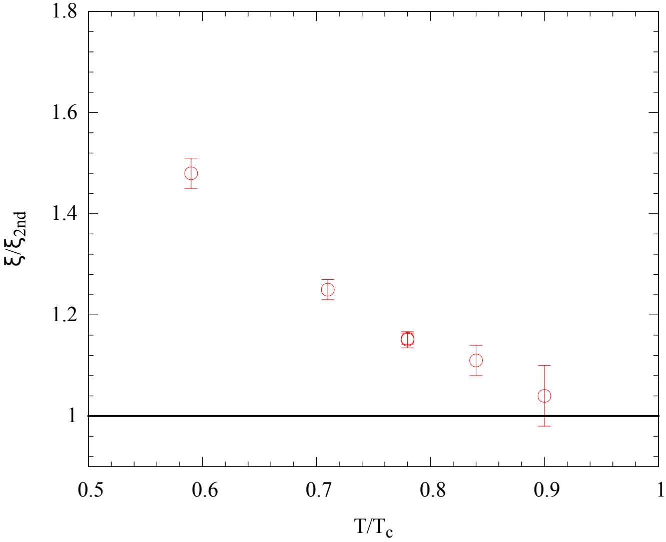
[1] M. Caselle, A. Nada, Phys. Rev. D 96, 074503 (2017), arXiv:1707.02164

The $d = 3 + 1$ SU(2) LGT

Results:

T/T_c	L	ξ/a	ξ_{2nd}/a	$\frac{\xi}{\xi_{2nd}}$
0.59	32	1.31(2)	0.887(8)	1.48(3)
0.71	32	2.31(4)	1.842(15)	1.25(2)
0.78	32	2.56(2)	2.22(1)	1.153(11)
0.78	32	3.08(4)	2.67(2)	1.151(16)
0.84	64	3.05(6)	2.74(4)	1.11(3)
0.90	64	6.9(2)	6.6(3)	1.04(6)

The $d = 3 + 1$ SU(2) LGT



The $d = 3 + 1$ SU(2) LGT

We see two main features

- for $T/T_c \to 1$ ξ/ξ_{2nd} is very close to 1.
  Same as what happens in the high-T phase of the 3d Ising model, in agreement with Svetitsky-Yaffe conjecture
- ξ/ξ_{2nd} increases dramatically as T/T_c decreases.
  This increase suggests that as T/T_c decreases the states of the spectrum coalesce toward the ground state, exactly as it happens in the $d = 2$ Ising model below T_c;

A very useful tool to understand both these features is the effective string description of the Polyakov loop correlators which indeed predicts, as a consequence of the "string" nature of the color flux tube, a rich spectrum of excitations.

This effective model will allow us to have a quantitative estimate of the energy of these excited states and of their T dependence.

Effective string action

Confinement is usually associated to the creation (via a mechanism which still has to be understood) of a thin flux tube joining the quark antiquark pair. (Nielsen-Olesen, 't Hooft, Wilson, Polyakov, Nambu ....).
If we accept this picture we cannot neglect quantum fluctuations of this flux tube. The area law is thus only the classical contribution to the interquark potential and we should expect quantum corrections to its form. The theory which describes these quantum fluctuations is known as "effective string theory".
The simplest choice for the effective string action is to describe the quantum fluctuations of the flux tube as free massless bosonic degrees of freedom

$$S = S_{cl} + \frac{\sigma}{2} \int d^2\xi \, [\partial_\alpha X \cdot \partial^\alpha X] \ ,$$

where:

- S_{cl} describes the usual ("classical") area-perimeter term.
- $X_i(\xi_0, \xi_1)$ $(i = 1, \ldots, d-2)$ parametrize the displacements orthogonal to the surface of minimal area representing the configuration around which we expand
- ξ_0, ξ_1 are the world-sheet coordinates.

The Lüscher term.

- The first quantum correction to the interquark potential is obtained summing over all the possible string configuration compatible with the Polyakov loop correlator (i.e. with Dirichlet boundary conditions along the Polyakov loops and periodic b.c in the compactified "temperature" direction).
- This is equivalent to the sum over all the possible surfaces borderd by the Polyakov loops i.e. to the partition function

$$< W(R,T) >= \int e^{-\sigma RT - \frac{\sigma}{2}\int d^2\xi X^i(-\partial^2)X^i}$$

- The functional integration is a trivial gaussian integral, the result is

$$V(R) = \sigma R - \frac{(d-2)\pi}{24R} + c$$

- This quantum correction is known as "Lüscher term" and is universal i.e. it does not depend on the ultraviolet details of the gauge theory but only on the geometric properties of the flux tube.

The Lüscher term.

This correction is in remarkable agreement with numerical simulations. First high precision test in d=4 SU(3) LGT more than fifteen years ago. [1]

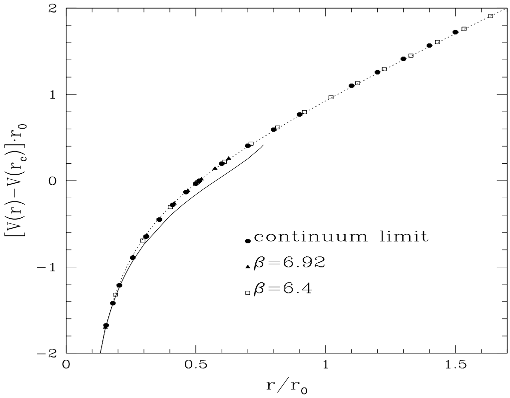


Figure : The static potential. The dashed line represents the bosonic string model and the solid line the prediction of perturbation theory.

[1] S. Necco and R. Sommer, Nucl.Phys. B622 (2002) 328

The Lüscher term.

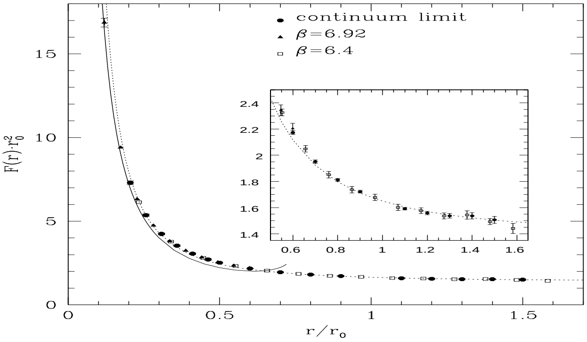


Figure : The force in the continuum limit and for finite resolution, where the discretization errors are estimated to be smaller than the statistical errors. The full line is the perturbative prediction. The dashed curve corresponds to the bosonic string model normalized by $r_0^2 F(r_0) = 1.65$.

The Nambu-Goto action.

- Evaluation of higher order quantum corrections requires further hypothesis on the nature of the flux tube. The simplest choice is the Nambu-Goto string in which quantum corrections are evaluated summing over all the possible surfaces bordered by the two Polyakov loops with a weight proportional to their area.

$$S = \sigma \int d^2\xi \sqrt{det(\eta_{\alpha\beta} + \partial_\alpha X \cdot \partial_\beta X)}$$

$$\sim \sigma RL + \frac{\sigma}{2} \int d^2\xi \left[\partial_\alpha X \cdot \partial^\alpha X + \frac{1}{8}(\partial_\alpha X \cdot \partial^\alpha X)^2 - \frac{1}{4}(\partial_\alpha X \cdot \partial_\beta X)^2 + \dots \right] ,$$

Interquark potential for the Nambu-Goto action.

The Nambu-Gotō action is simple enough to be exactly solvable. The large distance expansion of the Polyakov loop correlator in D space-time dimensions is[1]:

$$\langle P(x)^* P(y) \rangle = \sum_{n=0}^{\infty} w_n \frac{2r\sigma L}{E_n} \left(\frac{\pi}{\sigma}\right)^{\frac{1}{2}(D-2)} \left(\frac{E_n}{2\pi r}\right)^{\frac{1}{2}(D-1)} K_{\frac{1}{2}(D-3)}(E_n r)$$

where $r = |y - x|$ is the interquark distance, w_n denotes the multiplicity of the state, L the size of the lattice in the compactified time direction and E_n the closed-string energies which are given by

$$E_n = \sigma L \sqrt{1 + \frac{8\pi}{\sigma L^2} \left[-\frac{1}{24}(D-2) + n \right]}.$$

In the $D = 3 + 1$ case, thanks to the identity $K_{\frac{1}{2}}(z) = \sqrt{\frac{\pi}{2z}} e^{-z}$ we have:

$$\langle P(x)^* P(y) \rangle = \sum_{n=0}^{\infty} \frac{L}{2r} w_n e^{-E_n r}$$

which represents a collection of free particles of mass E_n.

[1] M. Lüscher and P. Weisz, JHEP 07 (2004) 014 hep-th/0406205
M. Billó and M. Caselle, JHEP 07 (2005) 038, hep-th/0505201

Interquark potential for the Nambu-Goto action.

In the large r limit the interquark potential is dominated by the lowest state which can be interpreted as a temperature dependent string tension

$$\frac{E_0}{L} = \sigma(T) = \sigma\sqrt{1 - \frac{\pi}{3\sigma L^2}} = \sigma\sqrt{1 - \frac{\pi T^2}{3\sigma}}.$$

- Performing an "open-closed string" transformation one can rewrite the energies in a form amenable for a low temperature expansion

$$\tilde{E}_n(R) = \sqrt{\sigma^2 R^2 + 2\pi\sigma\left(n - \frac{D-2}{24}\right)}$$

  where $\tilde{E}_n$ denotes the "dual" energy levels
- In particular $\tilde{E}_0(R)$ corresponds to the interquark potential

$$V(R) = \tilde{E}_0(R) = \sqrt{\sigma^2 R^2 - 2\pi\sigma\frac{D-2}{24}},$$

$$V(R) \sim \sigma R - \frac{\pi(D-2)}{24R} - \frac{1}{2\sigma R^3}\left(\frac{\pi(D-2)}{24}\right)^2 + O(1/R^5),$$

Effective string description: fixing T/T_c

In the framework of the Nambu-Gotō approximation one can also derive an estimate of the critical temperature T_c measured in units of the square root of the string tension $\sqrt{\sigma}$

$$\frac{T_c}{\sqrt{\sigma}} = \sqrt{\frac{3}{\pi(D-2)}}$$

given by the value of the ratio $\frac{T_c}{\sqrt{\sigma}}$ for which the lowest mass E_0 vanishes. We can thus rewrite the energy levels as a function of T/T_c; setting $D = 3+1$ we find

$$E_n = \frac{2\pi T_c^2}{3T} \left\{ 1 + 12 \frac{T^2}{T_c^2} \left[n - \frac{1}{12} \right] \right\}^{1/2} .$$

The gap between the different states, *decreases* as T/T_c decreases and all the states tend to accumulate toward the lowest state!

It is exactly this behaviour which leads in the other string channel to the appearence of the $1/R$ Lüscher term in the potential.

Comparison with the effective string description

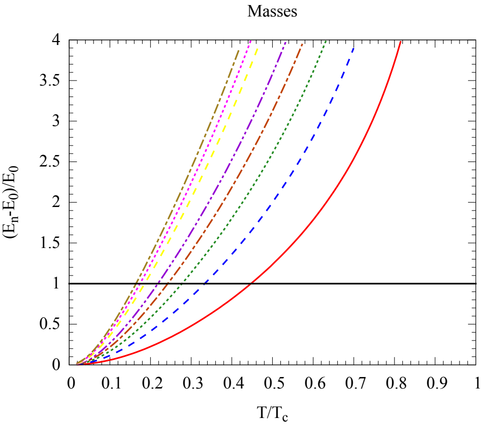


Figure : $(E_n - E_0)/E_0$ as a function of T/T_c for the first ten states. The black horizontal line represents the two particle threshold.

How much of this picture survives in real QCD?

Maybe more than what we expect.

- Greensite and Hollwieser[1] studied the ξ/ξ_{2nd} ratio in an Effective Polyakov Loop action with long range couplings, derived using the relative weights method on a $\mathrm{SU}(3)$ gauge theory with dynamical staggered fermions of mass 695 MeV.

  They found $\xi/\xi_{2nd} = 1.27(3)$ which is indeed compatible with a rich string-like spectrum

[1] J. Greensite, R. Hollwieser (2017), arXiv:1708.08031

Bound states in the deconfined phase and conformal perturbation

- Renormalization Group arguments (the "Svetitsky-Yaffe conjecture") suggest that the deconfinement transition of a $SU(N)$ LGT in $d+1$ dimensions belongs to the same universality class of the magnetization transition of a spin model in d dimension with symmetry group Z_N.
- In this framework the Polyakov loop is mapped into the spin and the deconfined phase of the LGT is mapped into the broken symmetry phase (low T) of the spin model.
- Since the low T phase of Z_N spin models is characterized by a rich spectrum of bound states we expect the same spectrum content also in the deconfined phase of a LGT, at least in the scaling region near the deconfinement point,

Bound states in the deconfined phase and conformal perturbation

To test this picture we focused on the $3+1$ SU(2) LGT and performed a two steps analysis

- We studied the short distance behaviour of the two point function in the scaling region using conformal perturbation in order to find the signatures of this bound state
- We performed a set of high precision simulations in the $3+1$ $SU(2)$ model looking for the same signatures.

Thermal Perturbation Theory: the 3d Ising Model[1]

In the Ising case we have only two relevant operators σ and ϵ, with scaling dimensions $\Delta_\sigma = 0.5181489(10)$ and $\Delta_\epsilon = 1.412625(10)$.
We find for the first three orders of perturbed two-point function of σ:

$$\langle \sigma(r)\sigma(0)\rangle_t = C^1_{\sigma\sigma}(0,r) + C^\epsilon_{\sigma\sigma}(0,r)\langle\epsilon\rangle_t + t\partial_t C^1_{\sigma\sigma}(0,r) + \ldots$$

To make contact with the usual definition for the structure constants we factorize the r dependence in the Wilson coefficients:

$$C^1_{\sigma\sigma}(0,r) = \frac{1}{r^{2\Delta_\sigma}}, \qquad C^\epsilon_{\sigma\sigma}(0,r) = C^\epsilon_{\sigma\sigma} r^{\Delta_\epsilon - 2\Delta_\sigma}$$

where we have chosen the usual normalization $C^1_{\sigma\sigma} = 1$.
Defining $\Delta_t = 3 - \Delta_\epsilon$, the perturbed one-point functions is:

$$\langle\epsilon\rangle_t = A^\pm |t|^{\frac{\Delta_\epsilon}{\Delta_t}}$$

Where $A^\pm$ are non-universal amplitudes. Introducing the scaling variable $s = tr^{\Delta_t}$ we end up with the following expression for the perturbed two-point function:

$$r^{2\Delta_\sigma}\langle \sigma(r)\sigma(0)\rangle_t = 1 + C^\epsilon_{\sigma\sigma} A^\pm |s|^{\frac{\Delta_\epsilon}{\Delta_t}} + t\partial_t C^1_{\sigma\sigma}(0,r) + \ldots$$

[1] M. Caselle, G. Costagliola, N. Magnoli, Phys. Rev. D 94, 026005 (2016), arXiv:1605.0513

Thermal Perturbation Theory: Results for the 3d Ising Model.

Inserting the known values of the structure constants, of their derivatives and of $A^{\pm}$ for the 3d Ising model we end up with:

$$r^{2\Delta_\sigma}\langle\sigma(r)\sigma(0)\rangle_t = 1 - 51.2(3)|s|^{\frac{\Delta_\epsilon}{\Delta_t}} + 65.7762..s \quad (t>0) + ...$$

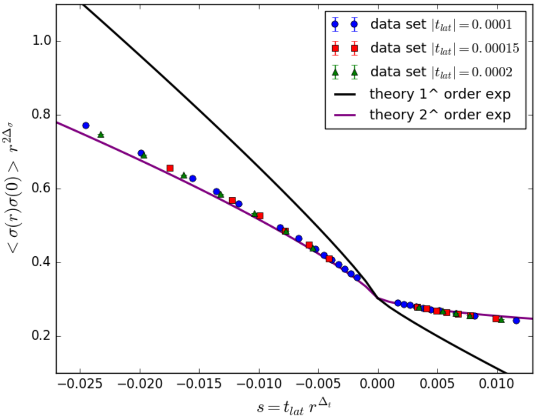
$$r^{2\Delta_\sigma}\langle\sigma(r)\sigma(0)\rangle_t = 1 + 95.6(6)|s|^{\frac{\Delta_\epsilon}{\Delta_t}} + 65.7762..s \quad (t<0) + ...$$

Notice that:

- The second and the third term are of the same size for $t>0$ and almost cancel, while this is not true for $t<0$ thus the behaviour is completely different in the $t>0$ and $t<0$ cases.
- The presence of a bound state only in the $t<0$ phase can be traced back to this different behaviour.

Comparison with Montecarlo Simulations of the 3d Ising Model

Simulations with a standard Metropolis updating with multispin coding on a cubic lattice of size $L = 300$ with periodic boundary condition.
$t_{lat} \equiv \beta_c - \beta$ with $\beta_c = 0.22165462$



Comparison with Montecarlo Simulations of the $3+1$ $SU(2)$ LGT[1]

β	$N_s^3 \times N_t$	T/T_c	n_{conf}	ξ
2.55	10×80^3	0.90	10^5	~ 11
2.569	10×80^3	0.96	10^5	~ 28
2.572	10×80^3	0.97	10^5	~ 55
2.58101	10×80^3	1	10^5	
2.58984	10×80^3	1.02	10^5	~ 30
2.59271	10×80^3	1.05	10^5	~ 25
2.61	10×80^3	1.10	10^5	~ 15

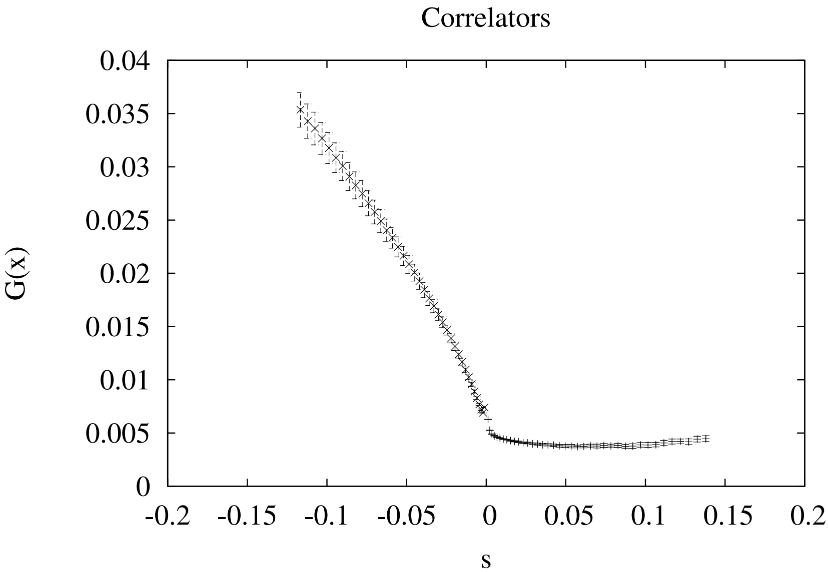
Table : Setup of the lattice simulations performed for the $\mathrm{SU}(2)$ gauge theory.

β	T/T_c	R_{max}	ξ	a	b
2.55	0.90	$[7-8]$	~ 11	-0.169(1)	0.099(1)
2.569	0.96	$[11-14]$	~ 28	-0.067(2)	0.037(1)
2.572	0.97	$[12-21]$	~ 55	-0.048(3)	0.026(2)
2.58984	1.02	$[25-30]$	~ 30	0.071(1)	- 0.022(1)
2.59271	1.05	$[20-25]$	~ 25	0.089(2)	- 0.023(2)
2.61	1.05	$[10-15]$	~ 15	0.230(10)	- 0.085(8)

Table : Results for the two coefficents of the conformal perturbation expansion.

[1] M. Caselle, N. Magnoli, A. Nada, M. Panero and M. Scanavino, in preparation

Comparison with Montecarlo Simulations of the $3+1$ $SU(2)$ LGT[1]



[1] M. Caselle, N. Magnoli, A. Nada, M. Panero and M. Scanavino, in preparation

Conclusions

- The ξ/ξ_{2nd} is a simple and "easy to evaluate" observable to test existing proposal for Effective Polyakov loop actions
- In $d = 3 + 1$ $SU(2)$ LGT we find values of the ratio larger and larger as the temperature decreases.
- These values have a natural interpretation in terms of effective string description of the Polyakov loop correlators. They are due to the presence of an infinite number of excited states in the spectrum, which become denser and denser as T decreases and to the exponential increase of the weights.
- Using conformal perturbation and Montecarlo simulations we find evidences of a bound state in the high T deconfiend phase of the $(3 + 1)$ $SU(2)$ LGT.

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