Tetraquarks, pentaquarks and the like
Old and new views

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Outline of the talk

Pre-history (from 1968 to 2003: hints for tetra- & penta-quarks)

- Motivation & Background
  - Duality (Rosner, 1968)
    - $MM \to MM$, $MB \to MB$, $B\bar{B} \to B\bar{B}$
  - Large N-expansions
    - $1/N_c$ @ $g^2 N_c = const$ ('t Hooft, 1973)
    - $1/N_f$ @ $g^2 N_c = const$ and $N_f/N_c = const$ (Veneziano, 1975)
  - Experiments (1975 -1980 & around 2003)
    - LEAR - S [$M \sim 1936$, $\Gamma \sim 4 - 8$ MeV] & other candidates
- A theoretical picture emerged from QCD predicting
  - "hidden baryon number" states $\to$ Baryonium (Rossi, Veneziano, 1977)

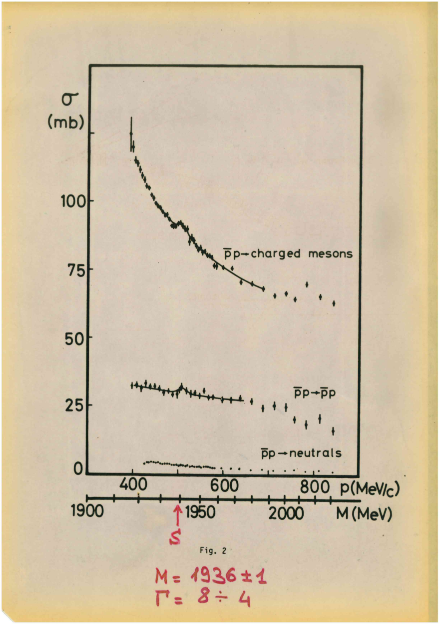
History (from 2003 to today)

- More "stable" experimental data (after 2011)
- A better understanding of Baryonium (Rossi, Veneziano, 2015)
- Phenomenology of tetra-quarks, penta-quarks, . . . (Yaffe, 1977 - Large number of papers . . . 2004 - 2018)

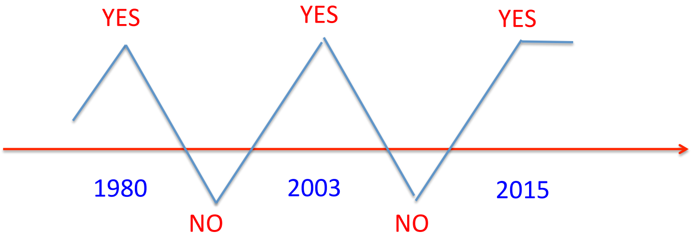
This talk is mainly based on

- G.C. Rossi and G. Veneziano, "A Possible Description of Baryon Dynamics in Dual and Gauge Theories," Nucl. Phys. B **123** (1977) 507.
- G.C. Rossi and G. Veneziano, "Electromagnetic Mixing of Narrow Baryonium States," Phys. Lett. **70B** (1977) 255.
- L. Montanet, G.C. Rossi and G. Veneziano, Phys. Rept. **63** (1980) 149.
- G.C. Rossi and G. Veneziano, "Isospin mixing of narrow pentaquark states," Phys. Lett. B **597** (2004) 338.
- G.C.Rossi and G. Veneziano, "The string-junction picture of multiquark states: an update," JHEP **1606** (2016) 041.
- The slides of my 1977 CERN seminar

From where everything starterd

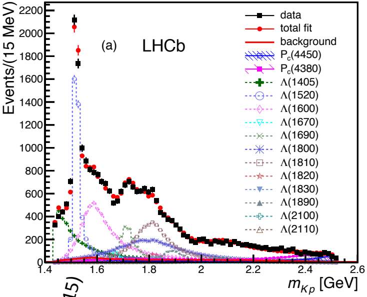


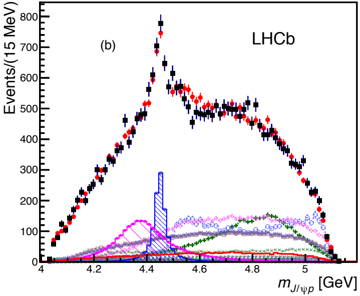
The multi-quark states saga



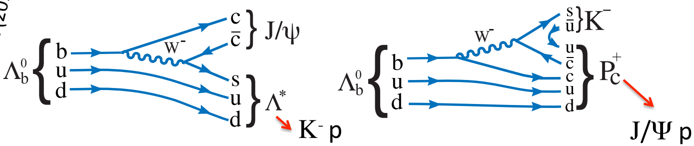
4q's & 5q's discovery history

The experimental evidence for 5q's states





Phys. Rev. Lett. 115, 072001 (2015)



Feynman diagrams for (a) $\Lambda_b^0 \to J/\psi \Lambda^*$ and (b) $\Lambda_b^0 \to P_c^+ K^-$ decay.

$M(P_{Ic}^+) = 4450$ $J^P = 5/2^+$ $\Gamma = 39$ MeV

$M(P_{IIc}^+) = 4380$ $J^P = 3/2^-$ $\Gamma = 205$ MeV

The emergence of Baryonium

- *S*-matrix approach
  - Duality in $MM \to MM$, $MB \to MB$, $B\bar{B} \to B\bar{B}$ amplitudes
  - Regge trajectories
  - Exchange degeneracy violation $\to$ multi-quark states

- Field theoretical approach
  - The physically interesting limit of QCD is g^2 & $\lambda = g^2 N_c$ small
  - More or less good control of the theory (see figure)
    - in perturbation theory: $g^2 \to 0$ @ N_c fixed
    - in the 't Hooft limit: $1/N_c \to \infty$ @ $\lambda = g^2 N_c$ fixed
    - in the strong coupling limit: $1/g^2 \to 0$ @ N_c fixed (possibly large)
    - in the AdS/CFT limit: $1/N_c \to 0$ @ λ fixed and large
  - As we shall see, "naturally"
    - mesons appear in the 't Hooft and strong coupling limit
    - baryons & baryonia in the strong coupling limit
  - The key question is: can we get to real physics?

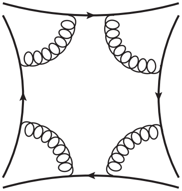
S-matrix approach & duality

- The problem
  - find a flavour and crossing symmetric simultaneous solution (for coupling and masses) to the duality constraints coming from MM, MB and $B\bar{B}$ scattering
    - starting from some lowest order (planar) approx. with no loops
    - including higher order (non-planar) terms and loops
  - understand EXD breaking for both mesons and baryons
  - construct a Topological Expansion for strong amplitudes
- We shall briefly examine
  - Duality in $MM \to MM$ amplitudes
  - Duality in $MB \to MB$ amplitudes
  - Duality in $B\bar{B} \to B\bar{B}$ amplitudes

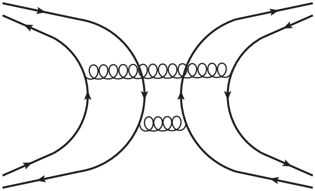
Duality in $MM \to MM$ amplitudes

$MM \to MM$ amplitudes

• At planar level one has duality between planar *s*- and *t*-channels, exact EXD and no exotic intermediate states (diagram (a))
• Understood in terms of large-N expansions (either 't Hooft, 1974 or topological Veneziano, 1974)
• Topological Expansion Veneziano, 1974 can be used to relate EXD breaking to non-planar corrections



(a)



(b)

• Duality connects glueball (Pomeron) exchange to a non-resonant non-planar two-meson background (diagram (b))

Okubo-Zweig-Iizuka rule

The OZI rule emerges

- Large-N (and strong coupling) expansions support the usual OZI rule suppressing decays that proceed via $q\bar{q}$ annihilation wrt decays by string breaking (with creation of a q-$\bar{q}$ pair)
- It suppresses flavour mixing in the mass matrix
- It is well obeyed in vector mesons ("ideal" mixing)
- Badly broken in light pseudoscalar sector. Reasons
  - Light masses of (pseudo)NG bosons
  - Large anomaly contribution (WV solution of the $U_A(1)$ problem)
  - OZI preserving decay of heavy quarkonia is often not allowed kinematically, so the lightest ones ($J/\psi, \Upsilon, \ldots$) are narrow

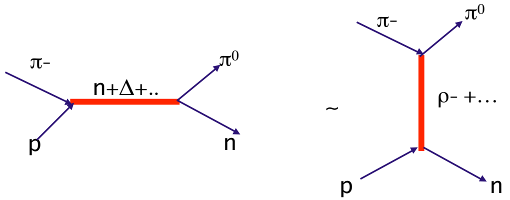
Duality in $MB \to MB$ amplitudes

Dolen-Horn-Smit duality - 1968

Dolen-Horn-Schmit duality

s-and t-channel descriptions of pion-nucleon charge exchange are, on average, equivalent, complementary,

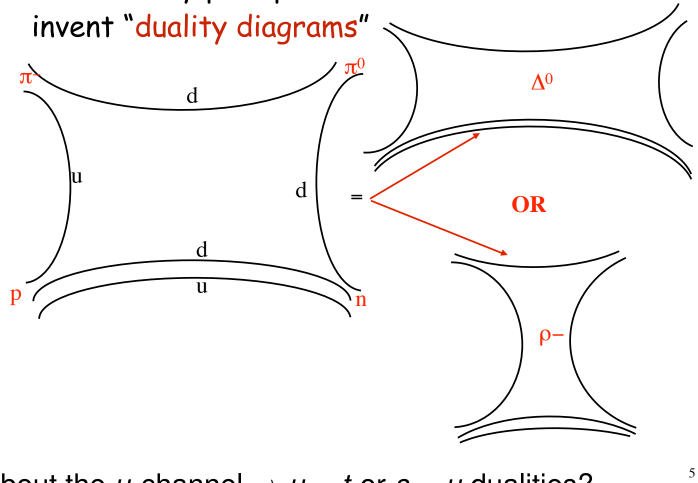
DUAL



More precisely, DHS suggested duality between Regge poles in the t-channel and resonances in s- and u-channel

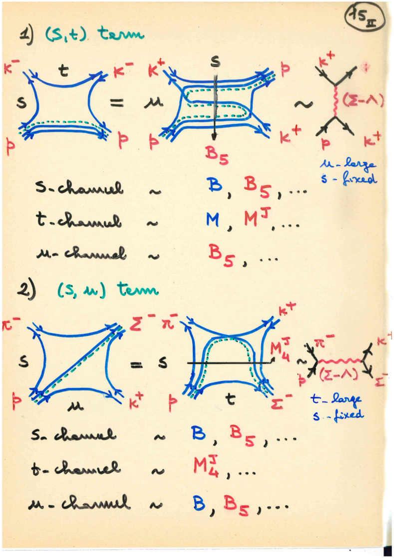
$s - t$ duality

DHS duality prompted Harari and Rosner to invent "duality diagrams"



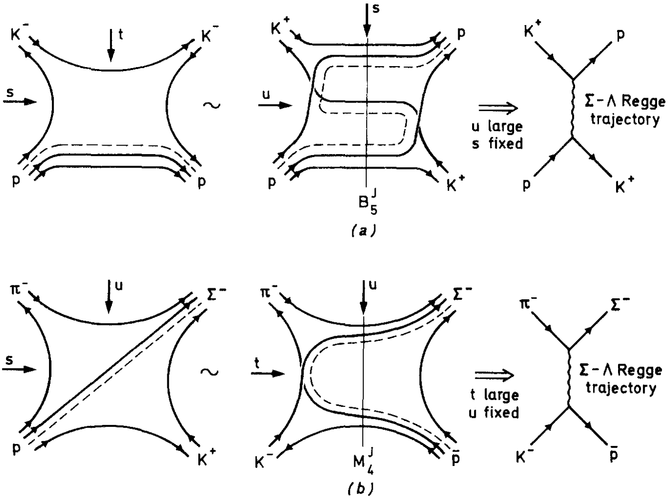
What about the u-channel $\rightarrow u - t$ or $s - u$ dualities?

Anticipating, in green the colour flow of the baryon (junction)



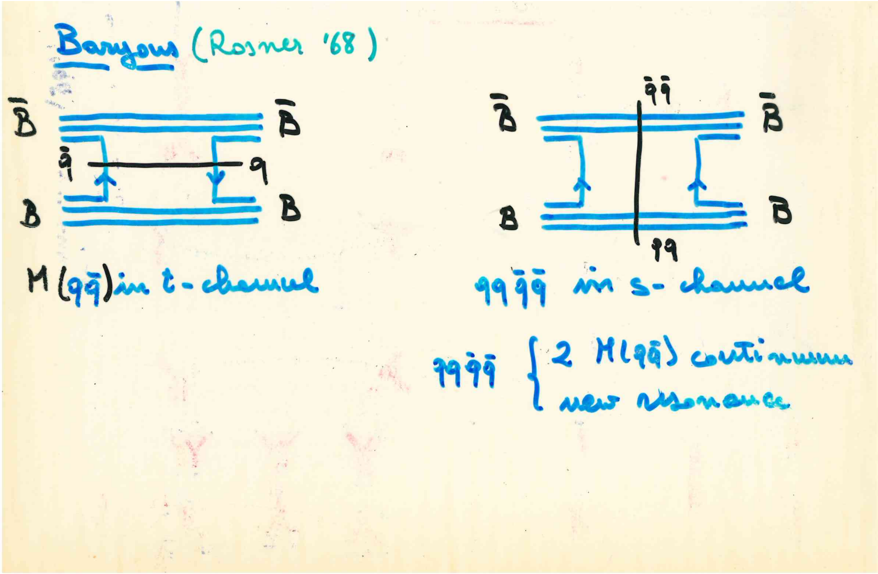
a slide from my 1977 CERN seminar

From the 1980 Rossi–Veneziano Physics Report



Multiquark states are needed already at “planar level”

Duality in $B\bar{B} \to B\bar{B}$ amplitudes


Baryons (Rosner '68)
$\bar{B}$
$\bar{q}$
q
B
$M(q\bar{q})$ in t-channel
$\bar{q}\bar{q}$
qq
$qq\bar{q}\bar{q}$ in s-channel
$qq\bar{q}\bar{q}$ { 2 $M(q\bar{q})$ continuum
new resonance

The systematics of

- hadronic states
- amplitudes

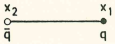
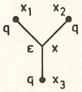
(in the QCD string language)

Hadronic states - I

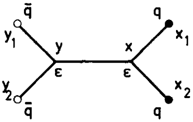
Hadronic states $\to$ irreducible gauge invariant operators in QCD

Table IIa

Simplest mesons and baryons : colour structure and string picture

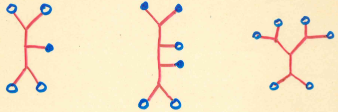
HADRON	GAUGE INVARIANT OPERATOR	STRING PICTURE
$M_2 = q\bar{q}$ meson	$\bar{q}^{j_2}(x_2)\left[P\exp\left(ig\int_{x_1}^{x_2}A_\mu\, dx^\mu\right)\right]_{j_2}^{j_1} q_{j_1}(x_1)$	
M_o = quarkless meson	$\mathrm{Tr}\left[P\exp\left(ig\oint A_\mu\, dx^\mu\right)\right]$	
$B_3 = qqq$ baryon	$\epsilon^{j_1 j_2 j_3}\left[P\exp\left(ig\int_{x_1}^{x}A_\mu\, dx^\mu\right)q(x_1)\right]_{j_1}$ $\left[P\exp\left(ig\int_{x_2}^{x}A_\mu\, dx^\mu\right)q(x_2)\right]_{j_2}\left[P\exp\left(ig\int_{x_3}^{x}A_\mu\, dx^\mu\right)q(x_3)\right]_{j_3}$	

The three ($N_c = 3$) baryonium families: colour structure and string picture. The symbol $\exp \int_x^y$ is a shorthand for the path ordered exponential used in table 2a

Hadron	Gauge invariant operator	String picture
M_4^J = baryonium with $qq\bar{q}\bar{q}$ quantum numbers	$\varepsilon_{j_1 j_2 j_3}\varepsilon^{k_1 k_2 k_3}\left[\bar{q}(y_1)\exp\int_y^{y_1}\right]^{j_1}\left[\bar{q}(y_2)\exp\int_y^{y_2}\right]^{j_2} \times\left[\exp\int_x^y\right]^{j_3}_{k_1}\left[\exp\int_{x_1}^x q(x_1)\right]_{k_2}\left[\exp\int_{x_2}^x q(x_2)\right]_{k_3}$	
M_2^J = baryonium with $q\bar{q}$ quantum numbers	$\varepsilon_{j_1 j_2 j_3}\varepsilon^{k_1 k_2 k_3}\left[\bar{q}(y_1)\exp\int_y^{y_1}\right]^{j_1} \times\left[\exp\int_x^y\right]^{j_2}_{k_1}\left[\exp\int_x^y\right]^{j_3}_{k_2}\left[\exp\int_{x_1}^x q(x_1)\right]_{k_3}$	
M_0^J = quarkless baryonium	$\varepsilon_{j_1 j_2 j_3}\varepsilon^{k_1 k_2 k_3}\left[\exp\int_x^y\right]^{j_1}_{k_1}\left[\exp\int_x^y\right]^{j_2}_{k_2}\left[\exp\int_x^y\right]^{j_3}_{k_3}$	

Hadronic states - III

According to our rules one can formally construct other exotic states beside $M^J_{0,2,4}$:



$B_5 \to B M^J_4$ $\quad$ $E_6 \to M^J_4 M^J_4$ $\quad$ $E'_6 \to BBB\bar{B}$

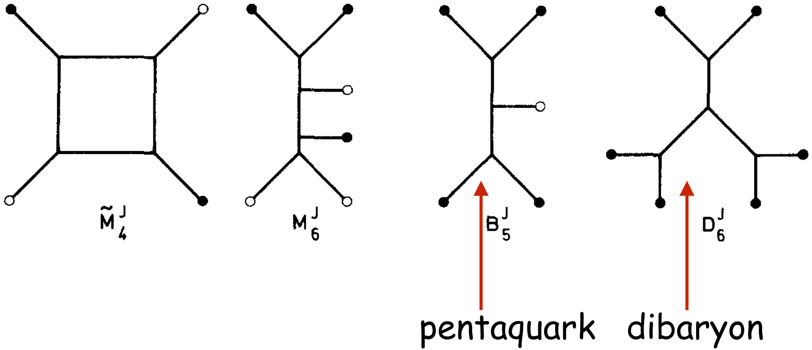
Important for duality in 2 body reactions.

For instance: existence of M^J_4 and B_5 modifies the pattern of EXD for baryon trajectories, as obtained from the absence of exotics in $B\bar{B} \to MM$ and $MB \to BM$.

This is very welcome, because such a pattern is in conflict with experimental evidence.

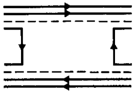
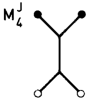
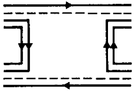
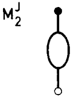
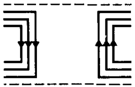
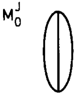
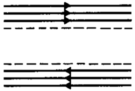
Other multiquark states

(from G. C. Rossi & GV, Phys. Rep. 1982)



$B\bar{B} \to B\bar{B}$ amplitudes - Scattering

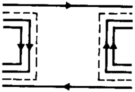
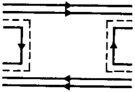
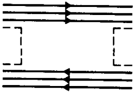
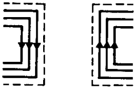
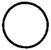
Contributions to $B\bar{B}$ scattering ($N_c = 3$)

$B\bar{B} \to B\bar{B}$ Junction duality diagrams annihilation		s-channel formation	Multiplicity[(a)]	t-channel[(b)] exchange	Slope
1			$\bar{n}(s') \simeq \bar{n}_{e^+e^-}(s')$	$s^{\alpha_R - 1} \sim s^{-1/2}$ Regge pole	α'_R
2			$\bar{n}(s') \simeq 2\bar{n}_{e^+e^-}(s'/4)$	$s^{2\alpha_R - 2} \sim s^{-1}$ 2-Reggeon cut	$\frac{1}{2}\alpha'_R$
3			$\bar{n}(s') \simeq 3\bar{n}_{e^+e^-}(s'/9)$	$s^{3\alpha_R - 3} \sim s^{-3/2}$ 3-Reggeon cut	$\frac{1}{3}\alpha'_R$
4		Non-resonant two jet background	$\bar{n}(s') \simeq 2\bar{n}_{e^+e^-}(s'/4)$	$s^{\alpha_P - 1} \sim s^0$ Pomeron	$\frac{1}{2}\alpha'_R$

[(a)] s' is the invariant mass of the final state excluding the leading baryons.
[(b)] To estimate the s-behaviour we have taken $\alpha_R = 0.5$.

$B\bar{B} \to B\bar{B}$ amplitudes - Annihilation

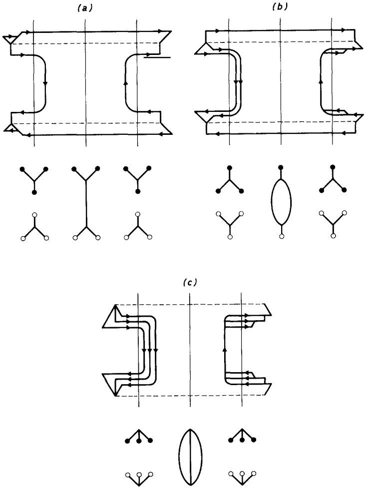
Contribution to $B\bar{B}$ annihilation ($N_c = 3$)

$B\bar{B} \to B\bar{B}$ Junction duality diagrams annihilation	s-channel formation	Multiplicity	t-channel[a] exchange	Slope
1 	$1q\bar{q}$ – jet	$\bar{n}(s) \simeq \bar{n}_{e^+e^-}(s)$	$s^{\alpha(M_4^J)-1} \sim s^{-3/2}$ Regge pole	$\alpha'(M_4^J) \sim \alpha'_R$
2 	$2q\bar{q}$ – jets	$\bar{n}(s) \simeq 2\bar{n}_{e^+e^-}(s/4)$	$s^{\alpha(M_2^J)-1} \sim s^{-1}$ Regge pole	$\alpha'(M_2^J) \sim \frac{1}{2}\alpha'_R$
3 	$3q\bar{q}$ – jets	$\bar{n}(s) \simeq 3\bar{n}_{e^+e^-}(s/9)$	$s^{\alpha(M_0^J)-1} \sim s^{-1/2}$ Regge pole	$\alpha'(M_0^J) \sim \frac{1}{3}\alpha'_R$
4 	M_0 	$\bar{n}(s) \simeq 2\bar{n}_{e^+e^-}(s/4)$	$s^{2\alpha_B-2} \sim s^{-2}$ 2-Reggeon cut	$\frac{1}{2}\alpha'_R$

[a]To estimate the s-behaviour we have taken $\alpha_B \simeq 0$.

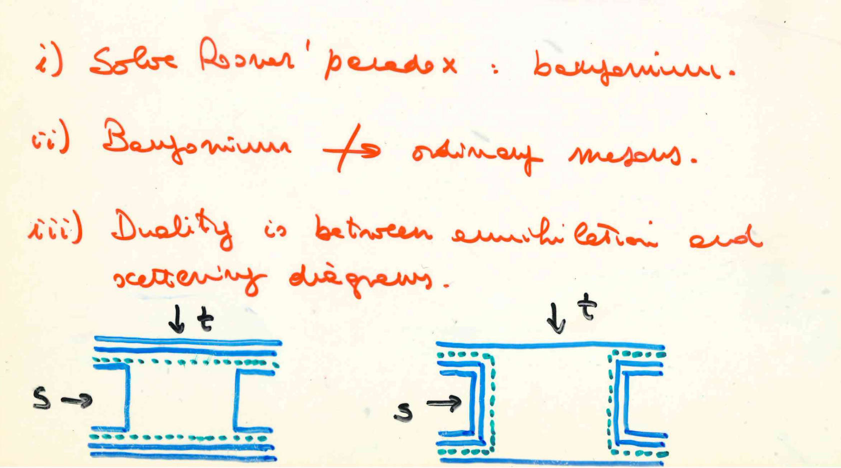
$B\bar{B} \to B\bar{B}$ scattering - an artistic view

Keep track of the junction/colour/baryon number flow
Neglect for the moment meson and baryon loops



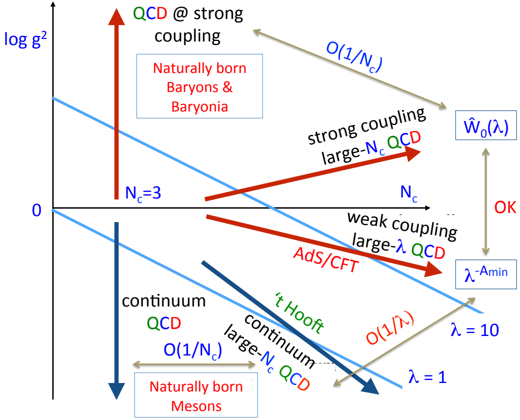
$B\bar{B} \to B\bar{B}$ annihilation

Solution of Rosner' paradox $\to$ just add 90° rotated diagrams!



Field theory

- The limits of QCD
- from large g^2 & fixed N_c ...
  - Meson propagator and amplitudes
  - Baryon propagator and amplitudes
- ... to small g^2 & N_c continuum QCD



We focus on the limits (we have good theoretical control of)

- 't Hooft large-N limit (fixed N_f and fixed $\lambda = g^2 N$)
- (lattice) strong coupling limit $g^2 \to \infty$, fixed N
- large λ limit with N large and g^2 possibly small (AdS/CFT)

Meson propagator and amplitudes

$q\bar{q}$ mesons are intermediate states in the gauge invariant correlator

$$G_{\mathcal{M}}(\mathcal{C}_{t'},\mathcal{C}_t) = \langle \mathcal{M}(\mathcal{C}_{t'})\mathcal{M}^\dagger(\mathcal{C}_t)\rangle$$

where

$$\mathcal{M}(\mathcal{C}_t) = \frac{1}{\sqrt{N_c}}\bar{q}(\vec{r},t)U[\mathcal{C}_t]q(\vec{s},t)\,, \quad U[\mathcal{C}_t] = \mathcal{P}\exp\left[ig\int_{\vec{r}}^{\vec{s}} d\vec{x}\,\vec{A}(\vec{x},t)\right]$$

$\mathcal{C}_t$ is a line joining the point $(\vec{r}, t)$ with $(\vec{s}, t)$

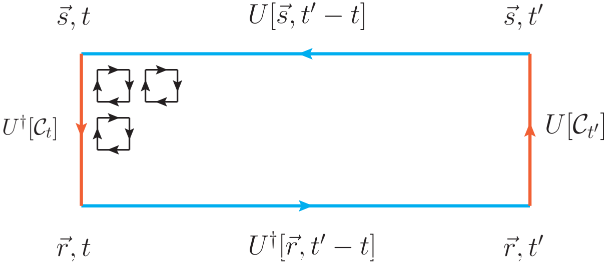
Contracting the quark fields, one finds

$$G_{\mathcal{M}}(\mathcal{C}_{t'},\mathcal{C}_t) = \frac{1}{N_c}\frac{\int\prod_i dU_i\,\mathrm{Tr}\Big(U^\dagger[\mathcal{C}_t]S_F(\vec{r},t;\vec{r},t')U[\mathcal{C}_{t'}]S_F(\vec{s},t';\vec{s},t)\Big)e^{-\frac{1}{g^2}S_{LYM}(U)}}{\int\prod_j dU_j e^{-\frac{1}{g^2}S_{LYM}(U)}}$$

In the static limit we replace the quark propagator with

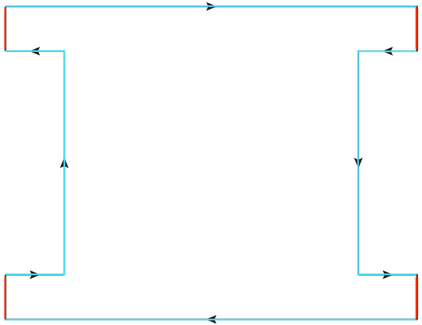
$$S_F(\vec{s},t';\vec{s},t) \to U[\vec{s},t'-t] = \prod_{\tau\in[t,t']} U[\vec{s},\tau]$$

$$G_{\mathcal{M}}(\mathcal{C}_{t'},\mathcal{C}_t) = \frac{1}{N_c}\frac{\int \prod_i dU_i \,\mathrm{Tr}\Big(U^\dagger[\mathcal{C}_t]U^\dagger[\vec{r},t-t']U[\mathcal{C}_{t'}]U[\vec{s},t'-t]\Big)e^{-\frac{1}{g^2}S_{LYM}(U)}}{\int \prod_j dU_j e^{-\frac{1}{g^2}S_{LYM}(U)}}$$



- $G_M(\mathcal{C}_{t'},\mathcal{C}_t) = \langle$ Wilson loop with sides $|\vec{s}-\vec{r}| \times |t'-t| \rangle$
- string tension $\kappa = a^{-2}\log g^2 N$

Dual s-t cross symmetric planar amplitude



$$\mathcal{M}_{MM\to MM}(\lambda) = \sum_h W_h(\lambda) N^{-2h} \propto \lambda^{-A_{min}}\,, \qquad \lambda = g^2 N$$

Interestingly, for mesons the SCLE can be argued to become, at large-N, a large $\lambda = g^2 N$ expansion, valid even at small g^2 Zuber

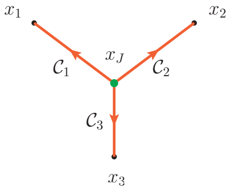
Baryon propagator and amplitudes

In SU(N_c) QCD the (normalized) wave-function of the baryon reads

$$B(\mathcal{C}_1,\mathcal{C}_2,\ldots,\mathcal{C}_{N_c}) = \frac{1}{\sqrt{N_c!}}\epsilon_{i_1 i_2 \ldots i_{N_c}} U[\mathcal{C}_1]^{i_1}_{j_1} q(x_1)^{j_1}\; U[\mathcal{C}_2]^{i_2}_{j_2} q(x_2)^{j_2} \ldots U[\mathcal{C}_{N_c}]^{i_{N_c}}_{j_{N_c}} q(x_{N_c})^{j_{N_c}}$$

$$U[\mathcal{C}_k]^{i_k}_{j_k} = \mathcal{P}\exp\left[ig\int_{\mathcal{C}(x_J,x_k)} dy^\mu A_\mu(y)\right]^{i_k}_{j_k}, \quad k=1,2,\ldots,N_c$$

with $\mathcal{C}(x_J,x_k)$ a curve joining the point x_J to x_k.



We want to compute in the strong coupling limit the correlator

$$G_B(\{\vec{r}_k, k=1,2,\ldots,N_c\},\vec{r}_J; t'-t) = \langle B(\mathcal{C}_1,\mathcal{C}_2,\ldots,\mathcal{C}_{N_c}) B^\dagger(\mathcal{C}'_1,\mathcal{C}'_2,\ldots,\mathcal{C}'_{N_c})\rangle$$

The computational strategy outlined for the meson propagator leads to a kind of “book” with pages sewed by a Levi-Civita symbol

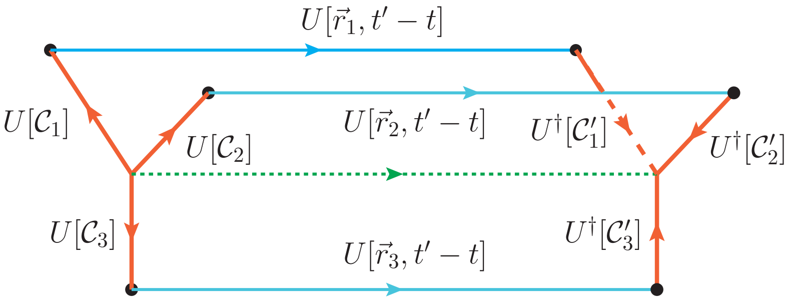


Figure: The $N_c = 3$ baryon propagator.

Tiling the pages of the book with plaquettes from the action

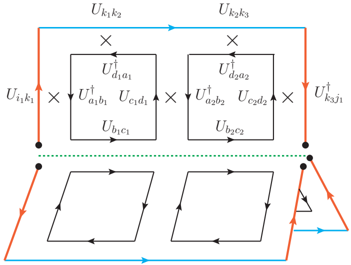


Figure: Tiling each of the the $N_c = 3$ pages with two plaquettes.

The group integral on the links along the dotted lines gives

$$\sum_{\ell_k} \int dU U_{i_1\ell_1} U_{i_2\ell_2} \dots U_{i_{N_c}\ell_{N_c}} \int dU U_{j_1\ell_1} U_{j_2\ell_2} \dots U_{j_{N_c}\ell_{N_c}} =$$

$$= \frac{1}{N_c!^2} \epsilon_{i_1 i_2 \dots i_{N_c}} \epsilon_{j_1 j_2 \dots j_{N_c}} \sum_{\ell_k} \epsilon_{\ell_1\ell_2\dots\ell_{N_c}} \epsilon_{\ell_1\ell_2\dots\ell_{N_c}} = \frac{1}{N_c!} \epsilon_{i_1 i_2 \dots i_{N_c}} \epsilon_{j_1 j_2 \dots j_{N_c}}$$

Scattering

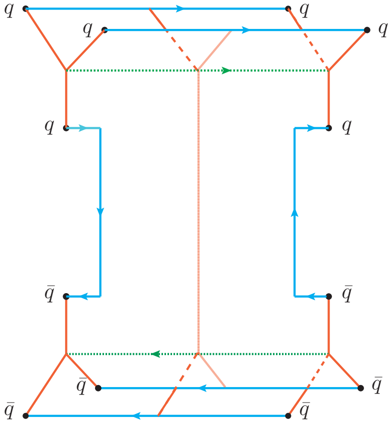


Figure: $B\bar{B}$ scattering at large λ, showing an s-channel M_4^J baryonium (tetraquark) state dual to a t-channel $q\,\bar{q}$ meson.

Annihilation

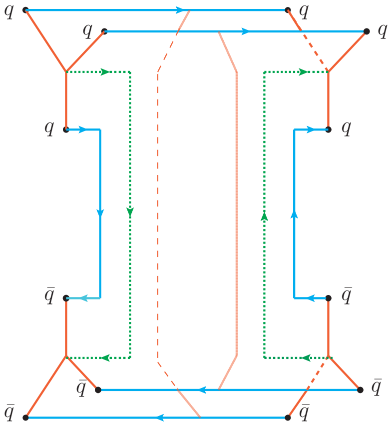
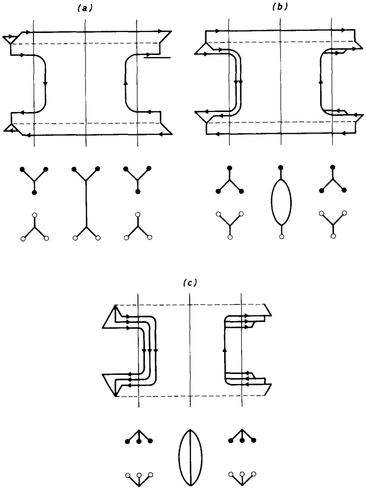


Figure: $B\bar{B}$ annihilation at large λ, showing a pair of s channel $q\,\bar{q}$ mesons dual to a t-channel M_2^J baryonium.

$B\bar{B} \to B\bar{B}$ amplitudes

We recover the diagrams of the previous topologically inspired picture of $B\bar{B} \to B\bar{B}$ amplitudes. Recall our picture of scattering amplitudes

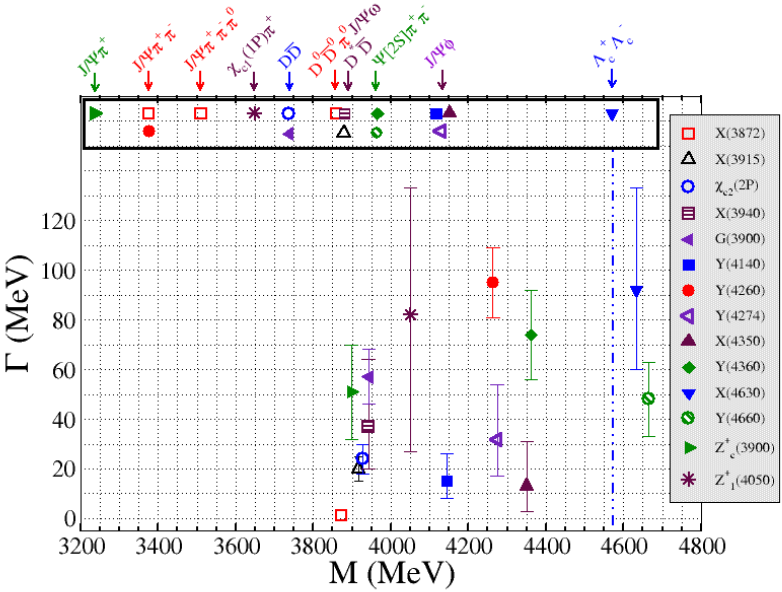


A few phenomenological observations

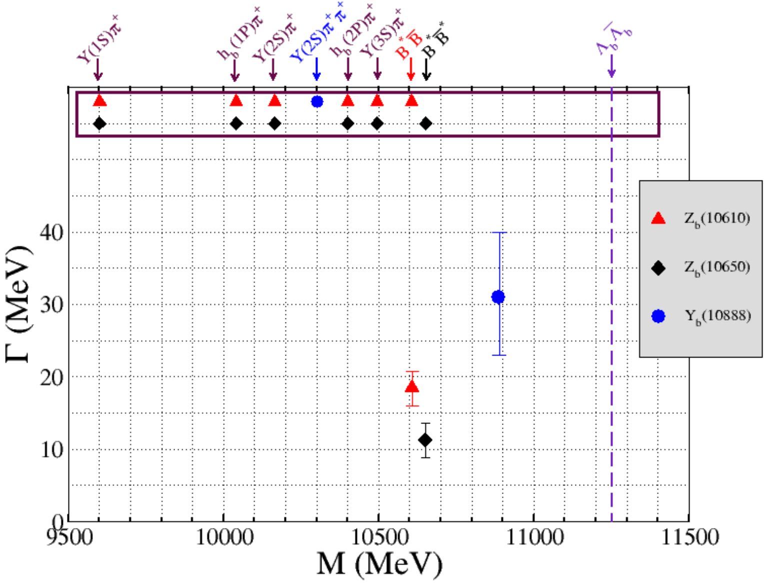
I - JOZI rule

- Large-N (Witten,1979) and strong coupling (GCR & GV, 2016) expansions support a Junction OZI (JOZI) rule, suppressing decays that proceed via junction-antijunction annihilation (leading to mesonic decays) wrt decays by string breaking (leading to baryonic decays)
- We have “mesophobic tetraquarks” (unlike molecular tetraquarks that should love mesons ... do we have a clear-cut distinction?)
- We might expect tetraquark states lying below threshold for baryonic decays to be unusually narrow.
- Peculiar tetraquark coupling to meson pairs
- A systematic analysis is missing (at least from our side)

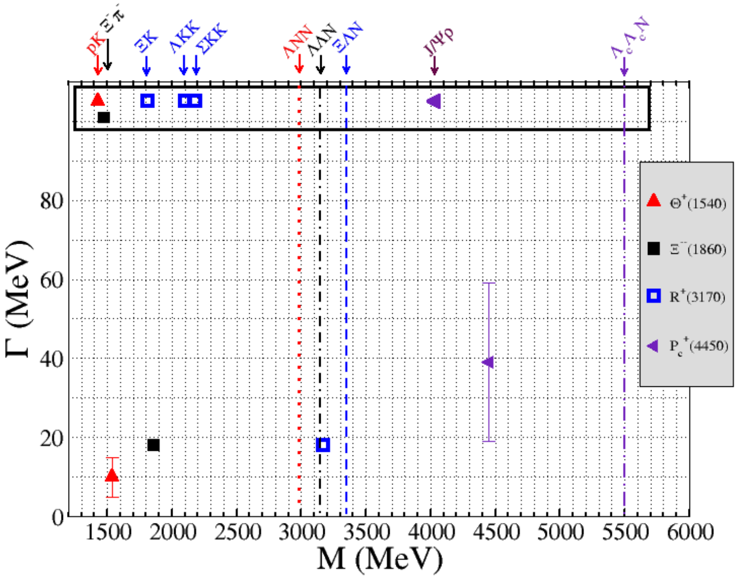
II - Hidden charm tetraquarks: a (dated) compilation



III - Hidden bottom tetraquarks: a (dated) compilation

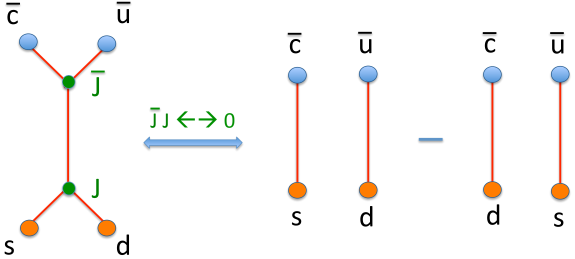


IV - Pentaquarks: a (dated) compilation



$$\bar q^{f_1}_{i_1}\bar q^{f_2}_{i_2}[\sum_k \epsilon_{k i_1 i_2}\epsilon_{k j_1 j_2}] q^{g_1}_{j_1} q^{g_2}_{j_2} = \bar q^{f_1}_{i_1}\bar q^{f_2}_{i_2}[\delta_{i_1 j_1}\delta_{i_2 j_2} - \delta_{i_1 j_2}\delta_{i_2 j_1}] q^{g_1}_{j_1} q^{g_2}_{j_2} =$$

$$= [\bar q^{f_1}_i q^{g_1}_i]\times[\bar q^{f_2}_j q^{g_2}_j] - [\bar q^{f_1}_i q^{g_2}_i]\times[\bar q^{f_2}_j q^{g_1}_j]$$



$B^J_4[\bar c\ \bar u$ - s d] ←→ $[D^-_s\ \pi^-]$ - $[D^-\ K^-]$

Conclusions

Conclusions - I

- The prediction of baryonium states is almost 50 years old
- We presented a Topological Expansion for baryon amplitudes supported by (old) large-N and (recent) large-g^2 arguments
- Duality in $B\bar{B}$ amplitudes is between annihilation & scattering
- i.e. annihilation between 1, 2, 3 $q\bar{q}$-jets and baryonia with $qq\bar{q}\bar{q}$, $q\bar{q}$ and no-quark content, respectively ($N = 3$)
- Duality diagrams displaying "flavour flow" must be augmented by "junction flow" $\rightarrow$ impact on "exchange degeneracy" of baryons

Conclusions - II

- Including quark loops makes baryonia to decay into baryons plus mesons, but not just into mesons $\to$ JOZI-rule
- Tetraquark states can be unusually narrow if near $B\bar{B}$ threshold
- Including baryon loops makes baryonia to mix with ordinary mesons $\to$ JOZI-rule violation
- Baryonium Regge trajectory intercepts, slopes and mixings can be estimated
- $B\bar{B}$ annihil. dominated by a $I=0$ flat trajectory with final states
  - consisting of three $q\bar{q}$-jets,
  - multiplicity $n^{p\bar{p}}_{ann} : n^{pp}_{scat} : n^{e^+e^-}_{ann} = 3 : 2 : 1$
- More work needed to separate baryonia from other exotic multiquark states (e.g. molecules)

Thanks for your attention