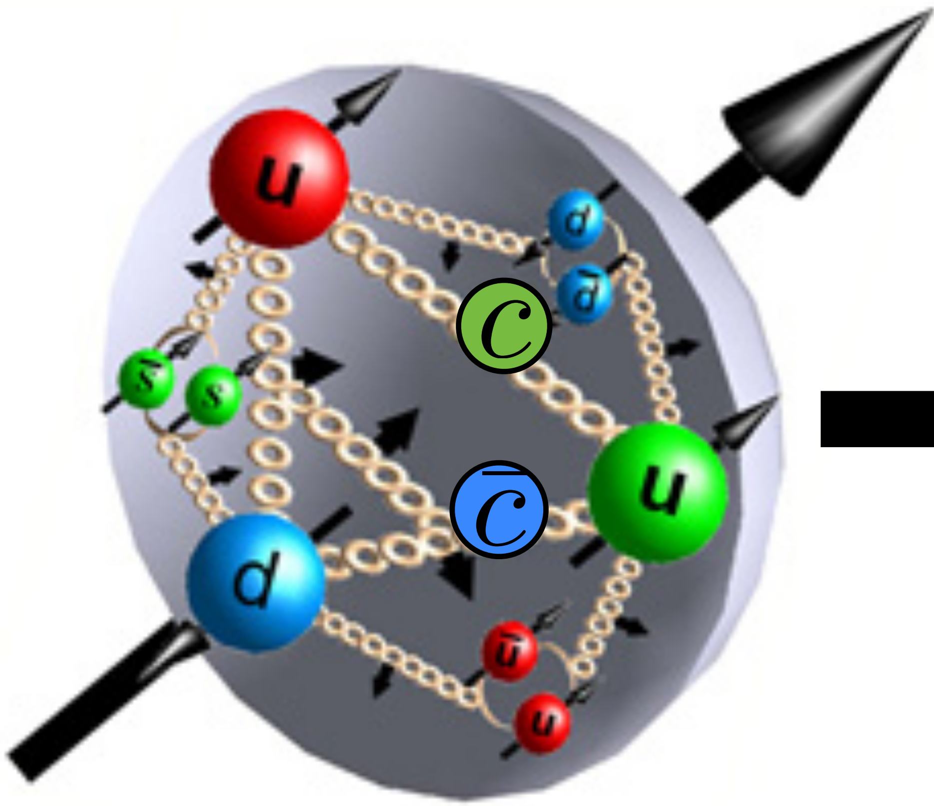
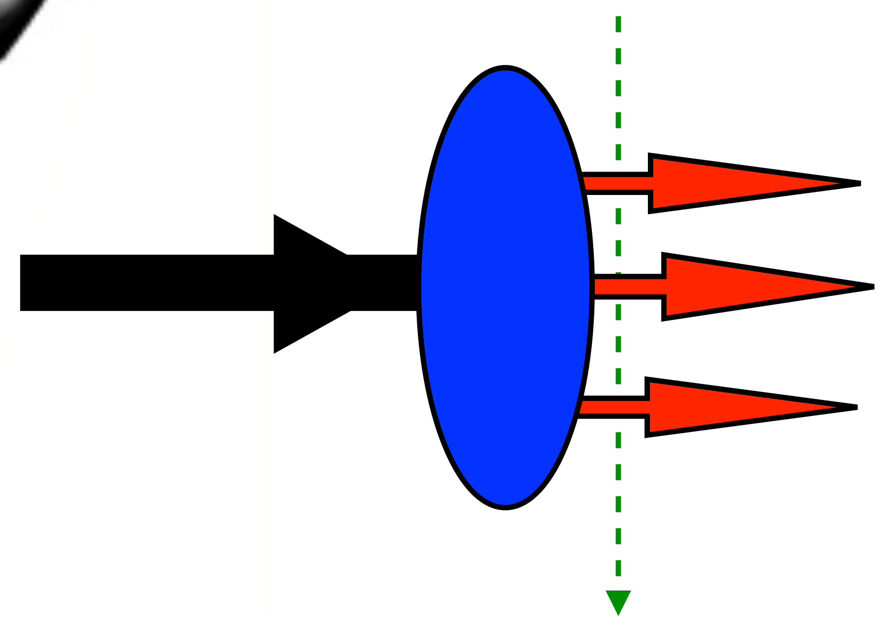
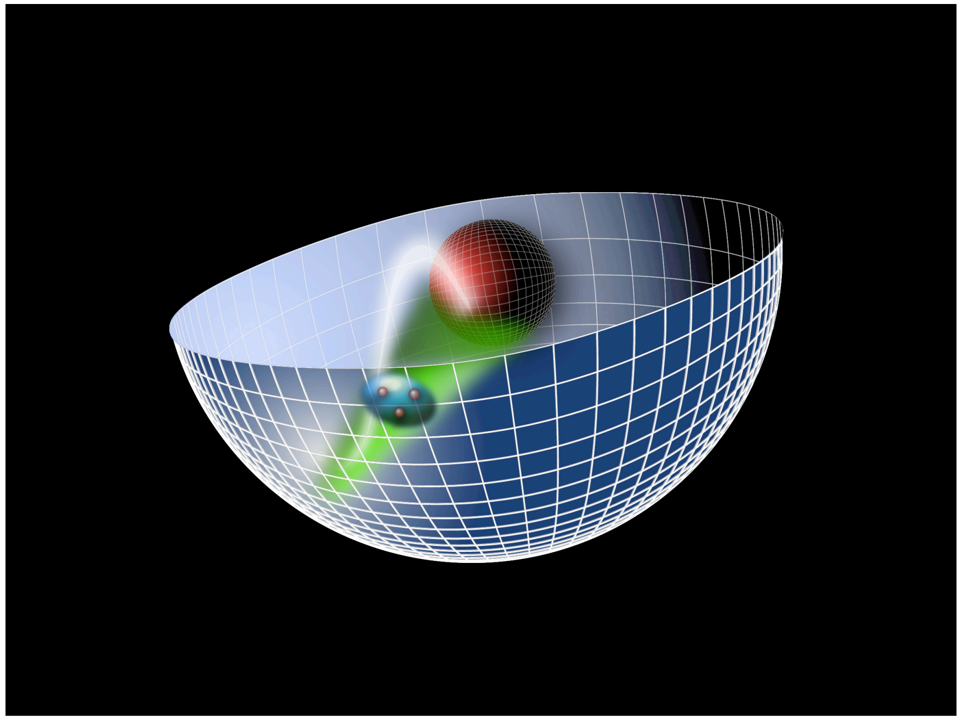
Supersymmetric Features of Hadron Physics and other Novel Features of QCD from Light-Front Holography and Superconformal Quantum Mechanics



Fixed $\tau = t + z/c$



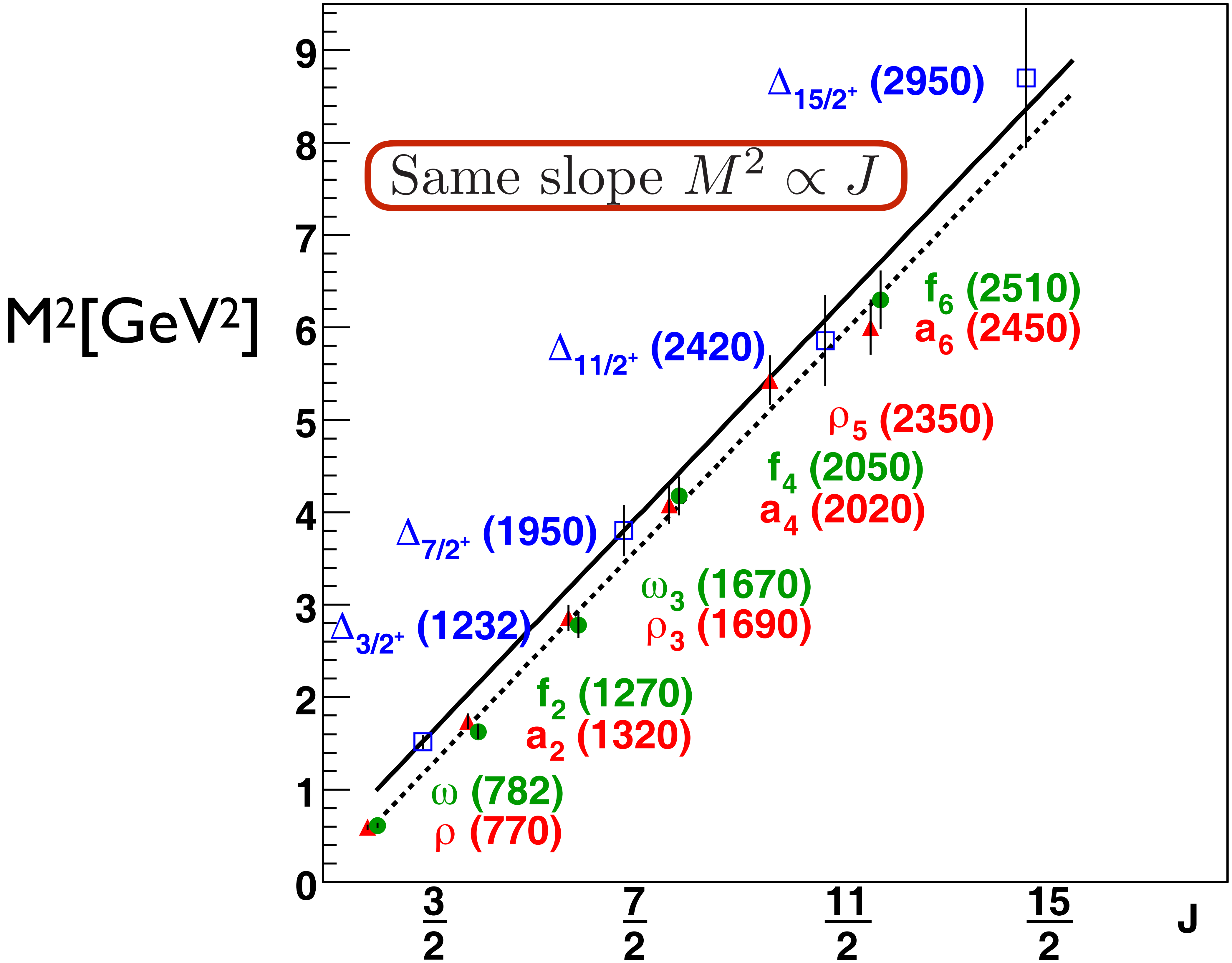


Bound States in Strongly Coupled Systems March 12, 2018

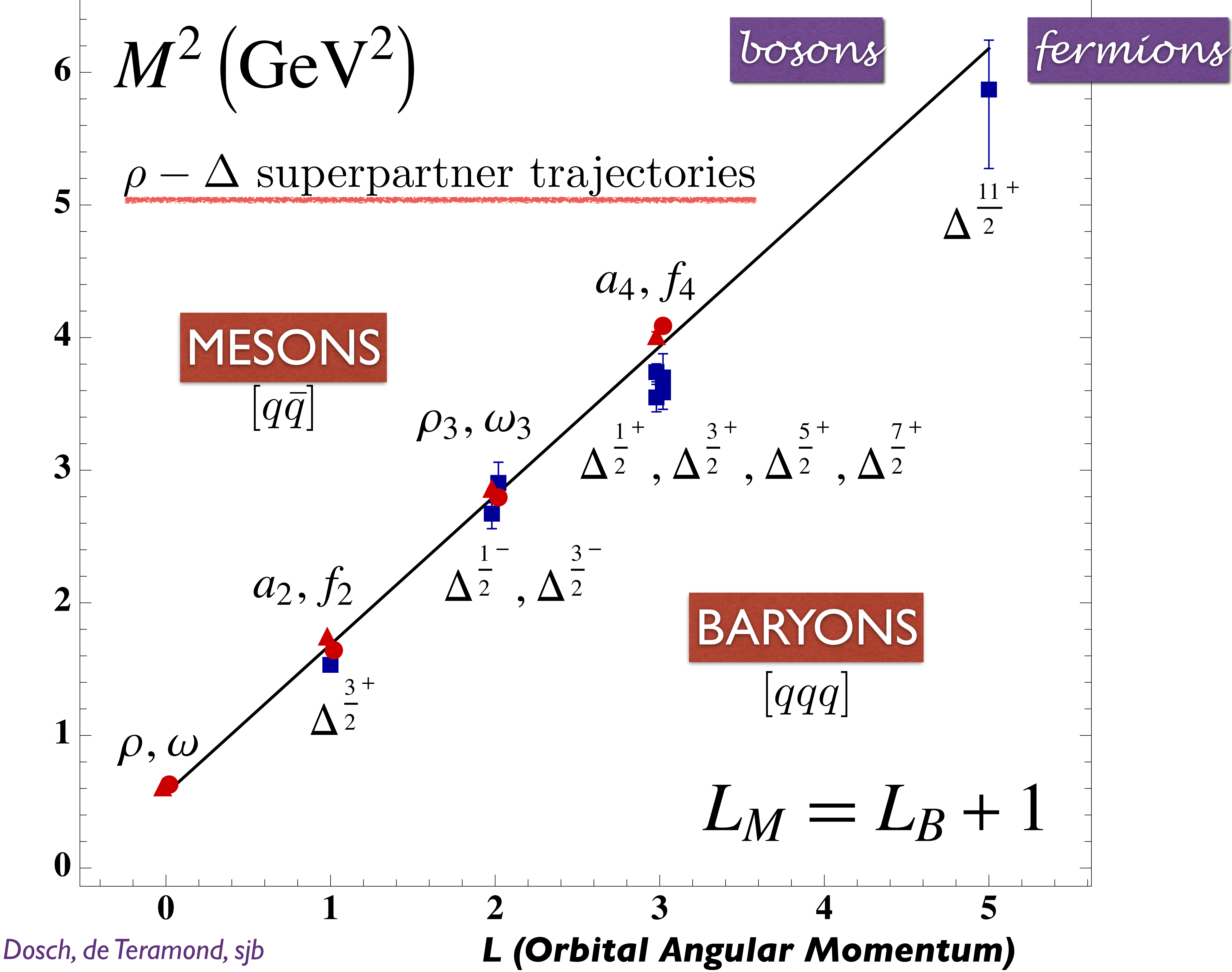
Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

with Guy de Tèramond, Hans Günter Dosch, C. Lorcè, M. Nielsen, K. Chiu, R. S. Sufian, A. Deur



The leading Regge trajectory: Δ resonances with maximal J in a given mass range.
Also shown is the Regge trajectory for mesons with J = L+S.



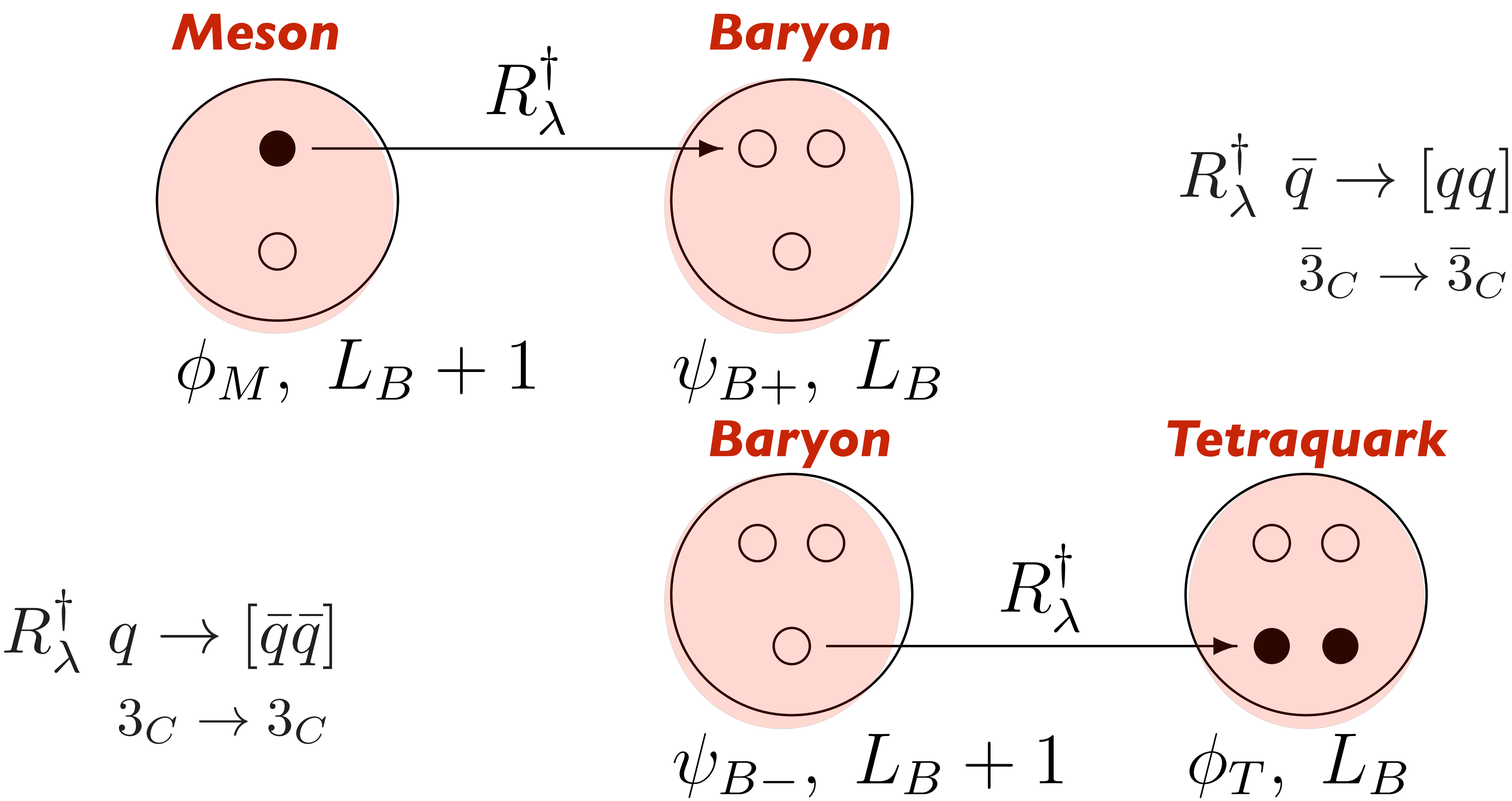
$M^2\,(\mathrm{GeV}^2)$
bosons
fermions
$\rho - \Delta$ superpartner trajectories
MESONS
$[q\bar{q}]$
BARYONS
$[qqq]$
ρ, ω
a_2, f_2
ρ_3, ω_3
a_4, f_4
$\Delta \frac{3}{2}^+$
$\Delta \frac{1}{2}^-, \Delta \frac{3}{2}^-$
$\Delta \frac{1}{2}^+, \Delta \frac{3}{2}^+, \Delta \frac{5}{2}^+, \Delta \frac{7}{2}^+$
$\Delta \frac{11}{2}^+$
$L_M = L_B + 1$
0
1
2
3
4
5
6
L (Orbital Angular Momentum)
Dosch, de Teramond, sjb

Guy de Tèramond, Hans Günter Dosch, sjb

Superconformal Algebra

2X2 Hadronic Multiplets: 4-Plet

Bosons, Fermions with Equal Mass!

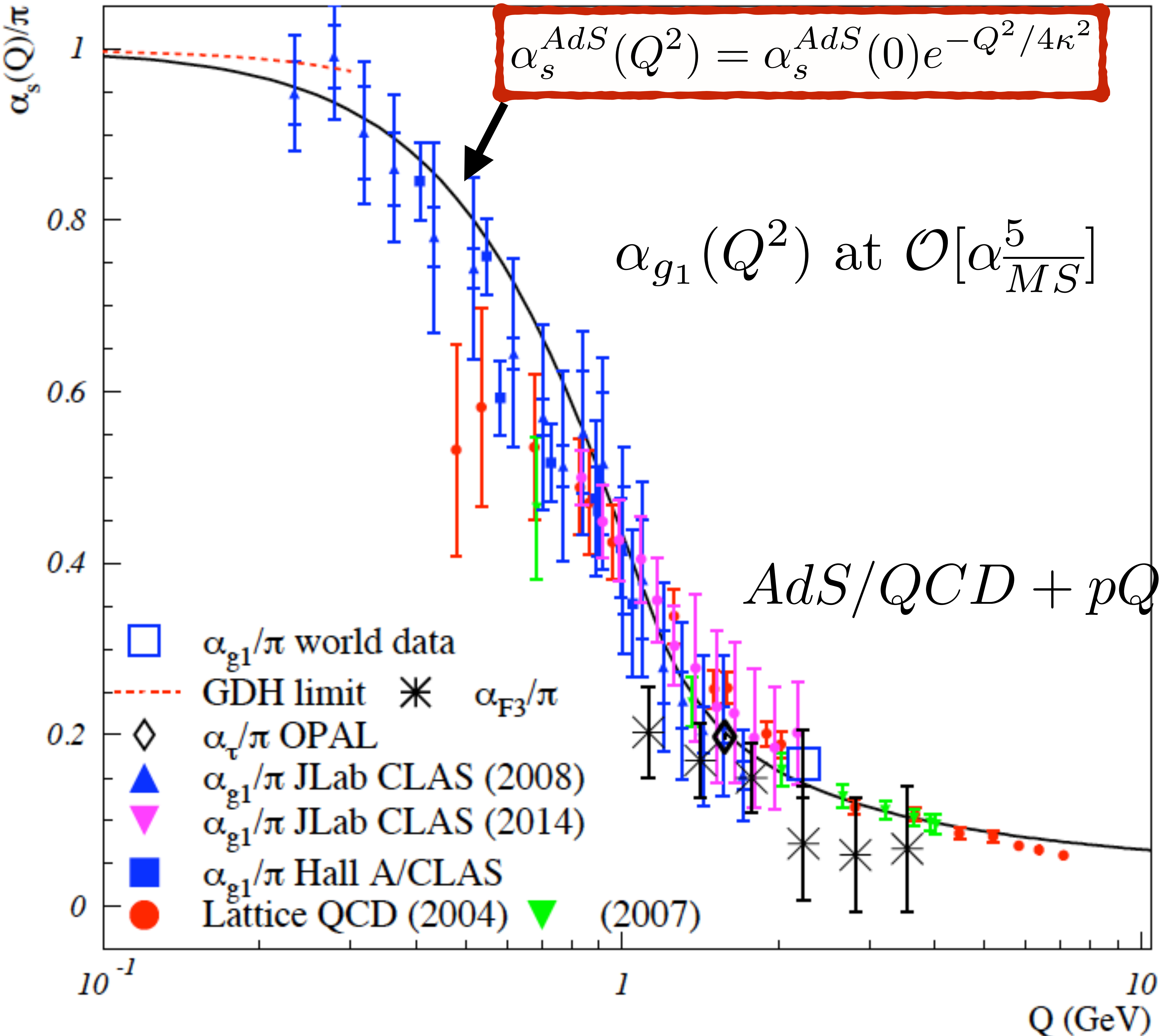


Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

Fundamental Question: Origin of the QCD Mass Scale

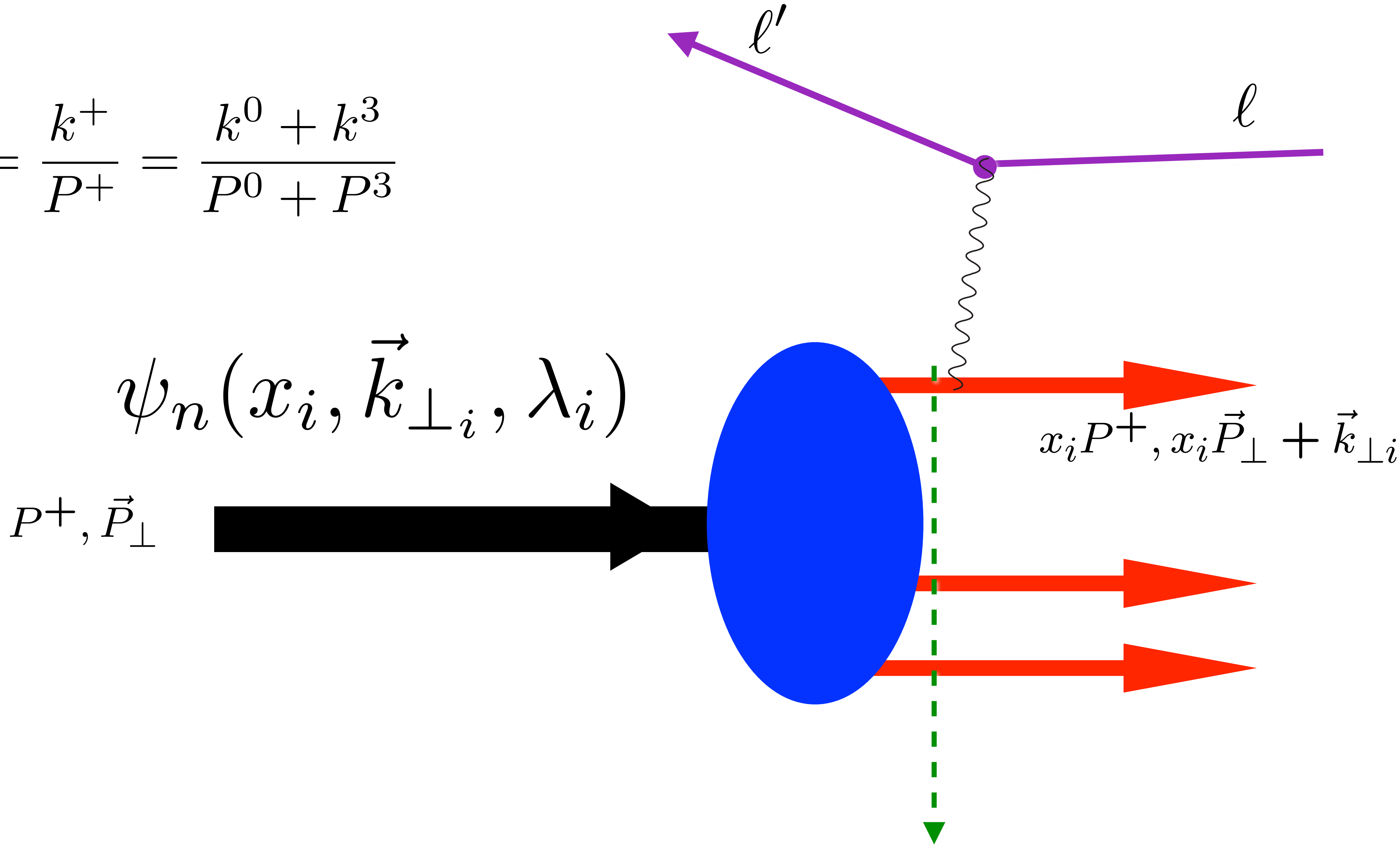
- ***Pion massless for m_q=0***
- ***What sets the mass of the proton when m_q=0 ?***
- ***QCD: No knowledge of MeV units:
Only ratios of masses can be predicted***
- ***Novel proposal by de Alfaro, Fubini, and Furlan (DAFF):
Mass scale κ can appear in Hamiltonian leaving the action conformal!***
- ***Unique Color-Confinement Potential*** $\kappa^4 \zeta^2$
- ***Eigenstates of Light-Front Hamiltonian determine hadronic mass spectrum and LF wavefunctions*** $\psi_H(x_i, \vec{k}_{\perp i}, \lambda_i)$
- ***Superconformal algebra: Degenerate meson, baryon, and tetraquark mass spectrum***
- ***Running QCD Coupling at all scales:*** $\text{Predict } \frac{\Lambda_{\overline{MS}}}{m_p}$

$$\Lambda_{\overline{MS}} = 0.5983\kappa = 0.5983\frac{m_\rho}{\sqrt{2}} = 0.4231 m_\rho = 0.328\ GeV$$



Deur, de Teramond, sjb

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Measurements of hadron LF wavefunction are at fixed LF time

Fixed $\tau = t + z/c$

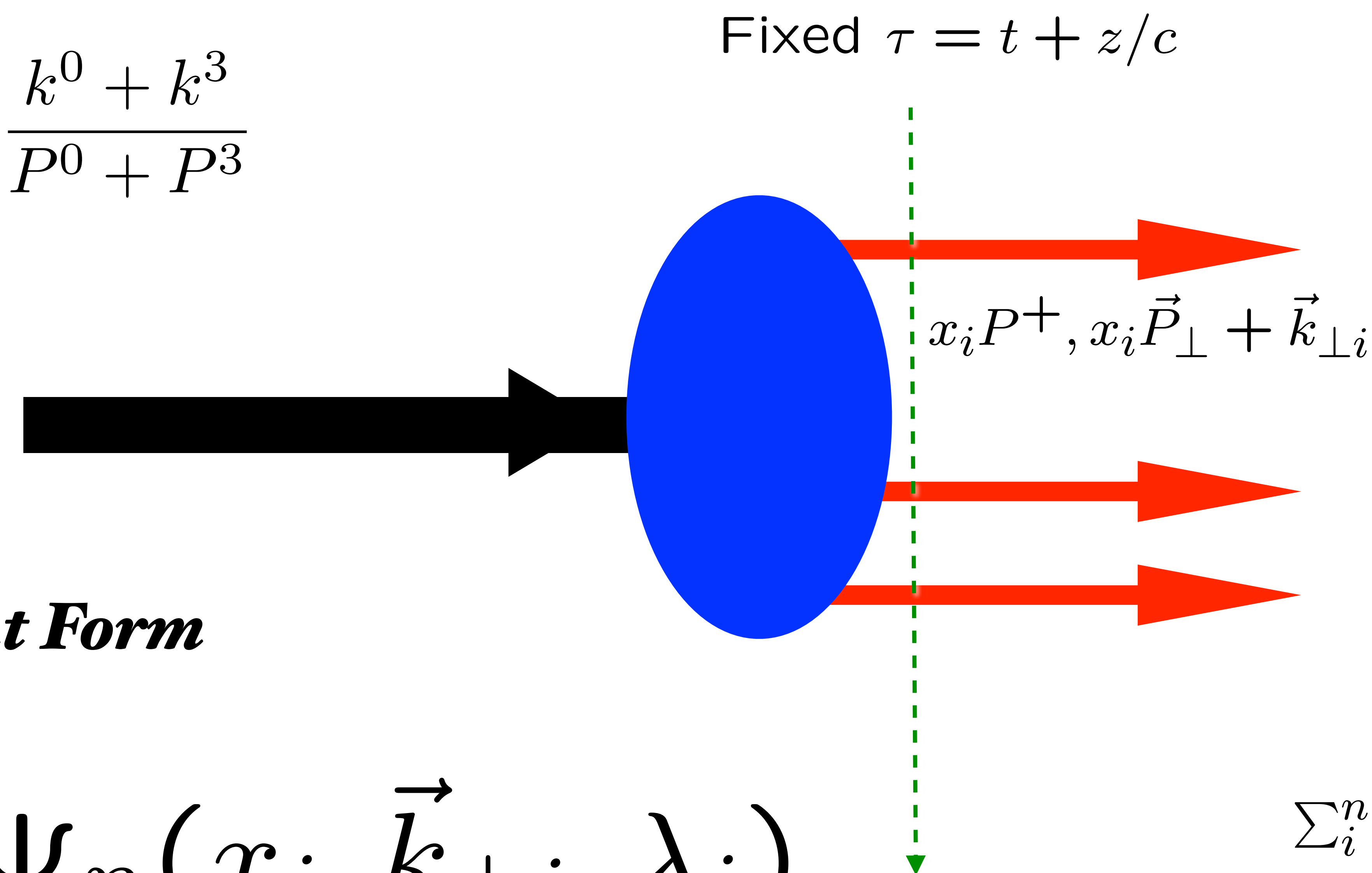
Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$P^+, \vec{P}_\perp$



Dirac: Front Form

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

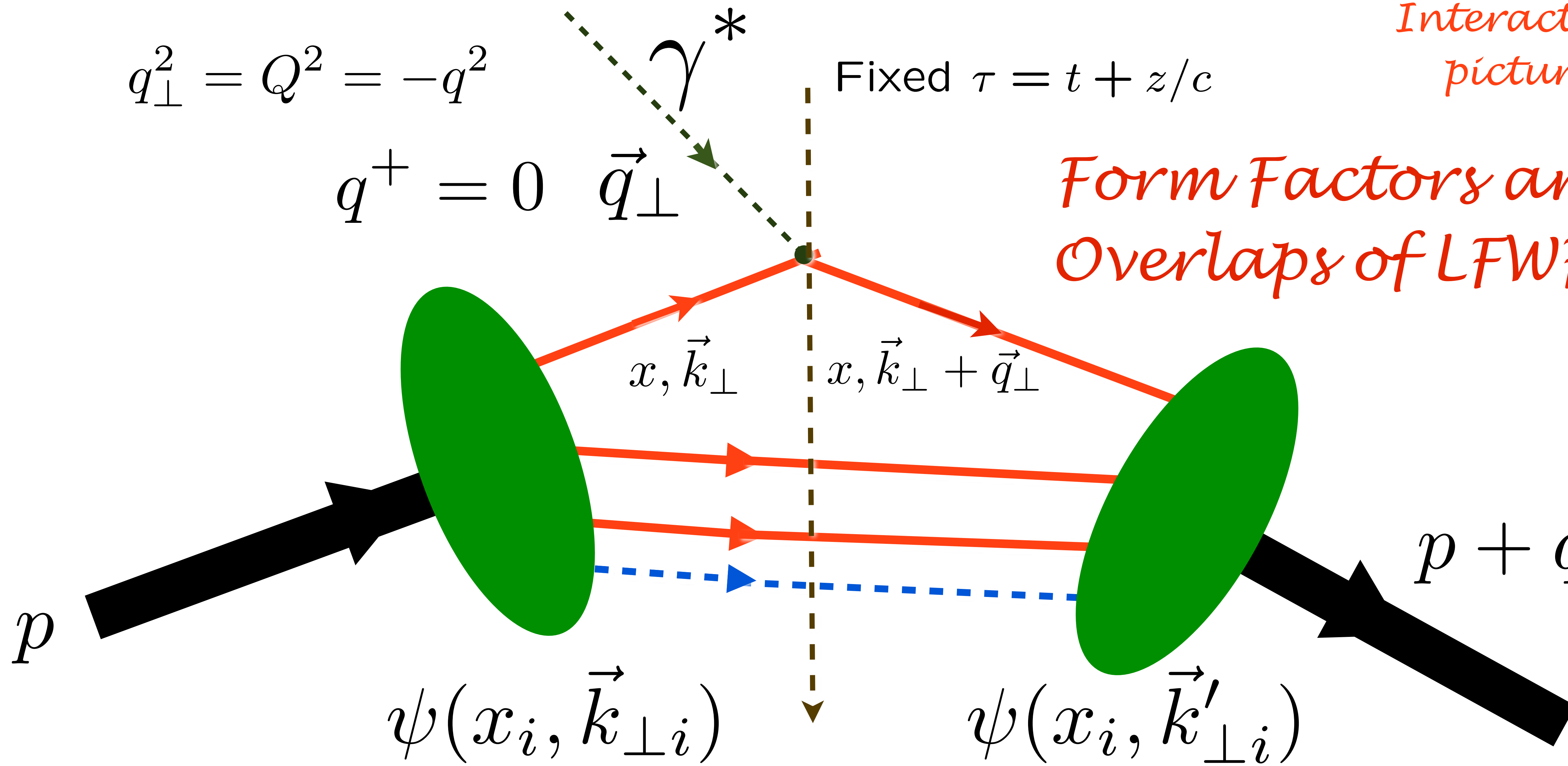
Causal, Frame-independent, Simple Vacuum, Current Matrix Elements are overlap of LFWFS

$$< p+q|j^+(0)|p >= 2p^+ F(q^2)$$

Front Form

Interaction picture

Form Factors are Overlaps of LFWFs



struck $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_\perp$

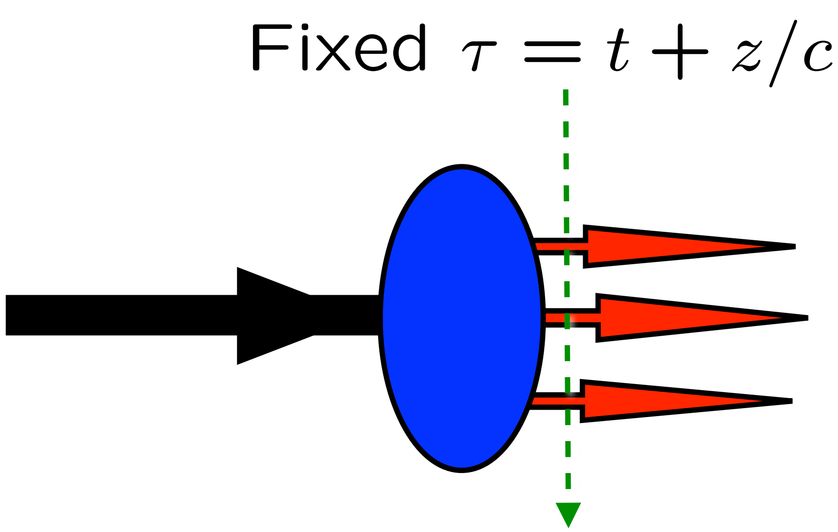
Drell &Yan, West
Exact LF formula!

Drell, sjb

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

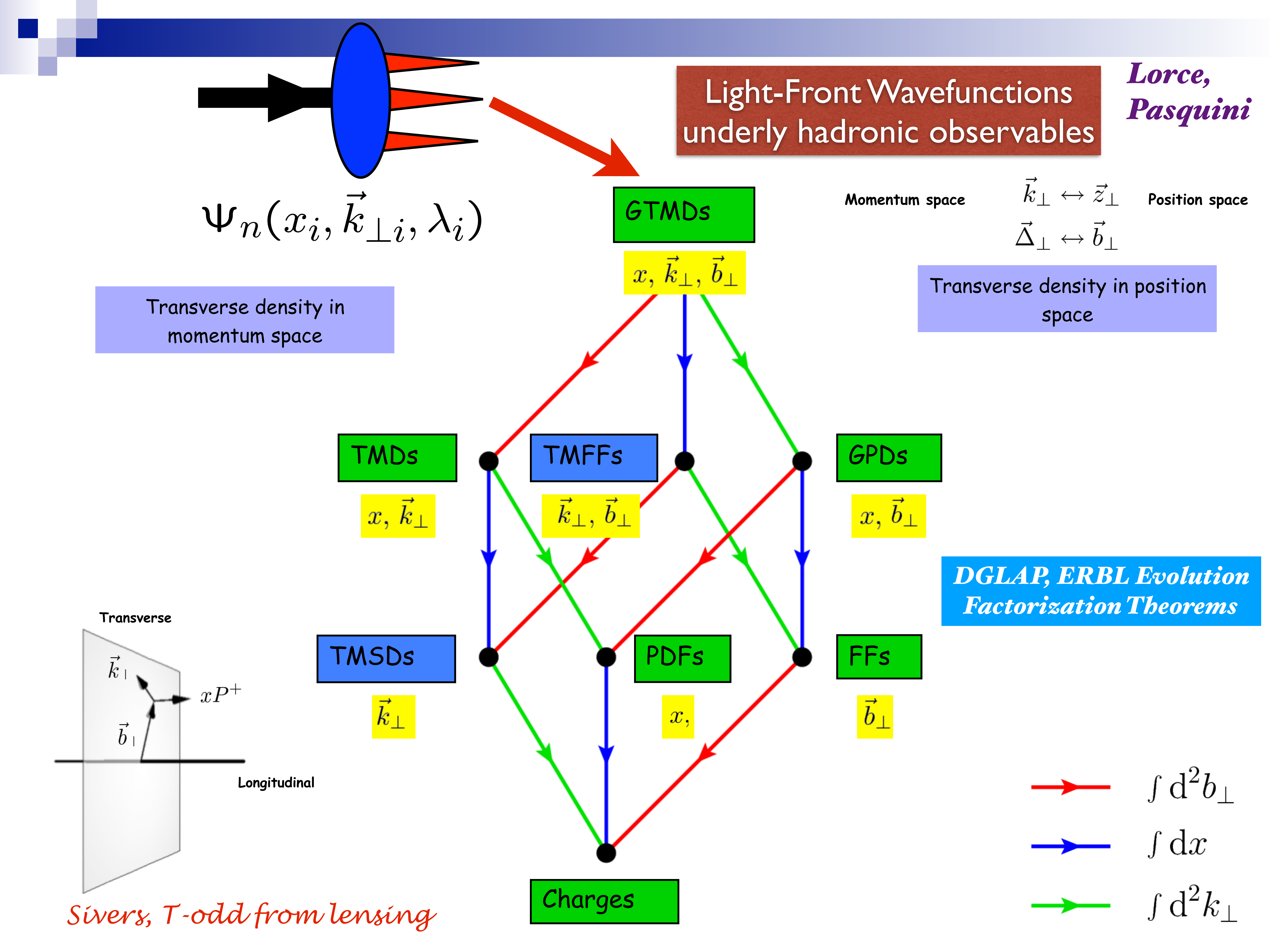
Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD} |\psi> = M^2 |\psi>$$

Direct connection to QCD Lagrangian

Off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space


Light-Front Wavefunctions underly hadronic observables
Lorce, Pasquini
$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$
Momentum space
$\vec{k}_\perp \leftrightarrow \vec{z}_\perp$
Position space
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$
GTMDs
$x,\, \vec{k}_\perp,\, \vec{b}_\perp$
Transverse density in momentum space
Transverse density in position space
TMDs
$x,\, \vec{k}_\perp$
TMFFs
$\vec{k}_\perp,\, \vec{b}_\perp$
GPDs
$x,\, \vec{b}_\perp$
DGLAP, ERBL Evolution
Factorization Theorems
Transverse
$\vec{k}_\perp$
xP^+
$\vec{b}_\perp$
Longitudinal
TMSDs
$\vec{k}_\perp$
PDFs
$x,$
FFs
$\vec{b}_\perp$
$\int \mathrm{d}^2 b_\perp$
$\int \mathrm{d}x$
$\int \mathrm{d}^2 k_\perp$
Charges
Sivers, T-odd from lensing

Single-spin asymmetries

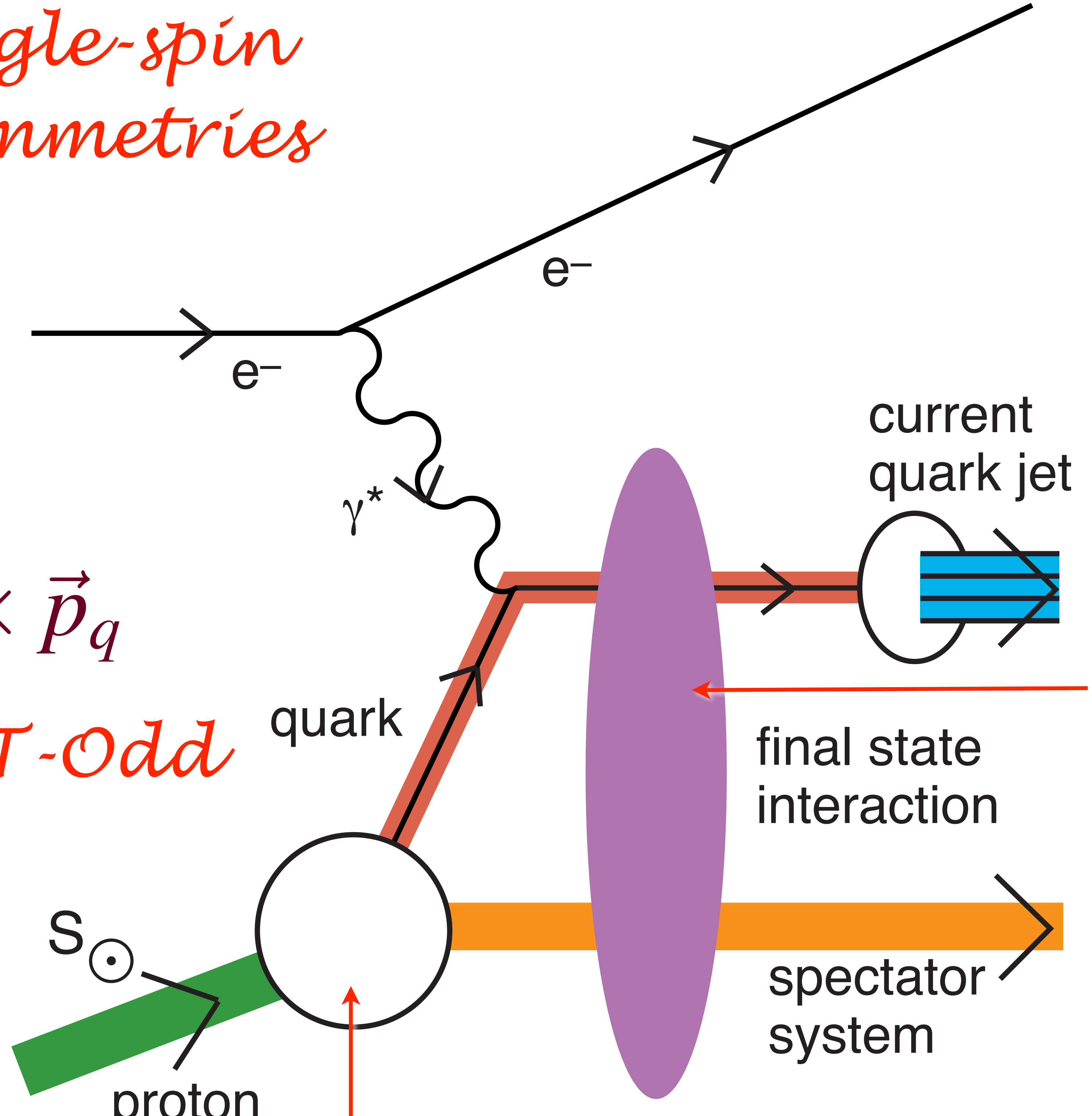
Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...

i $\vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Pseudo- T-Odd



QCD S- and P- Coulomb Phases --Wilson Line

"Lensing Effect"

"Lensing" involves soft scales

Light-Front Wavefunction S and P- Waves!

Leading-Twist Rescattering Violates pQCD Factorization!

Sign reversal in DY!

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \xcancel{\sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale come from?

QCD does not know what MeV units mean!
Only Ratios of Masses Determined

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique confinement potential!

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

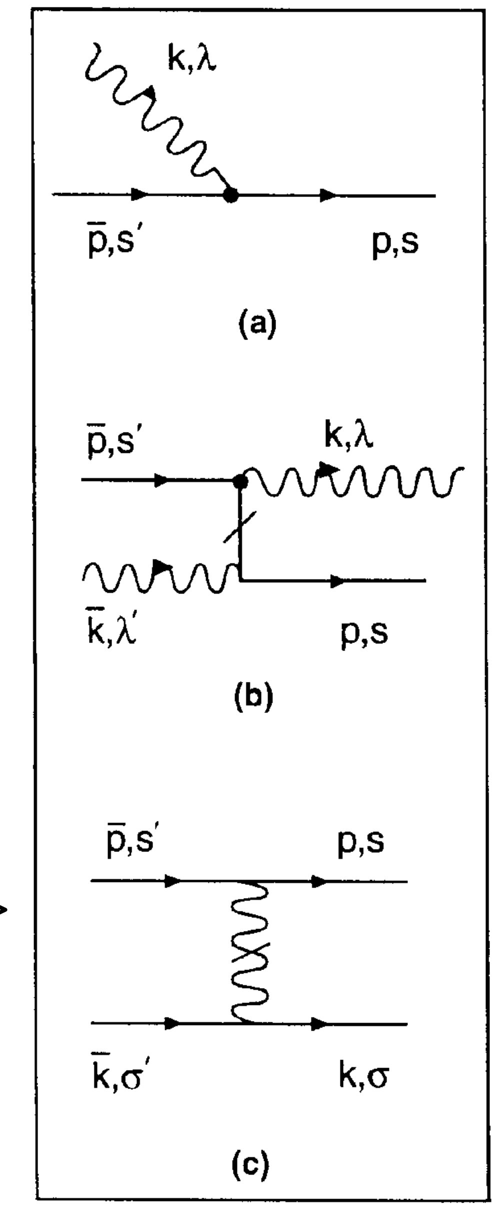
$$L^{QCD} \to H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

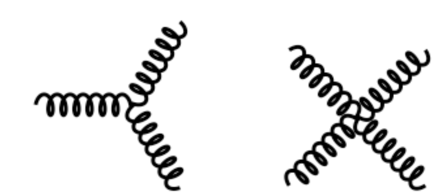
H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

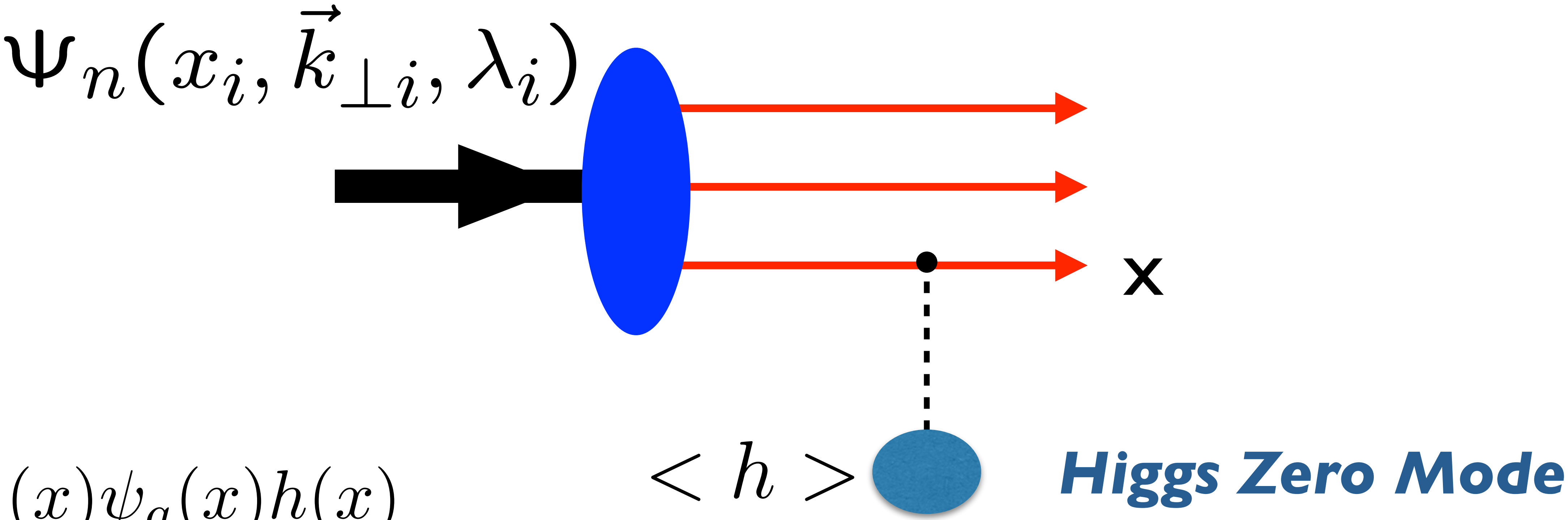


Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions



LFWFs: Off-shell in P- and invariant mass

$$H^{int}_{LF}$$



$$g_q \bar{\psi}_q(x) \psi_q(x) h(x)$$

Yukawa Higgs coupling of confined quark to Higgs zero mode gives

$$\bar{u}u \; g_q < h >= \frac{m_q}{x_q} m_q = \frac{m_q^2}{x_q}$$

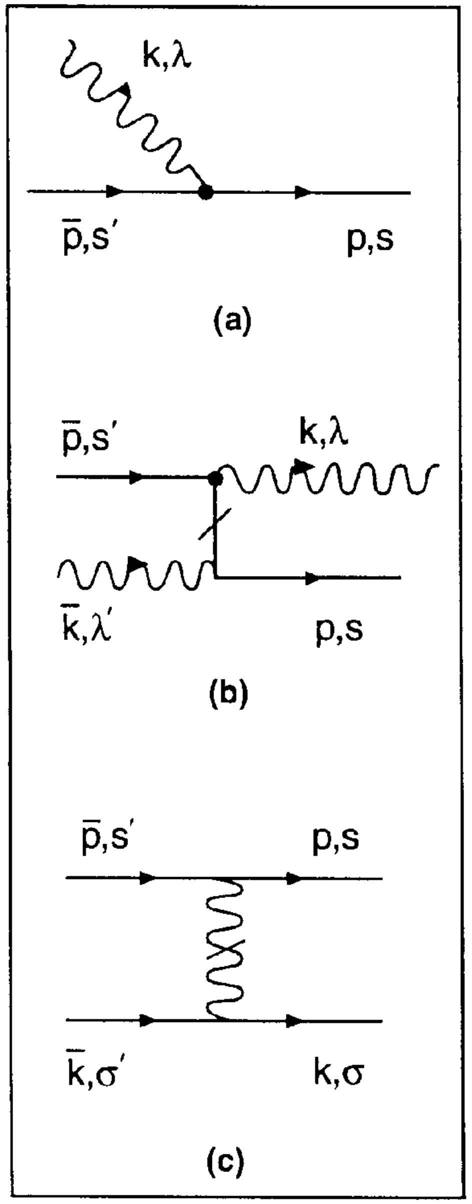
$$H_{LF} = \sum_q \frac{k_{\perp q}^2 + m_q^2}{x_q}$$

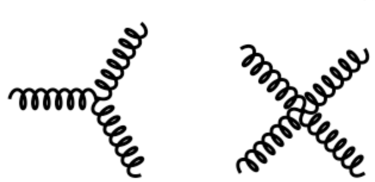
Light-Front QCD Heisenberg Equation

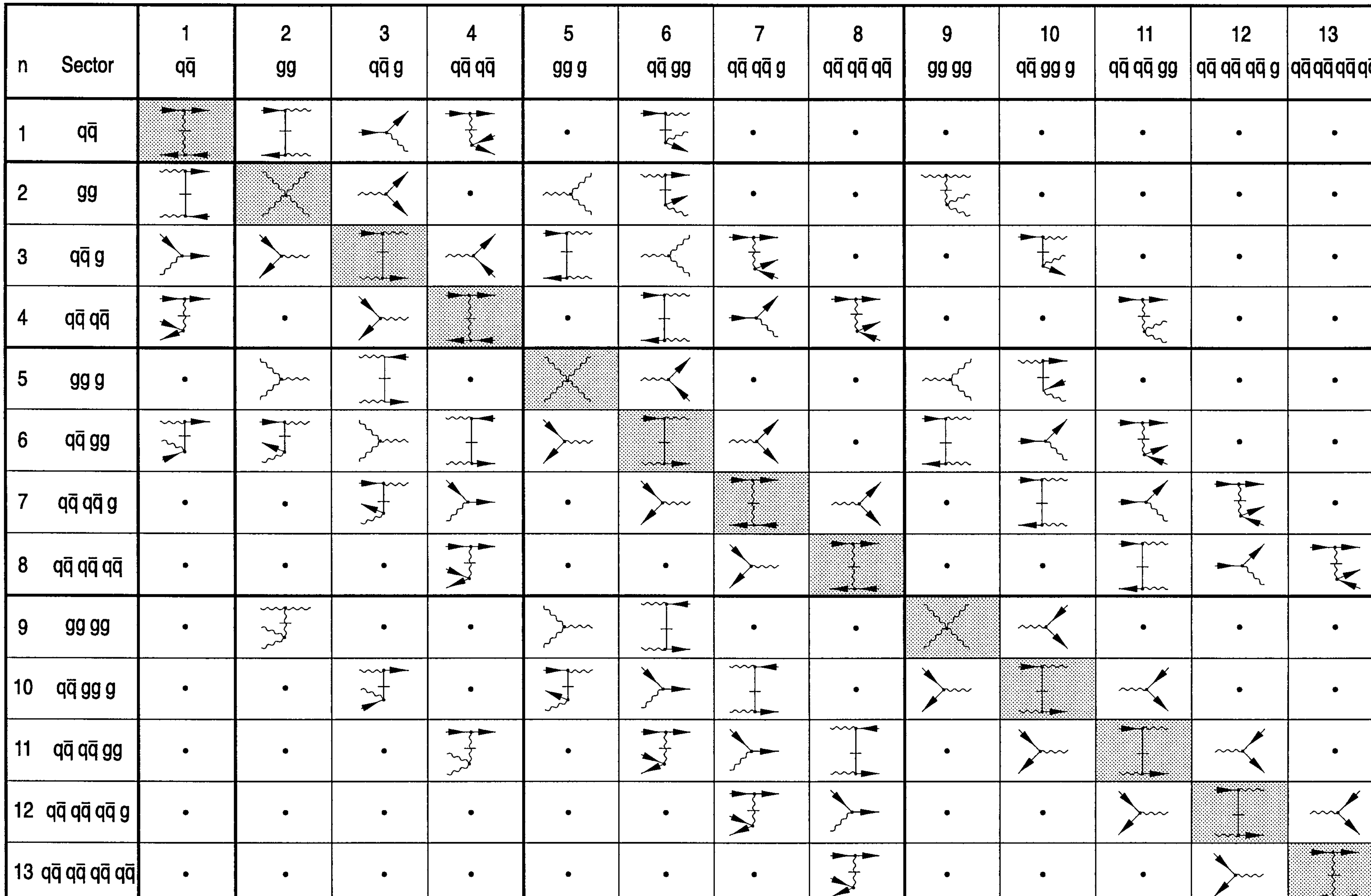
$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\,|\Psi_h\rangle$$

DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb







Minkowski space; frame-independent; no fermion doubling; no ghosts trivial vacuum

$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$

$$|p,S_z> = \sum_{n=3} \Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)|n;\vec{k}_{\perp_i},\lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)$$

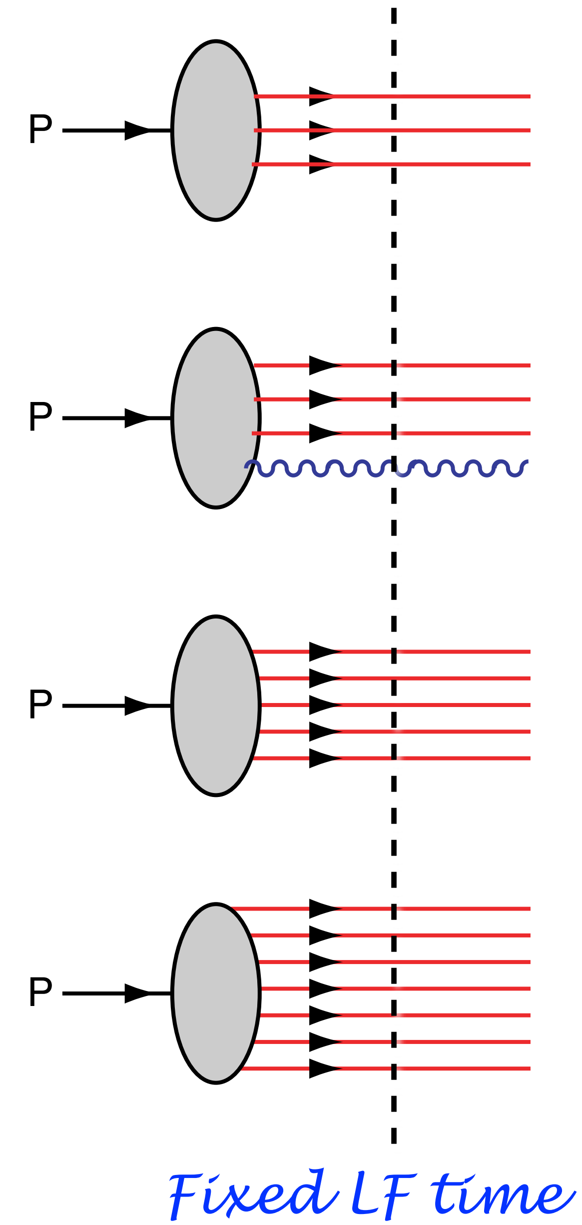
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fractions

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0+k_i^z}{P^0+P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \sum_i^n x_i = 1, \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Intrinsic heavy quarks
s(x), c(x), b(x) at high x !

$$\bar{s}(x) \neq s(x)$$
$$\bar{u}(x) \neq \bar{d}(x)$$

Hidden Color

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S}, \vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r, S, \ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

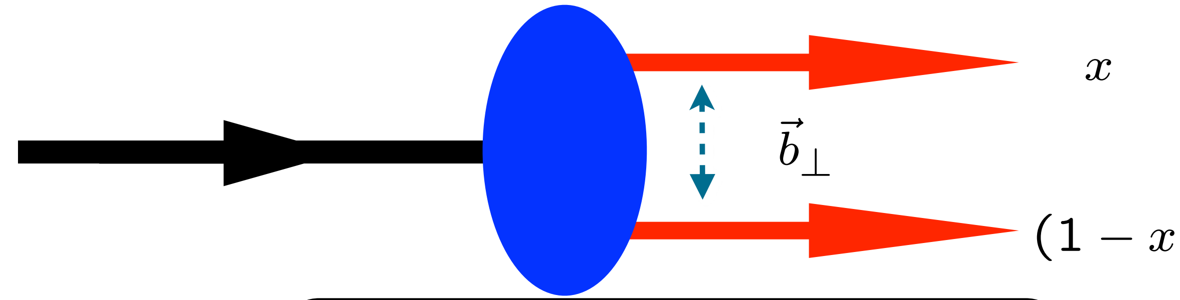
Semiclassical first approximation to QED

Schrödinger Eq.

Light-Front QCD

$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x)b^2_\perp$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$\left[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\text{eff}}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Azimuthal Basis

ζ, ϕ

$m_q = 0$

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

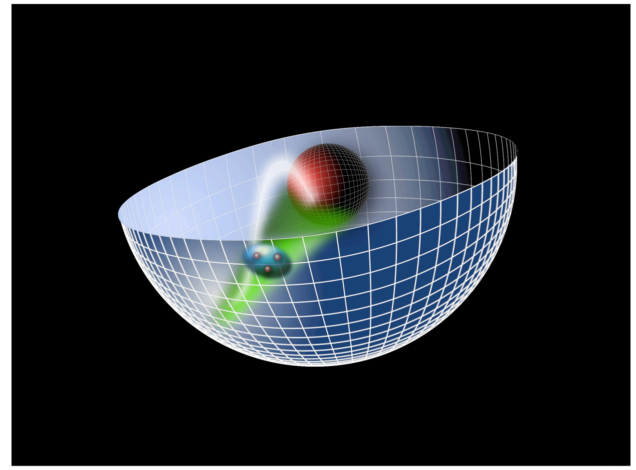
Confining AdS/QCD potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD *de Tèramond, Dosch, sjb*

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = M^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Single variable ζ

Unique Confinement Potential!

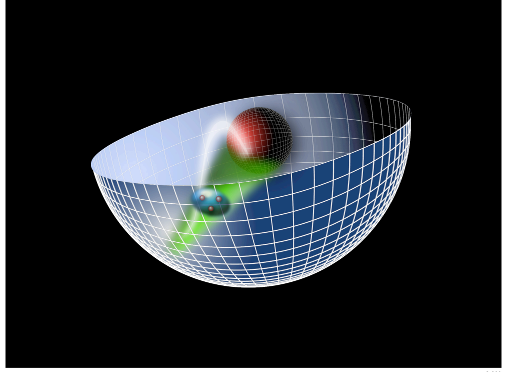
Conformal Symmetry of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

AdS$_5$



- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

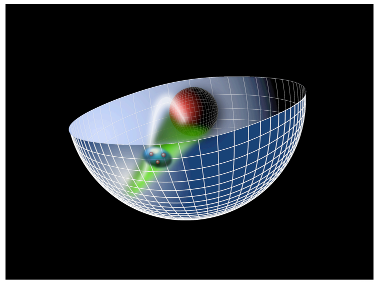
$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

AdS/CFT

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- Soft-wall dilaton profile breaks conformal invariance $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- Color Confinement in z
- Introduces confinement scale κ
- Uses AdS_5 as template for conformal theory

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Single-Variable Light-Front Bound State Equation in ζ!

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

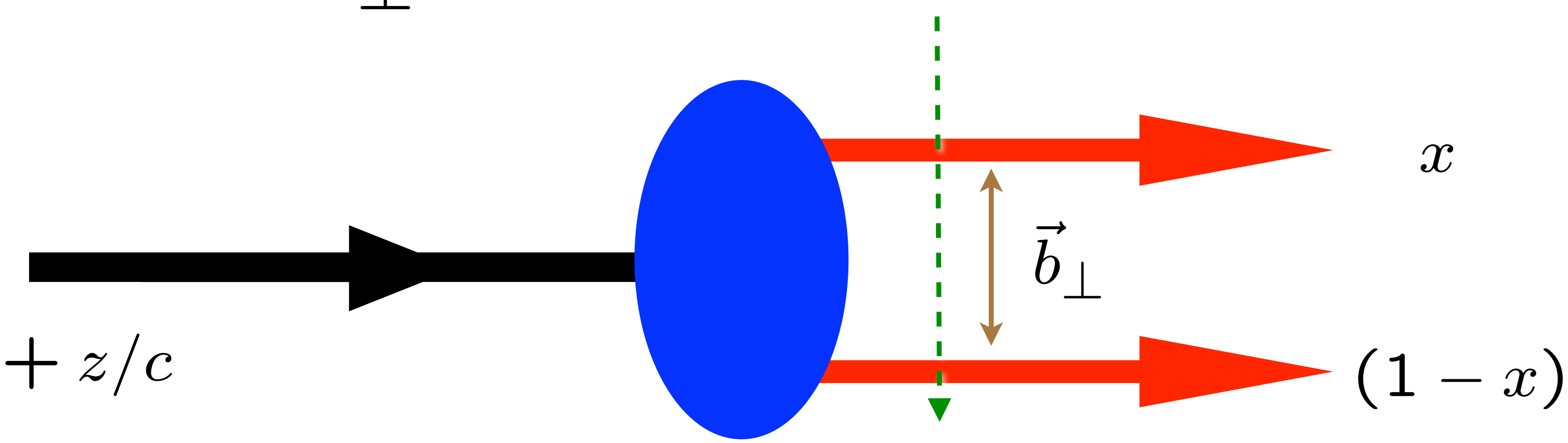
de Teramond, sjb

LF(3+1) ⟷ AdS_5

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



Fixed $\tau = t + z/c$

$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

Massless pion!

Meson Spectrum in Soft Wall Model

$$m_\pi = 0 \text{ if } m_q = 0$$

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

$$\vec{\zeta}^2 = \vec{b}^2_\perp x(1-x)$$

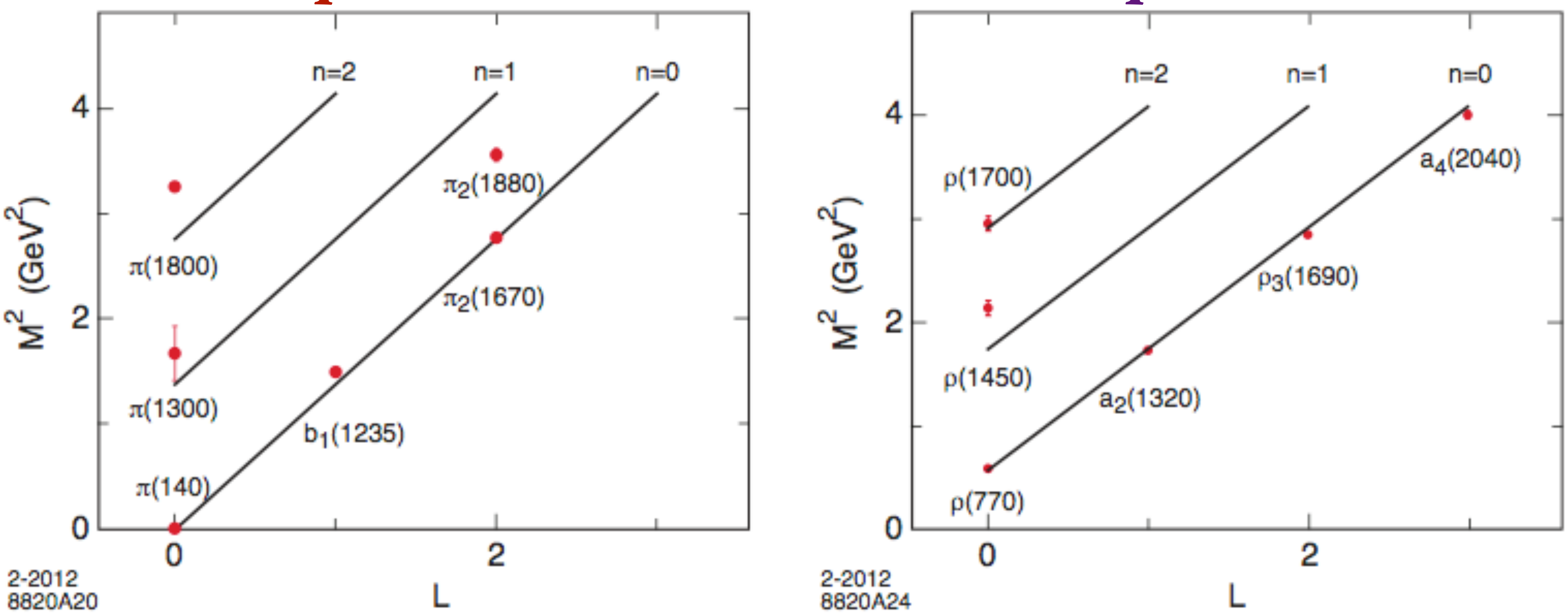
G. de Teramond, H. G. Dosch, sjb

- $J = L + S$, $I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2 (n + L + S/2)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

$m_q = 0$

Massless pion in Chiral Limit! ***Same slope in n and L!***



I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1$, $L = 1$, $J = 0, 1, 2$, vector meson a-states

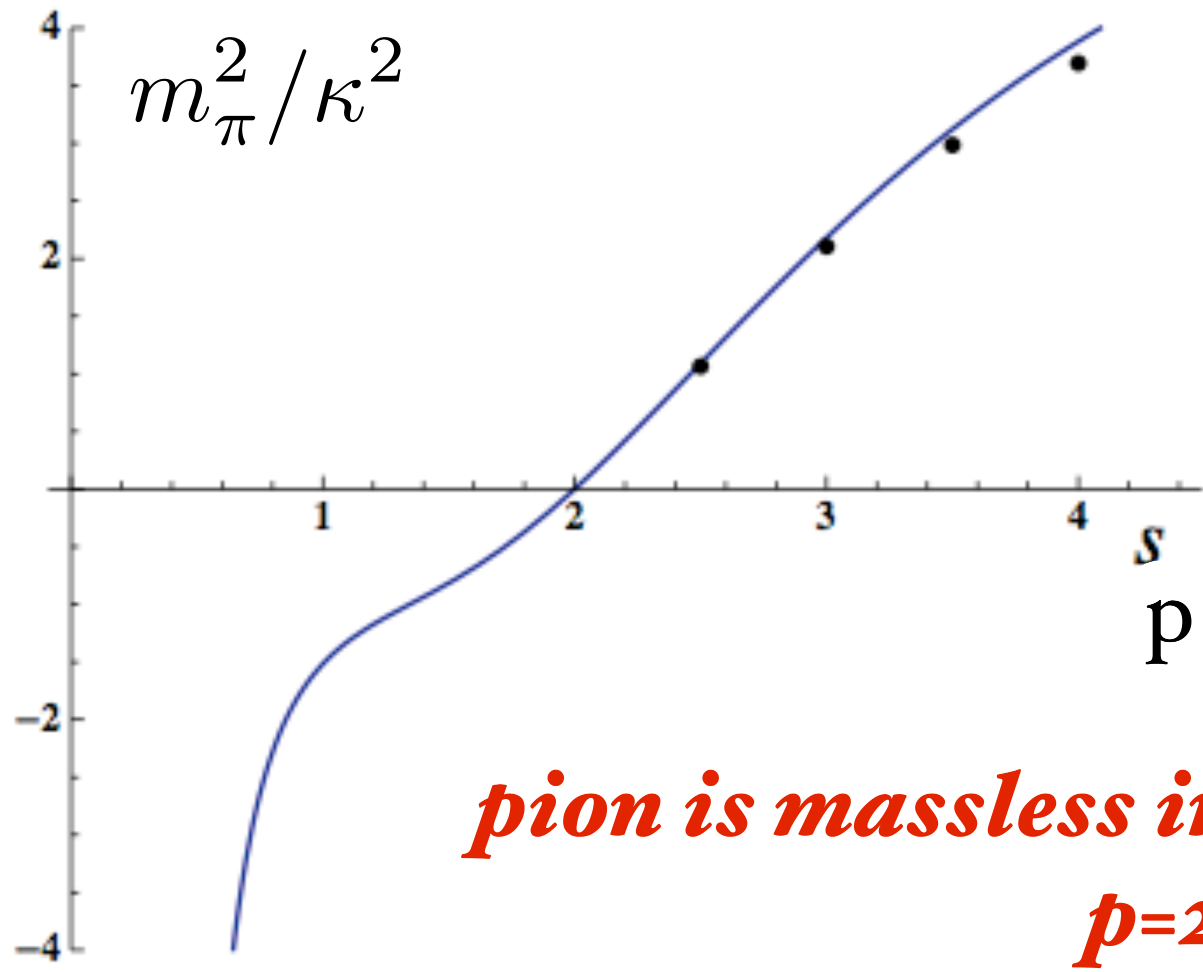
$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

G. de Teramond, H. G. Dosch, sjb

Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$



pion is massless in chiral limit iff p=2!

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **Dosch, de Tèramond, sjb**

Quark separation increases with L

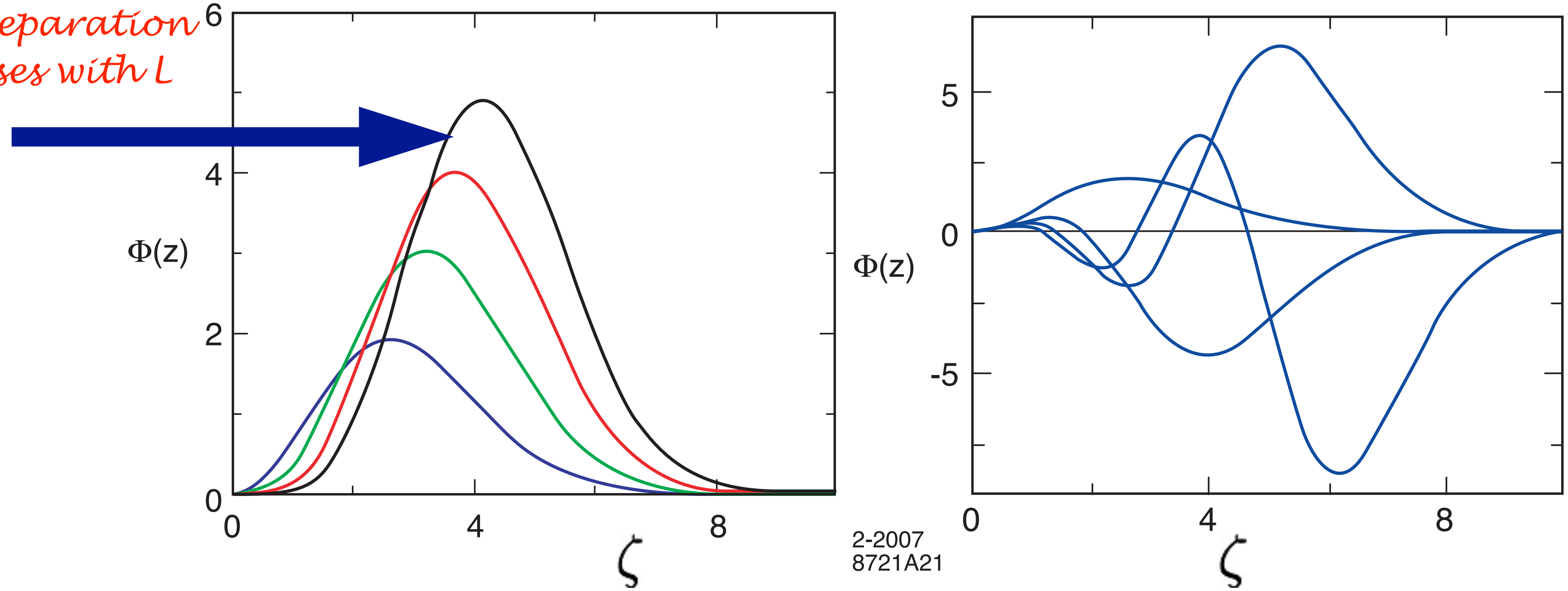
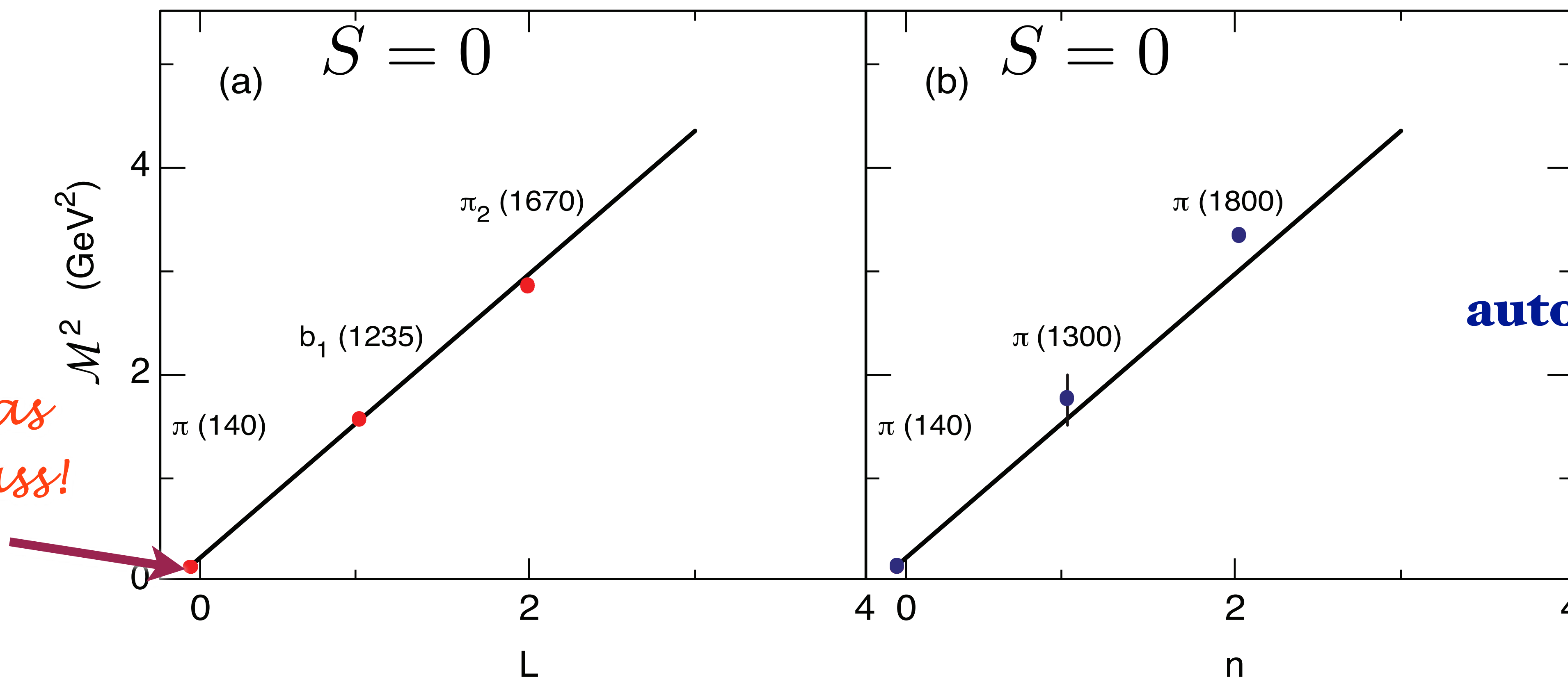


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model

Same slope in n and L!

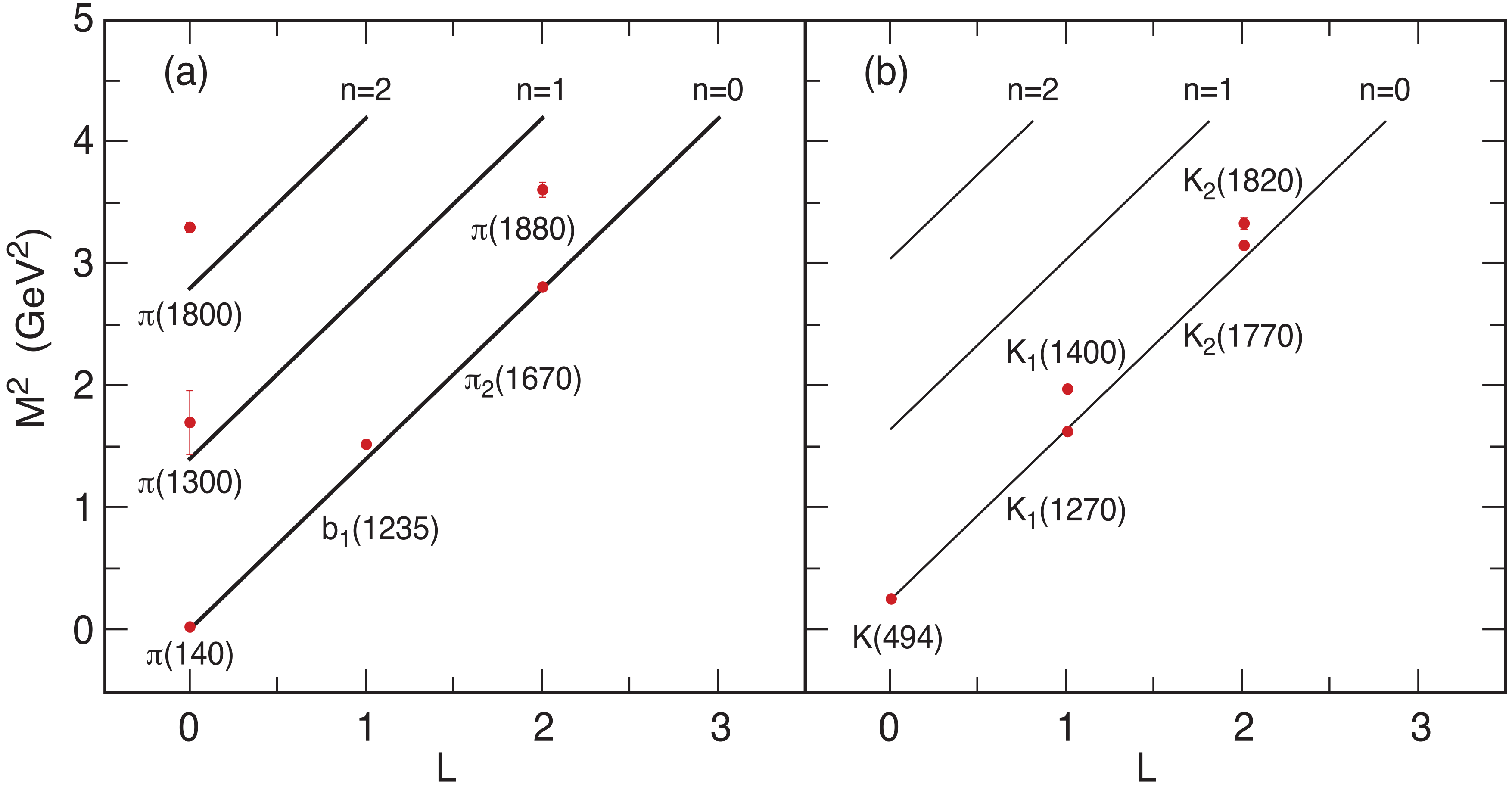
Pion has zero mass!

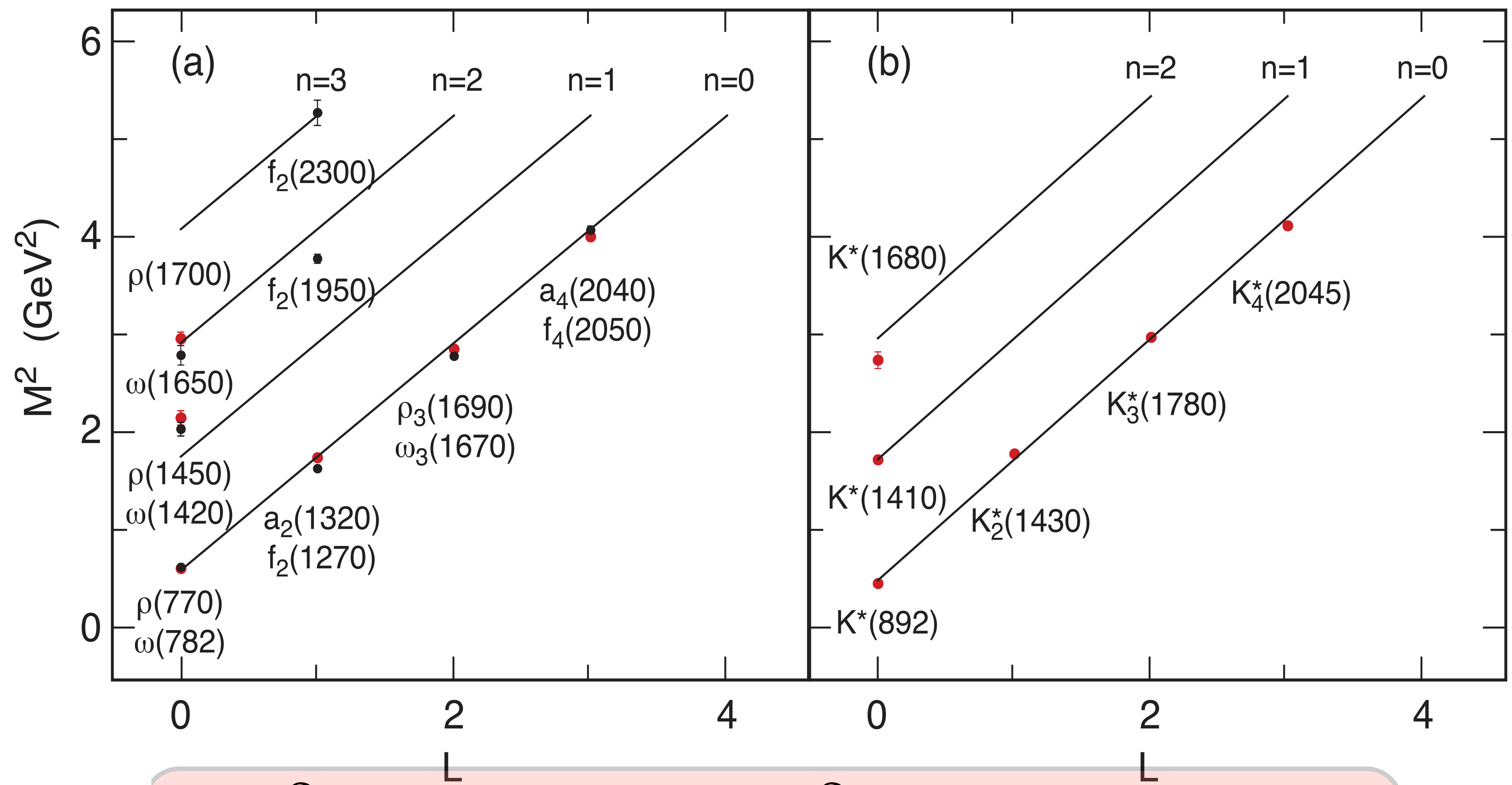


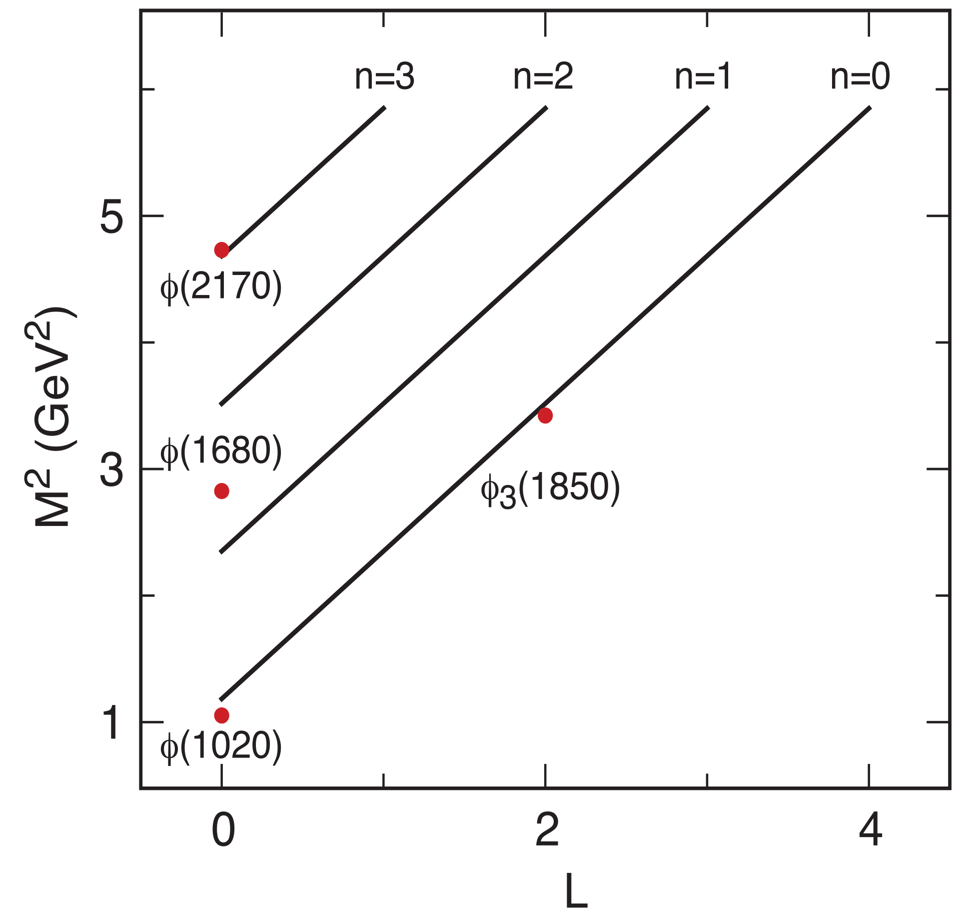
Pion mass automatically zero!

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.







$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

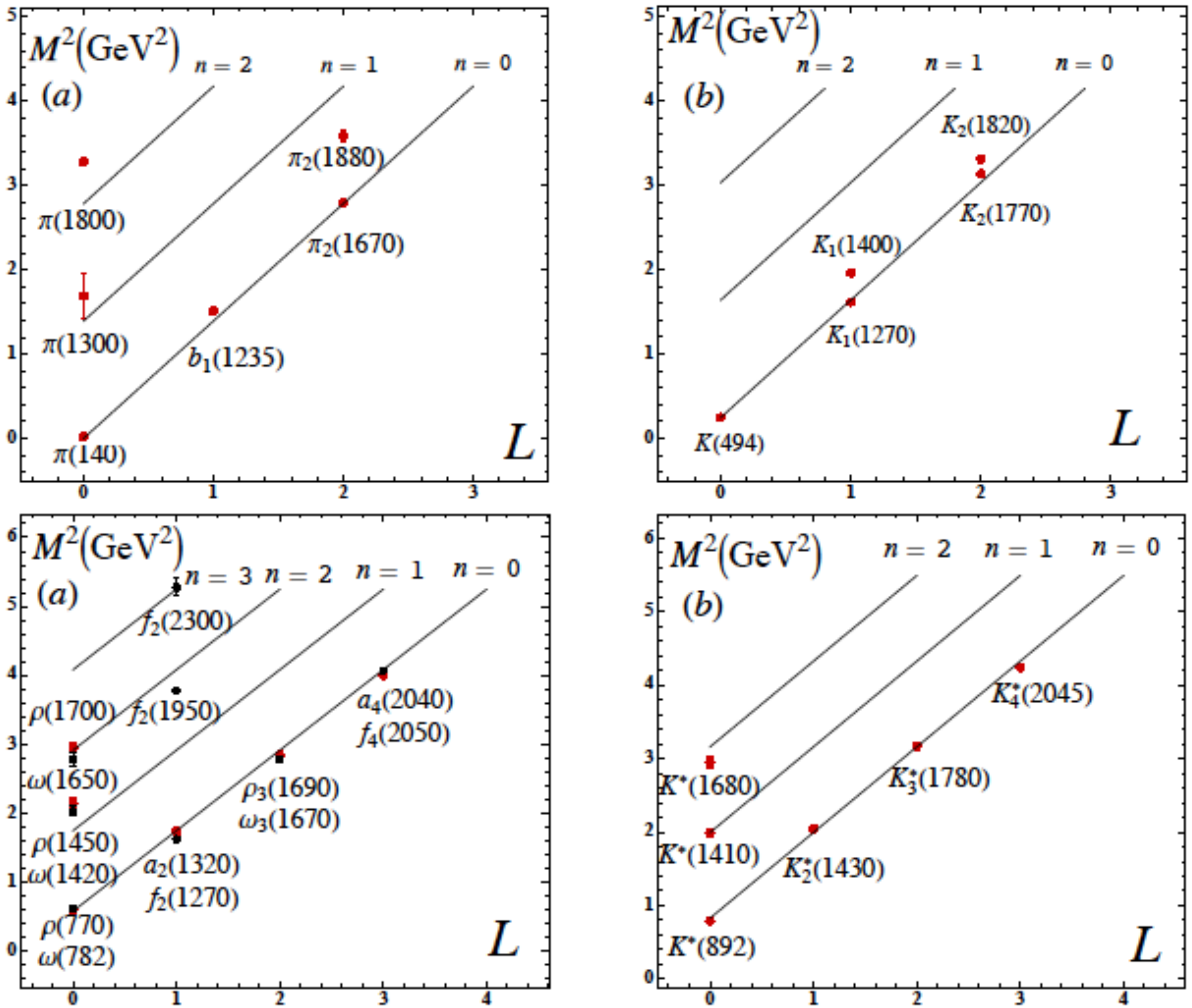
Equal Slope in n and L

De Tèramond, Dosch, sjb

$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar q}^2}{1-x} \right| X \right\rangle$$

from LF Higgs mechanism



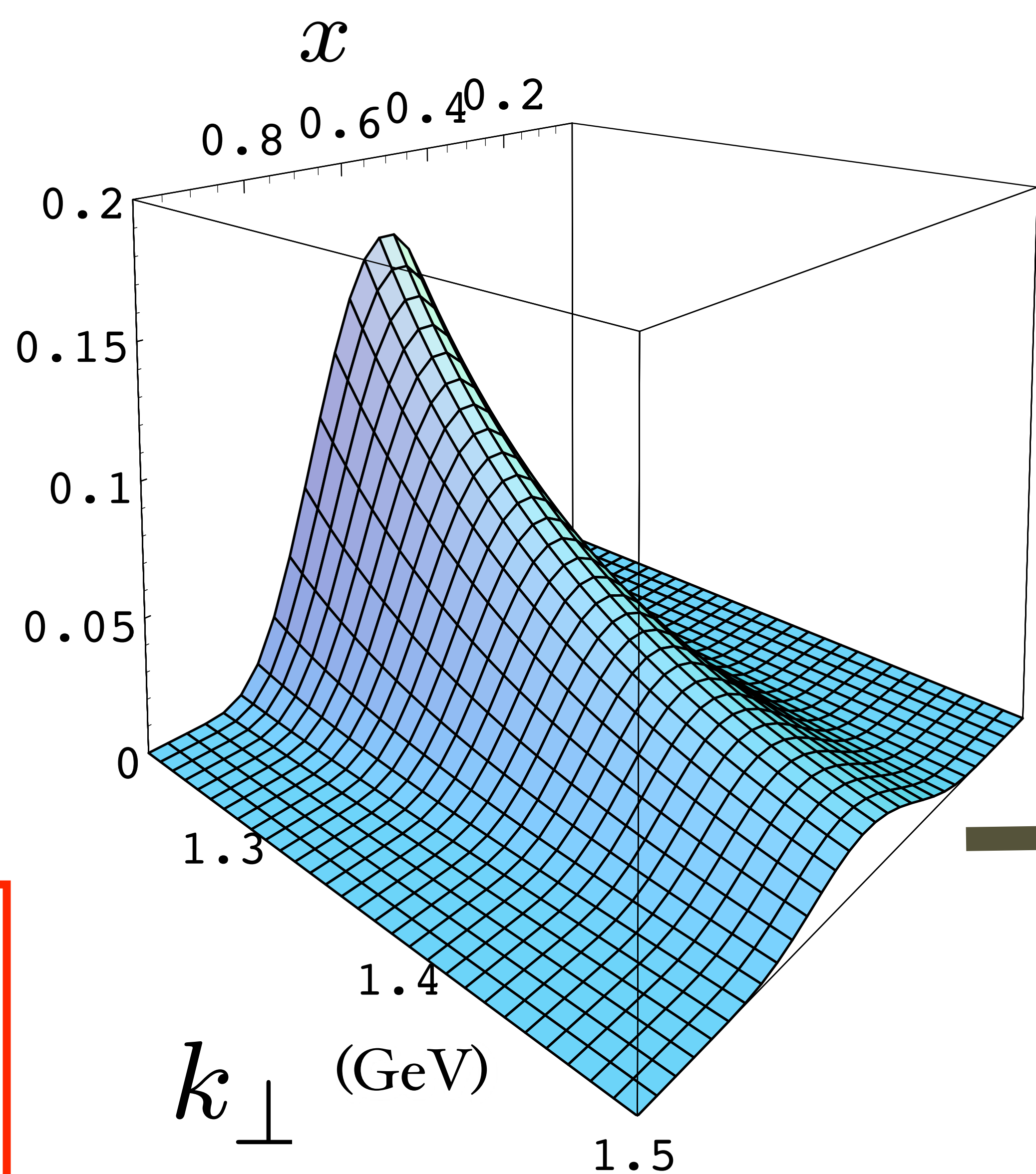
Effective mass from $m(p^2)$

Tandy, Roberts, et al

Prediction from AdS/QCD: Meson LFWF

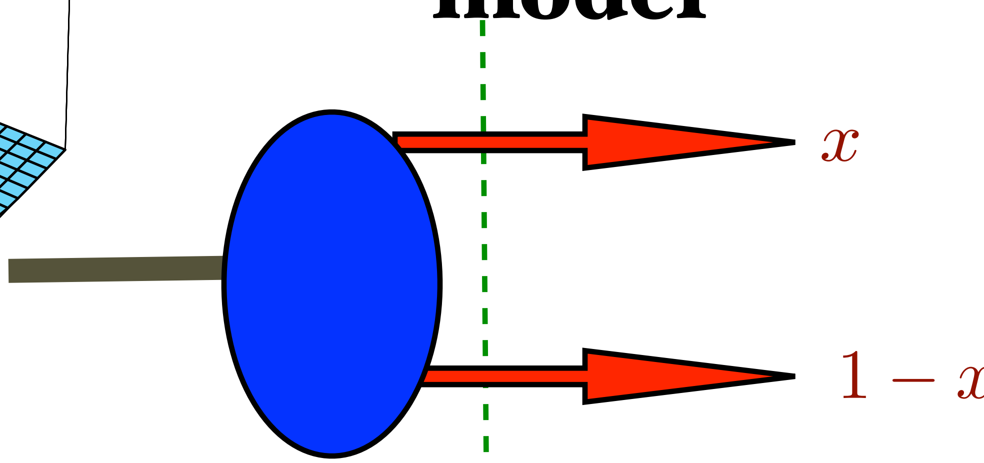
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

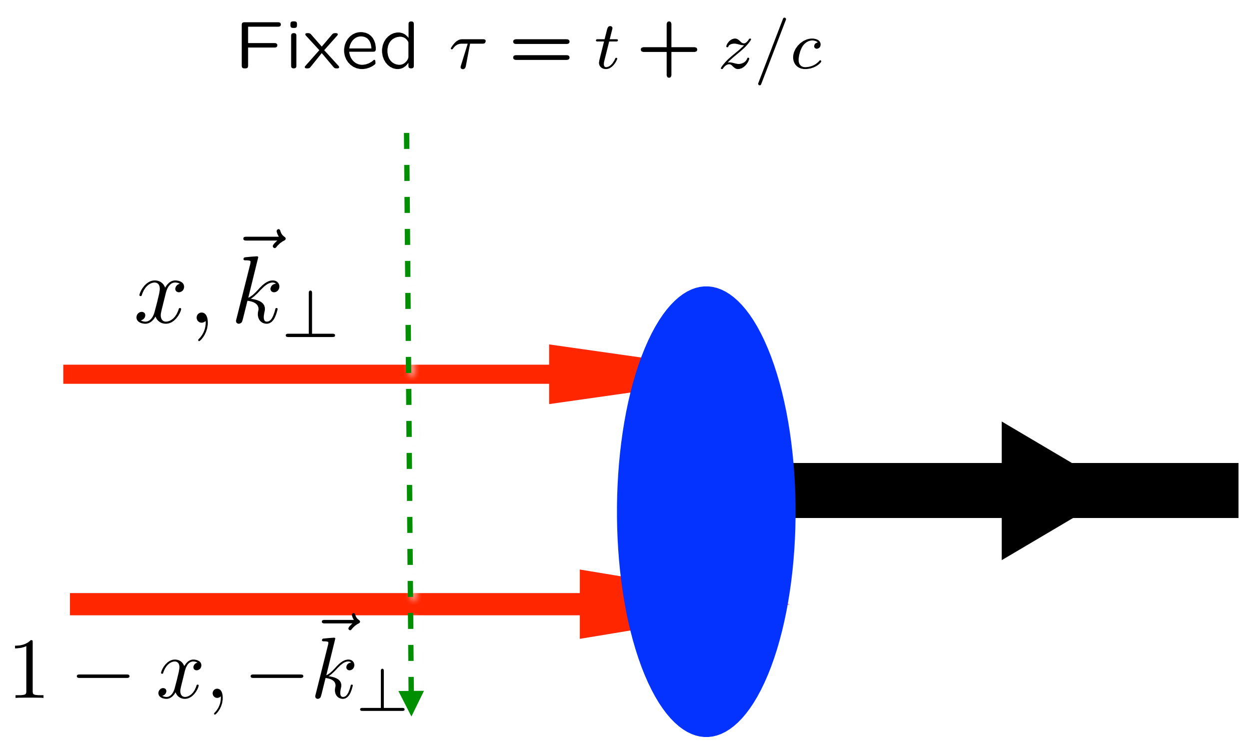
$$f_\pi = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

Same as DSE! **C. D. Roberts et al.**

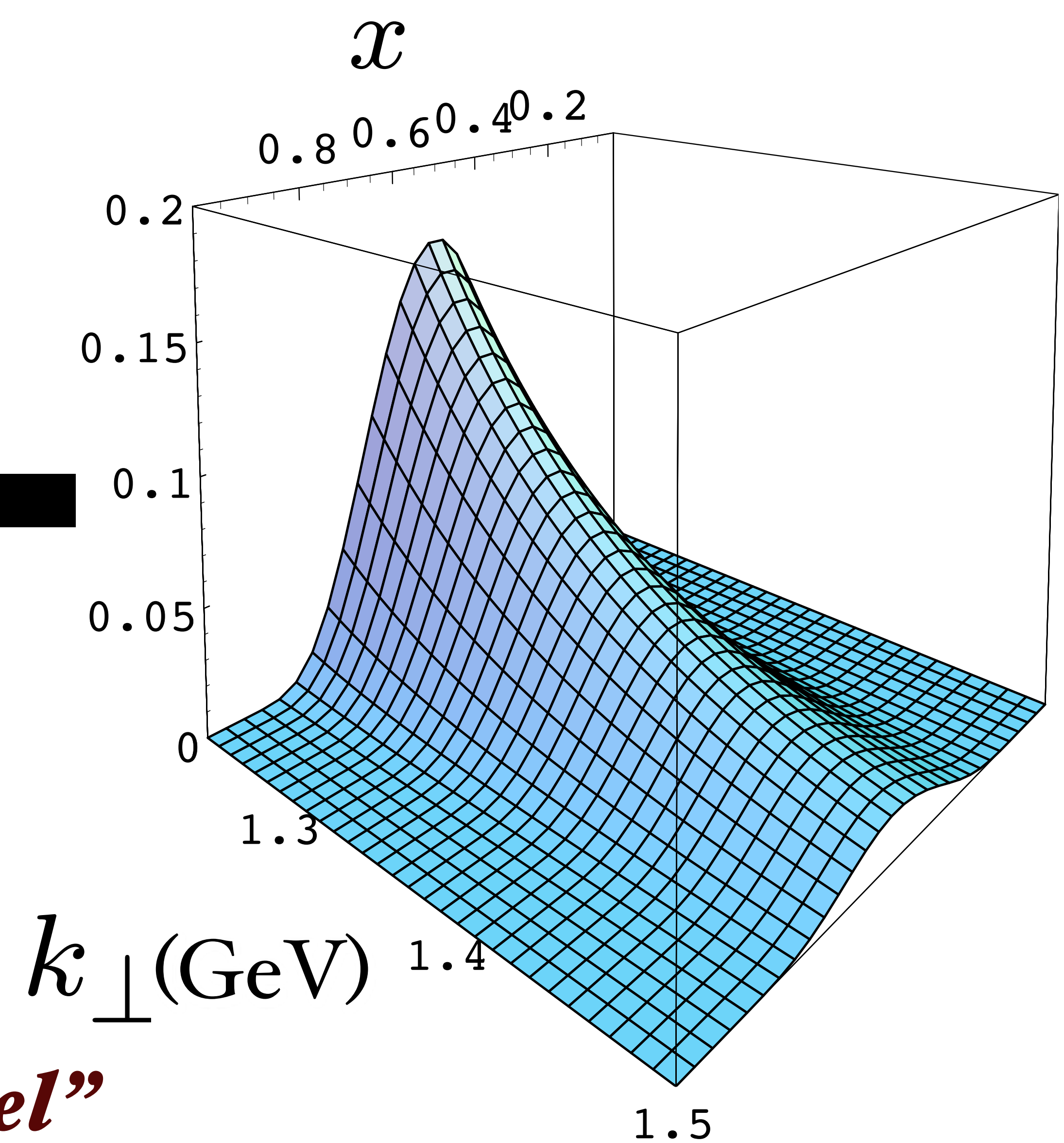
Provides Connection of Confinement to Hadron Structure

- *Light Front Wavefunctions:* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

off-shell in P^- and invariant mass $\mathcal{M}^2_{q\bar{q}}$



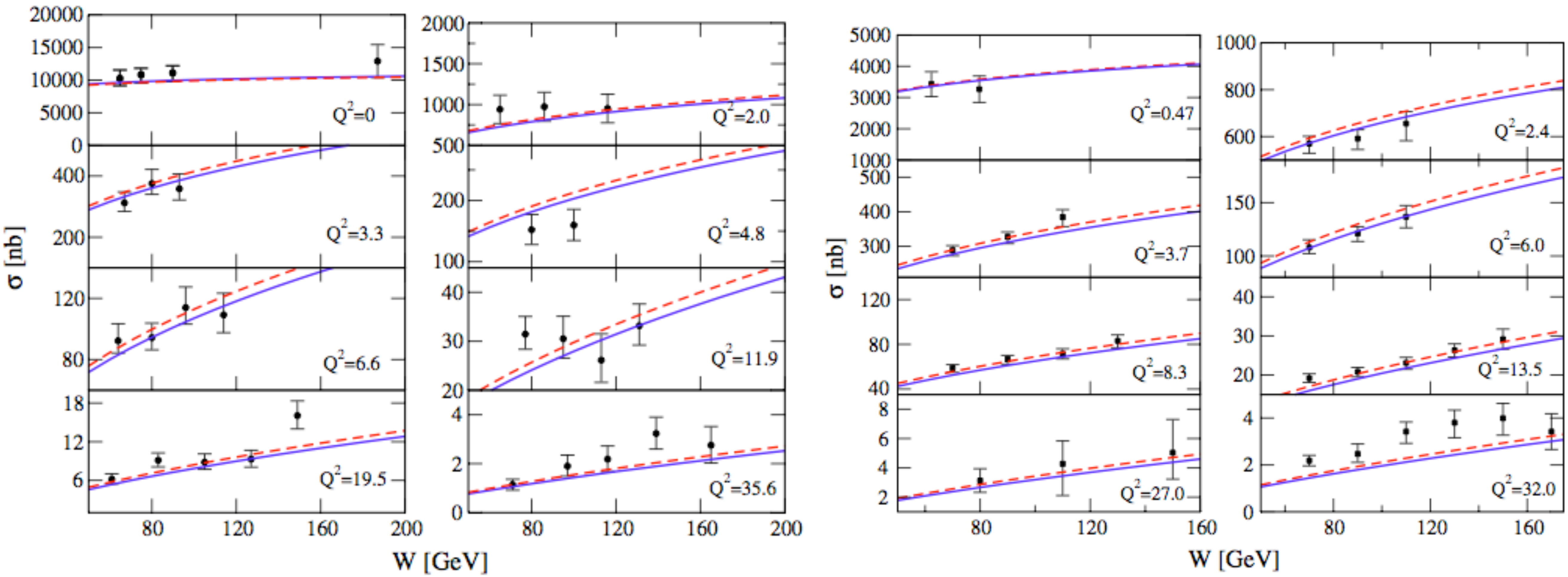
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



"Hadronization at the Amplitude Level"

Boost-invariant LFWF connects confined quarks and gluons to hadrons

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

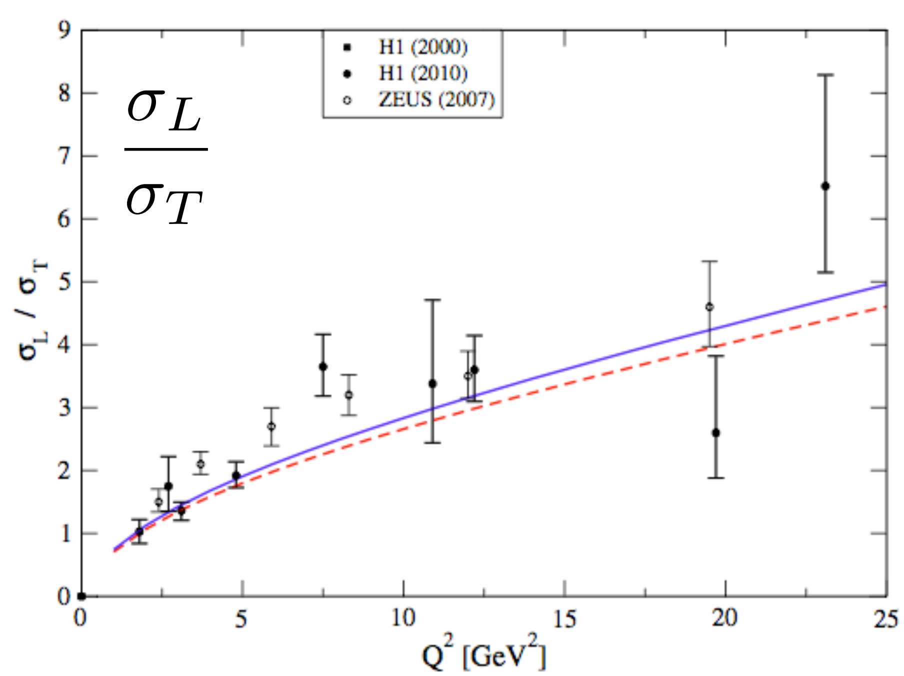


(a) H1

(b) ZEUS

J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$



$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale come from?

QCD does not know what MeV units mean!
Only Ratios of Masses Determined

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique confinement potential!

- **de Alfaro, Fubini, Furlan** (dAFF)

$$G|\psi(\tau)>= i\frac{\partial}{\partial\tau}|\psi(\tau)>$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

- **Dosch, de Teramond, sjb**

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**
- **Finite range**
- **Measure in Double-Parton Processes**

Retains conformal invariance of action despite mass scale!

Haag, Lopuszanski, Sohnius (1974)

Superconformal Quantum Mechanics

$$\{\psi, \psi^{+}\} = 1 \qquad B = \frac{1}{2}[\psi^{+}, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^{+} = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^{+}[-\partial_x + \frac{f}{x}], \quad Q^{+} = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^{+}x, \quad S^{+} = \psi x$$

$$\{Q, Q^{+}\} = 2H, \quad \{S, S^{+}\} = 2K$$

$$\{Q, S^{+}\} = f - B + 2iD, \quad \{Q^{+}, S\} = f - B - 2iD$$

generates conformal algebra

[H,D]= i H, [H, K] =2 i D, [K, D] = - i K

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

Supersymmetric Superconformal QM

(Fubini & Rabinovici, NPB245 (84) 17)

graded algebra of two fermionic operators (super charges) $Q,\ Q^\dagger$

$\{Q,Q\}=0,\ \{Q^\dagger,Q^\dagger\}=0$ with $H=\{Q,Q^\dagger\}$ → $[Q,H]=0,\ [Q^\dagger,H]=0$

minimum conformal realization -> particle with 2 degrees of freedom with:

$$Q=\psi^\dagger\left(-\frac{\partial}{\partial x}+\frac{f}{x}\right),\quad Q^\dagger=\psi\left(\frac{\partial}{\partial x}+\frac{f}{x}\right)$$

$\psi,\ \psi^\dagger$ spinor operators with

$\{\psi^\dagger,\psi\}=I,\ [\psi^\dagger,\psi]=\sigma_3$

in matrix notation

$$Q=\begin{pmatrix}0 & -\partial_x+\frac{f}{x}\\ 0 & 0\end{pmatrix},\quad Q^\dagger=\begin{pmatrix}0 & 0\\ \partial_x+\frac{f}{x} & 0\end{pmatrix}$$ →

$$H=\begin{pmatrix}-\partial_x^2+\frac{f^2+f}{x^2} & 0\\ 0 & -\partial_x^2+\frac{f^2-f}{x^2}\end{pmatrix}$$

H operates on two component states

$$|\phi\rangle=\begin{pmatrix}\phi_M\\ \phi_B\end{pmatrix}$$

with same eigenvalue

Superconformal Quantum Mechanics

Baryon Equation $Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$

Consider $R_w = Q + wS;$

w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2 K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \left(-\partial_x^2 + w^2 x^2 + 2wf - w + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2} \right)$$

$$G_{22} = \left(-\partial_x^2 + w^2 x^2 + 2wf + w + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2} \right)$$

Identify $f - \frac{1}{2} = L_B \,, \quad w = \kappa^2$

$\lambda = \kappa^2$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Baryon Equation

Superconformal Quantum Mechanics

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\Big)\psi_J^+ = M^2\psi_J^+$$

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\Big)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

Meson Equation

$$\lambda = \kappa^2$$

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\Big)\phi_J = M^2\phi_J$$

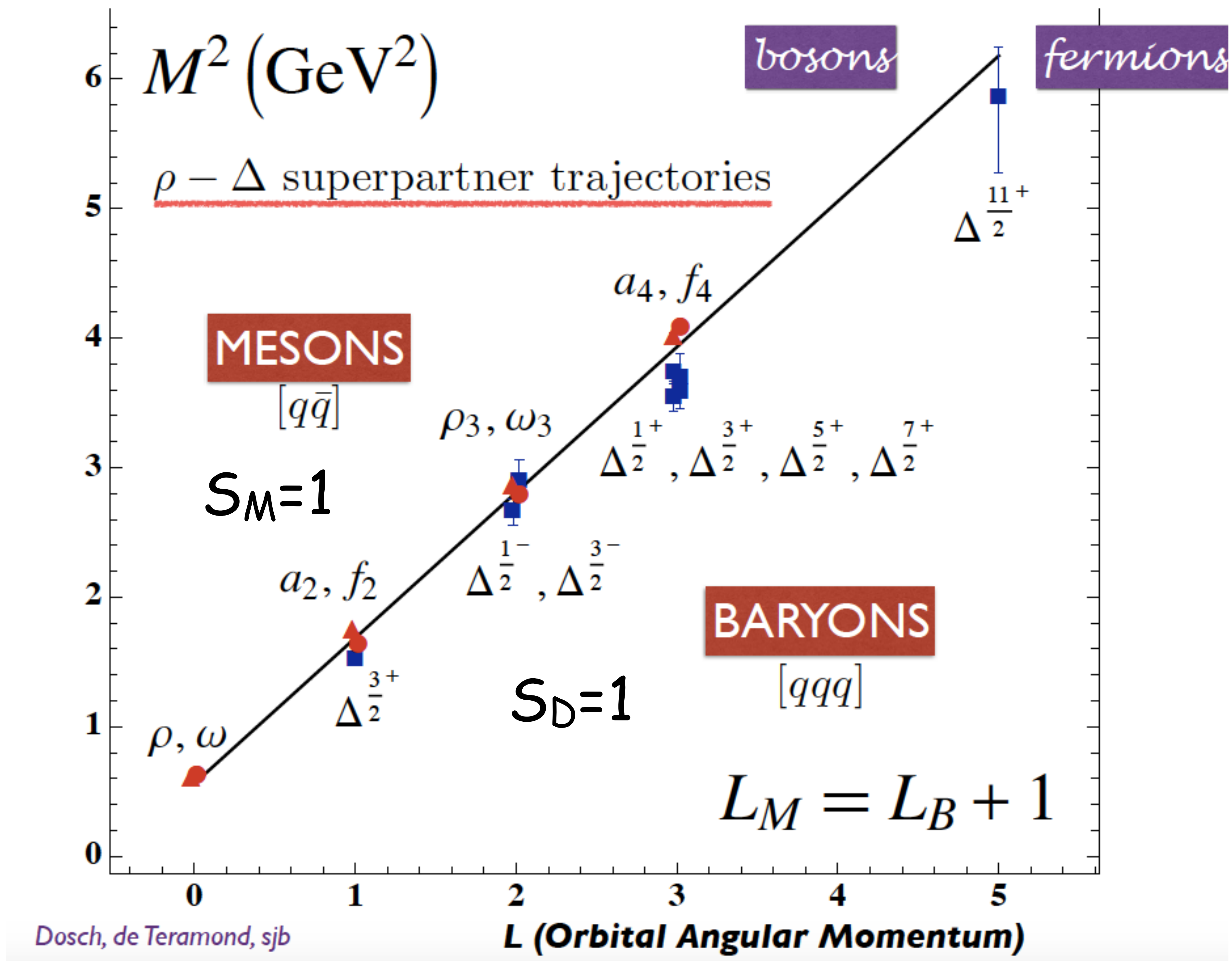
S=0, P=+

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ *!*

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M = L_B + 1$



$$M_M^2 = 4\lambda\left(n + L_M + \frac{S_M}{2}\right)$$

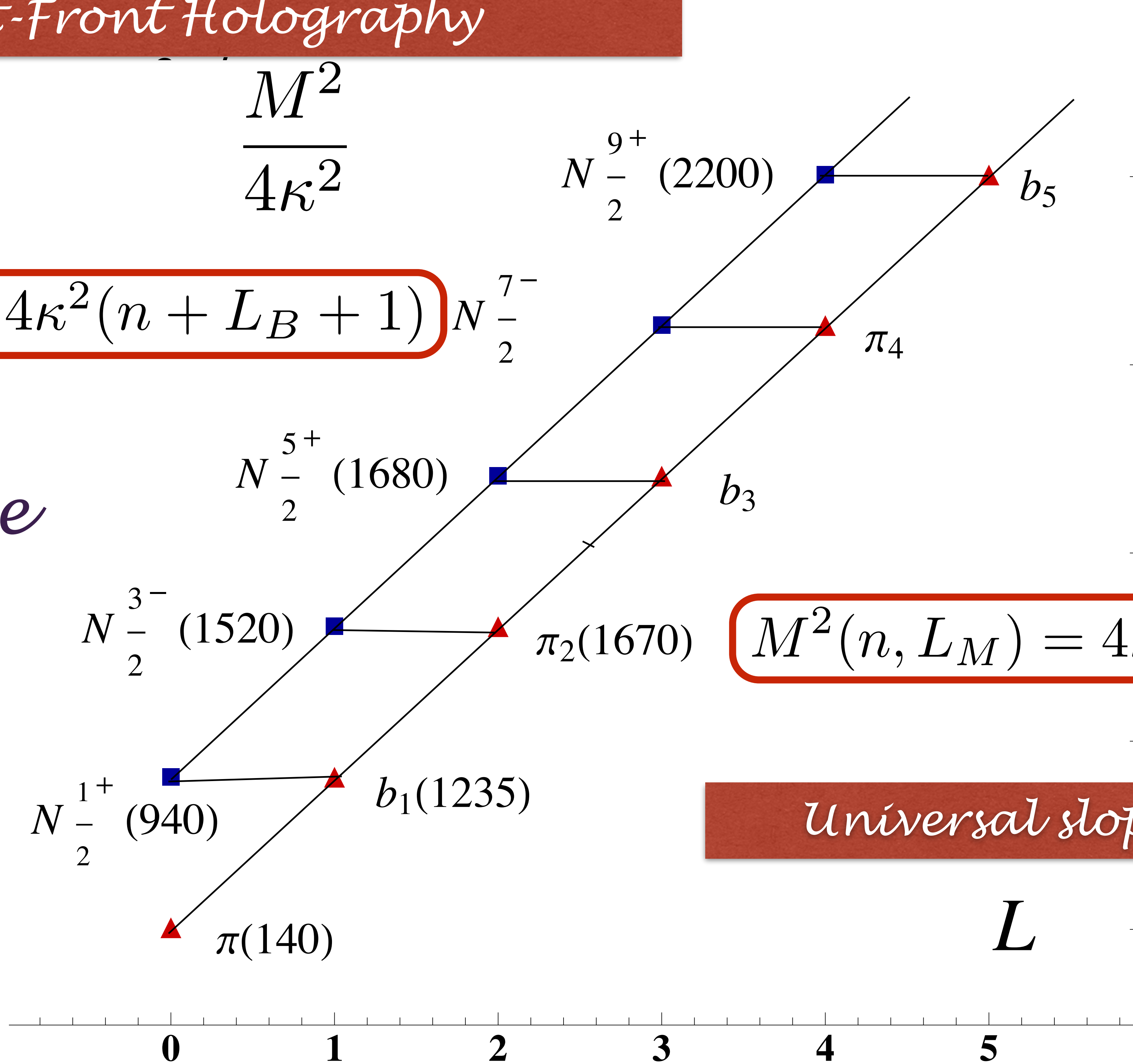
mesons with L_M=0 have no superpartners

$$M_B^2 = 4\lambda\left(n + L_B + \frac{S_D}{2} + 1\right)$$

π (L_M=S_M=0) $\Rightarrow$ M_π=0 in the chiral limit

Superconformal Quantum Mechanics
Light-Front Holography

de Tèramond, Dosch, Lorce, sjb



$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

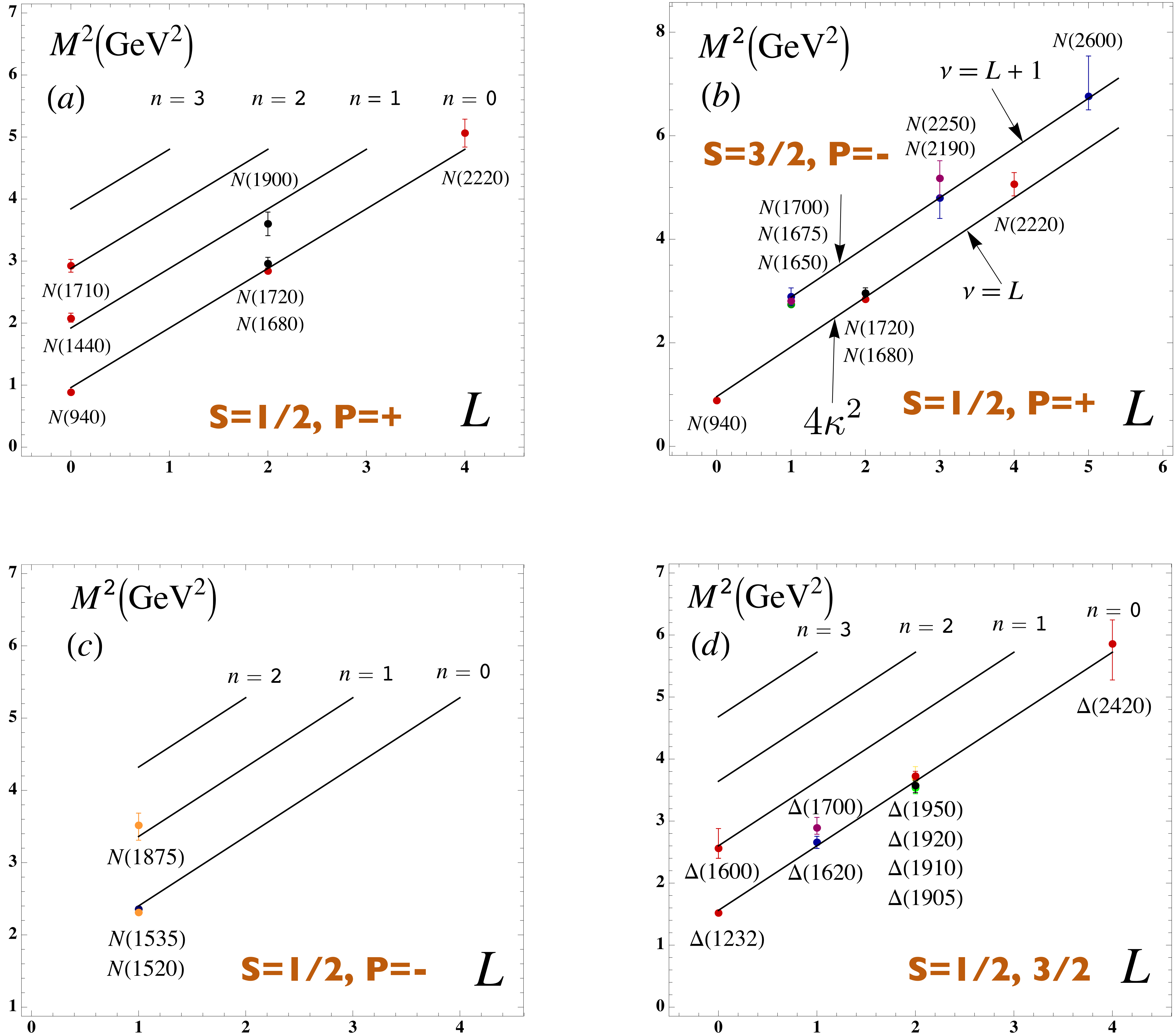
Same slope

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Universal slopes in n, L

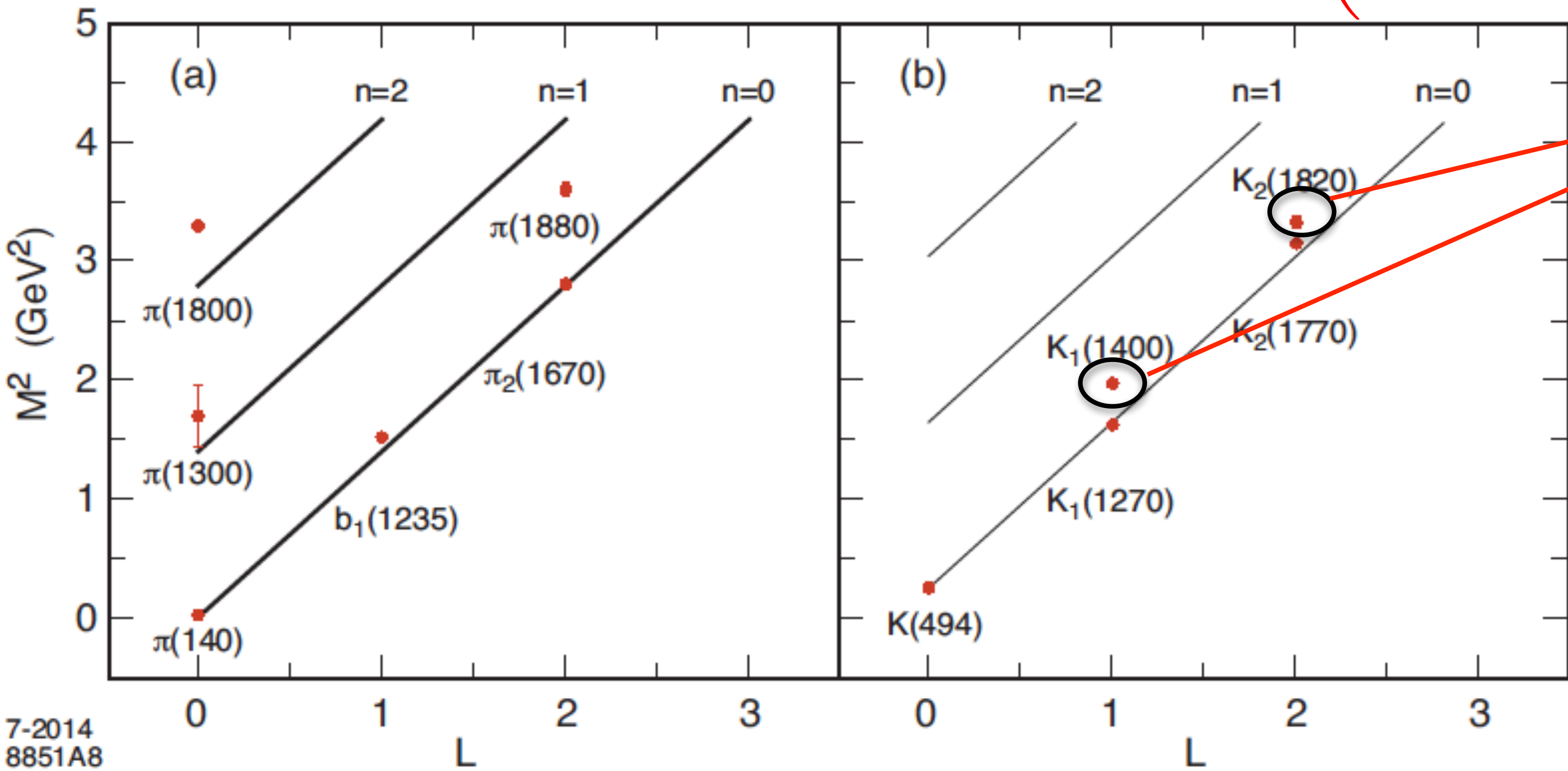
$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$



$M^2(\mathrm{GeV}^2)$
(a)
$n = 3$
$n = 2$
$n = 1$
$n = 0$
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(b)
$\nu = L + 1$
N(2600)
N(2250)
N(2190)
S=3/2, P=-
N(1700)
N(1675)
N(1650)
N(2220)
$\nu = L$
N(1720)
N(1680)
N(940)
$4\kappa^2$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(c)
$n = 2$
$n = 1$
$n = 0$
N(1875)
N(1535)
N(1520)
S=1/2, P=-
L
$M^2(\mathrm{GeV}^2)$
(d)
$n = 3$
$n = 2$
$n = 1$
$n = 0$
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
S=1/2, 3/2
L

$$\text{for } J = L + S \Rightarrow M^2_{n,L,S} = 4\lambda\left(n + L + \frac{S}{2}\right)$$



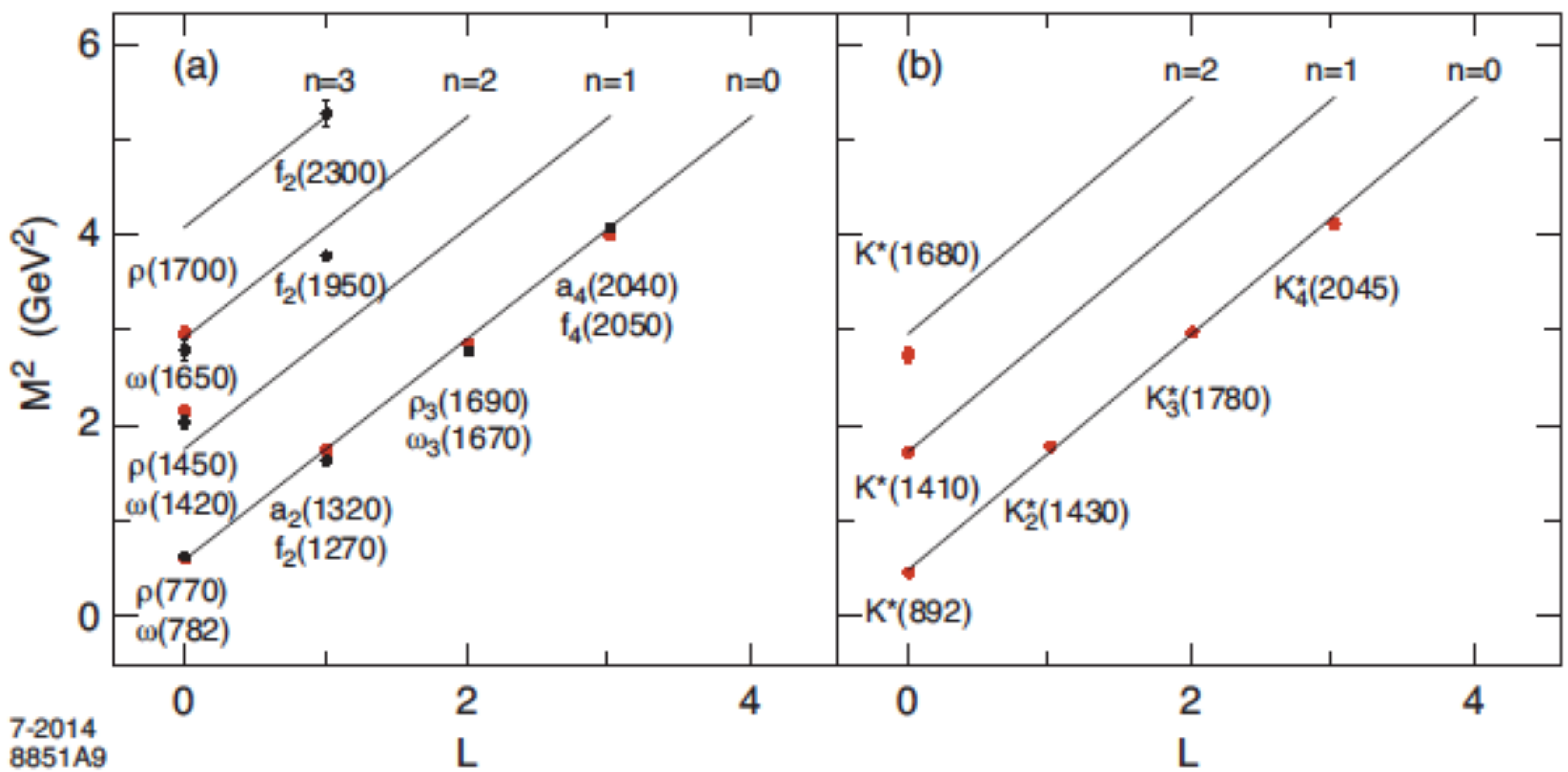
poor agreement!

Tetraquark candidates

(S=0)

$\sqrt{\lambda} = 0.59$ GeV

quark mass correction was included!



(S=1)

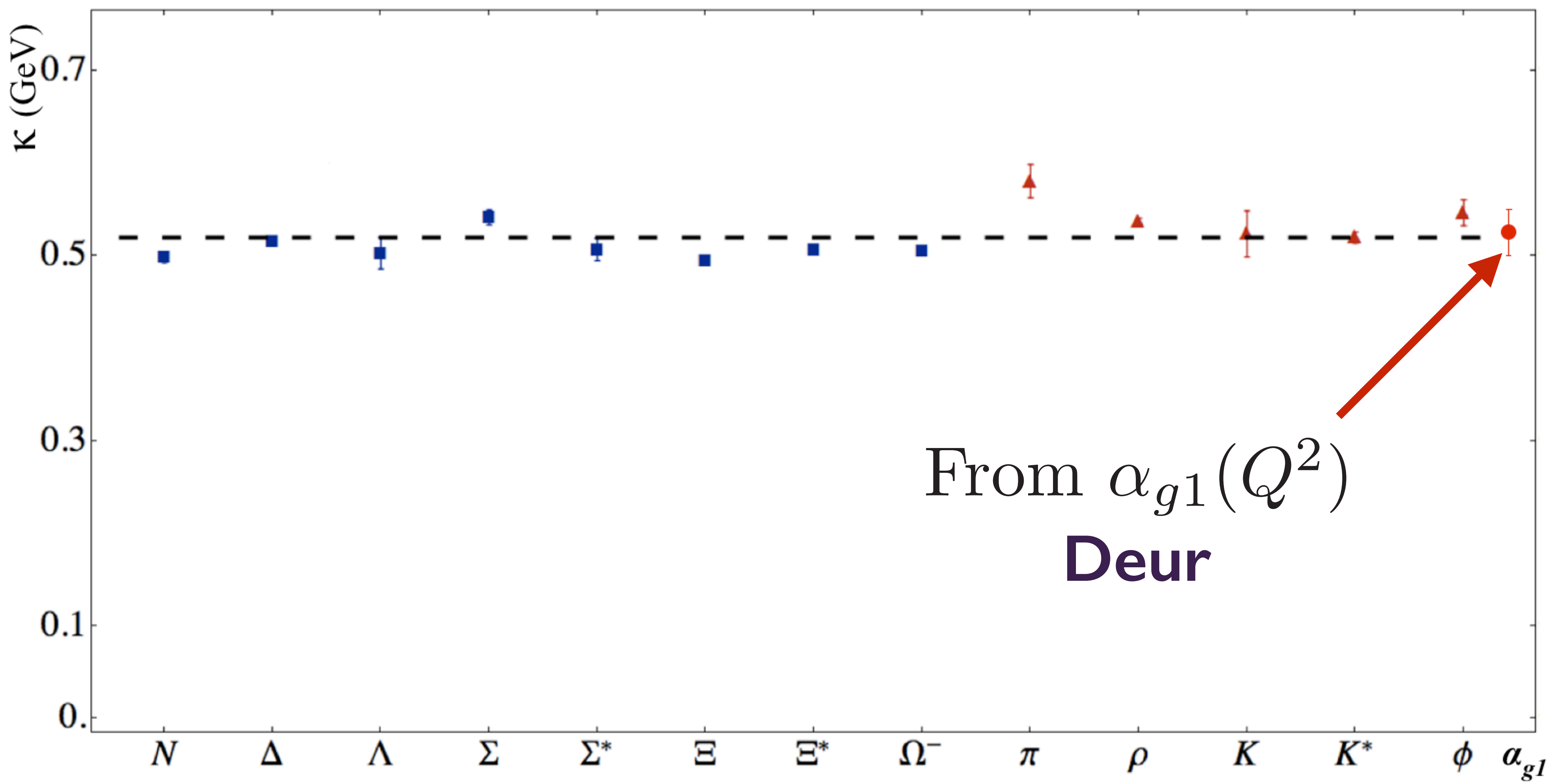
$\sqrt{\lambda} = 0.54$ GeV

$\lambda = \kappa^2$

de Tèramond, Dosch, Lorce', sjb

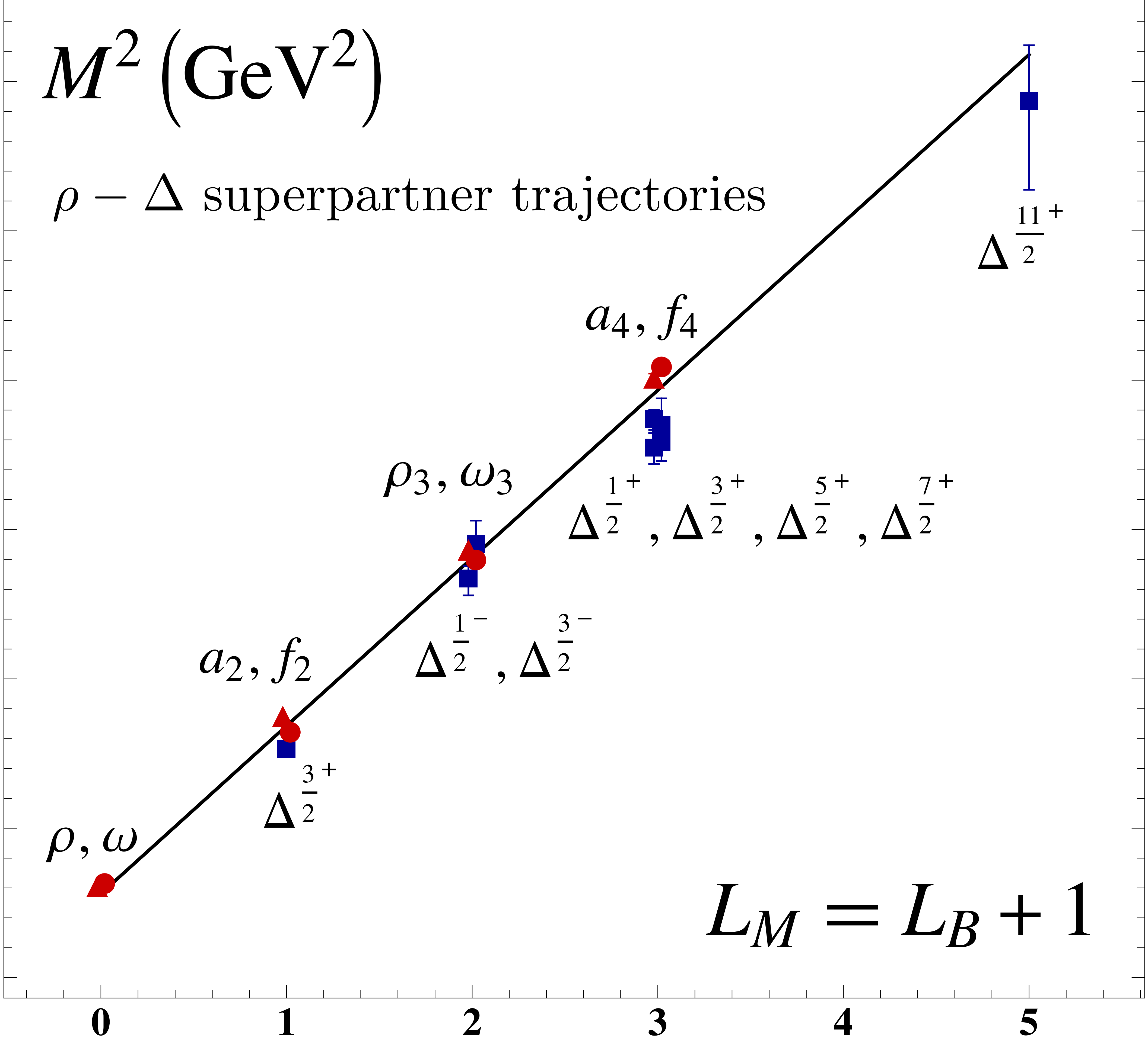
$$m_u = m_d = 46 \text{ MeV}, \; m_s = 357 \text{ MeV}$$

κ



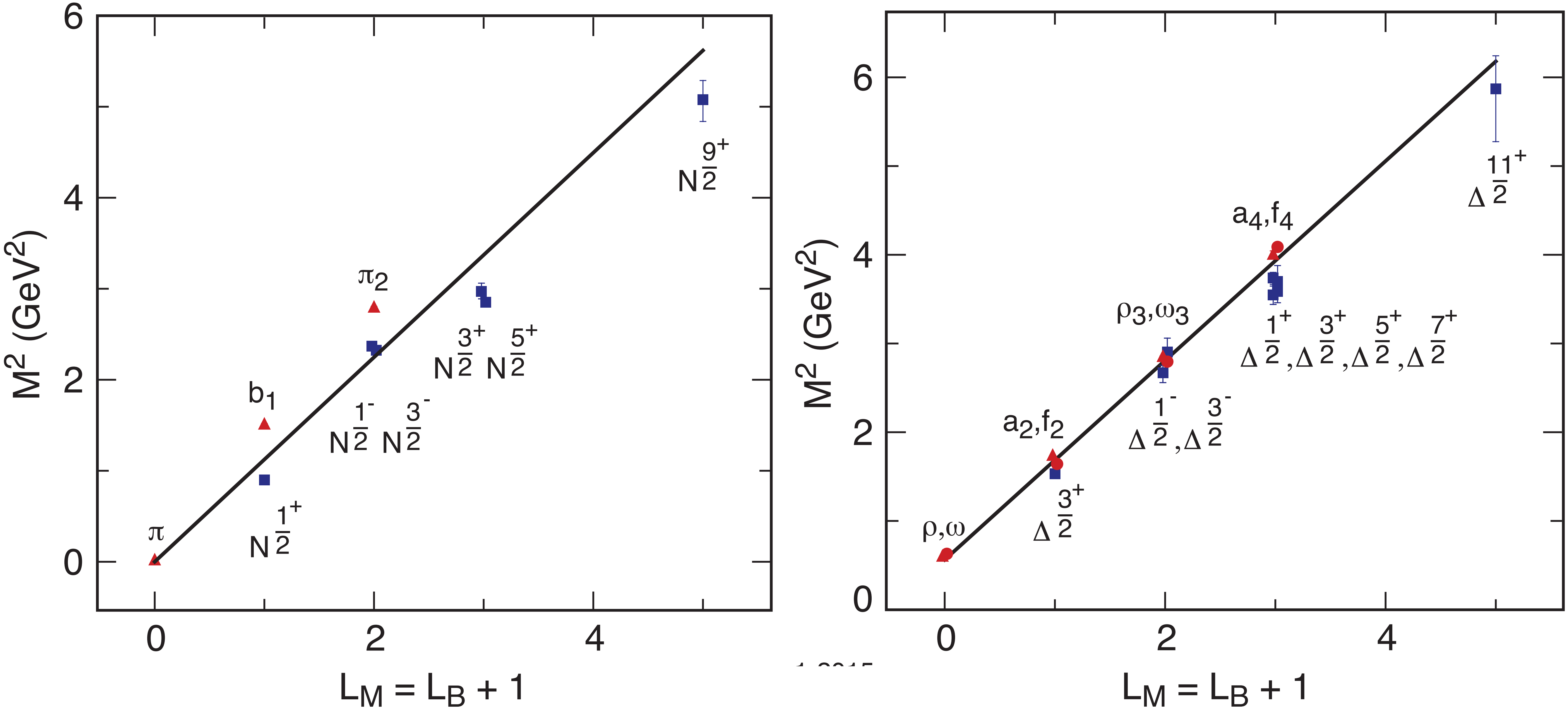
Fit to the slope of Regge trajectories, including radial excitations

Same Regge Slope for Meson, Baryons: Supersymmetric feature of hadron physics



de Tèramond, Dosch, Lorce' , sjb

Solid line: $\kappa = 0.53$ GeV



Superconformal meson-nucleon partners

de Tèramond, Dosch, Lorce', sjb

Universal Hadronic Features

- ***Universal quark light-front kinetic energy***

$$\Delta\mathcal{M}^2_{LFKE} = \kappa^2(1 + 2n + L)$$

Equal: Virial Theorem!

- ***Universal quark light-front potential energy***

$$\Delta\mathcal{M}^2_{LFPE} = \kappa^2(1 + 2n + L)$$

- ***Universal Constant Term***

$$\mathcal{M}^2_{spin} = 2\kappa^2(S + L - 1 + 2n_{diquark})$$

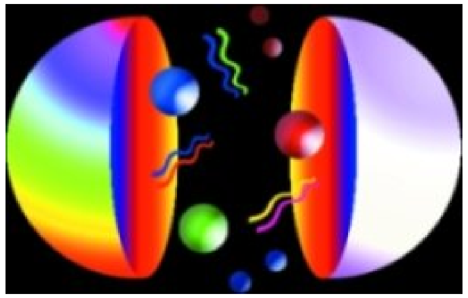
$$M^2 = \Delta\mathcal{M}^2_{LFKE} + \Delta\mathcal{M}^2_{LFPE} + \Delta\mathcal{M}^2_{spin}$$

$$+ < \sum_i \frac{m_i^2}{x_i} >$$

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int_0^\infty d\zeta \int_0^1 dx\psi_+^2(\zeta^2,x) = \int_0^\infty d\zeta \int_0^1 dx\psi_-^2(\zeta^2,x) = \frac{1}{2}$$

Quark Chiral Symmetry of Eigenstate!

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- "Chiral partners"

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Nucleon: Equal Probability for L=0,1

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No vacuum condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S_q^z >= 0$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

No mass -degenerate parity partners!

Features of Supersymmetric Equations

- J =L+S baryon simultaneously satisfies both equations of G with L , L+1 with same mass eigenvalue

- $J^z = L^z + 1/2 = (L^z + 1) - 1/2$ $\quad S^z = \pm 1/2$

- Proton spin carried by quark L^z

$$< J^z >= \frac{1}{2}(S_q^z = \frac{1}{2}, L^z = 0) + \frac{1}{2}(S_q^z = -\frac{1}{2}, L^z = 1) =< L^z >= \frac{1}{2}$$

- Mass-degenerate meson "superpartner" with $L_M = L_B + 1$. *"Shifted meson-baryon Duality"*

Mesons and baryons have same κ !

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$
\begin{aligned}
F_+(Q^2) &= g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_-(Q^2) &= g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2,
\end{aligned}
$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$
\begin{aligned}
F_1^p(Q^2) &= \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_1^n(Q^2) &= -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],
\end{aligned}
$$

where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} \, V(Q,z)\Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2}\sqrt{\frac{2n!}{(n+L)!}}\, z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4} \, \Psi_+^2(z) = 1$$

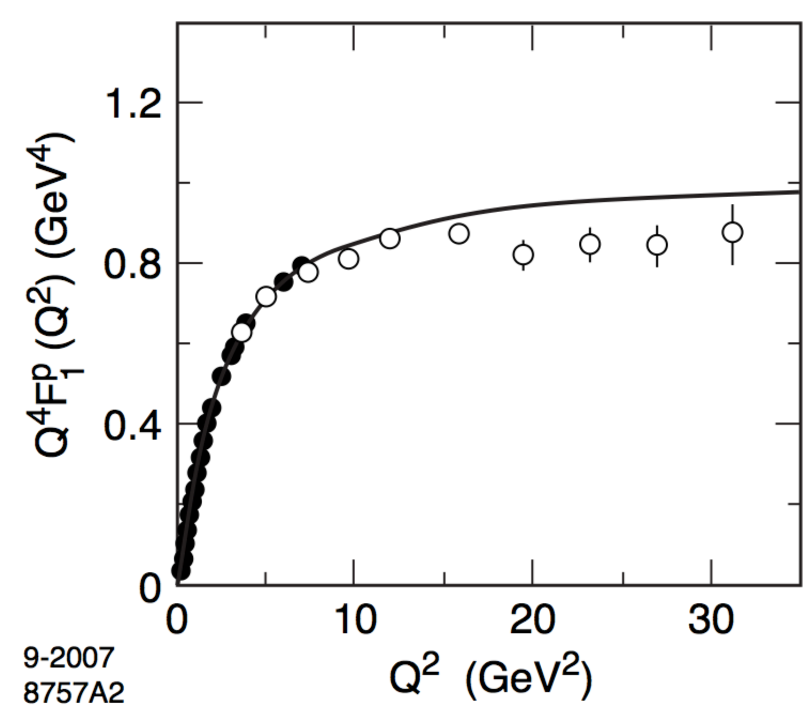
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} \, x^{\frac{Q^2}{4\kappa^2}} \, e^{-\kappa^2 z^2 x/(1-x)}$$

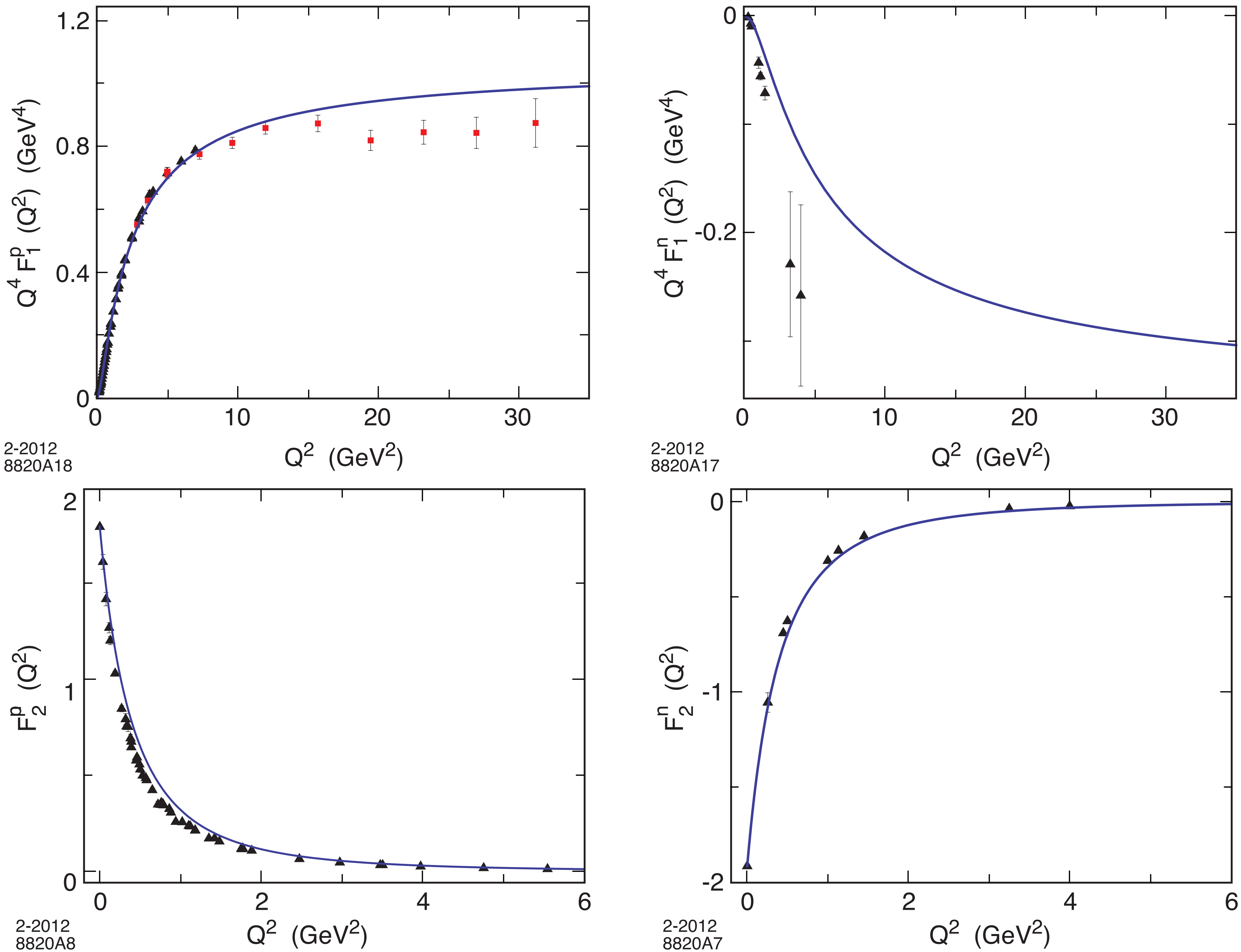
- Find

$$F_1^p(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}{}^2 \to 4\kappa^2(n+1/2)$

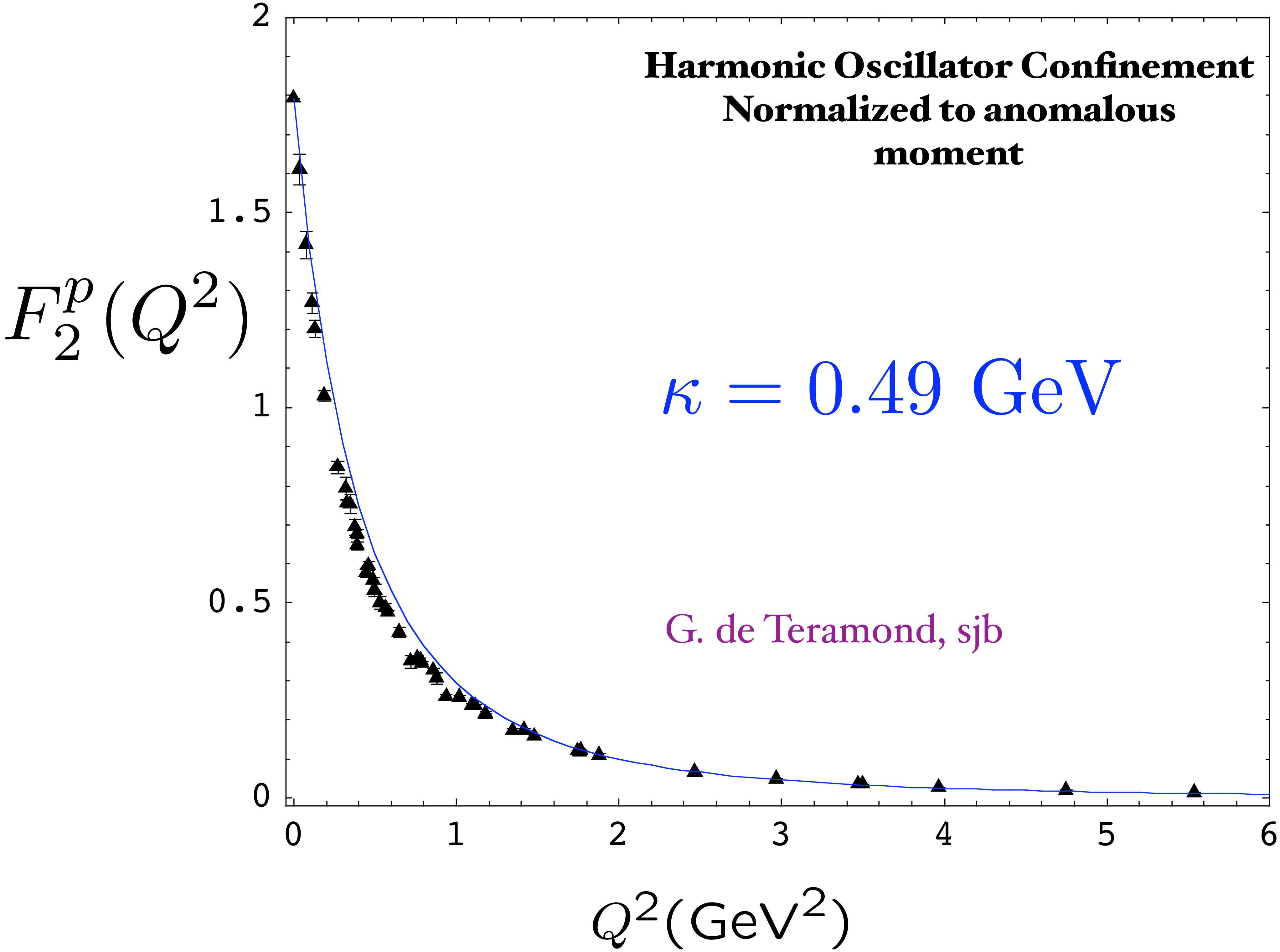


Using $SU(6)$ flavor symmetry and normalization to static quantities



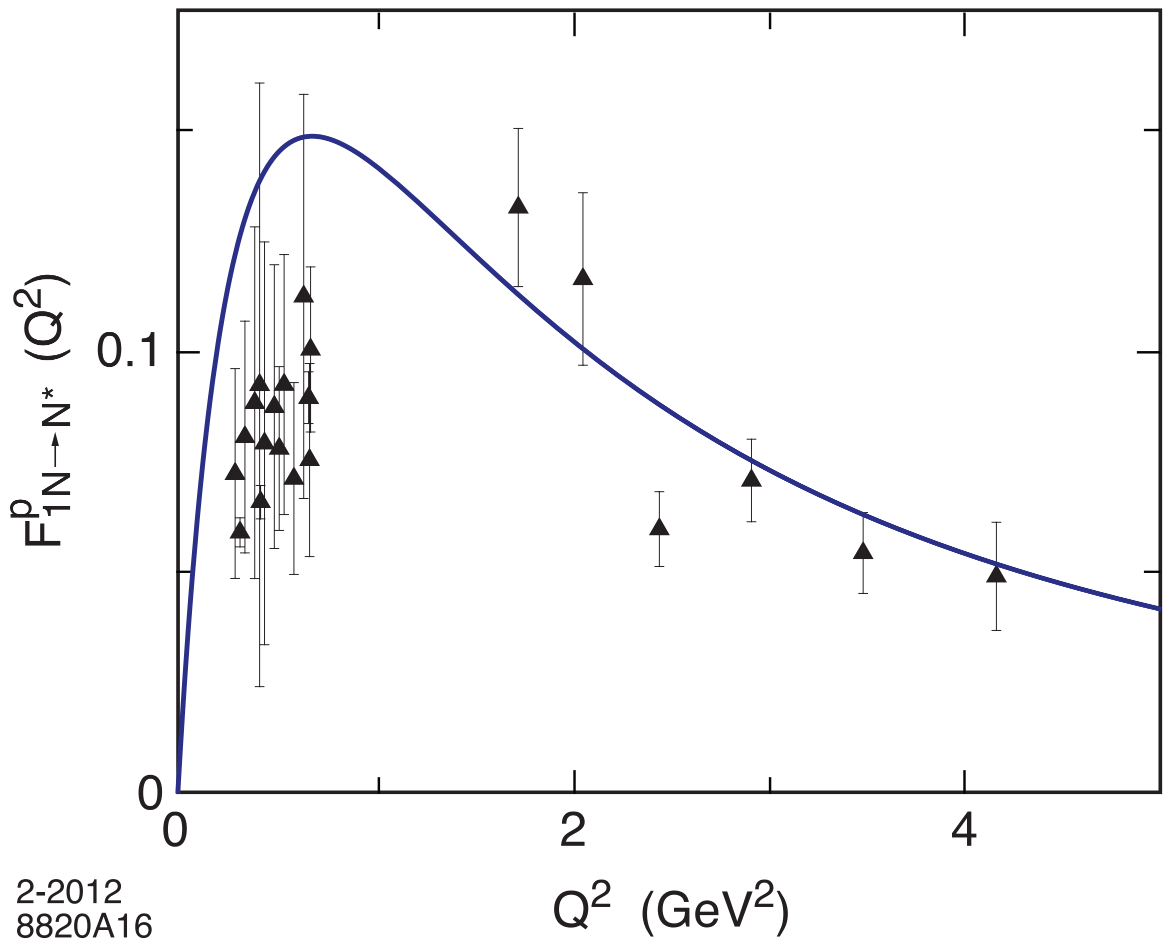
Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



Nucleon Transition Form Factors

$$F^p_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3}\frac{\frac{Q^2}{\mathcal{M}^2_\rho}}{\left(1+\frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho''}}\right)}.$$



Proton transition form factor to the first radial excited state. Data from JLab

Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \rightarrow N^*(1440)$: $\Psi_+^{n=0,L=0} \rightarrow \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_{1\,N\rightarrow N^*}^{\,p}(Q^2) = R^4 \int \frac{dz}{z^4} \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\left(F_{1\,N\rightarrow N^*}^{\,p}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4} \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

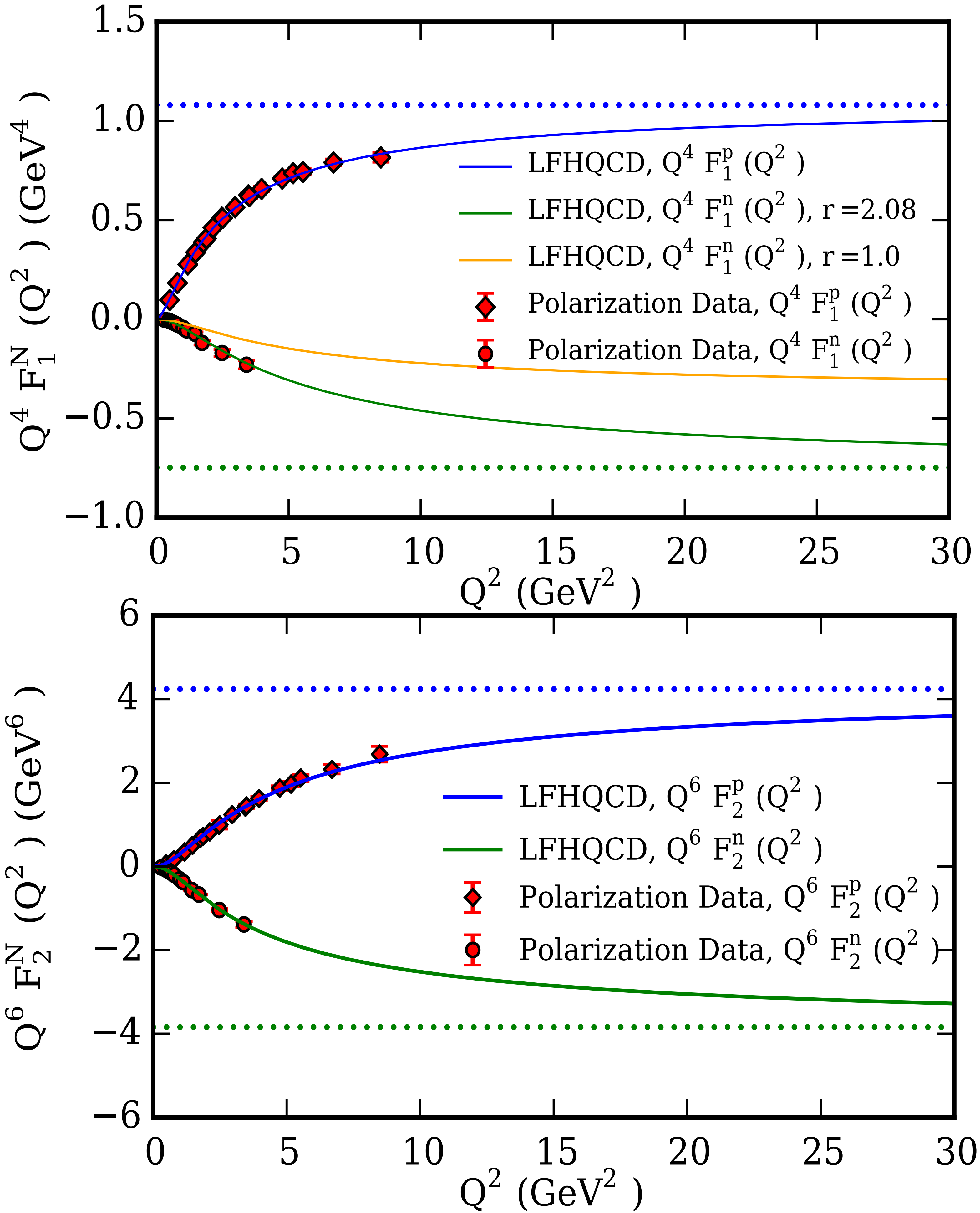
$$F_{1\,N\rightarrow N^*}^{\,p}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

with ${\mathcal{M}_{\rho_n}}^2 \rightarrow 4\kappa^2(n+1/2)$

de Teramond, sjb

Consistent with counting rule, twist 3

Sufian, de Teramond, Deur, Dosch, sjb



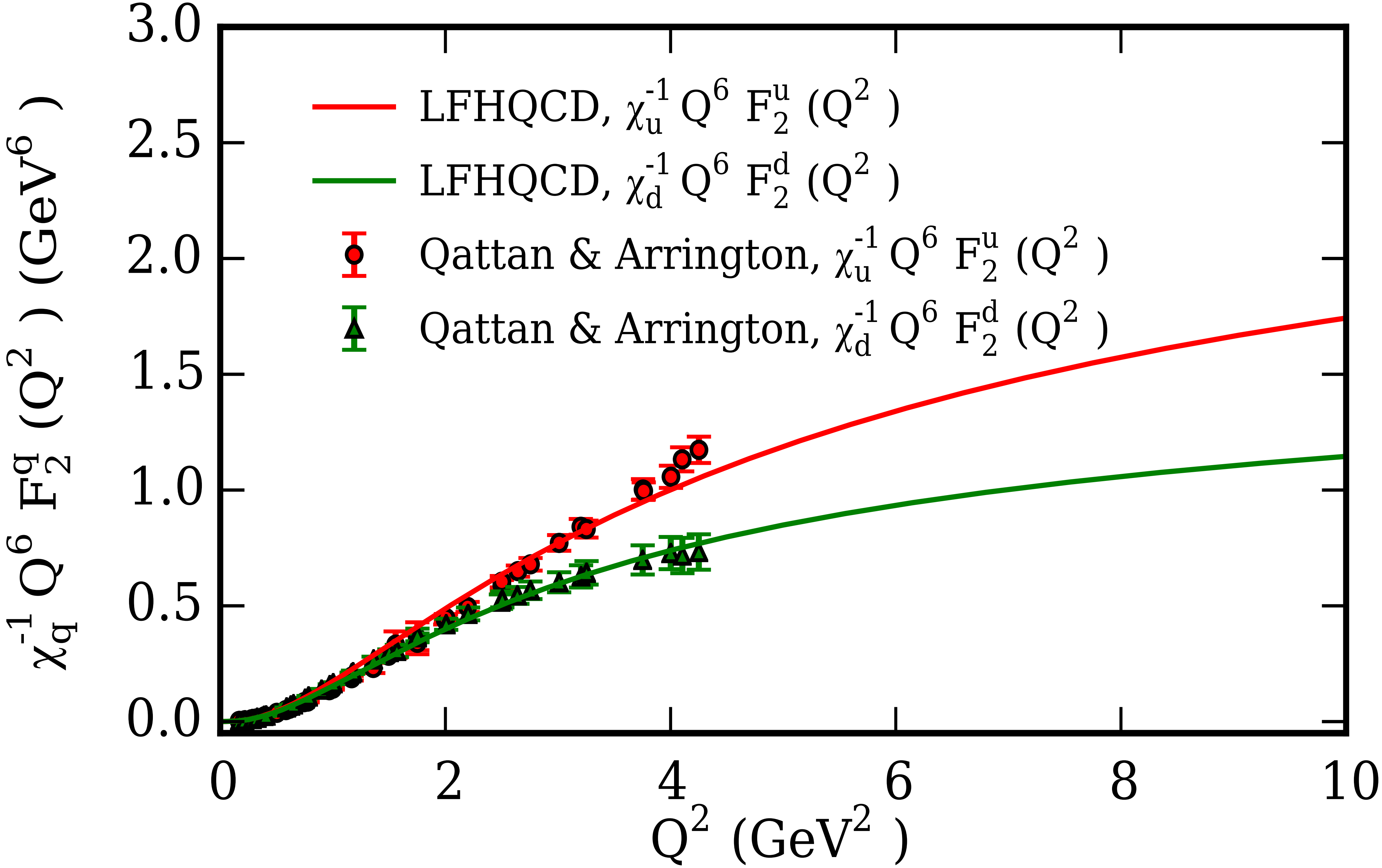
$Q^4 F_1^p(Q^2)$

$Q^4 F_1^n(Q^2)$

Includes 5-quark Fock states

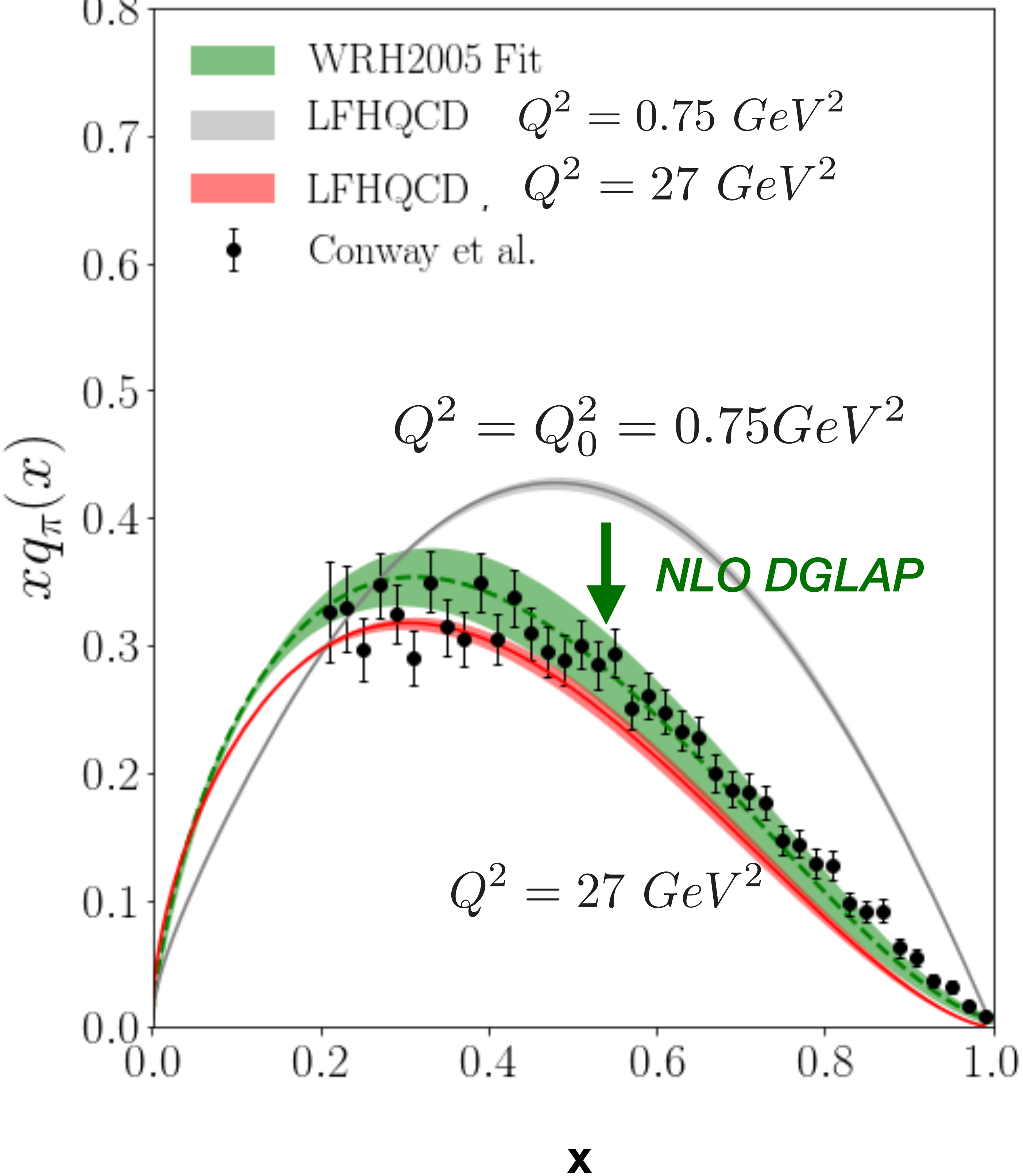
$Q^6 F_2^p(Q^2)$

$Q^6 F_2^n(Q^2)$



Flavor Dependence of $Q^6 F_2(Q^2)$

Sufian, de Teramond, Deur, Dosch, sjb



Start DGLAP evolution at transition scale Q^2_0

T. Liu, G. de Tèramond, G. Dosch, A. Deur, R.S. Sufian, sjb

$$q_\pi(x, Q^2 < Q_0^2) = \int d^2\vec{k}_\perp |\psi_\pi(x, \vec{k}_\perp)|^2$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

"No parameters"

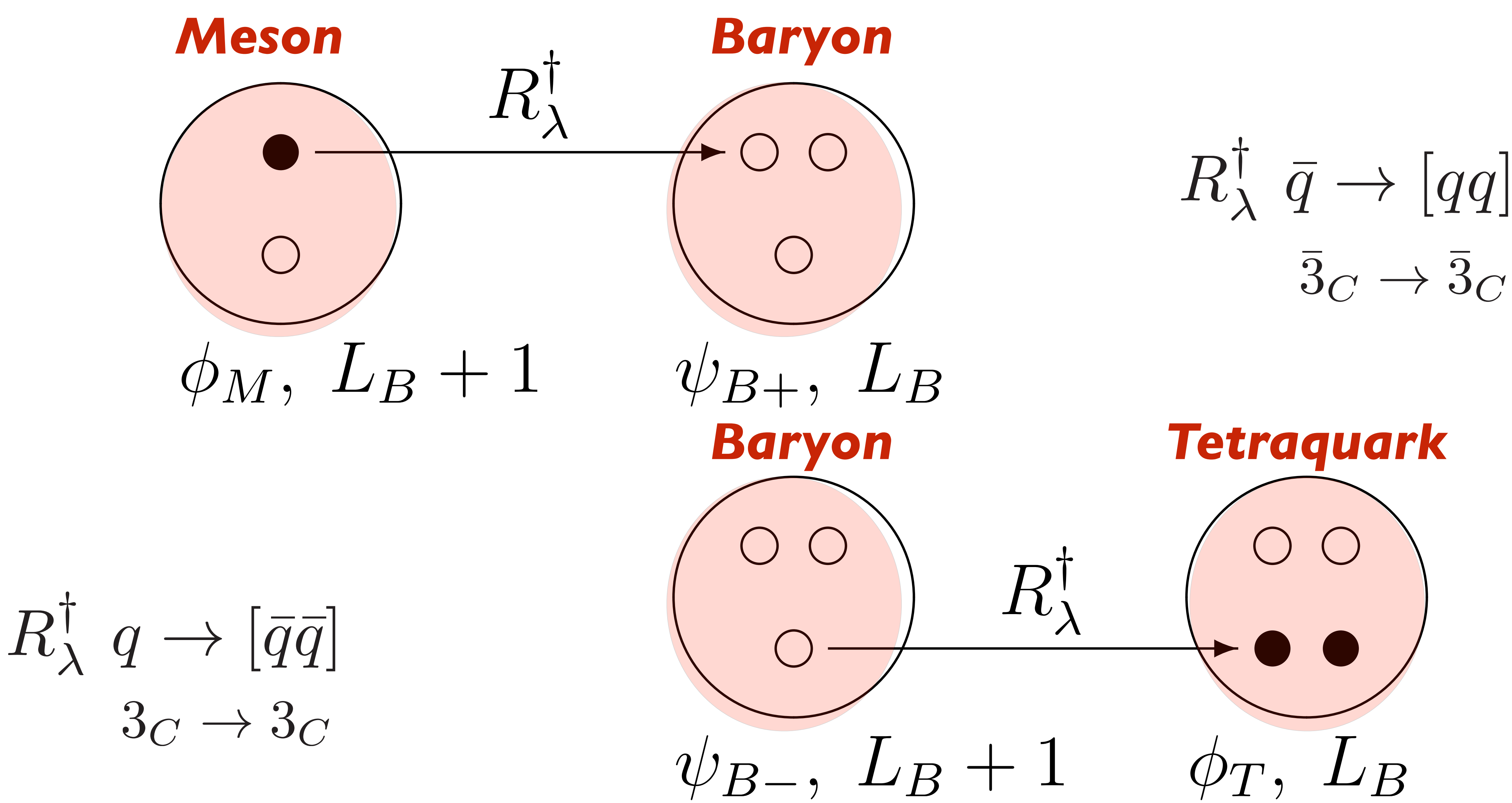
Includes AdS "dressed current" (Radyushkin) — VM poles in timelike form factor

Guy de Tèramond, Hans Günter Dosch, sjb

Superconformal Algebra

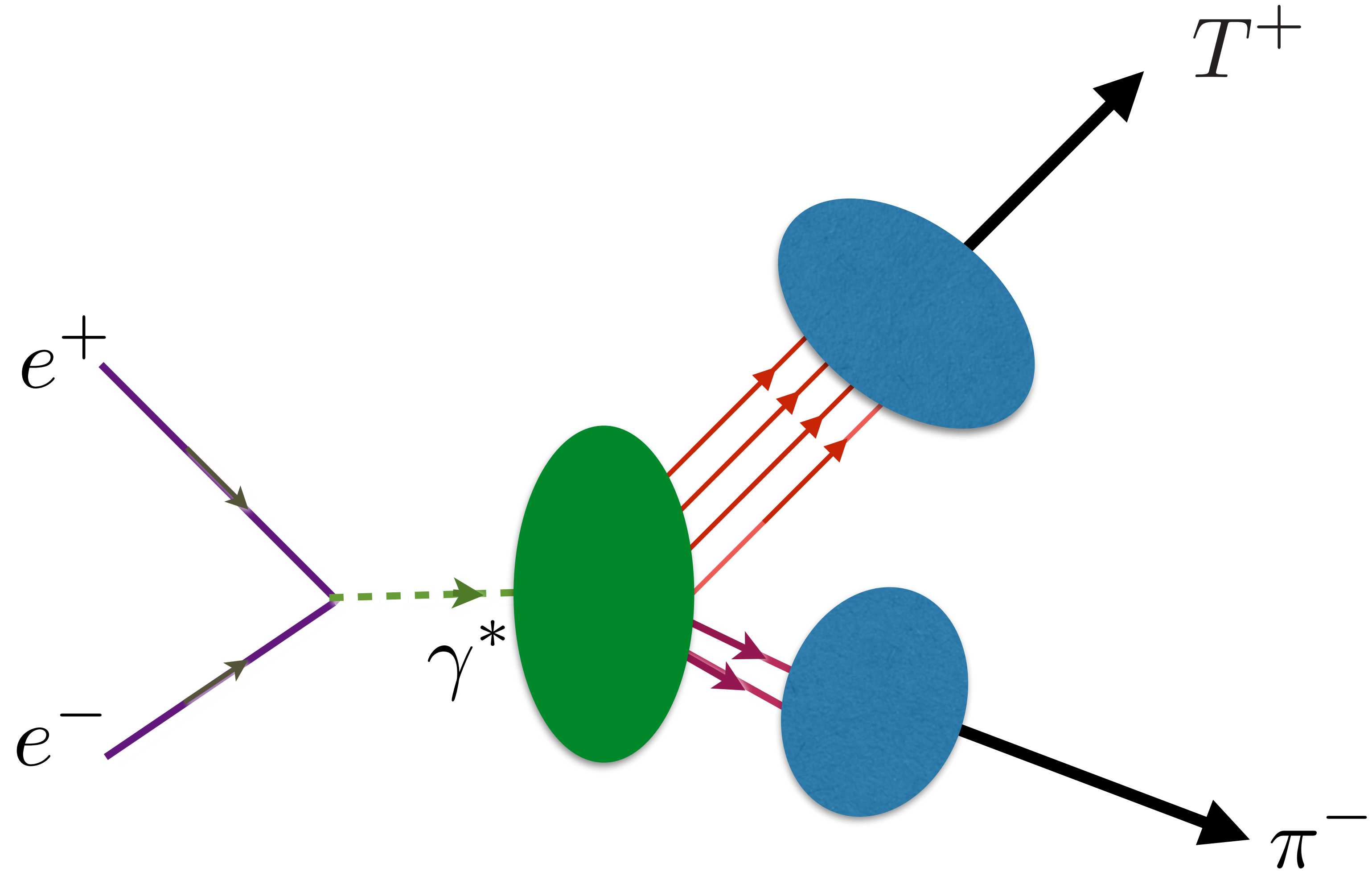
2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

$$\sigma(e^+e^- \to MT) \propto \frac{1}{s^{N-1}} \quad N=6$$



Use counting rules to identify composite structure

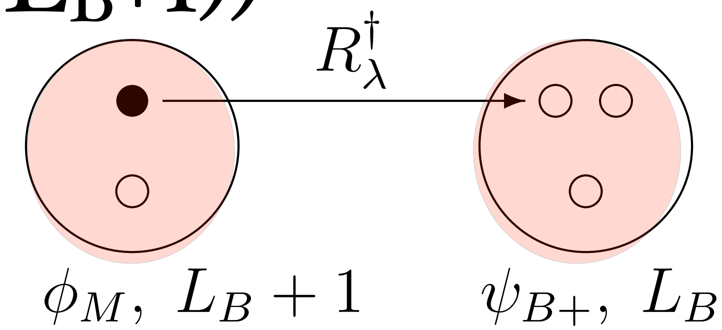
de Tèramond, Dosch, Lorce, sjb

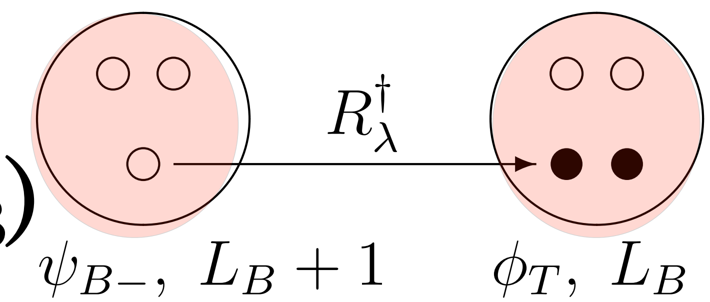
Superconformal Algebra

2X2 Hadronic Multiplets

$$\begin{pmatrix} \phi_M(L_M = L_B + 1) & \psi_{B-}(L_B + 1) \\ \psi_{B+}(L_B) & \phi_T(L_T = L_B) \end{pmatrix}$$

- quark-antiquark meson ($L_M = L_B$+1))
- quark-diquark baryon (L_B)
- quark-diquark baryon (L_B+1)
- diquark-antidiquark tetraquark ($L_T = L_B$)
- Universal Regge slopes $\lambda = \kappa^2$





Same Twist!

$$M_H^2/\lambda = \overbrace{\underbrace{(2n + L_H + 1)}_{kinetic} + \underbrace{(2n + L_H + 1)}_{potential}}^{contribution\ from\ 2\text{-}dim\ light\text{-}front\ harmonic\ oscillator} + \overbrace{2(L_H + s) + 2\chi}^{contribution\ from\ AdS\ and\ superconformal\ algebra} + < \sum_i \frac{m_i^2}{x_i} >$$

$\chi(mesons) = -1$ $\quad$ $\chi(baryons, tetraquarks) = +1$

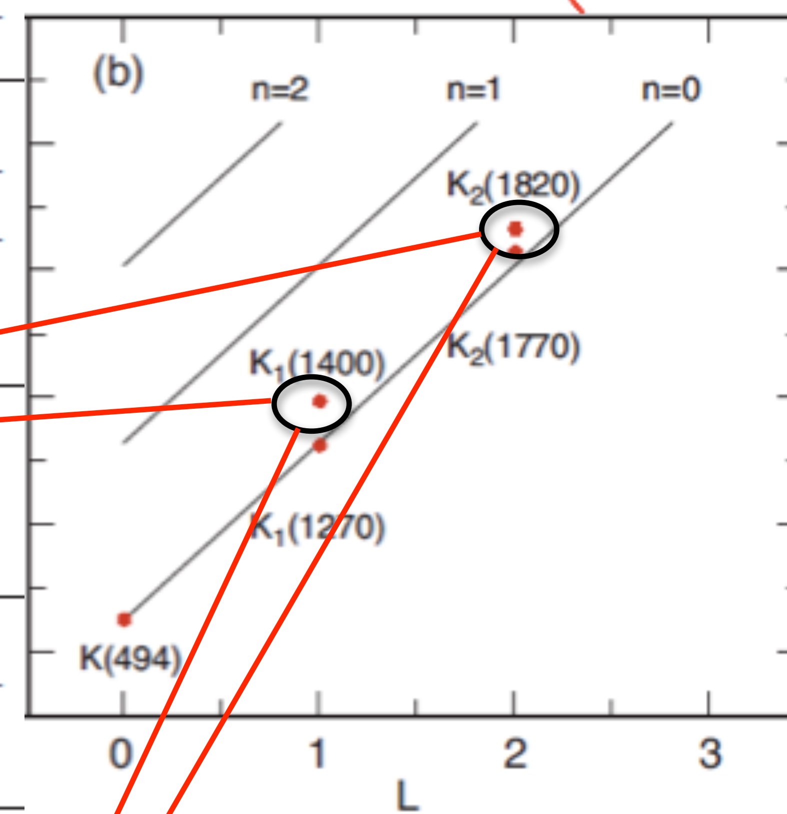
Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	1^{+-}	$b_1(1235)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	0^{++}	$f_0(980)$
$\bar{q}q$	2^{-+}	$\pi_2(1670)$	$[ud]q$	$(1/2)^-$	$N_{\frac{1}{2}^-}(1535)$	$[ud][\bar{u}\bar{d}]$	1^{-+}	$\pi_1(1400)$
				$(3/2)^-$	$N_{\frac{3}{2}^-}(1520)$			$\pi_1(1600)$
$\bar{q}q$	1^{--}	$\rho(770)$, $\omega(780)$	—	—	—	—	—	—
$\bar{q}q$	2^{++}	$a_2(1320)$, $f_2(1270)$	$[qq]q$	$(3/2)^+$	$\Delta(1232)$	$[qq][\bar{u}\bar{d}]$	1^{++}	$a_1(1260)$
$\bar{q}q$	3^{--}	$\rho_3(1690)$, $\omega_3(1670)$	$[qq]q$	$(1/2)^-$	$\Delta_{\frac{1}{2}^-}(1620)$	$[qq][\bar{u}\bar{d}]$	2^{--}	$\rho_2(\sim 1700)$?
				$(3/2)^-$	$\Delta_{\frac{3}{2}^-}(1700)$			
$\bar{q}q$	4^{++}	$a_4(2040)$, $f_4(2050)$	$[qq]q$	$(7/2)^+$	$\Delta_{\frac{7}{2}^+}(1950)$	$[qq][\bar{u}\bar{d}]$	3^{++}	$a_3(\sim 2070)$?
$\bar{q}s$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar{q}s$	$1^{+(-)}$	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	$0^{+(+)}$	$K_0^*(1430)$
$\bar{q}s$	$2^{-(+)}$	$K_2(1770)$	$[ud]s$	$(1/2)^-$	$\Lambda(1405)$	$[ud][\bar{s}\bar{q}]$	$1^{-(+)}$	$K_1^*(\sim 1700)$?
				$(3/2)^-$	$\Lambda(1520)$			
$\bar{s}q$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	$1^{+(-)}$	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$a_0(980)$
								$f_0(980)$
$\bar{s}q$	$1^{-(-)}$	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	$2^{+(+)}$	$K_2^*(1430)$	$[sq]q$	$(3/2)^+$	$\Sigma(1385)$	$[sq][\bar{q}\bar{q}]$	$1^{+(+)}$	$K_1(1400)$
$\bar{s}q$	$3^{-(-)}$	$K_3^*(1780)$	$[sq]q$	$(3/2)^-$	$\Sigma(1670)$	$[sq][\bar{q}\bar{q}]$	$2^{-(-)}$	$K_2(\sim 1700)$?
$\bar{s}q$	$4^{+(+)}$	$K_4^*(2045)$	$[sq]q$	$(7/2)^+$	$\Sigma(2030)$	$[sq][\bar{q}\bar{q}]$	$3^{+(+)}$	$K_3(\sim 2070)$?
$\bar{s}s$	0^{-+}	$\eta(550)$	—	—	—	—	—	—
$\bar{s}s$	1^{+-}	$h_1(1170)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$f_0(1370)$
								$a_0(1450)$
$\bar{s}s$	2^{-+}	$\eta_2(1645)$	$[sq]s$	$(?)^?$	$\Xi(1690)$	$[sq][\bar{s}\bar{q}]$	1^{-+}	$\Phi'(1750)$?
$\bar{s}s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	2^{++}	$f_2'(1525)$	$[sq]s$	$(3/2)^+$	$\Xi^*(1530)$	$[sq][\bar{s}\bar{q}]$	1^{++}	$f_1(1420)$
$\bar{s}s$	3^{--}	$\Phi_3(1850)$	$[sq]s$	$(3/2)^-$	$\Xi(1820)$	$[sq][\bar{s}\bar{q}]$	2^{--}	$\Phi_2(\sim 1800)$?
$\bar{s}s$	2^{++}	$f_2(1950)$	$[ss]s$	$(3/2)^+$	$\Omega(1672)$	$[ss][\bar{s}\bar{q}]$	$1^{+(+)}$	$K_1(\sim 1700)$?

Meson Baryon Tetraquark

New Organization of the Hadron Spectrum

M. Nielsen

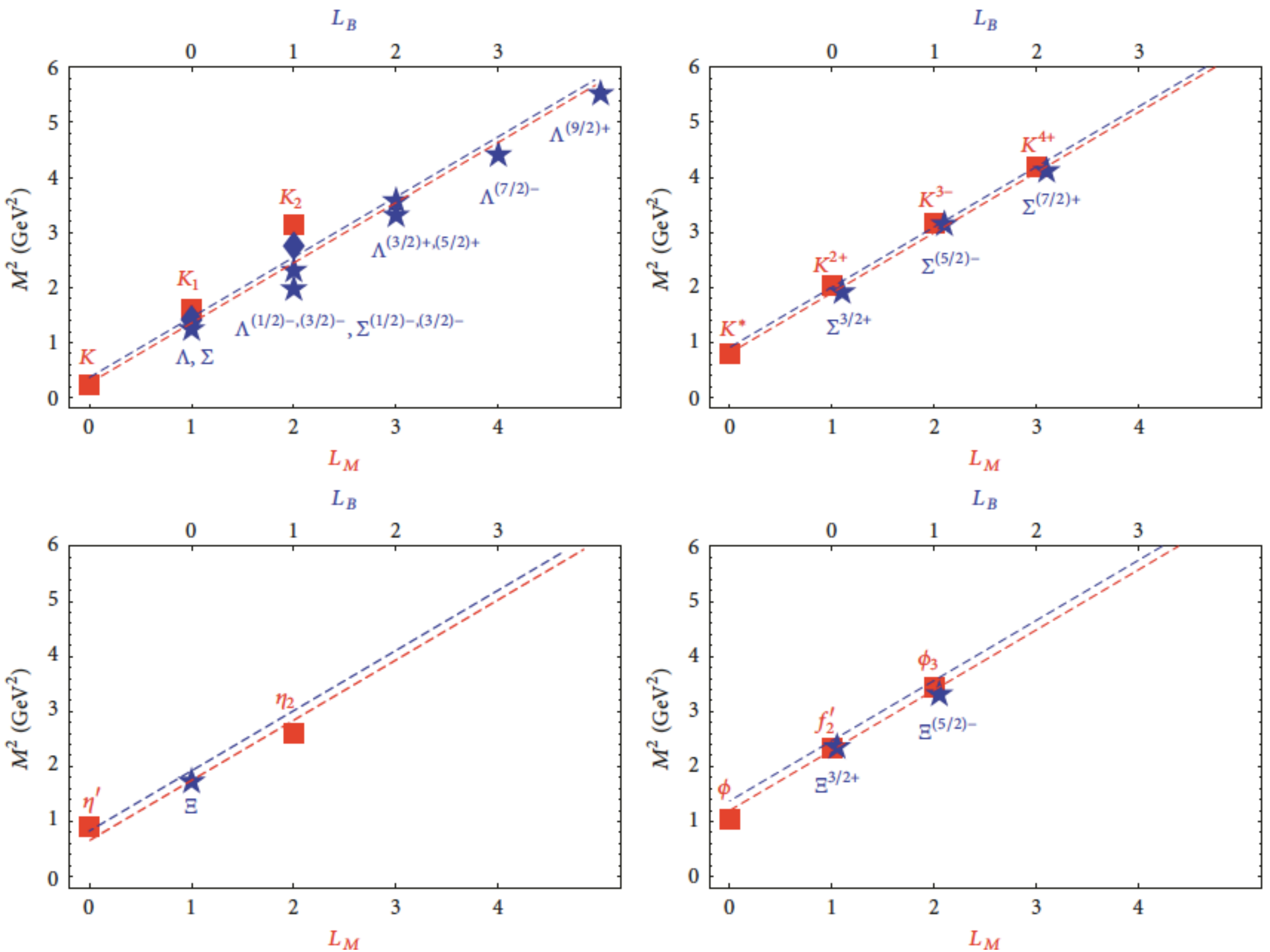
Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar q q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar q q$	1^{+-}	$h_1(1170)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar u\bar d]$	0^{++}	$\sigma(500)$
$\bar q q$	2^{-+}	$\eta_2(1645)$	$[ud]q$	$(3/2)^-$	$N_{\frac{3}{2}^-}(1520)$	$[ud][\bar u\bar d]$	1^{-+}	
$\bar q q$	1^{--}	$\rho(770),\ \omega(780)$	—	—	—	—	—	
$\bar q q$	2^{++}	$a_2(1320),\ f_2(1270)$	$(qq)q$	$(3/2)^+$	$\Delta(1232)$	$(qq)[\bar u\bar d]$	1^{++}	
$\bar q q$	3^{--}	$\rho_3(1690),\ \omega_3(1670)$	$(qq)q$	$(3/2)^-$	$\Delta_{\frac{3}{2}^-}(1700)$	$(qq)[\bar u\bar d]$	1^{-+}	
$\bar q q$	4^{++}	$a_4(2040),\ f_4(2050)$	$(qq)q$	$(7/2)^+$	$\Delta_{\frac{7}{2}^+}(1950)$	$(qq)[\bar u\bar d]$	—	
$\bar q s$	0^-	$\bar K(495)$	—	—	—	—		
$\bar q s$	1^+	$\bar K_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][s\bar q]$	0^+	
$\bar q s$	2^-	$\bar K_2(1770)$	$[ud]s$	$(3/2)^-$	$\Lambda(1520)$	$[ud][s\bar q]$	1^-	
$\bar s q$	0^-	$K(495)$	—	—	—	—	—	
$\bar s q$	1^+	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][s\bar q]$	0^{++}	
$\bar s q$	1^-	$K^*(890)$	—	—	—	—	—	—
$\bar s q$	2^+	$K_2^*(1430)$	$(sq)q$	$(3/2)^+$	$\Sigma(1385)$	$(sq)[\bar u\bar d]$	1^+	$K_1(1400)$
$\bar s q$	3^-	$K_3^*(1780)$	$(sq)q$	$(3/2)^-$	$\Sigma(1670)$	$(sq)[\bar u\bar d]$	2^-	$K_2(1820)$
$\bar s q$	4^+	$K_4^*(2045)$	$(sq)q$	$(7/2)^+$	$\Sigma(2030)$	$(sq)[\bar u\bar d]$	—	—
$\bar s s$	0^{-+}	$\eta(550),\ \eta'(958)$	—	—	—	—	—	—
$\bar s s$	1^{+-}	$h_1(1380)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar s\bar q]$	0^{++}	$f_0(1370)$ $a_0(1450)$
$\bar s s$	2^{-+}	$\eta_2(1870)$	$[sq]s$	$(3/2)^-$	$\Xi(1620)$	$[sq][\bar s\bar q]$	1^{-+}	—
$\bar s s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar s s$	2^{++}	$f_2'(1525)$	$(sq)s$	$(3/2)^+$	$\Xi^*(1530)$	$(sq)[\bar s\bar q]$	1^{++}	$f_1(1420)$ $a_1(1420)$
$\bar s s$	3^{--}	$\Phi_3(1850)$	$(sq)s$	$(3/2)^-$	$\Xi(1820)$	$(sq)[\bar s\bar q]$	2^{--}	—
$\bar s s$	2^{++}	$f_2(1640)$	$(ss)s$	$(3/2)^+$	$\Omega(1672)$	$(ss)[\bar s\bar q]$	1^+	$K_1(1650)$



I=0, 1 states

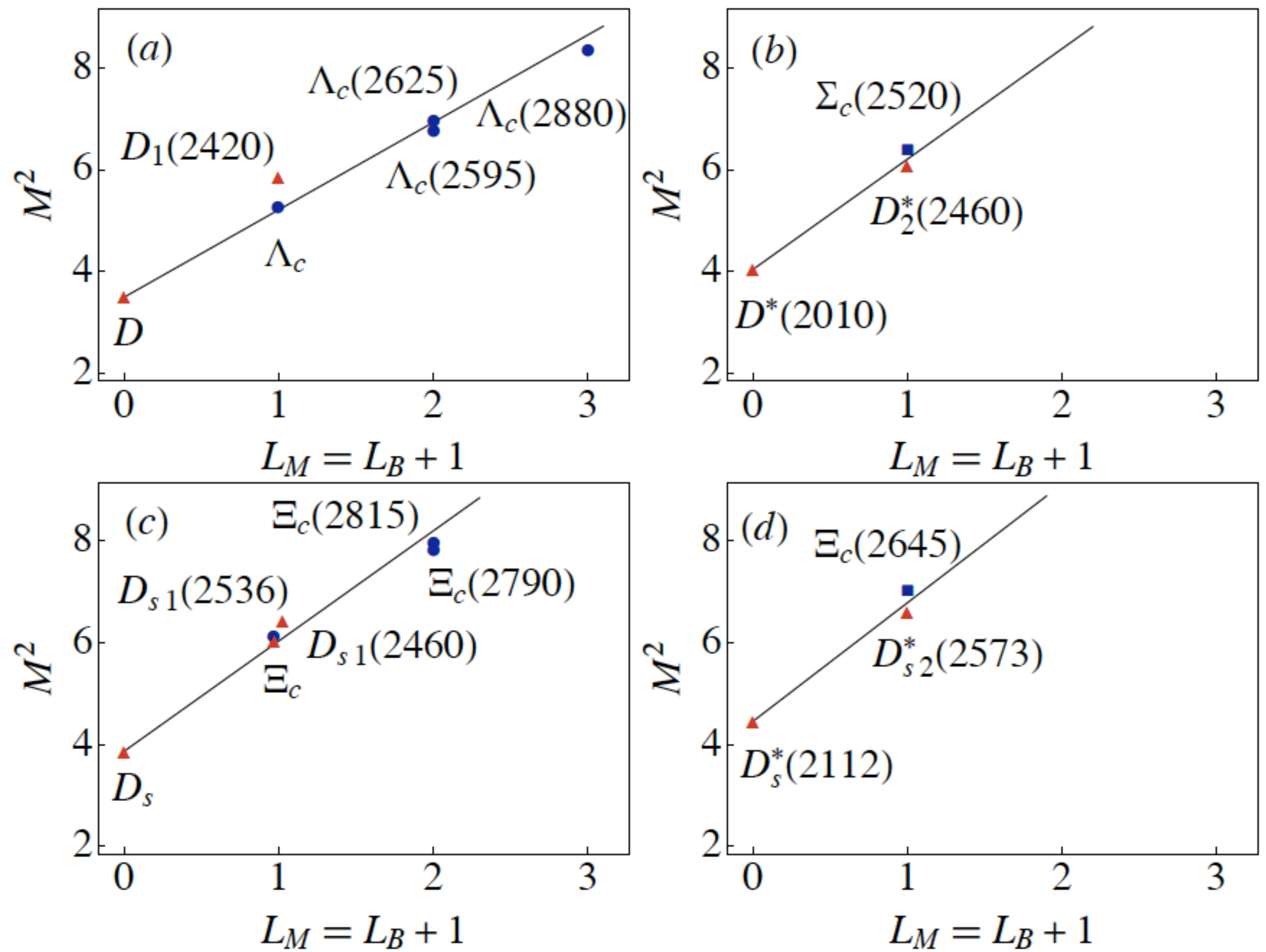
Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy charm quark mass does not break supersymmetry

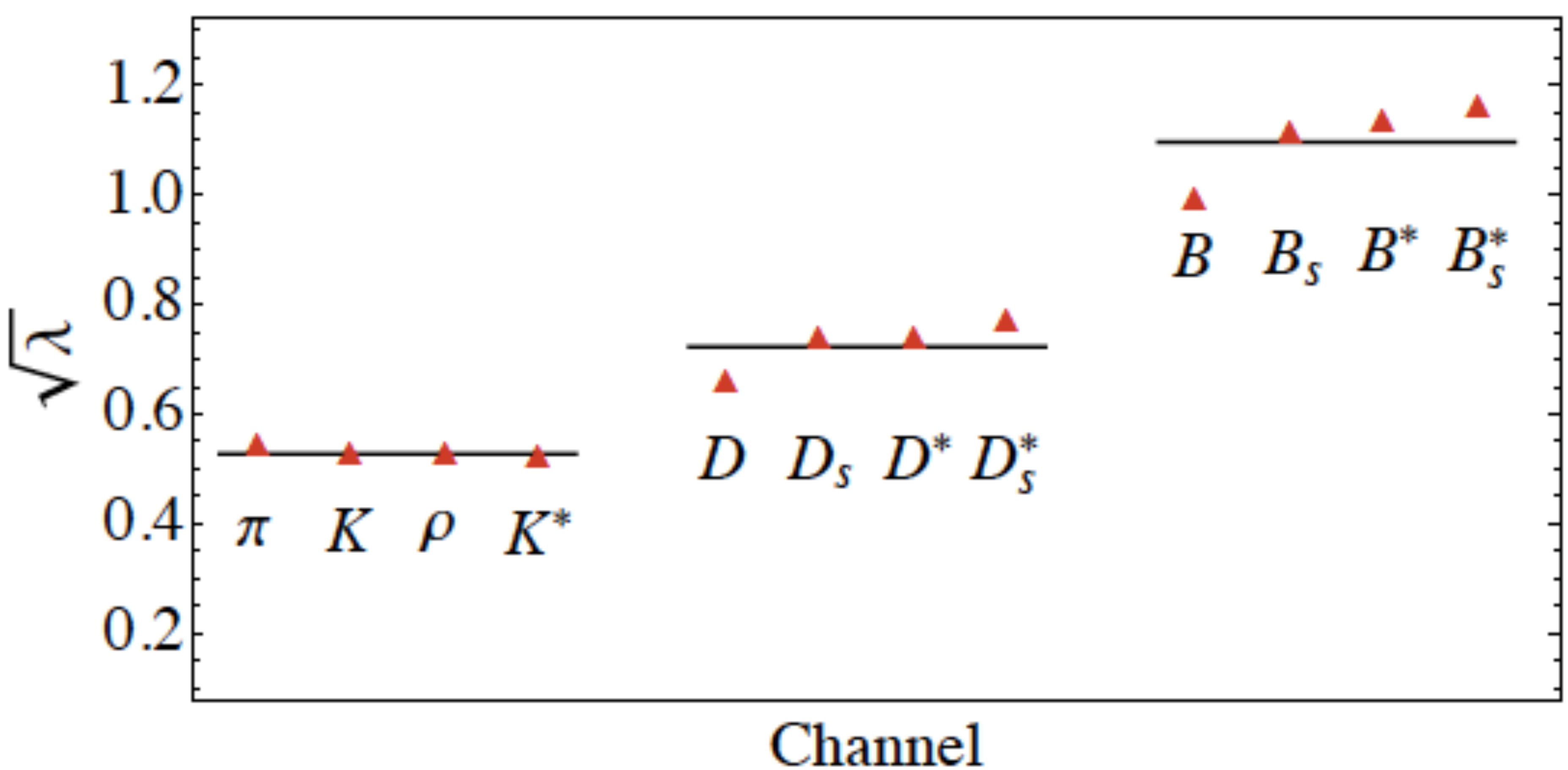
Extension to Heavy-Light sector

DdTB, arXiv:1611.02370

it was shown that the LF potential in the heavy-light sector, even for strongly broken conformal invariance, has the same quadratic form as the one dictated by the conformal algebra: $\varphi(\zeta) = \frac{1}{2}\lambda A \zeta^2$, A arbitrary constant

$$G_{SUSY} = \{R_\lambda, R_\lambda^\dagger\} + \mu^2 \mathbf{I}, \;\; \mu^2 = 2\lambda \mathcal{S} + \Delta M^2[m_1, ...m_N] \quad \lambda \to \lambda_Q = \frac{1}{2}\lambda A$$

quark mass correction



Superpartners for states with one c quark

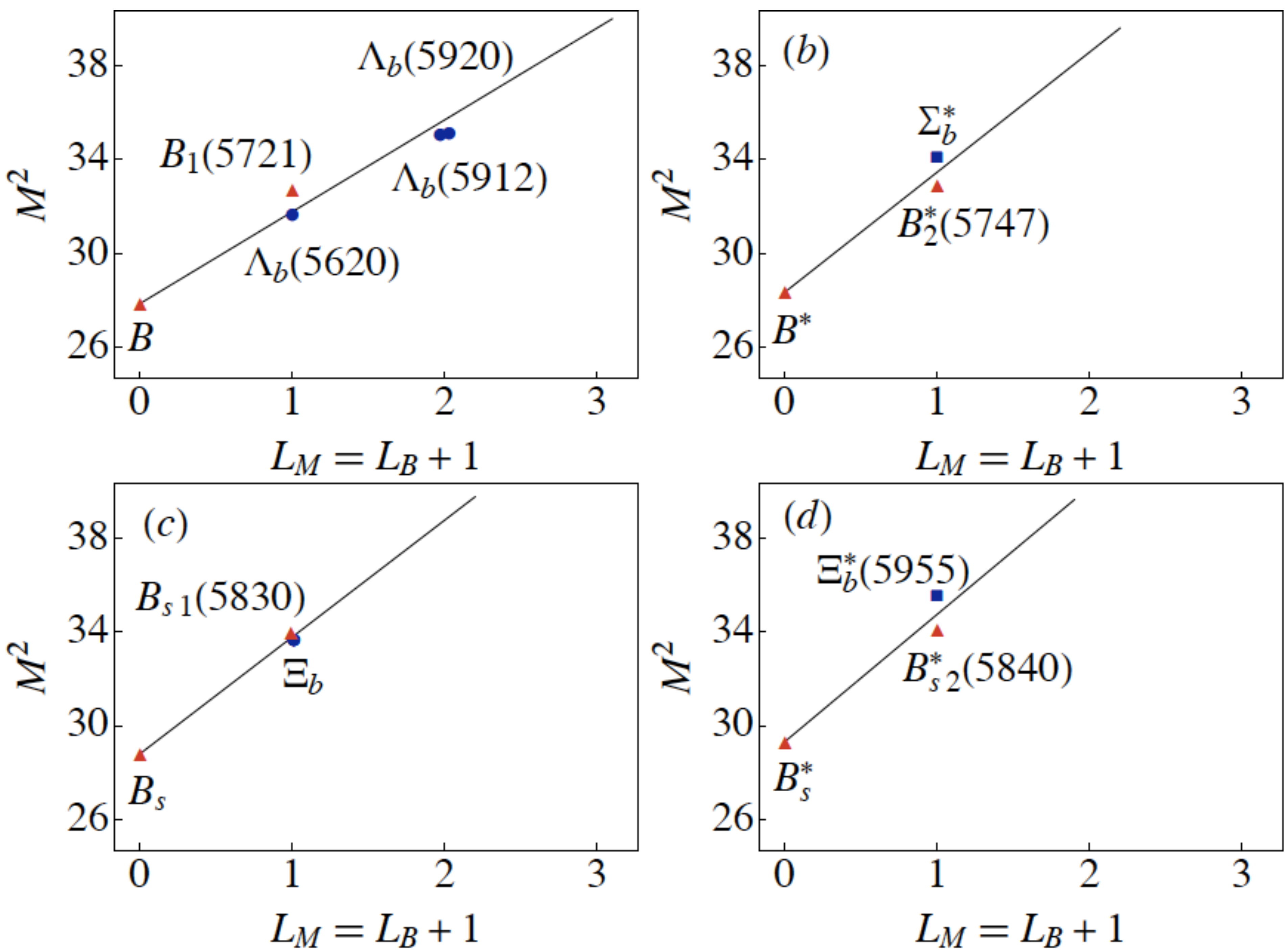
Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}c$	0^-	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	1^+	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	0^+	$\bar{D}_0^*(2400)$
$\bar{q}c$	2^-	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	1^-	—
$\bar{c}q$	0^-	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	1^+	$\bar{D}_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	0^+	$D_0^*(2400)$
$\bar{q}c$	1^-	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	2^+	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	1^+	$D(2550)$
$\bar{q}c$	3^-	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	0^-	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	1^+	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	0^+	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	2^-	$D_{s2}(\sim 2860)?$	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	1^-	—
$\bar{s}c$	1^-	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	2^+	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	1^+	$D_{s1}(2536)$
$\bar{c}s$	1^+	$D_{s1}(\sim 2700)?$	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	0^+	??
$\bar{s}c$	2^+	$D_{s2}^*(\sim 2750)?$	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	1^+	??

predictions

beautiful agreement!

Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy bottom quark mass does not break supersymmetry

Superpartners for states with one b quark

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}b$	0^-	$\bar{B}(5280)$	—	—	—	—	—	—
$\bar{q}b$	1^+	$\bar{B}_1(5720)$	$[ud]b$	$(1/2)^+$	$\Lambda_b(5620)$	$[ud][\bar{b}\bar{q}]$	0^+	$B_J(5732)$
$\bar{q}b$	2^-	$\bar{B}_J(5970)$	$[ud]b$	$(3/2)^-$	$\Lambda_b(5920)$	$[ud][\bar{b}\bar{q}]$	1^-	—
$\bar{b}q$	0^-	$B(5280)$	—	—	—	—	—	—
$\bar{b}q$	1^+	$B_1(5720)$	$[bq]q$	$(1/2)^+$	$\Sigma_b(5815)$	$[bq][\bar{u}\bar{d}]$	0^+	$\bar{B}_J(5732)$
$\bar{q}b$	1^-	$B^*(5325)$	—	—	—	—	—	—
$\bar{q}b$	2^+	$B_2^*(5747)$	$(qq)b$	$(3/2)^+$	$\Sigma_b^*(5835)$	$(qq)[\bar{b}\bar{q}]$	1^+	$B_J(5840)$
$\bar{s}b$	0^-	$B_s(5365)$	—	—	—	—	—	—
$\bar{s}b$	1^+	$B_{s1}(5830)$	$[qs]b$	$(1/2)^+$	$\Xi_b(5790)$	$[qs][\bar{b}\bar{q}]$	0^+	**$\bar{B}_{s0}^*(\sim 5800)$?**
$\bar{s}b$	1^-	$B_s^*(5415)$	—	—	—	—	—	—
$\bar{s}b$	2^+	$B_{s2}^*(5840)$	$(sq)b$	$(3/2)^+$	$\Xi_b^*(5950)$	$(sq)[\bar{b}\bar{q}]$	1^+	**$B_{s1}(\sim 5900)$?**
$\bar{b}s$	1^+	**$B_{s1}(\sim 6000)$?**	$[bs]s$	$(1/2)^+$	$\Omega_b(6045)$	$[bs][\bar{s}\bar{q}]$	0^+	??

predictions

States with two heavy quarks

Trawinski, Stanislaw, Glazek, Brodsky, De Te'ramond, Dosch, PRD90(2104)

quadratic potential in FF for light quarks ⟷ linear potential in IF V=Cr

Cornell potential for heavy quarks

I=0, I=1?

The LF confinement potential for systems containing two heavy quarks will be modified. Therefore the extension of superconformal algebra to such states is somewhat speculative. However...

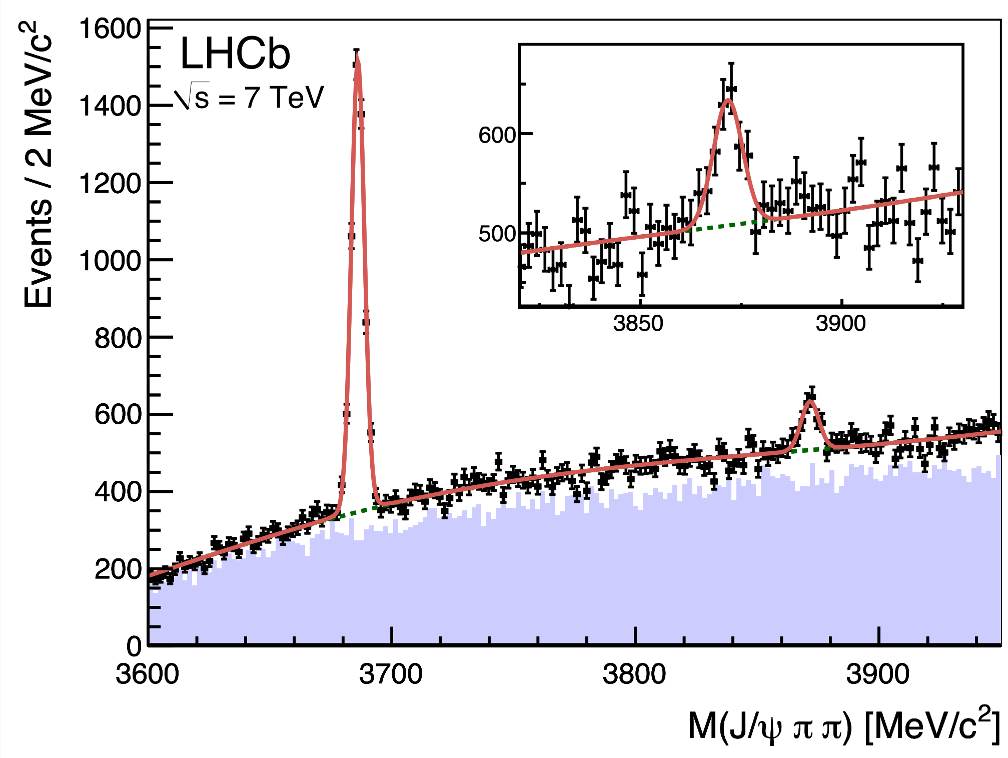
Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{c}c$	0^{-+}	$\eta_c(2984)$	—	—	—	—	—	—
$c\bar{c}$	1^{+-}	$h_c(3525)$	$[cq]c$	$(1/2)^+$	$\Xi_{cc}^{SELEX}(3520)$, $\Xi_{cc}^{LHCb}(3620)$	$[cq][\bar{c}\bar{q}]$	0^{++}	$\chi_{c0}(3415)$
$\bar{c}c$	1^{--}	$J/\psi(3096)$	—	—	—	—	—	—
$\bar{c}c$	2^{++}	$\chi_{c2}(3556)$	$(cq)c$	$(3/2)^+$	$\Xi_{cc}^{LHCb}(3620)$	$(cq)[\bar{c}\bar{q}]$	1^{++}	$\chi_{c1}(3510)$

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{c}c$	1^{--}	$J/\psi(3096)$	—	—	—	—	—	—
$\bar{c}c$	2^{++}	$\chi_{c2}(3556)$	$(cq)c$	$(3/2)^+$	$\Xi_{cc}^{LHCb}(3620)$	$(cq)[\bar{c}\bar{q}]$	1^{++}	$\chi_{c1}(3510)$
		n=1						
$\bar{c}c$	1^{--}	$\psi'(3686)$	—	—	—	—	—	—
$\bar{c}c$	2^{++}	$\chi_{c2}(3927)$	$(cq)c$	$(3/2)^+$	$\Xi_{cc}^{*}(\sim 3900)?$	$(cq)[\bar{c}\bar{q}]$	1^{++}	$X(3872)$
							1^{+-}	$Z_c(3900)$

New charmonium states, $Z_c(3900)$ ⇒ charged state, I=1!!!!!!!

X(3872) Belle II @ KEK (PRL91(2003))

M_X = (3872.20 ± 0.39) MeV
Γ < 1.2 MeV

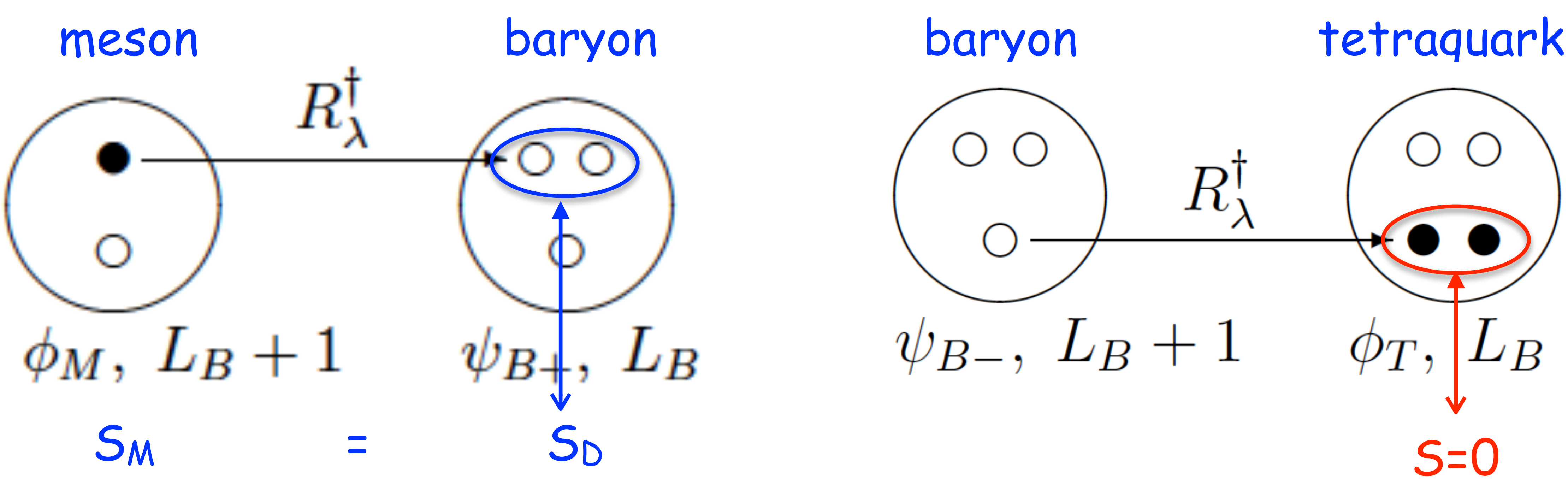


cc spec. for $J^{PC} = 1^{++}$ (Barnes & Godfrey, PRD69 (2004))

$2\,^3P_1\ (3990)$

$3\,^3P_1\ (4290)$

SUSY-LFHQCD ➜ linear Regge trajectories for mesons, baryons, tetraquarks



$$M_M^2 = 4\lambda_Q(n + L_M + \frac{S_M}{2}) + \Delta M^2[m_1, m_2],$$

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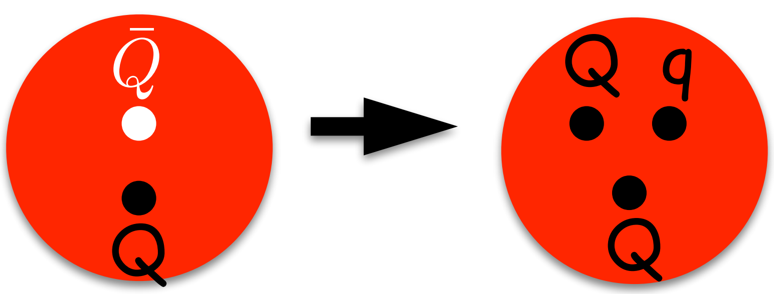
Predictions

R_λ^T: constituent into cluster(2) ➜ pentaquarks are molecular states

all $J^{PC} = 0^{++}, 1^{++}, 1^{-+}$ states ➜ tetraquark states



➜ no baryonic bound states with 3 heavy quarks

SUSY in superconformal QM ➜ symmetry properties of hadrons, not to quantum fields no need to introduce new supersymmetric fields or particles such as squarks or gluinos

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$e^{\phi(z)} = e^{+\kappa^2 z^2} \qquad\qquad S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\boxed{\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.}$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

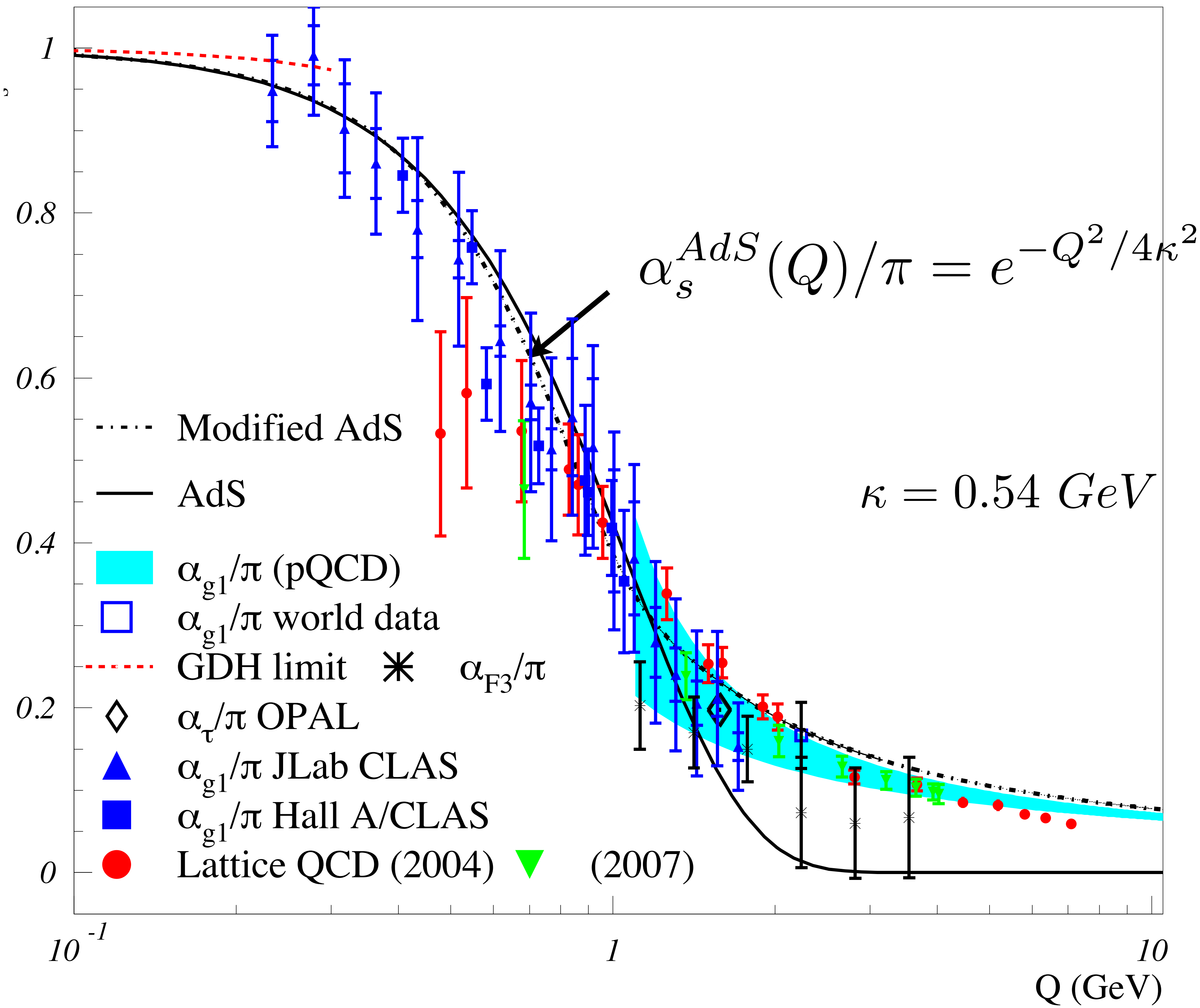
Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

Analytic, defined at all scales, IR Fixed Point

$$\frac{\alpha_s(Q)}{\pi}$$

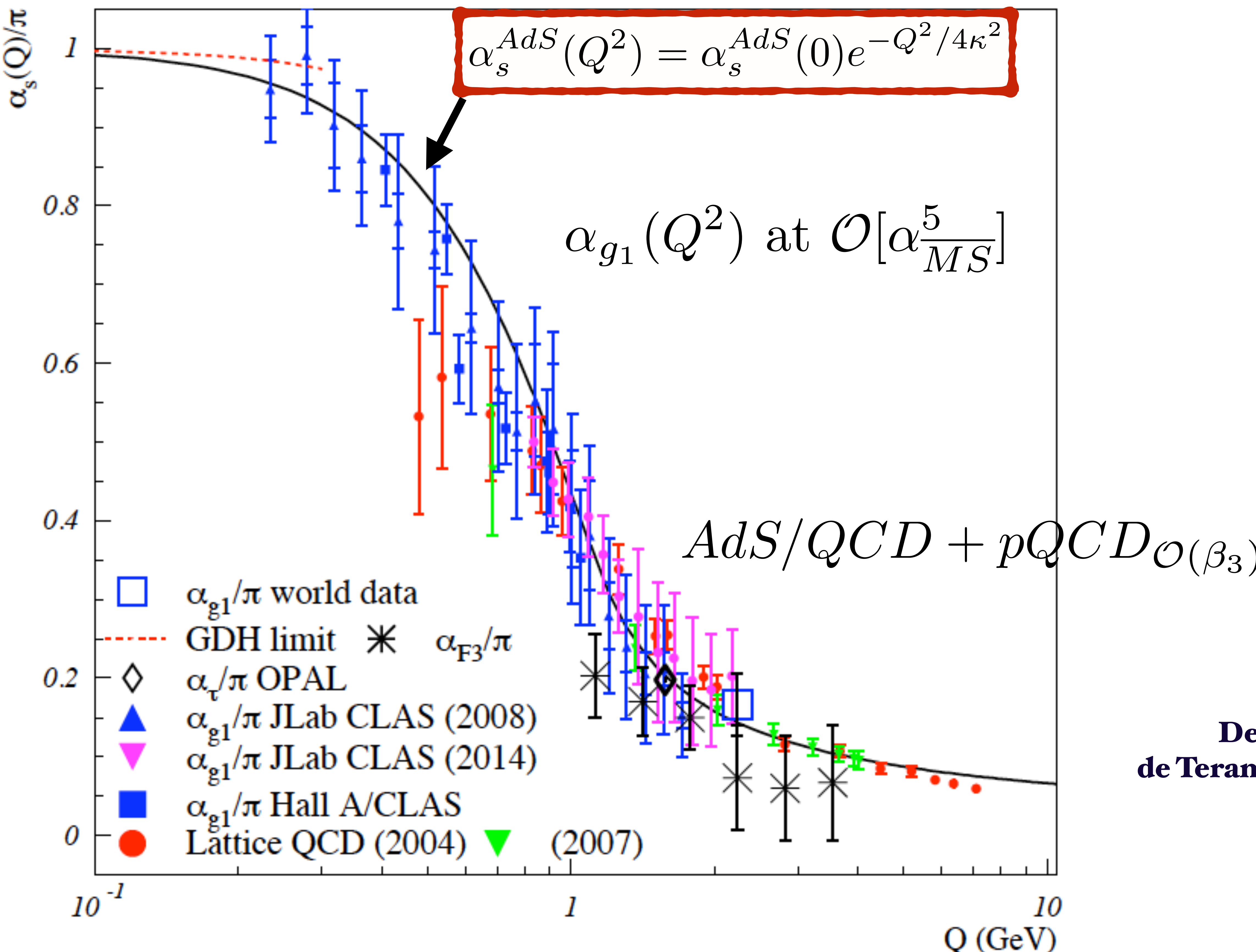


AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

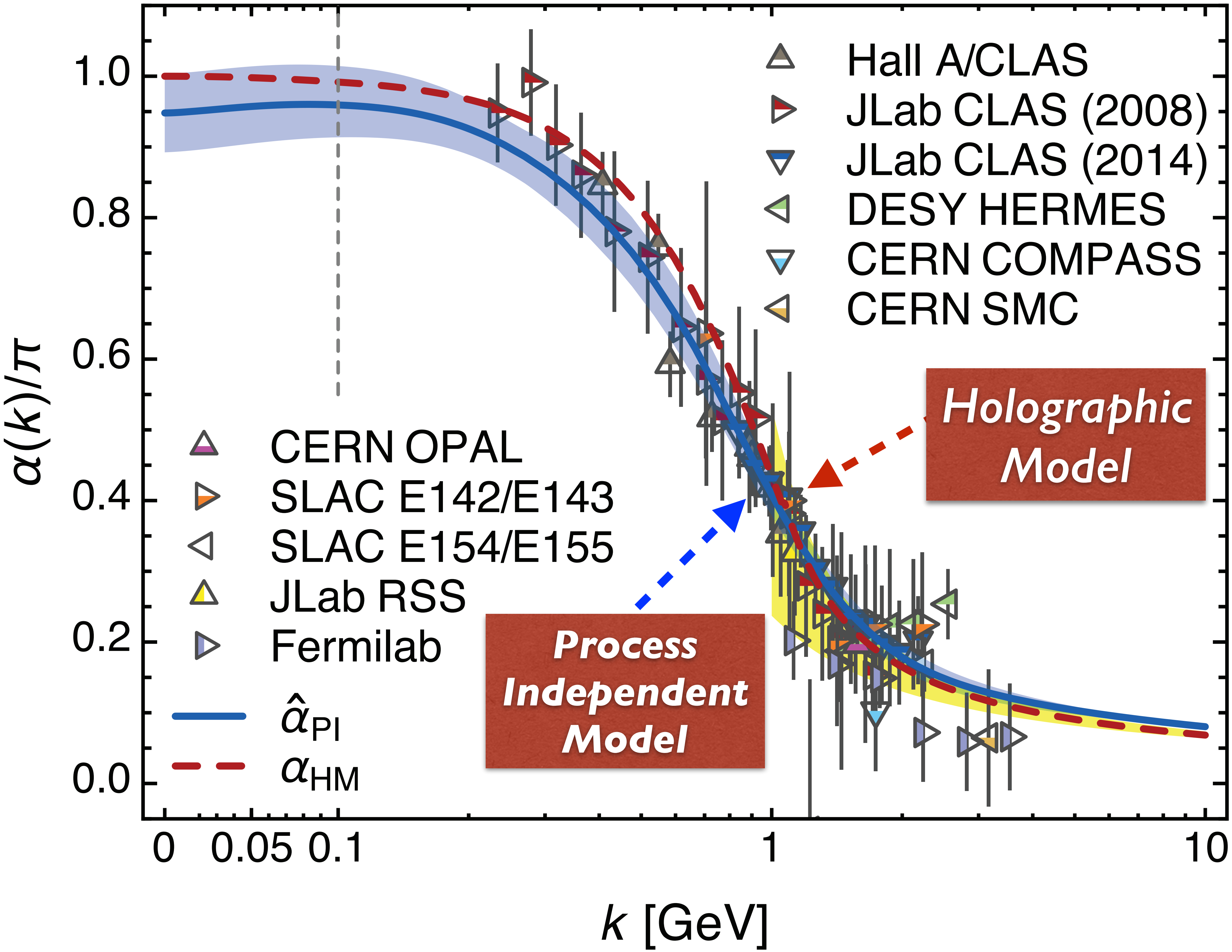
$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

$$\Lambda_{\overline{MS}} = 0.5983\kappa = 0.5983\frac{m_\rho}{\sqrt{2}} = 0.4231 m_\rho = 0.328\ GeV$$



Deur,
de Teramond, sjb



Process-independent strong running coupling

Daniele Binosi,[1] Cédric Mezrag,[2] Joannis Papavassiliou,[3] Craig D. Roberts,[2] and Jose Rodríguez-Quintero[4]

$m_\rho = \sqrt{2}\kappa$

$m_p = 2\kappa$

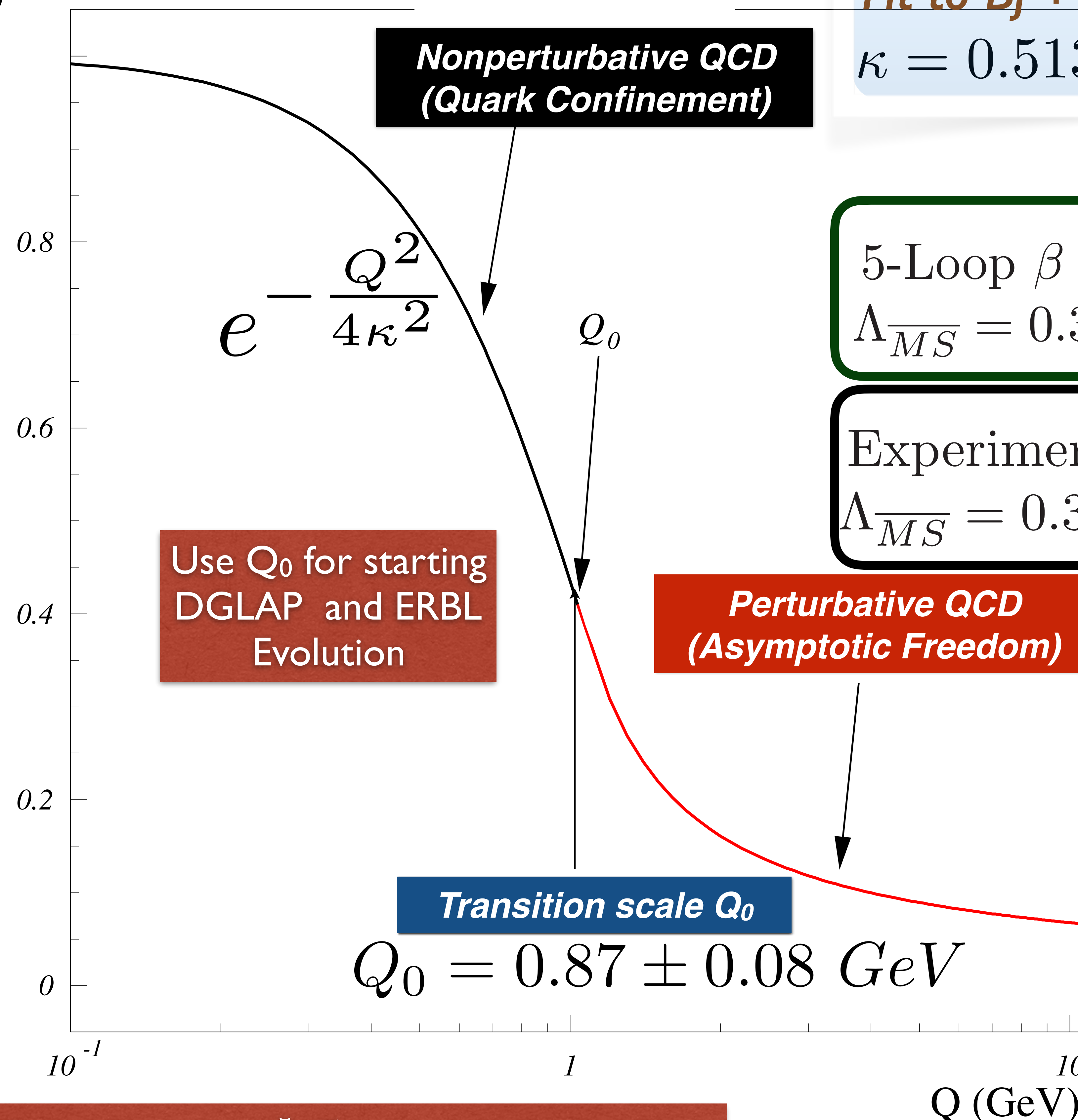
All-Scale QCD Coupling

Deur, de Tèramond, sjb

Fit to Bj + DHG Sum Rules:

$\kappa = 0.513 \pm 0.007\ GeV$

$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$



5-Loop β Prediction:

$\Lambda_{\overline{MS}} = 0.339 \pm 0.019\ GeV$

Experiment:

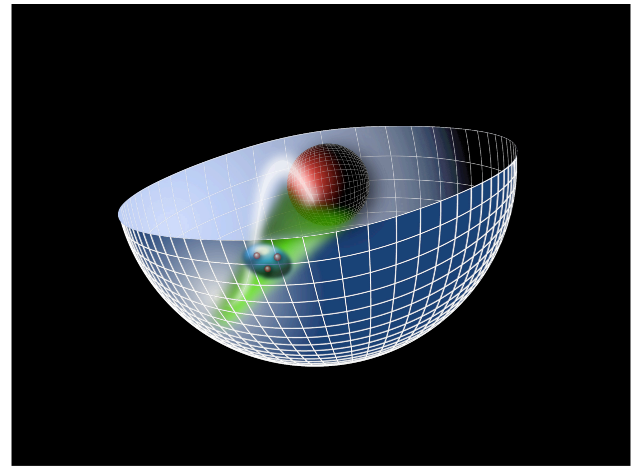
$\Lambda_{\overline{MS}} = 0.332 \pm 0.017\ GeV$

$\lambda \equiv \kappa^2$

Reverse Dimensional Transmutation!

$\overline{MS}$ *scheme*

de Tèramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = M^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Single variable ζ

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

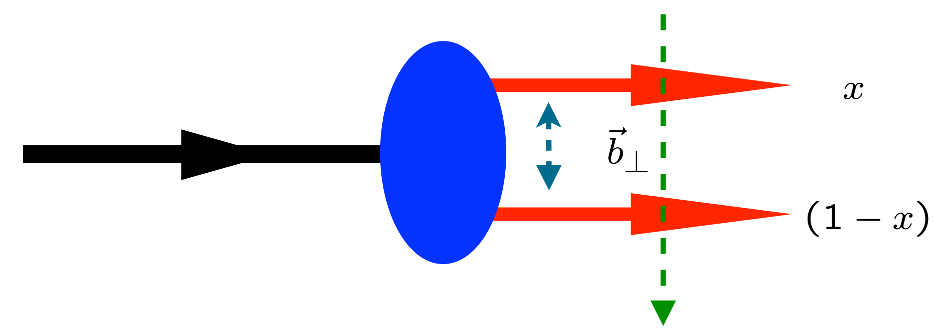
- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Underlying Principles

- *Poincarè Invariance: Independent of the observer's Lorentz frame*
- *Quantization at Fixed Light-Front Time* τ
- *Causality: Information within causal horizon*
- *Light-Front Holography: AdS₅ = LF (3+1)*

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_\perp^2 x(1-x)$$



$$\zeta^2 = x(1-x)b_\perp^2$$

- *Single fundamental hadronic mass scale κ: but retains the Conformal Invariance of the Action (dAFF)!*
- *Unique color-confining LF Potential!* $U(\zeta^2) = \kappa^4 \zeta^2$
- *Superconformal Algebra: Mass Degenerate 4-Plet:*

$$\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]$$

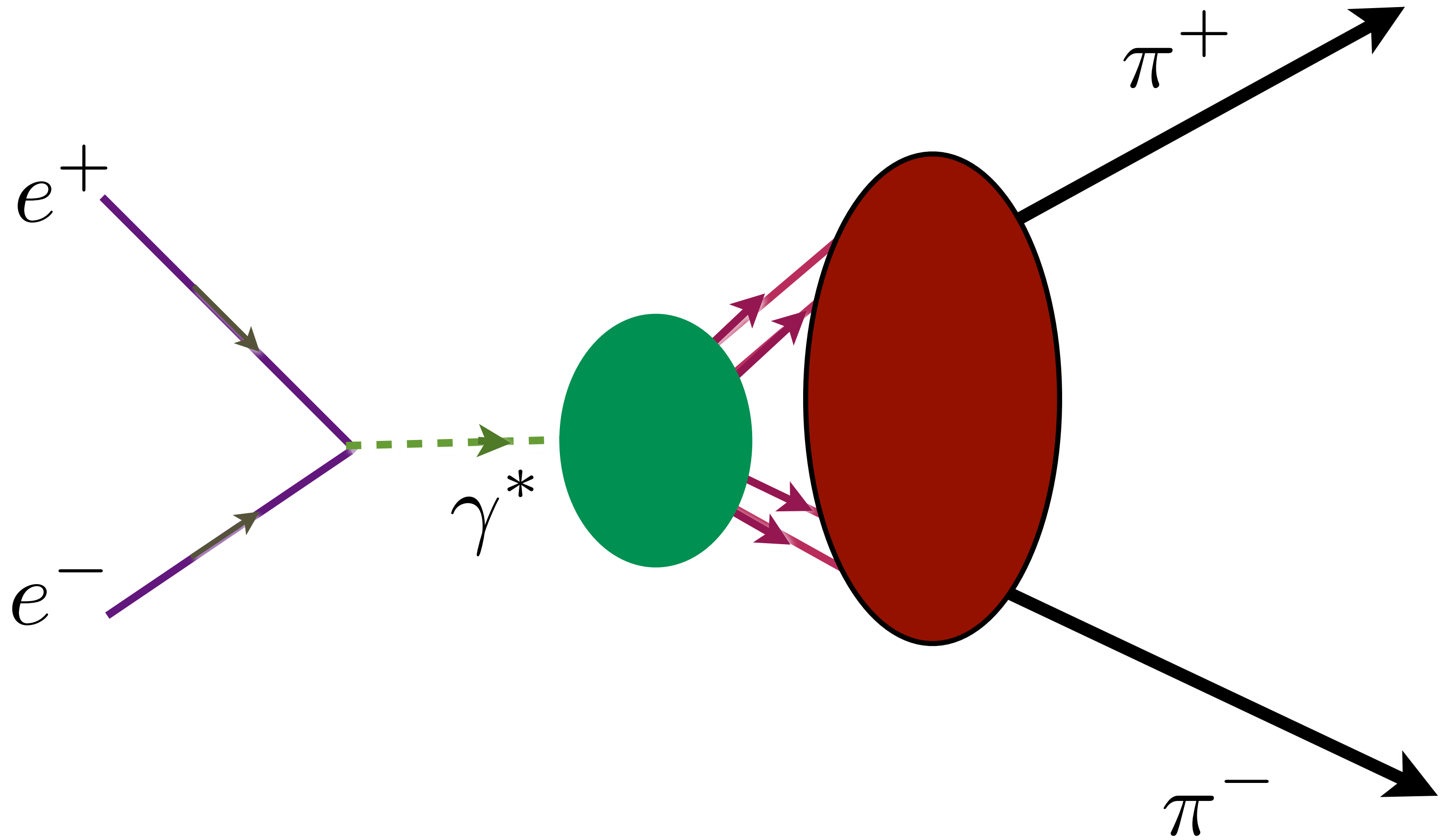
Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

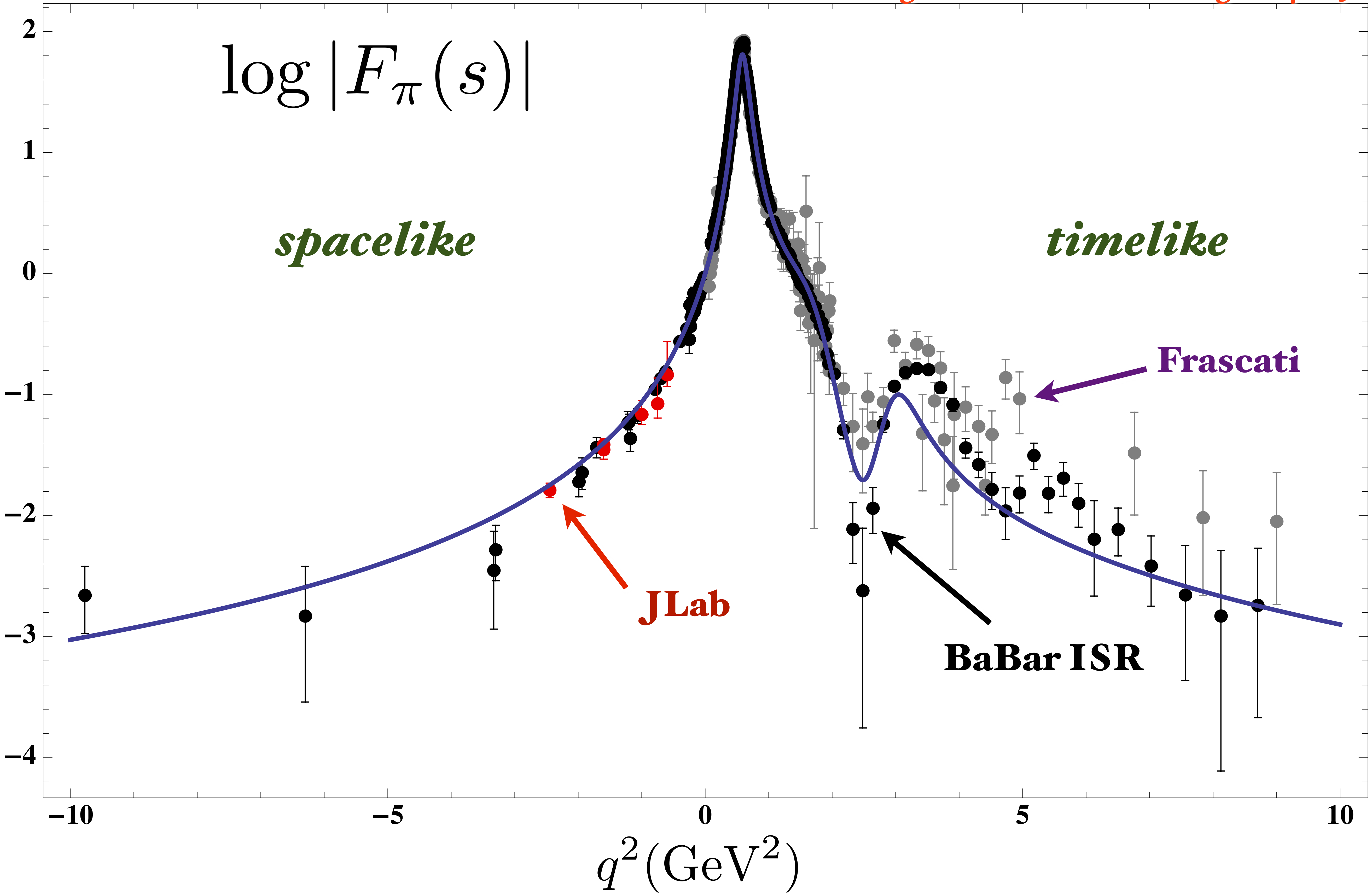
Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

Dressed soft-wall current brings in higher Fock states and more vector meson poles



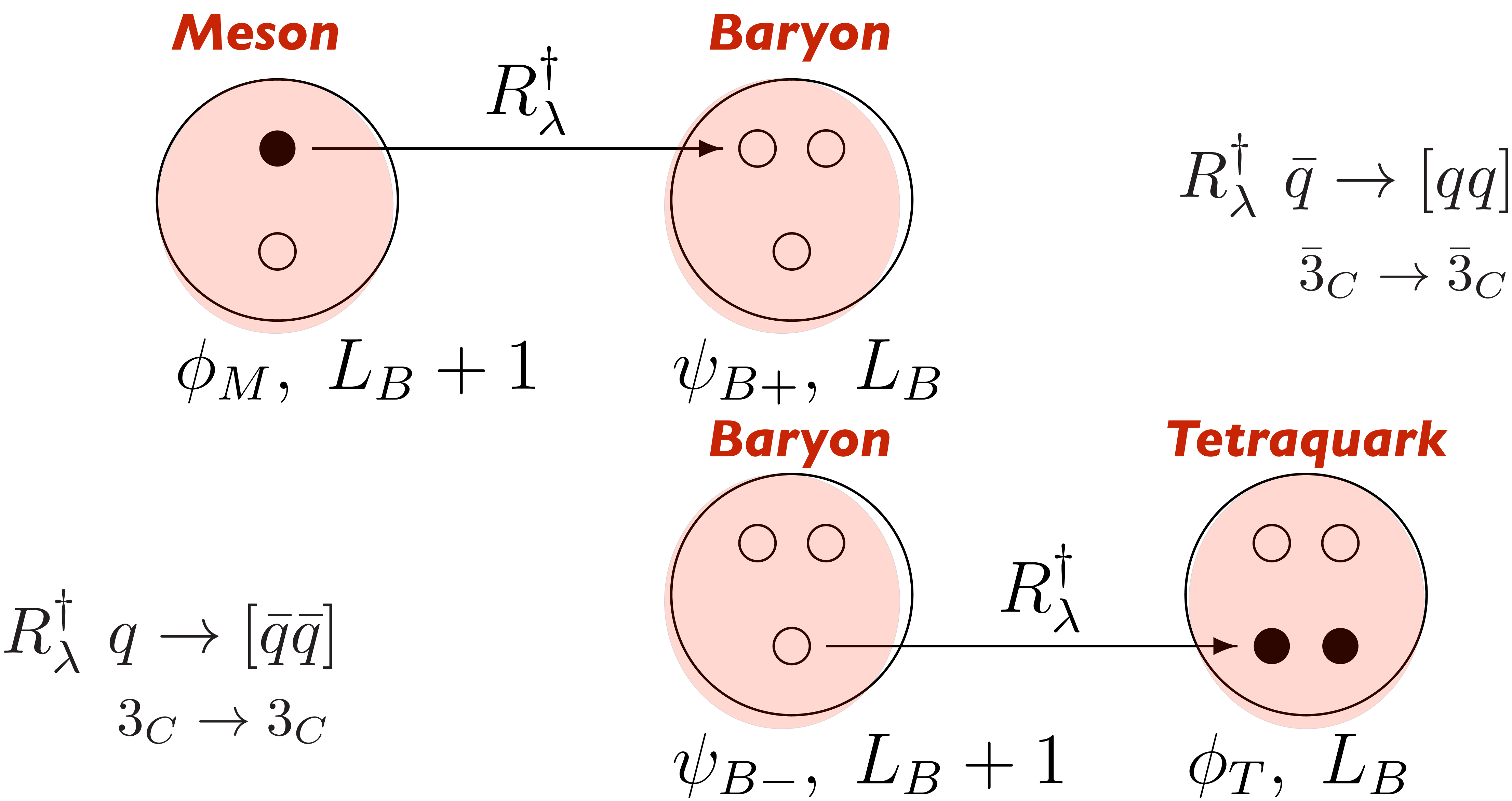
Pion Form Factor from AdS/QCD and Light-Front Holography



Superconformal Algebra

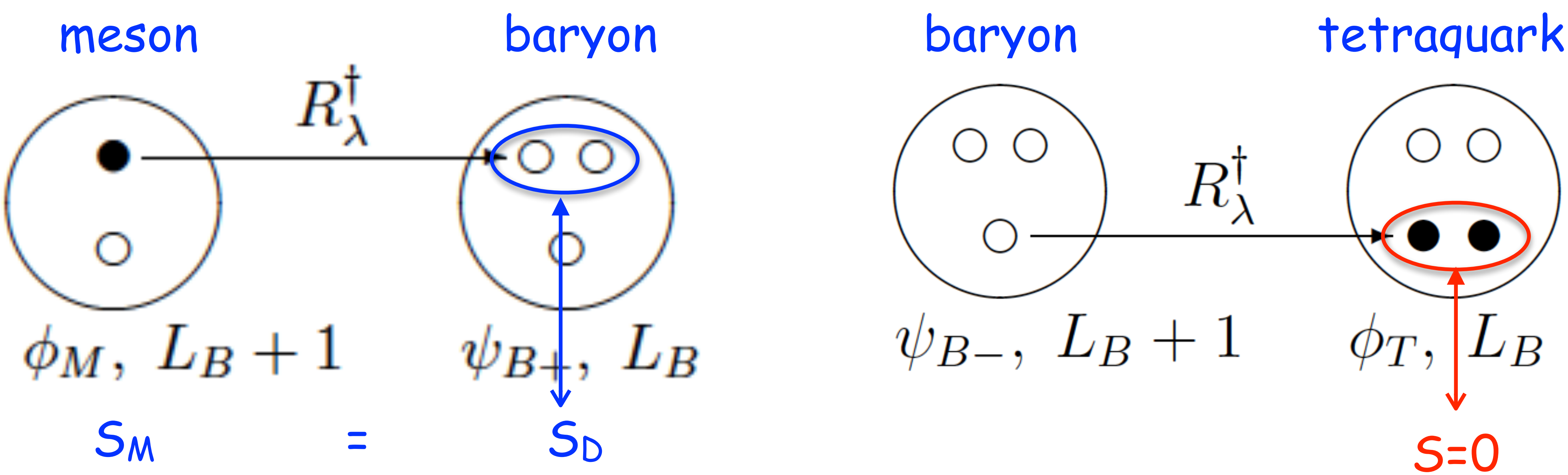
2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

SUSY-LFHQCD ➜ linear Regge trajectories for mesons, baryons, tetraquarks



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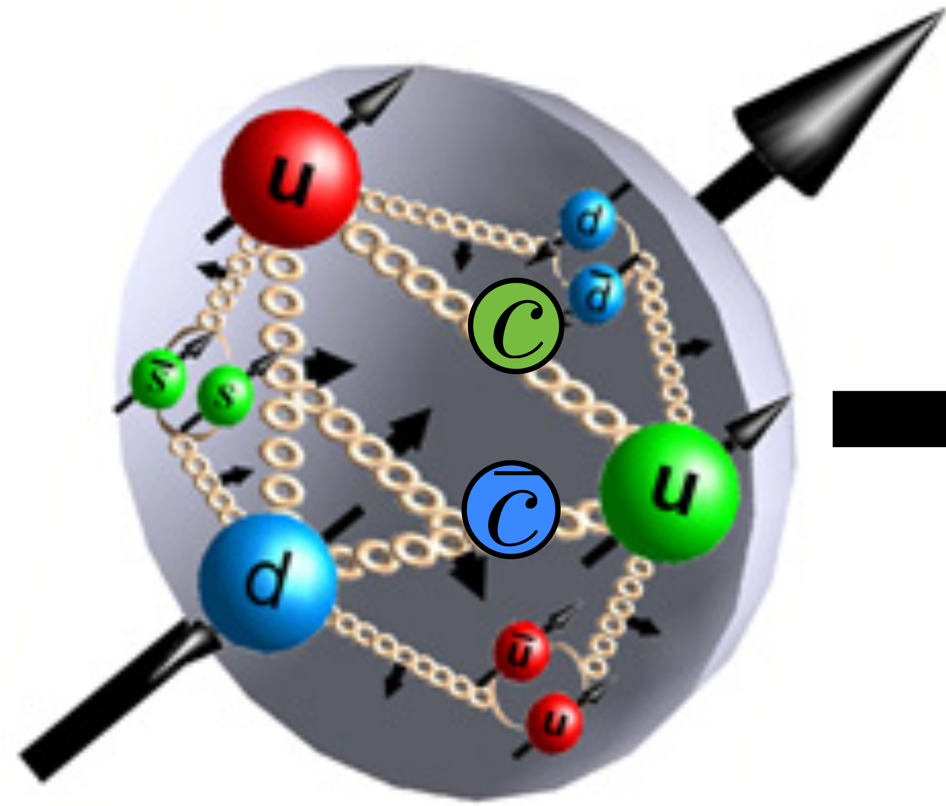
Features of LF Holographic QCD

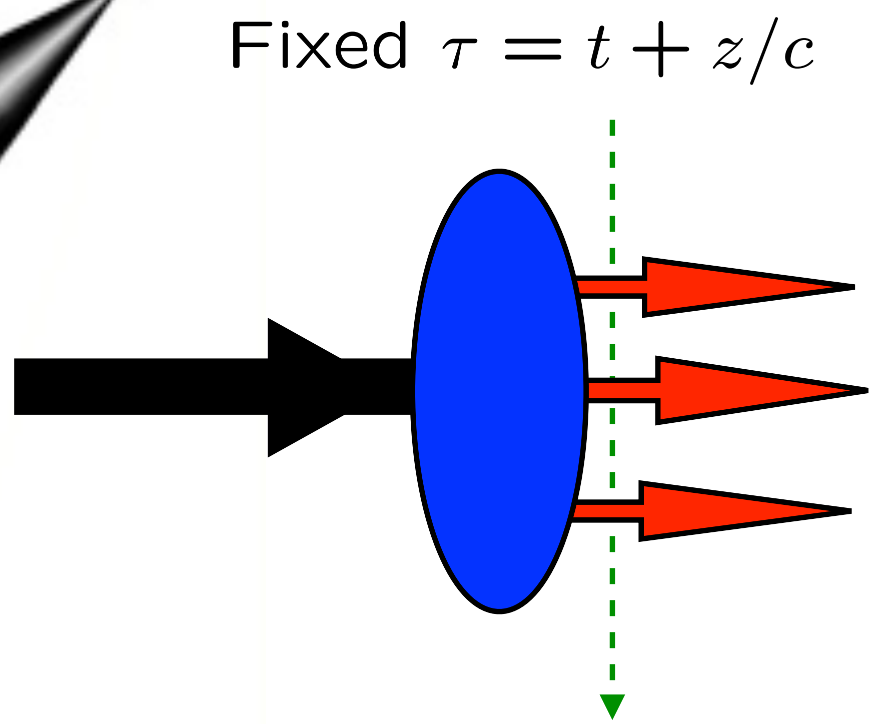
- Color Confinement, Analytic form of confinement potential
- Massless pion bound state in chiral limit
- QCD coupling at all scales
- Connection of perturbative and nonperturbative mass scales
- Poincare Invariant
- Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L
- Supersymmetric 4-Plet: Meson-Baryon Tetraquark Symmetry
- Light-Front Wavefunctions
- Form Factors, Structure Functions, Hadronic Observables
- Constituent Counting Rules
- Hadronization at the Amplitude Level
- Analytic First Approximation to QCD
- Systematically improvable BLFQ

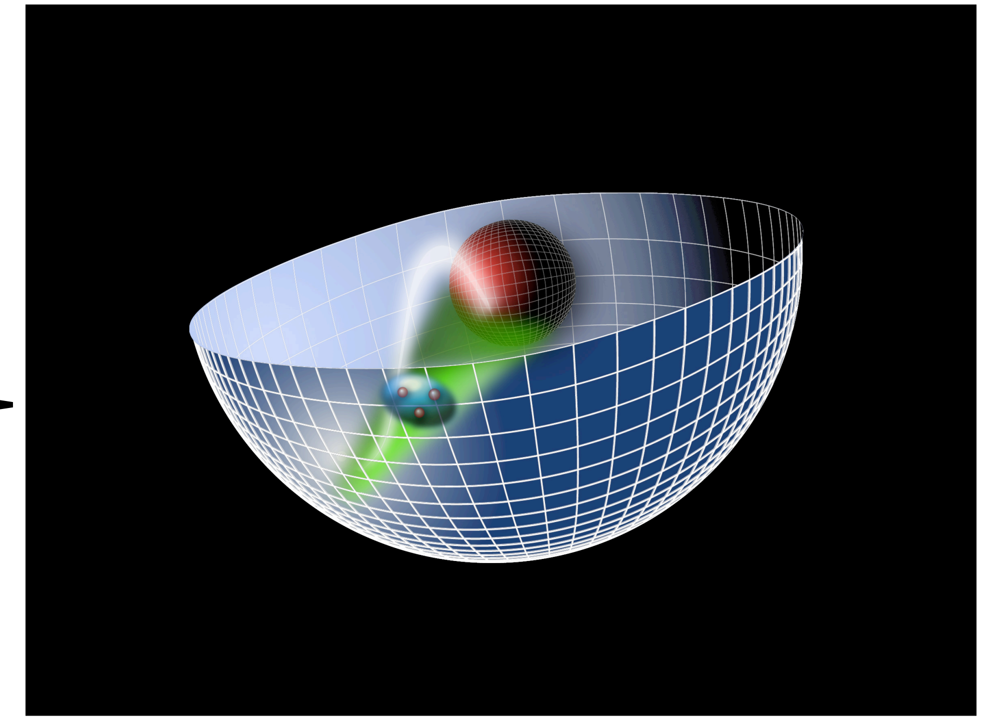
Invariance Principles of Quantum Field Theory

- *Polncarè Invariance: Physical predictions must be independent of the observer's Lorentz frame: Front Form*
- *Causality: Information within causal horizon: Front Form*
- *Gauge Invariance: Physical predictions of gauge theories must be independent of the choice of gauge*
- *Scheme-Independence: Physical predictions of renormalizable theories must be independent of the choice of the renormalization scheme — Principle of Maximum Conformality (PMC)*
- *Mass-Scale Invariance: Conformal Invariance of the Action (DAFF)*

Supersymmetric Features of Hadron Physics and other Novel Features of QCD from Light-Front Holography and Superconformal Quantum Mechanics







Bound States in Strongly Coupled Systems March 12, 2018

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

with Guy de Tèramond, Hans Günter Dosch, C. Lorcè, M. Nielsen, K. Chiu, R. S. Sufian, A. Deur