Resonant scattering amplitudes from lattice QCD

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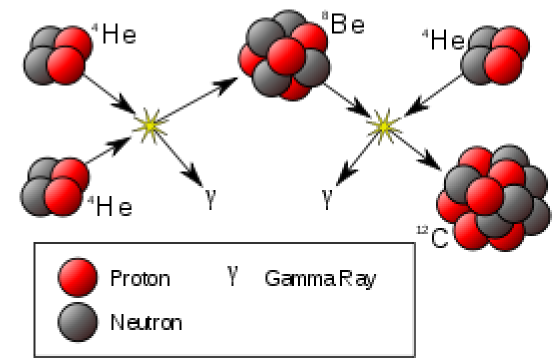
Why study hadron-hadron scattering amplitudes?

GeV

0.1 → Low energy pion, nucleon scattering:

$$\pi\pi \to \pi\pi,\ p\pi \to p\pi \quad \Rightarrow$$



1.0 → Hadron-photon scattering:

$$p\gamma \to p + X, \quad \gamma \to \pi\pi \text{ for } (g-2)_\mu$$

10 → Precision Standard Models tests and Exotic hadrons:

$$B \to K^* \ell^+ \ell^-, \quad X(3872), Z^+(3900), \ldots$$

1000 → QCD-like Beyond-the-Standard Model theories

$$f_0(500) \Rightarrow H(125), \quad \pi \Rightarrow G, \quad \rho \Rightarrow \tilde{\rho}$$

Scattering amplitudes in lattice QCD

- In imaginary time, $\langle 0|T[\hat{\mathcal{O}}'(x')\hat{\mathcal{O}}^\dagger(x)]|0\rangle$ generally contains no info about on-shell amplitudes. L. Maiani, M. Testa, *Phys. Lett.* **B245** (1990) 585

- Finite volume method: below $n \geq 3$ hadron thresholds:

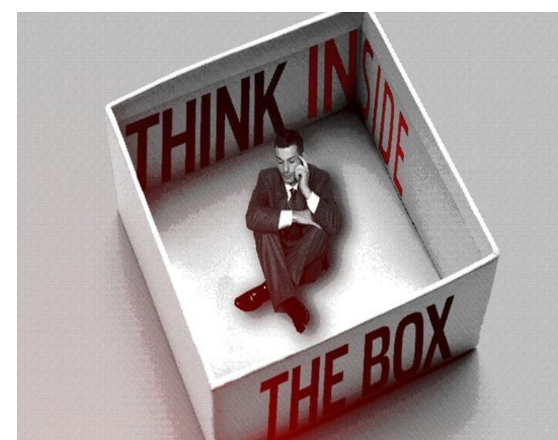
$$\det[1 - K(E_{\rm cm})B(L\mathbf{q}_{\rm cm})] + \mathrm{O}(\mathrm{e}^{-ML}) = 0$$

$$S = (1 - iK)^{-1}(1 + iK)$$

M. Lüscher, *Nucl. Phys.* **B354** (1991) 531

- Determinant over total angular momentum, channel, and total spin

- Algorithmic advances in quark propagation lead to improved statistical precision in $E_{\rm cm}$



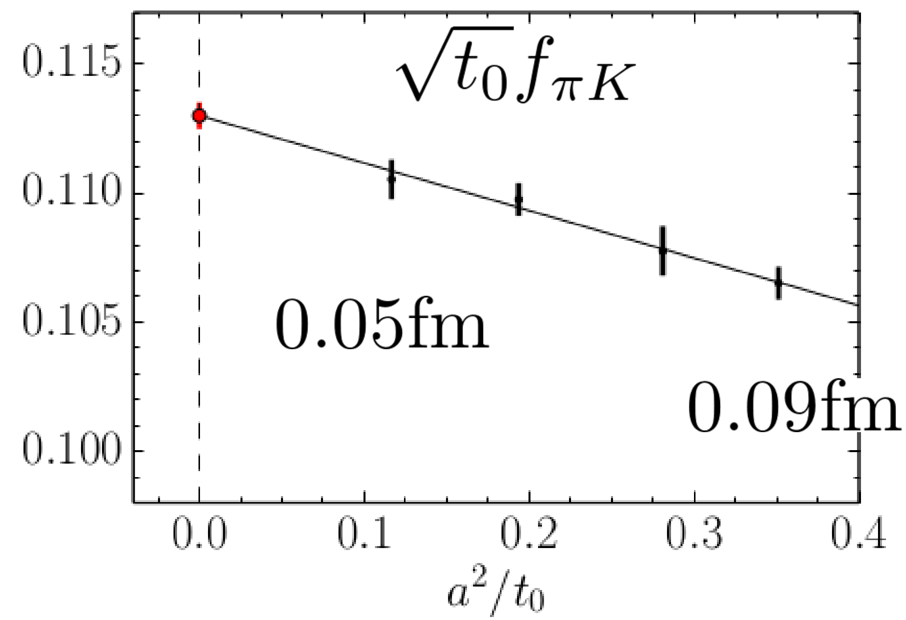
C. Morningstar, JB, J. Foley, K. Juge, D. Lenkner, M. Peardon, C. H. Wong, *Phys. Rev.* **D83** (2011) 114505;
M. Peardon, JB, J. Foley, C. Morningstar, J. Dudek, R. Edwards, B. Joo, H.W. Lin, D. Richards, K. Juge, *Phys. Rev.* **D80** (2009) 054506

Systematic errors in lattice energies

In order to provide QCD results, systematics must be assessed:

- Lattice Spacing:

$$E_{\mathrm{CM}}^{\mathrm{lat}} = E_{\mathrm{CM}}^{\mathrm{QCD}} + \mathrm{O}(a^2)$$

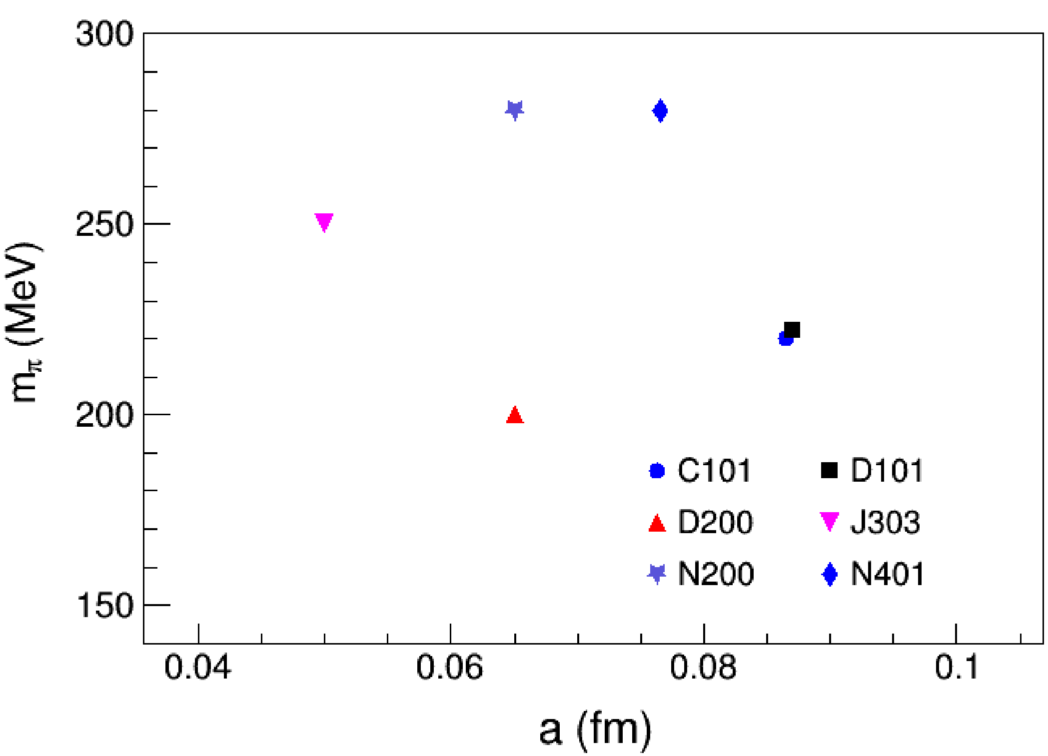


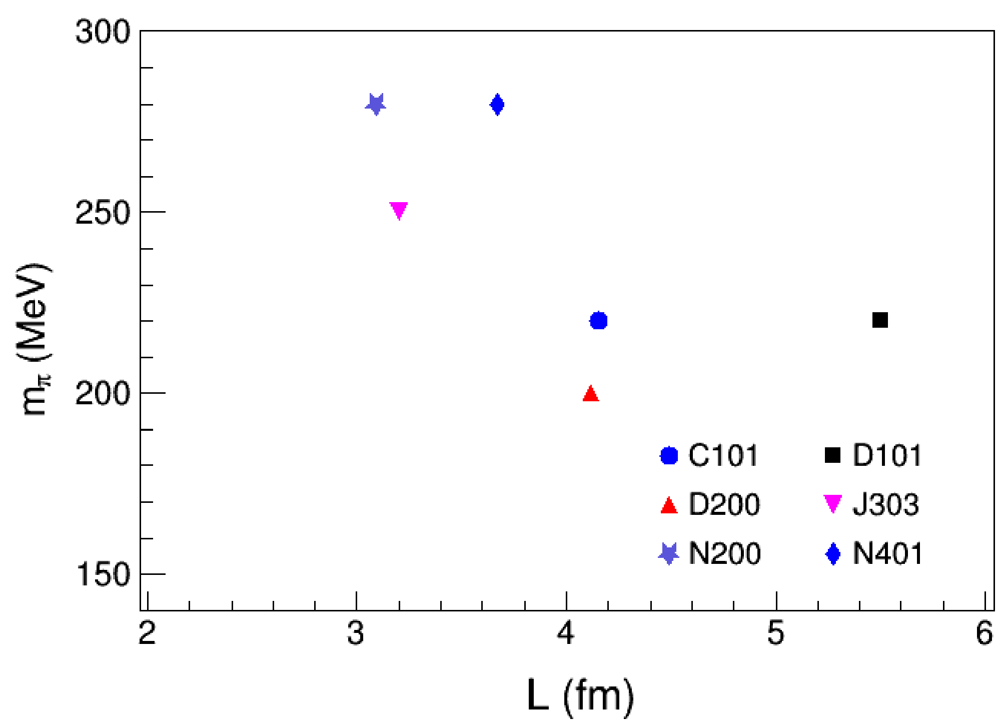
M. Bruno, T. Korzec, S. Schaefer, *Phys. Rev.* **D95** 074504 (2017)

- (Residual) Finite volume effects
- Unphysical quark masses (dependence on $m_{u,d}$, m_s also interesting)
- Higher partial waves in determinant: C. Morningstar, JB, B. Singha, R. Brett, J. Fallica, A. Hanlon, B. Hörz, Nucl. Phys. B924 (2017) 477

Many ensembles required

- Coordinated Lattice Simulations (CLS): broad EU effort
- 4 lattice spacings $a \geq 0.05\mathrm{fm}$, pion masses $m_\pi \gtrsim 190\mathrm{MeV}$
- Two $N_\mathrm{f} = 2+1$ chiral limits: $m_s = const.$ $\mathrm{Tr}M = const.$



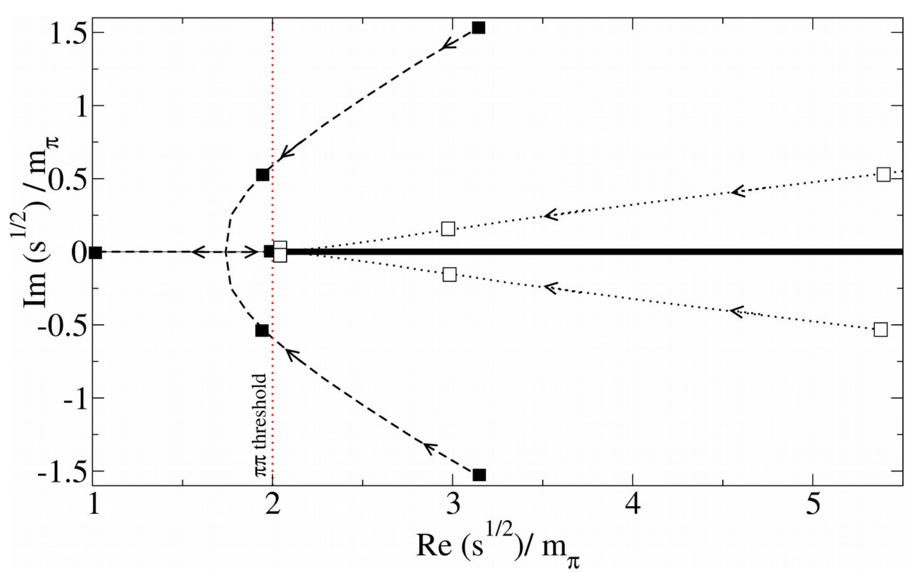


Elastic isovector pion-pion scattering

- Identical spinless particles
- Well-understood low-lying resonance:

$$\rho(770),\ (I^G)J^P = (1^+)1^-$$

- Pheno. interest: test of chiral effective theory and extensions



C. Hanhart, J.R. Pelaez, G. Rios
Phys.Lett. B739 (2014) 375-382

Symmetries of the finite-volume

- More total momenta => more amplitude points
- Finite volume symmetry groups $O_h^D,\ C_{4v}^D,\ C_{2v}^D,\ C_{3v}^D$ for (resp.) $\frac{L}{2\pi}\mathbf{P}_{\rm tot} = (0,0,0),\ (0,0,n),\ (0,n,n),\ (n,n,n)$
- Relevant irreps:

mom.	irrep	partial waves
$(0,0,0)$	T_{1u}^+	$1, 3, 5^2, \ldots$
$(0,0,n)$	A_1^+	$1, 3, 5^2, \ldots$
	E^+	$1, 3^2, 5^3, \ldots$
$(0,n,n)$	A_1^+	$1, 3^2, 5^3, \ldots$
	B_1^+	$1, 3^2, 5^3, \ldots$
	B_2^+	$1, 3^2, 5^3, \ldots$
(n,n,n)	A_1^+	$1, 3^2, 5^2, \ldots$
	E^+	$1, 3^2, 5^4, \ldots$

Elastic pion-pion workflow

- Determine several $E_{\mathrm{cm}} < 4m_\pi$ in each irrep.
- Neglect $\ell \geq 3$. Each energy gives

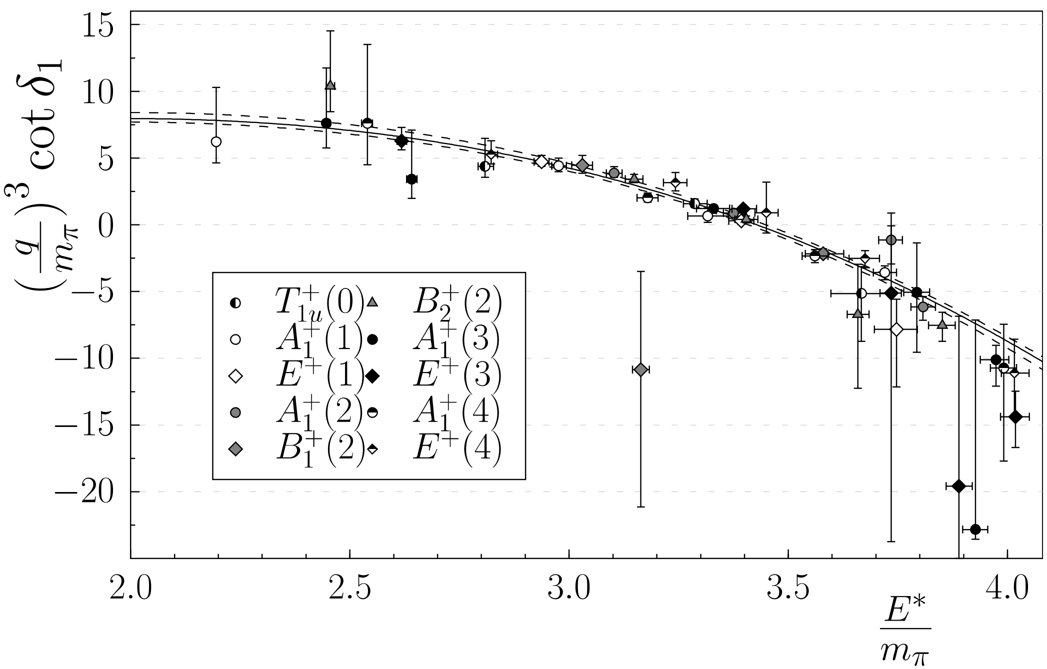
$$\hat{K}_{11}^{-1} = q_{\mathrm{cm}}^3 K_{11}^{-1} = q_{\mathrm{cm}}^3 \cot \delta_1(E_{\mathrm{cm}})$$

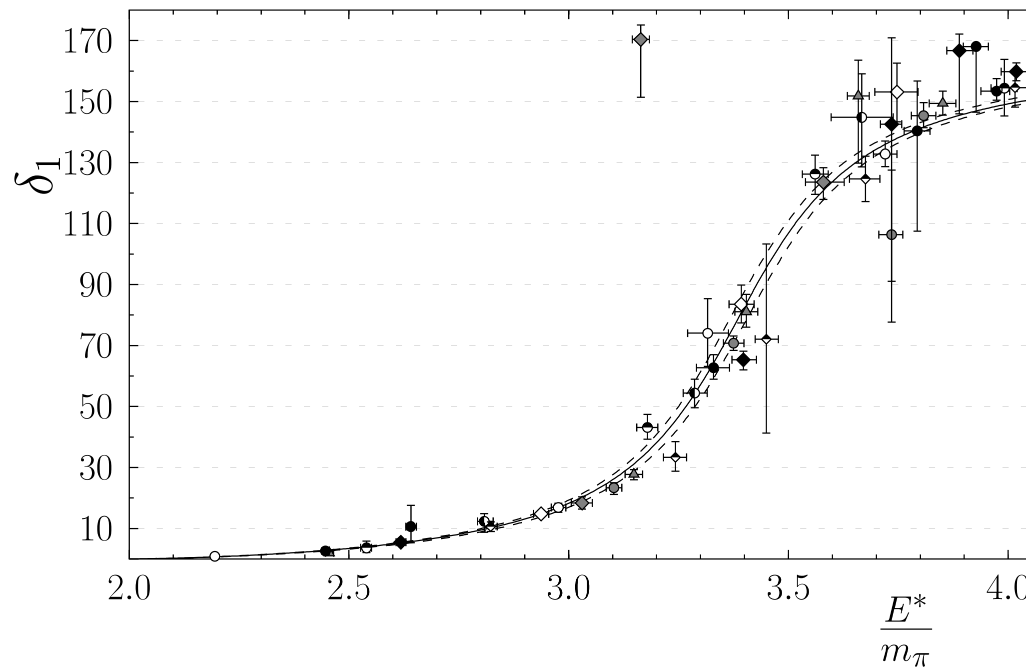
- Like experiment, fit points to functional form:
  Relativistic Breit-Wigner

$$\left[\frac{q_{\mathrm{cm}}}{m_\pi}\right]^3 \cot \delta_1(E_{\mathrm{cm}}) = \left(\frac{m_\rho^2}{m_\pi^2} - \frac{E_{\mathrm{cm}}^2}{m_\pi^2}\right) \frac{6\pi E_{\mathrm{cm}}}{g_{\rho\pi\pi}^2 m_\pi}$$

Isovector *p*-wave results: D101

$$(L = 5.53\mathrm{fm}, a = 0.086\mathrm{fm},\ m_\pi = 220\mathrm{MeV})$$





$$\frac{m_\rho}{m_\pi} = 3.42(2), \qquad g_{\rho\pi\pi} = 6.05(11), \qquad \frac{\chi^2}{d.o.f} = 0.9$$

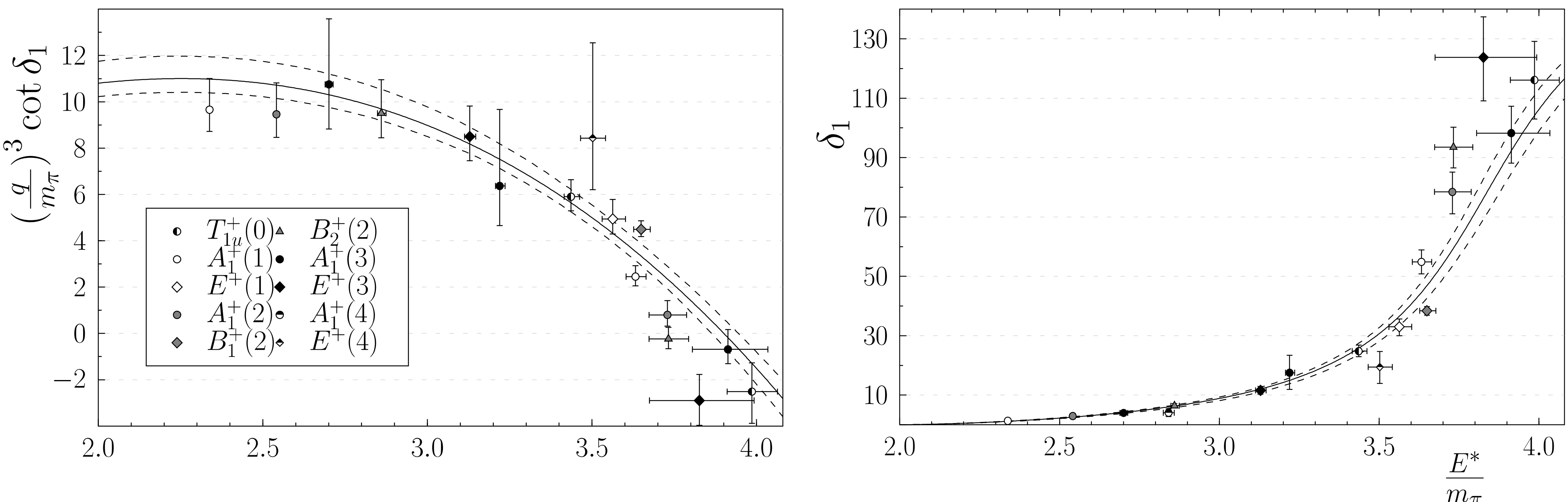
$$m_\rho = 763(9)\mathrm{MeV}$$

B. Hörz, *Ph.D. thesis*; B. Hörz, JB, C. Andersen, C. Morningstar, *in prep.*

Scale determination/uncertainties from M. Bruno, T. Korzec, S. Schaefer, *Phys. Rev.* **D95** 074504 (2017)

Isovector p-wave results: D200

$(L = 4.16\mathrm{fm}, a = 0.065\mathrm{fm},\ m_\pi = 200\mathrm{MeV})$



$$\frac{m_\rho}{m_\pi} = 3.901(41), \qquad g_{\rho\pi\pi} = 6.25(25), \qquad \frac{\chi^2}{d.o.f} = 1.34$$

$$m_\rho = 780(8)(8)\mathrm{MeV}$$

Higher partial waves

- Exhaustive determination of *B*-matrix elements

C. Morningstar, JB, B. Singha, R. Brett, J. Fallica, A. Hanlon, B. Hörz, Nucl. Phys. B924 (2017) 477

- All partial waves $\ell \leq 6$, all total spin $s \leq 7/2$, all irreps.

- Published C++ code for evaluation. Example *B*-matrix element:

$$B^{A_1,\mathrm{oa}}(\ell_1 = \ell_2 = 6, n_1 = n_2 = 1) = R_{00} - \frac{2\sqrt{5}}{55}R_{20} - \frac{96}{187}R_{40} - \frac{80\sqrt{13}}{3553}R_{60} + \frac{445\sqrt{17}}{3553}R_{80} + \frac{15\sqrt{24310}}{3553}R_{88} - \frac{498\sqrt{21}}{7429}R_{10,0} + \frac{6\sqrt{510510}}{7429}R_{10,8} + \frac{2178}{37145}R_{12,0} + \frac{66\sqrt{277134}}{37145}R_{12,8}$$

Higher partial waves

- Fit results w/o f-wave contribution: (aniso. data)

$$\frac{m_\rho}{m_\pi} = 3.354(24), \qquad g_{\rho\pi\pi} = 6.01(26), \qquad \frac{\chi^2}{d.o.f} = 1.02$$

- Fit results with f-wave contribution:

$$\frac{m_\rho}{m_\pi} = 3.353(24), \qquad g_{\rho\pi\pi} = 6.00(26),$$

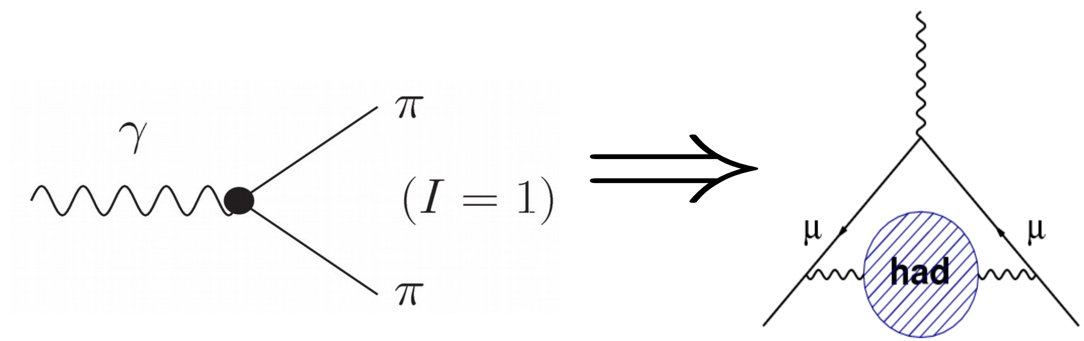
$$m_\pi^7 a_3 = -0.0003(24), \qquad \frac{\chi^2}{d.o.f} = 1.02$$

- Pheno. determination: $m_\pi^7 a_3 = 6.3(4) \times 10^{-5}$

Pelaez, Yndurian ‘05

Timelike pion form factor

- Low-energy hadron vacuum polarization $\Pi(q^2)$: important theoretical uncertainty in $(g-2)_\mu$



- Optical Theorem:

$$\operatorname{Im}\Pi(s) = \frac{\alpha(s)}{3} R(s)$$

$$R(s) = \sigma_{\mathrm{tot}}(\mathrm{e}^+\mathrm{e}^- \to \ \mathrm{hadrons}) \left(\frac{4\pi\alpha(s)^2}{3s}\right)^{-1}$$

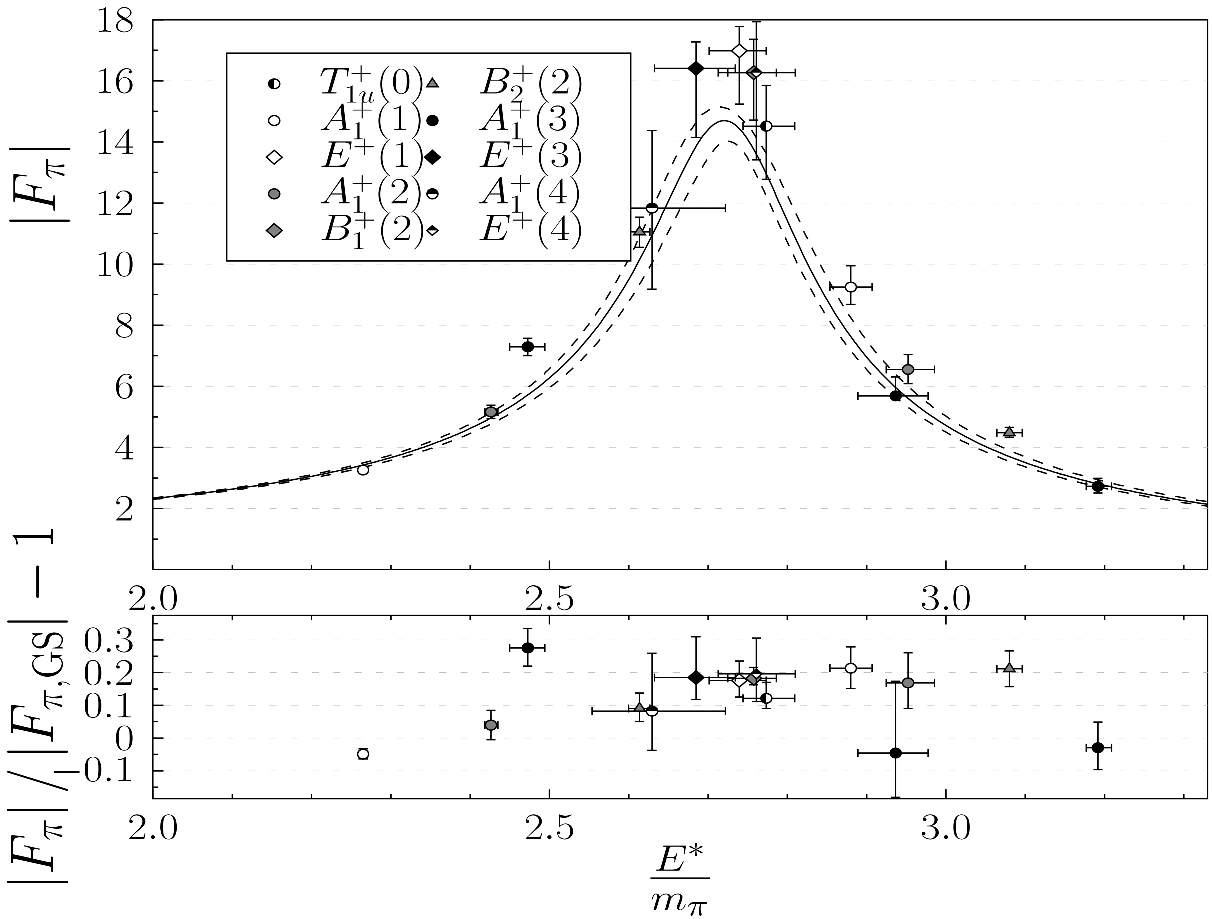
- At low energies, given by the time-like pion form-factor

$$R(s) = \frac{1}{4}\left(1 - \frac{4m_\pi^2}{s}\right)^{\frac{3}{2}} |F_\pi(s)|^2, \ 4m_\pi^2 < s < 9m_\pi^2$$

Jegerlehner and Nyffeler '09; Meyer '11;
Feng, et al. `15

Form factor results: N200

$(L = 3.12\text{fm}, a = 0.065\text{fm},\ m_\pi = 280\text{MeV})$

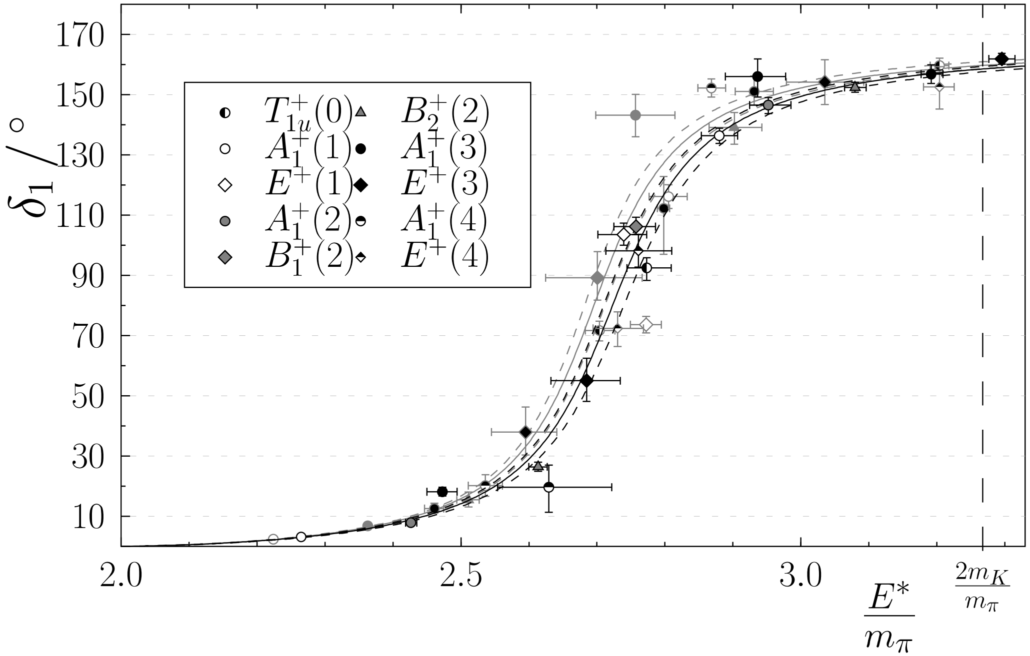


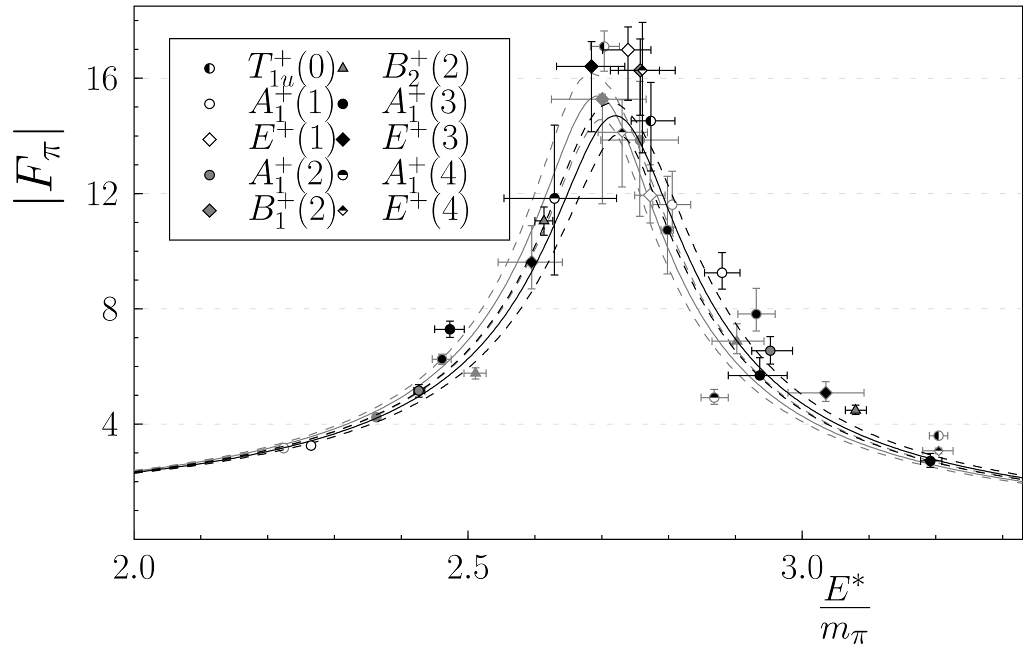
'Fit' is the Gounaris-Sakurai parametrization

Meyer, `11; Feng et al `15;

$$|F_\pi(E_{\rm cm})|^2 = \frac{2\pi E_{\rm cm}}{2L^3p^5} g(\gamma) \left(q\phi'(q) + p\frac{\partial\delta_1}{\partial p} \right) |\langle 0|\hat{j}_{\rm em}|\pi(\vec{p}_1)\pi(\vec{p}_2)\rangle|^2$$

Finite volume/lattice spacing

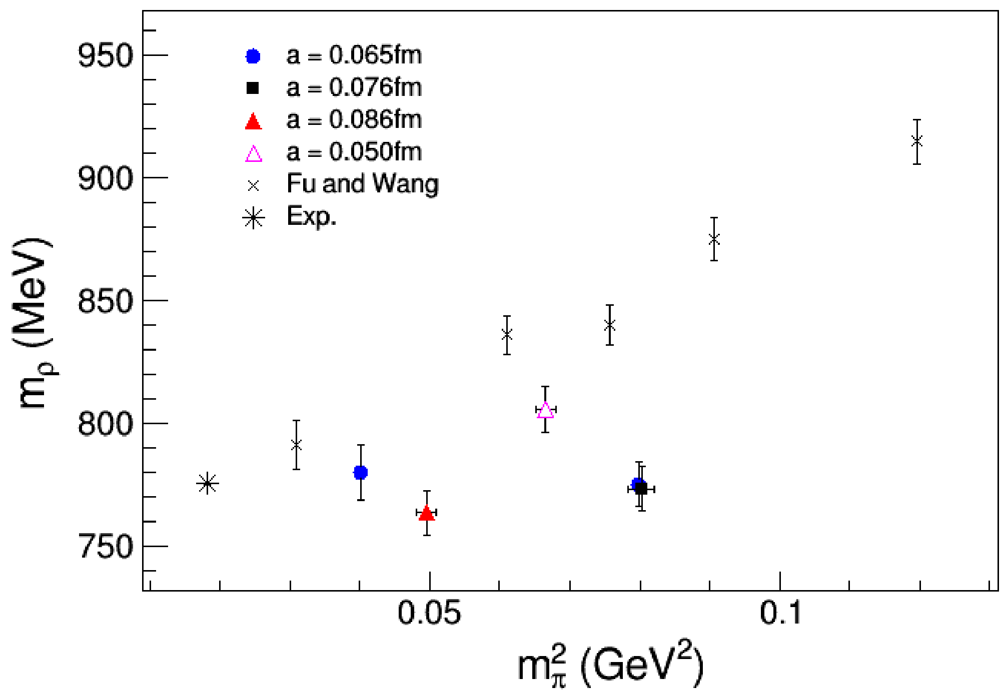


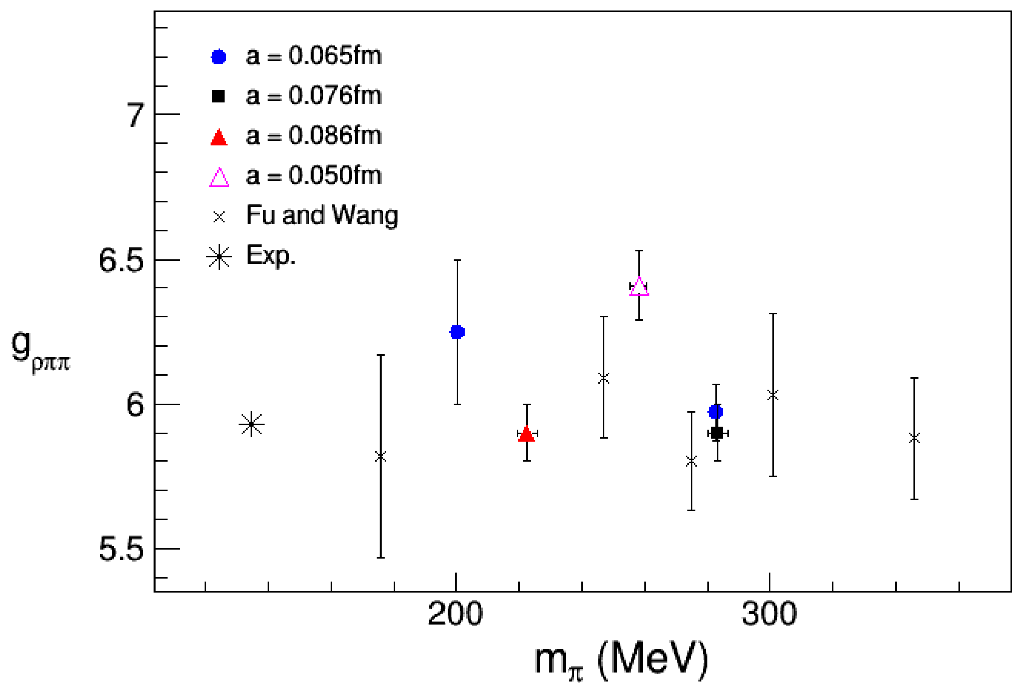


- Dark points: N200 $(L = 3.12\text{fm}, a = 0.065\text{fm},\ m_\pi = 280\text{MeV})$
- Gray points: N401 $(L = 3.65\text{fm}, a = 0.076\text{fm},\ m_\pi = 280\text{MeV})$

- Finite volume and cutoff effects not visible with our current statistics.

Mass/coupling summary





- Our 0.05fm point is preliminary (incomplete analysis)
- Gray points from: Z. Fu, L. Wang, Phys. Rev. D94 (2016) 034505
  - 3-flavor MILC ensembles, scale well-determined.
  - Different chiral trajectory: $m_s = const.$

Isodoublet kaon-pion scattering

- Two low-lying $I = 1/2$ resonances (with strangeness = 1):

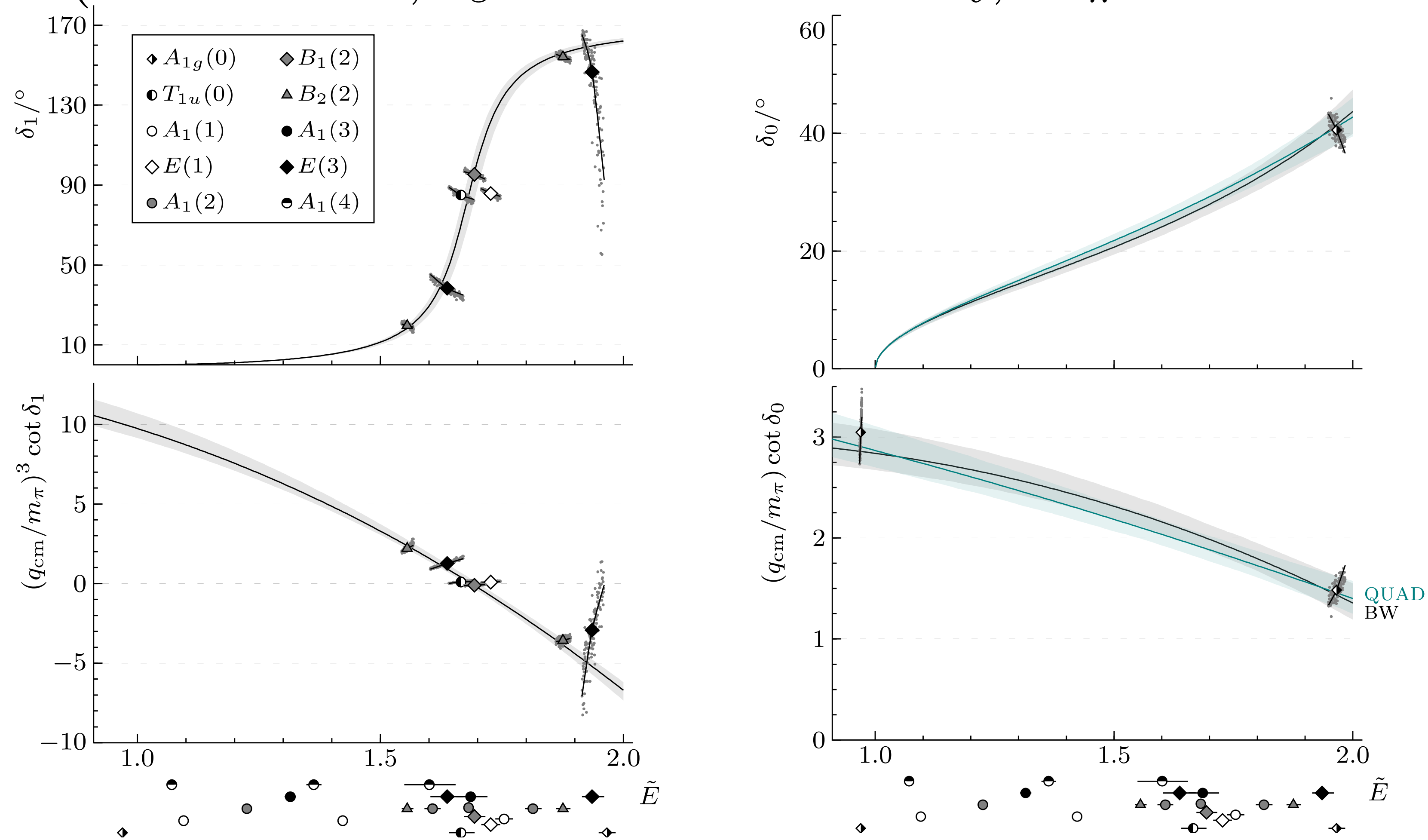
$$K^*(892) : \; J^P = 1^- \qquad\qquad K_0^*(800) : \; J^P = 0^+$$

- Non-identical particles: more partial wave mixing!
- No amplitude points for each energy: must fit simultaneously s-wave, p-wave (d-wave?).
- $K^*(892)$ important for BSM tests; nature of $K^*_0(800)$ unclear.

mom.	irrep	ℓ
0	A_{1g}	$0, 4, \ldots$
	T_{1u}	$1, 3, \ldots$
1	A_1	$0, 1, 2, \ldots$
	E	$1, 2, 3, \ldots$
2	A_1	$0, 1, 2, \ldots$
	B_1	$1, 2, 3, \ldots$
	B_2	$1, 2, 3, \ldots$
3	A_1	$0, 1, 2, \ldots$
	E	$1, 2, 3, \ldots$
4	A_1	$0, 1, 2, \ldots$

Results: s- and p-wave

$$(L = 3.84\text{fm}, a_s = 0.12\text{fm} = 3.5a_t,\ m_\pi = 240\text{MeV})$$



$$\frac{m_{K^*}}{m_\pi} = 3.808(18), \qquad g_{K^*K\pi} = 5.33(20), \qquad \frac{\chi^2}{d.o.f} = 1.42$$

A. Hanlon, *PhD thesis,* 2017; R. Brett, JB, J. Fallica, A. Hanlon, B. Hoerz, C. Morningstar, arXiv:1802.03100

Meson-baryon scattering

- Additional complication: non-zero spin!
- Signal-to-noise problem: difficult to attain statistical precision

- Examples:
  - Delta(1232):
    - benchmark baryon resonance calculation.
    - D(1232) form-factors of pheno. interest for DUNE, JLAB.

  - Lambda(1405):
    - Coupled channels: $\Sigma\pi,\ KN,\ \Lambda\eta$
    - Nature of pole(s) unsettled, relevant for nuclear matter.

Delta(1232) setup

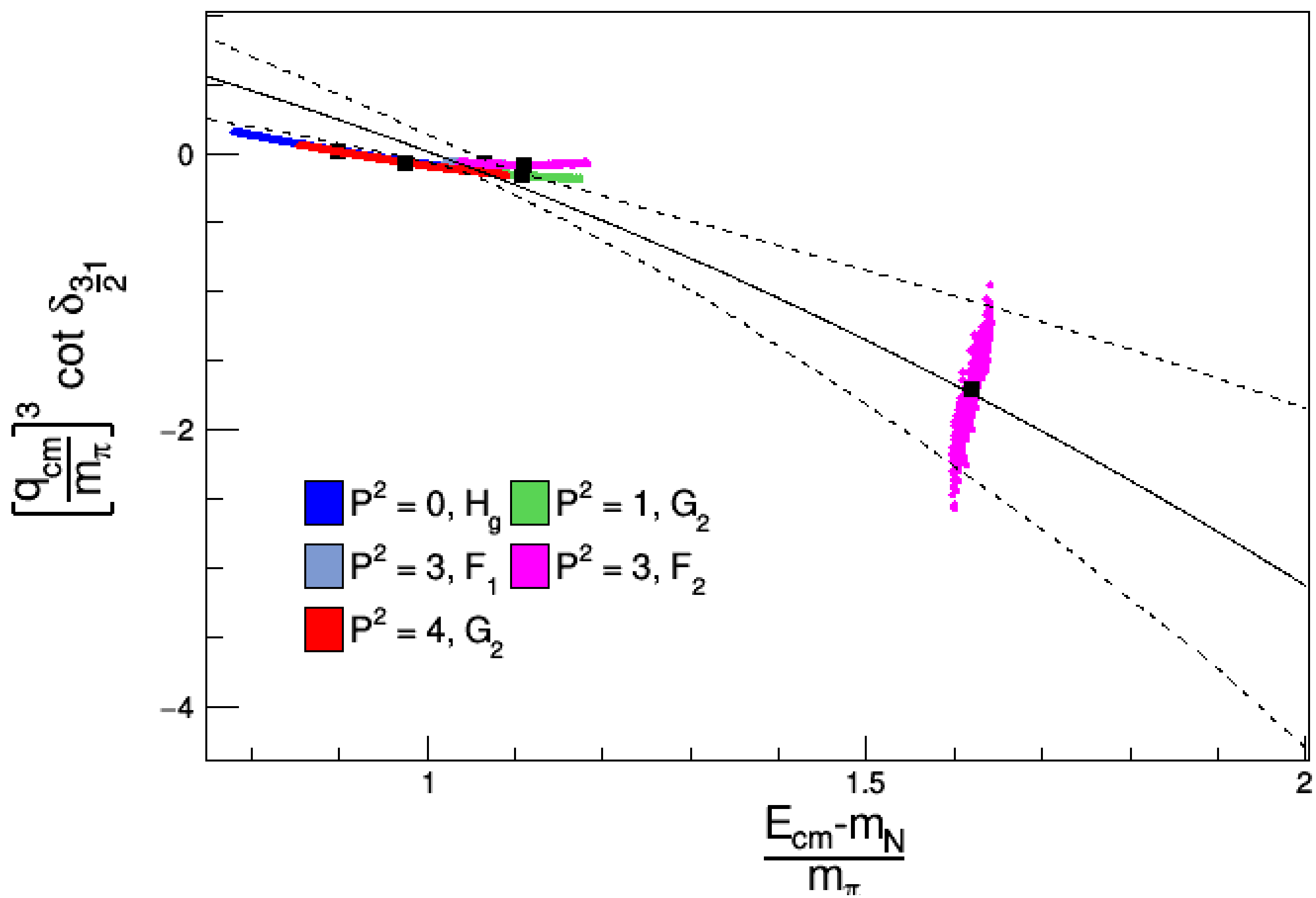
- Choose $I = 3/2$ irreps where $\ell(J^P) = 1(3/2^+)$ is the lowest partial wave

mom.	irrep	$\ell(J^p)$
$(0,0,0)$	H_g	$1(3/2^+), 3(5/2^+), \ldots$
	H_u	$2(3/2^-), 2(5/2^-), \ldots$
$(0,0,n)$	G_2	$1(3/2^+), 2(3/2^-), 2(5/2^-), \ldots$
(n,n,n)	F_1	$1(3/2^+), 2(3/2^-), 2(5/2^-), \ldots$
	F_2	$1(3/2^+), 2(3/2^-), 2(5/2^-), \ldots$

- Neglecting d-wave Delta(1700), relying on orbital angular momentum threshold suppression of d-wave.

Delta(1232) first results

$$(L = 3.6\text{fm}, a = 0.075\text{fm},\ m_\pi = 280\text{MeV})$$



$$\frac{m_\Delta}{m_\pi} = 4.738(47), \qquad g_{\Delta N\pi} = 19.0(4.7), \qquad \frac{\chi^2}{d.o.f} = 1.11$$

C. W. Andersen, JB, B. Hoerz, C. Morningstar, Phys. Rev. D97 (2018) 014506

Lambda(1405) setup

- In each irrep, need interpolators for $\Lambda,\ \Sigma-\pi,\ \bar{K}-N,\ \Lambda-\eta$

- Focus on (strangeness = -1) irreps containing $I(J^p)=0(1/2^-)$

mom.	irrep	$\ell(J^p)$
$(0,0,0)$	G_{1g}	$1(1/2^+), 3(7/2^+), \ldots$
	G_{1u}	$0(1/2^-), 4(7/2^-), \ldots$
	H_u	$2(3/2^-), 2(5/2^-), \ldots$
$(0,0,n)$	G_1	$0(1/2^-), 1(1/2^+), 1(3/2^+), \ldots$
	G_2	$1(3/2^+), 2(3/2^-), 2(5/2^-), \ldots$
$(0,n,n)$	G	$0(1/2^-), 1(1/2^+), 1(3/2^+), \ldots$
(n,n,n)	G	$0(1/2^-), 1(1/2^+), 1(3/2^+), \ldots$
	F_1	$1(3/2^+), 2(3/2^-), 2(5/2^-), \ldots$
	F_2	$1(3/2^+), 2(3/2^-), 2(5/2^-), \ldots$

- Resonances: $\Lambda(1405) : 1/2^-,\ \Lambda(1520) : 3/2^-,\ \Lambda(1600) : 1/2^+$

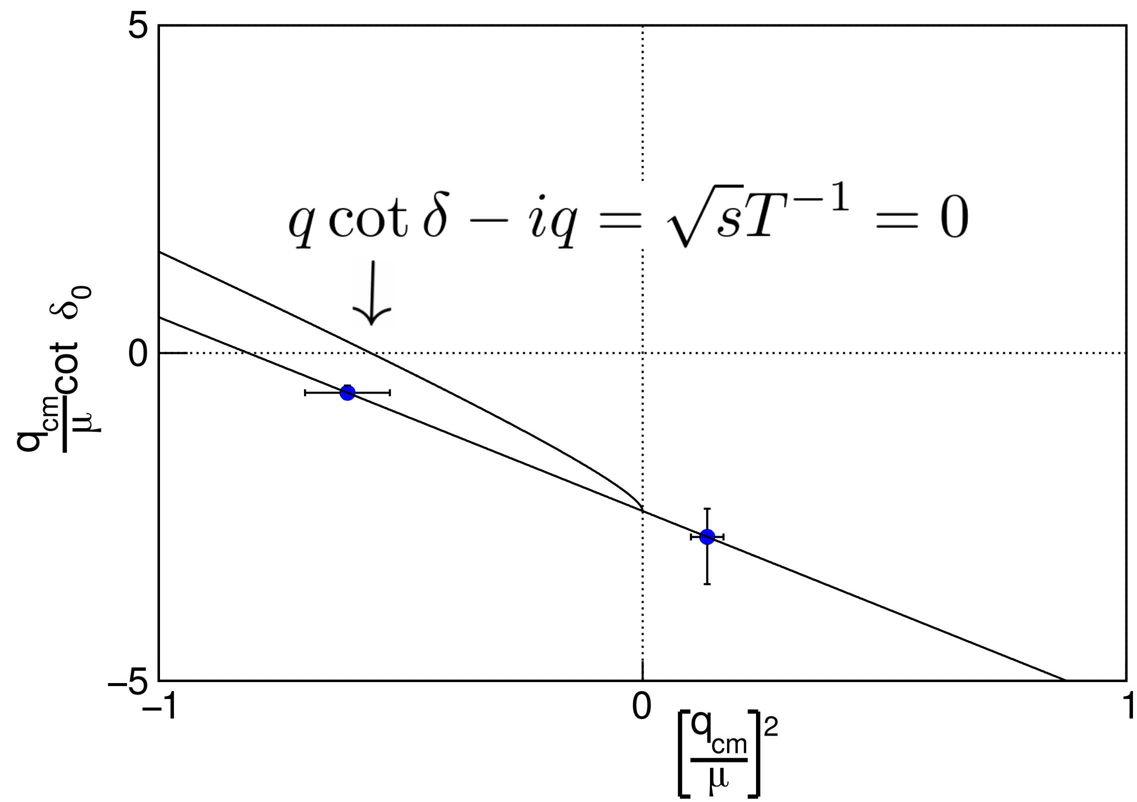
Preliminary elastic results

$$\Lambda(1405) \to \Sigma\pi \qquad (L = 3.12\text{fm}, a = 0.065\text{fm},\ m_\pi = 280\text{MeV})$$

- $G_{1u}(0)$ below inelastic threshold only.

- Fit form:

$$\frac{p_{\rm cm}}{\mu}\cot\delta_0 = \frac{1}{a_0\mu} + \frac{\mu r}{2}\left(\frac{p_{\rm cm}}{\mu}\right)^2$$



$$\frac{m_R}{\mu} = 6.143(77), \qquad \frac{1}{a\mu} = -2.41(57), \qquad \frac{\mu r}{2} = -2.9(1.1)$$

$$m_R = 1399(24)\text{MeV}$$

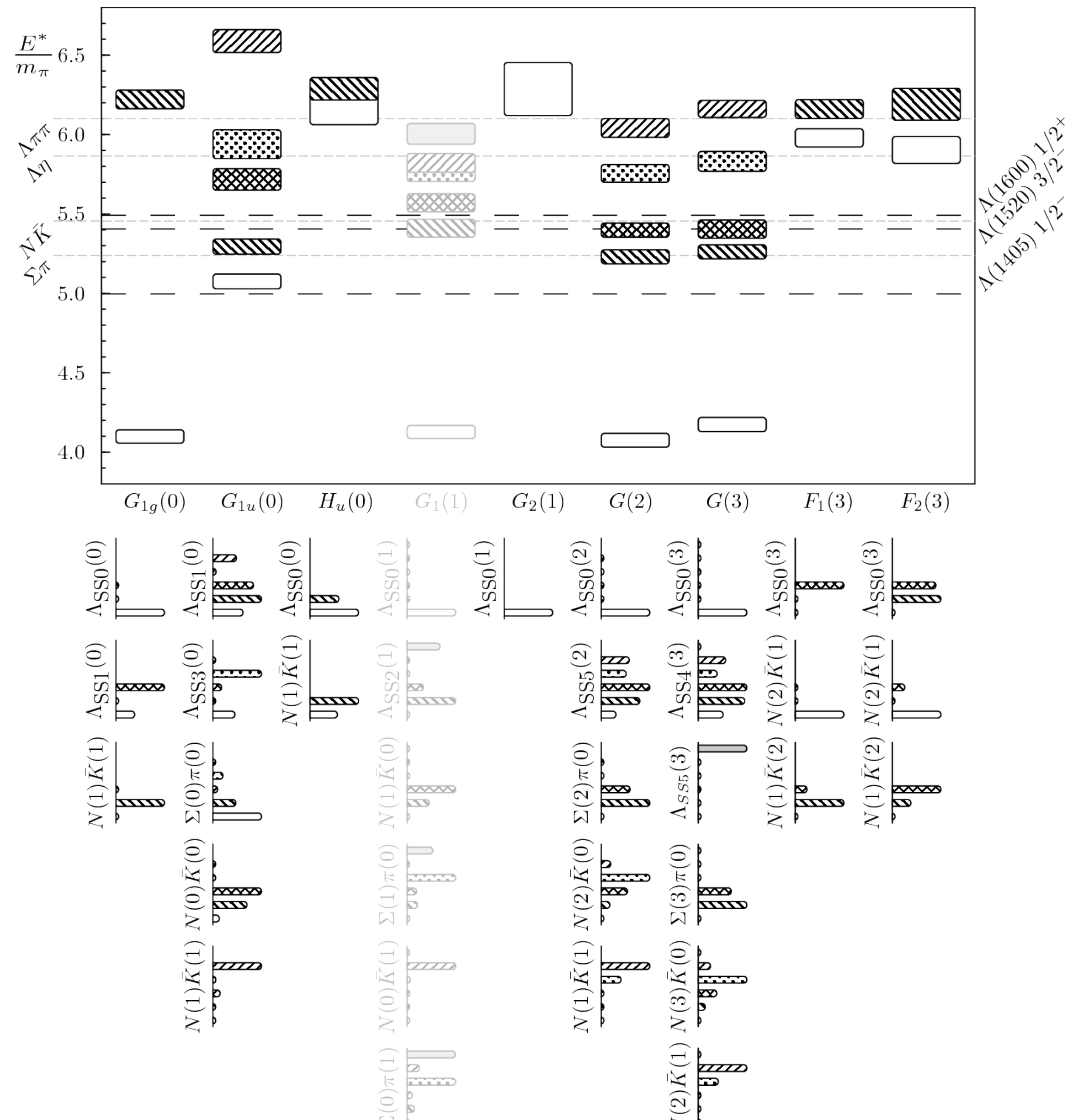
B. Hörz, C. Andersen, JB, M. Hansen, D. Mohler, C. Morningstar, H. Wittig, *in prep.*

Operator overlaps

- Qualitative information about the spectrum
- Definition:

$$A_{in} = |\langle 0|\mathcal{O}_i|n\rangle|$$

- Observations:
  - Ground state Lambda present as expected.
  - Where is Lambda(1405) in flight?
  - Where are Lambda(1520) and Lambda(1600)?



Conclusions

- Algorithmic advances enable precise finite-volume energies.
- CLS ensembles enable exploration of continuum, chiral, and infinite volume limits
- Simple resonance photoproduction amplitude: timelike pion form factor
- Cutoff, finite volume, and higher partial wave effects under control in pion-pion scattering.
- First progress in meson-baryon: Delta(1232), Lambda(1405). More data/other systems to come.