

# Excited mesons and resonances from lattice QCD – charm/charmonium

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UNIVERSITY OF  
CAMBRIDGE

had spec

# Excited lattice QCD spectroscopy

Finite-volume energy eigenstates from:

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Use many different interpolating operators



# Excited lattice QCD spectroscopy

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$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

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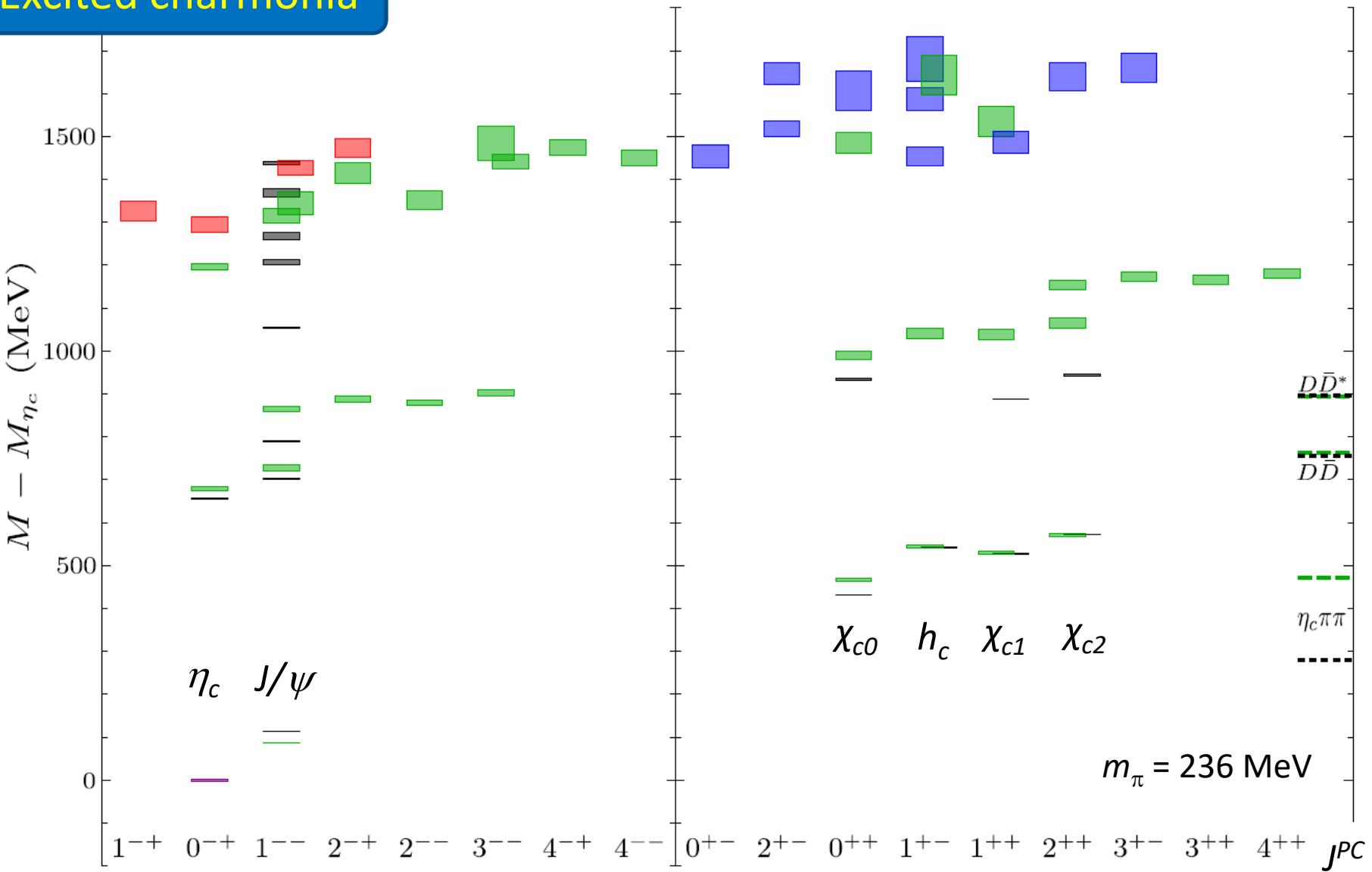
Use many different interpolating operators

Excited charmonia  
[JHEP 1612, 089 (2016)]

Large bases of fermion-bilinear  
operators  $\sim \bar{\psi} \Gamma D \dots \psi$

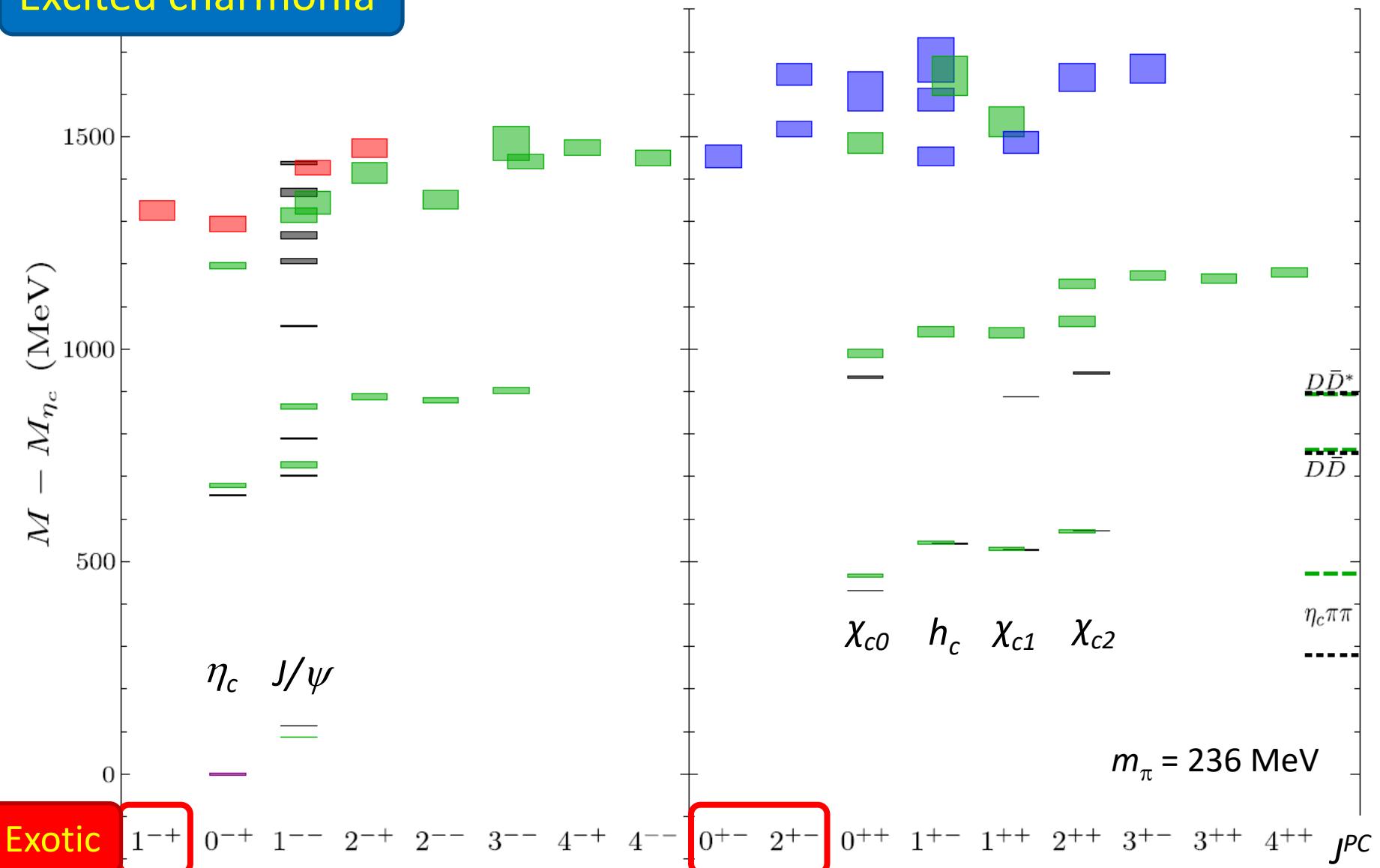


# Excited charmonia



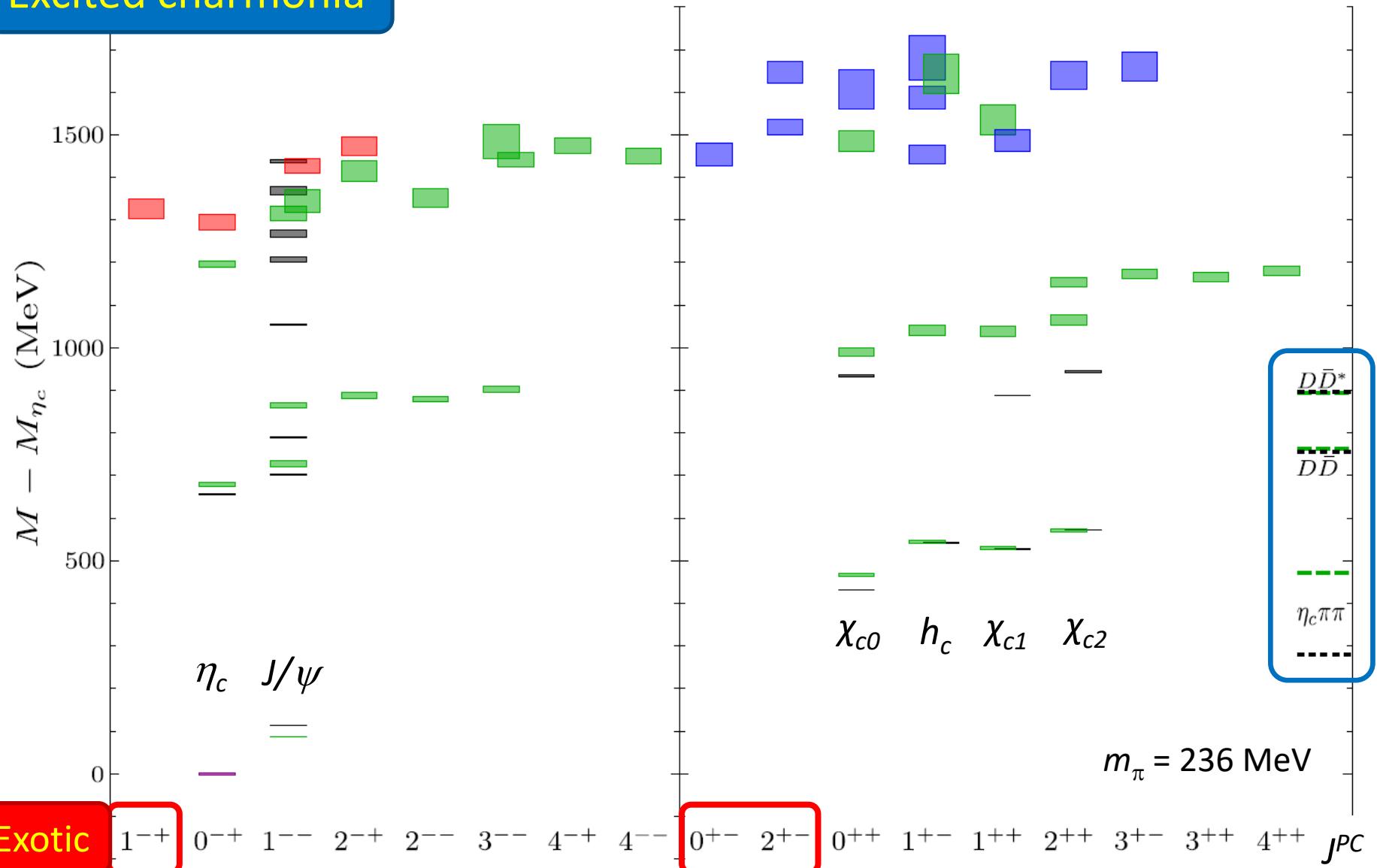
One lattice spacing and volume [Cheung *et al* (HadSpec), JHEP 1612, 089 (2016)]  
(similar pattern to older  $m_\pi = 391$  MeV, 1 lattice spacing and 3 volumes)

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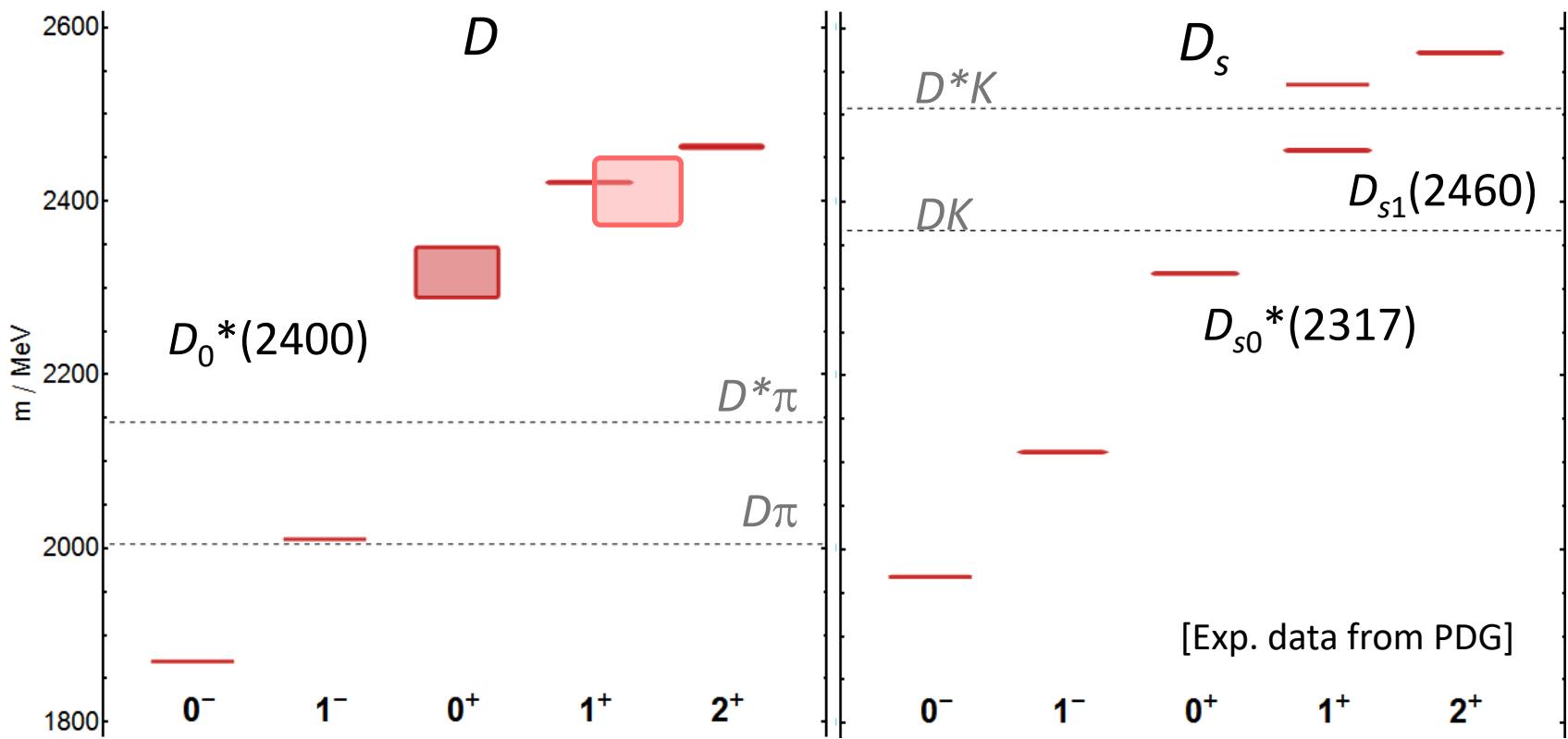
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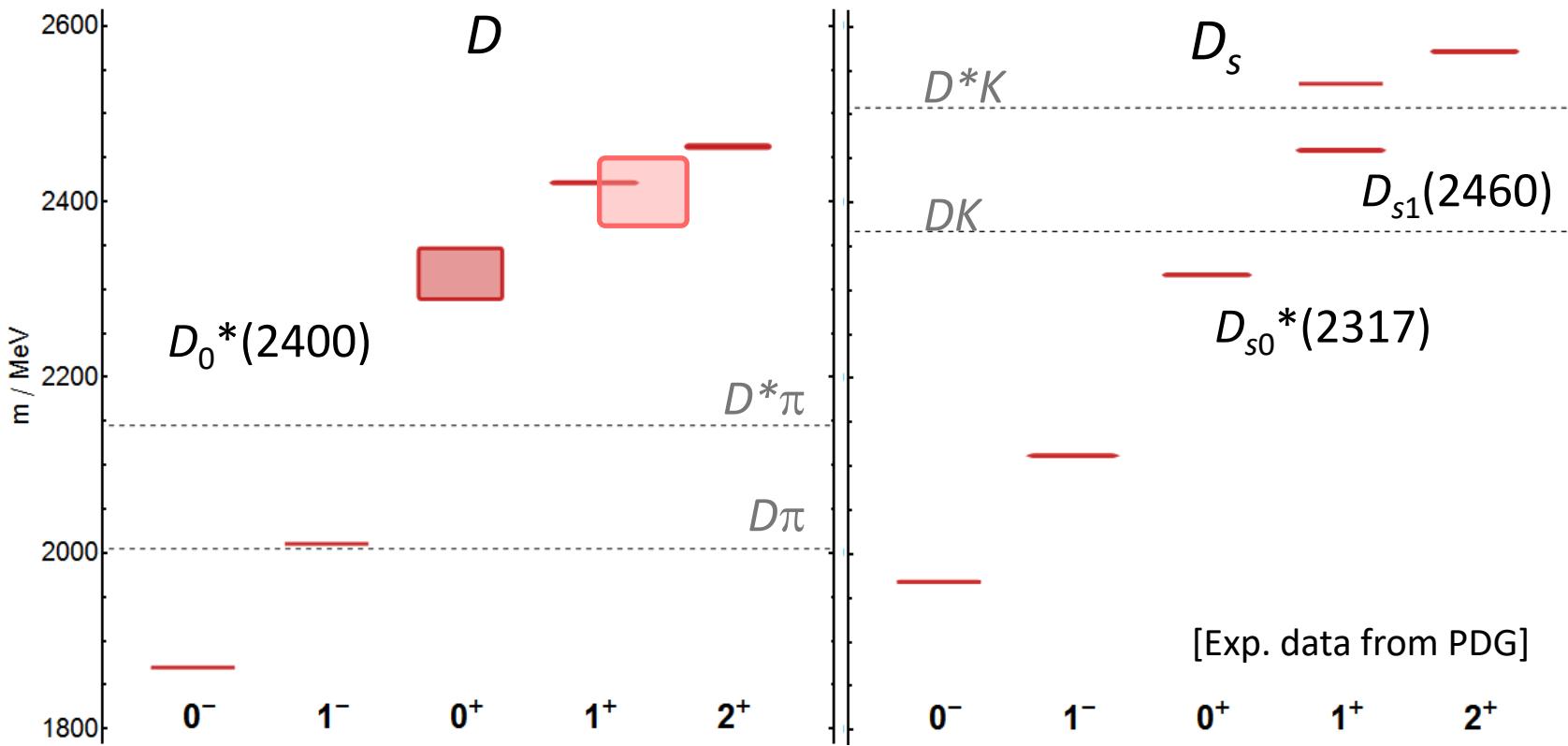


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# Charm-light ( $D$ ) and charm-strange ( $D_s$ ) mesons



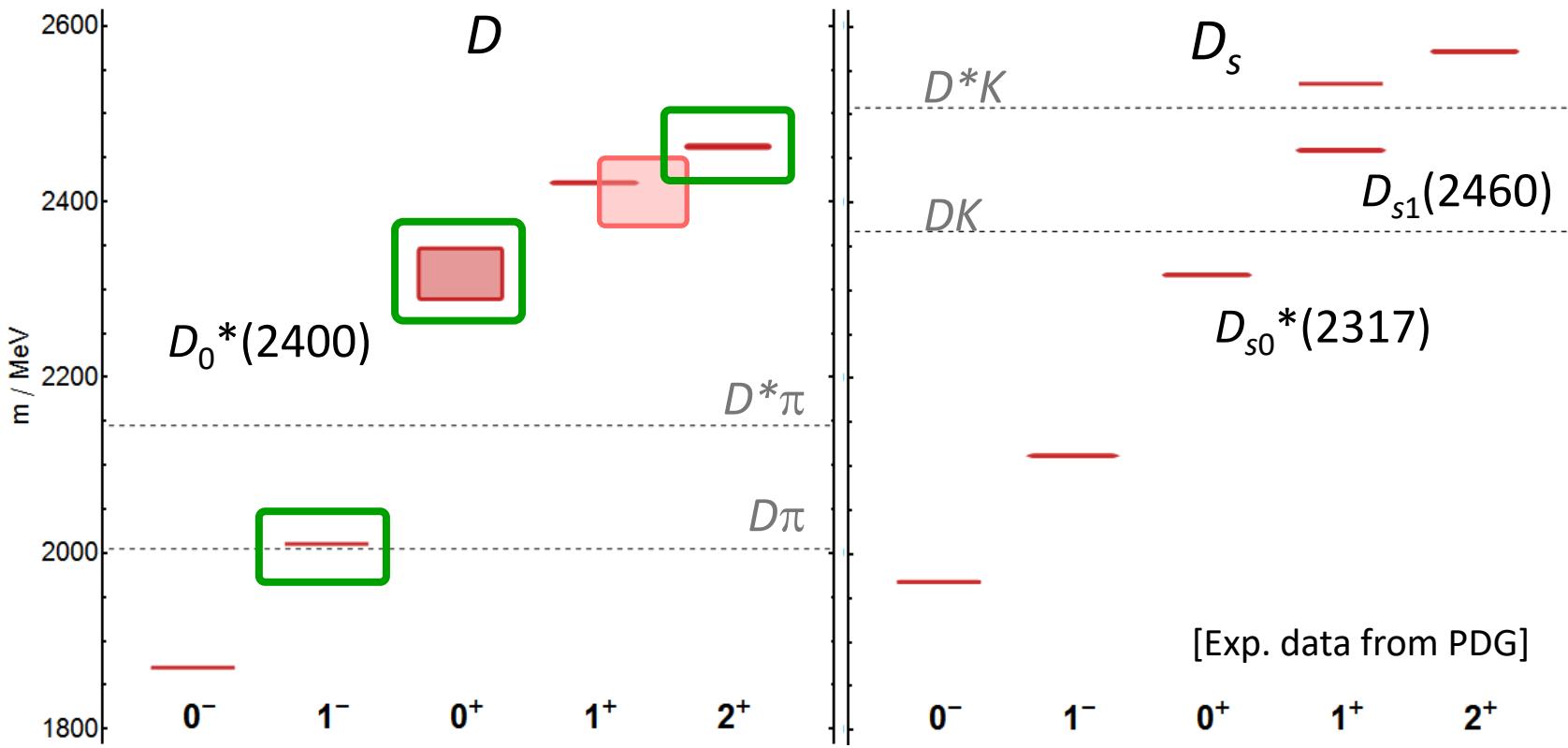
# Charm-light ( $D$ ) and charm-strange ( $D_s$ ) mesons



Some other LQCD studies:

- Mohler *et al* [PR D87, 034501 (2012)] –  $0^+ D \pi$  and  $1^+ D^* \pi$  resonances
- Mohler *et al* [PRL 111, 222001 (2013)] –  $0^+ D_{s0}(2317)$  below  $D K$  threshold
- Lang *et al* [PRD 90, 034510 (2014)] –  $0^+ D_{s0}(2317)$  and  $1^+ D_{s1}(2460), D_{s1}(2536)$
- Bali *et al* (RQCD) [PRD 96, 074501 (2017)] –  $0^+ D_{s0}(2317)$  and  $1^+ D_{s1}(2460)$

# Charm-light ( $D$ ) and charm-strange ( $D_s$ ) mesons



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- Mohler *et al* [PRL 111, 222001 (2013)] – 0<sup>+</sup>  $D_{s0}(2317)$  below  $D K$  threshold
- Lang *et al* [PRD 90, 034510 (2014)] – 0<sup>+</sup>  $D_{s0}(2317)$  and 1<sup>+</sup>  $D_{s1}(2460)$ ,  $D_{s1}(2536)$
- Bali *et al* (RQCD) [PRD 96, 074501 (2017)] – 0<sup>+</sup>  $D_{s0}(2317)$  and 1<sup>+</sup>  $D_{s1}(2460)$

$D\pi, D\eta, D_s\bar{K}$  ( $I=\frac{1}{2}$ )

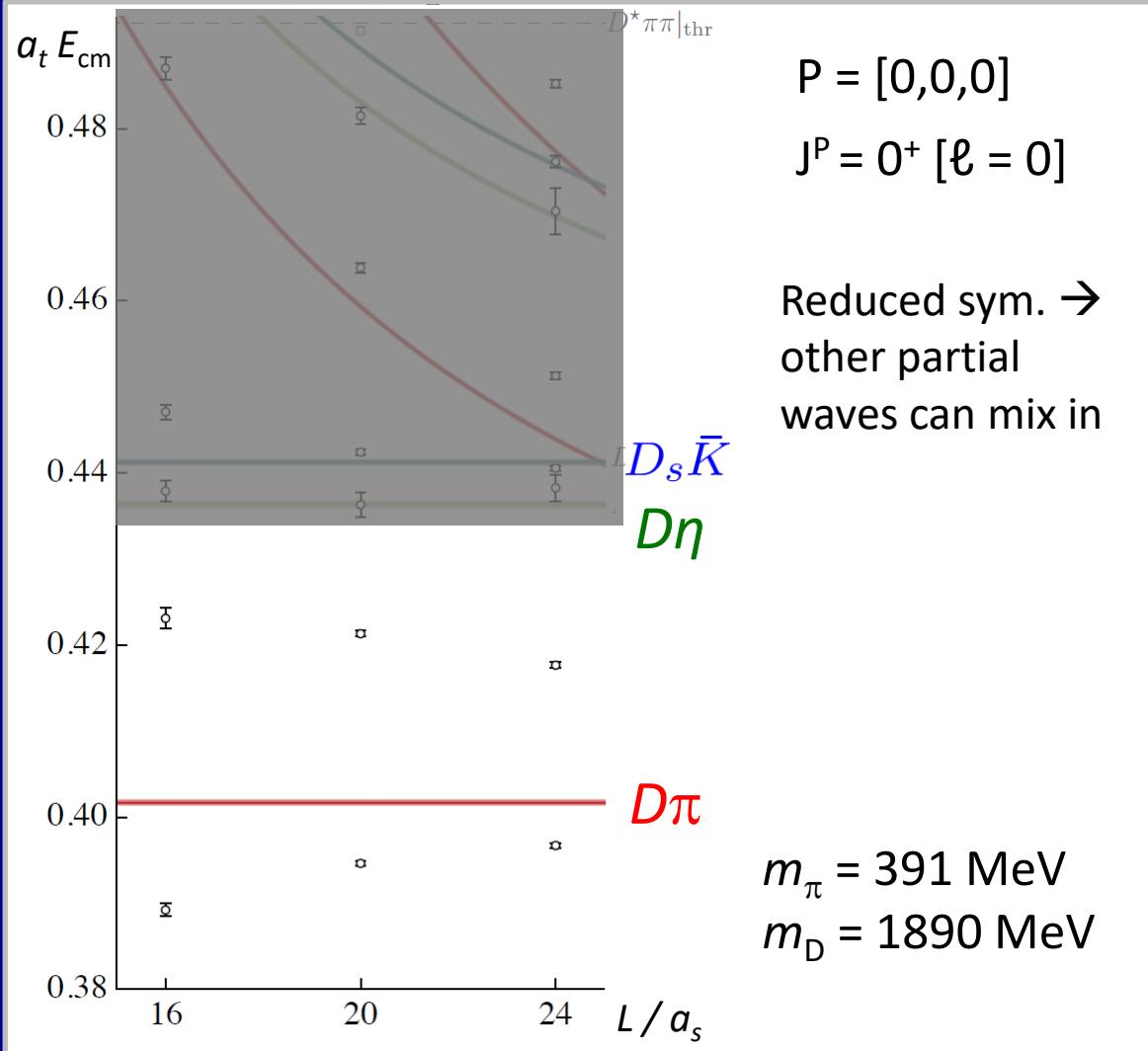
Isospin =  $\frac{1}{2}$   
Strangeness = 0  
Charm = 1

Use many different fermion-bilinear operators,

$$\sim \bar{\psi} \Gamma D \dots \psi$$

and  $D\pi, D\eta, D_s\bar{K}$   
'meson-meson'  
operators

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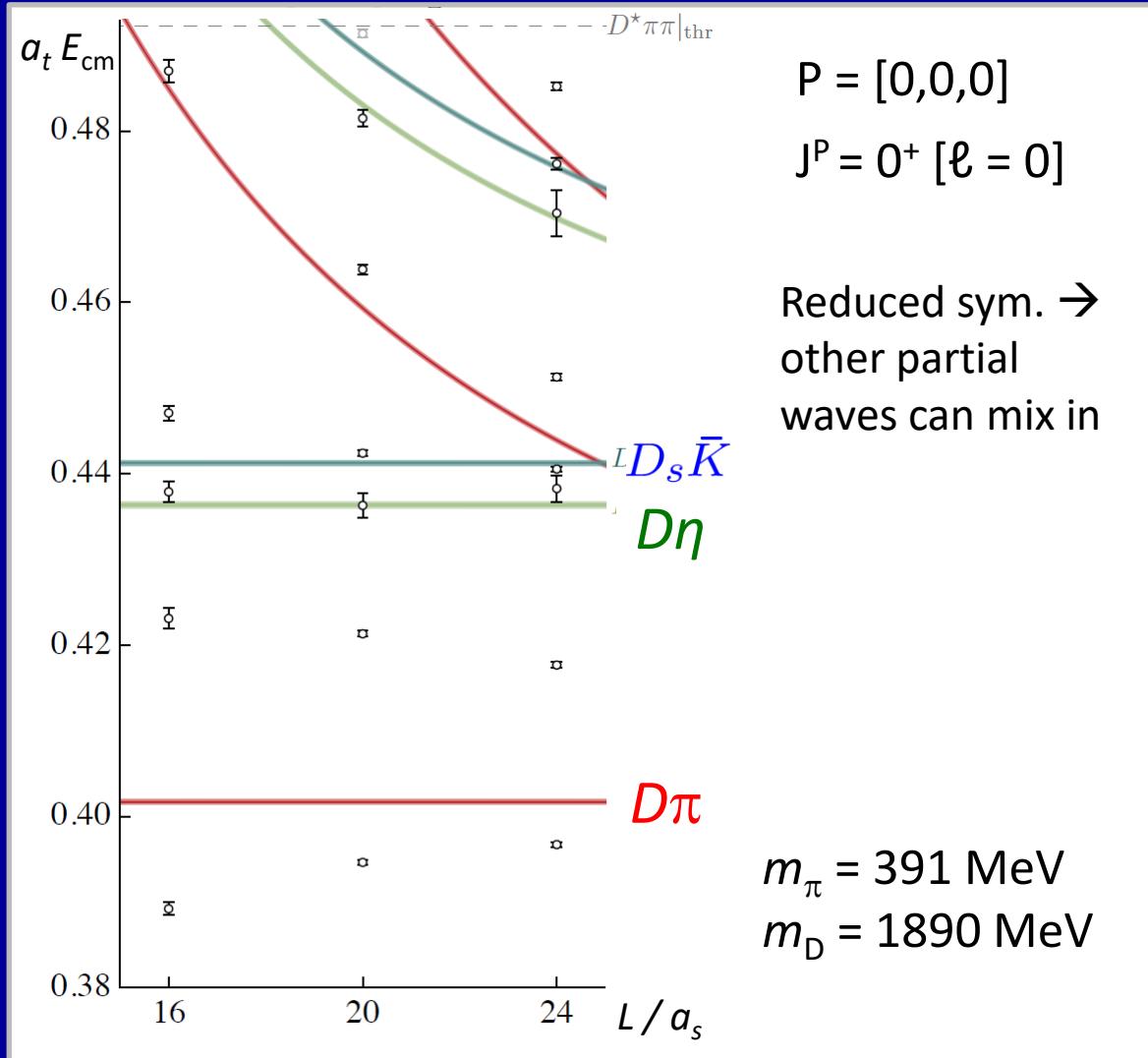
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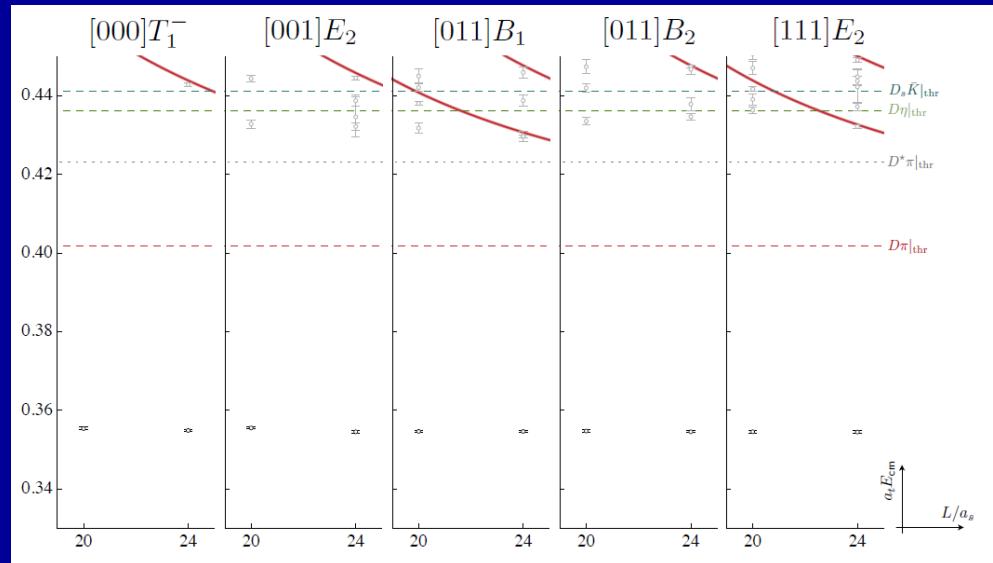
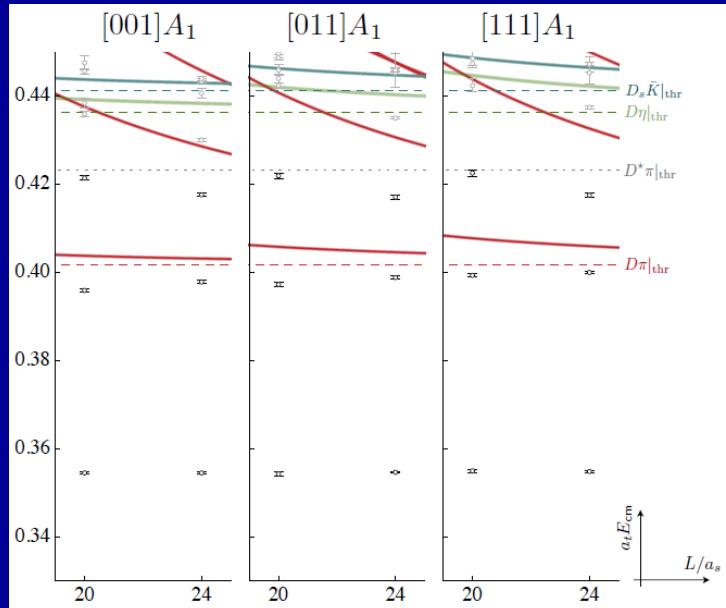
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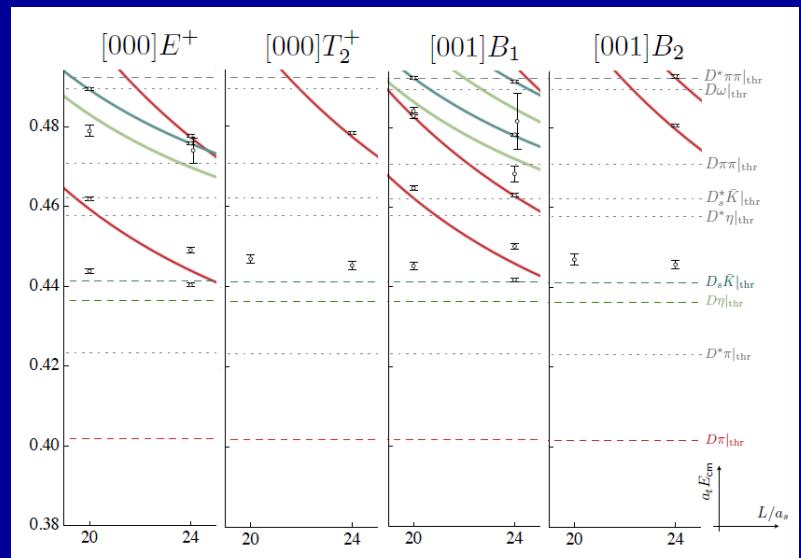
and  $D\pi, D\eta, D_s\bar{K}$   
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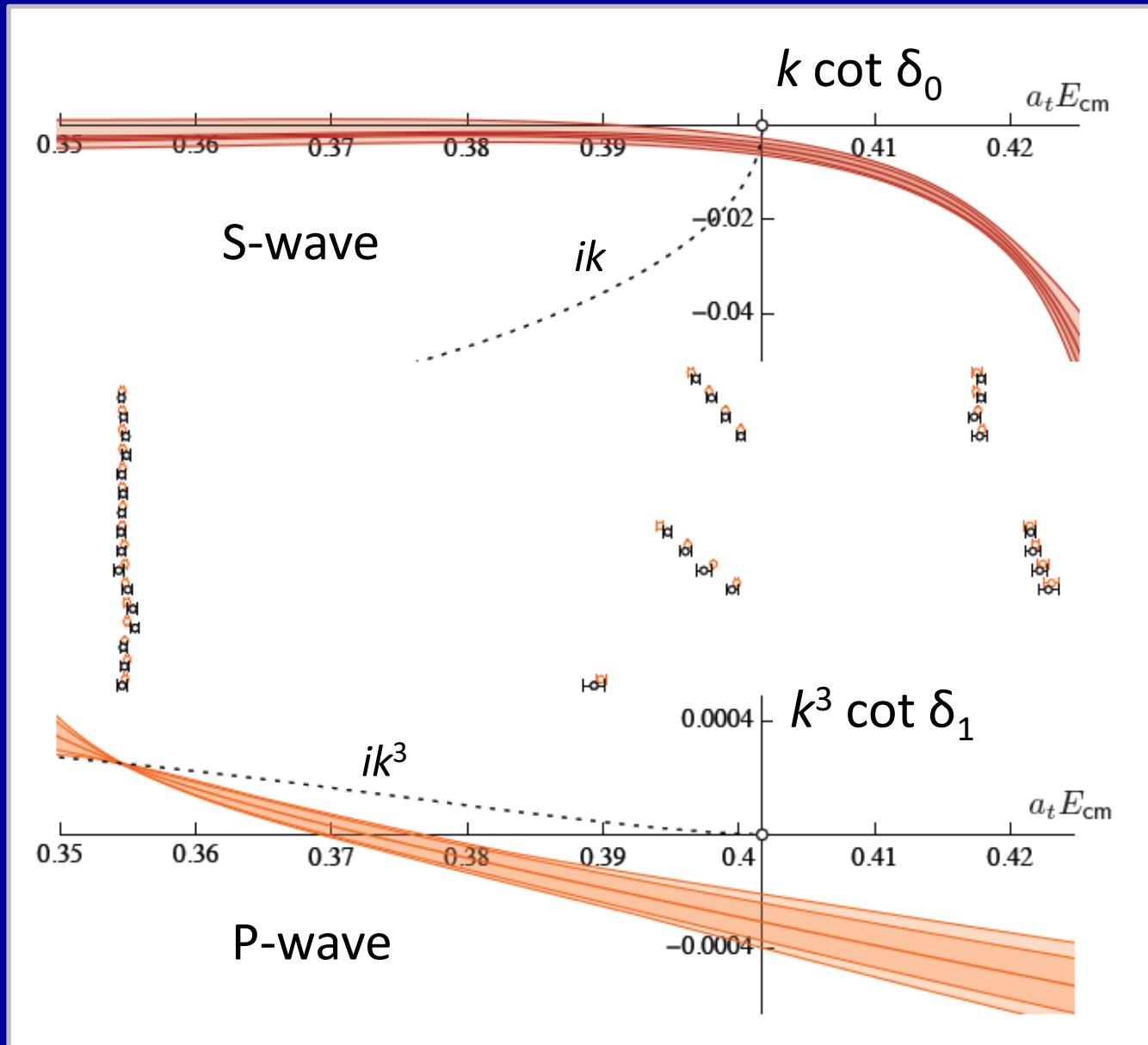
# $D\pi$ , $D\eta$ , $D_s\bar{K}$ ( $\ell=\frac{1}{2}$ ): spectra

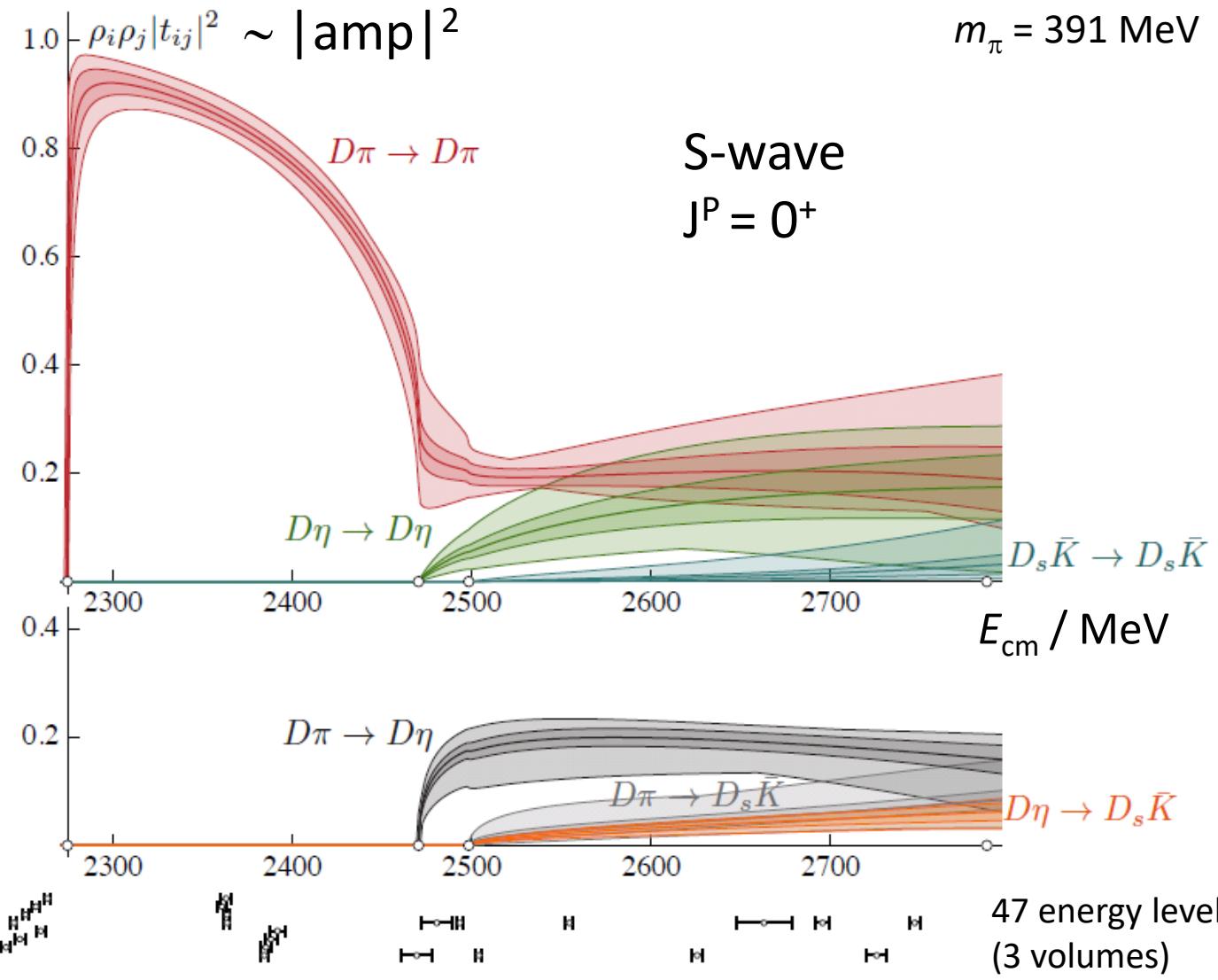
[JHEP 1610, 011 (2016)]

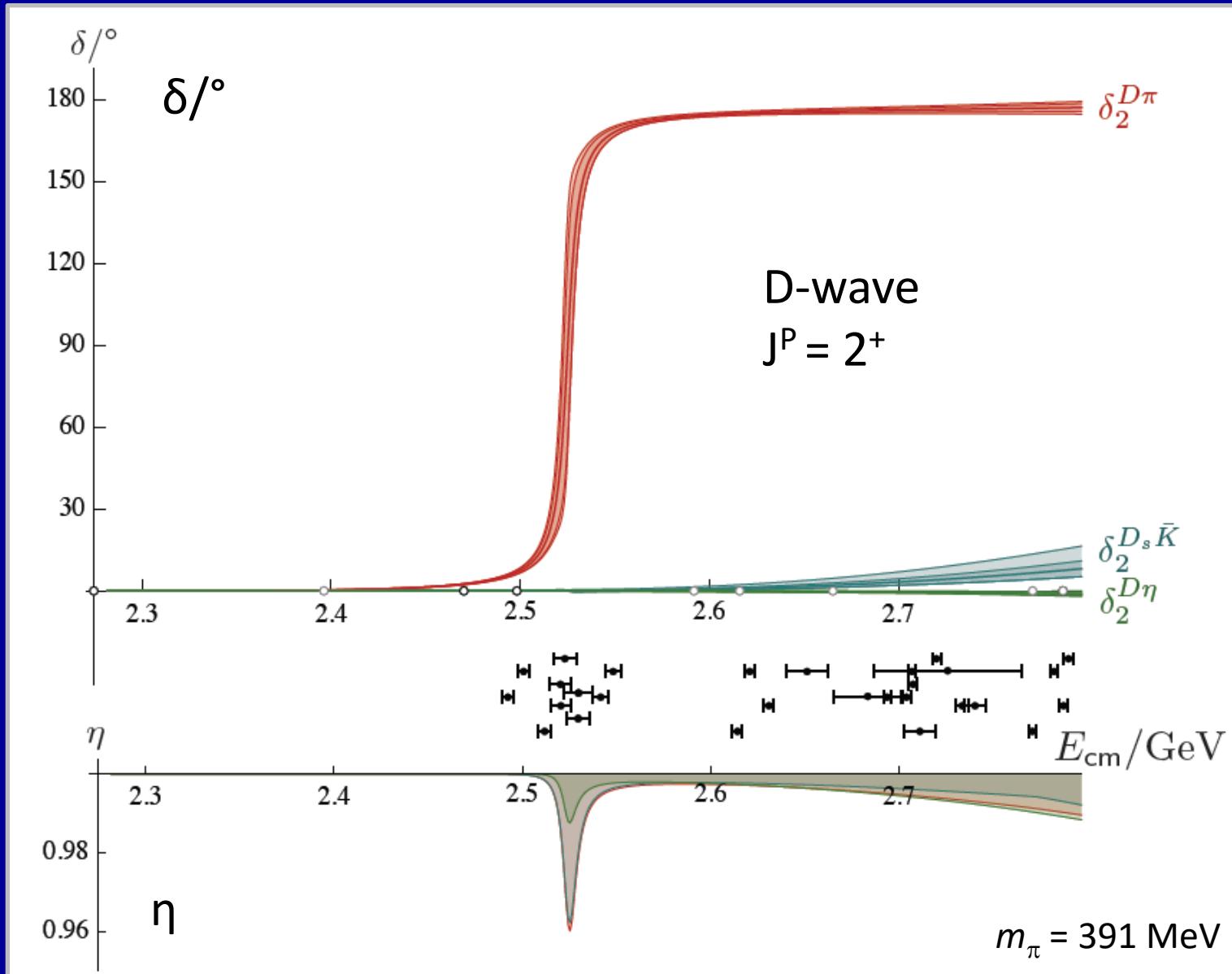


Use 47 energy levels for  $\ell = 0, 1$   
and 18 for  $\ell = 2$



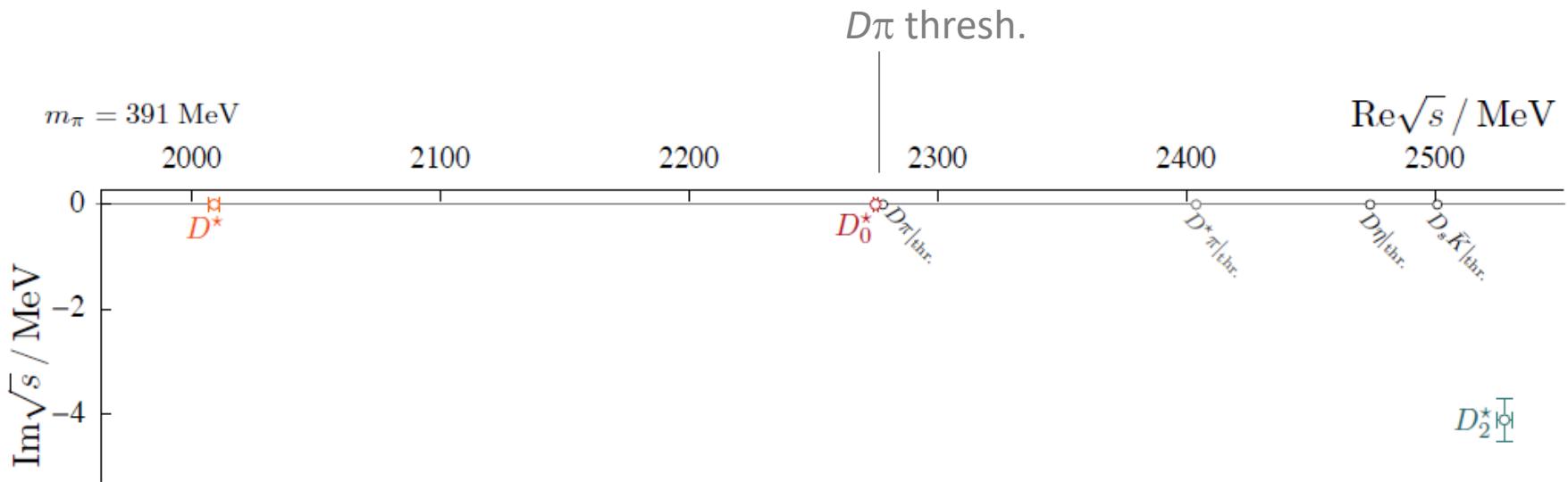


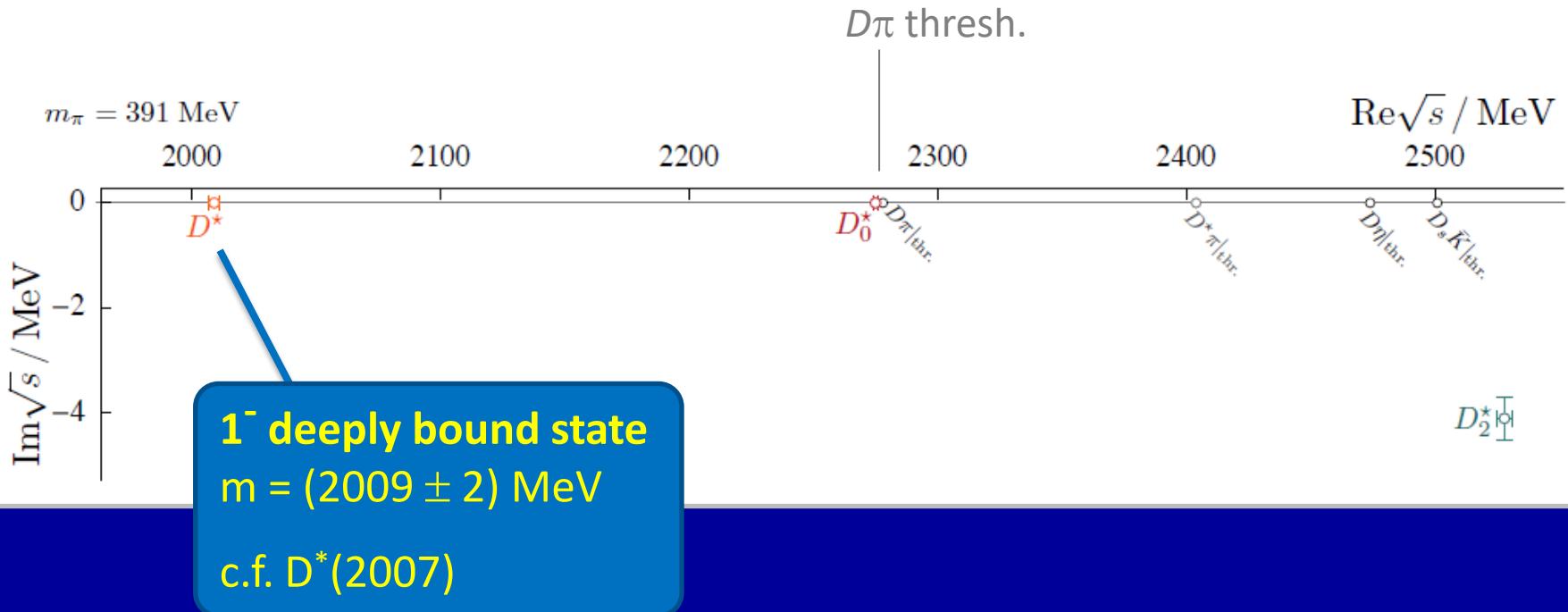




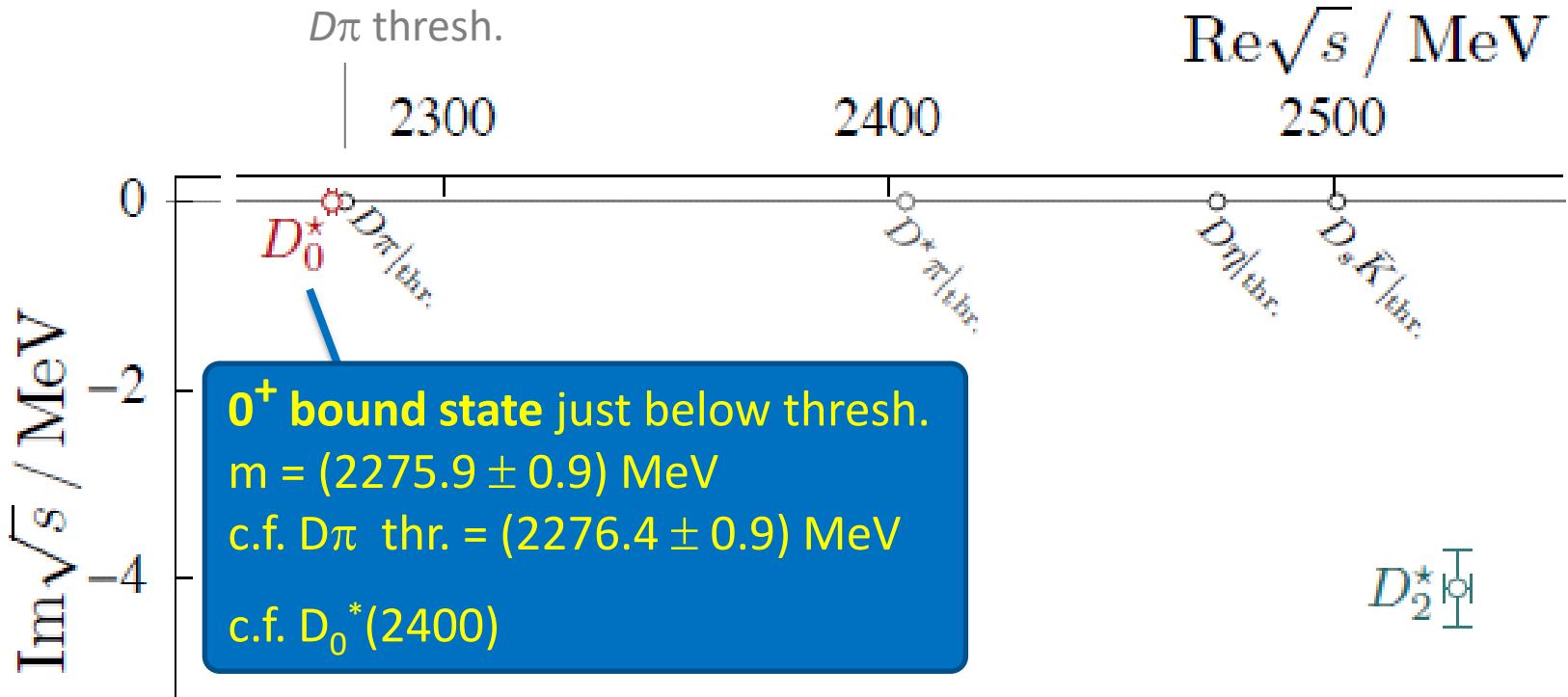
# $D\pi$ , $D\eta$ , $D_s\bar{K}$ ( $|l|=\frac{1}{2}$ ): poles of $t$ -matrix

[JHEP 1610, 011 (2016)]

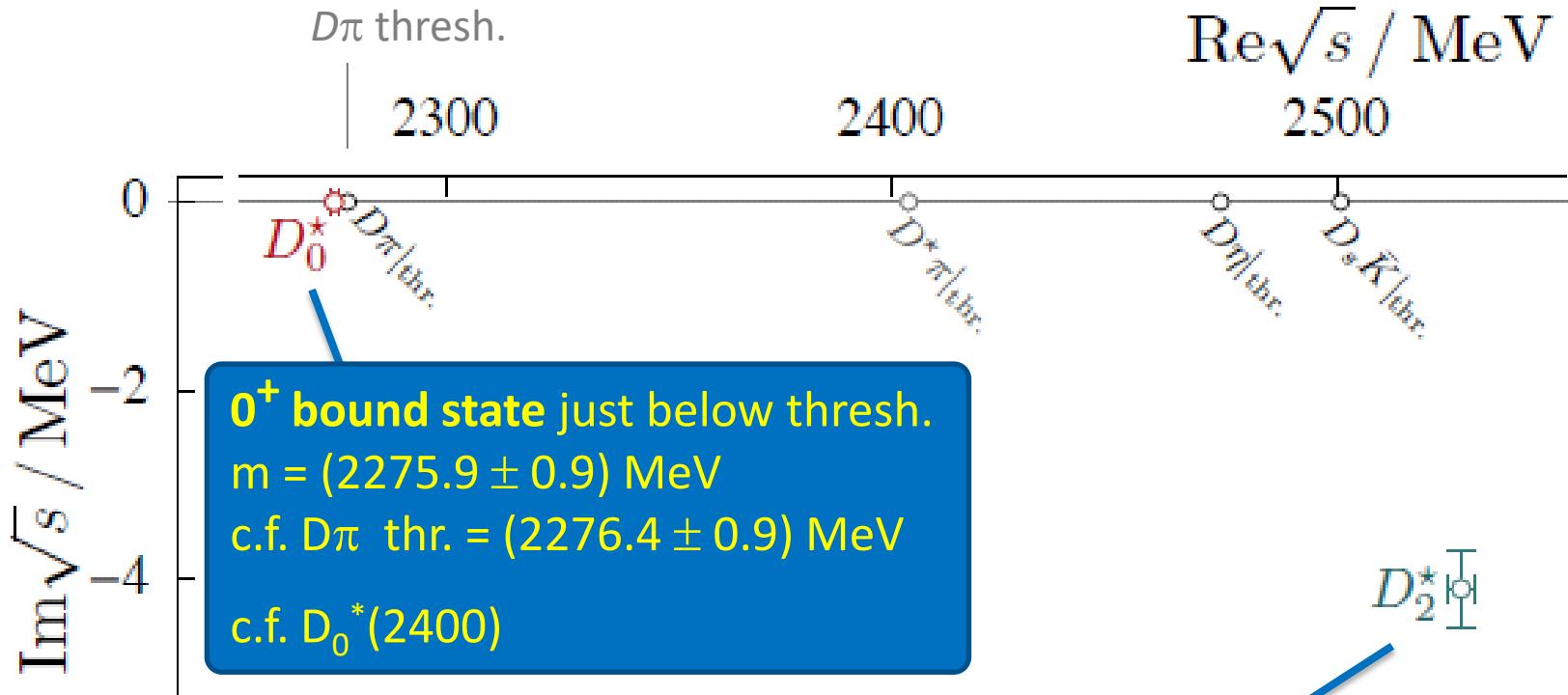




$m_\pi = 391$  MeV



$m_\pi = 391$  MeV



**2<sup>+</sup> narrow resonance**  
 $m = (2527 \pm 3)$  MeV  
 $\Gamma = (8.2 \pm 0.7)$  MeV  
 c.f.  $D_2^*(2460)$  (also couples to  $D^*\pi$ )

# Charm tetraquarks

Cheung, CT, Dudek, Edwards  
(HadSpec) [JHEP 1711, 033 (2017)]

Compute spectra in some **exotic-flavour** channels

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Use a range of '**meson-meson**' operators,

$$\sim \sum_{\hat{p}_1, \hat{p}_2} [\bar{q} \Gamma_1 q] (\vec{p}_1) [\bar{q} \Gamma_2 q] (\vec{p}_2)$$

and '**tetraquark**' (diquark-antidiquark) operators,

$$\sim \sum_{a,d} C_{ad} \left[ c_{abc} q_a^T \Gamma_1 q_b \right] \left[ c_{def} \bar{q}_e \Gamma_2 \bar{q}_f^T \right]$$

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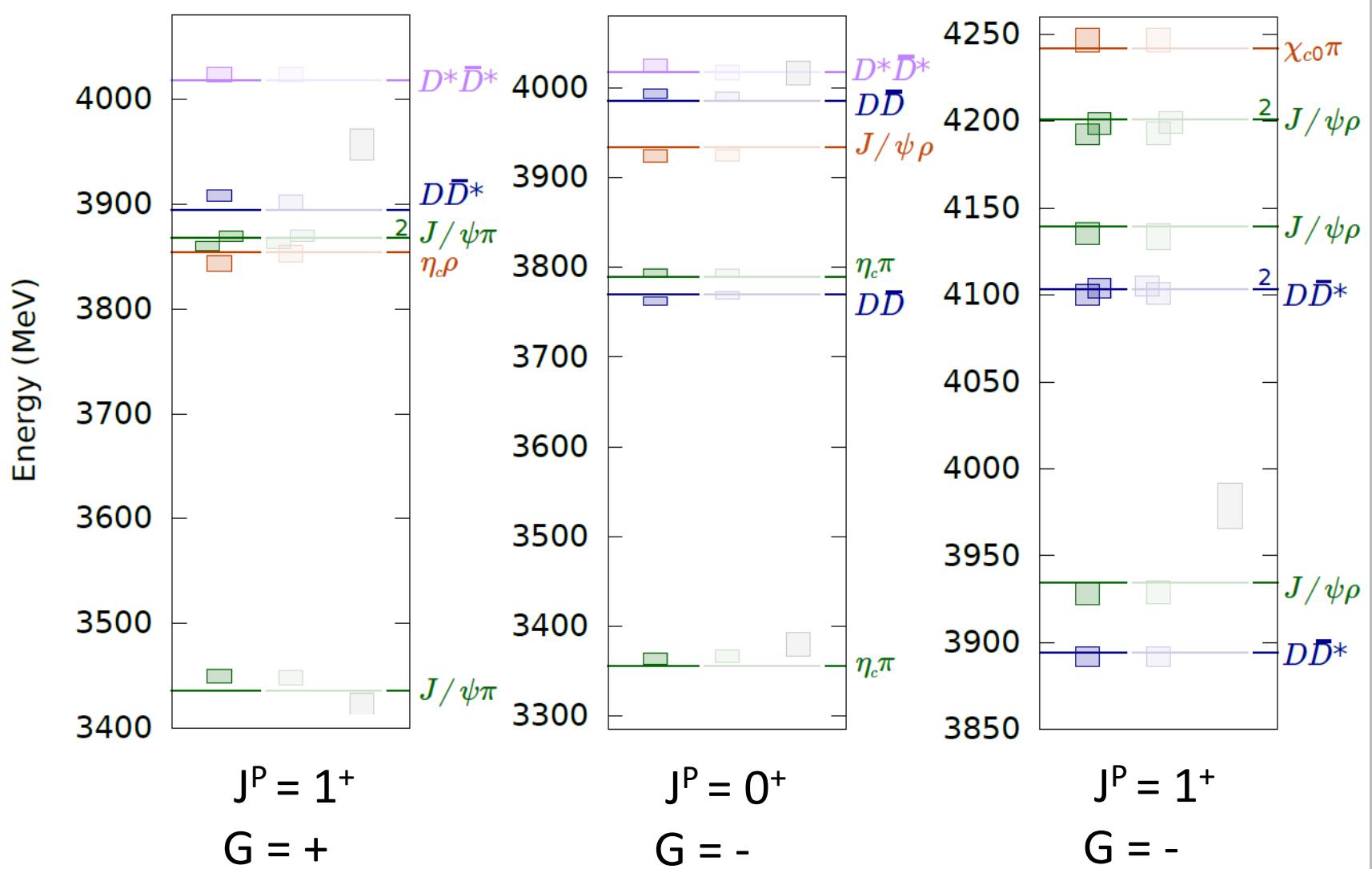
One lattice spacing

1 volume ( $\approx 2$  fm)

$m_\pi = 391$  MeV

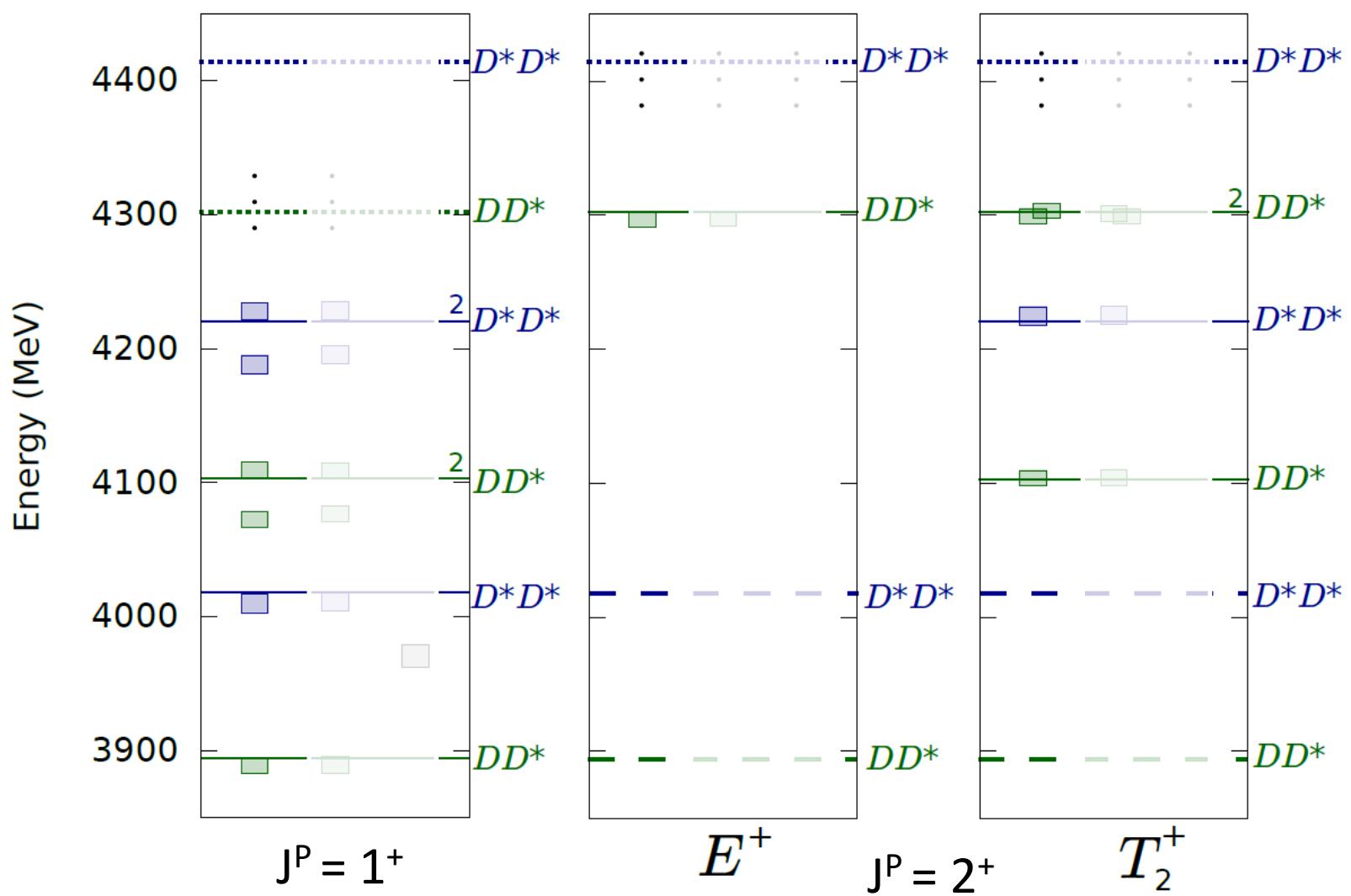
# Hidden-charm $|l|=1$ ( $c\bar{c}l\bar{l}$ )

[JHEP 1711, 033 (2017)]



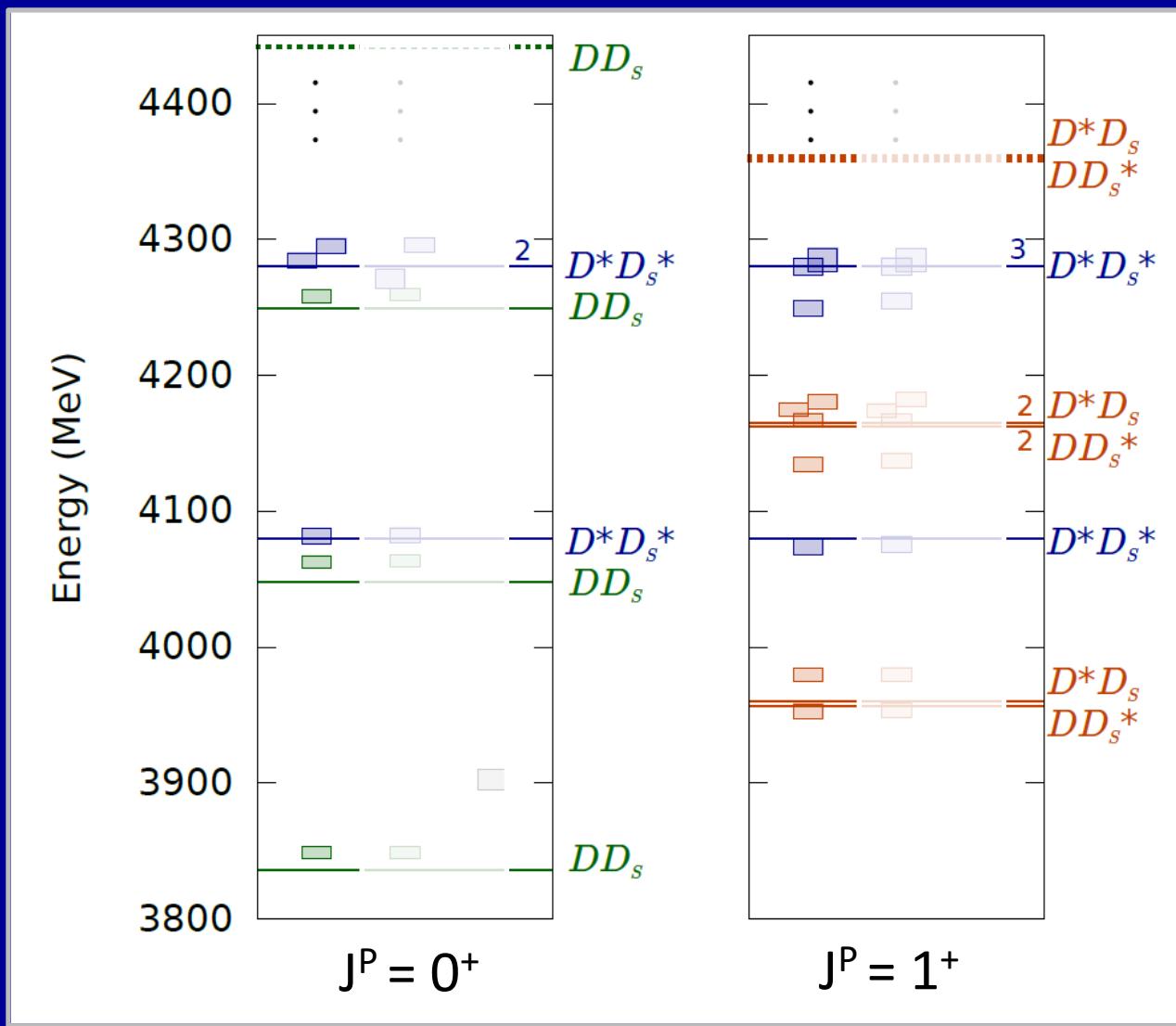
# Doubly-charmed $|l=0$ ( $c\bar{c}l\bar{l}$ )

[JHEP 1711, 033 (2017)]



# Doubly-charmed $|= \frac{1}{2}$ ( $cc\bar{l}\bar{s}$ )

[JHEP 1711, 033 (2017)]



# Summary

- Significant progress in LQCD calculations of excited hadrons, resonances, near-threshold states, etc.
- Examples of recent work (see also Raul's talk earlier):
  - $D\pi$ ,  $D\eta$ ,  $D_s\bar{K}$   $|l=1/2$  scattering (also  $|l=3/2$   $D\pi$ )
  - Exotic-flavour channels (tetraquarks)
- Work in progress on other channels and different  $m_\pi$
- Use  $m_\pi$  dependence as a tool to probe structure
- Ongoing work on formalism (e.g. 3-hadron scattering)

Jefferson Lab and surroundings, USA:

Raúl Briceño<sup>1</sup>, Jozef Dudek<sup>2</sup>, Robert Edwards, Bálint Joó,  
David Richards, Frank Winter, Bipasha Chakraborty  
(<sup>1</sup> and Old Dominion University, <sup>2</sup> and William & Mary)  
W&M: *Christopher Johnson, Archana Radhakrishnan*

Trinity College Dublin, Ireland:

Michael Peardon, Sinéad Ryan, David Wilson,  
*Cian O'Hara, David Tims*

University of Cambridge, UK:

CT, Graham Moir, *Gavin Cheung, Antoni Woss*

Tata Institute, India:

Nilmani Mathur

[www.hadspec.org](http://www.hadspec.org)

