An approach to QCD bound states

Bound states in strongly coupled systems

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Perturbative calculations of bound states requires NP input, even in QED

- The "lowest order" wave function already has all orders of α
- Free *in* and *out* states of *S*-matrix have no overlap with bound states
- The classical gauge field keeps the asymptotic states bound
- A boundary condition in the classical gluon field eqs gives Λ_{QCD}

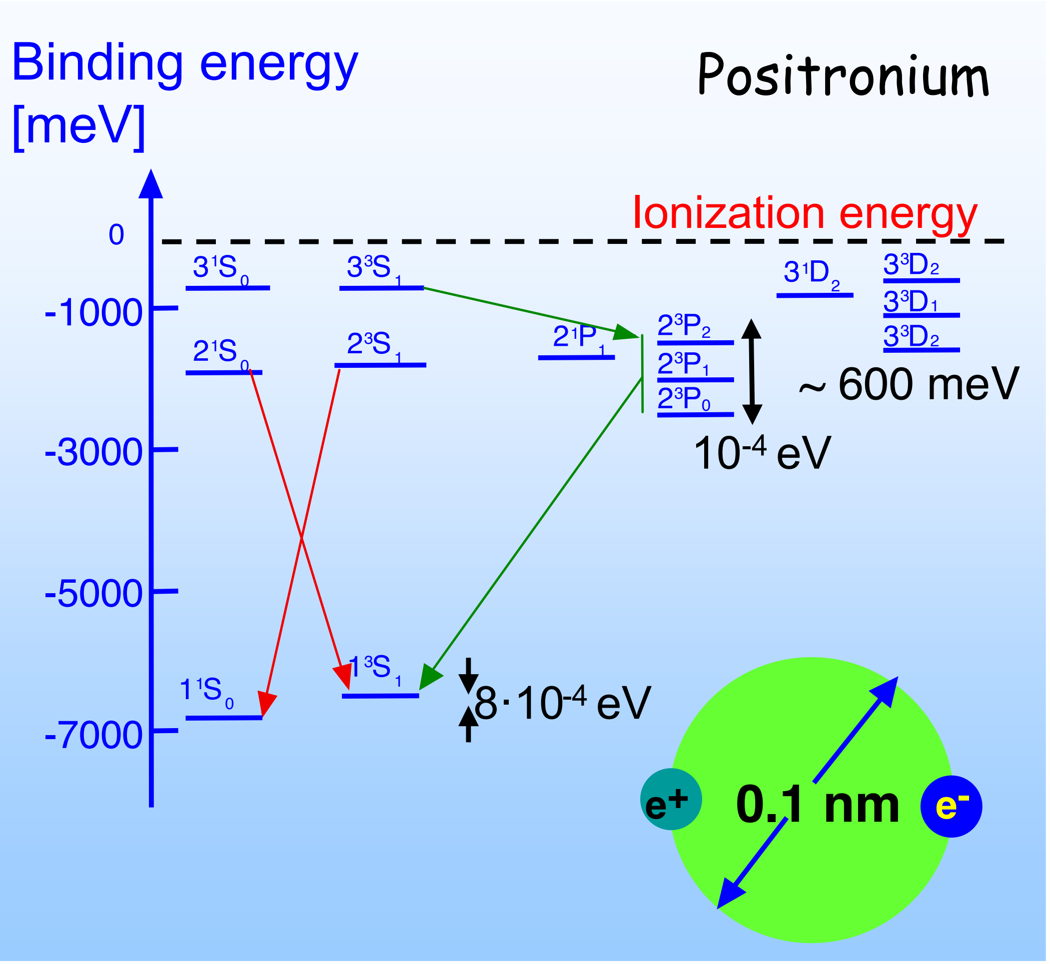
$q\bar{q}$ states bound by a classical gluon field may serve as hadrons at $\mathcal{O}(\alpha_s{}^0)$

- They incorporate confinement and chiral symmetry breaking
- Allow inclusion of higher orders in α_s via the perturbative *S*-matrix

PH 1605.01532 and to appear

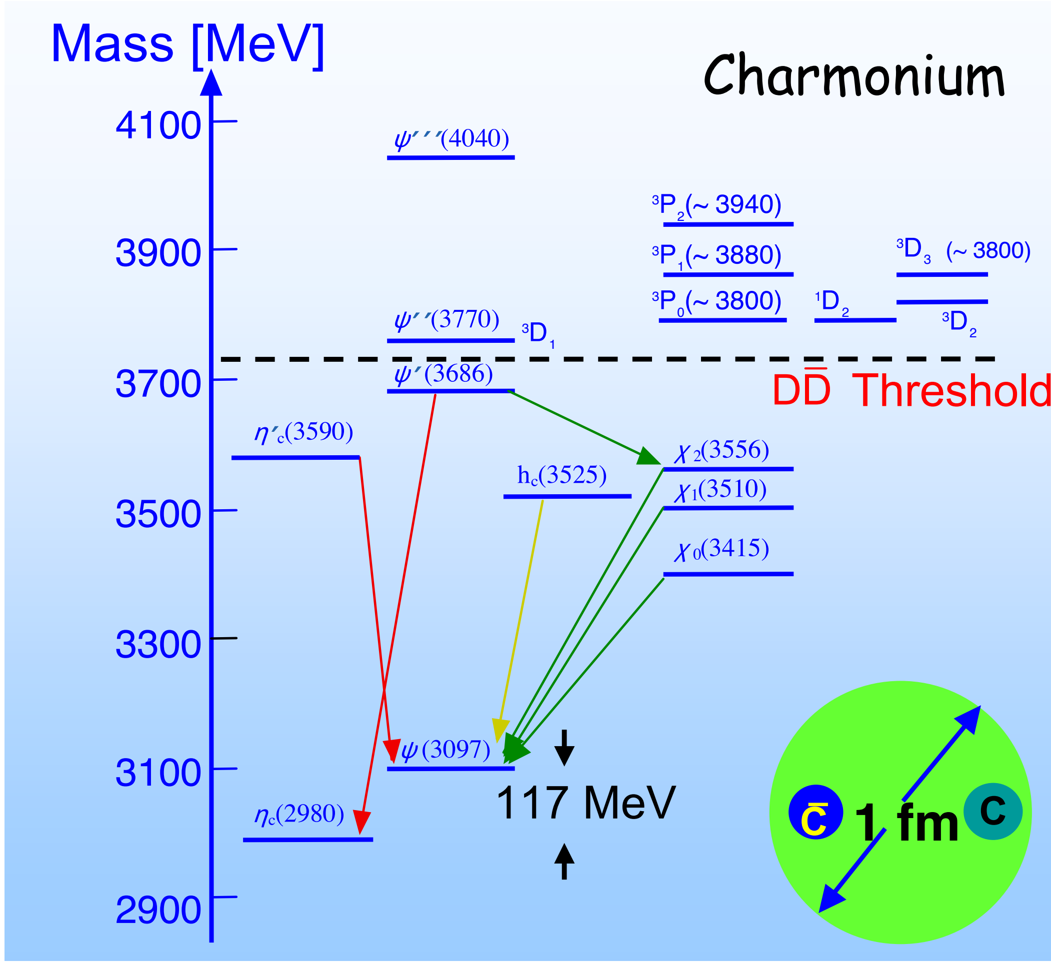
"The J/ψ is the Hydrogen atom of QCD"

QED



$$V(r) = -\frac{\alpha}{r}$$

QCD



$$V(r) = cr - \frac{4}{3}\frac{\alpha_s}{r}$$

QED works for atoms

Example: Hyperfine splitting in Positronium

G. S. Adkins,
Hyperfine Interact. **233** (2015) 59

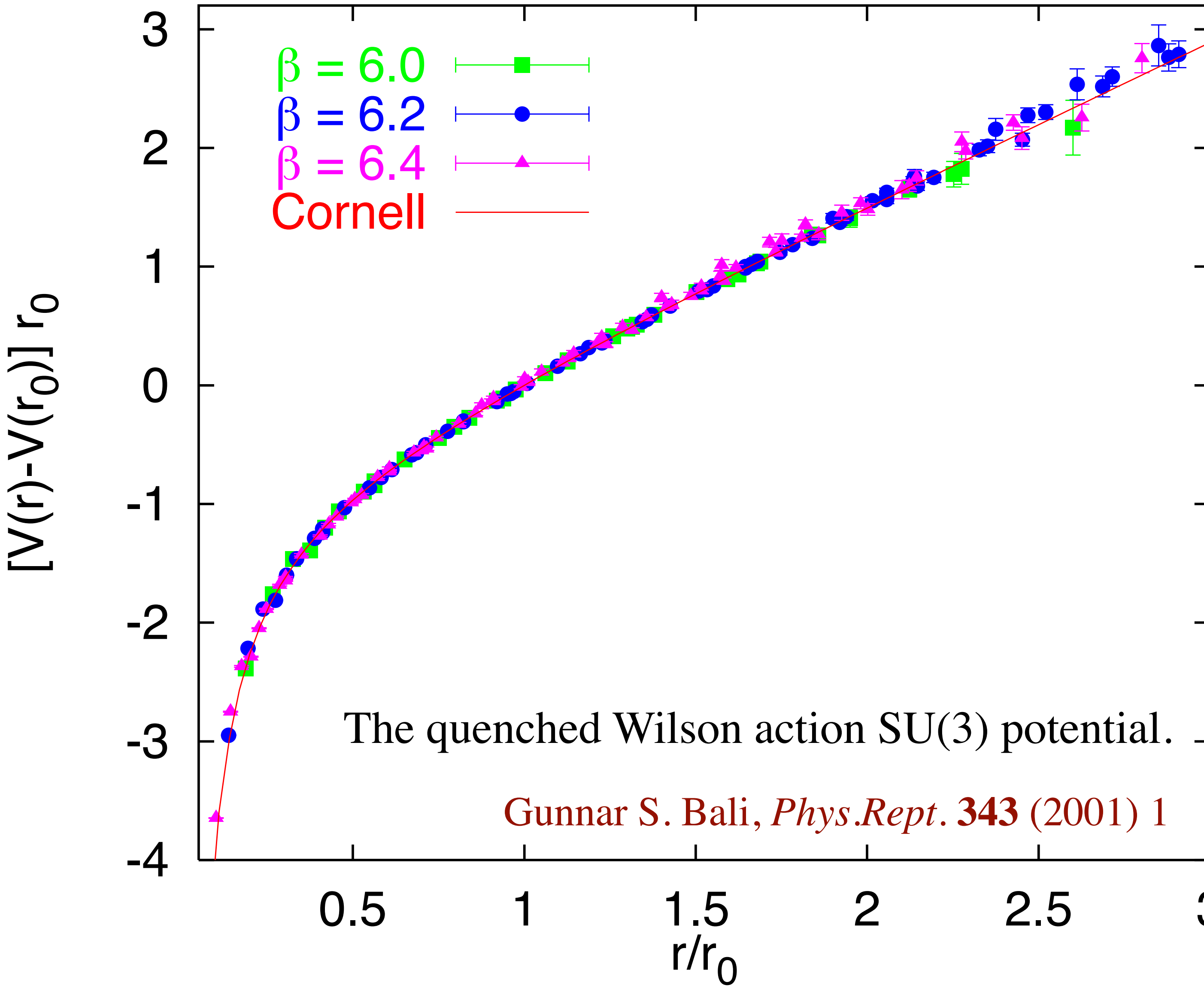
$$\Delta\nu_{QED} = m_e\alpha^4\left\{\frac{7}{12}-\frac{\alpha}{\pi}\left(\frac{8}{9}+\frac{\ln 2}{2}\right)\right.$$
$$+\frac{\alpha^2}{\pi^2}\left[-\frac{5}{24}\pi^2\ln\alpha+\frac{1367}{648}-\frac{5197}{3456}\pi^2+\left(\frac{221}{144}\pi^2+\frac{1}{2}\right)\ln 2-\frac{53}{32}\zeta(3)\right]$$
$$\left.-\frac{7\alpha^3}{8\pi}\ln^2\alpha+\frac{\alpha^3}{\pi}\ln\alpha\left(\frac{17}{3}\ln 2-\frac{217}{90}\right)+\mathcal{O}\left(\alpha^3\right)\right\} = 203.39169(41)\ \mathrm{GHz}$$

A. Ishida et al, 1310.6923 : $\Delta\nu_{EXP} = 203.3941 \pm .003$ GHz

- Binding energy is perturbative in α and $\log(\alpha)$ (measurable)
- Wave function $\psi(r) \propto \exp(-m\alpha r)$ is of $\mathcal{O}(\alpha^\infty)$ (used in NRQED)

There are many ways to (re)organize an expansion that starts with $\mathcal{O}(\alpha^\infty)$

Linear Cornell potential agrees with Lattice QCD



… and will here arise from a *classical* gluon field

Interaction Picture

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \qquad \mathcal{H}_0 \left|i\right\rangle_{in} = E_i \left|i\right\rangle_{in}$$

$$S_{fi} = {}_{out}\langle f, t \to \infty| \left\{ \mathrm{T} \exp \left[-i \int_{-\infty}^{\infty} dt\, H_I(t) \right] \right\} |i, t \to -\infty\rangle_{in}$$

Generates Feynman diagrams to arbitrary order for any scattering process

The *in*- and *out*-states at $t = \pm\infty$ must overlap the physical i, f states.

Bound states have no overlap with free *in*- and *out*-states at $t = \pm\infty$

No finite order Feynman diagram for $e^+e^- \to e^+e^-$ has a positronium pole.

$\Rightarrow$ **We need to expand around *in* and *out* states with their classical gauge field**

Define "Potential Picture"

$$\mathcal{H} = \mathcal{H}_V + \mathcal{H}_I \qquad\qquad \mathcal{H}_V = \mathcal{H}_0 + \mathcal{H}_I(A_{cl})$$

$$S_{fi} = {}_V\langle f, t \to \infty| \left\{ \mathrm{T}\exp\left[-i\int_{-\infty}^{\infty} dt\, H_I(t)\right]\right\} |i, t \to -\infty\rangle_V$$

$$\mathcal{H}_V\,|i\rangle_V = E_i\,|i\rangle_V$$

Perturbative expansion should be expanded around a stationary action

$$\int [dA^\mu]\, \exp\left(iS[A^\mu]/\hbar\right)$$

Classical field defined by stationary action

Dominates for $\hbar \to 0$

$$\frac{\delta S[A^\mu_{cl}]}{\delta A^\mu_{cl}} = 0$$

A proper derivation à la Interaction Picture required

Here: Stay at $\mathcal{H}_I^0$ level.

Two consequences of $\hbar \to 0$ in QCD

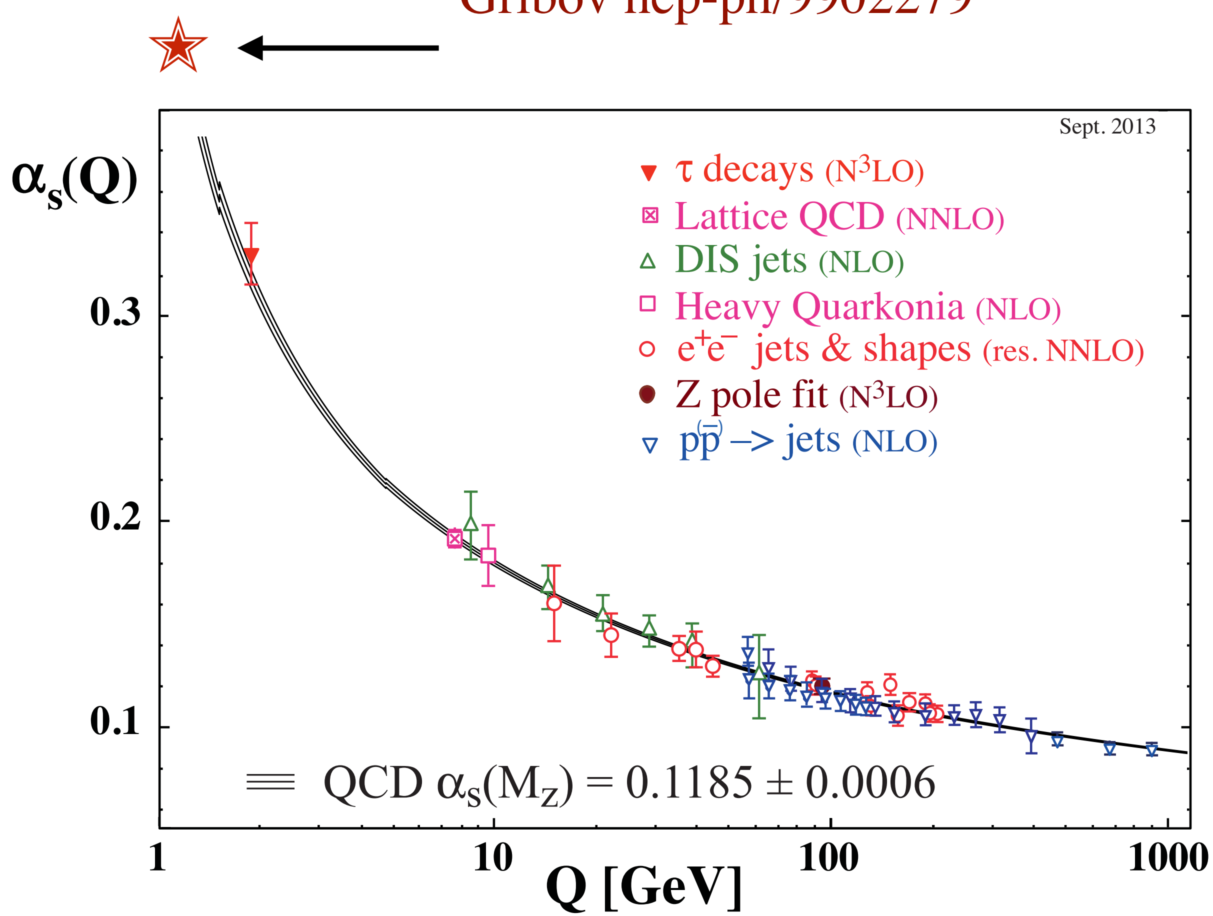
1. The suppression of loops, stops the running of α_s

Gribov's prediction agrees phenomenological estimates: $\alpha_s(0)/\pi \approx 0.14$

$\Rightarrow$ PQCD corrections to $\mathcal{O}(\hbar^0)$ can be relevant.

2. In the absence of loops, the QCD scale Λ_{QCD} cannot arise from renormalization.

$\alpha_s^{crit} \approx 0.43$ Gribov hep-ph/9902279



$\Rightarrow$ Λ_{QCD} must arise from a boundary condition on the classical field equations.

Illustration: Positronium at rest

$$|M\rangle_V = \int \frac{d\boldsymbol{k}}{(2\pi)^3}\,\phi(\boldsymbol{k})\,b^\dagger_{\boldsymbol{k},\lambda_1}\,d^\dagger_{-\boldsymbol{k},\lambda_2}\,|0\rangle$$

$\phi(\boldsymbol{k})$ is the Schrödinger wave function

$$= \int d\boldsymbol{x}_1\,d\boldsymbol{x}_2\,\overline{\psi}_\alpha(0,\boldsymbol{x}_1)\,\Phi_{\alpha\beta}(\boldsymbol{x}_1-\boldsymbol{x}_2)\,\psi_\beta(0,\boldsymbol{x}_2)\,|0\rangle$$

where Φ is given by the Schrödinger wave function as

$$\Phi_{\alpha\beta}(\boldsymbol{x}) = {}_\alpha\big[\gamma^0 u(-i\,\boldsymbol{\nabla},\lambda_1)\big]\big[\bar{v}(i\,\boldsymbol{\nabla},\lambda_2)\gamma^0\big]_\beta\,\phi(\boldsymbol{x})$$

Impose: $\mathcal{H}_V\,|M\rangle_V = M\,|M\rangle_V$

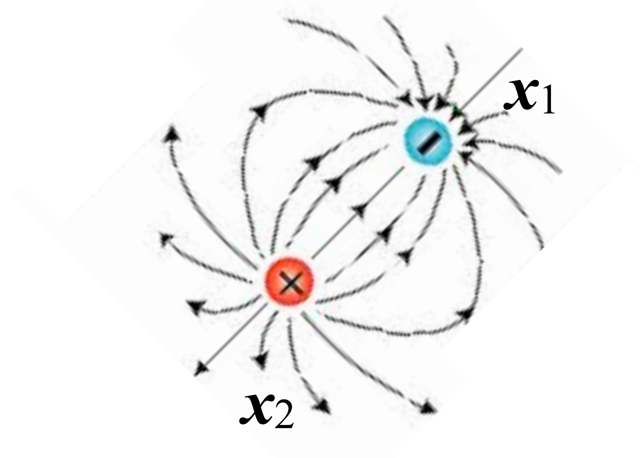
where $\mathcal{H}_V$ has the classical photon field.

The classical field for Positronium

For the component $|\boldsymbol{x}_1, \boldsymbol{x}_2\rangle = \bar{\psi}(t, \boldsymbol{x}_1)\psi(t, \boldsymbol{x}_2)\,|0\rangle$ of the Positronium state

the classical A^0 field is

$$eA^0(\boldsymbol{x}; \boldsymbol{x}_1, \boldsymbol{x}_2) = \frac{\alpha}{|\boldsymbol{x} - \boldsymbol{x}_1|} - \frac{\alpha}{|\boldsymbol{x} - \boldsymbol{x}_2|}$$



to be used in

$$\mathcal{H}_V(t; \boldsymbol{x}_1, \boldsymbol{x}_2) = \int d\boldsymbol{x}\, \psi^\dagger(t, \boldsymbol{x})\big[-i\boldsymbol{\nabla}\cdot\boldsymbol{\alpha} + m\gamma^0 + \tfrac{1}{2}eA^0(\boldsymbol{x}; \boldsymbol{x}_1, \boldsymbol{x}_2)\big]\psi(t, \boldsymbol{x})$$

Note: A^0 is determined instantaneously for all $\boldsymbol{x}$

It depends on $\boldsymbol{x}_1, \boldsymbol{x}_2$

$$eA^0(\boldsymbol{x}_1) = -eA^0(\boldsymbol{x}_2) = -\frac{\alpha}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|}$$ is the classical $-\alpha/r$ potential

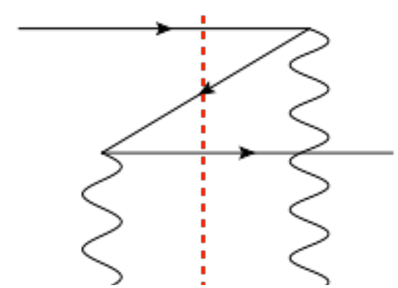
But: An external observer at $\boldsymbol{x}$ sees a dipole field

$|q\bar{q}\rangle_V$ states in QCD

qq̄ state at rest $$|M\rangle_V = \int d\boldsymbol{x}_1\, d\boldsymbol{x}_2\, \bar{\psi}_\alpha^A(t,\boldsymbol{x}_1)\, \Phi_{\alpha\beta}^{AB}(\boldsymbol{x}_1-\boldsymbol{x}_2)\, \psi_\beta^B(t,\boldsymbol{x}_2)\, |0\rangle$$

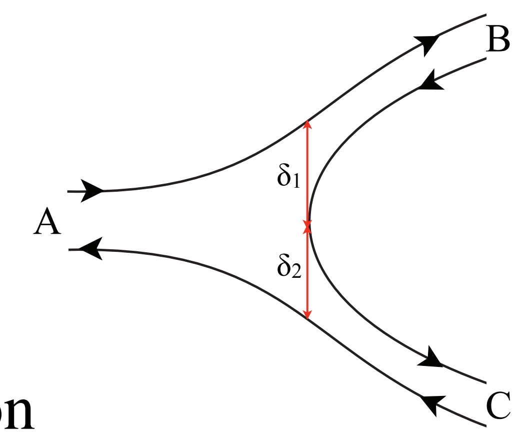
Color singlet wave function $$\Phi^{AB}(\boldsymbol{x}_1-\boldsymbol{x}_2) = \frac{1}{\sqrt{N_C}}\, \delta^{AB}\, \Phi(\boldsymbol{x}_1-\boldsymbol{x}_2)$$

Lesson from Dirac dynamics:
A strong potential creates pairs via *Z*-diagrams.
Those virtual pairs are included in $|M\rangle_V$



True particle production (string breaking)
is included iteratively, through the overlap
of the zero-width states

$$\langle B, C|A\rangle \;=$$



Duality allows qq̄ states to describe multiparticle production

Classical confining field in QCD

Consider a homogeneous solution of Gauss law, for each component $q^A(\boldsymbol{x}_1)\bar{q}^A(\boldsymbol{x}_2)$ of the state:

$$\boldsymbol{\nabla}_x^2 A_a^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2,A) = 0$$

Translation invariance requires a linear dependence on $\boldsymbol{x}$.

Universal field energy density determines dependence on $\boldsymbol{x}_1 - \boldsymbol{x}_2$

Color symmetry requires $A_a^0 \propto T_a^{AA}$

$$A_a^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2,A) = \left[\boldsymbol{x} - \tfrac{1}{2}(\boldsymbol{x}_1+\boldsymbol{x}_2)\right]\cdot\frac{\boldsymbol{x}_1-\boldsymbol{x}_2}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}\,T_a^{AA}\,6\Lambda^2$$

Unique?!

$$\sum_a \left[\boldsymbol{\nabla}_x A_a^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2,A)\right]^2 = 12\Lambda^4 \qquad \mathcal{O}\left(\alpha_s^0\right)$$

Universal field energy density determines Λ_{QCD}

$$\sum_A A_a^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2,A) \propto \mathrm{Tr}\,T^{AA} = 0$$

Another hadron feels no field at any $\boldsymbol{x}$

$\mathcal{A}_a^j$ is of $\mathcal{O}\left(g\right)$ Perturbative compared to $\mathcal{A}_a^0$

Bound state equation

$\mathcal{H}_V \, |M\rangle_V = M \, |M\rangle_V$ Bound state condition implies, with $\boldsymbol{x} = \boldsymbol{x}_1 - \boldsymbol{x}_2$

$$i\boldsymbol{\nabla} \cdot \left\{\gamma^0 \boldsymbol{\gamma}, \Phi(\boldsymbol{x})\right\} + m\left[\gamma^0, \Phi(\boldsymbol{x})\right] = \left[M - V(\boldsymbol{x})\right]\Phi(\boldsymbol{x})$$

$$V(\boldsymbol{x}_1 - \boldsymbol{x}_2) = \sum_a \tfrac{1}{2} g T_a^{AA} \left[A_a^0(\boldsymbol{x}_1) - A_a^0(\boldsymbol{x}_2)\right] = g\Lambda^2 |\boldsymbol{x}_1 - \boldsymbol{x}_2|$$

Expanding the 4 × 4 wave function in a basis of 16 Dirac structures $\Gamma_i(\boldsymbol{x})$

$$\Phi(\boldsymbol{x}) = \sum_i \Gamma_i(\boldsymbol{x}) F_i(r) Y_{j\lambda}(\hat{\boldsymbol{x}})$$

we may use rotational, parity and charge conjugation invariance to determine which $\Gamma_i(\boldsymbol{x})$ may occur for a state of given j^{PC}:

0^{-+} trajectory $[s=0,\ \ell=j]$: $-\eta_P = \eta_C = (-1)^j$ $\gamma_5,\ \gamma^0\gamma_5,\ \gamma_5\, \boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma_5\, \boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L}$

0^{--} trajectory $[s=1,\ \ell=j]$: $\eta_P = \eta_C = -(-1)^j$ $\gamma^0\gamma_5\, \boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma^0\gamma_5\, \boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \boldsymbol{\alpha}\cdot\boldsymbol{L},\ \gamma^0\, \boldsymbol{\alpha}\cdot\boldsymbol{L}$

0^{++} trajectory $[s=1,\ \ell=j\pm1]$: $\eta_P = \eta_C = +(-1)^j$ $1,\ \boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma^0\boldsymbol{\alpha}\cdot\boldsymbol{x},\ \boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \gamma^0\boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \gamma^0\gamma_5\, \boldsymbol{\alpha}\cdot\boldsymbol{L}$

0^{+-} trajectory [exotic]: $\eta_P = -\eta_C = (-1)^j$ $\gamma^0,\ \gamma_5\, \boldsymbol{\alpha}\cdot\boldsymbol{L}$

$\Longrightarrow$ There are no solutions for quantum numbers that would be exotic in the quark model (despite the relativistic dynamics)

Example: 0^{-+} trajectory wf's

$$\Phi_{-+}(\boldsymbol{x}) = \Big[\frac{2}{M-V}(i\boldsymbol{\alpha}\cdot\overrightarrow{\boldsymbol{\nabla}} + m\gamma^0) + 1\Big]\gamma_5\, F_1(r) Y_{j\lambda}(\hat{\boldsymbol{x}})$$

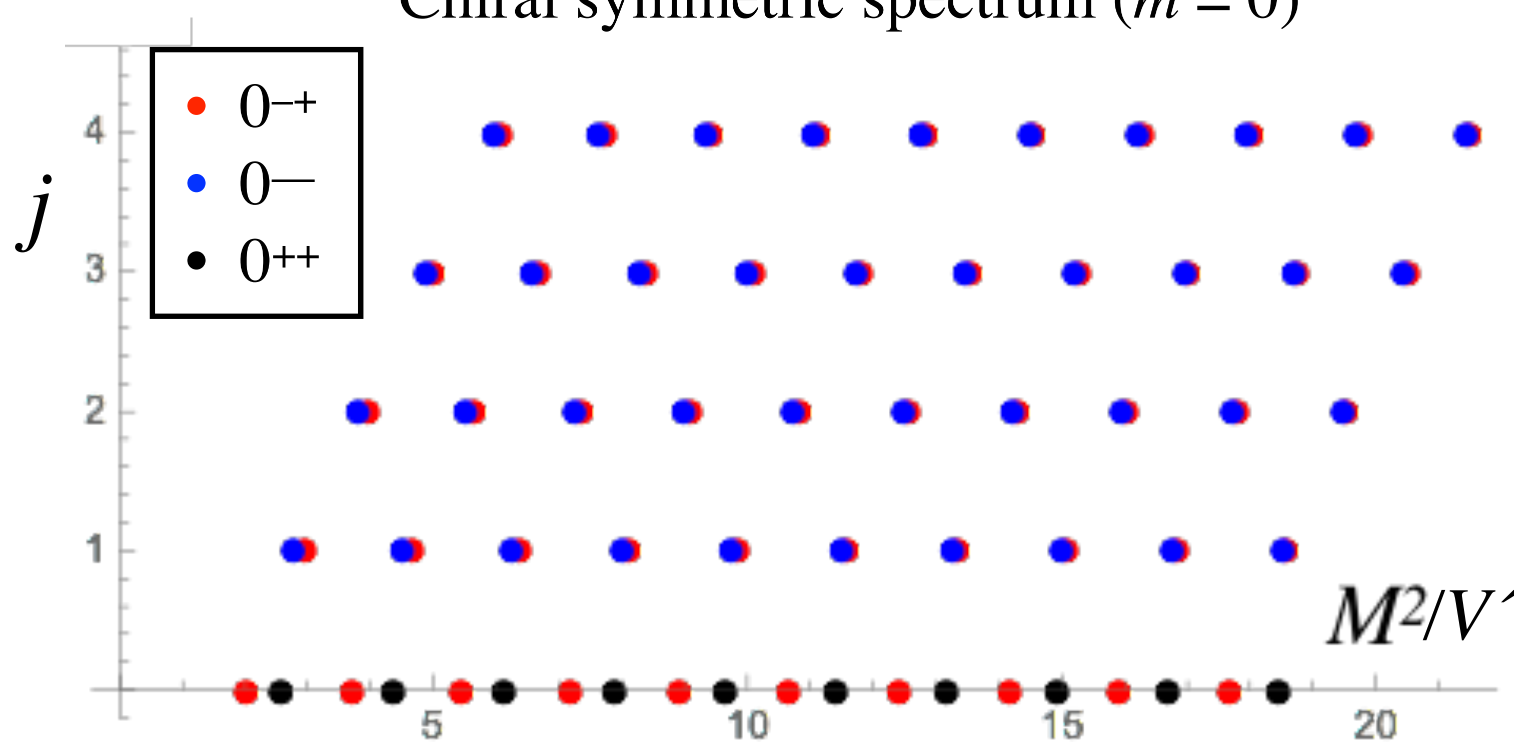
Radial equation: $F_1'' + \Big(\frac{2}{r} + \frac{V'}{M-V}\Big)F_1' + \Big[\tfrac{1}{4}(M-V)^2 - m^2 - \frac{j(j+1)}{r^2}\Big]F_1 = 0$

Local normalizability at $r = 0$ and at $V(r) = M$ determines the discrete M

Linear Regge trajectories with daughters

Spectrum similar to dual models

Chiral symmetric spectrum ($m = 0$)



Bound states in motion

A q$\bar{\text{q}}$ bound state with CM momentum $\boldsymbol{P}$ may be expressed as

$$|M,P\rangle_V \equiv \int d\boldsymbol{x}_1\, d\boldsymbol{x}_2\, \bar{\psi}(t=0,\boldsymbol{x}_1)\, e^{i\boldsymbol{P}\cdot(\boldsymbol{x}_1+\boldsymbol{x}_2)/2}\Phi^{(P)}(\boldsymbol{x}_1-\boldsymbol{x}_2)\psi(t=0,\boldsymbol{x}_2)\,|0\rangle$$

Note: In a Hamiltonian formulation states are at equal time in all frames.

Their boost covariance is not explicit: few (if any?) examples exist.

The potential Hamiltonian is

$$\mathcal{H}_V = \int d\boldsymbol{x}\, \psi^\dagger(t,\boldsymbol{x})\big[-i\boldsymbol{\alpha}\cdot\overrightarrow{\boldsymbol{\nabla}} + m\gamma^0 + \tfrac{1}{2}\gamma^0 g \not{A}_{(P)}\big]\psi(t,\boldsymbol{x})$$

What is the classical field $A^\mu_{(P)}$?

The answer depends on the frame of the observer.

1. The classical field is independent of P

The component $\bar{\psi}(\boldsymbol{x}_1)\psi(\boldsymbol{x}_2)\,|0\rangle$ specifies positions, not momenta.

It is independent of $\boldsymbol{P}$ and so is the instantaneous A^0 field.

The bound state equation has a $\boldsymbol{P}$-independent potential $V(\boldsymbol{x}) = V'|\boldsymbol{x}|$

$$i\boldsymbol{\nabla}\cdot\big\{\boldsymbol{\alpha},\Phi_1^{(P)}(\boldsymbol{x})\big\} - \tfrac{1}{2}\boldsymbol{P}\cdot\big[\boldsymbol{\alpha},\Phi_1^{(P)}(\boldsymbol{x})\big] + m\big[\gamma^0,\Phi_1^{(P)}(\boldsymbol{x})\big] = \big[E - V(\boldsymbol{x})\big]\Phi_1^{(P)}(\boldsymbol{x})$$

$\boldsymbol{P}$ breaks rotational symmetry: angular-radial separation is not possible.

An analytic solution for $\Phi_1^{(P)}(\boldsymbol{x})$ is found in D = 1+1 dimensions.

This provides a boundary condition at $\boldsymbol{x}_\perp = 0$, which ensures $E = \sqrt{\boldsymbol{P}^2 + M^2}$

The wave function $\Phi_1^{(P)}(\boldsymbol{x})$ is found numerically ($\boldsymbol{P}$-dependence analytically?)

$\Phi_1^{(P)}(\boldsymbol{x})$ determines the states with momentum $\boldsymbol{P}$ in the original frame.

2. The classical field is boosted to frame P

Gives the dynamics of the rest frame state as seen in a moving frame.

Define boost ξ taking $\boldsymbol{P} = (0, 0, P)$ along the z-axis: $P = M\sinh(\xi)$

In a moving frame the rest frame A^0 field appears as ($\boldsymbol{x} = \boldsymbol{x}_1 - \boldsymbol{x}_2$):

$$A^0_{(P)}(\boldsymbol{x}) = \cosh\xi\, A^0(\boldsymbol{x}_R) \qquad\qquad A^3_{(P)}(\boldsymbol{x}) = \sinh\xi\, A^0(\boldsymbol{x}_R)$$

where the rest frame (Lorentz dilated) separation is $\boldsymbol{x}_R = (x, y, z\cosh\xi)$

The P-dependence of this $\Phi_2^{(P)}(\boldsymbol{x})$ is found analytically from the BSE:

$$\Phi_2^{(P)}(\boldsymbol{x}) = e^{-\xi\gamma^0\gamma^3/2}\,\Phi^{(0)}(\boldsymbol{x}_R)\,e^{\xi\gamma^0\gamma^3/2}$$

The wave function is contracted and spin rotated (like the Dirac wf.)

Extra twist: The magnetic field $\boldsymbol{B} = \nabla\times\boldsymbol{A}$ causes the state to precess in time

States with P = M = 0

We required the wave function to be normalizable at $r = 0$ and $V'r = M$

For $M = 0$ the two points coincide. Regular, massless solutions are found.

The massless 0^{++} meson "σ" is particularly interesting: Having vacuum quantum numbers it can mix with the vacuum and break chiral invariance.

$$|\sigma\rangle = \int d\boldsymbol{x}_1\, d\boldsymbol{x}_2\, \bar{\psi}(\boldsymbol{x}_1)\, \Phi_\sigma(\boldsymbol{x}_1 - \boldsymbol{x}_2)\, \psi(\boldsymbol{x}_2)\, |0\rangle \equiv \hat{\sigma}\, |0\rangle$$

For $m = 0$: $\Phi_\sigma(\boldsymbol{x}) = N_\sigma \left[J_0(\tfrac{1}{4}r^2) + \boldsymbol{\alpha} \cdot \boldsymbol{x} \frac{i}{r} J_1(\tfrac{1}{4}r^2) \right]$

where J_0 and J_1 are Bessel functions.

$\hat{P}^\mu\, |\sigma\rangle = 0$ State has *vanishing four-momentum* in any frame

It is like a non-trivial condensate.

A chiral condensate

Since | σ ⟩ has vacuum quantum numbers and zero momentum it can mix with the perturbative vacuum without violating Poincaré invariance

Ansatz: $|\chi\rangle = \exp(\hat{\sigma})\,|0\rangle$ implies $\langle\chi|\bar{\psi}\psi\,|\chi\rangle = 4N_\sigma$

An infinitesimal chiral rotation of the condensate gives rise to a pion

$$U_\chi(\beta)\,|\chi\rangle = (1 - 2i\beta\,\hat{\pi}\,|\chi\rangle$$

where $\hat{\pi}$ is the massless 0^{-+} state with wave function $\Phi_\pi = \gamma_5\Phi_\sigma$

The massless pion is annihilated by the axial current:

$$\langle\chi|j_5^\mu(x)\hat{\pi}\,|\chi\rangle = iP^\mu f_\pi\, e^{-iP\cdot x} \quad = 0$$

Bound states built on $|\chi\rangle$

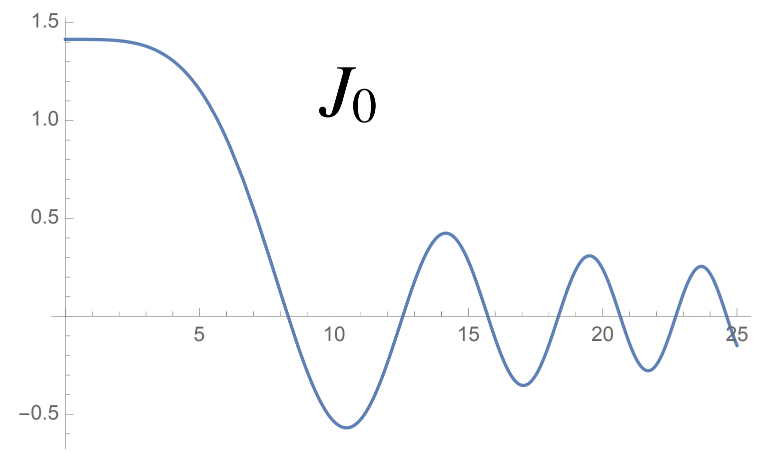
$$|M\rangle_\chi = \int d\boldsymbol{x}_1\, d\boldsymbol{x}_2\, \bar{\psi}(\boldsymbol{x}_1)\, \Phi(\boldsymbol{x}_1 - \boldsymbol{x}_2)\, \psi(\boldsymbol{x}_2)\, |\chi\rangle$$

The fields in $|\chi\rangle$ will break chiral invariance (no parity doublets).

For low momentum transfers Φ_σ may be approximated to be pointlike

$$\Phi_\sigma(\boldsymbol{x}) \to \Phi_{\sigma 0}(\boldsymbol{x}) = \delta^3(\boldsymbol{x})\phi_0$$

$$|\chi\rangle \to |\chi_0\rangle = \exp\left[\phi_0 \int d\boldsymbol{x}\, \bar{\psi}(\boldsymbol{x})\psi(\boldsymbol{x})\right] |0\rangle$$



The contractions of $\bar{\psi}(\boldsymbol{x}_1)\psi(\boldsymbol{x}_2)$ with $\bar{\psi}\psi$ in $|\chi\rangle$ will have the effect of a mass term in $\mathcal{H}_V$

$\Rightarrow$ Momentum dependent mass term as in the DSE approach?

Some topical issues

- Validity of perturbative S-matrix with bound asymptotic states

$$\mathcal{H} = \mathcal{H}_V + \mathcal{H}_I \qquad\qquad \mathcal{H}_V = \mathcal{H}_0 + \mathcal{H}_I(A_{cl})$$

- Equal-time bound states in motion
  - $\boldsymbol{P}$-dependence of wave function (fixed field)
  - Precession of state (in the magnetic field due to the boost)

- Phenomenology of chiral symmetry breaking with $m_u, m_d \neq 0$

- Hadron spectrum (including baryons)

- Duality and Parton distributions

- Hadron decays and scattering amplitudes (string breaking)

Back-up slides

Baryons

For baryons an analogous procedure gives the confining potential:

$$V_{\mathcal{B}}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) = \frac{g\Lambda^2}{\sqrt{2}}\sqrt{(\boldsymbol{x}_1-\boldsymbol{x}_2)^2+(\boldsymbol{x}_2-\boldsymbol{x}_3)^2+(\boldsymbol{x}_3-\boldsymbol{x}_1)^2}$$

It agrees with the meson potential when two quarks coincide:

$$V_{\mathcal{B}}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_2) = V_{\mathcal{M}}(\boldsymbol{x}_1-\boldsymbol{x}_2)$$

Translation invariance requires color singlet meson and baryon states.

The "external" color field vanishes also for the qqq states.

For SU(3) this type of solution only exists for $q\bar{q}$ and qqq states.

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The Dirac Electron in Simple Fields*

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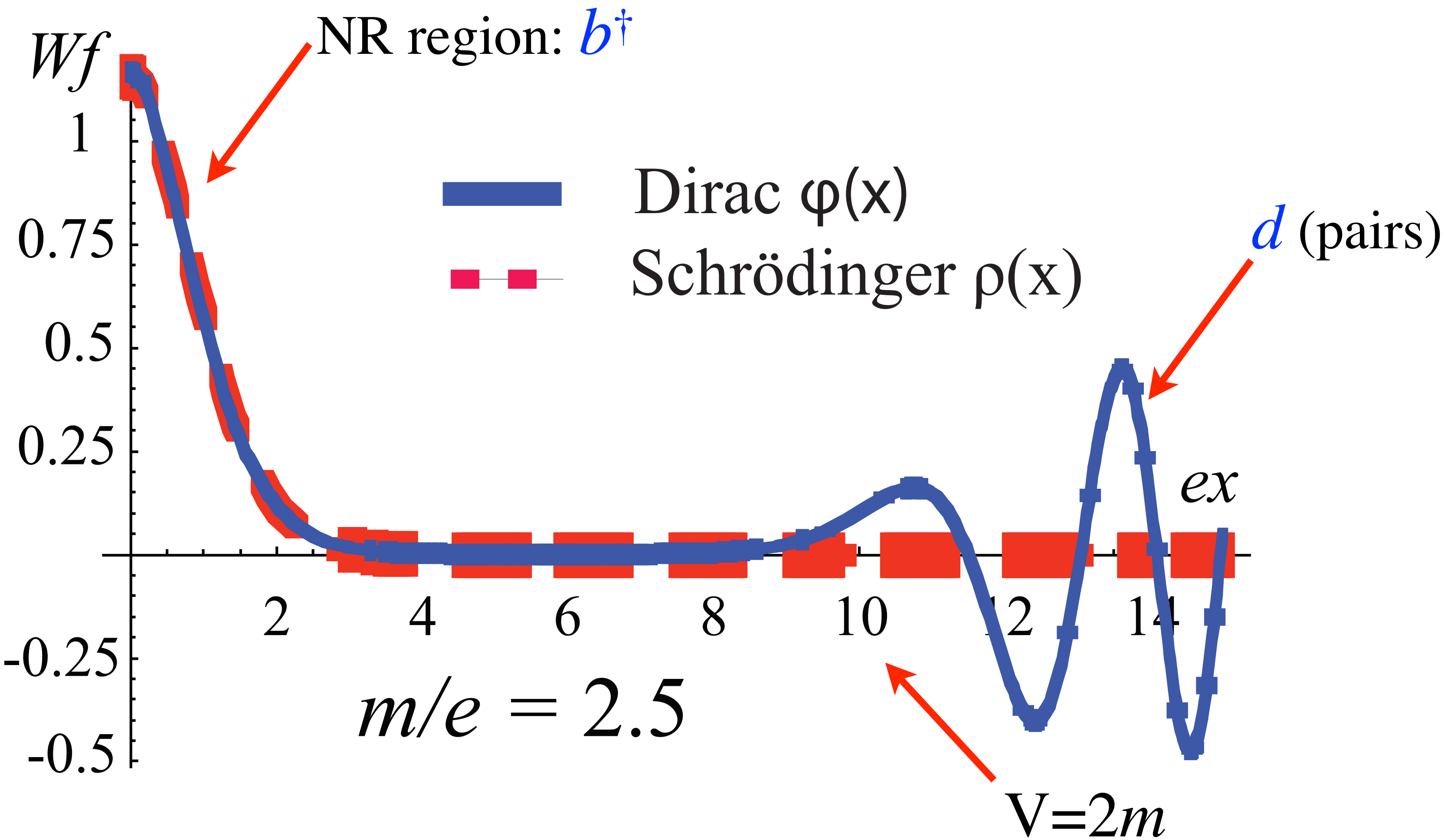
(Received June 6, 1932)

The relativity wave equations for the Dirac electron are transformed in a simple manner into a symmetric canonical form. This canonical form makes readily possible the investigation of the characteristics of the solutions of these relativity equations for simple potential fields. If the potential is a polynomial of any degree in x, a continuous energy spectrum characterizes the solutions. If the potential is a polynomial of any degree in $1/x$, the solutions possess a continuous energy spectrum when the energy is numerically greater than the rest-energy of the electron; values of the energy numerically less than the rest-energy are barred. When the potential is a polynomial of any degree in r, all values of the energy are allowed. For potentials which are polynomials in $1/r$ of degree higher than the first, the energy spectrum is again continuous. The quantization arising for the Coulomb potential is an exceptional case.

See also: E. C. Titchmarsh, Proc. London Math. Soc. (3) 11 (1961) 159 and 169; Quart. J. Math. Oxford (2), 12 (1961), 227.

Example of Dirac wave functions: $V(x) = \frac{1}{2}e^2|x|$ in D=1+1

$$|M \geq 0\rangle = \int \frac{dp}{2\pi 2E} \int dx \Big[b_p^\dagger u^\dagger(p) e^{-ipx} + d_p v^\dagger(p) e^{ipx}\Big] \left[\begin{array}{c} \varphi(x) \\ \chi(x) \end{array}\right] |\Omega\rangle$$

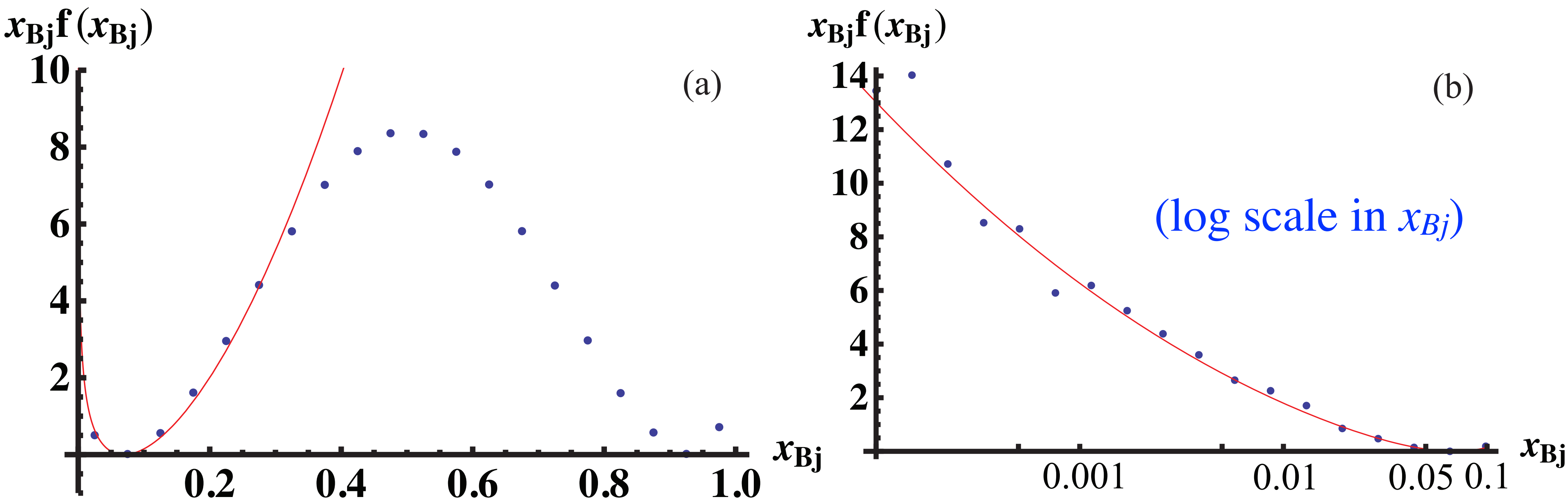


The “single particle” Dirac wave function contains pair contributions (duality)

Parton distributions have a sea component

The sea component is prominent at low m/e :

$$m/e = 0.1$$



The red curve is an analytic approximation, valid in the $x_{Bj} \to 0$ limit.

Note: Enhancement at low x is not due to Φ_A^{IMF} (valence wf.)

Plane waves in bound states

In the parton picture, high energy quarks can be treated as free constituents.
They are momentum eigenstates, described by plane waves.
How does this fit into the bound state wave functions?

Consider a highly excited state (P=0): $M \to \infty$, $V(x) << M$

$\sigma = (M\text{-}V)^2 \approx M^2 - 2MV \to \infty$

$$\Phi(\sigma \to \infty) \sim \exp(\pm i\sigma/2) = e^{\pm iM^2} \exp(\mp ix\, M/2)$$

Thus oscillations of the wf at large σ gives a plane wave with $p = \pm M/2$

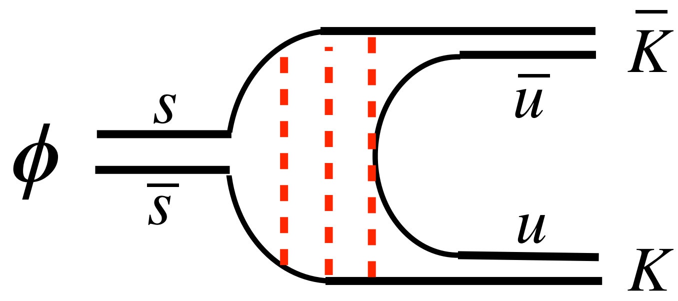
The operator expression for the state is in this limit:

$$|M, P=0\rangle = \frac{\sqrt{2\pi}}{2M}\big(b^\dagger_{M/2} d^\dagger_{-M/2} + b^\dagger_{-M/2} d^\dagger_{M/2}\big)|\Omega\rangle$$

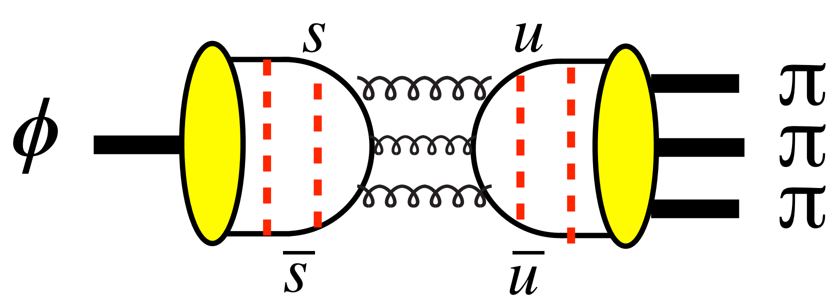
As in the parton picture, only *"valence"* particles appear (no *b* or *d* operators).

Rules of Thumb - e.g., OZI

Connected diagrams: Unsuppressed, string breaking from confining potential

		ΔE	Br
$\phi(1020) \to K\bar{K}$		26 MeV	83.1 %

Disconnected, perturbative diagrams are suppressed

$\phi(1020) \not\to \pi\pi\pi$		610 MeV	15.3 %

This suggests that perturbative corrections are small even in the soft regime.

$q\bar{q}$ wave functions

The separation of angular and radial coordinates in the BSE

$$i\boldsymbol{\nabla}\cdot\{\gamma^0\boldsymbol{\gamma},\Phi(\boldsymbol{x})\}+m\left[\gamma^0,\Phi(\boldsymbol{x})\right]=\left[E-V(r)\right]\Phi(\boldsymbol{x})$$

for any radial potential $V = V(r)$ and
equal fermion masses $m_1 = m_2 = m$ is in: Geffen and Suura, PRD 16 (1977) 3305

The solutions of given spin j and j_z are classified according to their charge conjugation C and parity P quantum numbers:

pion trajectory: $P=(-1)^{j+1}$ $C=(-1)^j$

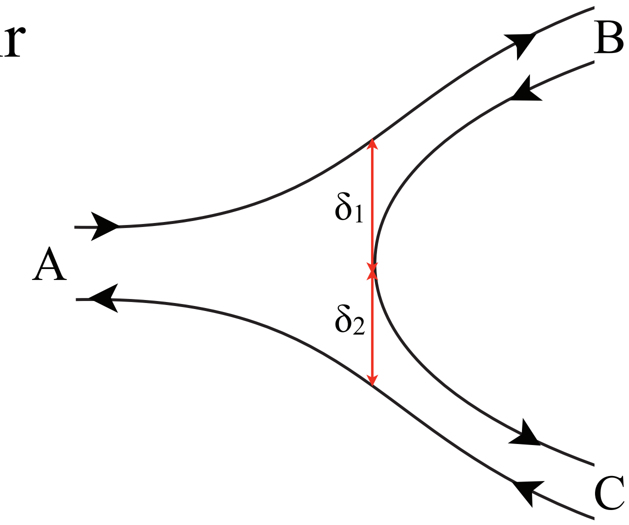
a_1 trajectory: $P=(-1)^{j+1}$ $C=(-1)^{j+1}$

rho trajectory: $P=(-1)^j$ $C=(-1)^j$

There are no “quark model exotics” with $P=(-1)^j$ and $C=(-1)^{j+1}$

String breaking: Pair production

The bound state equation was obtained neglecting pair production (string breaking).



There is an $\mathcal{O}\left(1/\sqrt{N_C}\right)$ coupling between the states:

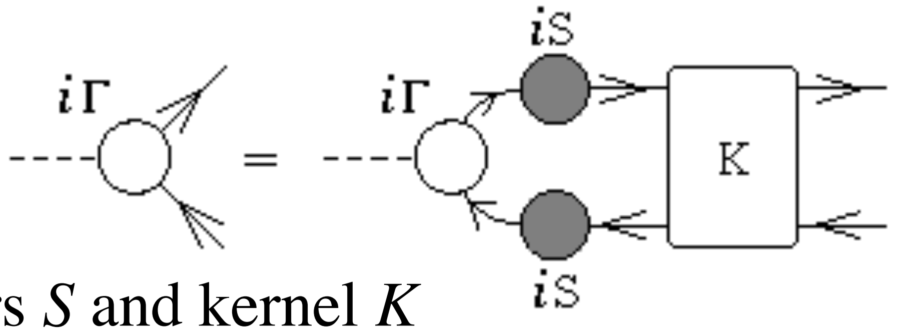
$$\langle B, C|A\rangle \ =$$

$$-\frac{(2\pi)^3}{\sqrt{N_C}}\delta^3(\boldsymbol{P}_A - \boldsymbol{P}_B - \boldsymbol{P}_C)\int d\boldsymbol{\delta}_1 d\boldsymbol{\delta}_2\, e^{i\boldsymbol{\delta}_1\cdot\boldsymbol{P}_C/2 - i\boldsymbol{\delta}_2\cdot\boldsymbol{P}_B/2}\mathrm{Tr}\left[\gamma^0\Phi_B^\dagger(\boldsymbol{\delta}_1)\Phi_A(\boldsymbol{\delta}_1+\boldsymbol{\delta}_2)\Phi_C^\dagger(\boldsymbol{\delta}_2)\right]$$

When squared, this gives a $1/N_C$ hadron loop unitarity correction.

Brief history of QFT bound states

1951: Salpeter & Bethe



Perturbatively expand propagators S and kernel K

Explicit Lorentz covariance ensured

1975: Caswell & Lepage: Not unique: ∞ # of equivalent equations, $S \leftrightarrow K$

1986: Caswell & Lepage NRQED: Effective NR field theory

Relativistic electrons are rare in atomic wave functions

Today: Accurate calculations of atomic properties use NRQED

Explicit Lorentz covariance is traded for physical arguments.
QED ensures validity of a rest frame calculation in any frame

NRQED chooses to start from Schrödinger atoms with $V(r) = -\alpha/r$