Companion poles in light and heavy mesonic sectors

Francesco Giacosa
UJK Kielce (Poland) **& Goethe U Frankfurt** (Germany)
In collaboration with
Milena Piotrowska (UJK Kielce), Susana Coito (UJK Kielce)
Thomas Wolkanowski (Goethe U, Ffm), Dirk H Rischke (Goethe U, Ffm)

Talk prepared for:
Bound states in strongly coupled systems
12-16/3/2018, Galileo Institute, Florence, Italy

Outline

From quarks and gluons to hadrons

A simple example: K*(892)

K0*(1430) and the light k

Ψ(3770)

a0(1450) and a0(980)

Outlook: Ψ(4040) and other states

Summary

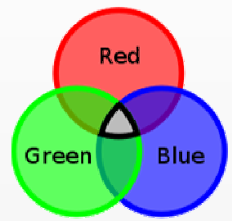
From QCD Lagrangian to baryons and mesons

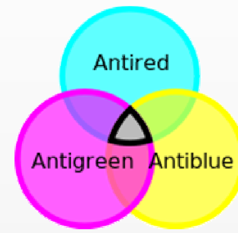
Born	Giuseppe Lodovico Lagrangia 25 January 1736 Turin
Died	10 April 1813 (aged 77) Paris

Francesco Giacosa

The QCD Lagrangian

Quark: u,d,s and c,b,t R,G,B





$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix} ; \; i = u,d,s \; ,...$$

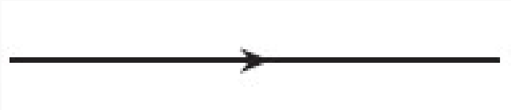
8 type of gluons (R$\overline{\text{G}}$,B$\overline{\text{G}}$,…)

$$\boxed{\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}}$$

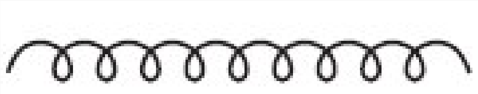
$$A_\mu^a \; ; \; a = 1,..., 8$$

Feynman diagrams of QCD

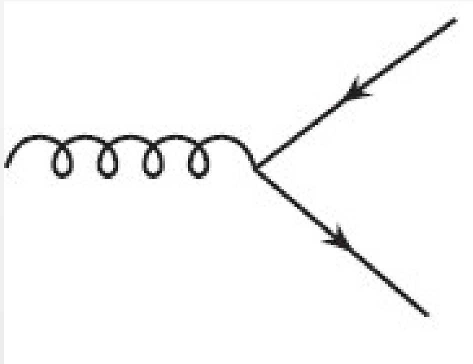
$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$



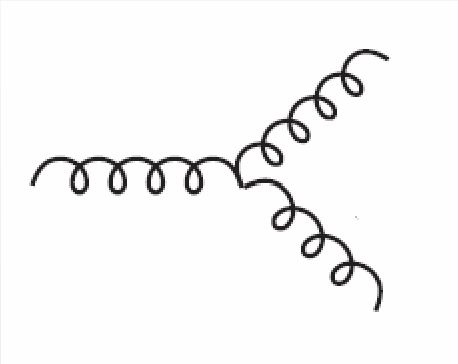
Quark



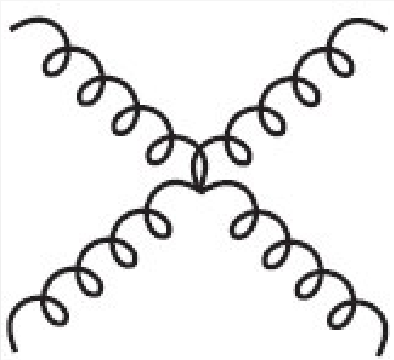
Gluon



Gluon-quark-antiquark vertex



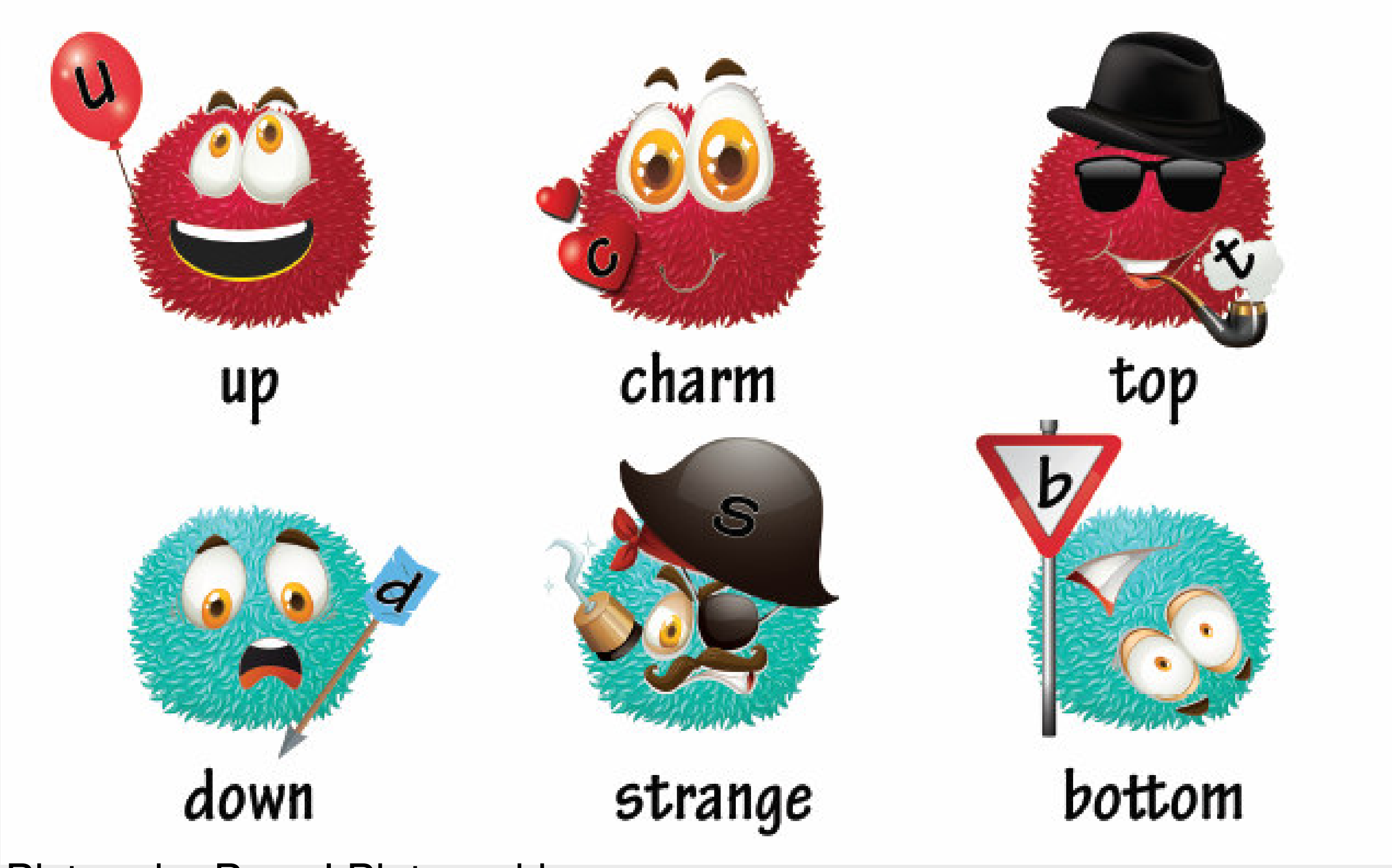
3-gluon vertex



4-gluon vertex

Already here one may ask: what about bound states of gluons???

Confinement: quarks never ‘seen’ directly. How they might look like ☺



Picture by Pawel Piotrowski

Symmetries of QCD and breakings

SU(3)color: exact. Confinement: you never see color, but only white states.

Dilatation invariance: holds only at a classical level and in the chiral limit. Broken by quantum fluctuations (**trace anomaly**) and by quark masses.

$SU(3)_R \times SU(3)_L$: holds in the chiral limit, but is broken by nonzero quark masses. Moreover, it is **spontaneously** broken to $U(3)_{V=R+L}$

$U(1)_{A=R-L}$: holds at a classical level, but is also broken by quantum fluctuations (**axial anomaly**)

Hadrons

The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‚colored‘ state has been seen.

Confinement: physical states are white and are called hadrons.

Hadrons can be:

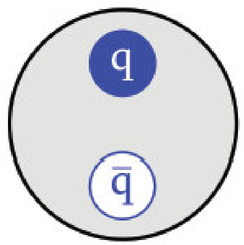
Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

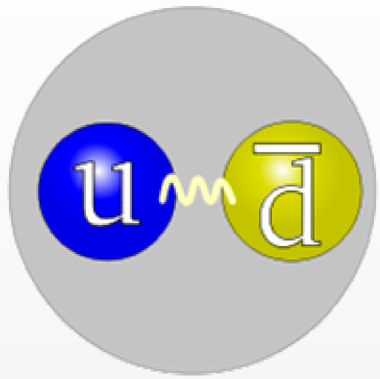
Conventional mesons

- Quark: u,d,s,... R,G,B
- Conventional mesons: quark − antiquark bound states.



- $|color>= \sqrt{1/3}(\bar{R}R + \bar{G}G + \bar{B}B)$
- With $q\bar{q}$ states we can understand a lot of QCD, but not everything.

Example of conventional quark-antiquark states: the ρ and the π mesons



$$m_u + m_d \approx 7 \ \text{MeV}$$

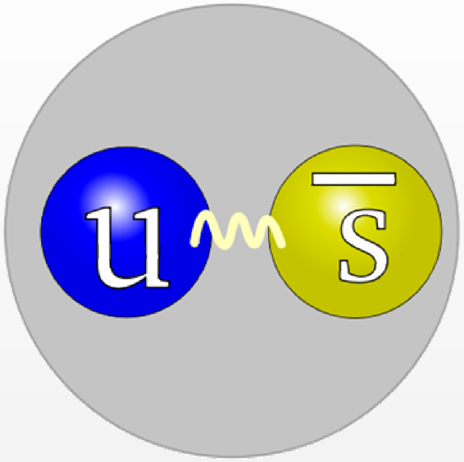
Vector channel: Rho-meson $\quad m_{\rho^+} = 775 \ \text{MeV}$

Mass generation in QCD is a nonpert. penomenon based on SSB

Pseudoscalar channel: Pion $\quad m_{\pi^+} = 139 \ \text{MeV}$

In the scalar channel, the situation is more complicated: a0(1450) and a0(980), see later on

Example of conventional quark-antiquark states: the K*(892) and the K mesons

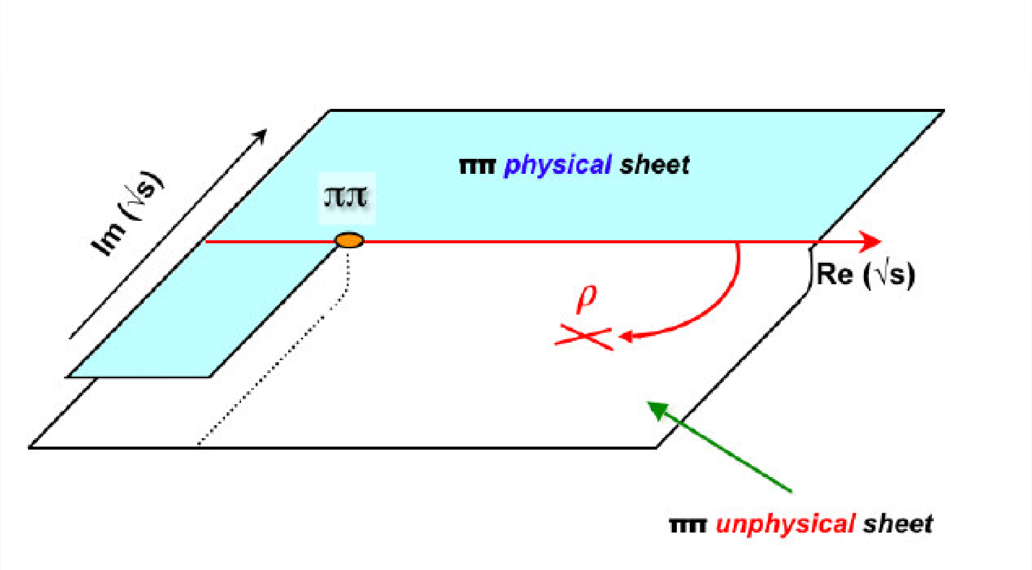


In the vector channel: K*(892) (brother of the rho meson)

In the pseudoscalar channel: positively charged kaon.

In the scalar channel, the situation is more complicated:
scalar kaons K0*(1430) and K0*(800) (see later).

Resonances: poles in complex plane



Classification of some conventional light mesons

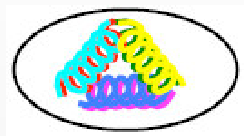
State	S	L	J	P	C	J^{PC}	Mesons	Name
$^1S_0$	0	0	0	-	+	0^{-+}	π η η' K	pseudoscalar
$^3S_1$	1	0	0	-	-	1^{--}	ρ ω ϕ K^*	vector
$^1P_1$	0	1	1	+	-	1^{+-}	b_1 h_1 h_1' K_1	pseudo-vector
$^3P_0$	1	1	0	+	+	0^{++}	a_0 f_0 f_0' K_0^*	scalar
$^3P_1$	1	1	1	+	+	1^{++}	a_1 f_1 f_1' K_1	axial vector
$^3P_2$	1	1	2	+	+	2^{++}	a_2 f_2 f_2' K_2^*	tensor

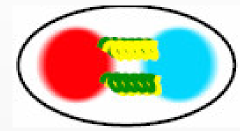
- Not all quantum numbers are permitted for a quark - antiquark states.

$$J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}, \cdots$$

are exotic quantum numbers.

Non-conventional mesons: theoretical expectations

1) Glueballs 

2) Hybrids 

3) Four-quark states

Compact diquark-antidiquark states 

Molecular states (a type of dynamical generation) 

Companion poles (another type of dynamical generation)

Companion poles

- In the following papers the idea of companion poles was discussed.

N. A. Törnqvist, *Understanding the scalar meson $q\bar{q}$ nonet*, *Z. Phys. C* **68** (1995) 647 [arXiv:hep-ph/9504372]; N. A. Törnqvist and M. Roos, *Confirmation of the Sigma Meson*, *Phys. Rev. Lett.* **76** (1996) 1575 [arXiv:hep-ph/9511210].

M. Boglione and M. R. Pennington, *Dynamical generation of scalar mesons*, *Phys. Rev. D* **65** 114010 (2002) [arXiv:hep-ph/0203149].

- In particular, the states K*(892) and a0(980) and represents a nice example (see later on).

- Related ideas studied by E. Oset, J. Pelaez, G. Rupp, Van Beveren,...

Loops in a simple ‘boring’ example: K*(892)

based on M. Soltysiak, T. Wolkanowski and F. G.,
Large-Nc pole trajectories of the vector kaon K*(892)
and of the scalar kaons K0*(800) and K0*(1430),"
Acta Phys. Polon. Supp.9 (2016) 321
[arXiv:1604.01636 [hep-ph]].

K*(892) from PDG

$K^*(892)$

$$I(J^P) = \tfrac{1}{2}(1^-)$$

$K^*(892)^{\pm}$ hadroproduced mass $m = 891.66 \pm 0.26$ MeV
$K^*(892)^{\pm}$ in τ decays mass $m = 895.5 \pm 0.8$ MeV
$K^*(892)^0$ mass $m = 895.81 \pm 0.19$ MeV (S = 1.4)
$K^*(892)^{\pm}$ hadroproduced full width $\Gamma = 50.8 \pm 0.9$ MeV
$K^*(892)^{\pm}$ in τ decays full width $\Gamma = 46.2 \pm 1.3$ MeV
$K^*(892)^0$ full width $\Gamma = 47.4 \pm 0.6$ MeV (S = 2.2)

$K^*(892)$ DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
$K\pi$	$\sim$ 100 %		289
$K^0\gamma$	$(2.46\pm0.21) \times 10^{-3}$		307
$K^{\pm}\gamma$	$(9.9 \pm0.9) \times 10^{-4}$		309
$K\pi\pi$	$< 7 \times 10^{-4}$	95%	223

A simple model for K*(892)

- Lagrangian:

$$\boxed{\mathcal{L}_v = cK^*(892)^+_\mu \partial^\mu K^- \pi^0 + \dots}$$

- Decay width:

$$\Gamma_{K^*}(m) = 3\frac{|\vec{k}_1|}{8\pi m^2}\frac{c^2}{3}\left[-M_\pi^2 + \frac{(m^2+M_\pi^2-M_K^2)^2}{4m^2}\right]F_\Lambda(m)$$

where:

$$F_\Lambda(m) = e^{-2\left|\vec{k}_1\right|^2/\Lambda^2}$$

$$\left|\vec{k}_1\right| = \frac{\sqrt{m^4+\left(M_K^2-M_\pi^2\right)^2-2\left(M_K^2+M_\pi^2\right)m^2}}{2m}\theta\left(m-M_K-M_\pi\right)$$

- The scalar part of the propagator of $K^*(892)$:

$\Delta_{K^*}(p^2=m^2) = \frac{1}{m^2-M_0^2+\Pi(m^2)+i\varepsilon}$ where M_0 is the bare mass of the vector

According to the optical theorem, $\operatorname{Im}\Pi(m) = m\Gamma_{K^*}(m)$

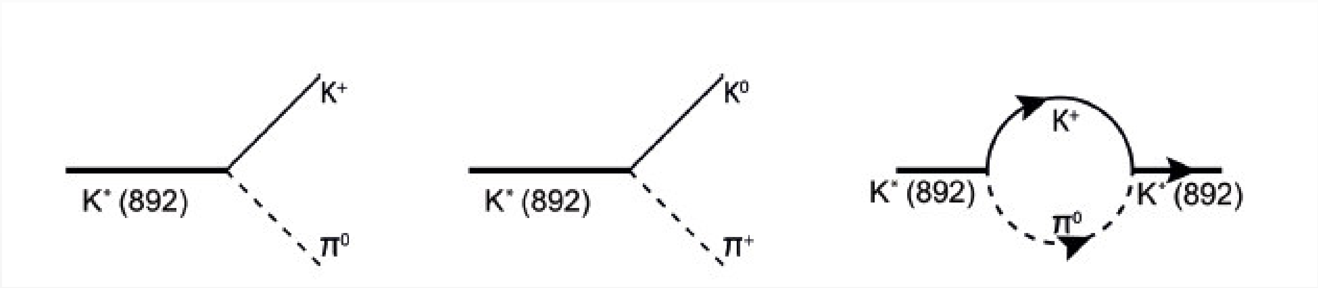
$$\operatorname{Re}\Pi(s) = \frac{1}{\pi}\fint ds' \,\frac{-\operatorname{Im}\Pi(s')}{s-s'}$$

Form factor: it can be included in the Lagrangian by making it nonlocal.
Even if it cuts the three-momentum, a covariant generalization is possible.

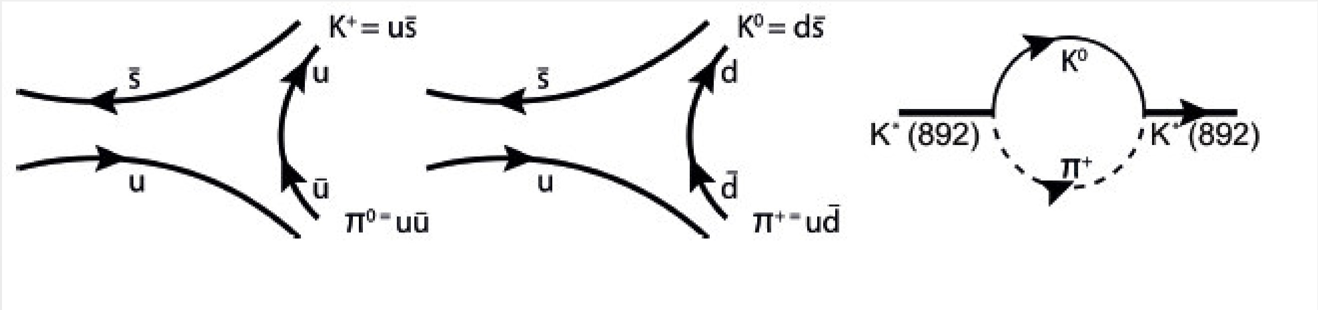
M. Soltysiak and F. Giacosa, ``A covariant nonlocal Lagrangian for the description of the scalar kaonic sector,''
Acta Phys.\ Polon.\ Supp.\ {\bf 9} (2016) 467 [arXiv:1607.01593 [hep-ph]].

Uniwersytet Jana Kochanowskiego w Kielcach

Tree-level and loops for K*(892)



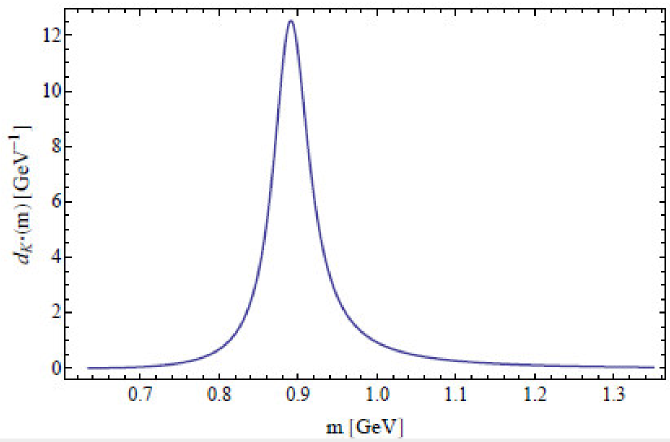
Microscopically, they correspond to:



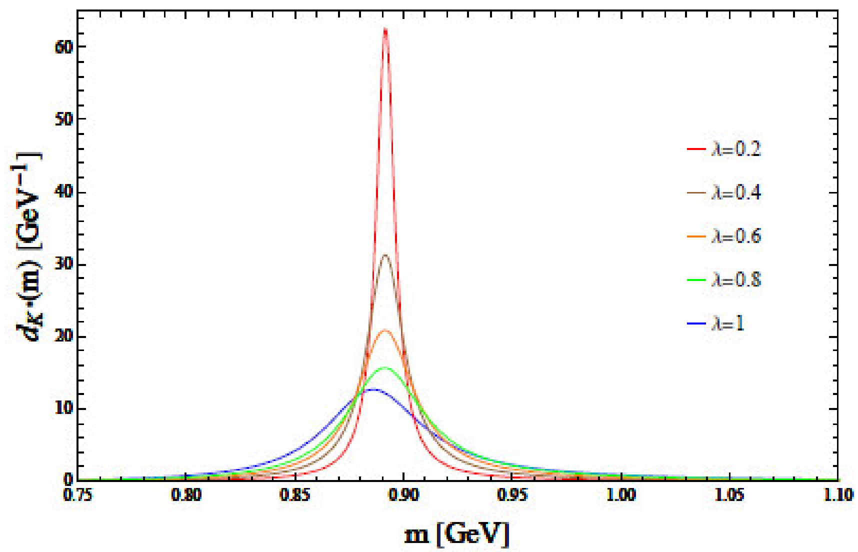
Spectral function

Spectral function $d_{K^*}(m)dm$ determines the probability that $K^*(892)$ has a mass beetwen m and $m + dm$.

- Specral function:
  $d_{K^*}(m) = \frac{2m}{\pi} |\operatorname{Im} \Delta_{K^*}(p^2 = m^2)|$
- normalization condition:
  $\int_0^\infty d_{K^*}(m)\mathrm{dm} = 1.$



Large-Nc study of K*(892)

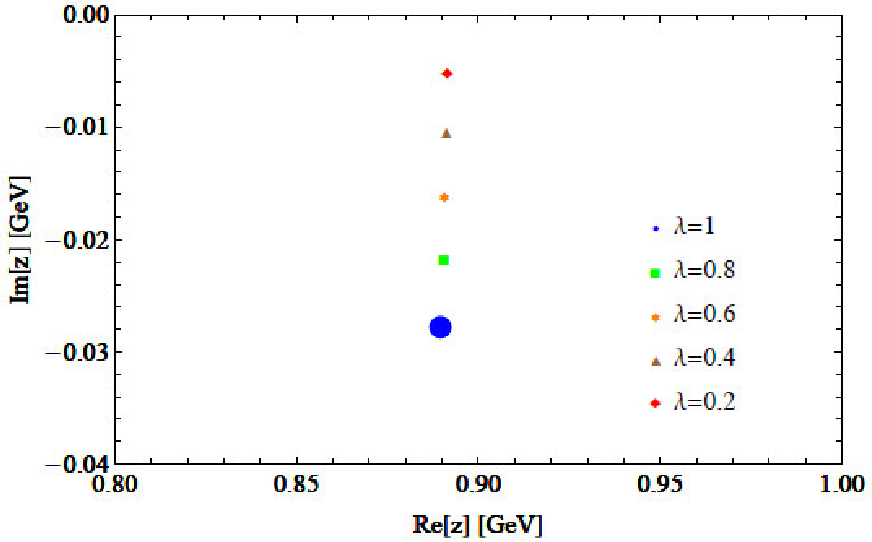


1604.01636

$c \to \sqrt{\lambda} c, \qquad \lambda \equiv \frac{3}{N_c} \qquad N_c$ is the number of colors

For large-N_c the spectral function tends to a Dirac$-\delta$, as expected.

Pole position of K*(892)



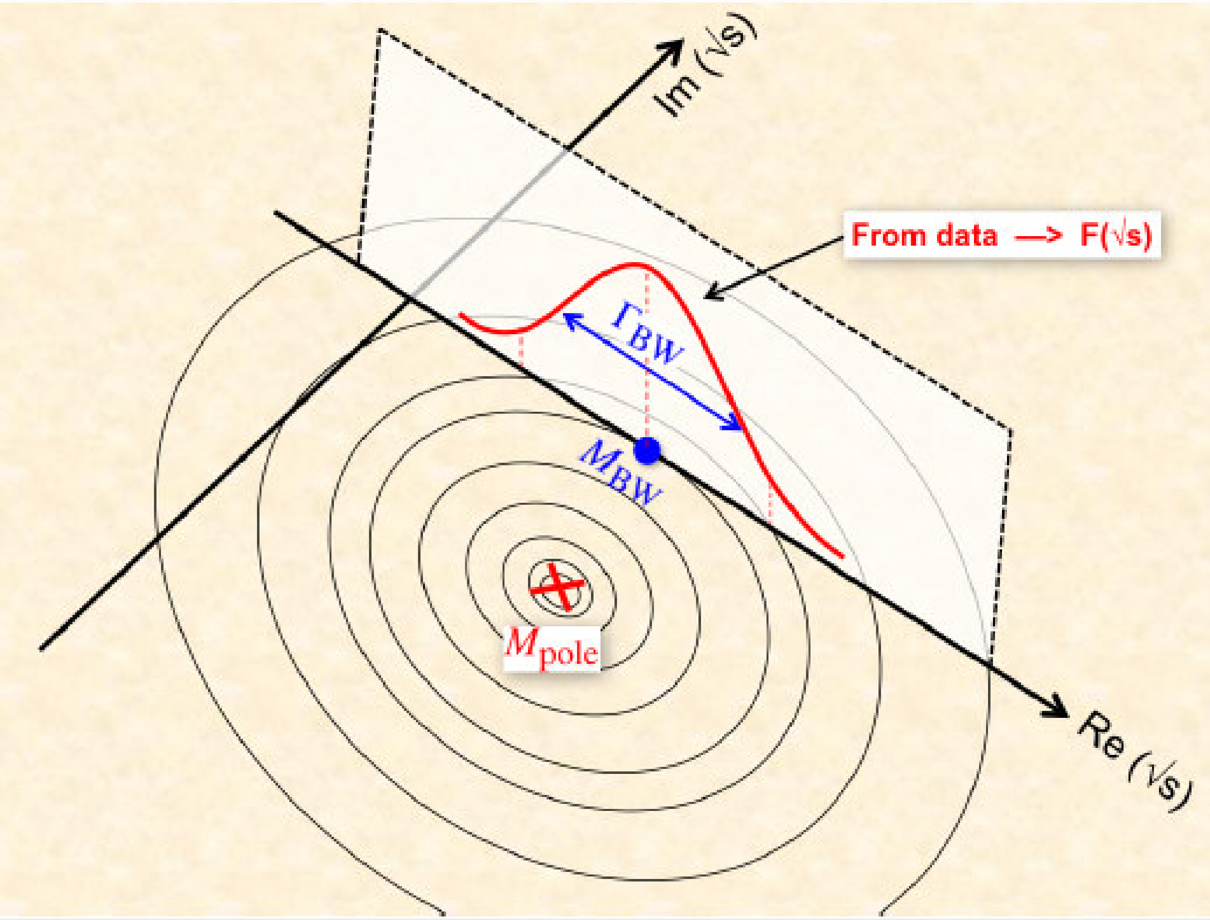
$K^*(892): 0.89 - 0.028i\ (GeV)$

For large N_c the pole tends to the real axis.

1604.01636

- It behaves like a Breit-Wigner resonance.
- one peak – one single pole.
- Large$-N_c$ in agreement with $q\bar{q}$.

Narrow state, nice corresponce



Short digression: non-exponential decay in QFT

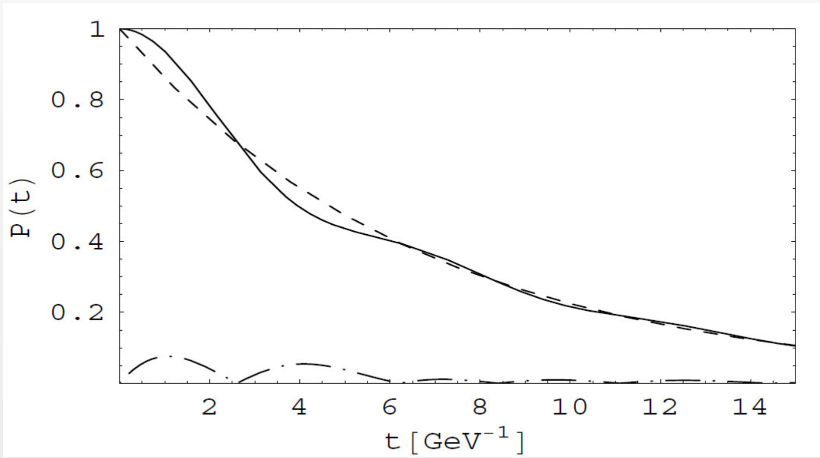
Survival probability amplitude:

$$\boxed{a(t) = \int_0^\infty dm\, d_S(m) e^{-imt}}$$

Just as in QM: non-trivial result!

No dep. on cutoff for a superrenormalizable field theory

Example: p(t) for the ρ meson



[arXiv:1005.4817 [hep-ph]]

More details in:

F. G., Non-exponential decay in quantum field theory and in quantum mechanics: the case of two (or more) decay channels, Found. Phys. 42 (2012) 1262 [arXiv:1110.5923].

K0*(800) as a companion pole of K0*(1430)

based on M. Soltysiak, T. Wolkanowski and F. G.,
K0*(800) as a companion pole of K0*(1430),'
Nucl. Phys. B 909 (2016) 418
[arXiv:1512.01071 [hep-ph]].

K0*(1430) from PDG

$K_0^*(1430)$ [nn] $\qquad I(J^P) = \frac{1}{2}(0^+)$

Mass $m = 1425 \pm 50$ MeV
Full width $\Gamma = 270 \pm 80$ MeV

$K_0^*(1430)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$K\pi$	(93 ± 10) %	619
$K\eta$	$(8.6^{+2.7}_{-3.4})$ %	486

K0*(800) from PDG

$K_0^*(800)$
or κ

$I(J^P) = \frac{1}{2}(0^+)$

OMITTED FROM SUMMARY TABLE

Needs confirmation. See the mini-review on scalar mesons under $f_0(500)$ (see the index for the page number).

$K_0^*(800)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
682 ±29 OUR AVERAGE		Error includes scale factor of 2.4. See the ideogram below.		

Simple Lagrangian for K0*(1430)

- Lagrangian:

$$\mathcal{L}_{int} = aK_0^{*+}K^-\pi^0 + bK_0^{*+}\partial_\mu K^-\partial^\mu\pi^0 + \ldots$$

- Decay width:

$$\Gamma_{K_0^*}(m) = 3\frac{\left|\vec{k}_1\right|}{8\pi m^2}\left[a - b\frac{m^2 - M_K^2 - M_\pi^2}{2}\right]^2 F_\Lambda(m)$$

where:

$$F_\Lambda(m) = e^{-2\left|\vec{k}_1\right|^2/\Lambda^2}$$

$$\left|\vec{k}_1\right| = \frac{\sqrt{m^4 + \left(M_K^2 - M_\pi^2\right)^2 - 2\left(M_K^2 + M_\pi^2\right)m^2}}{2m}\theta\left(m - M_K - M_\pi\right)$$

for $m = M_{K_0^*} \simeq 1.43$ GeV we have tree-level decay width $\Gamma_{K_0^*}^{tl} = \Gamma_{K_0^*}(M_{K_0^*})$.

Propagator of K0*(1430)

- Propagator of the scalar kaonic field:

  $\Delta_{K_0^*}(p^2=m^2)=\frac{1}{m^2-M_0^2+\Pi(m^2)+i\varepsilon}$ where M_0 is the bare mass of the scalar

- Specral function:

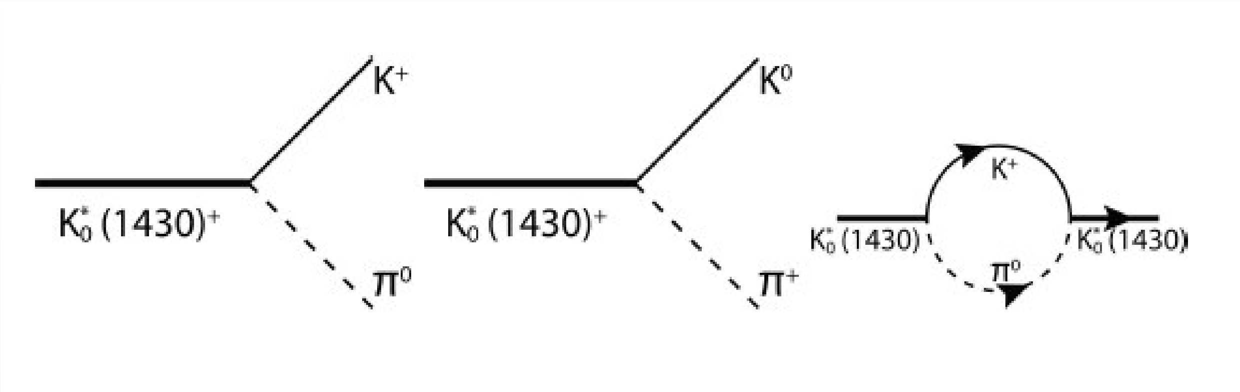
$$d_{K_0^*}(m)=\frac{2m}{\pi}|\,\mathrm{Im}\,\Delta_{K_0^*}(p^2=m^2)|$$

  normalization condition:

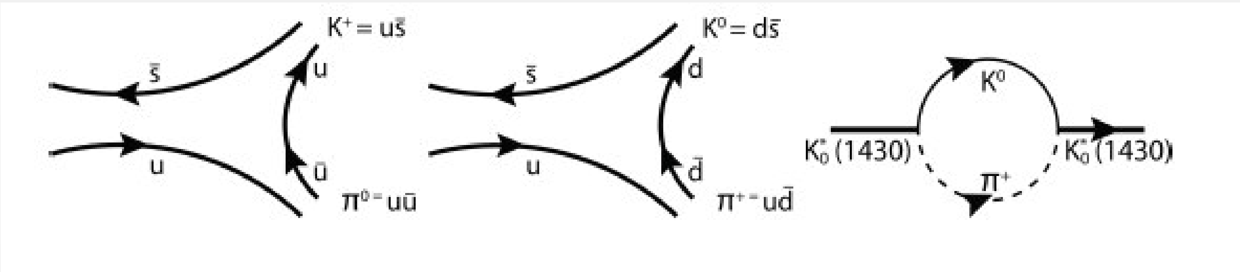
$$\int_0^\infty d_{K_0^*}(m)\mathrm{dm}=1.$$

  According to the optical theorem, $\mathrm{Im}\,\Pi(m)=m\Gamma_{K_0^*}(m)$.

Tree-level decays and loops

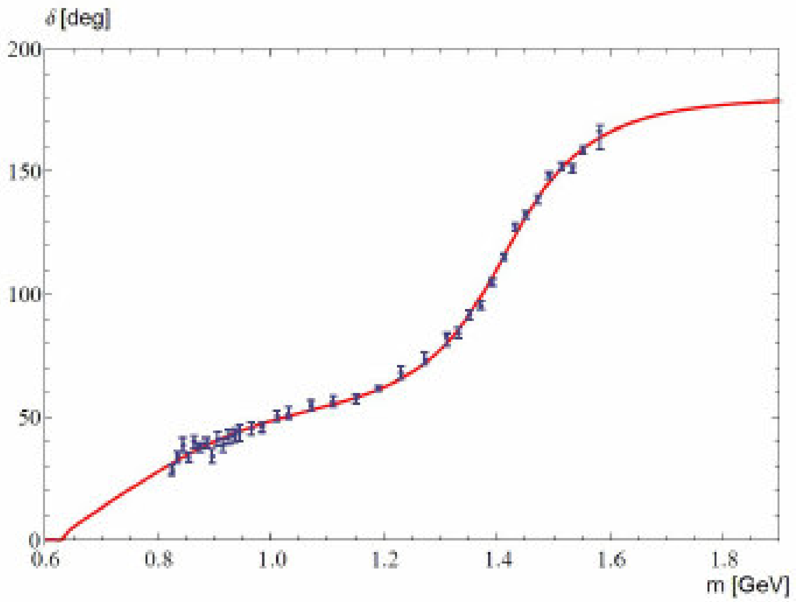


Microscopically, they correspond to:



Phase shift

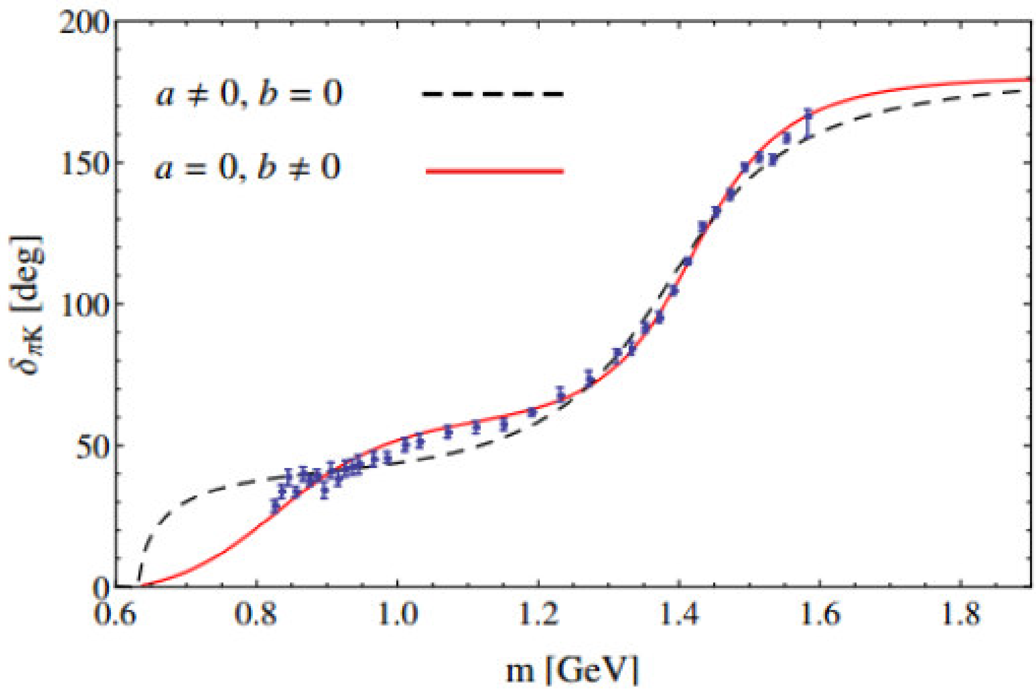
$$\delta(m) = \frac{1}{2}\arccos\left[1 - \pi\Gamma_{K_0^*}(m) d_{K_0^*}(m)\right] \ .$$



Fit		
	a	$1.60\pm\ 0.22\ GeV$
	b	$-11.16\pm0.82\ GeV^{-1}$
	Λ	$0.496\pm0.008\ GeV$
	M_0	$1.204\pm0.008\ GeV$

$\chi^2/d.o.f. = 1.25$

Phase shifts/2

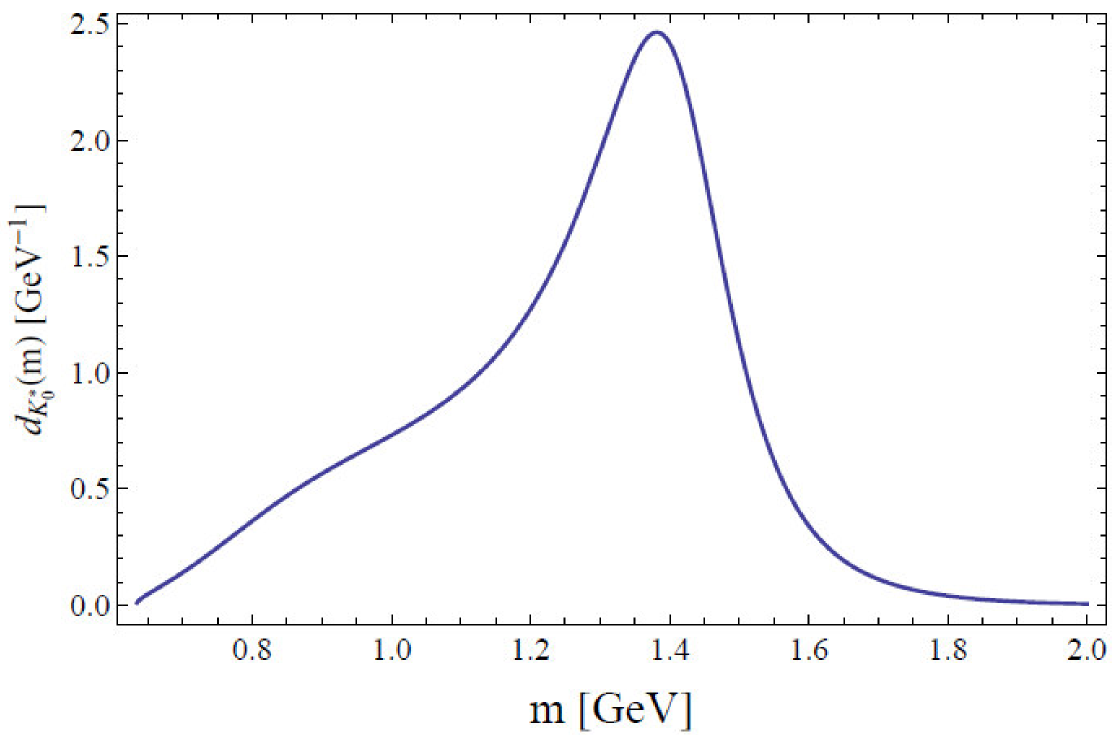


only nonderivative
$\mathcal{L}_{int} = aK_0^{*+}K^-\pi^0 + \ldots$

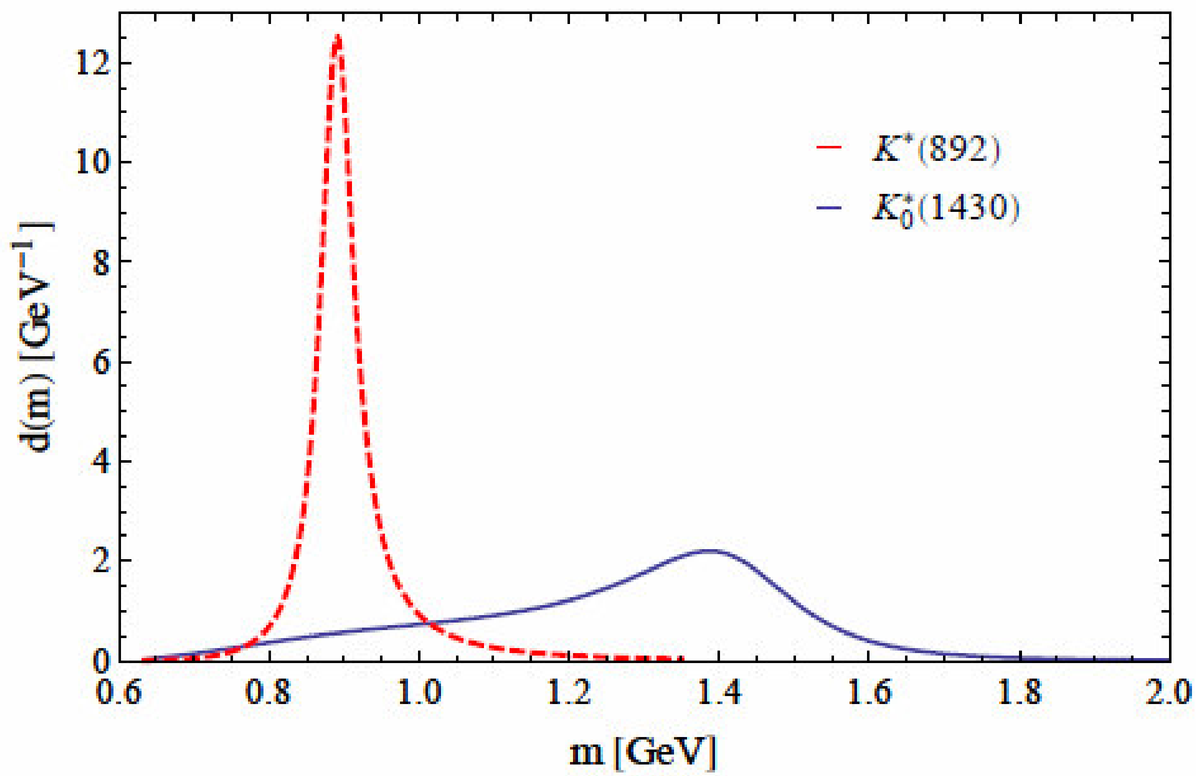
only derivative
$\mathcal{L}_{int} = bK_0^{*+}\partial_\mu K^-\partial^\mu\pi^0 + \ldots$

Spectral function

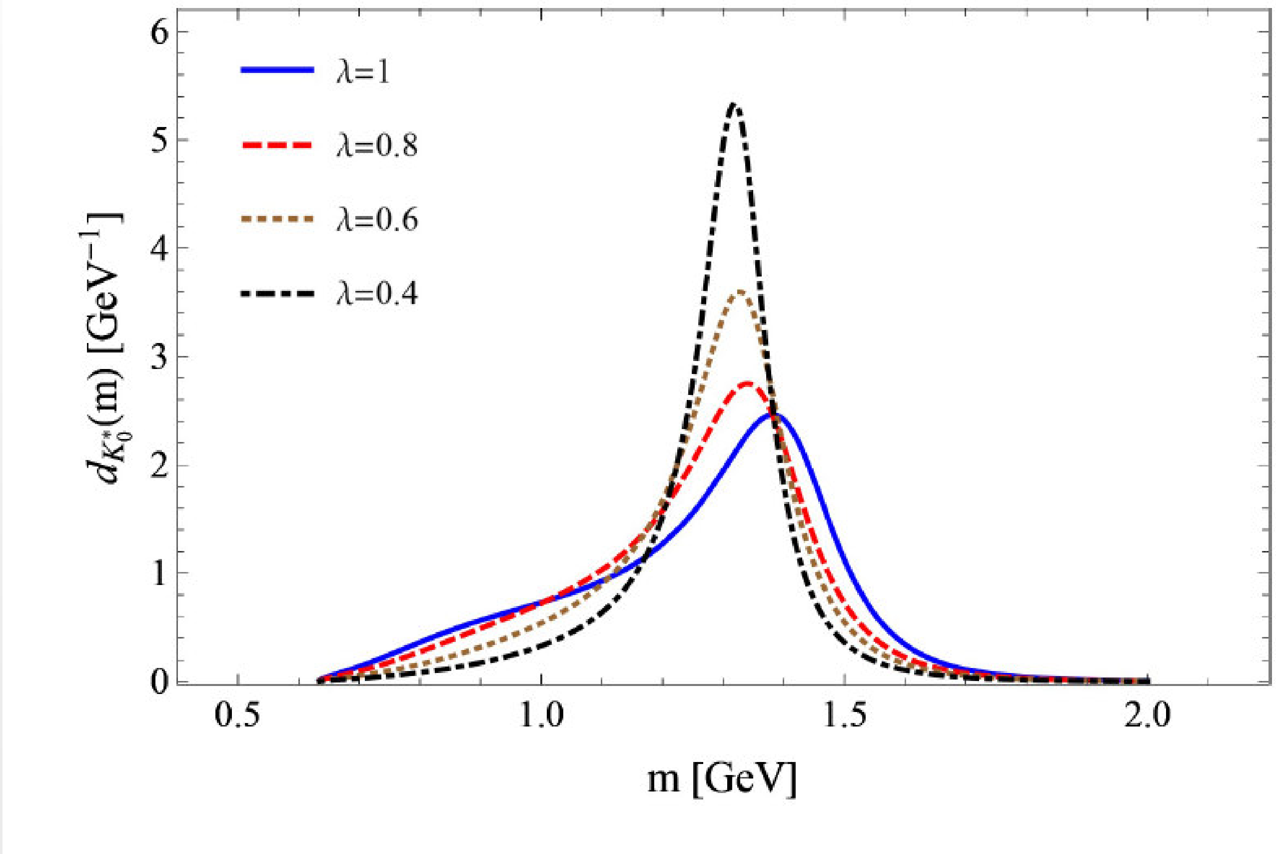
Is there a $K_0^*(800)$ or not?



Comparison of the spectral functions of K*(892) and K0*(1430)

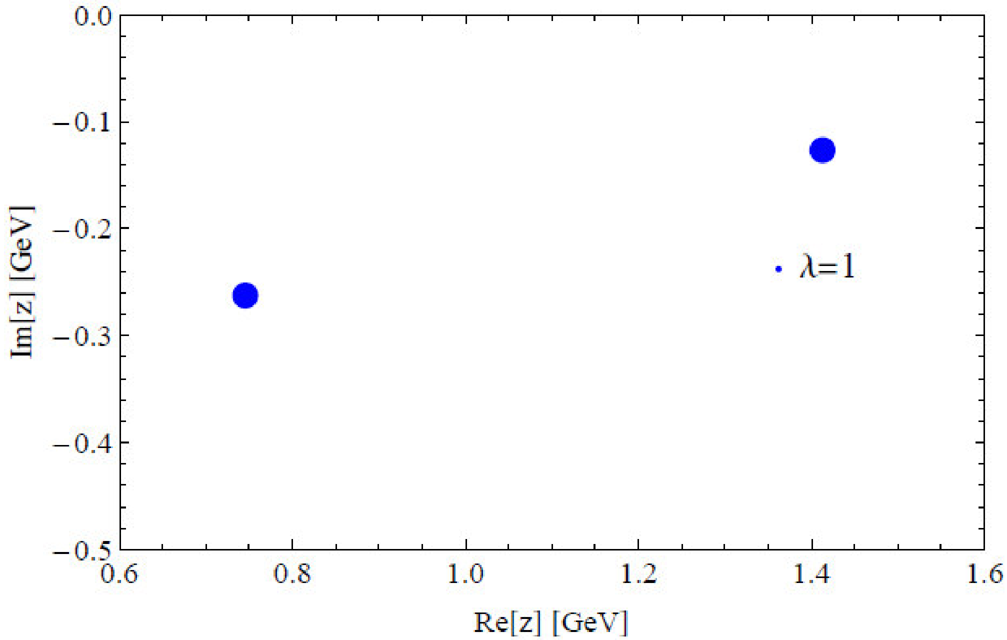


Large-Nc



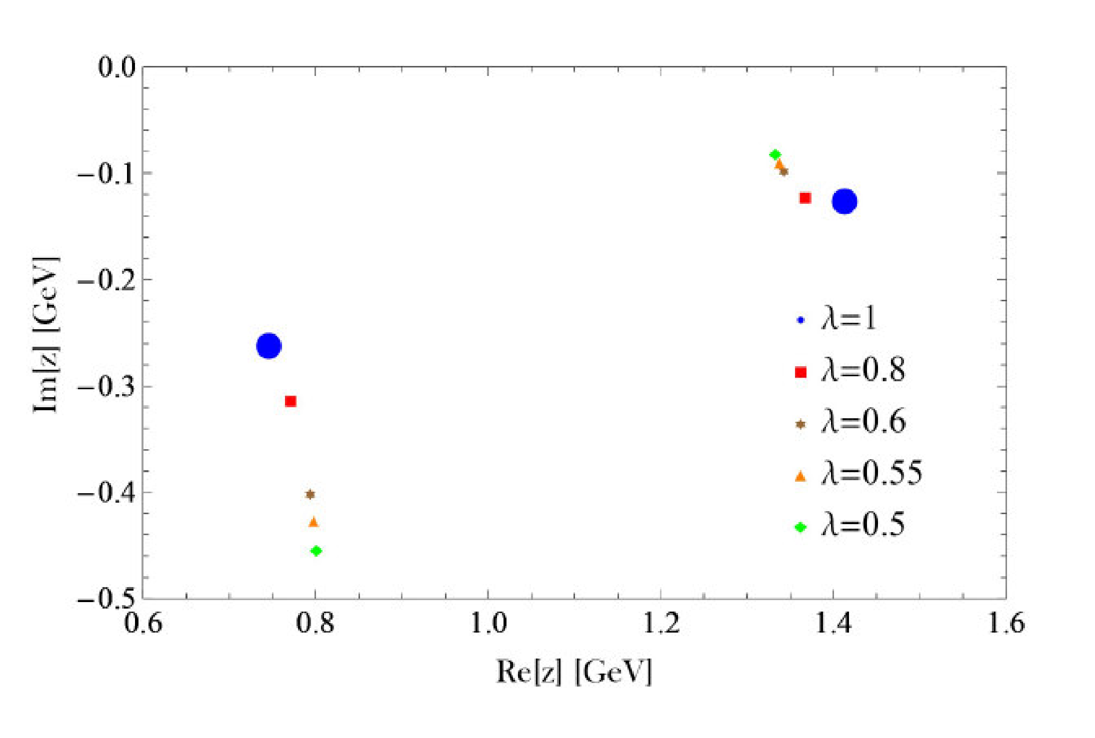
λ=1
λ=0.8
λ=0.6
λ=0.4
$d_{K_0^*}(m)$ [GeV^{-1}]
m [GeV]
0.5
1.0
1.5
2.0
0
1
2
3
4
5
6

Poles



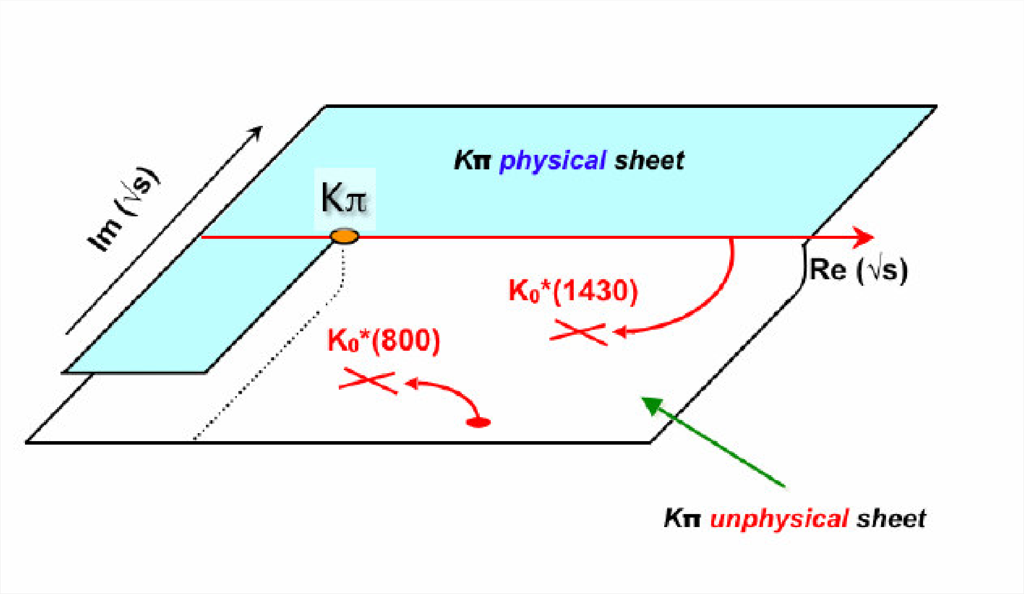
$$K_0^*(1430) : (1.413 \pm 0.057) - (0.127 \pm 0.011)\mathrm{i}\ (GeV)$$
$$K_0^*(800) : (0.745 \pm 0.029) - (0.263 \pm 0.027)\mathrm{i}\ (GeV)$$

Pole trajectories in the light scalar kaonic system



The additional companion pole on the left, corresponding to the light k, disappears for Nc larger then 12.4.

Pole trajectories/2



Considerations on the scalar kaonic system

- Scalar kaon: out of one "seed" state $\rightarrow$ 2 poles appear
  - $K_0^*(1430)$ corresponds to a peak
  - $K_0^*(800)$ "no peak" but there is a pole.
- We determined the position of the poles
  - for vector kaon ($0.89 - 0.028\mathrm{i}$ (GeV))
  - for scalar kaons
    $K_0^*(1430) : (1.413 \pm 0.057) - (0.127 \pm 0.011)\mathrm{i}$ (GeV)
    $K_0^*(800) : (0.745 \pm 0.029) - (0.263 \pm 0.027)\mathrm{i}$ (GeV)
- $K^*(892)$ is a quark-antiquark state.
- $K_0^*(1430)$ is predominantly a quark-antiquark state.
- $K_0^*(800)$ is a dynamically generated state.

Line-shape and poles of Ψ(3770)

based on S. Coito and F.G.,
Line-shape and poles of the psi(3770)
arXiv:1712.00969 [hep-ph].

General properties

- Ψ(3770)
- D-wave state, first charmonium above DD threshold.

$\psi(3770)$ $\quad I^G(J^{PC}) = 0^-(1^{--})$

$\psi(3770)$ MASS (MeV)

OUR FIT includes measurements of $m_{\psi(2S)}$, $m_{\psi(3770)}$, and $m_{\psi(3770)} - m_{\psi(2S)}$.

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
3773.13±0.35 OUR FIT Error includes scale factor of 1.1.				
3778.1 ±1.2 OUR AVERAGE				

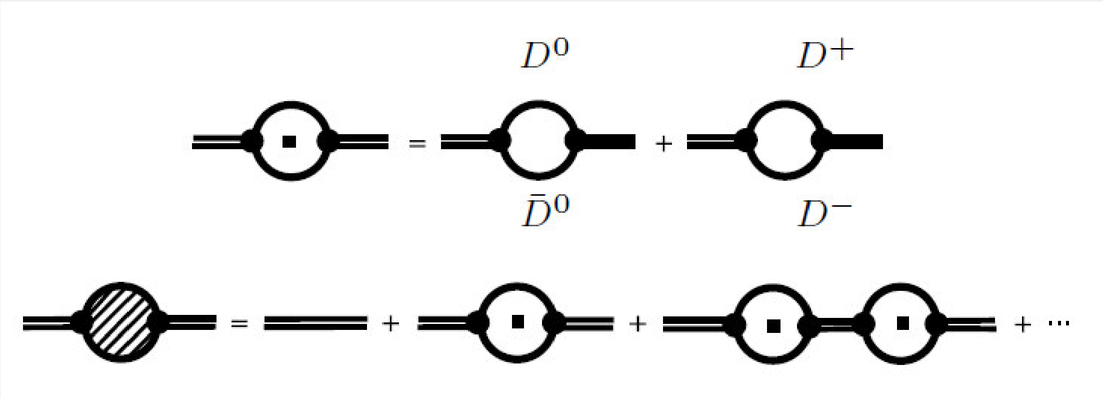
$\psi(3770)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*
27.2± 1.0 OUR FIT		
27.5± 0.9 OUR AVERAGE		

	Mode	Fraction (Γ_i/Γ)
Γ_1	$D\overline{D}$	$(93\ {}^{+8}_{-9})$ %
Γ_2	$D^0\overline{D}^0$	$(52\ {}^{+4}_{-5})$ %
Γ_3	D^+D^-	$(41\ \pm 4)$ %

Lagrangian and loops

$$\mathcal{L}_{\psi D\bar{D}} = i g_{\psi D^0 \bar{D}^0} \psi_\mu \left(\partial^\mu D^0 \bar{D}^0 - \partial^\mu \bar{D}^0 D^0 \right) + i g_{\psi D^+ D^-} \psi_\mu \left(\partial^\mu D^+ D^- - \partial^\mu D^- D^+ \right)$$



Fit to data

$$\sigma_{e^+e^-\to D\bar{D}} = \frac{\pi}{2E} g^2_{\psi e^+e^-} d_\psi(E)$$

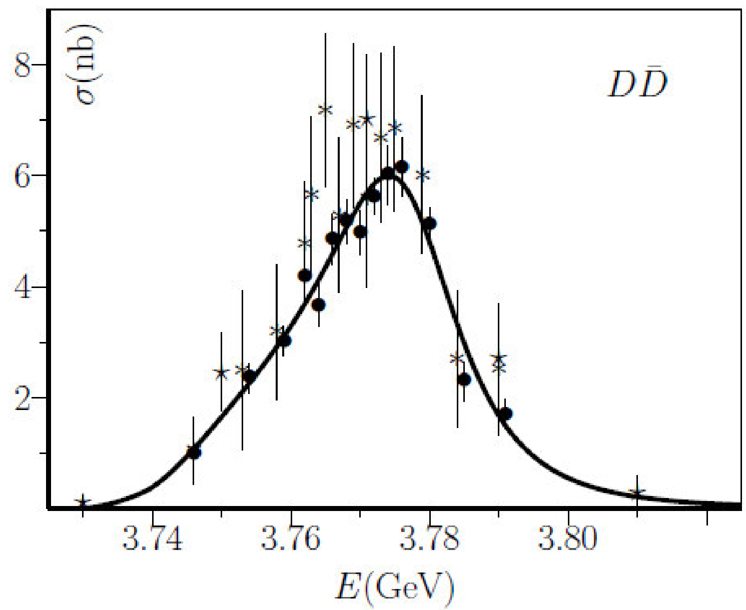
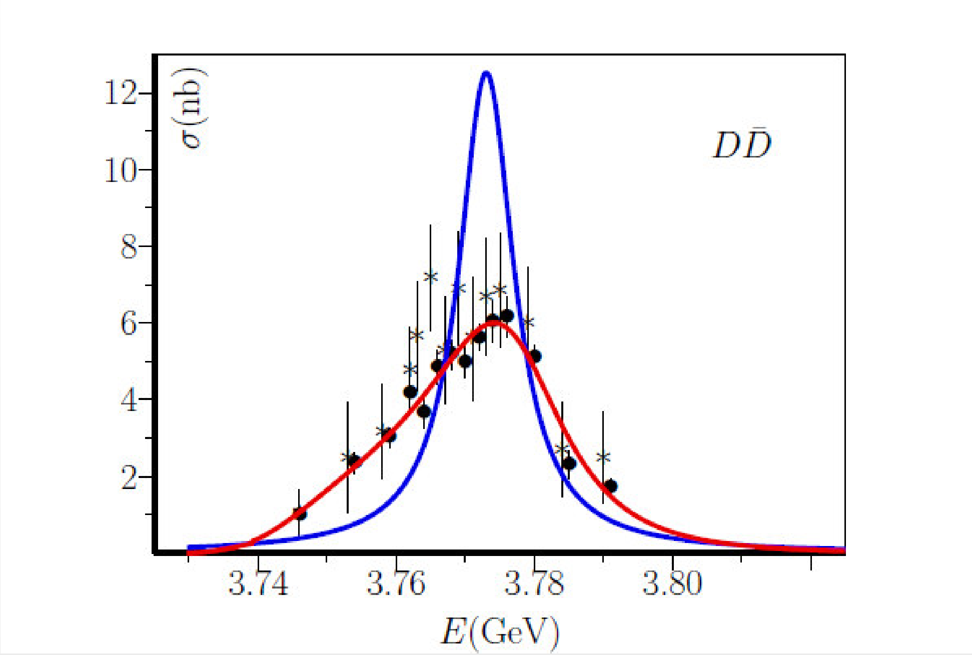


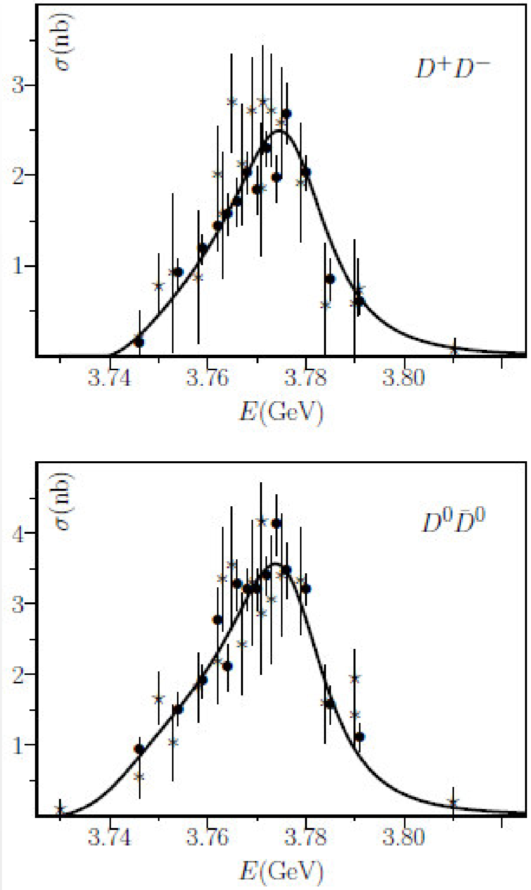
Fig. 5 Data: • BES [7], ∗ BES [5], ⋆ BaBar [6] (the latter not used in the fit). Solid line: our fit (cf. Table 1).

m_ψ (MeV)	3773.05 ± 0.95
Λ (MeV)	272.55 ± 1.17
$g_{\psi D\bar{D}}$	30.7 ± 4.8
$g_{\psi e^+e^-}$	$(1.062 \pm 0.032) \times 10^{-3}$
χ^2	20.52
$\chi^2/d.o.f$	0.86

Simple BW does not work ☺



Consequences of the fit



	$\Gamma(m_\psi^2)$
$\Gamma_{D^0\bar{D}^0}$ (MeV)	11.6 ± 1.2
$\Gamma_{D^+D^-}$ (MeV)	8.0 ± 0.8
$\Gamma_{D\bar{D}}$ (MeV)	19.5 ± 2.0
$\Gamma_{D^0\bar{D}^0}/\Gamma_{D^+D^-}$	1.45
$\Gamma_{D^0\bar{D}^0}/\Gamma_{D\bar{D}}$	0.59
$\Gamma_{D^+D^-}/\Gamma_{D\bar{D}}$	0.41

$\Gamma_{e^+e^-}$ (eV)	$\Gamma_{\mu^+\mu^-}$ (eV)	$\Gamma_{\tau^+\tau^-}$ (eV)
112.8 ± 6.8	112.8 ± 6.8	54.7 ± 3.3

Position(s) of the poles of Ψ(3770)

Two poles are present in the complex plane

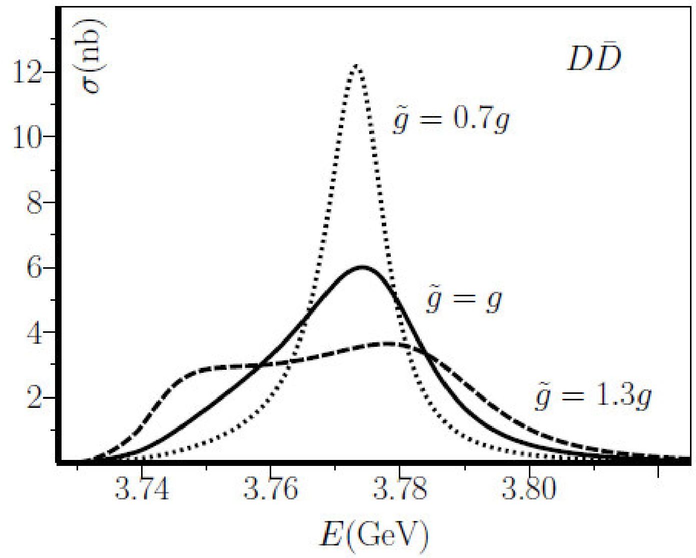
First pole: $E = 3776.8 - i12.3$ MeV,

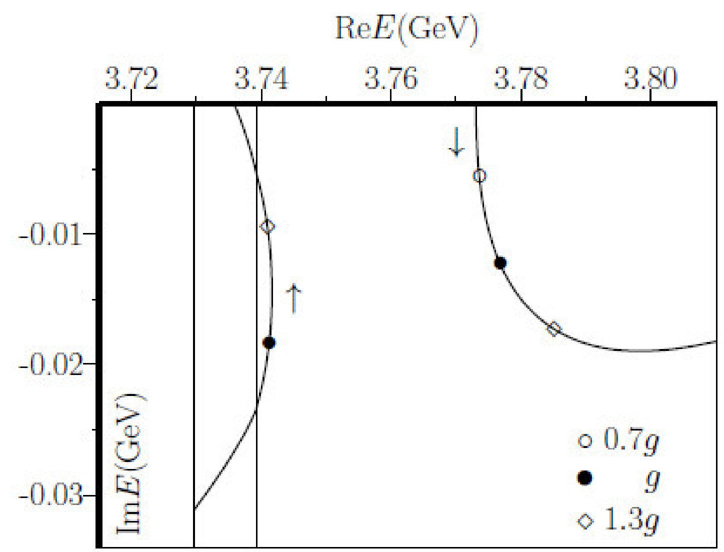
hence

$$m_\psi^{\rm pole} \simeq 3776.8 \pm 1.0 \text{ MeV and}$$
$$\Gamma_\psi^{\rm pole} \simeq 24.6 \pm 2.0 \text{ MeV},$$

Second pole: $E = 3741.2 - i18.5$ MeV

Pole trajectories of Ψ(3770)





Second pole: $E = 3741.2 - i18.5$ MeV

The additional companion pole disappears for Nc larger then 3.9.

a0(980) as a companion pole of a0(1450)

based on T. Wolkanowski, F.G. and D. H. Rischke,
a0(980) revisited,
Phys. Rev. D 93 (2016) no.1, 014002
[arXiv:1508.00372 [hep-ph]].

a0(1450) and a0(1470) from PDG

$a_0(1450)$

$I^G(J^{PC}) = 1^-(0^{++})$

See minireview on scalar mesons under $f_0(500)$.

$a_0(1450)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
1474 ±19 OUR AVERAGE				

Decays into ηπ, η'π, KK

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update

$a_0(980)$

$I^G(J^{PC}) = 1^-(0^{++})$

See our minireview on scalar mesons under $f_0(500)$. (See the index for the page number.)

$a_0(980)$ MASS

VALUE (MeV)	DOCUMENT ID
980±20 OUR ESTIMATE	Mass determination very model dependent

Decays into ηπ, KK

Re-analysis of TR (Tornqvist-Roos) and BP (Boglione-Pennington)

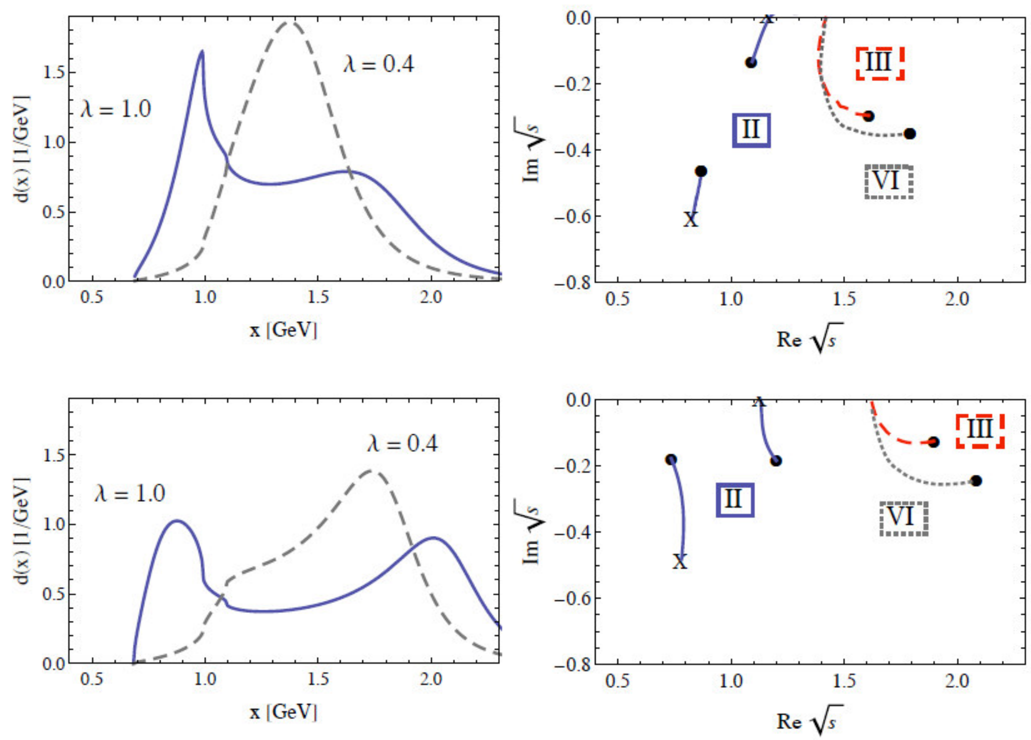


FIG. 1. Spectral functions (left panels) and positions of poles in the complex $\sqrt{s}$-plane (right panels) for the parameter sets of TR (upper row) and BP (lower row). Spectral functions are shown for $\lambda = 0.4$ (dashed grey lines) and $\lambda = 1.0$ (solid blue lines). The pole trajectories of the seed state are indicated by grey dotted or red dashed lines (for details, see the text) and the one for the dynamically generated resonance by solid blue lines. The roman numerals indicate the Riemann sheets where the respective poles can be found. Final pole positions ($\lambda = 1.0$) are indicated by solid black dots, pole positions at $\lambda_{c,i}$, *i.e.*, where the pole i first emerges, are indicated by X.

1508.00372

Two poles always present

Lagrangian for a0 system

$$\mathcal{L}_{a_0\eta\pi} = A_1 a_0^0 \eta \pi^0 + B_1 a_0^0 \partial_\mu \eta \partial^\mu \pi^0$$

$$\mathcal{L}_{a_0\eta'\pi} = A_2 a_0^0 \eta' \pi^0 + B_2 a_0^0 \partial_\mu \eta' \partial^\mu \pi^0$$

$$\mathcal{L}_{a_0 K\bar{K}} = A_3 a_0^0 (K^0 \bar{K}^0 - K^- K^+) + B_3 a_0^0 (\partial_\mu K^0 \partial^\mu \bar{K}^0 - \partial_\mu K^- \partial^\mu K^+)$$

The field a0 corresponds (roughly) to a0(1450)

$a_0(1450)$ this leads to

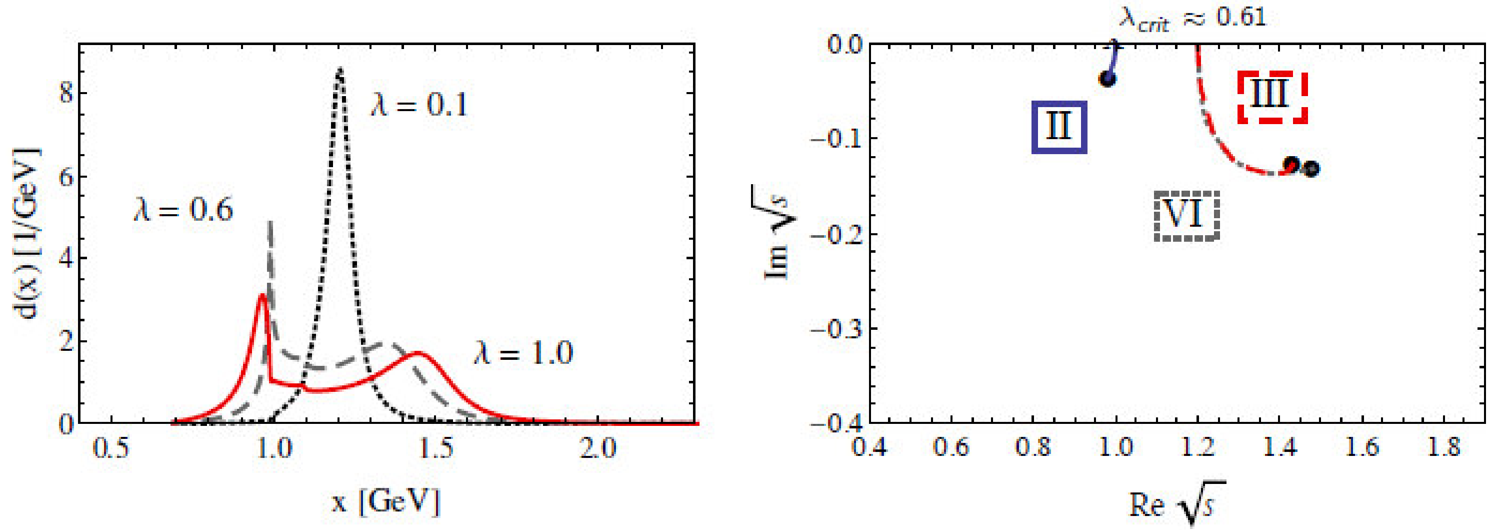
$$\frac{\Gamma^{\text{tree}}_{a_0\to\eta'\pi}}{\Gamma^{\text{tree}}_{a_0\to\eta\pi}} \simeq 0.44\ , \quad \frac{\Gamma^{\text{tree}}_{a_0\to K\bar{K}}}{\Gamma^{\text{tree}}_{a_0\to\eta\pi}} \simeq 0.96\ ,$$

which can be compared to the experimental values

$$\frac{\Gamma_{a_0\to\eta'\pi}}{\Gamma_{a_0\to\eta\pi}} = 0.35 \pm 0.16\ , \quad \frac{\Gamma_{a_0\to K\bar{K}}}{\Gamma_{a_0\to\eta\pi}} = 0.88 \pm 0.23$$

Spectral function, poles, and large-Nc

Uniwersytet



$$\text{Large-}N_c \text{ behavior:} \quad A_i^2 \to \lambda A_i^2 \ , \quad B_i^2 \to \lambda B_i^2 \ , \quad \lambda = \frac{3}{N_c}$$

$$a_0(1450) : \ 1.480 - i0.132 \text{ GeV} \ ,$$

$$a_0(980) \ : \ 0.981 - i0.037 \text{ GeV} \ ,$$

1508.00372

The additional companion pole on the left, corresponding to the a0(980), disappears for Nc larger then 4.9.

Branching ratios for a0(1450) and a0(980) in agreement with PDG

Comparison with TR and BP

9

	$a_0(980)$		$a_0(1450)$	
	m_{pole} [GeV]	Γ_{pole} [GeV]	m_{pole} [GeV]	Γ_{pole} [GeV]
TR [11]	1.084	0.270	1.566	0.578
BP [13]	1.186*	0.373*	1.896	0.250
Our results	0.969	0.090	1.450	0.270
PDG [1]	0.980 ± 0.020	0.050 to 0.100	1.474 ± 0.019	0.265 ± 0.013

*In order to compare to TR, the right pole on the second sheet was chosen.

TABLE I. Numerical results for the pole coordinates in the scalar–isovector sector in TR, BP, and our effective model, compared to the PDG values. In the case of the $a_0(1450)$, the poles listed for TR and BP are located on the third sheet, while our pole lies on the sixth sheet. All poles for the $a_0(980)$ are found on the second sheet. Note that all poles listed for BP were obtained performing the analytic continuation of the propagator given by BP.

Companion poles, ongoing and future studies

- $\Psi(4040)$ line shape and its companion poles
  (possible explanation of Y(4008)? Three channels (DD,DD*,D*D*, similar to a0)

- $\Psi(4160)$ line shape and its companion poles (eventually also above its mass)

- X(3872) as companion pole of a 1^++ charmonium state? A study needed.

- Ds(2317) as a companion pole?

Conclusions

- Emergence of companion poles is a viable mechanism to explain some resonances (or some features of known resonances)

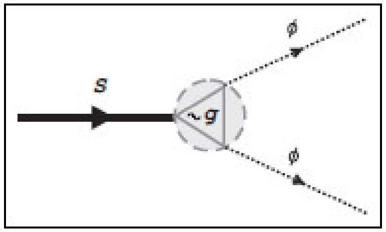
- The phenomenon is quite natural and takes place in light and as well as in heavy systems

- Various studies are ongoing

Thank You

Cutoff function/1

- the cutoff parameter Λ does **not** exist at the Lagrangian level
- it can be implemented by using a non-local interaction term (if $f_\Lambda(q) = f_\Lambda(|\mathbf{q}|)$), e.g.



$$\mathcal{L}_{\text{int}} = gS(x)\phi^2(x) \;\rightarrow\; \mathcal{L}_{\text{int}} = gS(x)\int d^4y\, \phi(x+y/2)\phi(x-y/2)\Phi(y)$$

- changes also the tree-level result for the decay width:

$$\Gamma^{\text{tree}}(s) \;\rightarrow\; \Gamma^{\text{tree}}(s)\cdot f_\Lambda^2(p_{S\phi\phi})$$

- our choice:

Regularization function in our case

$$f_\Lambda(q) = \exp\left(-|\mathbf{q}|^2/\Lambda^2\right)$$

Cutoff function/2

The contribution of the loop $\Pi(m^2)$ in which the particles φ_1 and φ_2 circulate as calculated from the original local Lagrangian (1) reads

$$\Pi(m^2) = \int \frac{d^4k_1}{(2\pi)^4} \frac{\left[a - b\,(k_1 \cdot k_2)\right]^2}{\left[k_1^2 - m_1^2 + i\varepsilon\right]\left[k_2^2 - m_2^2 + i\varepsilon\right]} \;, \tag{6}$$

where the constraint $k_2 = p - k_1$ is understood and p is the momentum of the unstable particle S. In its reference frame $p = (m, \vec{0})$. As mentioned above, this loop contribution is divergent (with Λ^4). The substitution (4) makes it convergent thanks to the form-factor:

$$\Pi(m^2) \to \Pi_\Lambda(m^2) = \int \frac{d^4k_1}{(2\pi)^4} \frac{\left[a - b\,(k_1 \cdot k_2)\right]^2 f_\Lambda^2(\vec{k}_1^2)}{\left[k_1^2 - m_1^2 + i\varepsilon\right]\left[k_2^2 - m_2^2 + i\varepsilon\right]} \;. \tag{7}$$

At this point, one may object that the form factor breaks covariance, since it depends on the three-momentum only. We will show in the next section that this is not necessarily the case. Once the form factor is in-

Cutoff function/3

Nonlocal extension 2: We aim to determine the Lagrangian which generates the vertex function (4) in a covariant manner [14]. We start from the general nonlocal expression:

$$aS\varphi_1\varphi_2 \to g\int d^4z d^4y_1 d^4y_1 S(x+z)\varphi_1(x+y_1)\varphi_2(x+y_2)\Phi(z,y_1,y_2)\ , \quad (10)$$

where the vertex-function $\Phi(z,y_1,y_2)$ in position space has been introduced. The case $\Phi(z,y_1,y_2)=\delta(z)\delta(y_1)\delta(y_2)$ delivers the local limit (1). The vertex function in momentum space is given by:

$$\varphi(p,k_1,k_2)=\int d^4z d^4y_1 d^4y_2 e^{ipz}e^{-ik_1y_1}e^{-ik_2y_2}\Phi(z,y_1,y_2)\ .$$

Here we *assume* that $\Phi(z,y_1,y_2)$ is such that

$$\varphi(p,k_1,k_2)=\varphi\left(p,q=\frac{k_1-k_2}{2}\right)=f_\Lambda\left(\frac{q^2p^2-(q\cdot p)^2}{p^2}\right)\ . \quad (11)$$

It respects covariance because the final form factor is a function of Lorentz-products. Nevertheless, in the rest frame of S one recovers the desired dependence:

$$\frac{q^2p^2-(q\cdot p)^2}{p^2}=\vec{k}_1^2 \text{ (for } p=(m,0)).$$

Parameters in the a0-system

There are eight parameters in our approach: m_0, $\Lambda = \sqrt{2}k_0$, and six coupling constants A_i, B_i $(i = 1, 2, 3)$. We vary the numerical values of m_0 and Λ within reasonable intervals $m_0 \in (0.8, 1.5)$ GeV and $\Lambda \in (0.4, 1.5)$ GeV and each time perform a fit of the six coupling constants to six experimental quantities: one pole in the PDG range for $a_0(980)$ (in our case $\sqrt{s} = (0.969 - i0.045)$ GeV) and one for $a_0(1450)$ (in our case $\sqrt{s} = (1.450 - i0.135)$ GeV), and the central values of the branching ratios of $a_0(1450)$ [see Eq. (22)]. By this, all six free parameters can be fixed.

It turns out that there is only a *narrow* range of suitable values of the parameters m_0 and Λ for which the fit of the six coupling constants is possible: approximately $m_0 \in (0.9, 1.2)$ GeV and $\Lambda \in (0.4, 0.9)$ GeV. Here, 'approximately' refers to the fact that, due to the interdependence of the parameters, the window is not rectangular. However, a small change in m_0 and/or Λ by 50 MeV near the borders of the quoted interval does not allow one to reproduce the data anymore. Thus, although we have eight parameters, we are severely constrained in their choice in order to describe the $I = 1$ resonance. As we will see below, the present parameters also explain why $a_0(980)$ couples strongly to kaons. The final values for the parameters and coupling constants are:

$$m_0 = 1.15 \text{ GeV} , \qquad \Lambda = 0.6 \text{ GeV} , \tag{17}$$

$$\begin{aligned} A_1 &= 2.52 \text{ GeV} , & B_1 &= -8.07 \text{ GeV}^{-1} , \\ A_2 &= 9.27 \text{ GeV} , & B_2 &= 9.25 \text{ GeV}^{-1} , \\ A_3 &= -6.56 \text{ GeV} , & B_3 &= -1.54 \text{ GeV}^{-1} . \end{aligned} \tag{18}$$

Meson

Definition(s):

1) A meson is a strongly interacting particle with integer spin.

2) A meson is a strongly interacting particle with zero baryon number.

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

Quark-antiquark states: the large-Nc limit

As Isgur-Godfrey have shown, the quark model works.
Theoretical justification relies on the large-Nc expansion.
G. 't Hooft, Nucl. Phys. B **72** (1974) 461

For comprehensive reviews on Nc:

Baryons in the 1/n Expansion
Edward Witten
Published in **Nucl.Phys. B160 (1979) 57**

S. R. Coleman, Lectures given at 17th International School of Subnuclear Physics: Pointlike Structures Inside and Outside Hadrons 31 Jul - 10 Aug 1979. Erice, Italy. SLAC-PUB-2484.
R. F. Lebed, Czech. J. Phys. **49** (1999) 1273 [nucl-th/9810080].

$$\left|\rho^{+}\right\rangle \propto \left|u\bar{d}\right\rangle + \frac{1}{N_c}\left(\left|\pi^{+}\pi^{0}\right\rangle + ...\right)$$

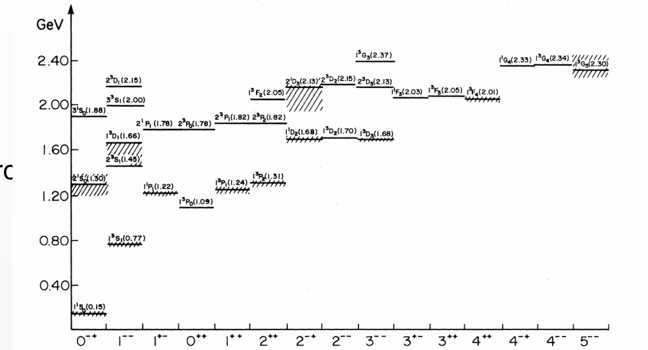
where

$$\left|u\bar{d}\right\rangle = \left|\text{valence } u + \text{valence } \bar{d} + \text{gluons}\right\rangle$$

Mesons beyond q-qbar: the first term in the first expansion is of non-quarkonium type

Quark model(s) and their QFT extensions

Mesons in a Relativized Quark Model with Chromodynamics
S. Godfrey, Nathan Isgur
Published in Phys.Rev. D32 (1985) **189-231**



Mesonic loops e.g. included into
A Low Lying Scalar Meson Nonet in a Unitarized Meson Model
E. van Beveren, T. A. Rijken, K. Metzger, C.~Dullemond, G.~Rupp and J. E.~Ribeiro
Z. Phys. C **30** (1986) 615
Meson spectroscopy: too much excitement and too few excitations
G. Rupp, S. Coito and E. van Beveren,
Acta Phys. Polon. Supp. 5 (2012) 1007

QCD phenomenology based on a chiral effective Lagrangian
Tetsuo Hatsuda, Teiji Kunihiro
Phys.Rept. **247** (1994) 221-367

NJL: quark-based model with
chiral symmetry and SSB
chiral condensate
Effective quark mass
Mesons as quarkonia (pion: ok)

The Infrared behavior of QCD Green's functions: Confinement dynamical symmetry breaking, and hadrons as relativistic bound states
Reinhard Alkofer, Lorenz von Smekal
Phys.Rept. **353** (2001) 281

Baryons as relativistic three-quark bound states
G. Eichmann, H.~ Sanchis-Alepuz, R. Williams, R. Alkofer and C. S. Fischer,
Progr. Part. Nucl. Phys. **91** (2016) 1

DS:
quarks and gluons propagators
from QCD
Condensates
Effective quark and gluon masses
Spectra of mesons as quarkonia
(pion: ok) and baryons as qqq states