Confinement criteria and the Higgs Mechanism

Jeff Greensite
San Francisco State University

Bound States in Strongly Coupled Systems
Galileo Galilei Institute
Firenze, Italia
March 2018

with Kazue Matsuyama, arXiv:1708.08979

What is "confinement"?

Suppose we have an SU(N) gauge theory with matter fields in the fundamental representation, e.g. QCD. Wilson loops have perimeter-law falloff asymptotically, Polyakov lines have a non-zero VEV, what does it mean to say such theories (QCD in particular) are confining?

Most people take it to mean “color confinement” or

C-confinement

There are only color neutral particles in the asymptotic spectrum.

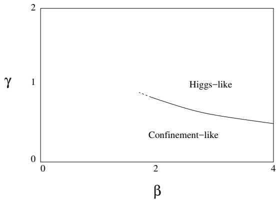
The problem with C-confinement is that it also holds true for gauge-Higgs theories, deep in the Higgs regime, where there are

- only Yukawa forces,
- no linearly rising Regge trajectories,
- no color electric flux tubes.

If C-confinement is “confinement,” then the Higgs phase is also confining.

How we know this:

1. **Elitzur's** Theorem: No such thing as spontaneous symmetry breaking of a local gauge symmetry.
2. The **Fradkin-Shenker-Osterwalder-Seiler** (FSOS) Theorem: There is no transition in coupling-constant space which isolates the Higgs phase from a confinement-like phase.
3. **Frölich-Morchio-Strocchi** (FMS) and also **'t Hooft** (1980): physical particles (e.g. W's) in the spectrum are created by gauge-invariant operators in the Higgs region.



FMS show how to recover the usual results of perturbation theory, starting from gauge-invariant composite operators.

Conclusion: If the confinement-like (QCD-like) region has a color neutral spectrum, then so does the Higgs-like region.

In a pure SU(N) gauge theory there is a different and stronger meaning that can be assigned to the word “confinement," which goes beyond C-confinement.

Of course the spectrum consists only of color neutral objects: glueballs.

But such theories *also* have the property that the static quark potential rises linearly or, equivalently, that large planar Wilson loops have an area-law falloff.

Is there any way to generalize this property to gauge theories with matter in the fundamental representation?

Separation-of-charge ("S") confinement

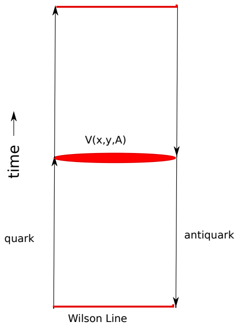
The Wilson area-law criterion for pure gauge theories is equivalent to "S-confinement."

A static $q\overline{q}$ pair, connected by a Wilson line, evolves in Euclidean time to some state

$$\Psi_V \equiv \overline{q}^a(\mathbf{x}) V^{ab}(\mathbf{x},\mathbf{y};A) q^b(\mathbf{y}) \Psi_0$$

where $V(\mathbf{x},\mathbf{y};A)$ is a gauge bi-covariant operator transforming as

$$V^{ab}(\mathbf{x},\mathbf{y};A) \rightarrow g^{ac}(\mathbf{x},t) V^{cd}(\mathbf{x},\mathbf{y};A) g^{\dagger db}(\mathbf{y},t)$$



The energy above the vacuum energy $\mathcal{E}_{vac}$ is

$$E_V(R) = \langle \Psi_V | H | \Psi_V \rangle - \mathcal{E}_{vac}$$

S-confinement, continued...

S-confinement

means that there exists an asymptotically linear function $E_0(R)$, i.e.

$$\lim_{R\to\infty} \frac{dE_0}{dR} = \sigma > 0$$

such that

$$E_V(R) \geq E_0(R)$$

for ***ANY*** choice of bi-covariant $V(\mathbf{x}, \mathbf{y}; A)$.

For an SU(N) pure gauge theory, $E_0(R)$ is the ground state energy of a static quark-antiquark pair, and σ is the string tension. This is equivalent to the Wilson area-law criterion.

Our proposal: S-confinement should also be regarded as the confinement criterion in gauge+matter theories. The crucial element is that the bi-covariant operators $V^{ab}(\mathbf{x}, \mathbf{y}; A)$ *must depend only on the gauge field A* at a fixed time, and not on the matter fields.

The idea is to study the energy $E_V(R)$ of physical states with large separations R of static color charges, *unscreened by matter fields*.

If $V^{ab}(\mathbf{x}, \mathbf{y}; A)$ would also depend on the matter field(s), then it is easy to violate the S-confinement criterion, e.g. let ϕ be a matter field in the fundamental representation, and

$$V^{ab}(\mathbf{x}, \mathbf{y}, \phi) = \phi^a(\mathbf{x})\phi^{\dagger b}(\mathbf{y})$$

Then

$$\Psi_V = \{\overline{q}^a(\mathbf{x})\phi^a(\mathbf{x})\} \times \{\phi^{\dagger b}(\mathbf{y})q^b(\mathbf{y})\}\Psi_0$$

corresponds to two color singlet (static quark + Higgs) states, only weakly interacting at large separations. Operators V of this kind, which depend on the matter fields, are excluded.

This also means that the lower bound $E_0(R)$, unlike in pure gauge theories, is *not* the lowest energy of a state containing a static quark-antiquark pair.

It is the lowest energy of such states when color screening by matter is excluded.

SU(2) doublet to group element

We consider a unimodular $|\phi| = 1$ Higgs field. In SU(2) the doublet can be mapped to an SU(2) group element

$$\vec{\phi} = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \Longrightarrow \phi = \begin{bmatrix} \phi_2^* & \phi_1 \\ -\phi_1^* & \phi_2 \end{bmatrix}$$

and the corresponding action is

$$S = \beta \sum_{plaq} \frac{1}{2} \text{Tr}[UUU^\dagger U^\dagger] + \gamma \sum_{x,\mu} \frac{1}{2} \text{Tr}[\phi^\dagger(x) U_\mu(x) \phi(x + \widehat{\mu})]$$

Tests of S-confinement

1. Does S-confinement exist *anywhere* in the $\beta - \gamma$ phase diagram, apart from pure gauge theory?

   It can be shown that gauge-Higgs theory is S-confining in the region

   $$\gamma \ll \beta \ll 1 \quad \text{and} \quad \gamma \ll \frac{1}{5}$$

   This is based on strong-coupling expansions and a theorem (Gershgorim) in linear algebra.

2. Then does S-confinement hold *everywhere* in the $\beta - \gamma$ phase diagram?

   If we can find *even one V* which violates the S-confinement criterion in some region of the phase diagram, then we know the theory is not S-confining in that region. The question is which V-operators might be useful for this purpose.

Candidate operators

For $V =$ Wilson line, $E_V(R) \propto R$ even for non-confining theories. Not useful!
Instead we consider

1. **The Dirac state**
   generalization of the lowest energy state with static charges in an abelian theory.

2. **Pseudomatter**
   Introduce fields built from the gauge field which transform like matter fields. See if these induce string-breaking.

3. **"Fat link" states**
   Wilson lines built from smoothed links.

The Calculation

In general

$$E_V(R) = -\lim_{t\to 0} \frac{d}{dt} \log \left[\frac{\langle \Psi_V | e^{-Ht} | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \right] - \mathcal{E}_{vac}$$

on the lattice

$$E_V(R) = -\log \left[\frac{\left\langle \text{Tr} \left[U_0(x,t) V(x,y,t+1) U_0^\dagger(y,t) V(y,x,t) \right] \right\rangle}{\langle \text{Tr} \left[V(x,y,t) V(y,x,t) \right] \rangle} \right]$$

and we will focus on the SU(2) gauge-Higgs action

$$S = \beta \sum_{plaq} \frac{1}{2} \text{Tr}[UUU^\dagger U^\dagger] + \gamma \sum_{x,\mu} \frac{1}{2} \text{Tr}[\phi^\dagger(x) U_\mu(x) \phi(x+\widehat{\mu})]$$

where ϕ is SU(2) group-valued.

The Dirac state

In an abelian theory, the gauge-invariant ground state with static $\pm$ electric charges is

$$\Psi_{\overline{q}q} = \{\overline{q}(\mathbf{x})G_C^\dagger(\mathbf{x};A)\} \times \{G_C(\mathbf{y};A)q(\mathbf{y})\}\Psi_0$$

where

$$G_C(\mathbf{x};A) = \exp\left[-i\int d^3z\, A_i(z)\partial_i \frac{1}{4\pi|\mathbf{x}-\vec{z}|}\right]$$

$G_C(\mathbf{x},A)$ is the gauge transformation $A \rightarrow$ Coulomb gauge. Non-abelian theory: define $V^{ab}(x,y;A) = G_C^{\dagger ac}(\mathbf{x};A)G_C^{cb}(\mathbf{y};A)$ and

$$\begin{aligned}\Psi_V &= \overline{q}^a(\mathbf{x})G_C^{\dagger ac}(\mathbf{x};A)G_C^{cb}(\mathbf{y};A)q^b(\mathbf{y})\Psi_0 \\ &= \overline{q}^c(\mathbf{x})q^c(\mathbf{y})\Psi_0 \quad \text{in Coulomb gauge}\end{aligned}$$

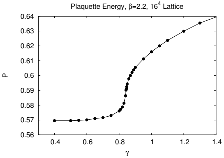
then compute in Coulomb gauge

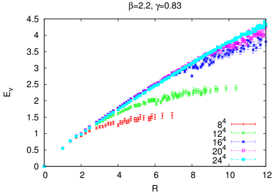
$$E_V(R) = -\log\left\langle \frac{1}{N}\mathrm{Tr}[U_0(\mathbf{0},0)U_0^\dagger(\mathbf{R},0)]\right\rangle$$

by lattice Monte Carlo.

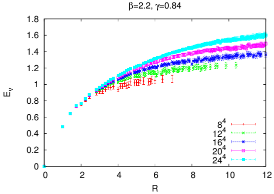
$E_V(R)$ in the Dirac state

There is a sharp thermodynamic crossover in the SU(2) gauge model at $\beta = 2.2, \gamma \approx 0.84$.

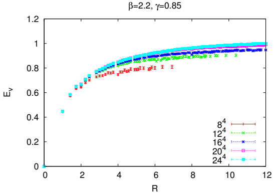




(a)



(b)

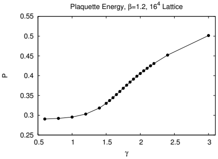


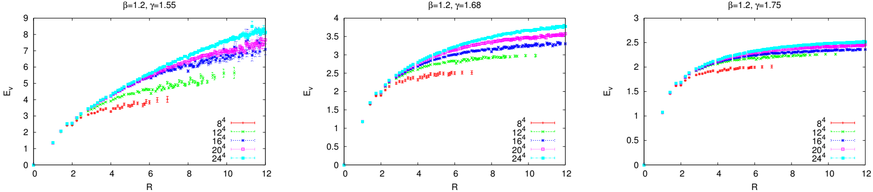
(c)

$E_V(R)$ rises linearly below the crossover, consistent with (but not a proof of) S-confinement in this region.

The theory appears to be in the C-confinement phase above the transition.

There is no thermodynamic transition or crossover in the SU(2) gauge model at $\beta = 1.2$.





(a) (b) (c)

$E_V(R)$ would appear to rise linearly below roughly $\gamma = 1.68$, at least in the large volume limit. This is consistent with the conjectured S-confinement at small γ.

The theory appears to be in the C-confinement phase at higher γ.

Remnant symmetry breaking

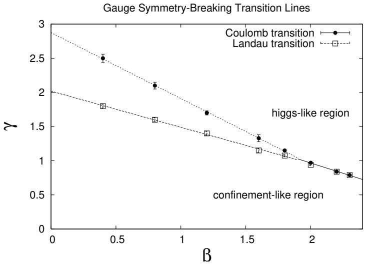
The transition in $E_V(R)$ coincides with the breaking of a remnant gauge symmetry $g(x,t) = g(t)$ that exists in Coulomb gauge. The appropriate order parameter for the symmetry breaking on a time slice is

$$u(t) = \frac{1}{\sqrt{2}V_3} \sum_{\mathbf{x}} U_0(\mathbf{x}, t)$$

and on the lattice we compute the susceptability

$$\chi = V_3(\langle |u|^2 \rangle - \langle |u| \rangle^2) \quad \text{where} \quad |u| = \sqrt{\frac{1}{N_t} \sum_{t=1}^{N_t} \mathrm{Tr}[u^\dagger(t) u(t)]}$$

Other gauges have other remnant symmetries. However, the transition lines for remnant-symmetry breaking are gauge-dependent.



Pseudomatter

A pseudomatter field is a field constructed from the gauge field which transforms like matter in the fundamental representation. Any example is any eigenstate

$$(-D_iD_i)^{ab}_{\mathbf{xy}}\varphi^b_n(\mathbf{y}) = \lambda_n\varphi^a_n(\mathbf{x})$$

of the covariant spatial Laplacian

$$(-D_iD_i)^{ab}_{\mathbf{xy}} = \sum_{k=1}^{3}\left[2\delta^{ab}\delta_{\mathbf{xy}} - U^{ab}_k(\mathbf{x})\delta_{\mathbf{y},\mathbf{x}+\hat{k}} - U^{\dagger ab}_k(\mathbf{x}-\hat{k})\delta_{\mathbf{y},\mathbf{x}-\hat{k}}\right]$$

We construct

$$V^{ab}(\mathbf{x},\mathbf{y};A) = \varphi^a_1(\mathbf{x})\varphi^{\dagger b}_1(\mathbf{y})$$

from the lowest-lying eigenstate, and compute $E_V(R)$ by lattice Monte Carlo.

Fat links

Let $V_{thin}(\mathbf{x}, \mathbf{y}; A)$ be a Wilson line running between $\mathbf{x}, \mathbf{y}$, and

$$\Psi_{thin}(R) = \overline{q}(x) V_{thin}(x, y; A) q(y)$$

Likewise, let $U_k^{(0)}(\mathbf{x}) = U_k(\mathbf{x}, t)$ and construct fat links by an iterative procedure

$$\begin{aligned} U_i^{(n+1)}(x) &= \mathcal{N}\Big\{\alpha U_i^{(n)}(x) + \sum_{j \neq i} \Big(U_j^{(n)}(x) U_i^{(n)}(x+\hat{j}) U_j^{\dagger}(x+\hat{i}) \\ &\qquad + U_j^{(n)\dagger}(x-\hat{j}) U_i^{(n)}(x-\hat{j}) U_j^{(n)}(x-\hat{j}+\hat{i}) \Big) \Big\} \end{aligned}$$

Denote the link variables after the last iteration as $U_i^{fat}(\mathbf{x})$ and define

$$\begin{aligned} V_{fat}(x, y; A) &= U_k^{fat}(x) U_k^{fat}(x+\hat{k}) \ldots U_k^{fat}(x+(R-1)\hat{k}) \\ \Psi_{fat}(R) &= \overline{q}(x) V_{fat}(x, y; A) q(y) \end{aligned}$$

We then compute $E_V(R)$ for $V = V_{thin}, V_{fat}$.

Results

- We find an S to C-confinement transition for the V operator constructed from pseudomatter fields. The transition line is close to (but a little below) the transition line for the Dirac state.

- The fat link state seems to be everywhere S-confining. This doesn't mean the gauge-Higgs theory is everywhere S-confining. It means instead that not every operator can detect the transition to C-confinement.

Other criteria for distinguishing the confinement from the Higgs phase have been proposed in the past:

1. the Kugo-Ojima criterion
2. Non-positivity/unphysical poles in quark/gluon propagators
3. the Fredenhagen-Marcu criterion

Kugo-Ojima

A condition for C-confinement, formulated in covariant gauges. Introduce

$$u^{ab}(p^2)\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) = \int d^4x\; e^{ip(x-y)} \langle 0|T[D_\mu c^a(x) g(A_\nu \times \overline{c})^b(y)|0\rangle$$

Then $\langle \text{phys}\, |Q^a| \text{phys}\rangle = 0$ providing that

- remnant symmetry with respect to spacetime-independent gauge transformations is unbroken
- $u^{ab}(0) = -\delta^{ab}$

The Neuberger 0/0 Problem

The problem with Kugo-Ojima is that remnant symmetry is broken in the Higgs region, yet C-confinement persists throughout the phase diagram. Somethings wrong...probably the assumption of unbroken BRST symmetry.

It was shown by Neuberger in 1986 that in a covariant gauge, the functional integral

$$Z = \int DA_\mu DcD\overline{c}\, \exp[-(S + S_{gf})]$$

vanishes. *Every expectation value has the form* $0/0$. The argument applies to any BRST invariant action.

Latttice simulations in, e.g. Landau gauge, avoid the 0/0 problem, but the gauge-fixing procedure also breaks BRST symmetry.

Formulations which depend on BRST symmetry are unreliable at the non-perturbative level.

Non-positivity & unphysical poles

The idea is that positivity violation and/or unphysical poles in covariant-gauge quark and gluon propagators would mean that such particles can't show up as asymptotic states in scattering amplitudes. Positivity violation has been observed in lattice calculations of the gluon propagator.

But what is actually done on the lattice is to compute

$$D^{ab}_{\mu\nu}(x-y) = \langle [G_L \circ A]^a_\mu(x)[G_L \circ A]^b_\nu(y)\rangle$$

where G_L is the gauge transformation taking configuration A to Landau gauge inside the first Gribov horizon. This avoids the $0/0$ problem, but the restriction also

- breaks BRST symmetry
- violates the condition needed for reflection positivity (G_L is non-local in time)

If one cannot rely on BRST arguments, then *there is no strong reason to suppose that quark and gluon operators in covariant gauges create physical states*, and there is one good reason to think otherwise, namely, the fact that correlators of such operators do not satisfy the requirements for reflection positivity.

Other criteria, continued

In covariant gauges:

- **BRST yes?** $\Longrightarrow$ Neuberger 0/0 problem!
- **BRST no?** $\Longrightarrow$ propagator-to-confinement connection is unclear.

Isolated quark/gluon operators in Landau gauge do not create physical states, but that doesn't mean that physical states with widely separated color charges do not exist!

The correct conclusion is that other types of operators must be employed to create such states.

Dirac, pseudomatter, fat link states are examples.

The real question is whether or not a wide separation of color charged objects in a physical state incurs a proportionally large cost in energy.

Brout-Englert-Higgs and symmetry breaking

Does the transition from S to C-confinement correspond to the spontaneous breaking of some symmetry in the gauge-Higgs theory?

Is there any gauge-invariant meaning to "spontaneous symmetry breaking" in the context of the Brout-Englert-Higgs mechanism?

It is well-known, in the SU(2) gauge-Higgs model, that the full symmetry of the Higgs action

$$S_H = \gamma \sum_{x,\mu} \frac{1}{2} \text{Tr}[\phi^\dagger(x) U_\mu(x) \phi(x+\widehat{\mu})]$$

is $\text{SU}(2)_{gauge} \times \text{SU}(2)_{global}$:

$$\begin{aligned} U_\mu(x) &\rightarrow L(x) U_\mu(x) L^\dagger(x+\hat{\mu}) \\ \phi(x) &\rightarrow L(x)\phi(x) R \end{aligned}$$

Global SU(2) symmetry

SU(2)$_{gauge}$ can't break spontaneously, but what about SU(2)$_{global}$? Note that Z is a sum of "spin systems"

$$Z(\beta,\gamma) = \int DU \, Z_{spin}(\gamma, U) e^{-S_W}$$

where

$$\begin{aligned} Z_{spin}(\gamma, U) &= \int D\phi \; e^{-S_H[\phi, U]} \\ &= e^{-\mathcal{F}_H[\gamma, U]} \end{aligned}$$

The only symmetry of the spin system, since $U_\mu(x)$ is fixed, is the SU(2)$_{global}$ symmetry $\phi(x) \rightarrow \phi(x)R$.

Question: Can we observe a spontaneous breaking of the SU(2)$_{global}$ (R-transformation) symmetry without recourse to gauge-fixing?

This might be a gauge-invariant version of the gauge-dependent statement that $\langle \phi \rangle \neq 0$.

Gauge-invariant order parameter for spontaneous symmetry breaking

Consider $\phi(x)$ fluctuating in a background gauge field U, which is held fixed. Denote its average value in this background as $\overline{\phi}(x;U)$.

In general, $\int dx\phi = 0$, because if no gauge is fixed, so $U_\mu(x)$ varies wildly in space, then $\phi(x)$ also varies wildly.

On the other hand, it could be that

$$\overline{\phi}(x;U) \equiv \langle \phi(x) \rangle_U \neq 0$$

at any given point x, even if the spatial average vanishes.

Since the action at fixed U_μ is invariant under $\phi(x) \rightarrow \phi(x)R$,
this would imply SSB of an SU(2)$_{global}$ symmetry.

So we introduce the following gauge-covariant operator:

$$\begin{aligned}\overline{\phi}(x;U) &= \frac{1}{Z[U]}\int D\phi'\ \phi'(x)\exp\left[\gamma\sum_{x,\mu}\frac{1}{2}\mathrm{Tr}[\phi'^{\dagger}(x)U_\mu(x)\phi'(x+\hat{\mu})]\right] \\ Z[U] &= \int D\phi'\ \exp\left[\gamma\sum_{x,\mu}\frac{1}{2}\mathrm{Tr}[\phi'^{\dagger}(x)U_\mu(x)\phi'(x+\hat{\mu})]\right]\end{aligned}$$

and compute the following gauge-invariant order parameter:

$$\begin{aligned}Q &= \left\langle\sqrt{\frac{1}{2}\mathrm{Tr}[\overline{\phi}^{\dagger}(x;U)\overline{\phi}(x;U)]}\right\rangle \\ &= \frac{1}{Z}\int DUD\phi\sqrt{\frac{1}{2}\mathrm{Tr}[\overline{\phi}^{\dagger}(x;U)\overline{\phi}(x;U)]}\,e^{S[U,\phi]}\end{aligned}$$

by a Monte Carlo-within-a-Monte Carlo. Of course, there is no spontaneous symmetry breaking on a finite lattice; any "broken" state is only metastable in time (just like a real magnet). "Time" in our case is the number of Monte Carlo sweeps n_{sw} used to compute $\overline{\phi'}(x;U)$.

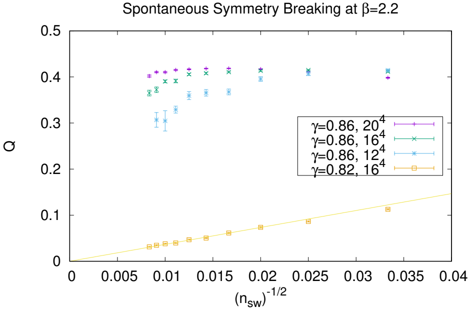
Results

In the unbroken phase we expect $Q \propto \frac{1}{\sqrt{n_{sw}}}$.

For the broken phase, we expect Q is roughly constant with n_{sw}. Eventually $Q \to 0$ in the broken phase, but only after a Monte Carlo time which increases with lattice volume.

And that's what we see.

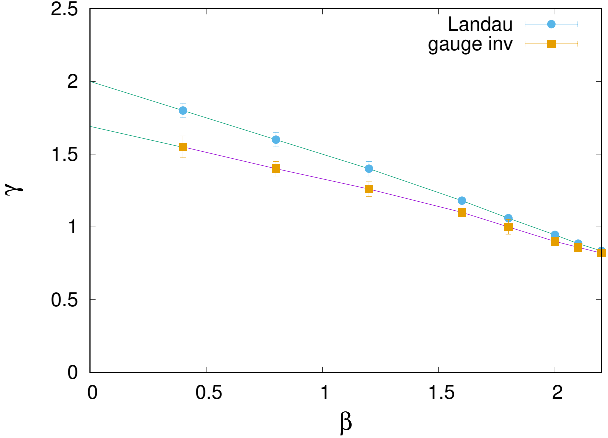
Here are the results at $\beta = 2.2$, above ($\gamma = 0.86$) and below ($\gamma = 0.82$) the thermodynamic crossover.



In this way we can map out the SSB transition line throughout the phase diagram.

Transition lines

Both the gauge-invariant transition line and the Landau gauge transition are shown; they are clearly not identical.



Hidden center symmetry

The global "R" symmetry in the SU(2) gauge-Higgs model is accidental. A Higgs field in SU(N) gauge-Higgs theory at $N > 2$ cannot be expressed as an SU(N) group element.

However, the SU(N>2) Higgs action

$$S_H[U,\phi] = \gamma \sum_{x,\mu} \mathrm{Re}[\phi^\dagger(x) U_\mu(x) \phi(x+\widehat{\mu})]$$

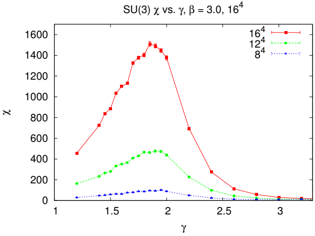
does have a discrete global symmetry

$$\phi(x) \rightarrow z\phi(x) \quad , \quad z = e^{2\pi i n/N}, \; n = 0, 1, 2, ..., N-1$$

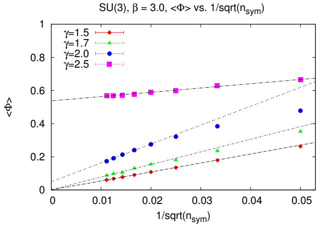
and this global symmetry can be spontaneously broken. The order parameter is the same as before

$$|\overline{\phi}(x;U)| = \sqrt{\overline{\phi}^\dagger(x;U)\overline{\phi}(x;U)}$$

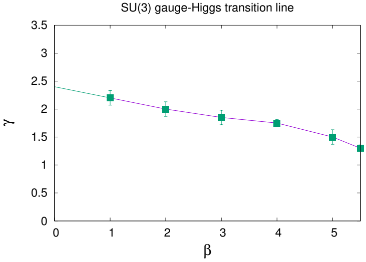
except that a dot product of color indices, rather than a trace, is implied



susceptibility



order parameter



transition line

Conclusions

We have

1. defined a generalization of the Wilson area law criterion, “S-confinement,” which is applicable to gauge theories with matter fields in the fundamental representation,
2. shown that in gauge-Higgs theories there must exist a transition between two physically distinct (S and C) types of confinement,
3. identified a “hidden” global symmetry in SU(N) gauge-higgs theories, and
4. shown that this symmetry breaks spontaneously, as detected by a gauge-invariant order parameter.

Our conjecture is that the S-to-C confinement transition and the gauge-invariant symmetry-breaking transition coincide.

EXTRA

SLIDES

Frölich, Morchio, and Strocchi (and independently 't Hooft) pointed out that physical particles in SU(2) gauge-Higgs theory are created by gauge-invariant composite operators.

As 't Hooft put it, a physical lepton can be thought of as a Higgs-fermion bound state.

$\varphi^*_\alpha \boldsymbol{\sigma}_{\alpha\beta}\varphi_\beta \cdot \boldsymbol{F}_{\mu\nu}$,	("W^0") ,
$\varphi_\alpha \varepsilon_{\alpha\gamma}\boldsymbol{\sigma}_{\gamma\beta}\varphi_\beta \cdot \boldsymbol{F}_{\mu\nu}$,	("W^+") ,
$\varphi^*_\beta \boldsymbol{\sigma}_{\beta\gamma}\varepsilon_{\gamma\alpha}\varphi^*_\alpha \cdot \boldsymbol{F}_{\mu\nu}$,	("W^-") ,
$\varphi^*_\alpha \psi_\alpha$,	("e") ,
$\varphi_\alpha \varepsilon_{\alpha\beta}\psi_\beta$,	("ν") ,
$\varphi^*_\alpha \varphi_\alpha$	("Higgs particle") ,

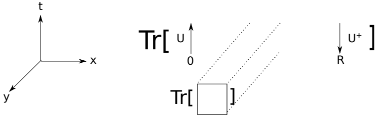
But there are other options for creating gauge-singlet physical states. Their properties have not been investigated, except in one case: The Dirac state for a static quark-antiquark pair in pure SU(2) gauge theory.

We want to calculate

$$\langle \Psi_V | \mathrm{Tr} E_x^2(p) | \Psi_V \rangle - \langle \Psi_0 | \mathrm{Tr} E_x^2 | \Psi_0 \rangle$$

in the Dirac state

$$\Psi_V = \overline{q}^a(\mathbf{x}) G_C^{\dagger ac}(\mathbf{x}; A) G_C^{cb}(\mathbf{y}; A) q^b(\mathbf{y}) \Psi_0$$



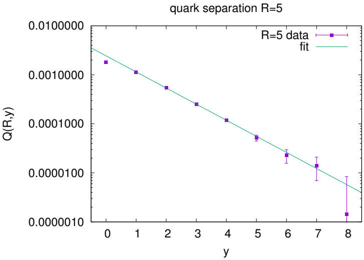
Let R be the quark-antiquark separation along the x-axis, and y a transverse distance from the axis. Then on the lattice we compute

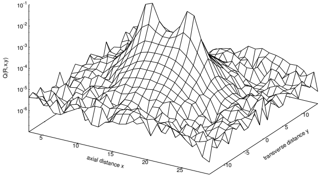
$$Q(R, y) = \frac{\langle \mathrm{Tr}[U_0(\mathbf{0}, 0) U_0^\dagger(\mathbf{R}_L, 0)] \frac{1}{2} \mathrm{Tr} U_P(\vec{p}, 0) \rangle}{\langle \mathrm{Tr}[U_0(\mathbf{0}, 0) U_0^\dagger(\mathbf{R}_L, 0)] \rangle} - \langle \frac{1}{2} \mathrm{Tr} U_P \rangle$$

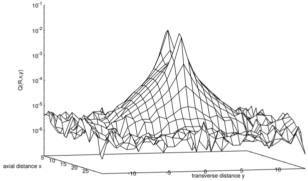
in Coulomb gauge.

Result: $Q(R, y)$ falls of exponentially with transverse distance y. *A flux tube.*

The data shown is for $\beta = 2.5$ in pure SU(2) lattice gauge theory. In that case we have calculated the color electric field distribution surrounding the quark-antiquark pair.
K. Chung and JG, arXiv:1704.08995.







It would be interesting to investigate the Dirac and pseudomatter states in the Higgs regime.