Associated Production of a Top Pair and a Heavy Colorless Boson at the LHC

Andrea Ferroglia

New York City College of Technology
and
The Graduate School and University Center
CUNY

Universita di Genova
November 20, 2017

Outline

- Top quarks and heavy colorless bosons at the LHC
- Factorization of the partonic cross section in the partonic threshold limit and resummation
- Results and plots for

$$p\,p \longrightarrow t\,\bar{t}\,W^{\pm}$$

$$p\,p \longrightarrow t\,\bar{t}\,H$$

$$p\,p \longrightarrow t\,\bar{t}\,Z$$

In collaboration with...

- Work on associated production of top pair and Higgs boson:

  A. Broggio, B.D. Pecjak, A. Signer, L.L. Yang

  JHEP 1603 (2016) 124 [arXiv:1510.01914]

  JHEP 1702 (2017) 127 [arXiv:1611.00049]

- Work on associated production of a top pair and W or Z boson

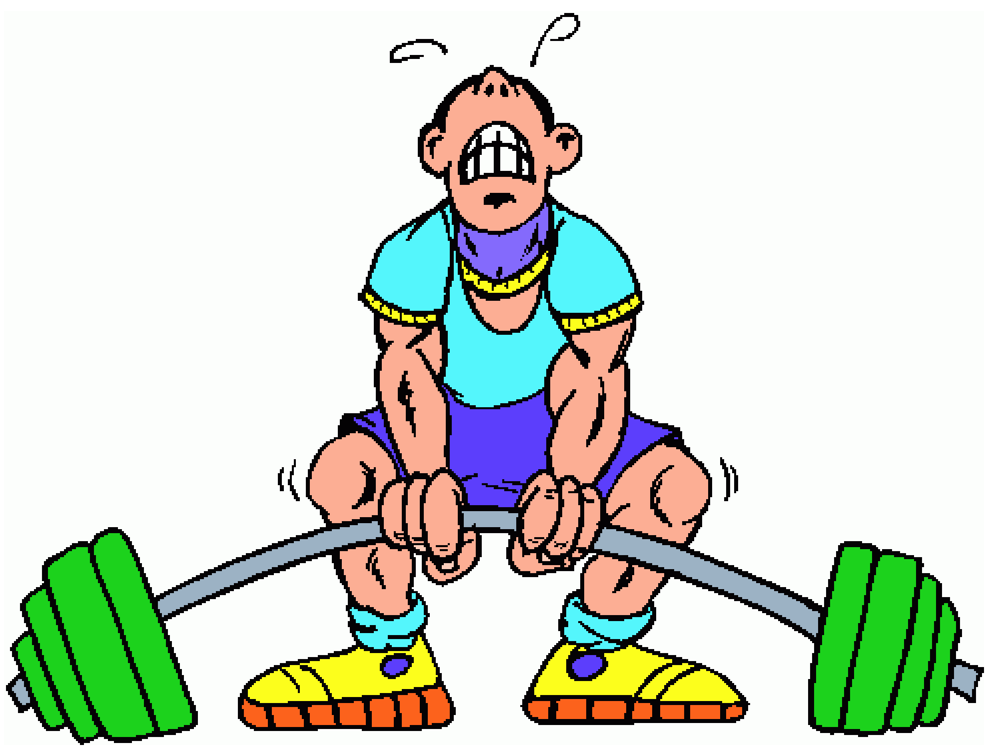
  A. Broggio, B.D. Pecjak, G. Ossola, R.D. Sameshima

  JHEP 1609 (2016) 089 [arXiv:1607.05303]

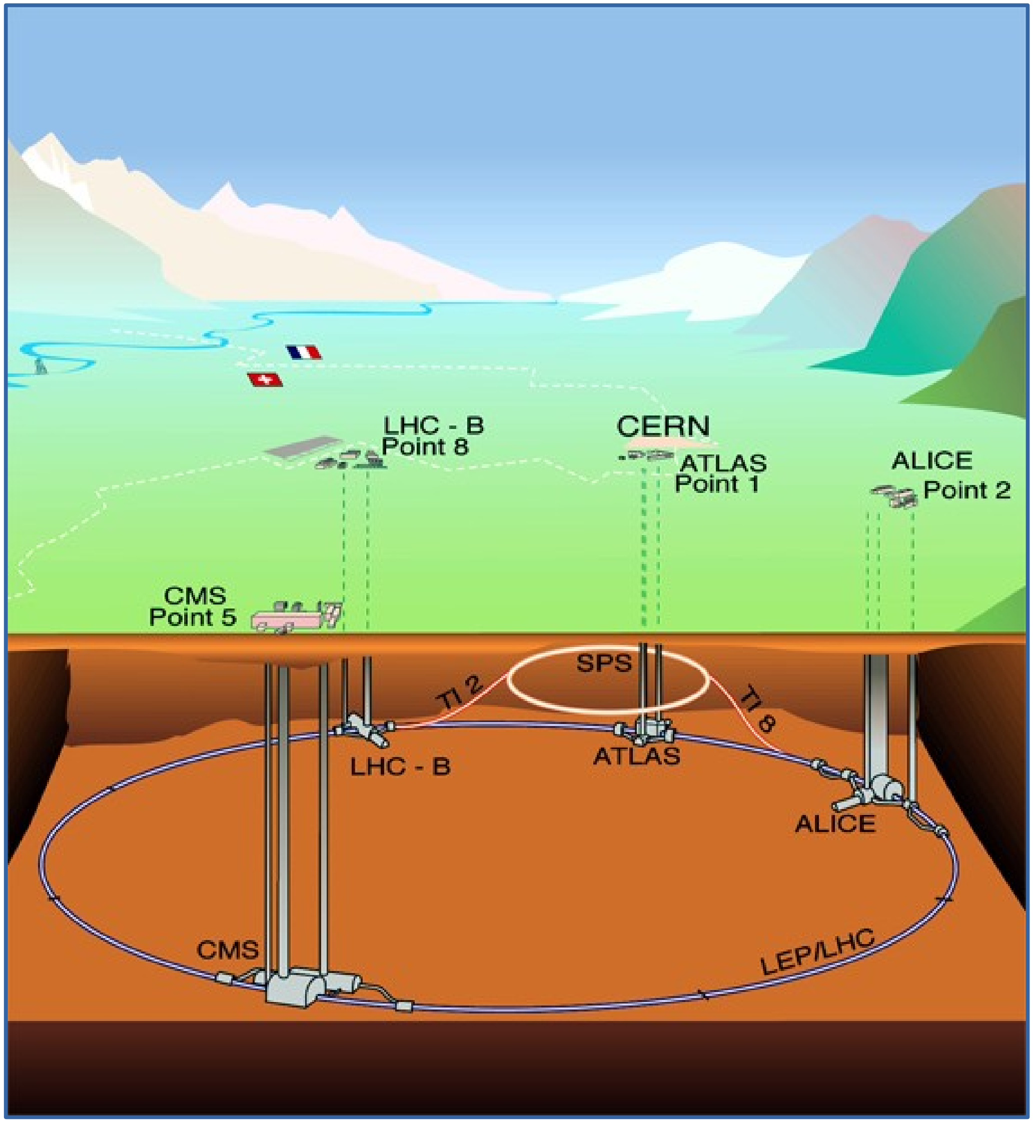
  JHEP 1704 (2017) 105 [arXiv:1702.00800]

Introduction:

Top quarks and Higgs (or Z, or W) bosons at the LHC



The Large Hadron Collider



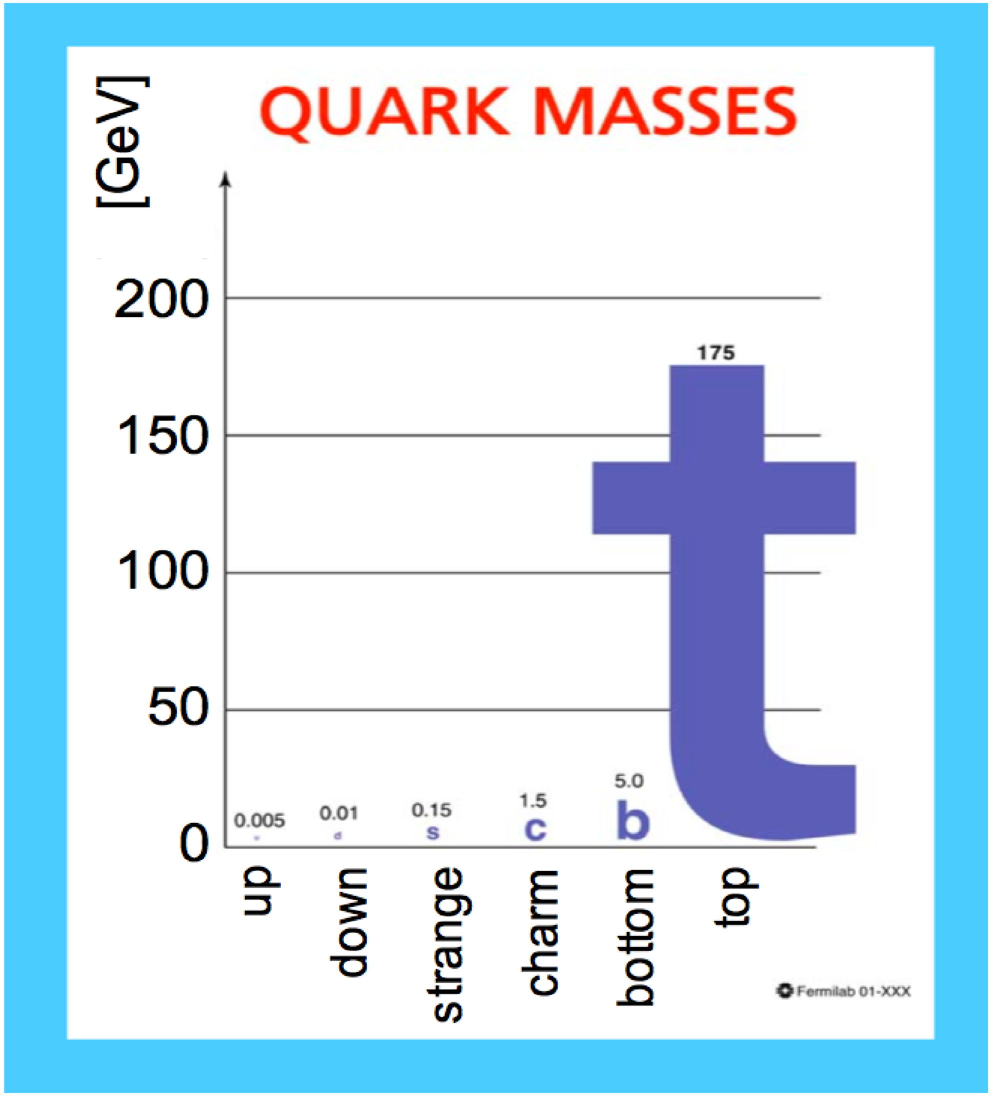
Proton-Proton collider @ CERN

Run I 2010-2013
c.o.m. energy 7-8 TeV

2012 Higgs boson discovered

Run II 2015 – present
c.o.m. energy 13 TeV

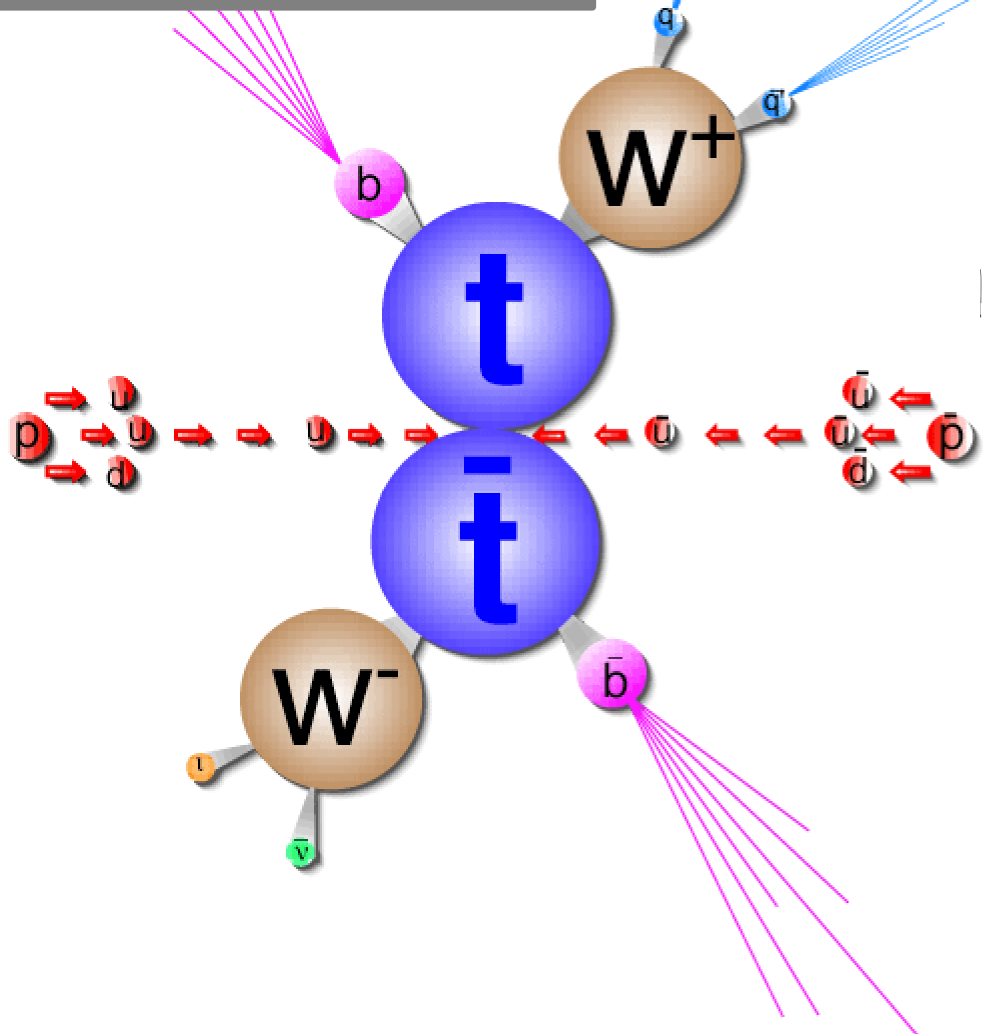
The heaviest particle in the SM



- Discovered in 1995 at Tevatron (> 20 years!)
- At the Tevatron ~ 10000 top quarks observed
- Mass measured to < 1 % accuracy
- At the LHC 13 TeV, already 33*10^6 top pair events produced (250*10^6 events expected at 300 1/fb of luminosity)
- According to the SM, the top quark is the elementary particle which couples most strongly to the Higgs boson

Top pair production

Top quarks are mainly produced in pairs with antitops



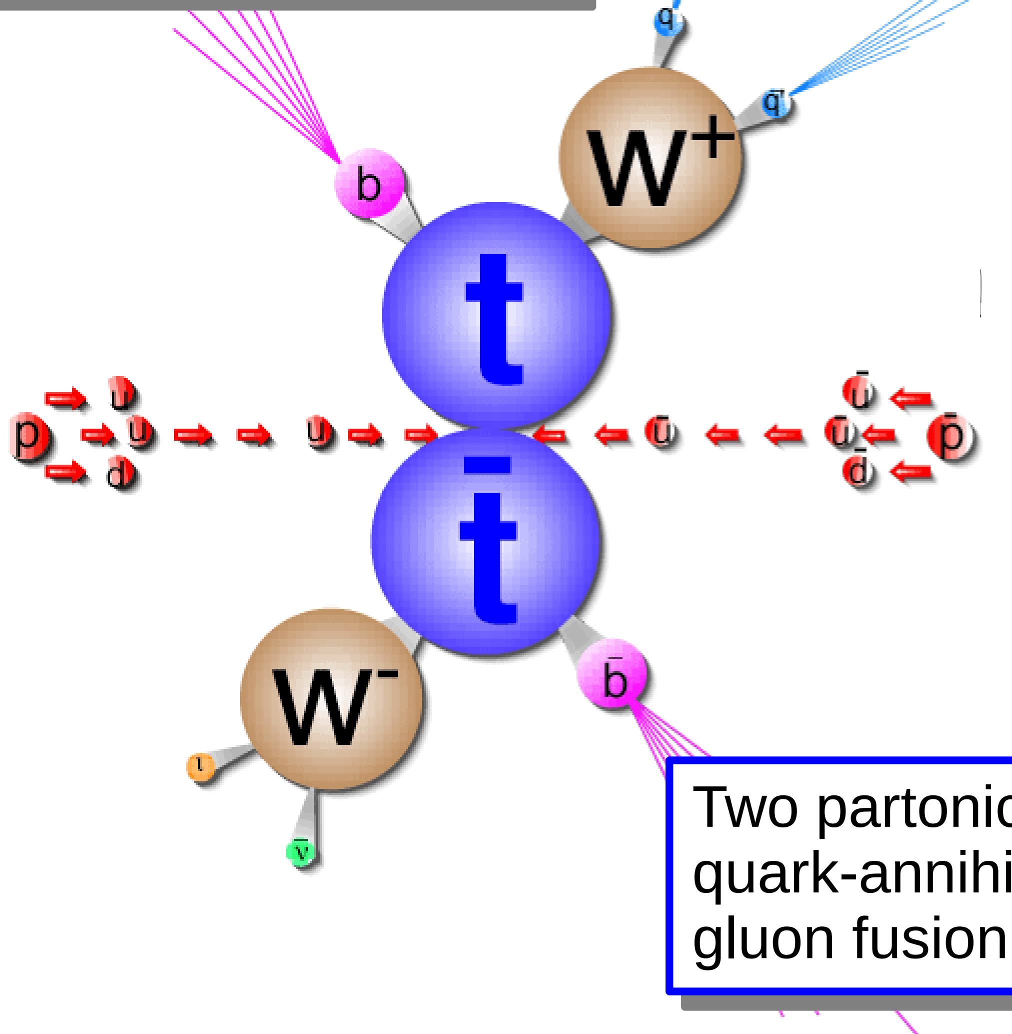
$$\sigma \sim 800 \text{ pb} \quad \text{at} \quad 13 \text{ TeV}$$

Complete NNLO calculation for on shell top quarks; total cross section and differential distributions

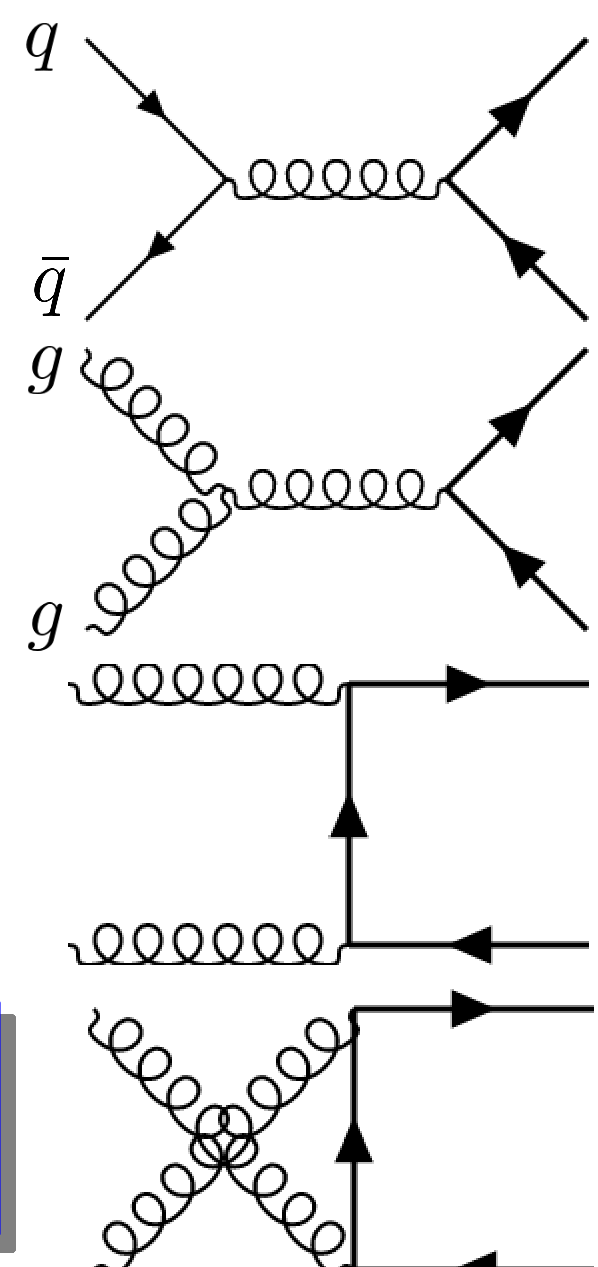
Czakon, Fiedler, Heimes, Mitov ('13–'17)

Top pair production

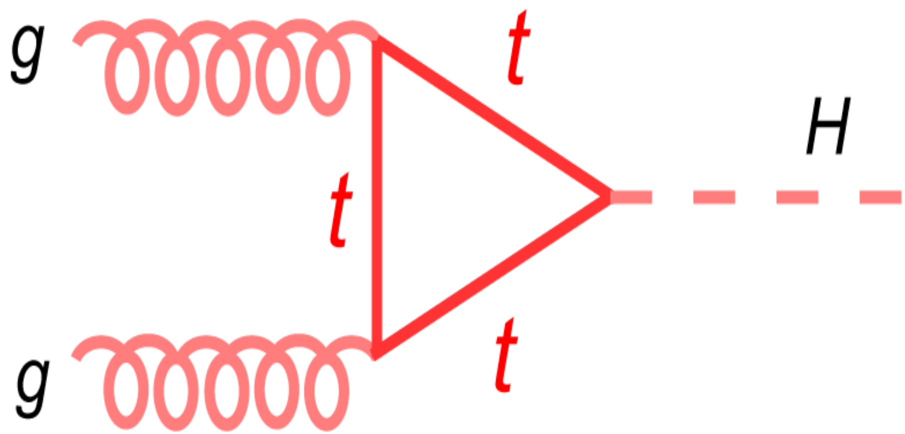
Top quarks are mainly produced in pairs with antitops

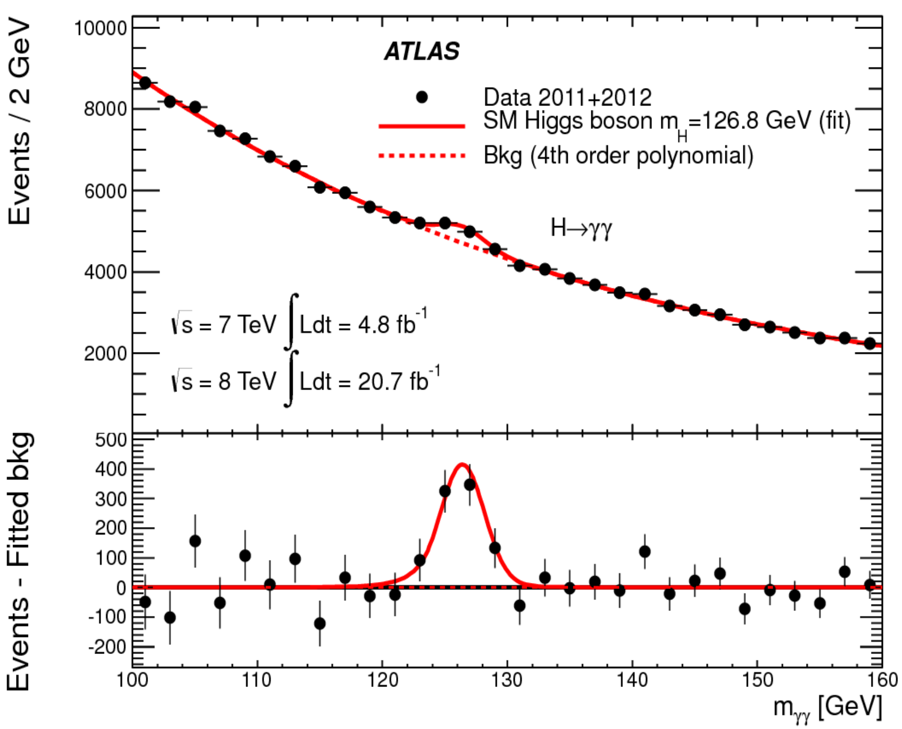


Two partonic channels: quark-annihilation and gluon fusion



Top quark and Higgs boson



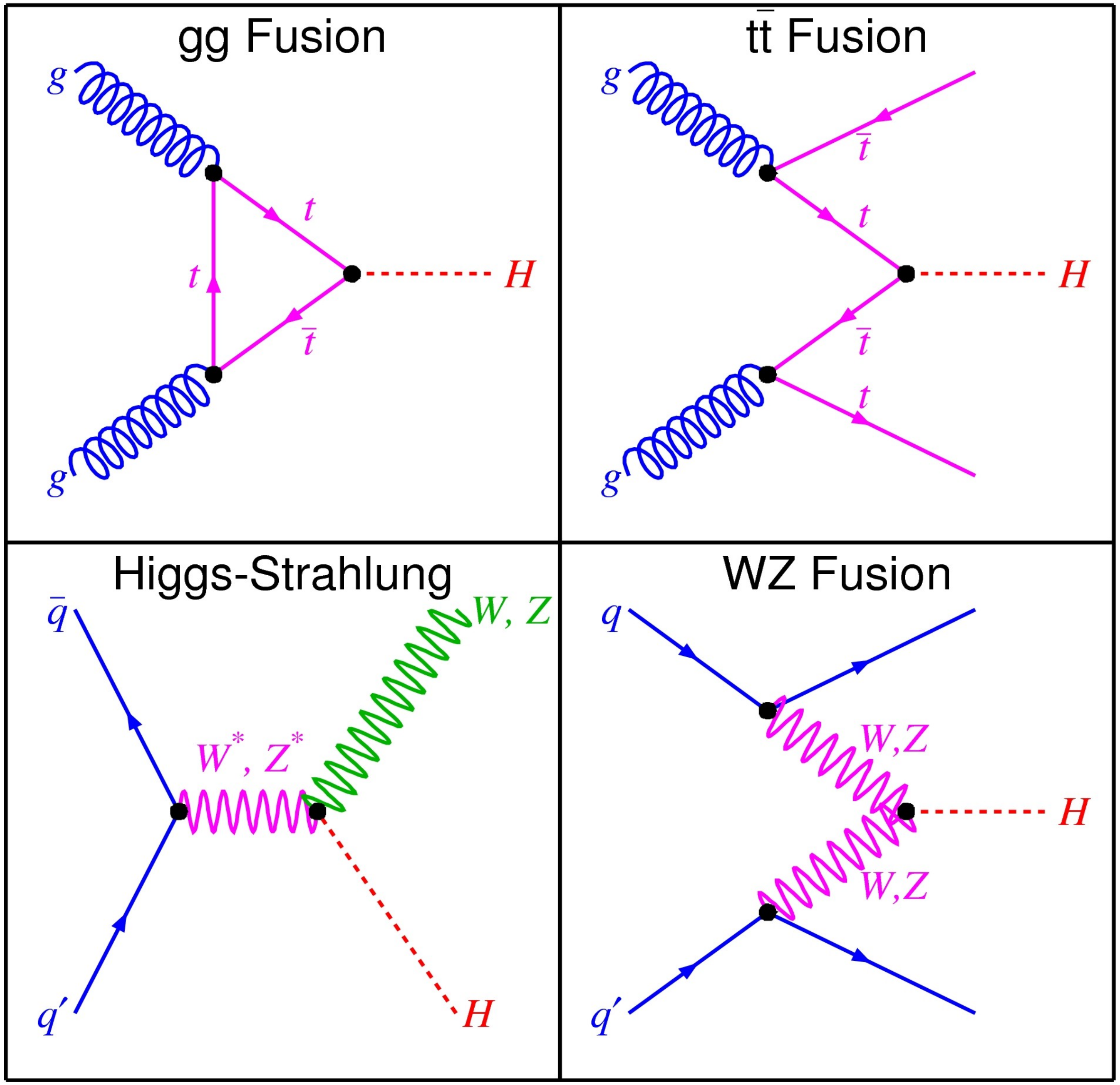


- The discovery of the Higgs boson was the main achievement of the Run I at the LHC
- The study of the properties of the Higgs boson are in many ways related to the study of the top quark
- While the gluon fusion channel provides the largest production cross section for the Higgs boson at the LHC, another production channel allows one to access directly the top-quark Yukawa coupling

Higgs boson production channels

$\sigma \sim 49\,\mathrm{pb}$

$\sigma \sim 0.6\,\mathrm{pb}$



$\sigma_W \sim 1.5\,\mathrm{pb}$

$\sigma_Z \sim 0.97\,\mathrm{pb}$

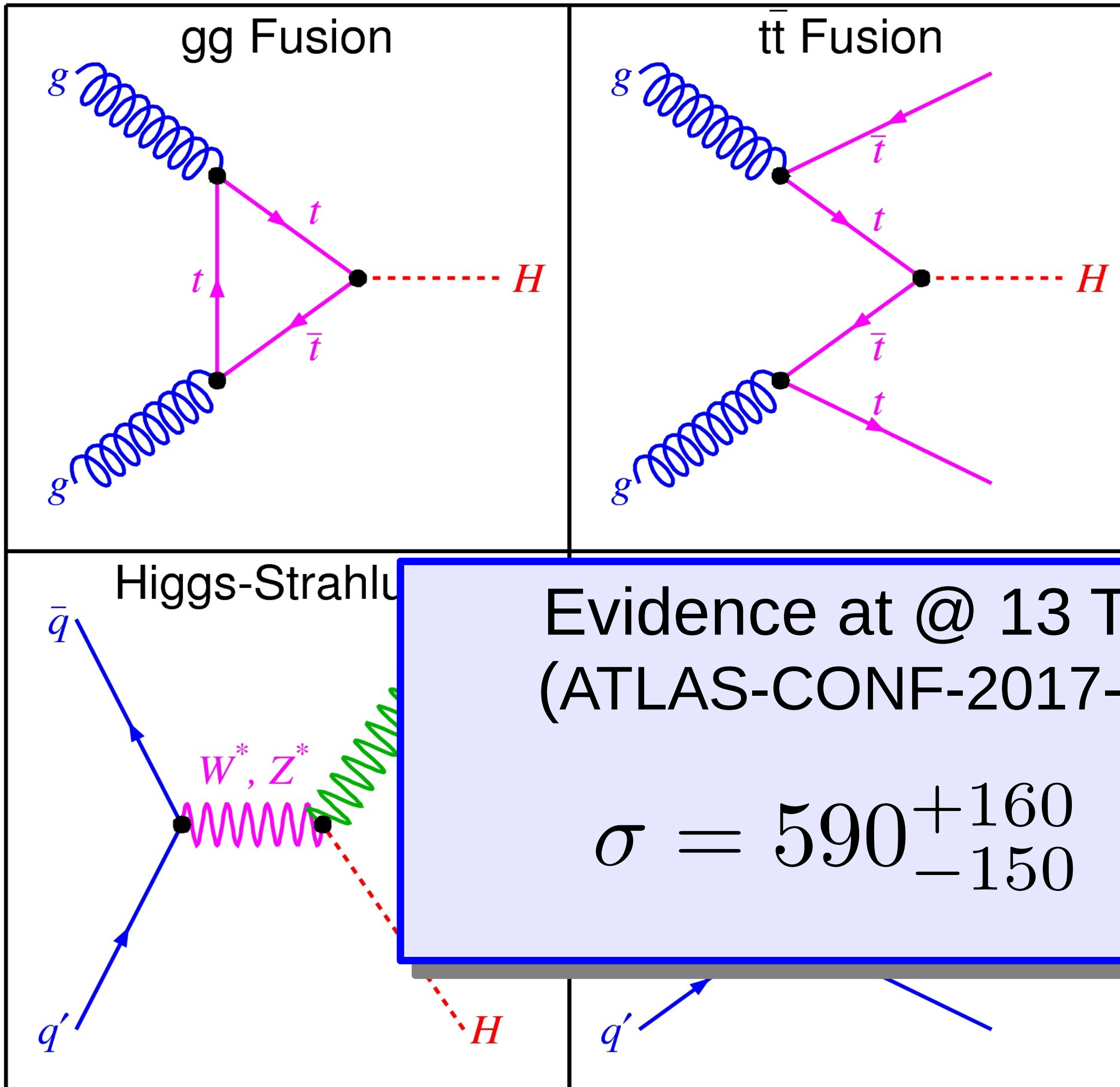
$\sigma \sim 4.2\,\mathrm{pb}$

LHC @ 14 TeV

Higgs boson production channels

$\sigma \sim 49\,\mathrm{pb}$

$\sigma \sim 0.6\,\mathrm{pb}$



$\sigma_W \sim 1.5\,\mathrm{pb}$

$\sigma_Z \sim 0.97\,\mathrm{pb}$

$2\,\mathrm{pb}$

Evidence at @ 13 TeV!

(ATLAS-CONF-2017-077)

$$\sigma = 590^{+160}_{-150}\ \mathrm{fb}$$

LHC @ 14 TeV

A (incomplete) history of top pair + Higgs calculations

- Cross section and some distributions evaluated to NLO QCD

  Beenakker, Dittmaier, Kraemer, Pluember, Spira, Zerwas ('01-'02)

  Dawson, Reina, Wackeroth, Orr,Jackson ('01,'03)

- In 2 → 3 processes ("multileg processes"), analytic NLO calculations become cumbersome: top pair + Higgs production was one of the first processes to be used to test automated tools

  Frixione et al ('11), Hirshi et al.('11)

  Garzelli et al.('11), Bevilacqua et al.('11)

- EW corrections to the parton level cross section are known

  Frixione, Hirshi,Pagani, Shao, Zaro ('14)

  Zhang, Ma, Chen, Guo ('14)

  Frixione, Hirshi, Pagani, Shao, Zaro ('15)

- NLO QCD corrections were interfaced with `SHERPA` and `POWHEG BOX`

  Gleisberg, Hoeche, Krauss, Schonherr, Schaumann ('09)

  Hartanto, Jaeger, Reina, Wackeroth ('15)

A (incomplete) history of top pair + Higgs calculations

- Cross section and some distributions evaluated to NLO QCD

Beenakker, Dittmaier, Kraemer, Pluember, Spira, Zerwas ('01–'02)
Dawson, Reina, Wackeroth, Orr, Jackson ('01,'03)

- In 2 ⟶ 3 processes ("multileg processes"), analytic NLO calculations become cumbersome: top pair + Higgs production was one of the first processes to be used to test automated tools

Frixione et al ('11), Hirshi et al.('11)
Garzelli et al.('11), Bevilacqua et al.('11)

- EW

aro ('14)
Guo ('14)
aro ('15)

NLL resummation at production threshold for the total cross section

NNLL resummation at partonic threshold for invariant mass distribution

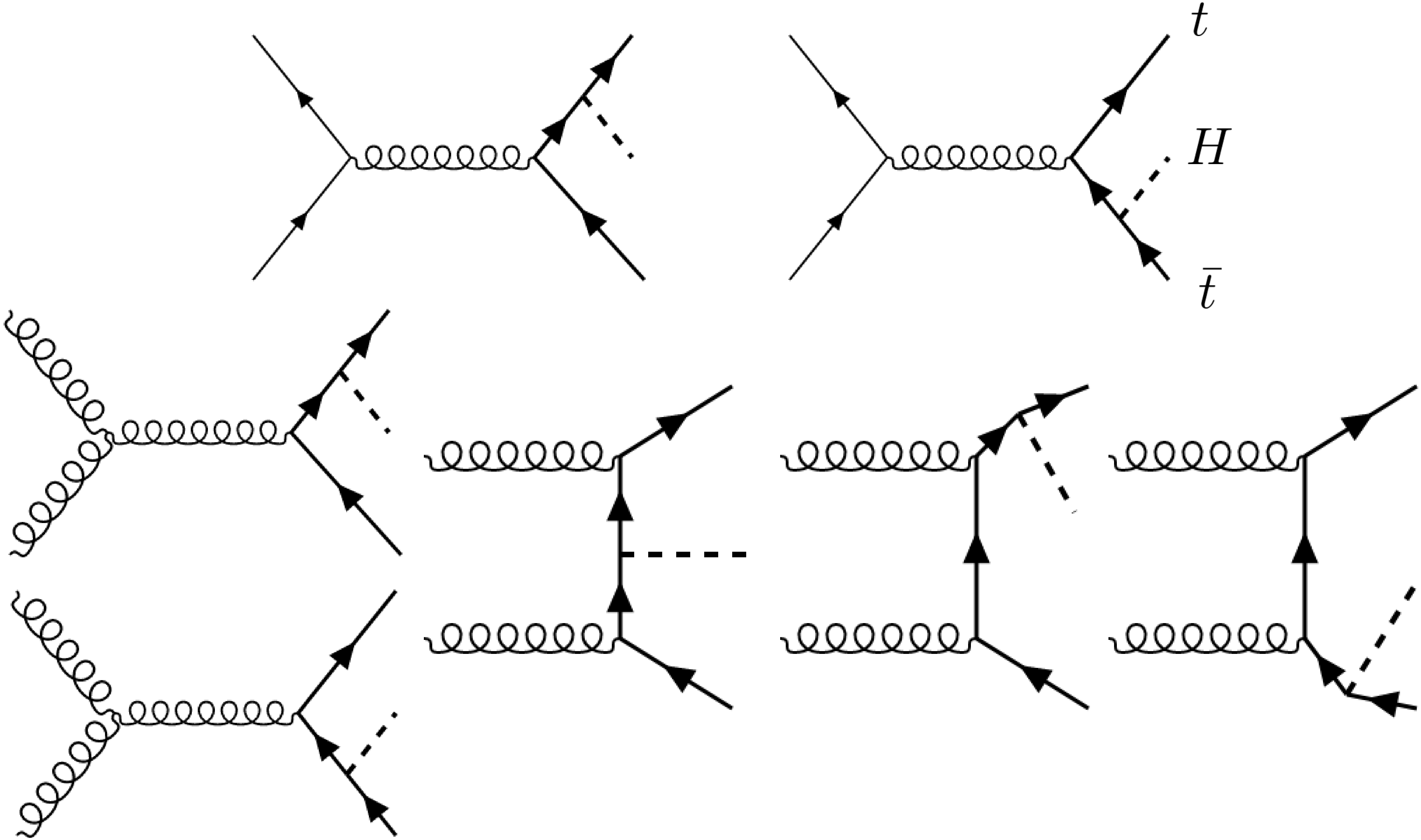
Kulesza, Motyka, Stebel, Theeuwes ('15,'16,'17)

- NLO

POWHEG BOX

Gleisberg, Hoeche, Krauss, Schonherr, Schaumann ('09)
Hartanto, Jaeger, Reina, Wackeroth ('15)

Top pair + H - tree level diagrams

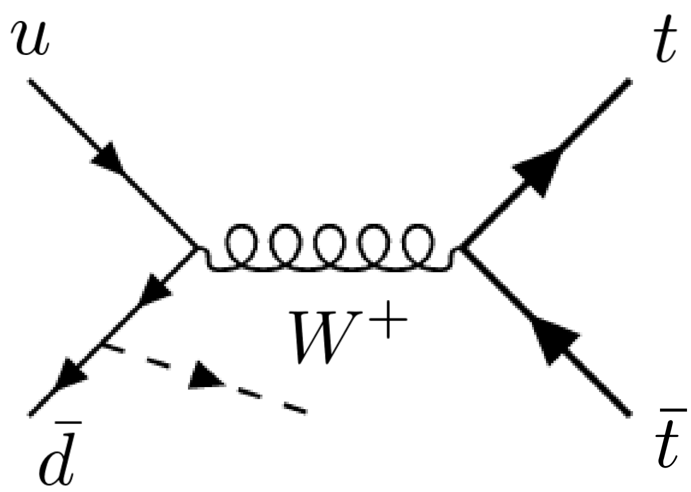


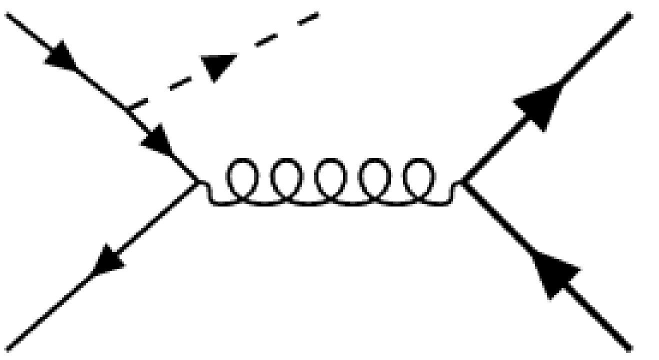
Top pair + W or Z boson

- $t\bar{t}W/t\bar{t}Z$ are the two heaviest set of particles observed at the LHC with c.o.m. energy of 7,8,13 TeV
- Important to detect anomalies in the top couplings of the Z boson
- Both processes would be altered by a variety of new physics models
- Can be considered background processes in new physics searches
- Both processes were calculated to NLO in QCD

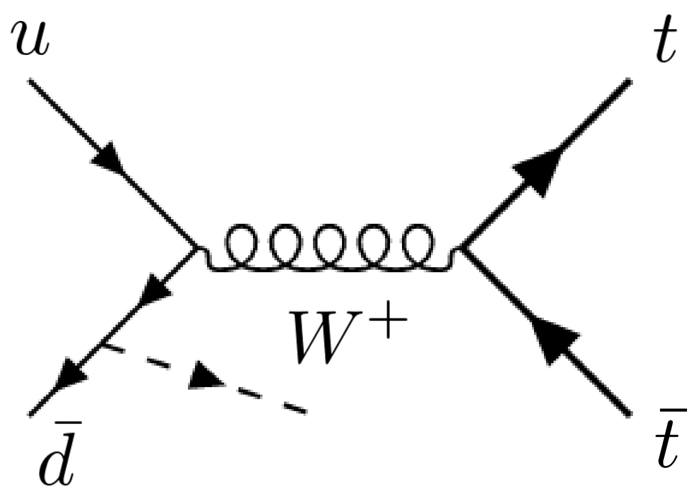
Garzelli, Kardos, Papadopoulos, Trocsanyi ('12)
Campbell, Ellis ('12)
Maltoni, Mangano, Tsinikos, Zaro ('11)

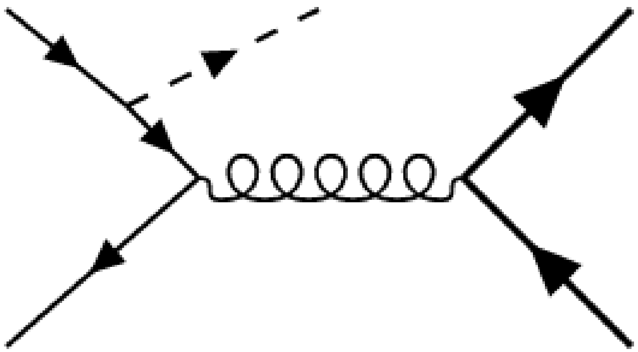
Top pair + W – tree level diagrams





Top pair + W – tree level diagrams





Top pair + Z at the lowest order involves all of the diagrams of the type present in the top pair + Higgs case (the Z boson can be emitted from the top quark line) and two diagrams of the type shown above (the Z boson can be emitted from the light quark line as well)

Top pair + W or Z boson

- Processes measured by CMS and ATLAS at 8 TeV

$$\sigma_{t\bar{t}W} = 382^{+117}_{-102}\ \mathrm{fb\ (CMS)} \qquad \sigma_{t\bar{t}W} = 369^{+100}_{-91}\ \mathrm{fb\ (ATLAS)}$$

$$\sigma_{t\bar{t}Z} = 242^{+65}_{-55}\ \mathrm{fb\ (CMS)} \qquad \sigma_{t\bar{t}Z} = 176^{+58}_{-52}\ \mathrm{fb\ (ATLAS)}$$

- W production measurements are in agreement with each other but about 1.5 σ larger than the NLO prediction

- Processes measured by CMS and ATLAS at 13 TeV (numbers from top 2017)

$$\sigma_{t\bar{t}W} = 800^{+120+130}_{-110-120}\ \mathrm{fb\ (CMS)} \qquad \sigma_{t\bar{t}W} = 1500^{+800}_{-800}\ \mathrm{fb\ (ATLAS)}$$

$$\sigma_{t\bar{t}Z} = 1000^{+90+120}_{-80-100}\ \mathrm{fb\ (CMS)} \qquad \sigma_{t\bar{t}Z} = 900^{+300}_{-300}\ \mathrm{fb\ (ATLAS)}$$

CMS PAS TOP 17-05 (stat,sys) Eur. Phys. J. C77 (2017)

Large logarithmic corrections

- The partonic cross section for top pair (+Higgs,W or Z) production receives potentially large corrections from soft gluon emission diagrams
- Schematically, the partonic cross section depends on logarithms of the ratio of two different scales:

$$L \equiv \ln\left(\frac{\text{“hard” scale}}{\text{“soft” scale}}\right)$$

- It can be that $\alpha_s L \sim 1$
- One needs to reorganize the perturbative series: Resummation
- The resummation of soft emission corrections can be carried out by means of effective field theory methods

Large logarithmic corrections

- The partonic cross section for top pair (+Higgs) production receives potentially large corrections from soft gluon emission dia
- So log
- It c
- On tion
- Th d out by means of effective field theory methods

Renormalization group improved perturbation theory schematically:

➔ Separation of scales ↔ factorization

➔ Evaluate each (single-scale) factor in fixed order perturbation theory at a scale for which it is free of large logs

➔ Use Renormalization Group Equations to evolve the factors to a common scale

Goal

We want to analyze the factorization properties of

$$p + p \longrightarrow t + \bar{t} + H(\text{or } W, Z) + X$$

in the soft emission limit in order to

i. Obtain NNLL resummation formulas for these processes
ii. Evaluate the total cross section and differential distributions depending on the 4-momenta of the final state particles
iii. Match NLO and NNLL calculations to obtain NLO+NNLL predictions

Goal

We want to analyze the factorization properties of

$$p + p \longrightarrow t + \bar{t} + H(\text{or } W, Z) + X$$

in the soft emission limit in order to

Additional final state radiation

i. Obtain NNLL resummation formulas for these processes

ii. Evaluate the total cross section and differential distributions depending on the 4-momenta of the final state particles

iii. Match NLO and NNLL calculations to obtain NLO+NNLL predictions

Soft limit & factorization

(for top pair + H production)

“Pair” Invariant Mass kinematics

- For top pair + Higgs production, we have two tree-level partonic processes

$$q(p_1) + \bar{q}(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$
$$g(p_1) + g(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$

- Define the invariants

$$\hat{s} = (p_1 + p_2)^2 \qquad M^2 = (p_3 + p_4 + p_5)^2$$

"Pair" Invariant Mass kinematics

- For top pair + Higgs production, we have two tree-level partonic processes

$$q(p_1) + \bar{q}(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$
$$g(p_1) + g(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$

- Define the invariants

$$\hat{s} = (p_1 + p_2)^2 \qquad M^2 = (p_3 + p_4 + p_5)^2$$

Partonic center of mass energy (squared)

Invariant mass of the heavy particles in the final state

“Pair” Invariant Mass kinematics

- For top pair + Higgs production, we have two tree-level partonic processes

$$q(p_1) + \bar{q}(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$
$$g(p_1) + g(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$

- Define the invariants

$$\hat{s} = (p_1 + p_2)^2 \qquad M^2 = (p_3 + p_4 + p_5)^2$$

If real radiation in the final state is present, $\hat{s} \neq M^2$

$$z = \frac{M^2}{\hat{s}}$$

“Pair” Invariant Mass kinematics

- For top pair + Higgs production, we have two tree-level partonic processes

$$q(p_1) + \bar{q}(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$
$$g(p_1) + g(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$

- Define the invariants

$$\hat{s} = (p_1 + p_2)^2 \qquad M^2 = (p_3 + p_4 + p_5)^2$$

Partonic threshold limit

$$z \longrightarrow 1$$

If real radiation in the final state is

$$z = \frac{M^2}{\hat{s}}$$

Factorization in a nutshell

Differential cross section:

$$\frac{d^2\sigma}{dM^2} = \mathit{ff}(z) \otimes C(z)$$

Factorization in a nutshell

Differential cross section:

$$\frac{d^2\sigma}{dM^2} = ff(z) \otimes C(z)$$

Partonic luminosity

$$ff = \int_y^1 \frac{dx}{x} f_{i/N_1}(x) f_{j/N_2}\left(\frac{y}{x}\right)$$

Factorization in a nutshell

Differential cross section:

$$\frac{d^2\sigma}{dM^2} = ff(z) \otimes C(z)$$

Partonic luminosity

Hard scattering kernel
(partonic cross section)

Factorization in a nutshell

Differential cross section:

$$\frac{d^2\sigma}{dM^2} = ff(z) \otimes C(z)$$

In the soft emission limit a clear scale hierarchy emerges:

$$\hat{s}, M^2, m_t^2 \gg \hat{s}(1-z)^2 \gg \Lambda_{\mathrm{QCD}}^2$$

Factorization in a nutshell

Differential cross section:

$$\frac{d^2\sigma}{dM^2} = ff(z) \otimes C(z)$$

In the soft emission limit a clear scale hierarchy emerges:

$$\underbrace{\hat{s}, M^2, m_t^2}_{\text{Hard scales}} \gg \underbrace{\hat{s}(1-z)^2}_{\text{Soft scale}} \gg \Lambda^2_{\mathrm{QCD}}$$

Hard scales

Soft scale

Factorization in a nutshell

Differential cross section:

$$\frac{d^2\sigma}{dM^2} = ff(z) \otimes C(z)$$

In the soft emission limit a clear scale hierarchy emerges:

$$\hat{s}, M^2, m_t^2 \gg \hat{s}(1-z)^2 \gg \Lambda^2_{\mathrm{QCD}}$$

In this limit, the partonic cross section factors into two parts:

$$C_{ij} = \mathrm{Tr}\left[\mathbf{H}_{ij}\left(M, \{p_i\}, \mu\right) \mathbf{S}_{ij}\left(\sqrt{\hat{s}}(1-z), \{p_i\}, \mu\right)\right]$$

Hard function
(virtual corrections)

Soft function
(real soft emission)

Factorization in a nutshell

Differential cross section:

$$\frac{d^2\sigma}{dM^2} = ff(z) \otimes C(z)$$

In the soft emission limit a clear scale hierarchy emerges:

$$\hat{s}, M^2, m_t^2 \gg \hat{s}(1-z)^2 \gg \Lambda_{\rm QCD}^2$$

In this limit, the partonic cross section factors into two parts:

$$C_{ij} = \mathrm{Tr}\left[\mathbf{H}_{ij}\left(M, \{p_i\}, \mu\right) \mathbf{S}_{ij}\left(\sqrt{\hat{s}}(1-z), \{p_i\}, \mu\right)\right]$$

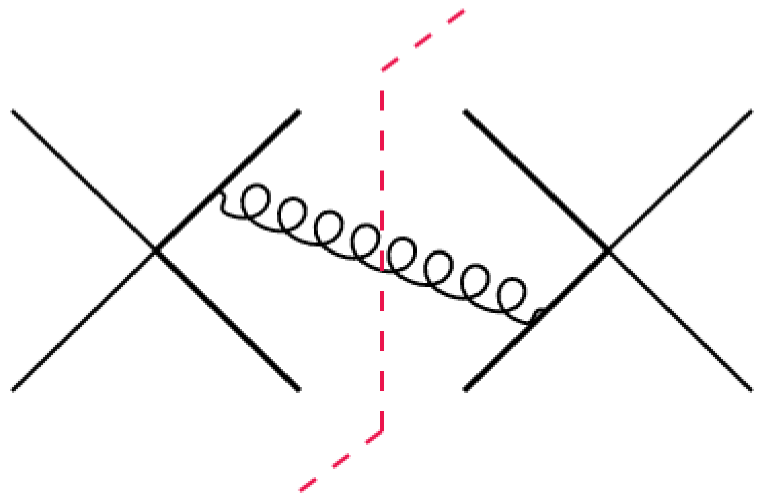
Hard function
(virtual corrections)

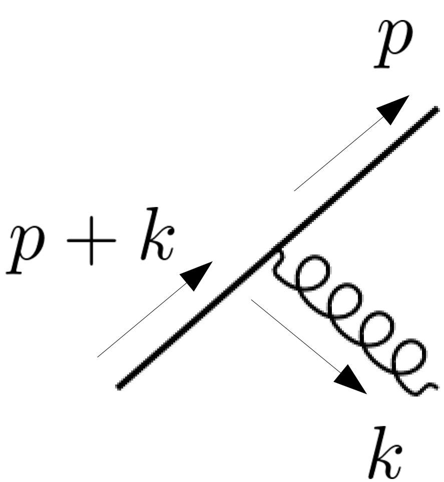
both are matrices
(in color space)

Soft function
(real soft emission)

Soft function at NLO

The soft function can be calculated by evaluating diagrams involving the emission of soft gluons from the external legs



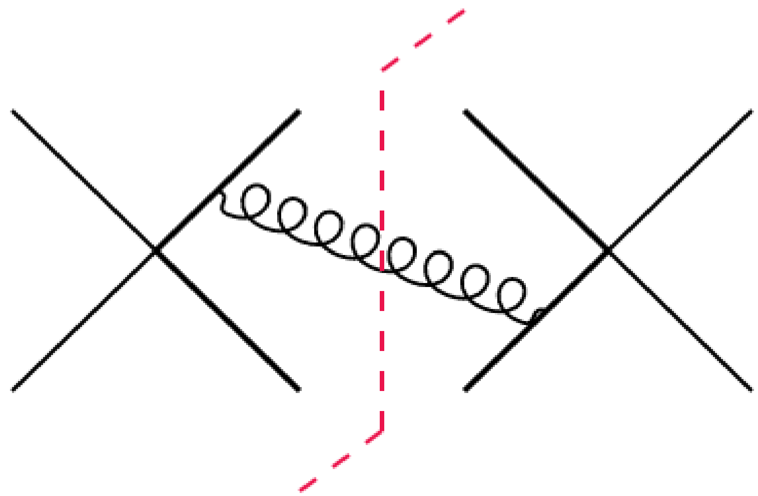


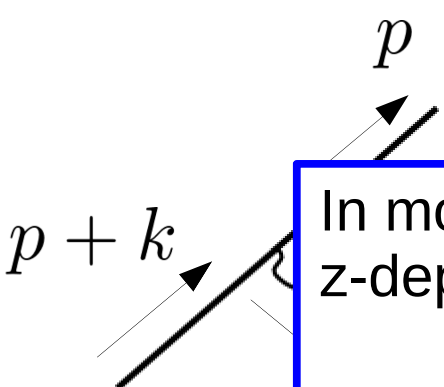
$$i \frac{\not{p} + \not{k} + m}{(p+k)^2 - m^2} \gamma^\mu \xrightarrow{k \to 0} i \frac{p^\mu}{p \cdot k}$$

Eikonal vertex

$$\mathcal{I}_{ij}(\epsilon, x_0, \mu) = -\frac{(4\pi\mu^2)^\epsilon}{\pi^{2-\epsilon}} v_i \cdot v_j \int d^d k \, \frac{e^{-ik^0 x_0}}{v_i \cdot k \, v_j \cdot k} (2\pi) \, \delta(k^2) \, \theta(k^0)$$

Soft function at NLO





konal
ertex

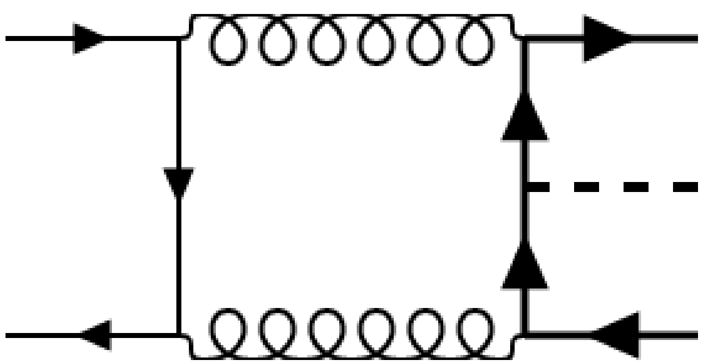
In momentum space the soft function depends on z-dependent plus distributions

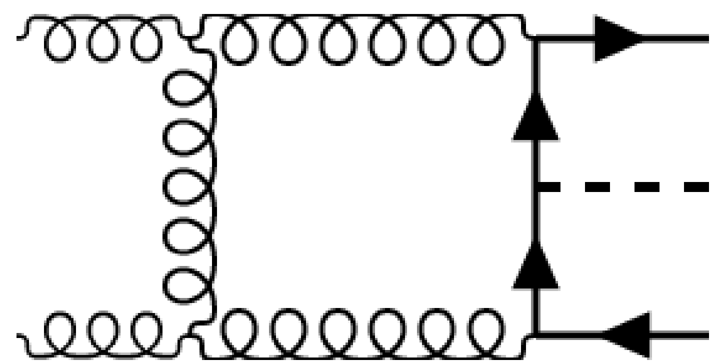
$$P'_n(z) \equiv \left[\frac{1}{1-z} \ln^n \left(\frac{M^2 (1-z)^2}{\mu^2 z} \right) \right]_+$$

$\mathcal{I}_{ij}(\epsilon, x_0$ … $^2)\, \theta(k^0)$

Hard function at NLO

In order to evaluate the NLO hard function one needs to calculate one-loop QCD amplitudes. In doing this one need to separate the various components of the amplitude in color space. Some examples are:





For top-quark pair production, the NLO matrix elements can still be calculated analytically: Things are more complicated for top-pair+Higgs, which is a 2 to 3 process

Top pair + Higgs: Hard function

- The calculation of the NLO hard function requires the evaluation of one loop amplitudes for a 2 $\longrightarrow$ 3 process (separating out the various color components)

- The evaluation of the NLO QCD corrections to $t\bar{t}H$ corrections was carried out with “traditional” (Passarino-Veltman like) reduction methods

  Beenakker, Dittmaier, et al. ('01-'02)
  Dawson, Reina, et al. ('01,'03)

- In order to calculate the NLO hard function, it is convenient to take advantage of the automated tools available on the market. However, to date, none provides hard functions out of the box and all require some level of customization

Solution:

Modified version of GoSam and Openloops (+ Collier)

Cullen, Greiner, Heinrich, Luisoni, Mastrolia, Ossola,et al. ('12-'14)
Cascioli, Maierhofer, Pozzorini ('12) Denner, Dittmaier, Hofer (‘16)

Renormalization group equations

- The hard and soft functions are free from large logarithms and can be evaluated in fixed order perturbation theory
- The hard and soft functions satisfy RGEs regulated by anomalous dimensions which can also be calculated up to a given order in the strong coupling constant α_s
- By solving the RGEs one can resum large corrections depending on the ratio of hard and soft scales

Renormalization group equations

- The hard and soft functions are free from large logarithms and can be evaluated in fixed order perturbation theory
- The hard and soft functions satisfy RGEs regulated by anomalous dimensions which can also be calculated up to a given order in the strong coupling constant α_s
- By solving the RGEs one can resum large corrections depending on the ratio of hard and soft scales
- In practice, it is more convenient to solve the RGEs in Laplace or Mellin space, where convolutions become regular products

$$\frac{d\sigma}{dM^2} \propto \int_\tau^1 \frac{dz}{z} ff\left(\frac{\tau}{z}\right) \mathrm{Tr}\left[\mathbf{H}\mathbf{S}(z)\right] \to \frac{d\tilde{\sigma}}{dM^2} \propto \tilde{ff}\,\mathrm{Tr}\left[\mathbf{H}\tilde{\mathbf{s}}\right]$$

$$[\tau = M^2/s\,, \quad s = (\text{collider energy})^2]$$

Renormalization group equations

- The hard and soft functions are free from large logarithms and can be evaluated in fixed order perturbation theory
- The hard and soft functions satisfy RGEs regulated by anomalous dimensions which can also be calculated up to a given order in the strong coupling constant α_s
- By solving the RGEs one can resum large corrections depending on the ratio of hard and soft scales
- In practice, it is more convenient to solve the RGEs in Laplace of Mellin space, where convolutions become regular products

$$\frac{d}{d\ln\mu}\tilde{\mathbf{s}} = -\mathbf{\Gamma}_s\tilde{\mathbf{s}} - \tilde{\mathbf{s}}\mathbf{\Gamma}_s^\dagger$$

Mellin space

- NNLL resummation in momentum space,(“SCET approach”), was already carried out by Li, Li, and Li (2014) for associated top pair + W production

- The resummation can also be carried out in Mellin space (by taking the Mellin transform of the factorized cross section), similar to “direct QCD” resummation

$$\tilde{c}(N,\mu) = \int_0^1 dz z^{N-1} \int d\text{PS}_{t\bar{t}H} \text{Tr}\left[\mathbf{H}\left(\{p\},\mu\right)\mathbf{S}\left(\sqrt{\hat{s}}(1-z),\{p\},\mu\right)\right]$$

- The total cross section can be then recovered with an inverse Mellin transform

$$\sigma = \frac{1}{2s}\int_{\tau_{\min}}^1 \frac{d\tau}{\tau}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN \tau^{-N} \widetilde{ff}(N,\mu) \int d\text{PS}_{t\bar{t}H}\,\widetilde{c}\,(N,\mu)$$

+ Minimal Prescription (Catani et al, ‘96) + Mellin space lum. (Bonvini and Marzani ‘12, ‘14)

Mellin space

- Hard and soft scales are evaluated at values of the scale where large corrections are absent

$$\mu_h = M \quad \mu_s = M/\bar{N}$$

- RG evolution is used to obtain $\widetilde{c}$ at the scale μ_f

$$\widetilde{c}(\mu_f) = \mathrm{Tr}\left[\widetilde{\mathbf{U}}(\mu_f, \mu_h, \mu_s)\,\mathbf{H}(\mu_h)\,\widetilde{\mathbf{U}}^\dagger(\mu_f, \mu_h, \mu_s)\widetilde{\mathbf{s}}\,(\mu_s)\right]$$

- By rewriting $\alpha_s(\mu_f)$ and $\alpha_s(\mu_s)$ as a function of $\alpha_s(\mu_h)$

$$\widetilde{\mathbf{U}} = \exp\left\{\frac{4\pi}{\alpha_s(\mu_h)} g_1\left(\lambda, \lambda_f\right) + g_2\left(\lambda, \lambda_f\right) + \frac{\alpha_s(\mu_h)}{4\pi} g_3\left(\lambda, \lambda_f\right) + \cdots\right\}$$

$$\times\, \mathbf{u}(\{p\}, \mu_h, \mu_s)$$

$$\lambda = \frac{\alpha_s(\mu_h)}{2\pi}\beta_0 \ln\frac{\mu_h}{\mu_s}\,, \qquad \lambda_f = \frac{\alpha_s(\mu_h)}{2\pi}\beta_0 \ln\frac{\mu_h}{\mu_f}$$

Dynamical threshold enhancement

- Soft limit = good approximation of the partonic cross section
- However, we are colliding protons: Does the soft limit provide reasonable results for hadronic observables? (ex. invariant mass distribution)

$$\frac{d\sigma}{dM} \propto \int_{\tau}^{1} \frac{dz}{z} \sum_{\text{channels}} ff\left(\frac{\tau}{z}, \mu\right) \underbrace{C(z, \cdots, \mu)}$$

Hard scattering kernel
Partonic cross section

Dynamical threshold enhancement

- Soft limit = good approximation of the partonic cross section
- However, we are colliding protons: Does the soft limit provide reasonable results for hadronic observables? (ex. invariant mass distribution)

$$\frac{d\sigma}{dM} \propto \int_\tau^1 \frac{dz}{z} \sum_{\text{channels}} ff\left(\frac{\tau}{z}, \mu\right) C(z, \cdots, \mu)$$

The soft limit (z → 1) after convolutions with the partonic luminosity provides a good approximation to the observable if:

- $\tau \sim 1$, but this situation is not interesting phenomenologically
- $ff \to 0 \quad z \to \tau$: Dynamical threshold enhancement

Dynamical threshold enhancement: tests

- Does dynamical threshold enhancement occur? We need to check if the approximate NLO predictions are reasonably close to the full NLO calculations
- In top pair production dynamical threshold enhancement does take place; we will see that the same is true for top pair + Higgs and top pair + W/Z
- Warning: approximate NLO formulas obtained from the soft limit ignore the contribution of the quark-gluon channel (which is formally subleading in the $z \to 1$ limit)

Complete NLO calculations

We need complete NLO results for the total cross section and the differential distributions we are interested in, both to validate the approximate formulas and to match results to the full NLO:

MadGraph5_aMC@NLO

Precise theoretical predictions are obtained by combining NNLL resummation and NLO calculation. The matching procedure allows one to avoid the double counting of terms included in both approaches

$$d\sigma^{\mathrm{NLO+NNLL}} = d\sigma^{\mathrm{NNLL}}\Big|_{\mu_h,\mu_s,\mu_f} + \left(d\sigma^{\mathrm{NLO}} - d\sigma^{\mathrm{NNLL}}\Big|_{\mu_s=\mu_h=\mu_f} \right)$$

Summary so far

- ✔ In the soft emission limit, the partonic cross section factors into a hard function and a soft function
- ✔ Hard functions and soft functions satisfy RGEs regulated by know anomalous dimensions
- ✔ If one is able to calculate the hard and soft functions at NLO and to solve the RGEs one can implement NNLL resummation
- ✔ Compare NLO and approx. NLO obtained by re-expanding the resummation formula. If the agreement is good (and it is) it makes sense to evaluate numerically the NNLL resummed cross section and match it to NLO calculations

Top pair + W:

Numerical evaluation of NNLL resummation formulas

Resummation: Top pair + W boson

- With respect to factorization in the soft limit, top pair + W behaves as top pair + Higgs

- At lowest order top pair + W receives contributions only from from the quark-annihilation channel, no contribution from the gluon-fusion channel (shorter running times for resummation)

- Among the various elements which contribute to the factorization formula up to NNLL, only the NLO hard function differs from the one needed for the production of top pair + Higgs and needs to be recalculated

Resummation: Top pair + W boson

- With respect to factorization in the soft limit, top pair + W behaves as top pair + Higgs

- At lowes … butions only fron … el, no contribut … shorter running t …

Top pair + W production is an ideal process to develop and test an in-house program which can evaluate resummation formulas to NNLL

- Among the various elements which contribute to the factorization formula up to NNLL, only the NLO hard function differs from the one needed for the production of top pair + Higgs and needs to be recalculated

Final state phase space

- The final state phase space is written as the convolution of two two-particle phase spaces:

$$\int d\Phi_{t\bar{t}H} = \int \frac{ds_{t\bar{t}}}{2\pi} \frac{1}{2M^2} \frac{d\Omega}{16\pi^2} K\left(M^2, s_{t\bar{t}}, m_H^2\right) \frac{1}{2s_{t\bar{t}}} \frac{d\Omega^*}{16\pi^2} K\left(s_{t\bar{t}}, m_t^2, m_t^2\right)$$

$$K(x, y, z) = \sqrt{x^2 + y^2 + z^2}$$

- Five integrations left in the final state phase space
- Three integrations for the initial state (τ, N, and the luminosity variable x)
- One needs to build a Monte Carlo integration over 8 variables
- The 8 integration variables determine the top, antitop, Higgs (or W/Z) and incoming parton momenta: one can bin events and plot distributions

Scale uncertainty

- In fixed order results, the scale uncertainty is evaluated by varying $\mu_f \in [\mu_{f,0}/2, 2\mu_{f,0}] \text{ with } \mu_{f,0} = M$

- For resummed results, we vary all scales (hard, soft and factorization) independently in the range $\mu_i \in [\mu_{i,0}/2, 2\mu_{i,0}]$

- For an observable O (the total cross section, or the value of a differential cross section in a given bin) one evaluates (for $i = s, f, h \text{ and } \kappa_i = \mu_i/\mu_{i,0}$)

$$\Delta O_i^+ = \max\{O(\kappa_i = 1/2), O(\kappa_i = 1), O(\kappa_i = 2)\} - O(\kappa_i = 1)$$

$$\Delta O_i^- = \min\{O(\kappa_i = 1/2), O(\kappa_i = 1), O(\kappa_i = 2)\} - O(\kappa_i = 1)$$

- The quantities $\Delta O_i^+ (\Delta O_i^-)$ are then combined in quadrature in order to obtain the scale uncertainty above (below) the central value

Total cross section @ 8 TeV

order	PDFs order	code	σ [fb]
NLO	NLO	MG5_aMC	$121.6^{+15.2}_{-14.0}$
NLO no qg	NLO	MG5_aMC	$118.1^{+10.3}_{-11.3}$
app. NLO	NLO	MC	$116.0^{+10.3}_{-11.6}$

Good agreement NLO
approx NLO

W^+production

MMHT 2014 PDFs here and in the following

Total cross section @ 8 TeV

order	PDFs order	code	σ [fb]
LO	LO	MG5_aMC	$82.0^{+21.2}_{-15.7}$
NLO	NLO	MG5_aMC	$121.6^{+15.2}_{-14.0}$
NLO+NLL	NLO	MC +MG5_aMC	$124.8^{+13.1}_{-8.0}$
NLO+NNLL	NNLO	MC +MG5_aMC	$128.7^{+5.5}_{-4.7}$

The NLO+NNLL cross section is slightly larger than the NLO cross section, the residual scale uncertainty is about 1/3 of the NLO one

W^+production

MMHT 2014 PDFs here and in the following

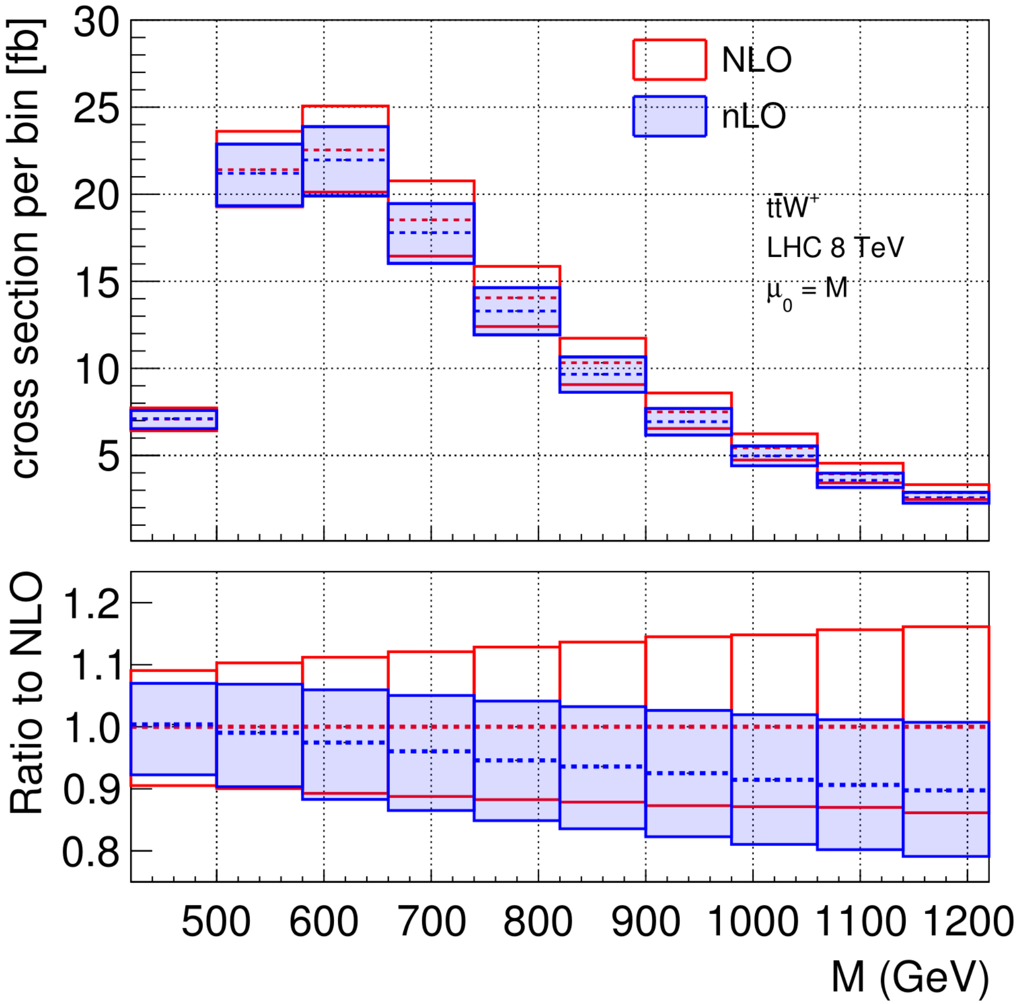
Total cross section @ 13 TeV

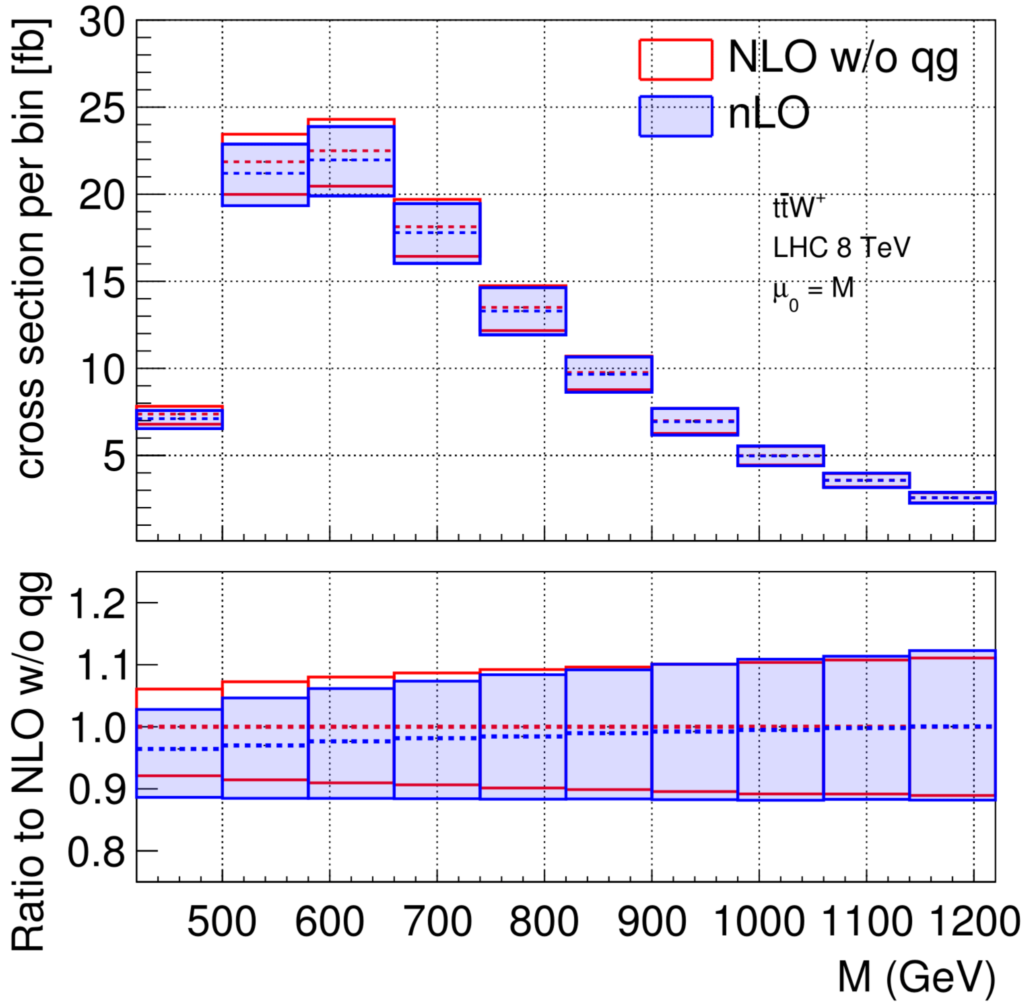
order	PDFs order	code	σ [fb]
LO	LO	MG5_aMC	$202.1^{+45.5}_{-34.9}$
NLO	NLO	MG5_aMC	$316.9^{+39.3}_{-34.9}$
NLO no qg	NLO	MG5_aMC	$293.3^{+19.3}_{-22.7}$
app. NLO	NLO	MC	$288.1^{+21.4}_{-23.8}$
nNLO (Mellin)	NNLO	MC +MG5_aMC	$330.5^{+26.2}_{-19.2}$
NLO+NNLL	NNLO	MC +MG5_aMC	$333.0^{+14.9}_{-12.4}$

W^+production

(Results for tTW^- can be found in the paper)

tTW distributions dynamical threshold enhancement

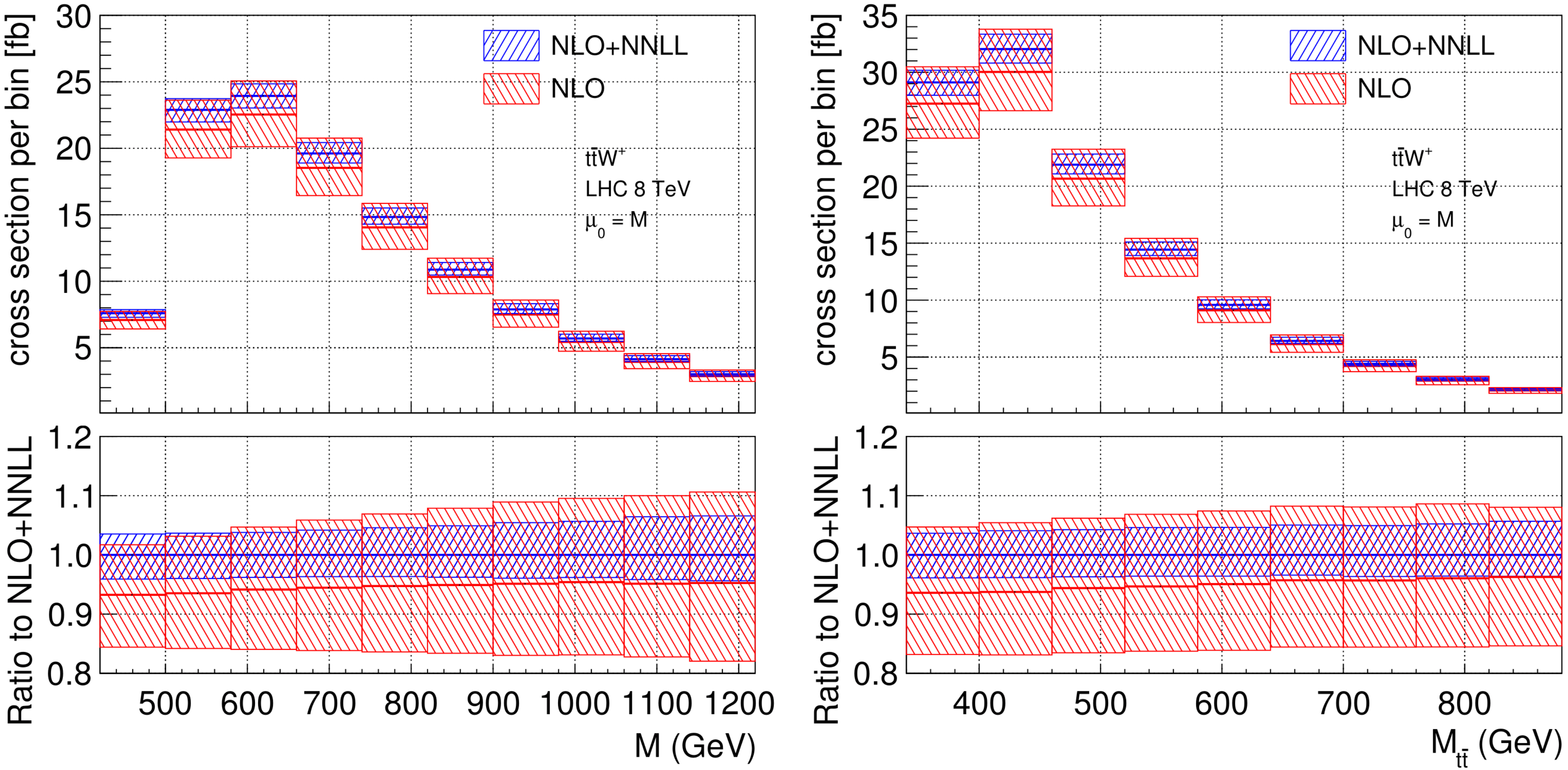




Good agreement NLO vs approx. NLO. The shape of the distribution is slightly different

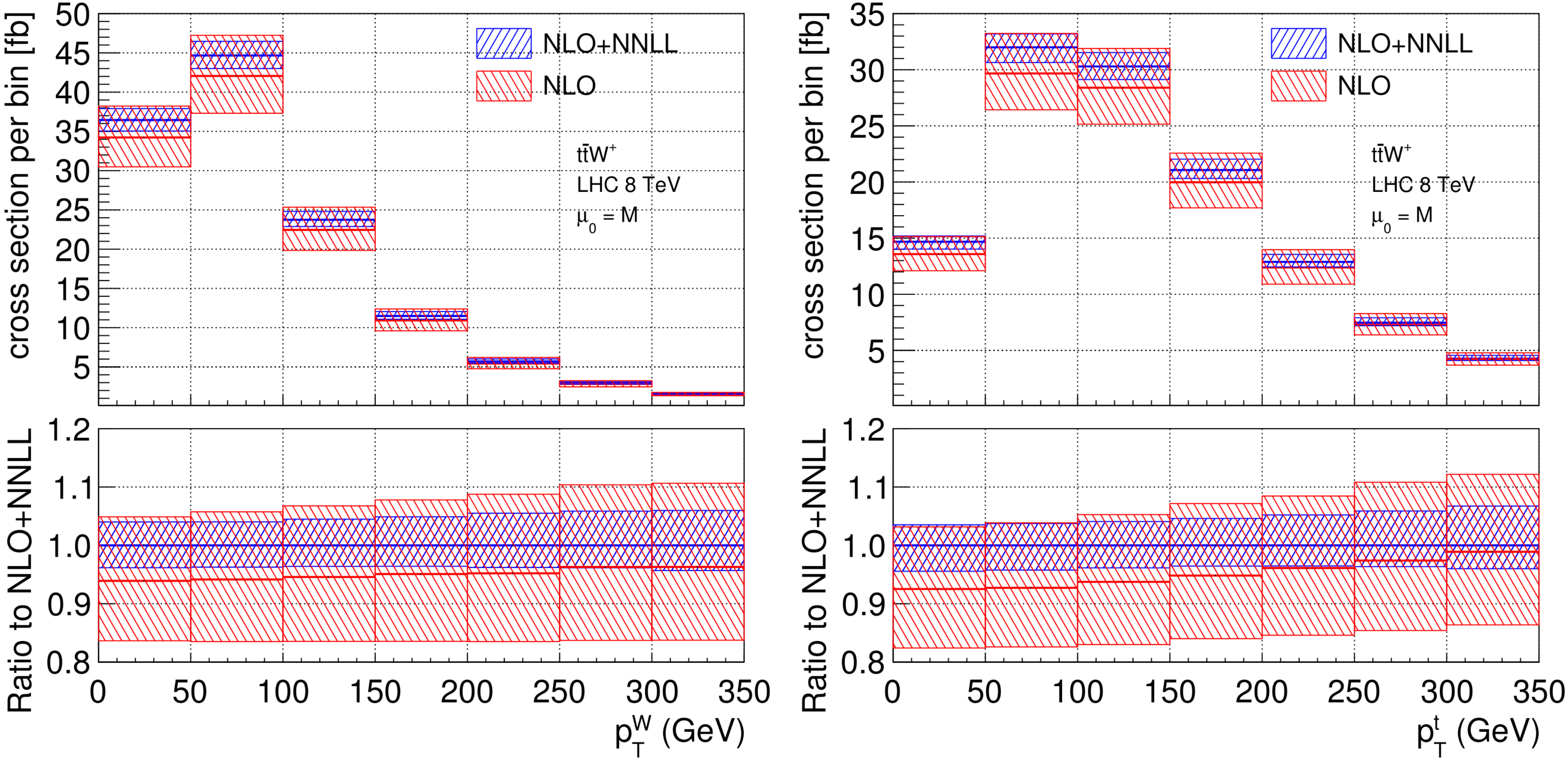
Approx NLO has the same shape as the NLO if one excludes the quark gluon channel contribution from the NLO

tTW distributions at NLO+NNLL



NLO+NNLL distributions overlap with the upper part of the NLO bands.
The NLO+NNLL bands are narrower than the NLO bands

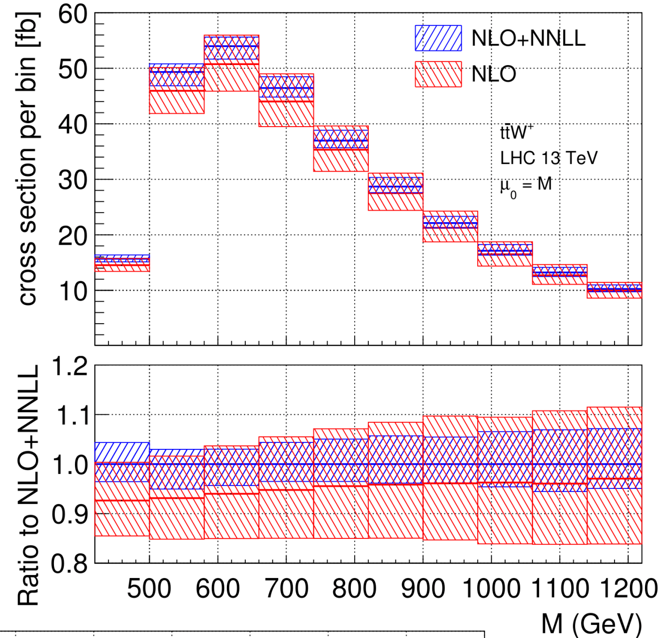
tTW distributions at NLO+NNLL

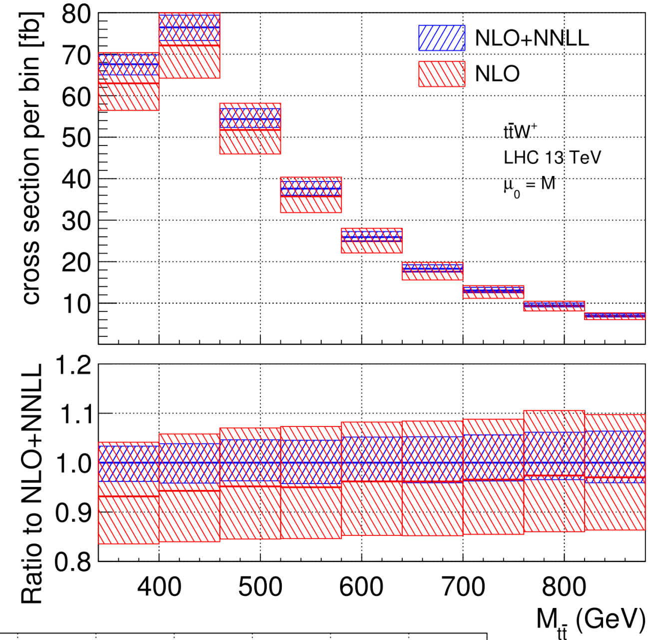


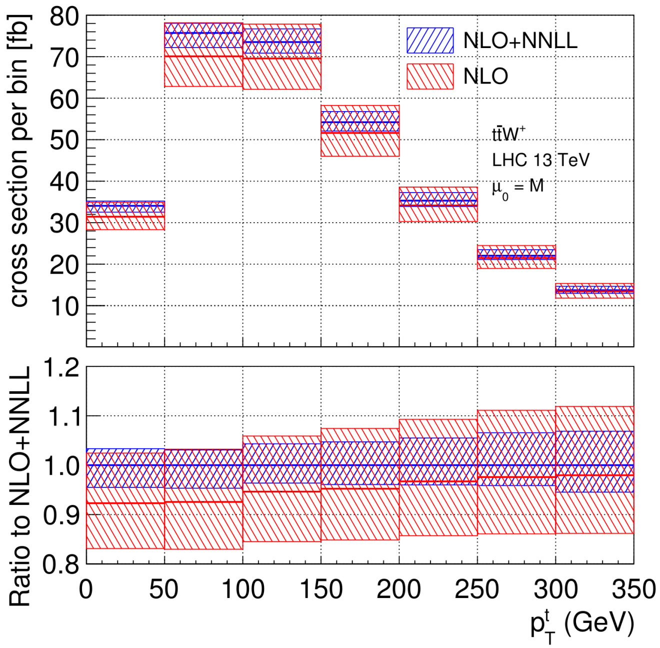
NLO+NNLL distributions overlap with the upper part of the NLO bands.

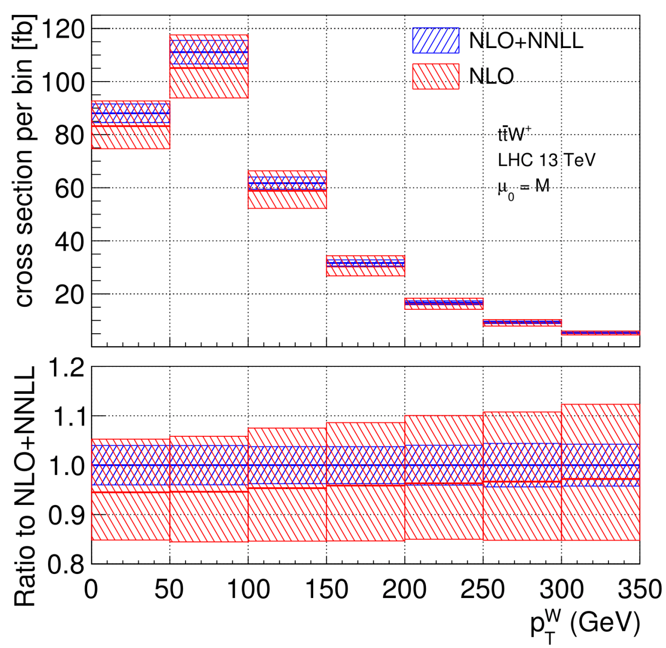
The NLO+NNLL bands are narrower than the NLO bands

tTW^+ @ 13 TeV: NLO+NNLL





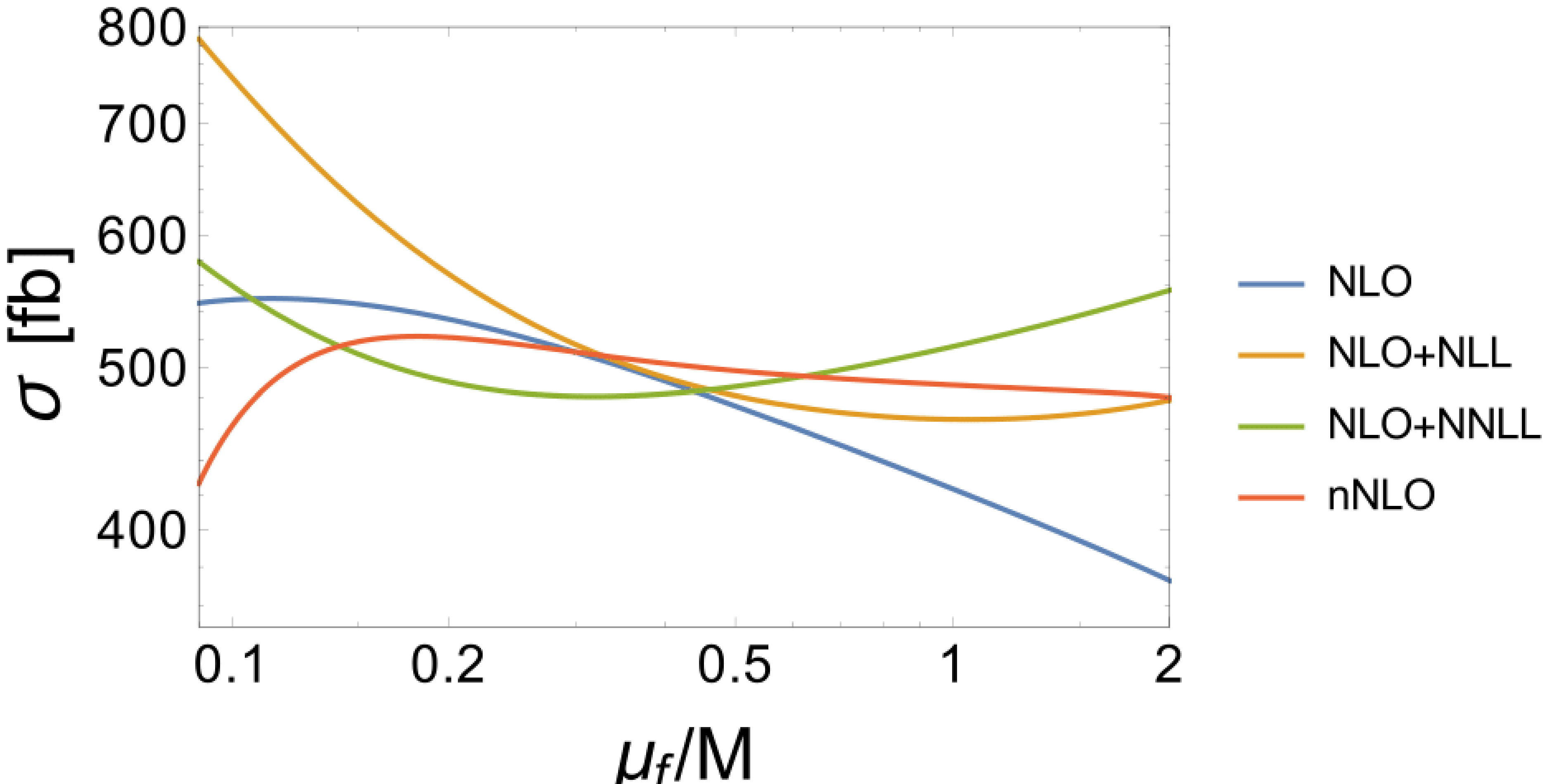




Top Pair + Higgs boson to NLO + NNLL accuracy

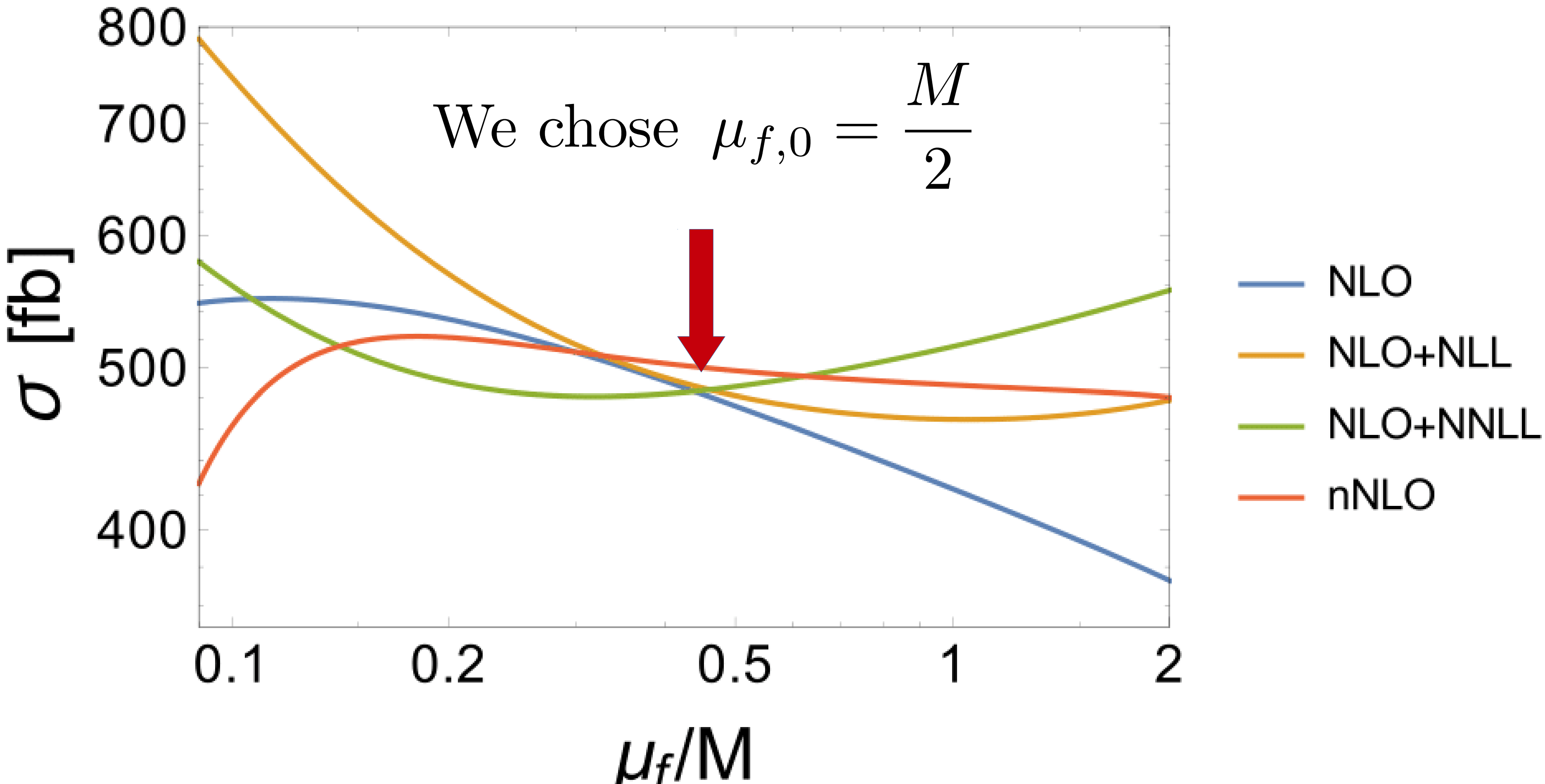
(Two channels, longer running times)

tTH production scale dependence



The factorization scale should be chosen such in such a way that logarithms of the ratio μ_f /M are not large. Since we are working in the partonic threshold limit it is natural to choose a dynamical value for the factorization scale which is correlated with M

tTH production scale dependence



The factorization scale should be chosen such in such a way that logarithms of the ratio µ_f /M are not large. Since we are working in the partonic threshold limit it is natural to choose a dynamical value for the factorization scale which is correlated with M

Total cross section @ 13 TeV

order	PDF order	code	σ [fb]
app. NLO	NLO	MC	$473.3^{+0.0}_{-28.6}$
NLO no qg	NLO	MG5_aMC	$482.1^{+10.9}_{-35.1}$
NLO	NLO	MG5_aMC	$474.8^{+47.2}_{-51.9}$

$t\bar{t}H$ boson production $\mu_f = M/2$

MMHT 2014 PDFs

App. NLO results include only the leading-power contributions from the gluon fusion and quark-annihilation channels in the soft limit

App NLO vs NLO no qg gives a measure of the power corrections away from the soft limit

Large contribution of the qg channel to the scale uncertainty.

Total cross section @ 13 TeV

order	PDF order	code	σ [fb]
app. NLO	NLO	MC	$473.3^{+0.0}_{-28.6}$
NLO no qg	NLO	MG5_aMC	$482.1^{+10.9}_{-35.1}$
NLO	NLO	MG5_aMC	$474.8^{+47.2}_{-51.9}$

$t\bar{t}H$ boson production $\mu_f = M/2$

MMHT 2014 PDFs

The fact that the leading terms in the soft limit make up the bulk of the NLO correction provides a strong motivation to resum these leading terms to all orders.

No information is lost by doing this, as both sources of power corrections are taken into account by matching with NLO as discussed above

Total cross section @ 13 TeV

order	PDF order	code	σ [fb]
LO	LO	MG5_aMC	$378.7^{+120.5}_{-85.2}$
NLO	NLO	MG5_aMC	$474.8^{+47.2}_{-51.9}$
NLO+NLL	NLO	MC +MG5_aMC	$480.1^{+57.7}_{-15.7}$
NLO+NNLL	NNLO	MC +MG5_aMC	$486.4^{+29.9}_{-24.5}$

$t\bar{t}H$ boson production $\mu_f = M/2$

The scale uncertainties get progressively smaller when moving from NLO to NLO+NLL to NLO+NNLL, and the higher-order results are roughly within the range predicted by the uncertainty bands of the lower-order ones.

Total cross section @ 13 TeV

order	PDF order	code	σ [fb]
LO	LO	MG5_aMC	$378.7^{+120.5}_{-85.2}$
NLO	NLO	MG5_aMC	$474.8^{+47.2}_{-51.9}$
NLO+NLL	NLO	MC +MG5_aMC	$480.1^{+57.7}_{-15.7}$
NLO+NNLL	NNLO	MC +MG5_aMC	$486.4^{+29.9}_{-24.5}$

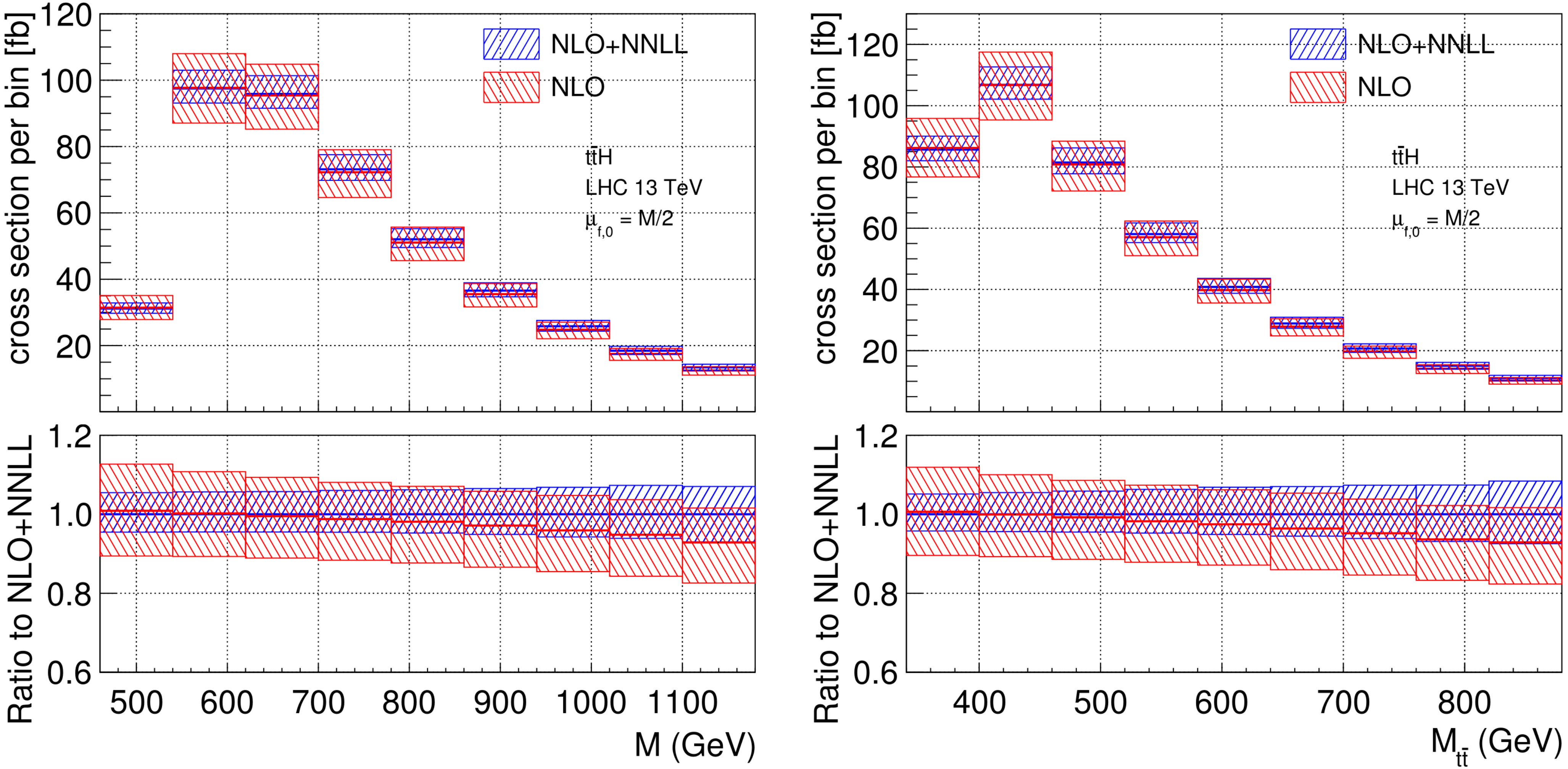
$t\bar{t}H$ boson product

ATLAS measured value

$$\sigma = 590^{+160}_{-150} \text{ fb}$$

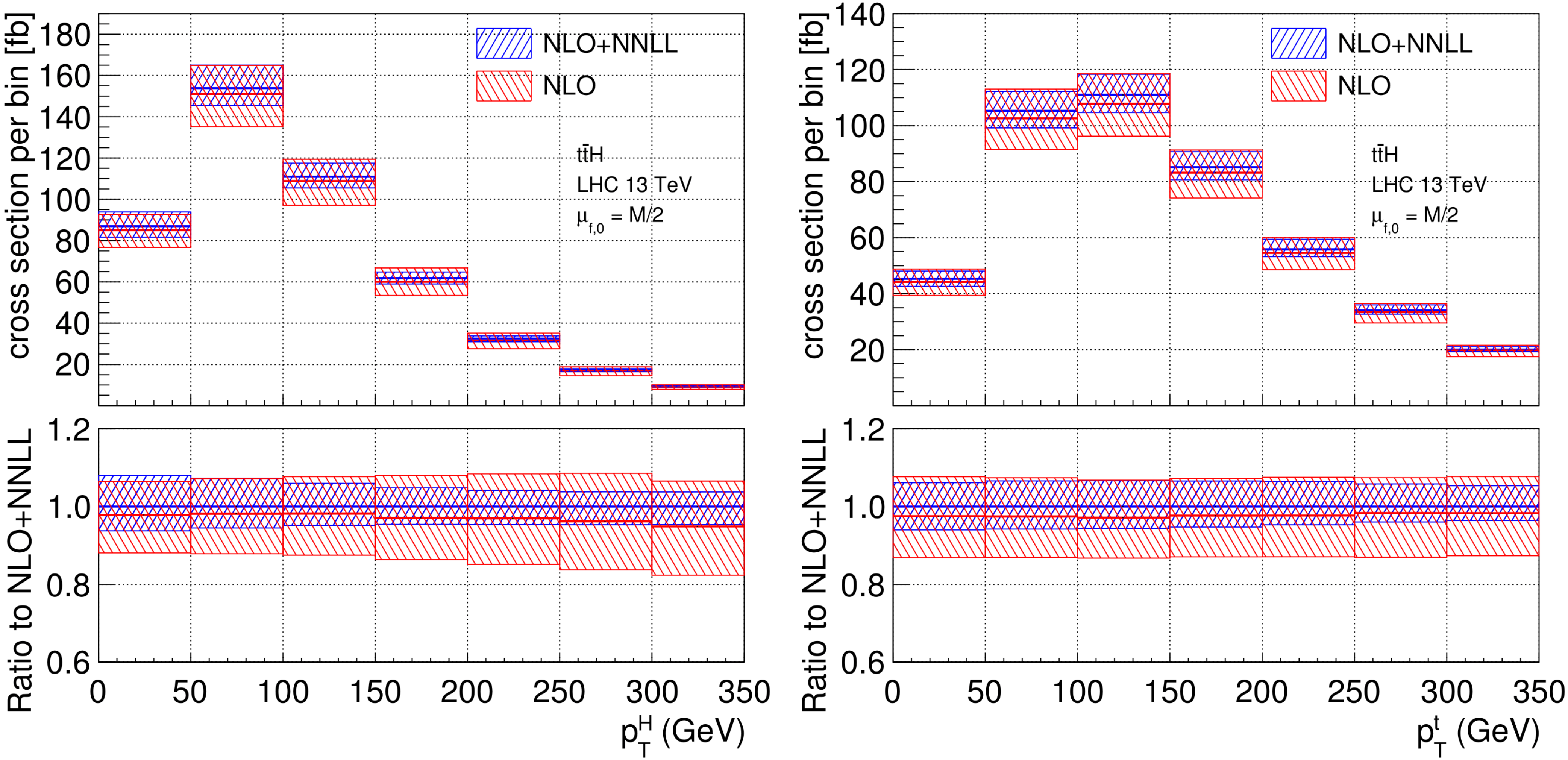
The scale uncertainties get
when moving from NLO to NL ,
and the higher-order results are roughly within the range predicted by the uncertainty bands of the lower-order ones.

tTH distributions at NLO+NNLL



NLO+NNLL distributions overlap with the upper part of the NLO bands.
The NLO+NNLL bands are narrower than the NLO bands

tTH distributions at NLO+NNLL



NLO+NNLL distributions overlap with the upper part of the NLO bands.
The NLO+NNLL bands are narrower than the NLO bands

Top Pair + Z boson to NLO + NNLL accuracy

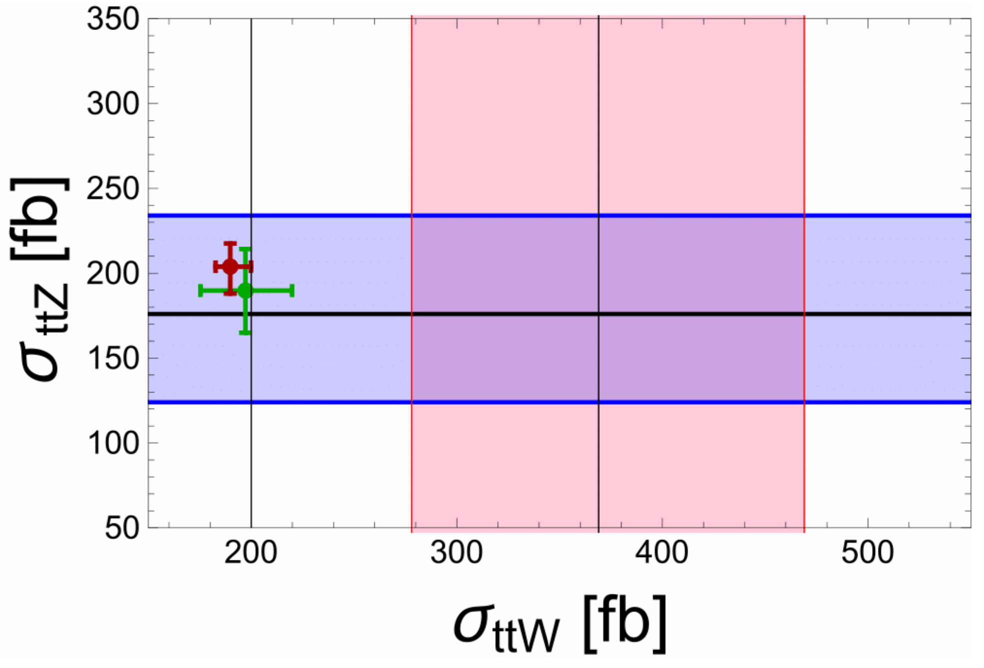
Total cross section @ 13 TeV

order	PDF order	code	σ [fb]
LO	LO	MG5_aMC	$521.4^{+165.4}_{-116.9}$
app. NLO	NLO	MC	$737.7^{+38.5}_{-64.5}$
NLO no qg	NLO	MG5_aMC	$730.5^{+41.2}_{-65.0}$
NLO	NLO	MG5_aMC	$728.3^{+93.8}_{-90.3}$
NLO+NLL	NLO	MC +MG5_aMC	$742.0^{+90.1}_{-30.3}$
NLO+NNLL	NNLO	MC +MG5_aMC	$777.8^{+61.3}_{-65.2}$
nNLO (Mellin)	NNLO	MC +MG5_aMC	$798.4^{+36.4}_{-23.3}$
$(\text{NLO+NNLL})_{\text{exp.}}$	NNLO	MC +MG5_aMC	$766.2^{+17.2}_{-50.1}$

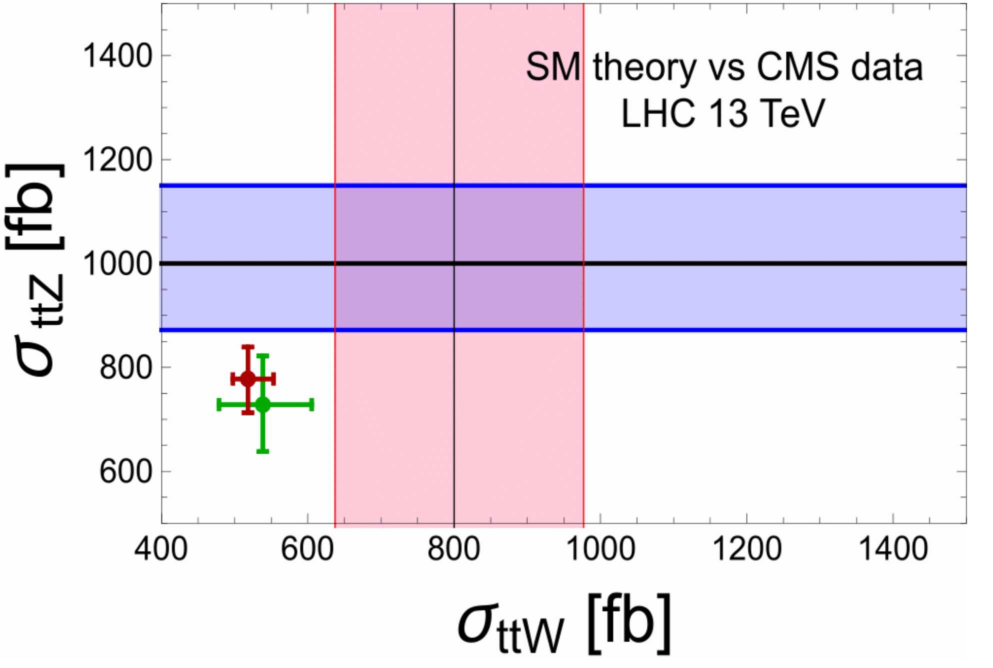
$t\bar{t}Z$ production $\mu_f = M/2$

tTZ/tTW cross section vs data

8 TeV - ATLAS data
arXiv:1509.05276

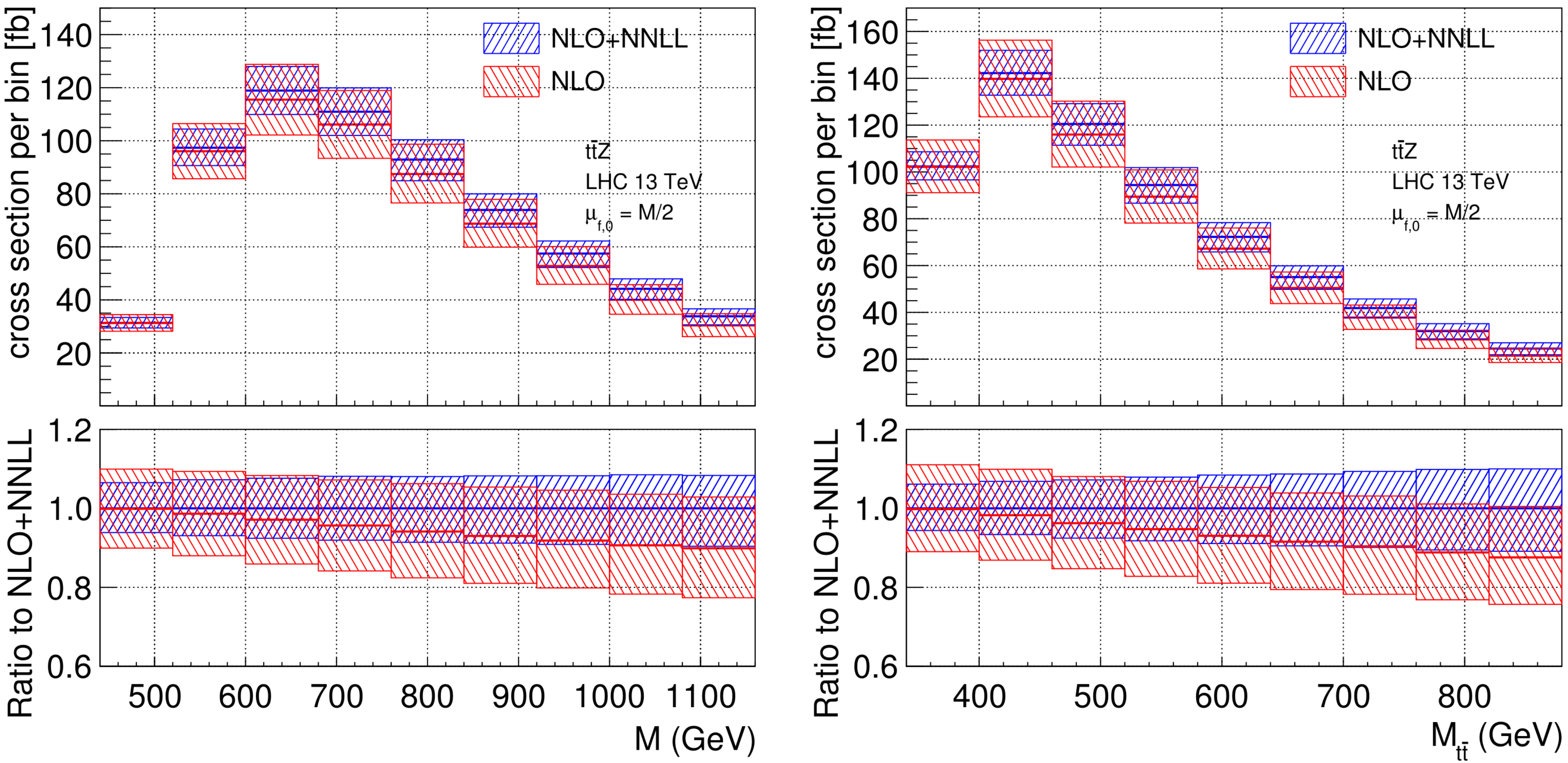


13 TeV - CMS data
CMS PAS TOP 17 05



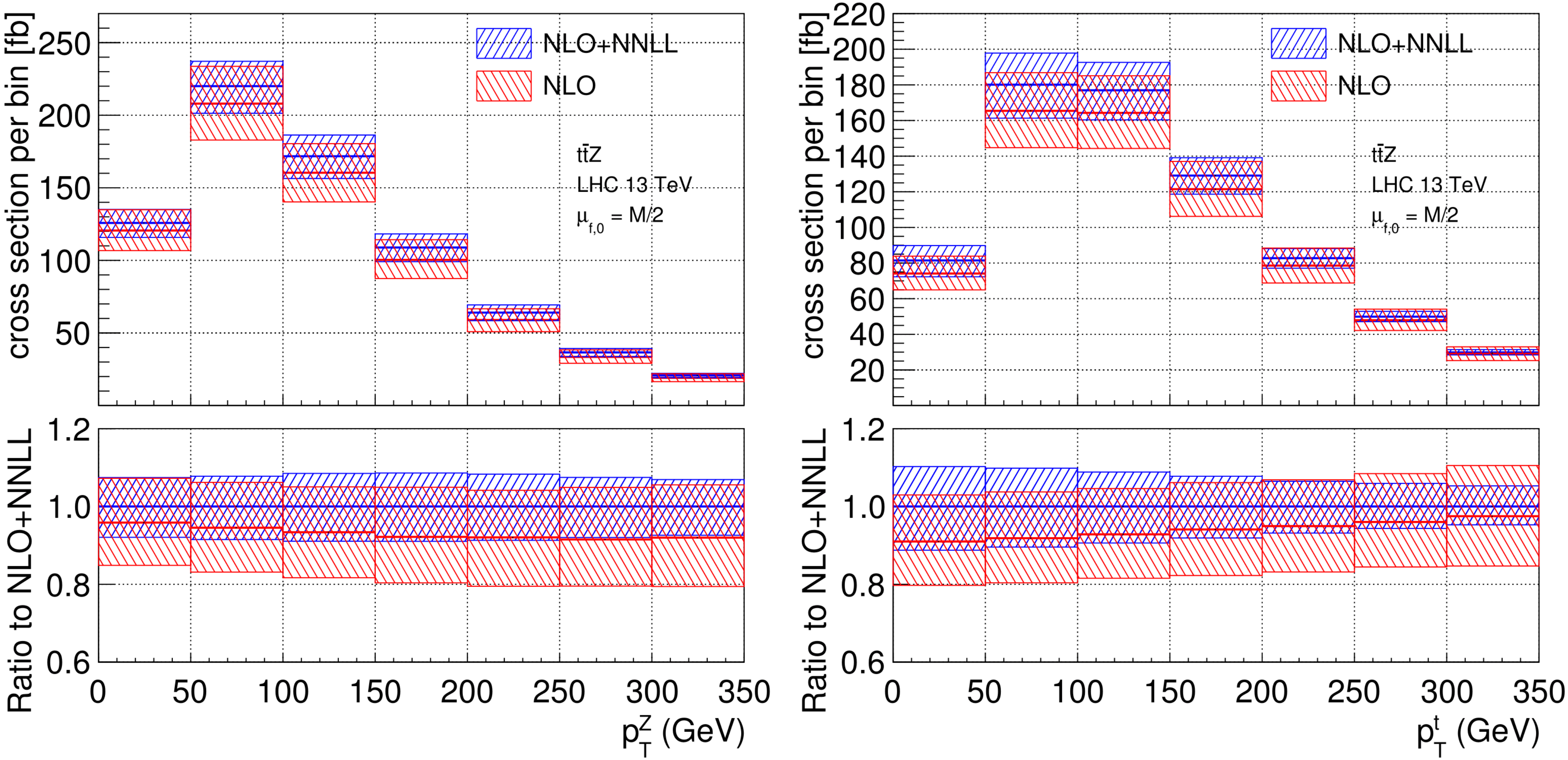
Total cross section at **NLO** (green cross) and **NLO+NNLL** (red cross) compared to the ATLAS measurement (8 TeV) and CMS measurement (13 TeV). The crosses reflect only the scale uncertainty, not the PDF uncertainty

tTZ distributions at NLO+NNLL



NLO+NNLL distributions overlap with the upper part of the NLO bands.
The NLO+NNLL bands are narrower than the NLO bands

tTZ distributions at NLO+NNLL



NLO+NNLL distributions overlap with the upper part of the NLO bands.

The NLO+NNLL bands are narrower than the NLO bands

Conclusions and Outlook

- We implemented a method to study partonic threshold corrections to top pair + H/W/Z boson production
- NLO+NNLL results are available for top pair + W production (total cross section + diff. distributions)
- NLO+NNLL results are available for top pair + H production (total cross section + diff. Distributions)
- NLO+NNLL results are available for top pair + Z production (total cross section + diff. Distributions)
- We evaluated top pair + H to NLO+NLL allowing for a pseudoscalar ttH coupling (with A. Broggio, M. Fiolhais and A. Onofre)

Conclusions and Outlook

- We implemented a method to study partonic threshold corrections to top pair + H/W/Z boson production
- NLO+NNLL results are available for top pair + W production (total cross section + diff. distributions)
- NLO+NNLL results are available for top pair + H production (total cross section + diff. Distributions)
- NLO+NNLL results are available for top pair + Z production (total cross section + diff. Distributions)
- We ev ng for a
  pseud
  Fiolha

In the process of combining NLO+NNLL resummation with NLO electroweak corrections corrections (in collaboration with R. Frederix, D. Pagani, I. Tsinikos, M. Zaro, ...)

Back up material

Channels

$$pp \to t\bar{t}W^+ \to bW^+\bar{b}W^-W^+ \to l^+l^+ + \text{jets}$$
$$pp \to t\bar{t}W^+ \to bW^+\bar{b}W^-W^+ \to l^+l^+l^- + \text{jets}$$

$$pp \to t\bar{t}Z \to bW^+\bar{b}W^-Z \to l^\pm l^\pm l^\mp + \text{jets}$$
$$pp \to t\bar{t}Z \to bW^+\bar{b}W^-Z \to l^+l^+l^-l^- + \text{jets}$$

$$pp \to t\bar{t}H \to 1-2l,\ H \to b\bar{b}$$

$$pp \to t\bar{t}H \to 1-2l,\ H \to WW^*, \tau\tau, ZZ^*$$

$$pp \to t\bar{t}H \to 0-2l,\ H \to \gamma\gamma$$

$$pp \to t\bar{t}H \to 0-2l,\ H \to ZZ^* \to 4l$$

Resummation

From a lecture by E. Laenen

("Direct QCD" approach)

Resummation = (re-)arrangement of large logarithms in perturbative expansion

$$\hat{O} = \overbrace{\underbrace{1 + \alpha_s(L^2 + L + 1)}_{}}^{\text{NLO}} + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \mathcal{O}(\alpha_s^3)$$

$$\hat{O} = \overbrace{1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1)}^{\text{NNLO}} + \mathcal{O}(\alpha_s^3)$$

$$= \exp\Big(\underbrace{\underbrace{Lg_1(\alpha_s L)}_{\text{LL}} + g_2(\alpha_s L)}_{\text{NLL}} + \alpha_s g_3(\alpha_s L) + \cdots\Big) \underbrace{C(\alpha_s)}_{\text{constants}}$$

$$+ \text{ suppressed terms}$$

$$Lg_1 \longrightarrow \alpha_s^n L^{n+1}\,, \quad g_2 \longrightarrow \alpha_s^n L^n\,, \quad \alpha_s g_3 \longrightarrow \alpha_s^{n+1} L^n$$

Resummation reduces the theoretical uncertainty on a given observable

Snippets from the modified code

```
+--------------------------------------------------------------------------+
|                                              GoSam                       |
|  / _)  / \  / __) (  ) ( \/ )          An Automated One-Loop            |
| ( (/\( () )\__ \ /__\  )    (         Matrix Element Generator          |
|  \__/ \__/ (___/(_)(_)(_/\/\_)        Version 2.0.1 Rev: 735            |
|                                                                          |
|                                 (c) The GoSam Collaboration 2011-2014   |
```

(code modified with the help of N. Greiner and G. Ossola)

Fix the phase space point:

```
Phase space point
vec_1   250.00000000000000        0.0000000000000000        0.0000000000000000        250.00000000000000
vec_2   250.00000000000000        0.0000000000000000        0.0000000000000000       -250.00000000000000
vec_3   187.73850389979739      -58.983229676638324      -41.627869607566652       -11.808257084878599
vec_4   184.32670746268576       39.113315116582932       50.299577830700244        -4.6932008990400051
vec_5   127.93478863751679       19.869914560055363       -8.6717082231335887       16.501457983918591
```

Output LO hard function and (UV renormalized) NLO hard function in the quark annihilation channel, after rotation to the desired color basis:

```
Print Tree Level Hard Function:
(-1.1117307432712692OE-021,  0.0000000000000000     ) (-3.8116482626443515 3E-021,  0.0000000000000000     )
(-3.3881317890172013 6E-021,  0.0000000000000000     ) ( 9.1403180646541744 1E-005,  0.0000000000000000     )

Print One Loop Hard Function:
( 1.4772254600114996 0E-018, 3.3881317890172013 6E-021) (-2.3089383098535146 6E-004, 1.4722285402959297 0E-004)
(-2.3089383098535146 6E-004,-1.4722285402959297 0E-004) (-2.4868451361348187 0E-004,  0.0000000000000000     )
```

Snippets from the modified code

```
GoSam
An Automated One-Loop
Matrix Element Generator
Version 2.0.1 Rev: 735
```

(code modified with the help of N. Greiner and G. Ossola)

The IR poles of the HF are can be subtracted by using the Becher-Neubert formula for the IR poles in QCD amplitudes

$$\mathbf{H}^{(1)} = \mathbf{H}^{(1)\mathrm{IR}} - \left(\mathbf{Z}^{(1)}\mathbf{H}^{(0)} - \mathbf{H}^{(0)}\mathbf{Z}^{(1)\dagger}\right)$$

The calculation of the hard function was also implemented by modifying MadLoop. The GoSam and MadLoop implementations are in agreement.

GoSam takes about 100ms to calculate the HF in a phase space point

```
(-3.38813178901720136E-021,  0.0000000000000000      ) ( 9.14031806465417441E-005,  0.0000000000000000      )

Print One Loop Hard Function:
( 1.47722546001149960E-018, 3.38813178901720136E-021) (-2.30893830985351466E-004, 1.47222854029592970E-004)
(-2.30893830985351466E-004,-1.47222854029592970E-004) (-2.48684513613481870E-004,  0.0000000000000000      )
```

Total cross section @ 8 TeV

order	PDFs order	code	σ [fb]
LO	LO	MG5_aMC	$82.0^{+21.2}_{-15.7}$
NLO	NLO	MG5_aMC	$121.6^{+15.2}_{-14.0}$
NLO no qg	NLO	MG5_aMC	$118.1^{+10.3}_{-11.3}$
app. NLO	NLO	MC	$116.0^{+10.3}_{-11.6}$
nNLO (momentum)	NNLO	MC + MG5_aMC	$127.7^{+9.2}_{-7.4}$
nNLO (Mellin)	NNLO	MC +MG5_aMC	$127.6^{+9.2}_{-7.4}$
NLO+NLL	NLO	MC +MG5_aMC	$124.8^{+13.1}_{-8.0}$
$(\mathrm{NLO+NNLL})_{\mathrm{NNLO\,exp.}}$	NNLO	MC +MG5_aMC	$126.7^{+5.0}_{-6.5}$
NLO+NNLL	NNLO	MC +MG5_aMC	$128.7^{+5.5}_{-4.7}$

W^+production

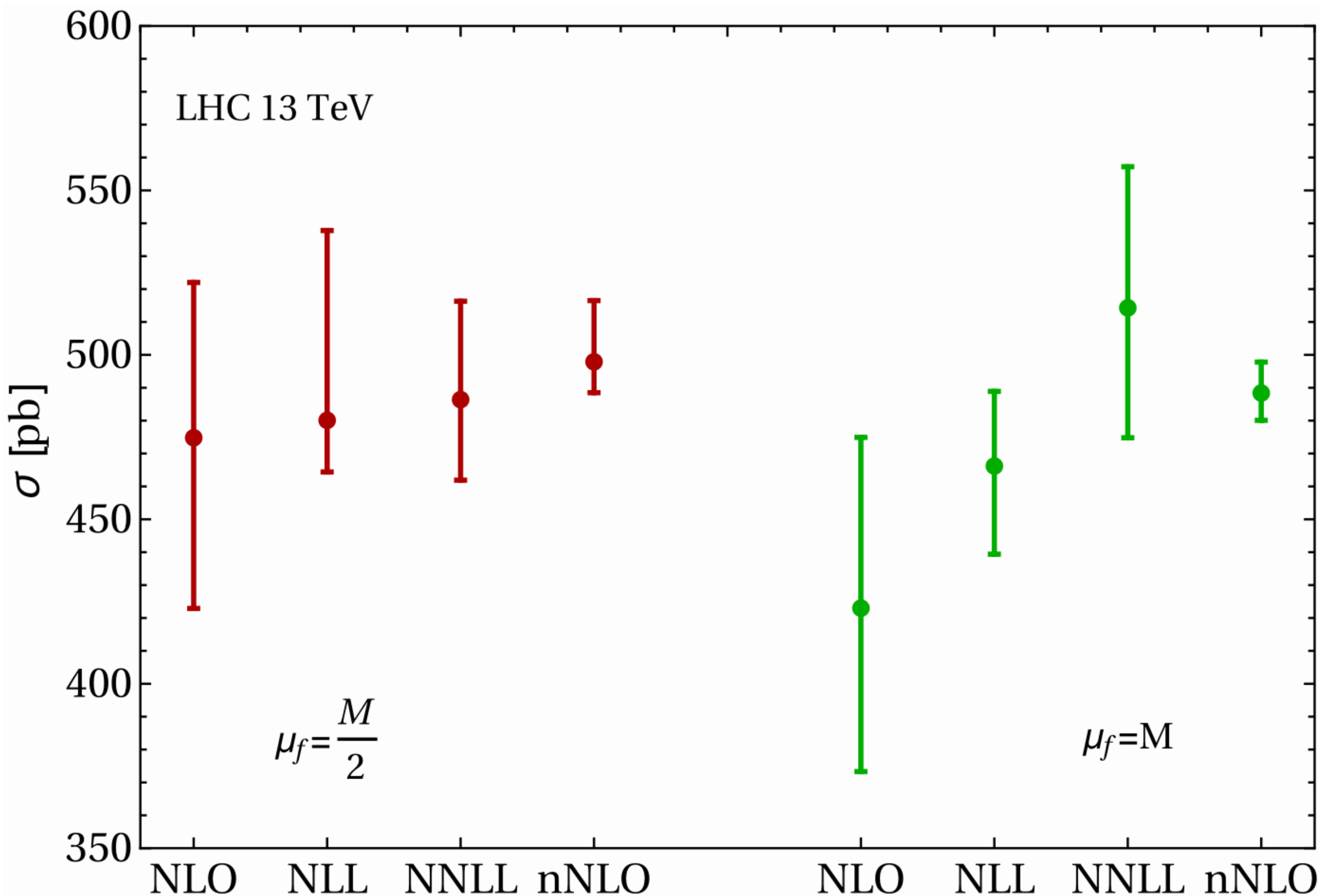
MMHT 2014 PDFs here and in the following

Total cross section @ 13 TeV

order	PDF order	code	σ [fb]
LO	LO	MG5_aMC	$378.7^{+120.5}_{-85.2}$
app. NLO	NLO	MC	$473.3^{+0.0}_{-28.6}$
NLO no qg	NLO	MG5_aMC	$482.1^{+10.9}_{-35.1}$
NLO	NLO	MG5_aMC	$474.8^{+47.2}_{-51.9}$
NLO+NLL	NLO	MC +MG5_aMC	$480.1^{+57.7}_{-15.7}$
NLO+NNLL	NNLO	MC +MG5_aMC	$486.4^{+29.9}_{-24.5}$
nNLO (Mellin)	NNLO	MC +MG5_aMC	$497.9^{+18.5}_{-9.4}$
$(\text{NLO+NNLL})_{\text{exp.}}$	NNLO	MC +MG5_aMC	$482.7^{+10.7}_{-21.1}$

$t\bar{t}H$ boson production $\mu_f = M/2$

Comparison among predictions at different factorization scales



Top pair + Z scale choice

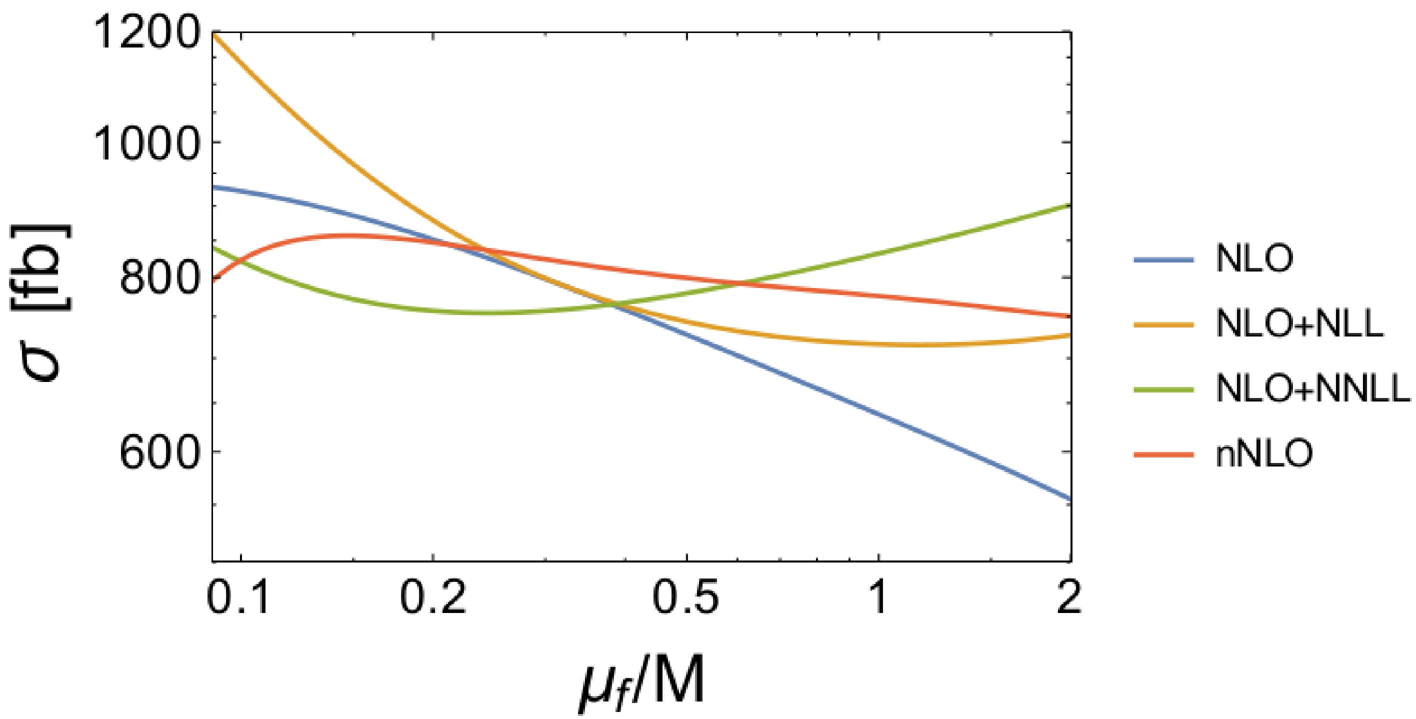
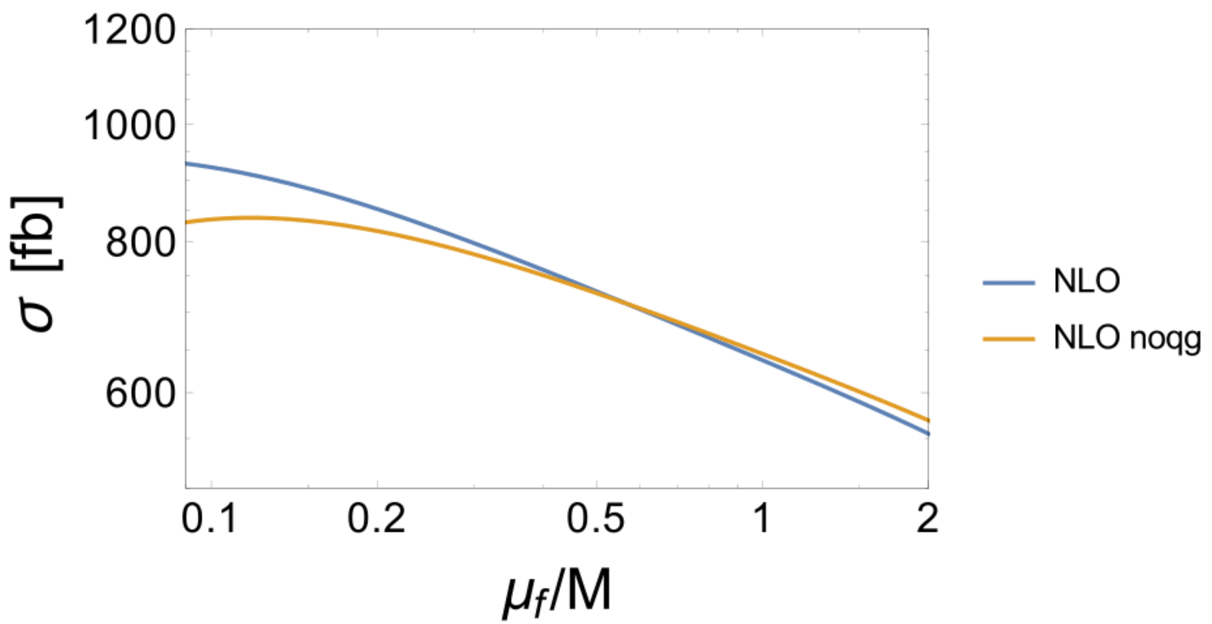
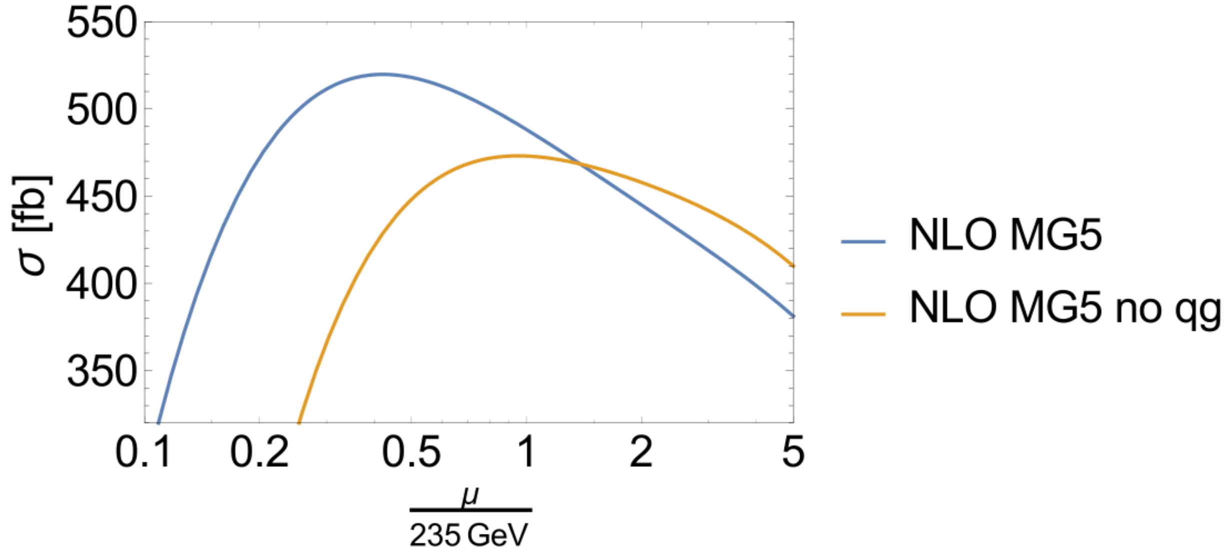


Figure 1. Factorization-scale dependence of the total $t\bar{t}Z$ production cross section at the LHC with $\sqrt{s} = 13\,\mathrm{TeV}$. The NLO and NLO+NLL curves are obtained using MMHT 2014 NLO PDFs, while the NLO+NNLL and nNLO curves are obtained using MMHT 2014 NNLO PDFs.

NLO vs NLO no qg



ttZ
dynamic
scale



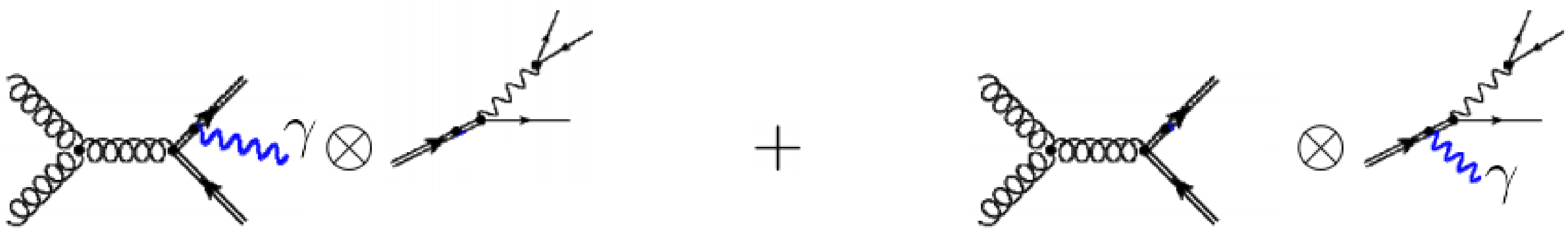
ttH
fixed
scale

Top pair + photon

Associated production: *ttbar+γ*

From M.Schulze slides, TOP 2017

- Feature: $t\bar{t}+\gamma$ introduces additional *radiative decays*



A) γ emission *before* top goes on-shell *B)* γ emission *after* top goes on-shell

- More than half of the total cross section from contribution *B)*

$$p_T^{\gamma} \geq 30\,\mathrm{GeV}$$

$$\sigma^{\mathrm{NLO}}_{\mathrm{prod}} = 61\,\mathrm{fb}$$
$$\sigma^{\mathrm{NLO}}_{\mathrm{decay}} = 77\,\mathrm{fb}$$
$$\sigma^{\mathrm{NLO}}_{\mathrm{total}} = 138\,\mathrm{fb}$$

Minimal Prescription

Minimal prescription

From M.Bonvini slides, 2009

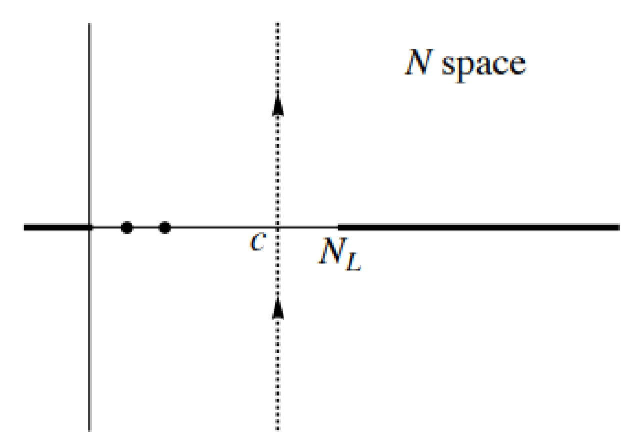
Proposed by S.Catani, M.Mangano, P.Nason, L.Trentadue:

$$\sigma^{\mathrm{MP}}(x,Q^2) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, x^{-N}\, \mathcal{L}(N,Q^2)\,\hat{\sigma}^{\mathrm{res}}\left(N,\alpha_s(Q^2)\right)$$

with $c < N_L$, as in the figure.

Good properties:

- well defined for all x
- exact for invertible functions
- asymptotic to the original divergent series



But...

- a non-physical region of the parton cross-section contributes
- problems in numerical implementation