Broadband Modeling of Blazar Spectra with a Turbulent Acceleration Model

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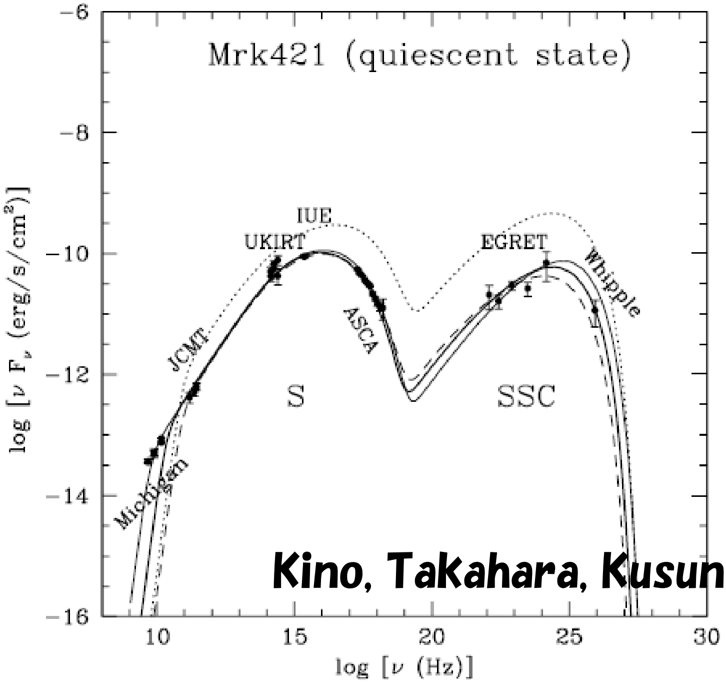
Open Problems in Blazar Emission

1. Index harder than 2 for the electron injection spectrum.



Kino, Takahara, Kusunose 2002

FIG. 4.—One-zone SSC model spectra for the steady state emission of Mrk 421. The thick solid line shows the best-fit spectrum where the adopted parameters are $\delta = 12$, $R = 2.8 \times 10^{16}$ cm, $B = 0.12$ G, $\gamma_{\rm max} = 1.5 \times 10^5$, $q_e = 9.6 \times 10^{-6}$ cm^{-3} s^{-1}, $s = 1.6$, and $u_e/u_B = 5$. The dotted line shows the spectrum obtained using the analytic estimates for Mrk 421. The thin solid and dashed lines show the spectra of low and high injection models, respectively, to indicate the uncertainty range of the spectral fitting.

2. Lower maximum energy

Mrk 421 :

B=38mG
If $\eta = 1$, **even for** $\beta_{\rm sh} = 0.1$,
$E_{\rm max} = 7$ TeV.
But actual Max. Energy 50 GeV

The Bohm factor should be $\sim 10^4$

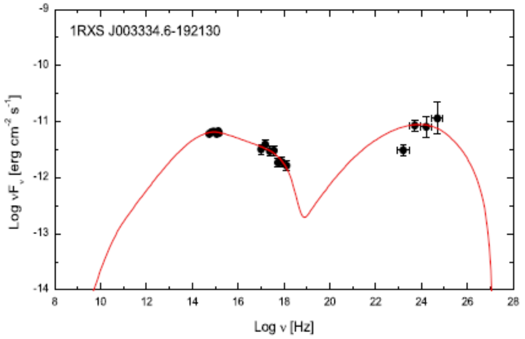
Inoue & Takahara 2002

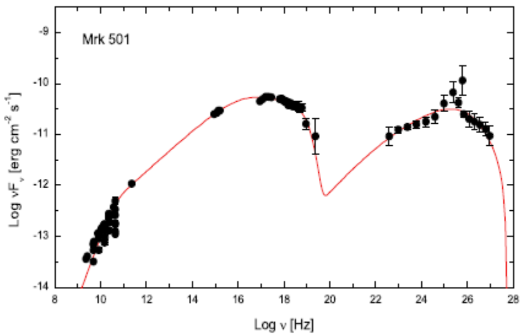
Supernova remnants and pulsar wind nebulae indicate $\eta = 1$.

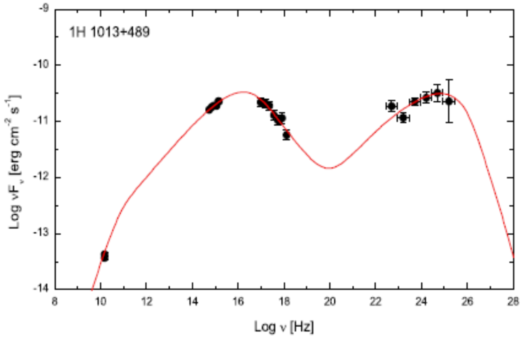
3. Too sharp break in the electron spectrum.

Compared to the cooling break, the difference in the indices seems large.

Electron Index in Fermi BL Lacs

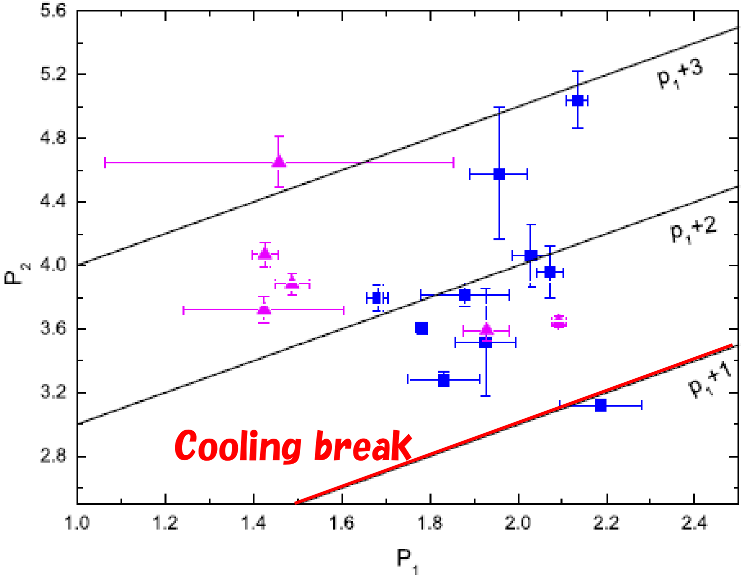




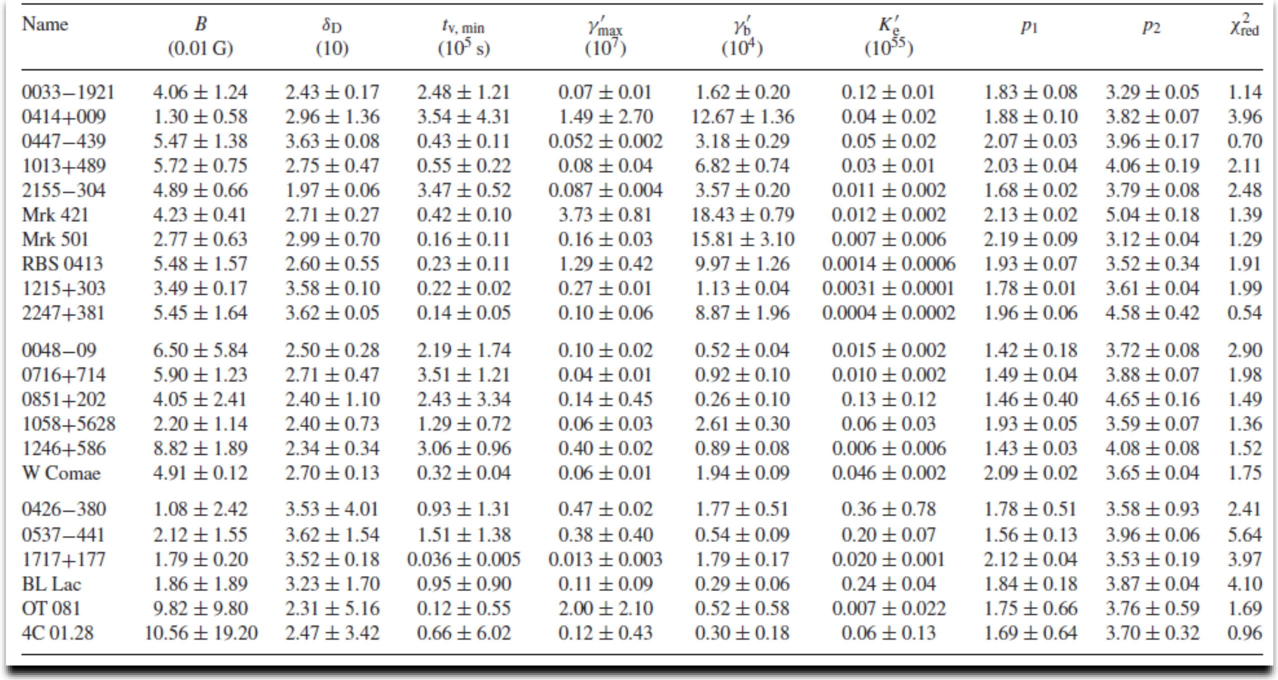


Yan+

MNRAS 439, 2933–2942 (2014)



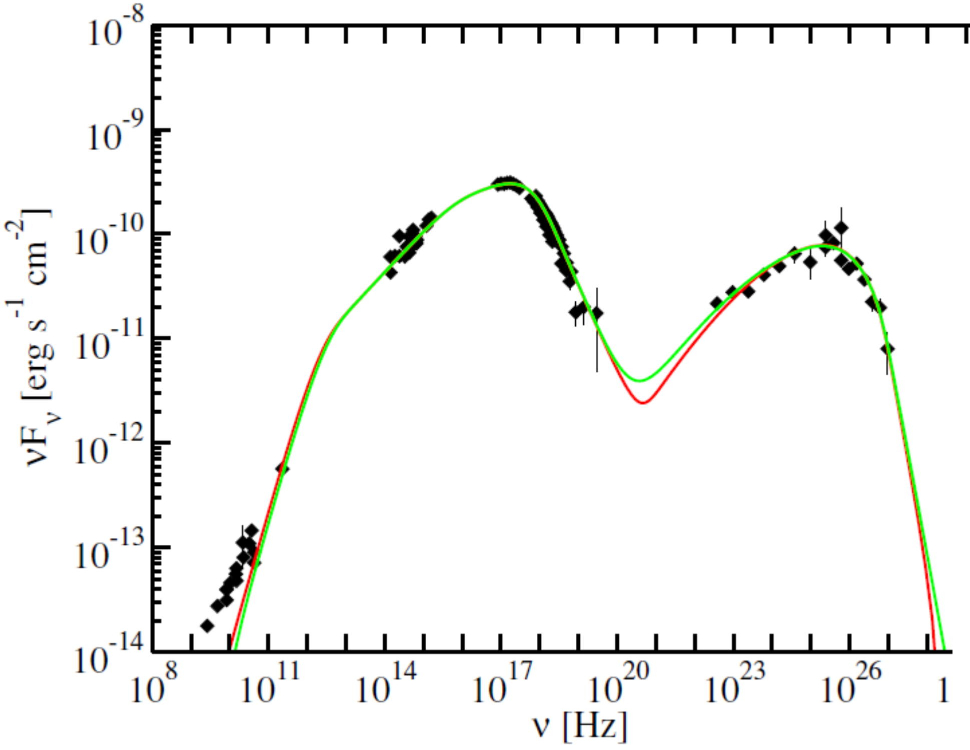
Cooling break

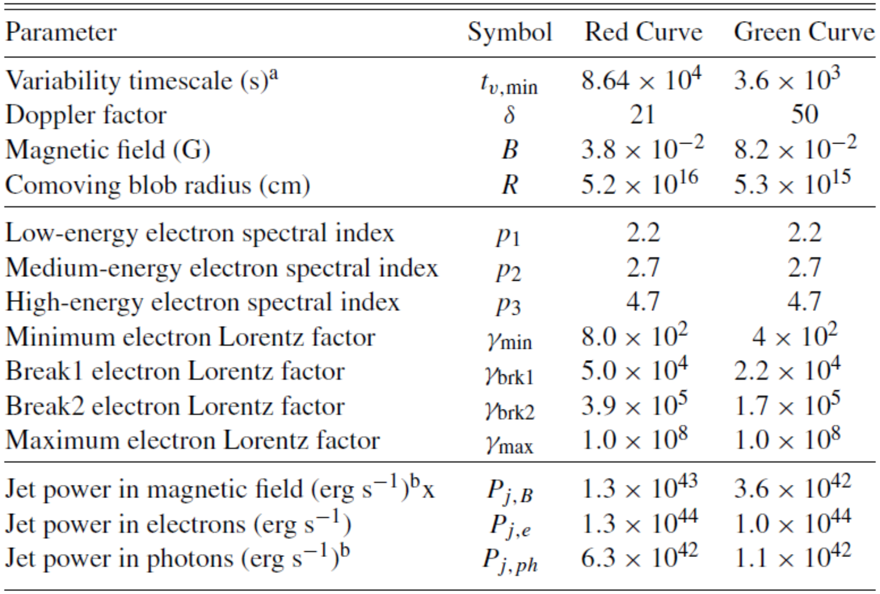


Name	B (0.01 G)	δ_D (10)	$t_{v,\,\min}$ (10^5 s)	$\gamma'_{\max}$ (10^7)	γ'_b (10^4)	K'_e (10^{55})	p_1	p_2	$\chi^2_{\rm red}$
0033−1921	4.06 ± 1.24	2.43 ± 0.17	2.48 ± 1.21	0.07 ± 0.01	1.62 ± 0.20	0.12 ± 0.01	1.83 ± 0.08	3.29 ± 0.05	1.14
0414+009	1.30 ± 0.58	2.96 ± 1.36	3.54 ± 4.31	1.49 ± 2.70	12.67 ± 1.36	0.04 ± 0.02	1.88 ± 0.10	3.82 ± 0.07	3.96
0447−439	5.47 ± 1.38	3.63 ± 0.08	0.43 ± 0.11	0.052 ± 0.002	3.18 ± 0.29	0.05 ± 0.02	2.07 ± 0.03	3.96 ± 0.17	0.70
1013+489	5.72 ± 0.75	2.75 ± 0.47	0.55 ± 0.22	0.08 ± 0.04	6.82 ± 0.74	0.03 ± 0.01	2.03 ± 0.04	4.06 ± 0.19	2.11
2155−304	4.89 ± 0.66	1.97 ± 0.06	3.47 ± 0.52	0.087 ± 0.004	3.57 ± 0.20	0.011 ± 0.002	1.68 ± 0.02	3.79 ± 0.08	2.48
Mrk 421	4.23 ± 0.41	2.71 ± 0.27	0.42 ± 0.10	3.73 ± 0.81	18.43 ± 0.79	0.012 ± 0.002	2.13 ± 0.02	5.04 ± 0.18	1.39
Mrk 501	2.77 ± 0.63	2.99 ± 0.70	0.16 ± 0.11	0.16 ± 0.03	15.81 ± 3.10	0.007 ± 0.006	2.19 ± 0.09	3.12 ± 0.04	1.29
RBS 0413	5.48 ± 1.57	2.60 ± 0.55	0.23 ± 0.11	1.29 ± 0.42	9.97 ± 1.26	0.0014 ± 0.0006	1.93 ± 0.07	3.52 ± 0.34	1.91
1215+303	3.49 ± 0.17	3.58 ± 0.10	0.22 ± 0.02	0.27 ± 0.01	1.13 ± 0.04	0.0031 ± 0.0001	1.78 ± 0.01	3.61 ± 0.04	1.99
2247+381	5.45 ± 1.64	3.62 ± 0.05	0.14 ± 0.05	0.10 ± 0.06	8.87 ± 1.96	0.0004 ± 0.0002	1.96 ± 0.06	4.58 ± 0.42	0.54
0048−09	6.50 ± 5.84	2.50 ± 0.28	2.19 ± 1.74	0.10 ± 0.02	0.52 ± 0.04	0.015 ± 0.002	1.42 ± 0.18	3.72 ± 0.08	2.90
0716+714	5.90 ± 1.23	2.71 ± 0.47	3.51 ± 1.21	0.04 ± 0.01	0.92 ± 0.10	0.010 ± 0.002	1.49 ± 0.04	3.88 ± 0.07	1.98
0851+202	4.05 ± 2.41	2.40 ± 1.10	2.43 ± 3.34	0.14 ± 0.45	0.26 ± 0.10	0.13 ± 0.12	1.46 ± 0.40	4.65 ± 0.16	1.49
1058+5628	2.20 ± 1.14	2.40 ± 0.73	1.29 ± 0.72	0.06 ± 0.03	2.61 ± 0.30	0.06 ± 0.03	1.93 ± 0.05	3.59 ± 0.07	1.36
1246+586	8.82 ± 1.89	2.34 ± 0.34	3.06 ± 0.96	0.40 ± 0.02	0.89 ± 0.08	0.006 ± 0.006	1.43 ± 0.03	4.08 ± 0.08	1.52
W Comae	4.91 ± 0.12	2.70 ± 0.13	0.32 ± 0.04	0.06 ± 0.01	1.94 ± 0.09	0.046 ± 0.002	2.09 ± 0.02	3.65 ± 0.04	1.75
0426−380	1.08 ± 2.42	3.53 ± 4.01	0.93 ± 1.31	0.47 ± 0.02	1.77 ± 0.51	0.36 ± 0.78	1.78 ± 0.51	3.58 ± 0.93	2.41
0537−441	2.12 ± 1.55	3.62 ± 1.54	1.51 ± 1.38	0.38 ± 0.40	0.54 ± 0.09	0.20 ± 0.07	1.56 ± 0.13	3.96 ± 0.06	5.64
1717+177	1.79 ± 0.20	3.52 ± 0.18	0.036 ± 0.005	0.013 ± 0.003	1.79 ± 0.17	0.020 ± 0.001	2.12 ± 0.04	3.53 ± 0.19	3.97
BL Lac	1.86 ± 1.89	3.23 ± 1.70	0.95 ± 0.90	0.11 ± 0.09	0.29 ± 0.06	0.24 ± 0.04	1.84 ± 0.18	3.87 ± 0.04	4.10
OT 081	9.82 ± 9.80	2.31 ± 5.16	0.12 ± 0.55	2.00 ± 2.10	0.52 ± 0.58	0.007 ± 0.022	1.75 ± 0.66	3.76 ± 0.59	1.69
4C 01.28	10.56 ± 19.20	2.47 ± 3.42	0.66 ± 6.02	0.12 ± 0.43	0.30 ± 0.18	0.06 ± 0.13	1.69 ± 0.64	3.70 ± 0.32	0.96

Model with many parameters.

Mrk 421 Spectral Fit

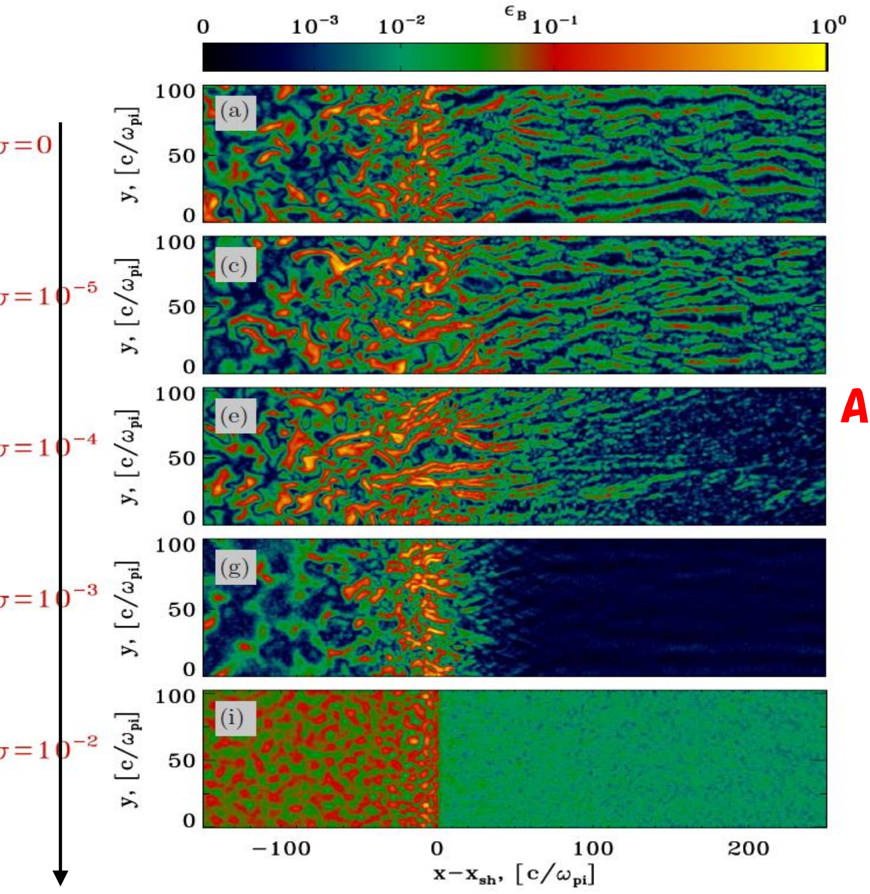




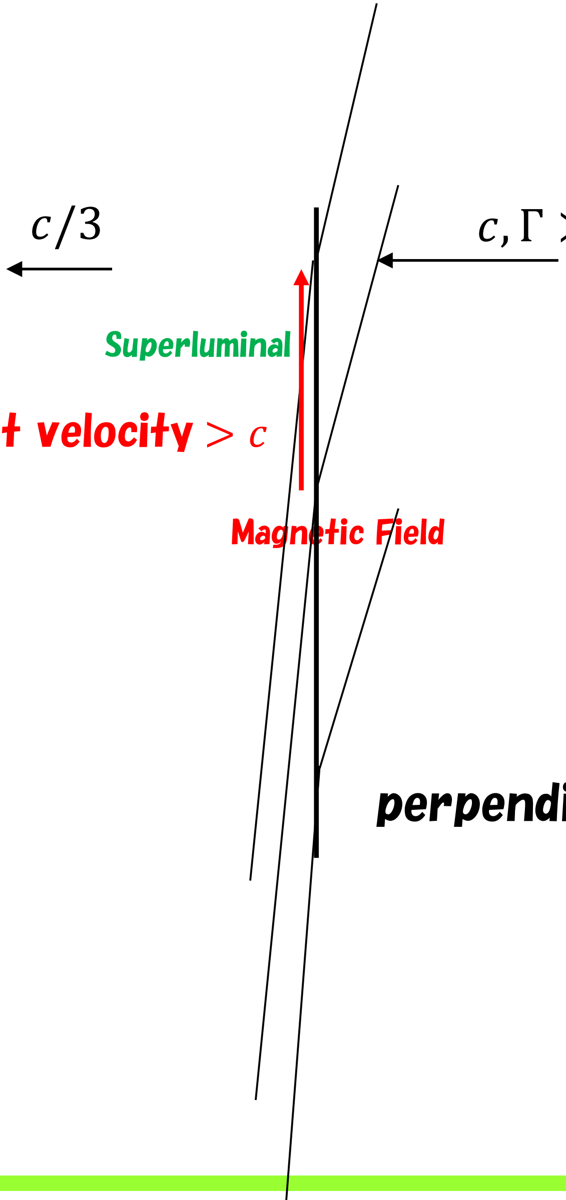
Parameter	Symbol	Red Curve	Green Curve
Variability timescale (s)[a]	$t_{v,\min}$	8.64×10^4	3.6×10^3
Doppler factor	δ	21	50
Magnetic field (G)	B	3.8×10^{-2}	8.2×10^{-2}
Comoving blob radius (cm)	R	5.2×10^{16}	5.3×10^{15}
Low-energy electron spectral index	p_1	2.2	2.2
Medium-energy electron spectral index	p_2	2.7	2.7
High-energy electron spectral index	p_3	4.7	4.7
Minimum electron Lorentz factor	$\gamma_{\min}$	8.0×10^2	4×10^2
Break1 electron Lorentz factor	γ_{brk1}	5.0×10^4	2.2×10^4
Break2 electron Lorentz factor	γ_{brk2}	3.9×10^5	1.7×10^5
Maximum electron Lorentz factor	$\gamma_{\max}$	1.0×10^8	1.0×10^8
Jet power in magnetic field (erg s^{-1})[b]x	$P_{j,B}$	1.3×10^{43}	3.6×10^{42}
Jet power in electrons (erg s^{-1})	$P_{j,e}$	1.3×10^{44}	1.0×10^{44}
Jet power in photons (erg s^{-1})[b]	$P_{j,ph}$	6.3×10^{42}	1.1×10^{42}

Phenomenological model requires many parameters.

Difficulties in Shock Acceleration



magnetized

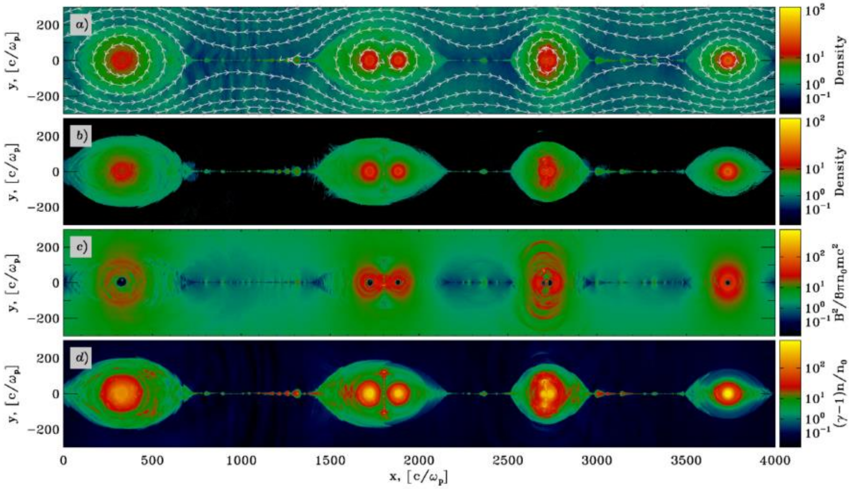


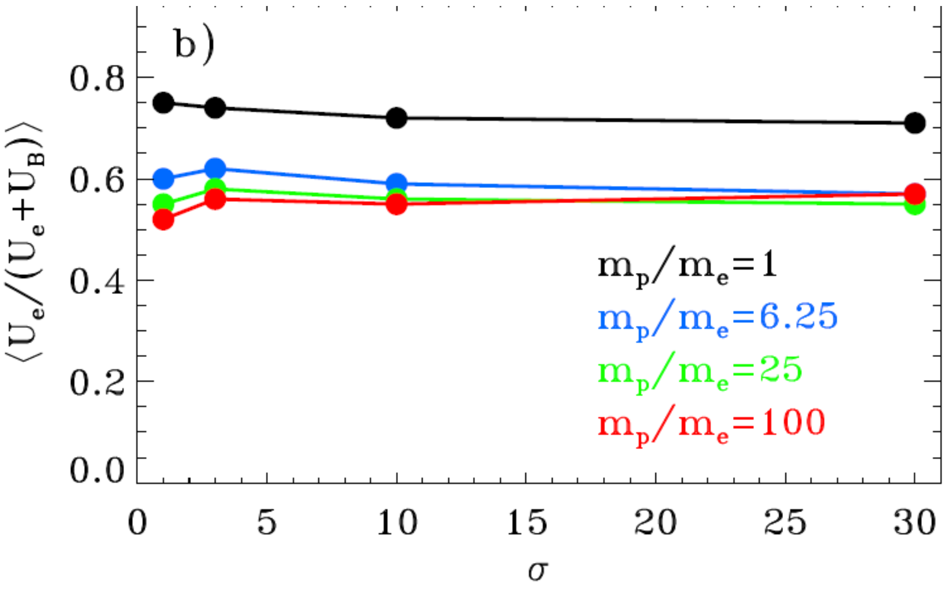
Apparent velocity $> c$

perpendicular is likely

Alternative: Magnetic Recconection

Sironi+ 2015

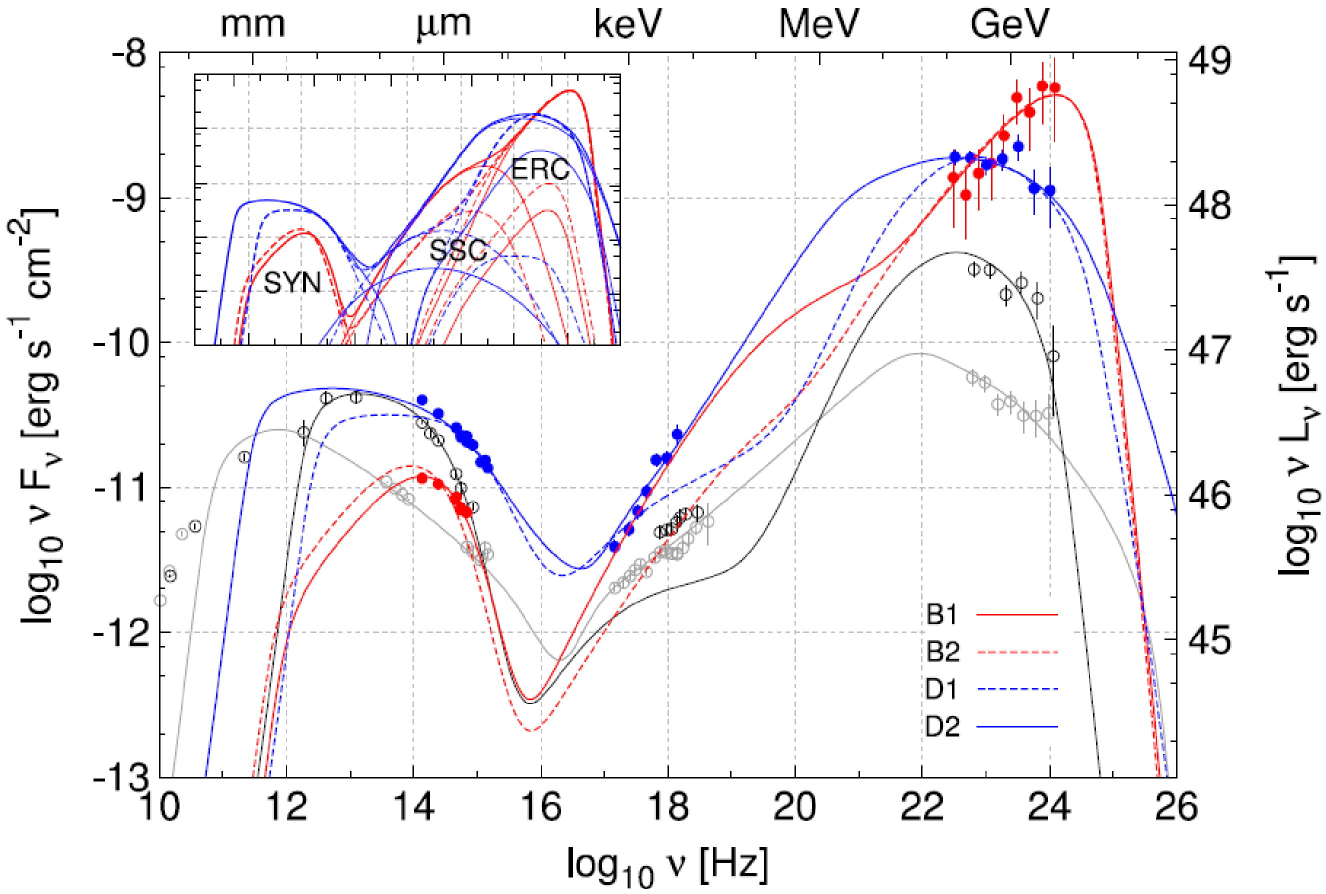




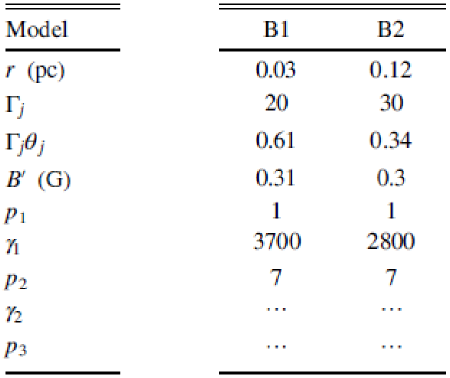
Equipartition

But in Mrk 421, Magnetic Energy is 3-10% of Electrons'
In 1ES 0229+200, only 0.3%.

Very Hard Spectrum in 3C 279



Broken power-law Parameters

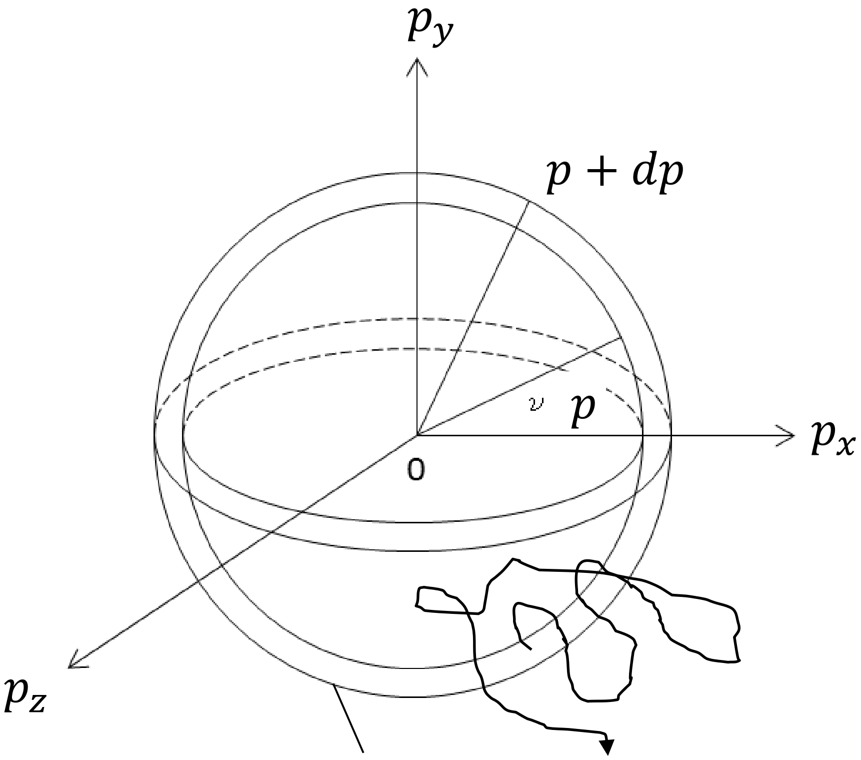


Model	B1	B2
r (pc)	0.03	0.12
Γ_j	20	30
$\Gamma_j\theta_j$	0.61	0.34
B' (G)	0.31	0.3
p_1	1	1
γ_1	3700	2800
p_2	7	7
γ_2	…	…
p_3	…	…

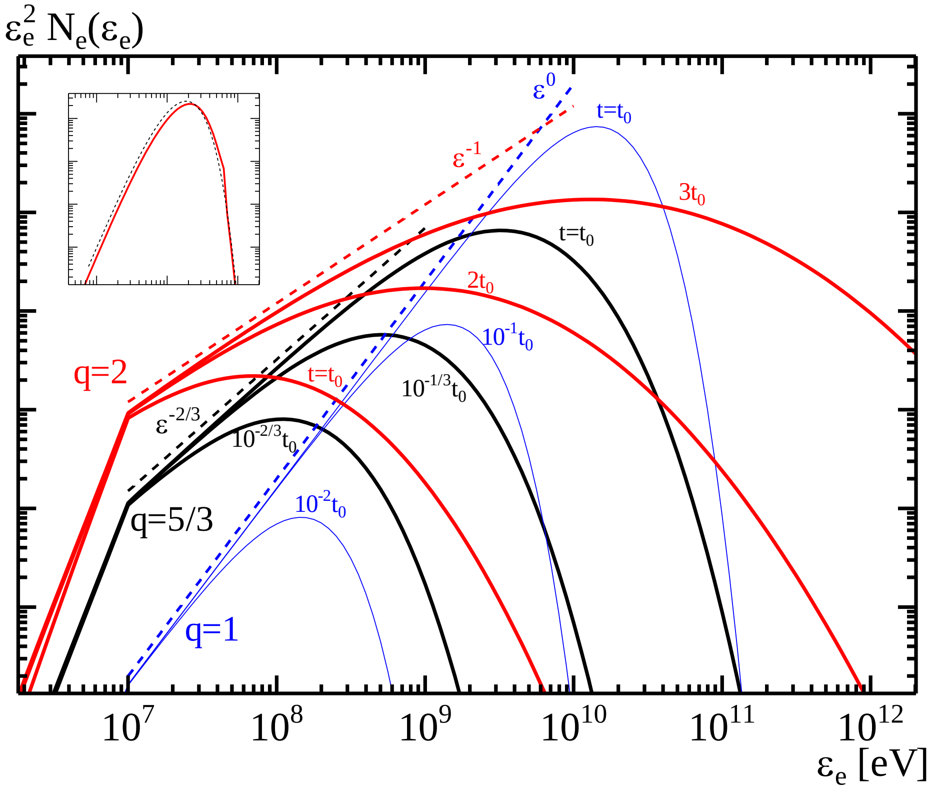
$$L_B/L_j \lesssim 10^{-4}$$

Very hard electron spectrum, but very low magnetic field.

Alternative: stochastic acceleration by turbulece



Energy=const.

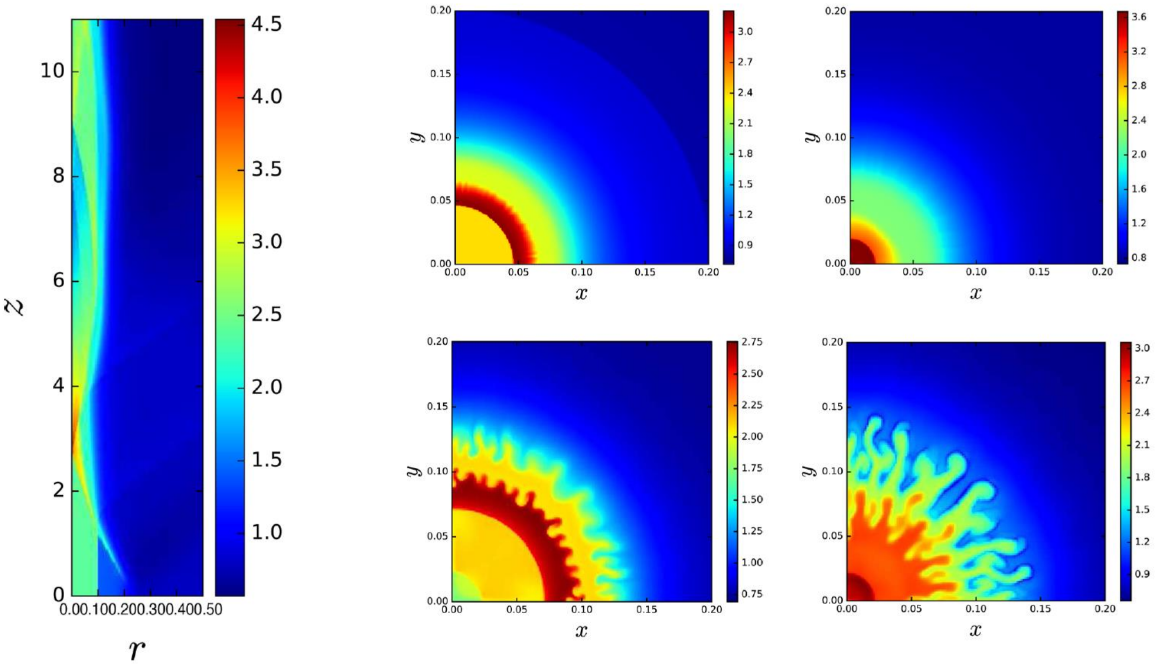


Outer: larger volume → Higher acceleration

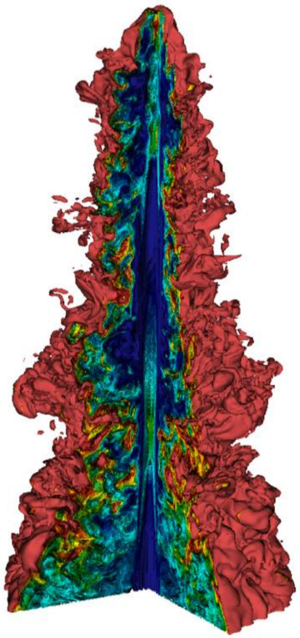
$$\frac{\partial N_{\rm e}(\varepsilon,t)}{\partial t} = \frac{\partial}{\partial E}\left[D_{EE}\frac{\partial N_{\rm e}(E,t)}{\partial E}\right] - \frac{\partial}{\partial E}\left[\left(\frac{2D_{EE}}{E} - \langle \dot{E}_{\rm cool}\rangle\right) N_{\rm e}(E,t)\right] + \dot{N}_{\rm e,inj}(E,t)$$

Diffusion **Acceleration** **Cooling** **Injection**

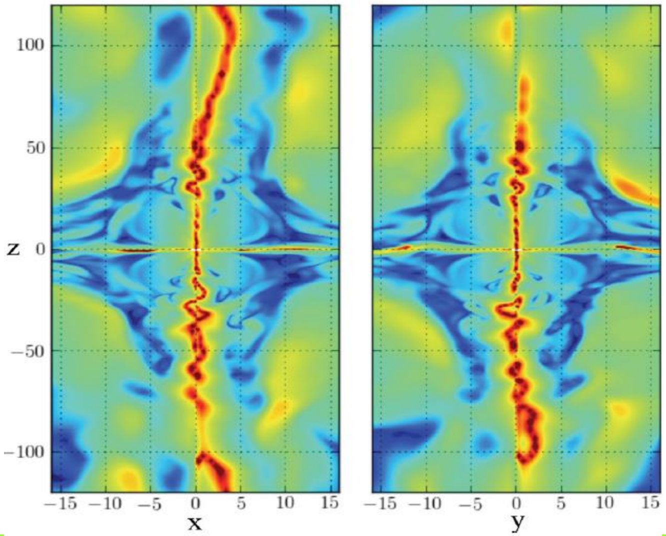
Excitation of Turbulence



Rayleigh-Taylor
(Toma+ 2017)



Kelvin-Helmholtz
(Beckwith+ 2011)

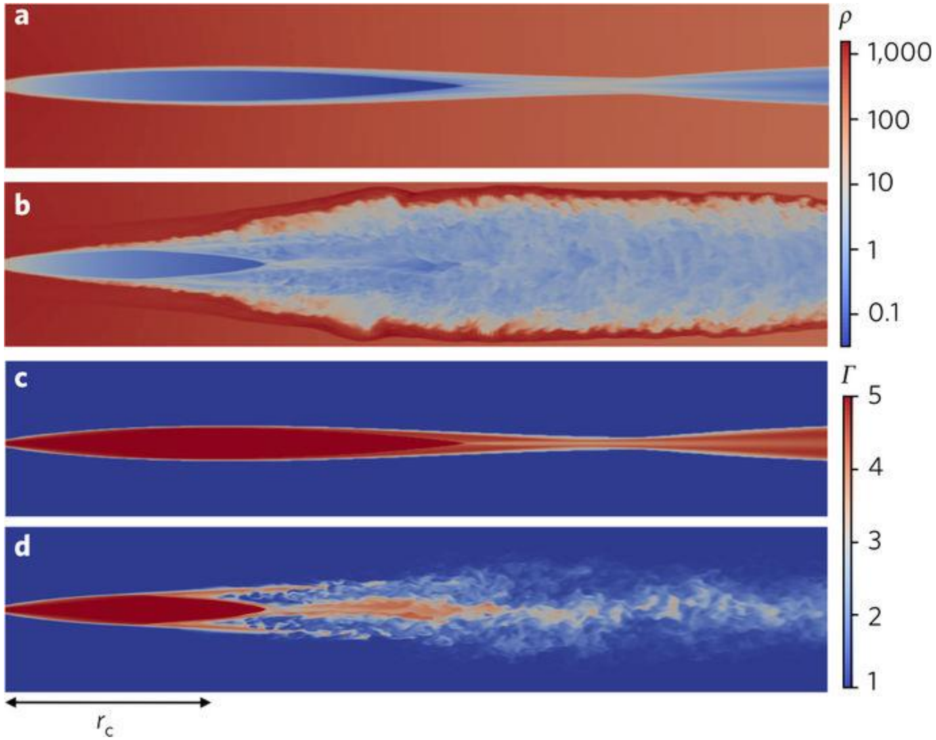


Kink
(Bromberg & Tchekhovskoy)
2016

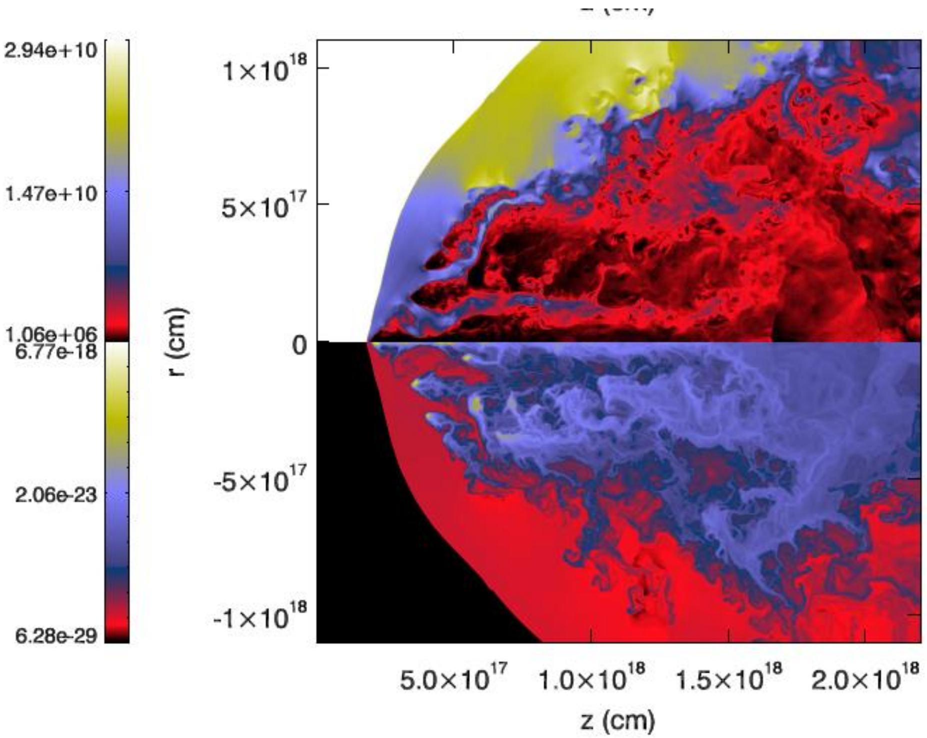
Instability 2

Reconfinement induced one.

Star-jet interaction

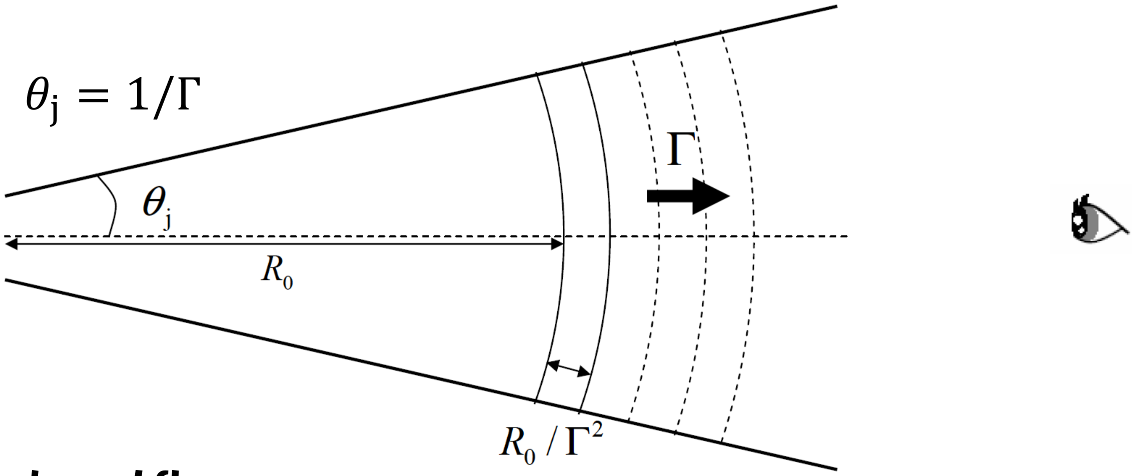


Gourgouliatos & Komissarov 2018



Perucho+ 2017

Our Model



$$B' = B_0 \left(\frac{R}{R_0}\right)^{-1}$$

$$D(\varepsilon) = K\varepsilon^q$$

- ***Steady outflow***
- ***Continuous shell ejection with a width of R_0/Γ in commoving frame***
- ***Electron injection from R=R_0 to 2R_0 with stochastic acceleration***
- ***Both injection and acceleration stop at R=2R_0***

Physical Processes

- *Electron injection*
- *Stochastic acceleration*
- *Synchrotron emission and cooling*
- *Inverse Compton emission and cooling*
- *Adiabatic cooling (V∝R^2)*
- *Photon escape*
- *No electron escape!*

Main Parameters:

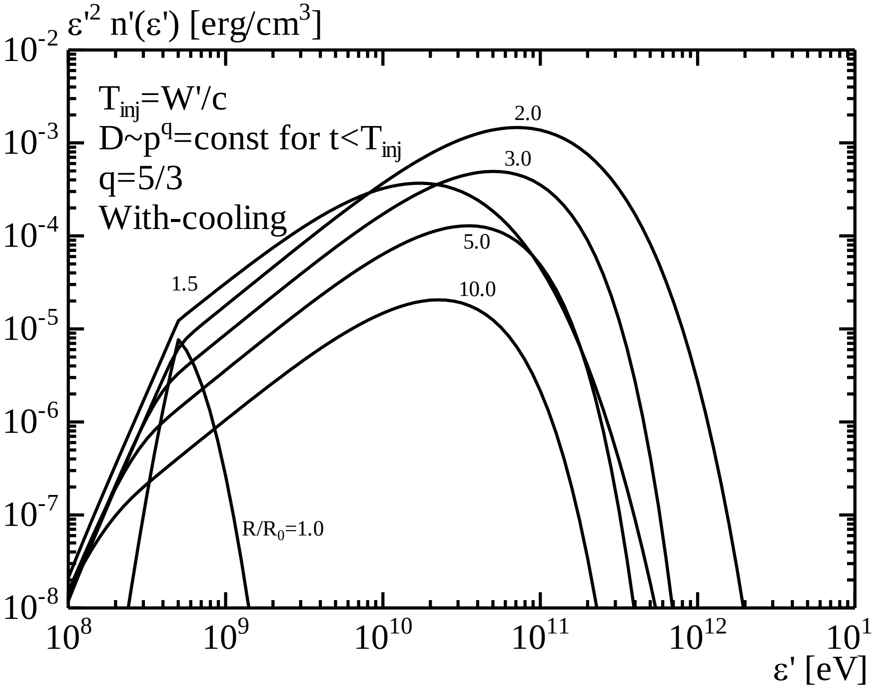
R_0, Γ, B_0, K

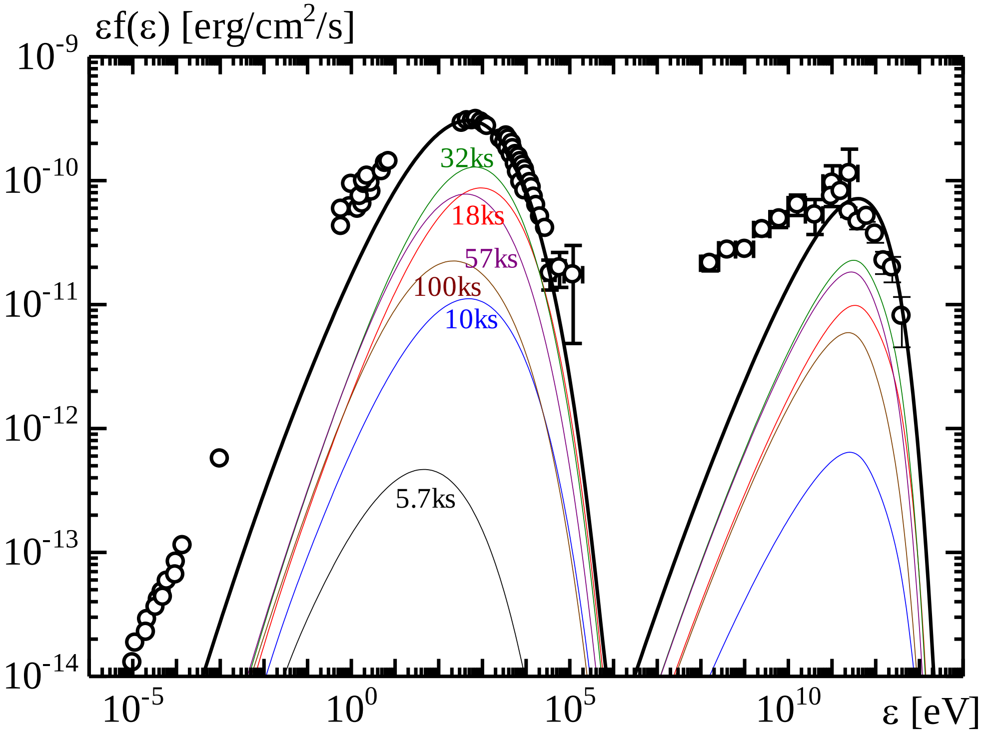
The others are $q, \gamma_{\rm inj}$, and injection rate.

Mrk421+Kolmogorov

$$t_{\mathrm{coll}}^{-1} = \nu \equiv \frac{\pi}{4}\frac{k|\delta B^2|_k}{B^2}\Omega \qquad \delta B^2(k) \propto k^{-q} \rightarrow \nu \propto k^{2-q} \propto E^{q-2}$$

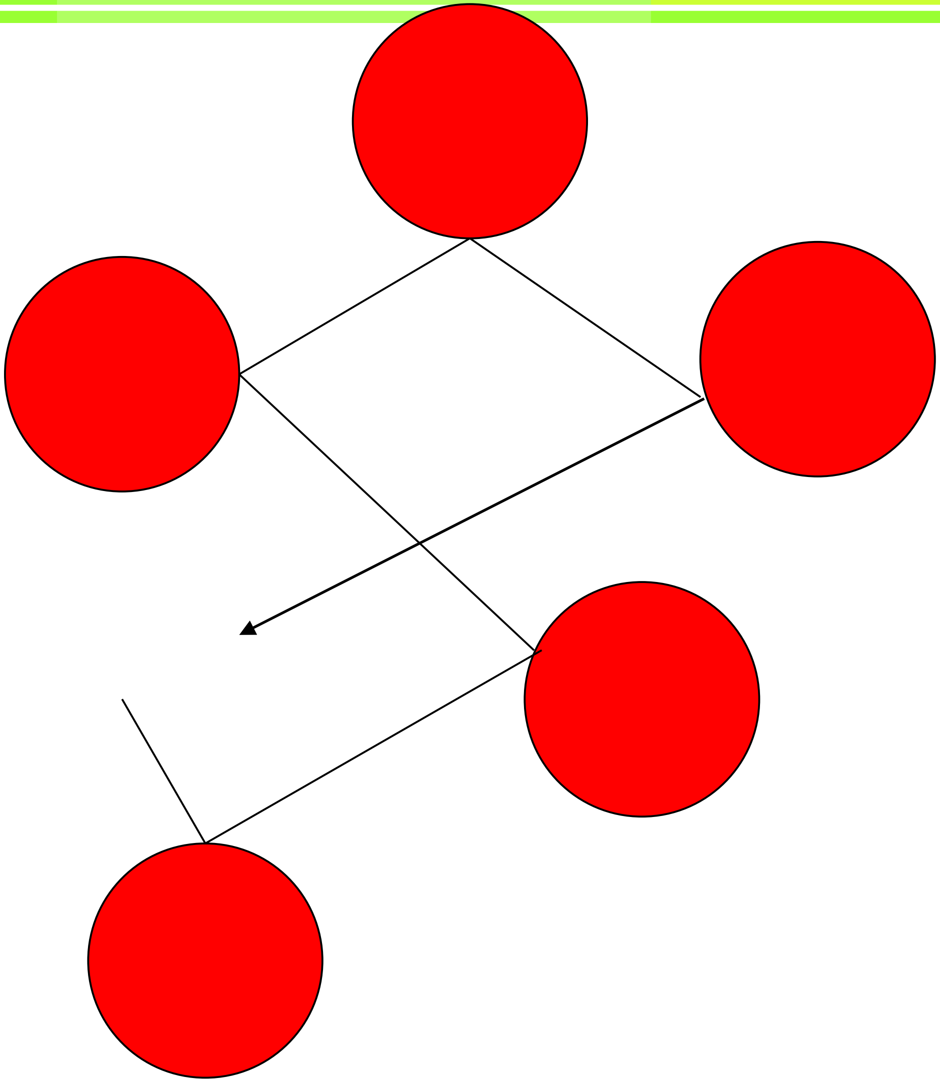
$D_{EE} \propto \nu E^2 \propto E^q$ **Kolmogorov case** $q = 5/3$





The spectrum becomes too hard. **Asano+ 14**

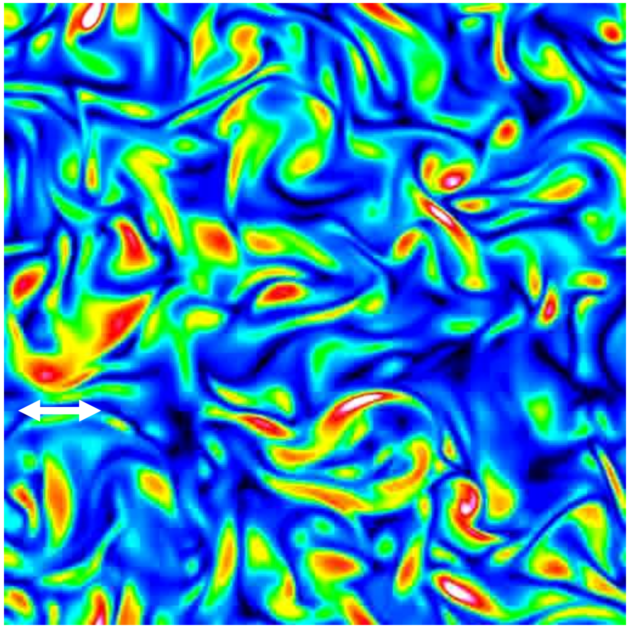
Hard-sphere



Scattering frequency is independent of energy.

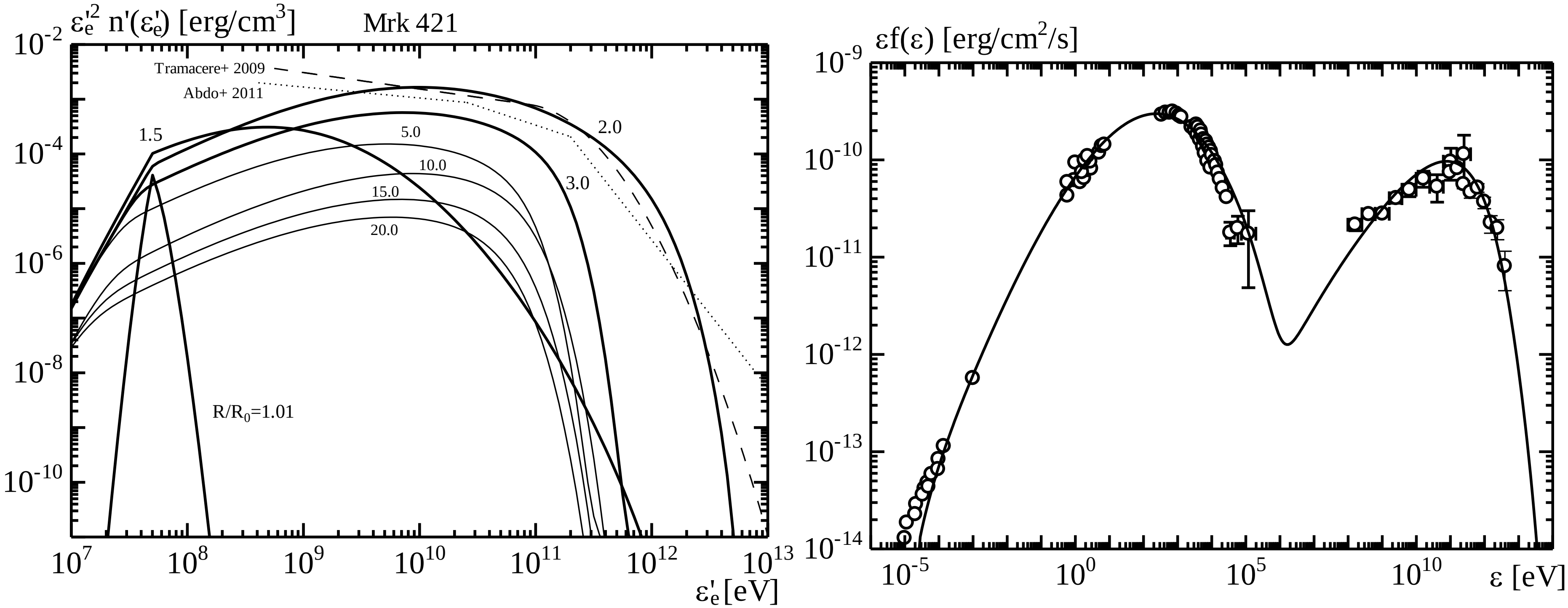
$$D(\varepsilon) = K\varepsilon^2$$

$$t_{\mathrm{acc}} \propto \varepsilon^0$$



Typical eddy size may correspond to the sphere radius.

Mrk421+Hard Sphere



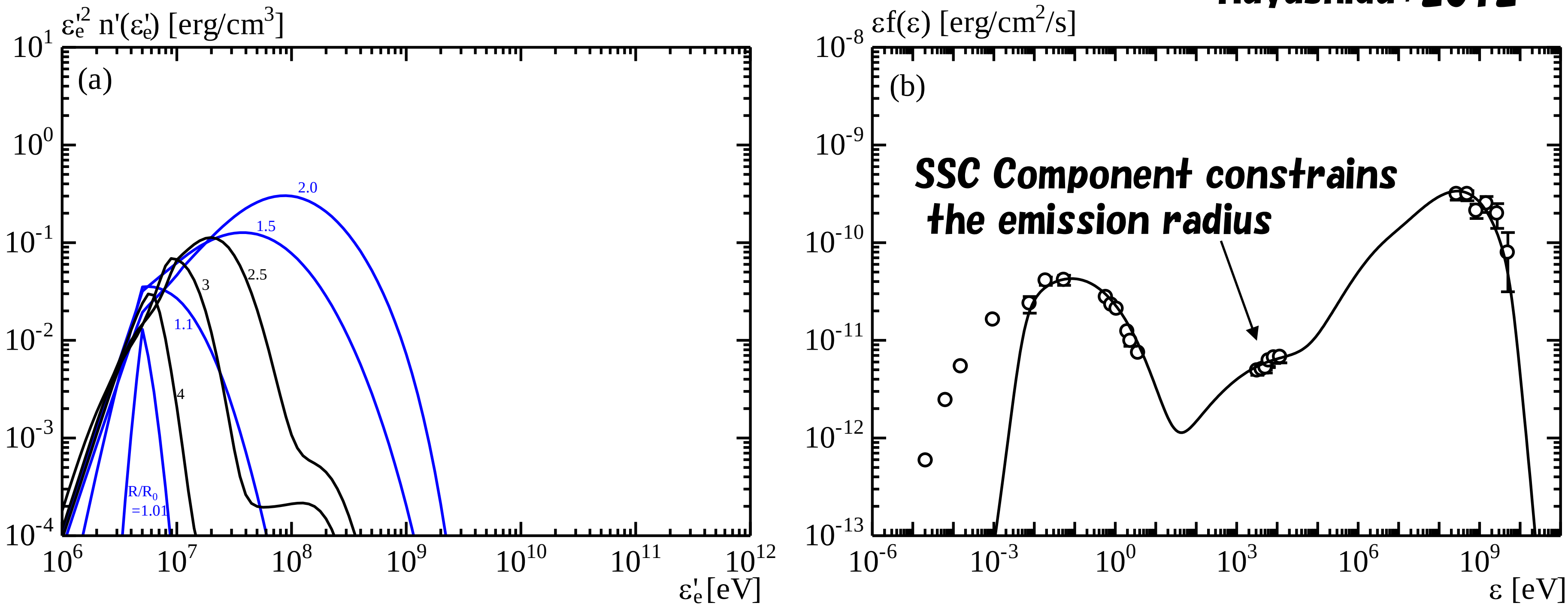
$$\Gamma = 15, B_0 = 0.16\mathrm{G}, W' = \frac{R_0}{\Gamma} = 10^{16}\mathrm{cm}, K = 3.7 \times 10^{-6}\mathrm{s}^{-1}, \dot{N} = 9.8 \times 10^{46}\mathrm{s}^{-1}$$

$$t_{\mathrm{coll}} \propto E^0,\ \ D_{EE} = KE^2$$

3C 279 +Hard Sphere

Model for the Steady State

Asano & Hayashida 2015
Hayashida+2012



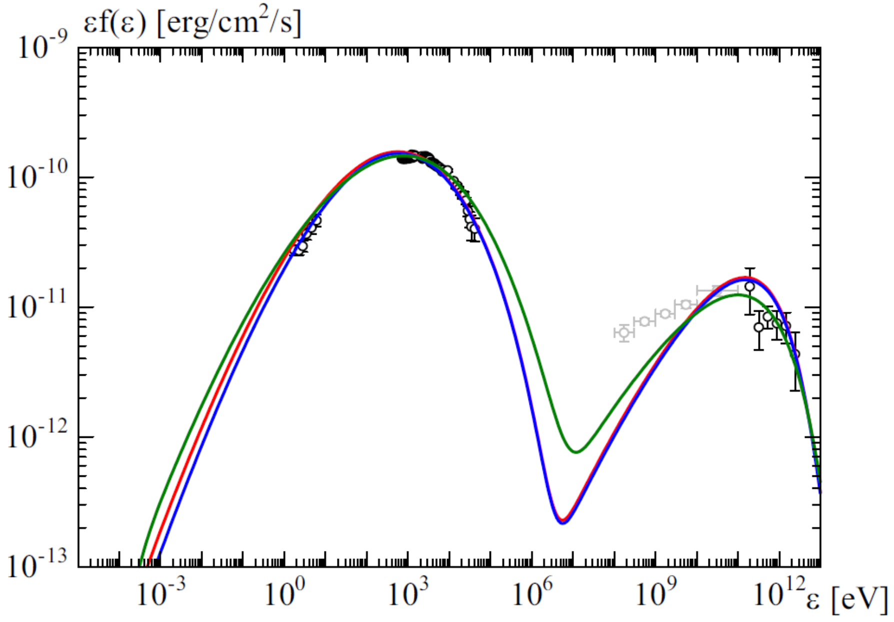
SSC Component constrains the emission radius

$R_0 = 0.023$ pc, $\Gamma = 15$, $K' = 9 \times 10^{-6}$ s^{-1} ($t_{\rm acc} = 1/(2\,K') = 0.35 W'/c$), $\dot{N}'_{\rm e} = 7.8 \times 10^{49}$ s^{-1} ($\dot{n}'_{\rm e} = 0.26 (R/R_0)^{-2}$ cm^{-3} s^{-1}), and $B_0 = 7$ G.

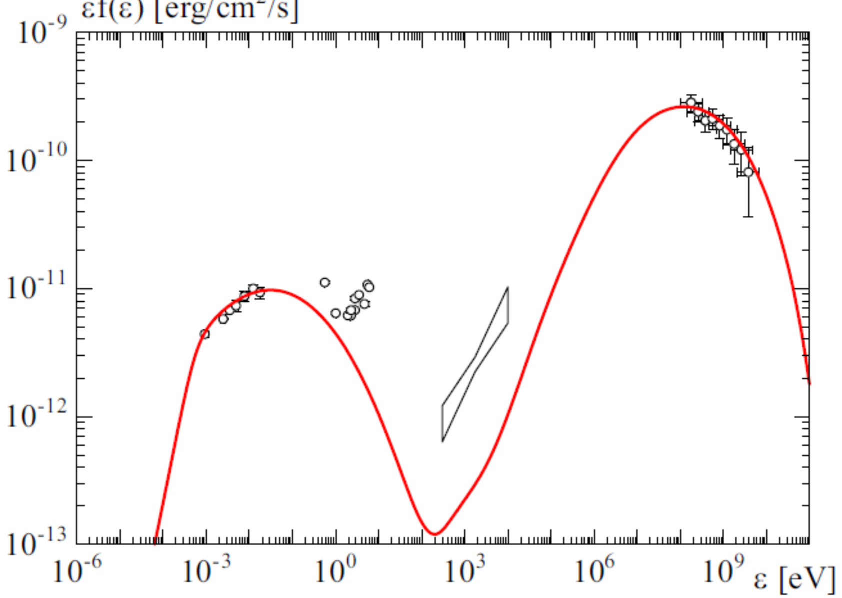
External photons: $T'_{\rm UV} = 10\Gamma {\rm eV}, U'_{\rm UV} = 8\left(\frac{\Gamma}{15}\right)^2 {\rm erg\ cm^{-3}}$

Other Blazars

1ES 1959+650.

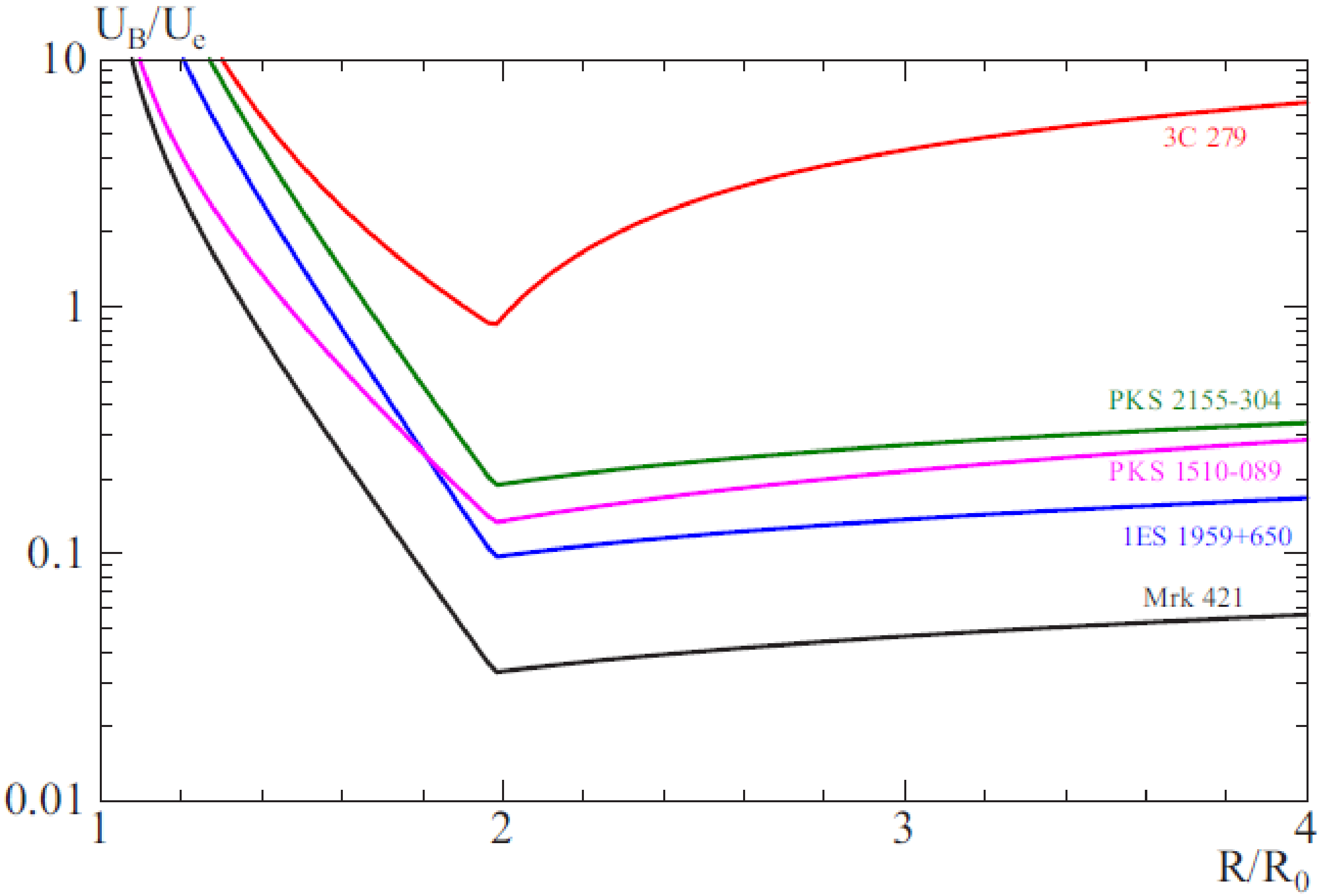


PKS 1510-089



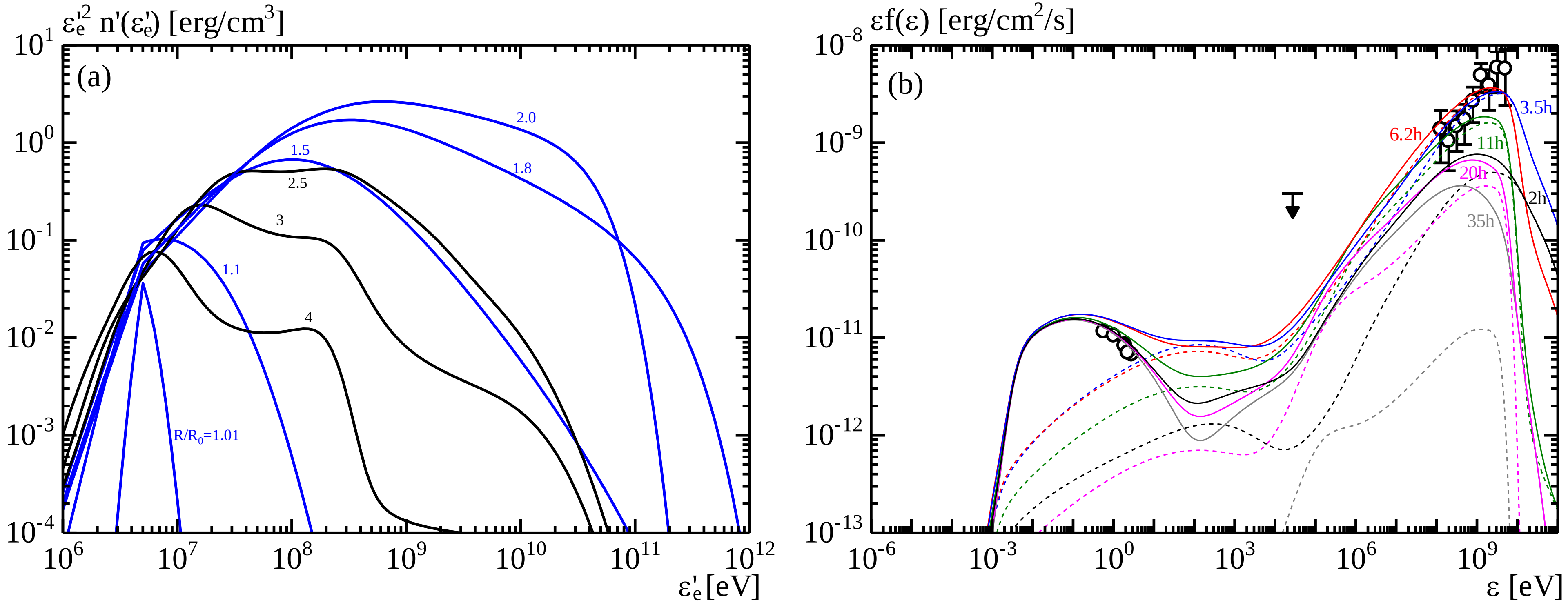
		$\gamma'_{\rm inj}$	R_0	$R_{\rm out}/R_0$	Γ	B_0	K	$\dot{N}'$	$L_{\rm D}$	UV/IR
			cm			G	$\rm s^{-1}$	$\rm s^{-1}$	$\rm erg\ s^{-1}$	
Mrk 421	A	10	1.5×10^{17}	30	15	0.18	4.8×10^{-6}	2.4×10^{47}	—	—
	B	100	1.5×10^{17}	30	15	0.16	3.7×10^{-6}	9.8×10^{46}	—	—
1ES 1959+650	A	100	1.6×10^{17}	30	20	0.18	5.0×10^{-6}	3.7×10^{46}	—	—
	B	100	1.6×10^{17}	10	20	0.18	5.0×10^{-6}	3.7×10^{46}	—	—
	C	10	4.0×10^{16}	30	40	0.5	4.3×10^{-5}	1.5×10^{47}	—	—
PKS 2155–304		10	6.0×10^{16}	30	20	1.2	1.2×10^{-5}	1.5×10^{48}	—	—
3C 279		10	7.1×10^{16}	30	15	8.0	9.5×10^{-6}	7.3×10^{49}	6.0×10^{45}	UV
PKS 1510–089		10	6.0×10^{17}	30	20	0.38	9.0×10^{-7}	7.3×10^{49}	5.0×10^{45}	IR

Evolution of Energy Densities



Flare Model

The same radius, Lorentz factor, and UV field as those in the steady model.



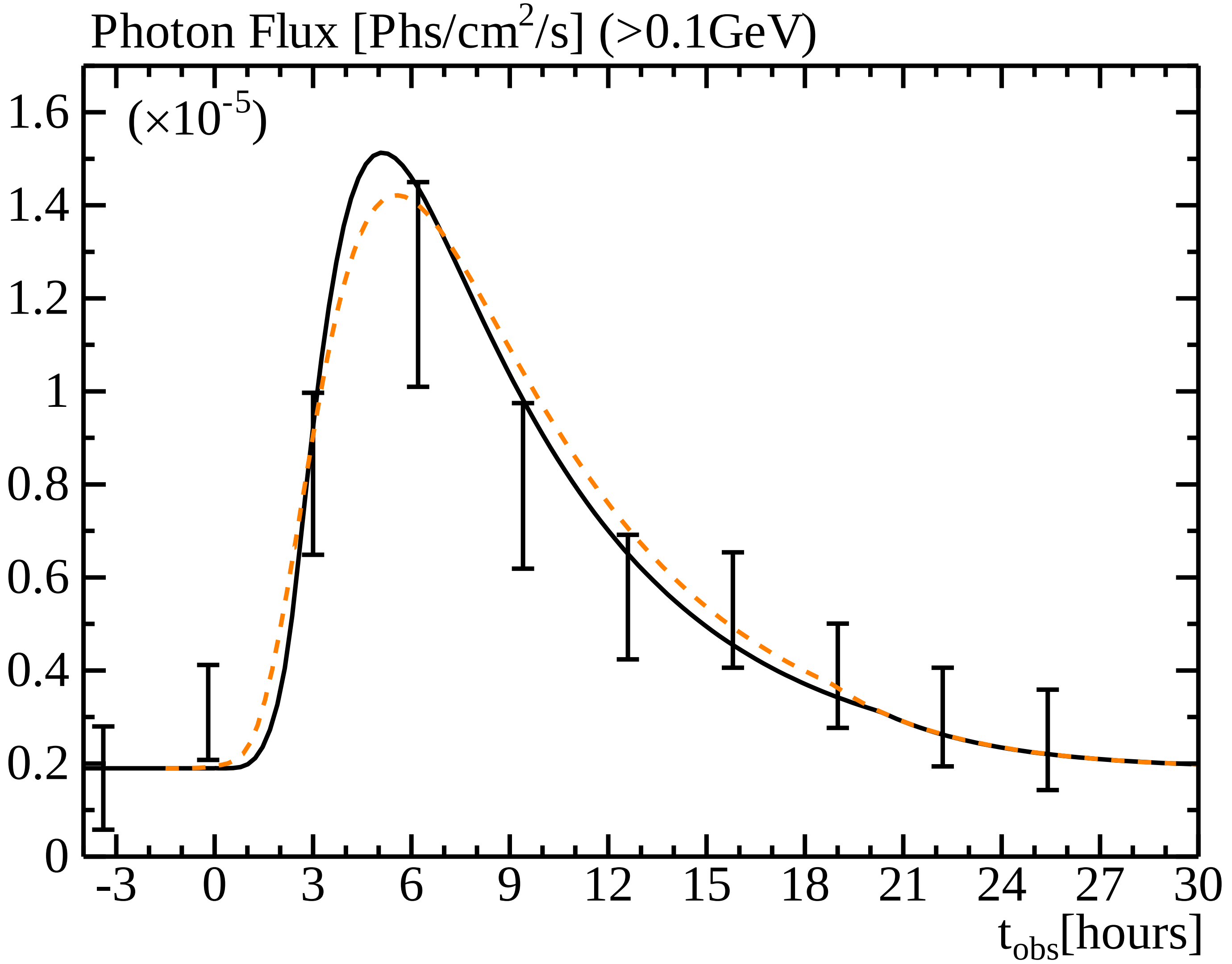
$q = 2$

$$K' = 1.3 \times 10^{-5}\ \mathrm{s}^{-1} \quad (t_{\mathrm{acc}} = 1/(2\,K') = 0.25W'/c),$$

$$\dot{N}'_{\mathrm{e}} = 2.5 \times 10^{50}\ \mathrm{s}^{-1} \quad (\dot{n}'_{\mathrm{e}} = 0.85(R/R_0)^{-2}\ \mathrm{cm}^{-3}\ \mathrm{s}^{-1}),$$

$$B_0 = 0.25\ \mathrm{G}.$$

Light curve


Photon Flux [Phs/cm^2/s] (>0.1GeV)
(×10^{-5})
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-3
0
3
6
9
12
15
18
21
24
27
30
t$_{obs}$[hours]

Diffusion Coefficient

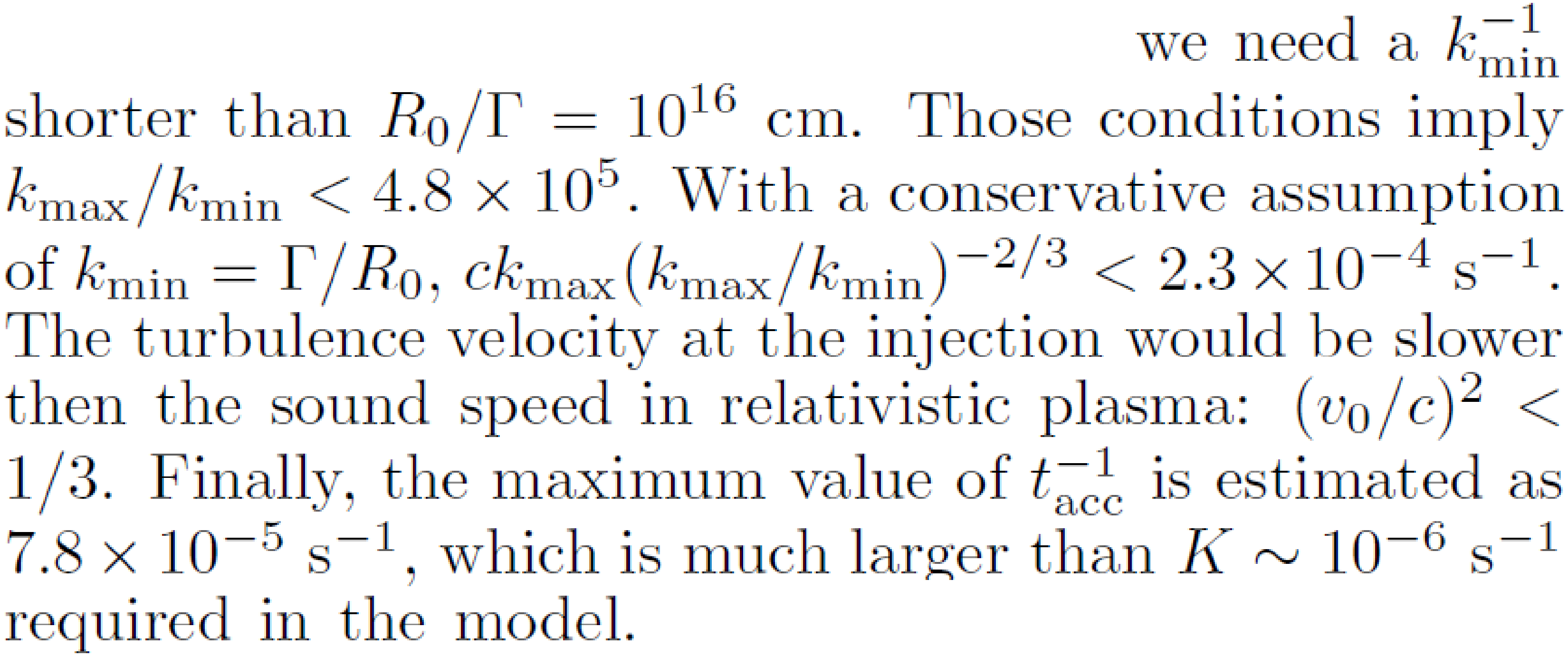
Mrk 421

$B_0 = 0.16\mathrm{G} \rightarrow r_L = 2.1 \times 10^{10}\mathrm{cm@TeV}$

$D_{EE} = KE^2, \qquad K = 3.7 \times 10^{-6}\mathrm{s}^{-1}$

For $\nu = 5/3$

$$K = \left(\frac{\delta v}{c}\right)^2 ck_{\max}\frac{2}{3}\left(\frac{k_{\max}}{k_{\min}}\right)^{-2/3}$$



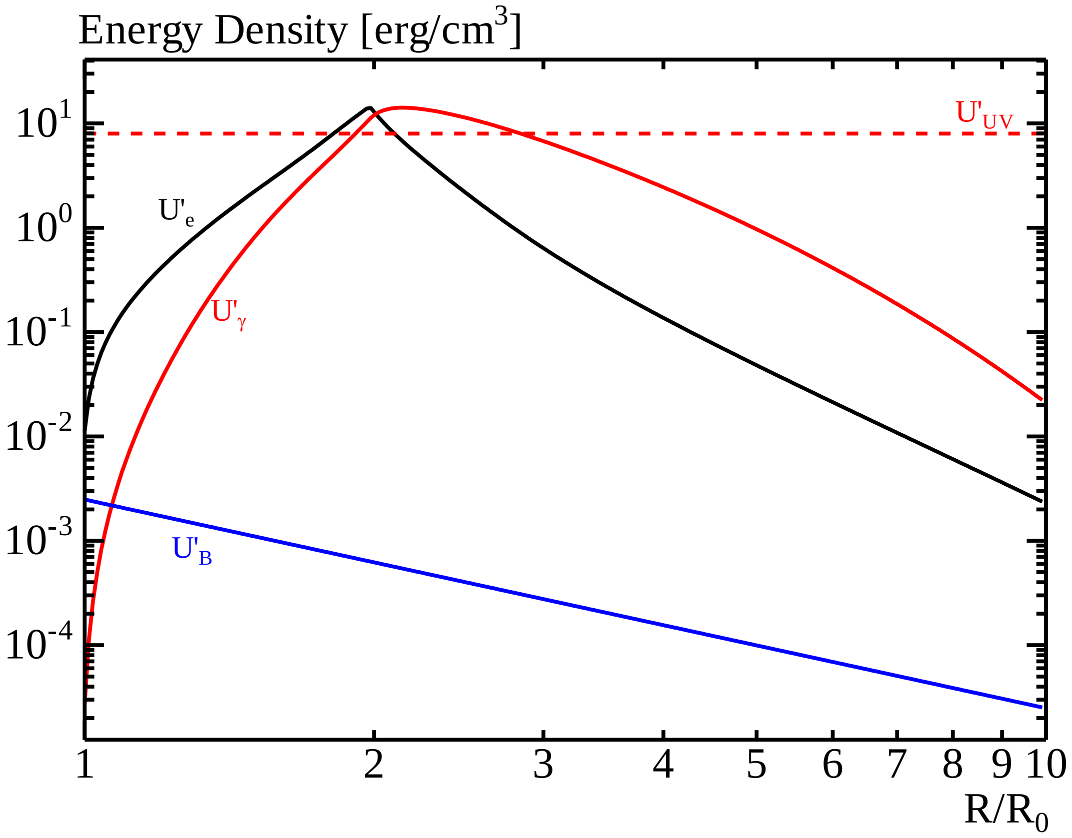
we need a $k_{\min}^{-1}$ shorter than $R_0/\Gamma = 10^{16}$ cm. Those conditions imply $k_{\max}/k_{\min} < 4.8 \times 10^5$. With a conservative assumption of $k_{\min} = \Gamma/R_0$, $ck_{\max}(k_{\max}/k_{\min})^{-2/3} < 2.3 \times 10^{-4}\ \mathrm{s}^{-1}$. The turbulence velocity at the injection would be slower then the sound speed in relativistic plasma: $(v_0/c)^2 < 1/3$. Finally, the maximum value of t_{acc}^{-1} is estimated as $7.8 \times 10^{-5}\ \mathrm{s}^{-1}$, which is much larger than $K \sim 10^{-6}\ \mathrm{s}^{-1}$ required in the model.

Summary

- The turbulence acceleration is an alternative model for the electron acceleration in blazars.
- However, the gyro-resonant-like acceleration in Kolmogorov turbulence seems not adequate.
- Particles interacting compressible waves are accelerated via TTD resonance.
- Hard-sphere-like acceleration in this case agree with the observed spectra of blazars.

予備スライド

Energy Density



Very Weak Magnetic Field.

Negative for
the magnetic reconnection,
and
the jet acceleration by magnetic field.

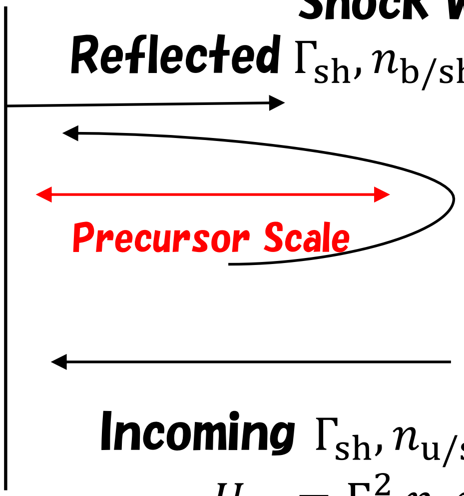
$$\Gamma \sim \left(\frac{\underline{R}}{3r_g}\right)^{0.4} \simeq 8.8 < \underline{15}$$

Assumed Values

Alfven wave seems not responsible for the acceleration.
Fast wave (kinetic>>magnetic) can be the energy source.
There may be the shortest scale of turbulence, which is the dominant scatterer.

Suppresion of Turbulence in the upstream

Reflected particles



Shock wave rest frame

Reflected $\Gamma_{\rm sh}, n_{\rm b/sh}, U_{b/\rm sh} = \Gamma_{\rm sh} n_{\rm b/sh} m_{\rm p} c^2$

Incoming $\Gamma_{\rm sh}, n_{\rm u/sh} = \Gamma_{\rm sh} n_{\rm u}$

$$U_{\rm sh} = \Gamma_{\rm sh}^2 n_{\rm u} m_{\rm p} c^2$$

$$\xi \equiv \frac{U_{b/\rm sh}}{U_{\rm sh}} = \frac{n_{\rm b/sh}}{n_{\rm u/sh}} = \frac{n_{\rm b/u}}{\Gamma_{\rm sh}^2 n_{\rm u}} < 1$$

Magnetization parameter

$$\sigma = \frac{B_{\rm u}^2}{4\pi n_{\rm u} m_{\rm p} c^2}$$

Turbulence is required to scatter particles.

Precursor Scale in the upstream

$$\Delta l = (c - v_{\rm sh})\Delta t_{\rm ret} \simeq \frac{c}{\Gamma_{\rm sh}^2} \frac{r_{\rm L/u}}{c\Gamma_{\rm sh}} = \frac{m_{\rm p} c^2}{\Gamma_{\rm sh} e B_{\rm u}}$$

Plasma frequency in the upstream

$$\omega_{\rm pb/u} = \sqrt{\frac{4\pi n_{\rm b/u} e^2}{\Gamma_{\rm b} m_{\rm p}}} = \sqrt{\frac{4\pi n_{\rm u} e^2}{m_{\rm p}} \frac{n_{\rm b/u}}{\Gamma_{\rm sh}^2 n_{\rm u}}} = \xi^{1/2} \omega_p$$

should be larger than $c/\Delta l$
to induce turbulence.

Then, $\sigma < \xi \Gamma_{\rm sh}^{-2}$

Only for low magnetized case,
the particle acceleration is possible.

Hard-Sphere Model

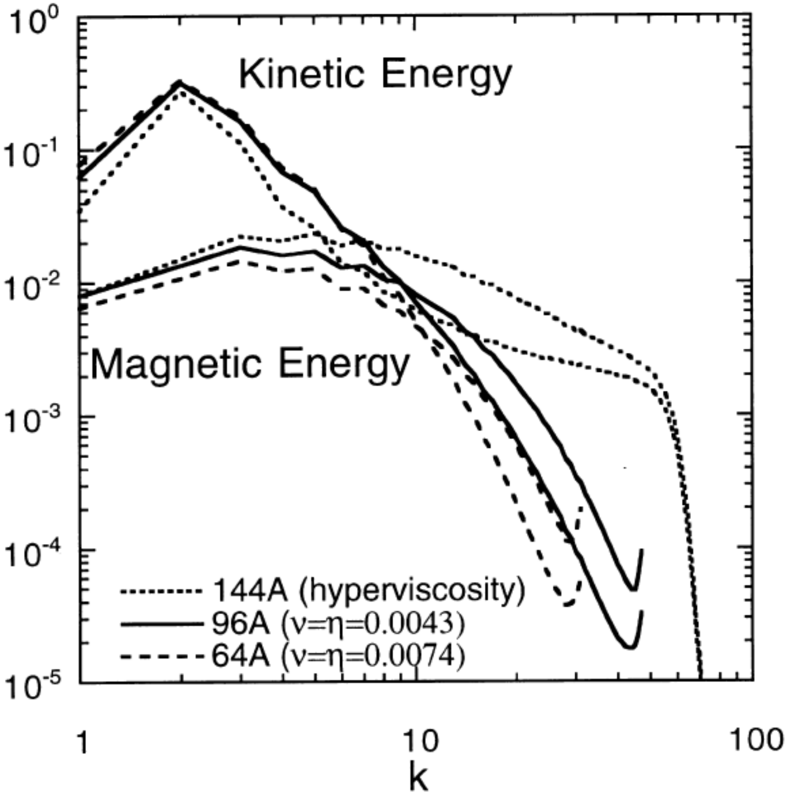
Supposing the compressional waves are responsible, we model as following.

$$D_{\varepsilon\varepsilon} = K\varepsilon^2$$

Parameters are

$$\underbrace{R_0, \Gamma, B_0, \dot{N}}_{\text{Required for any model}}, \underbrace{K}_{\text{Only one peculiar parameters}}$$

$$B = B_0 \left(\frac{R}{R_0}\right)^{-1}$$

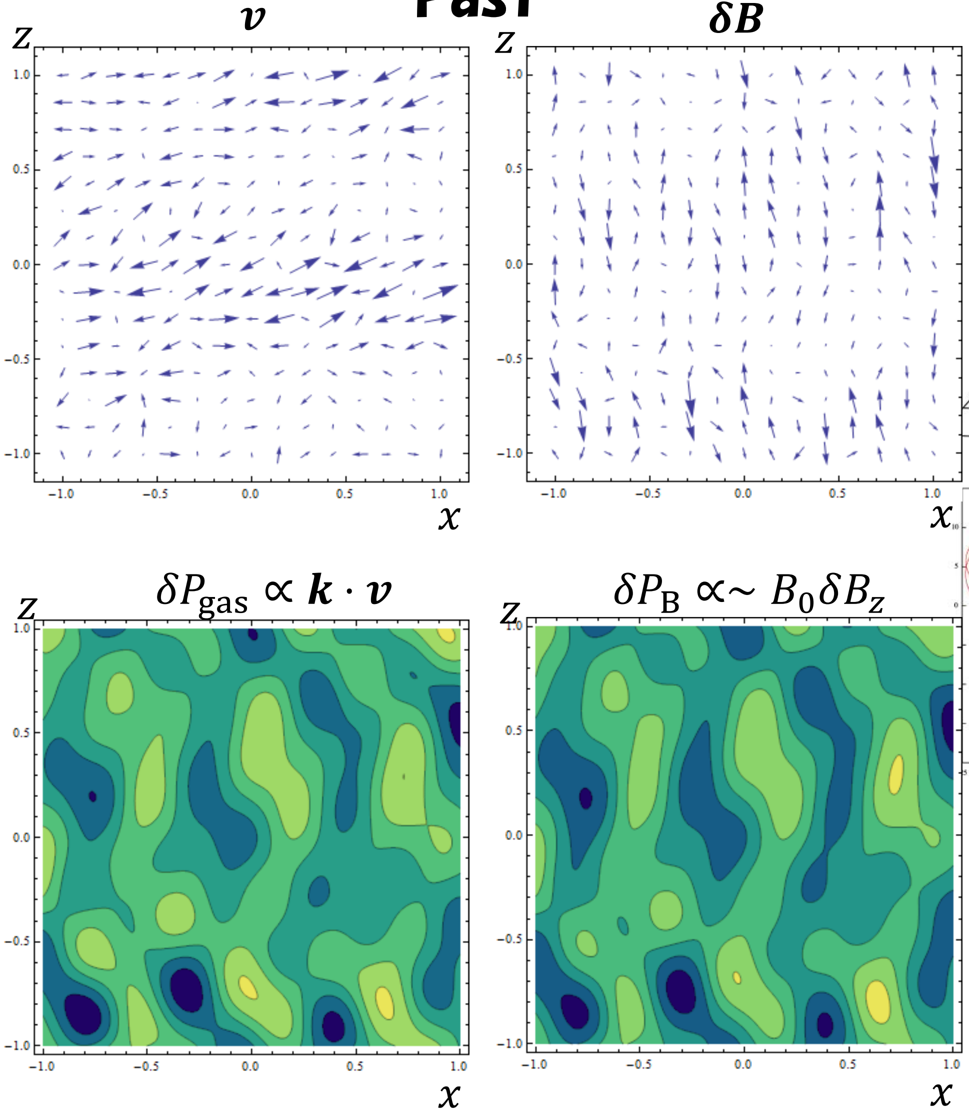


Cho & Vishniac 2000

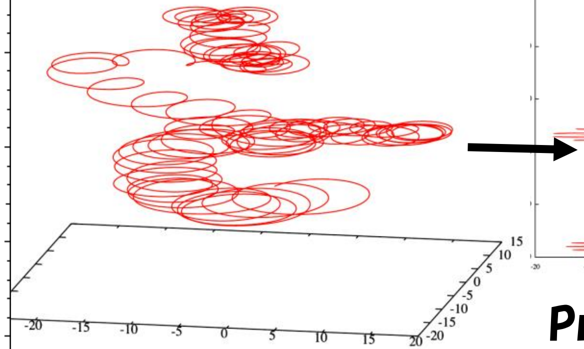
Parameters are constant during the dynamical time scale $R_0/(c\Gamma)$, and injection and acceleration suddenly shutdown after that.

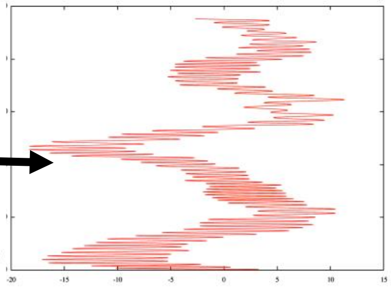
Test particle simulation in pure linear Waves

Fast

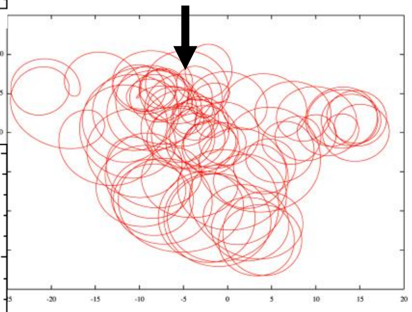


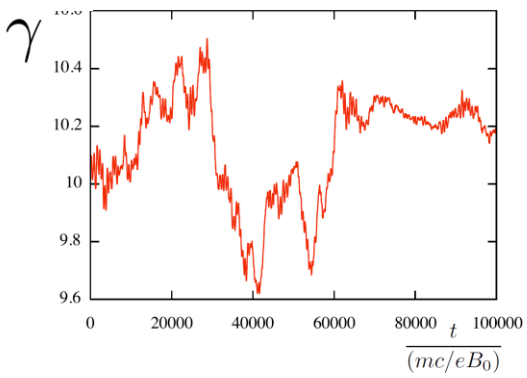
High-beta plasma





Projection



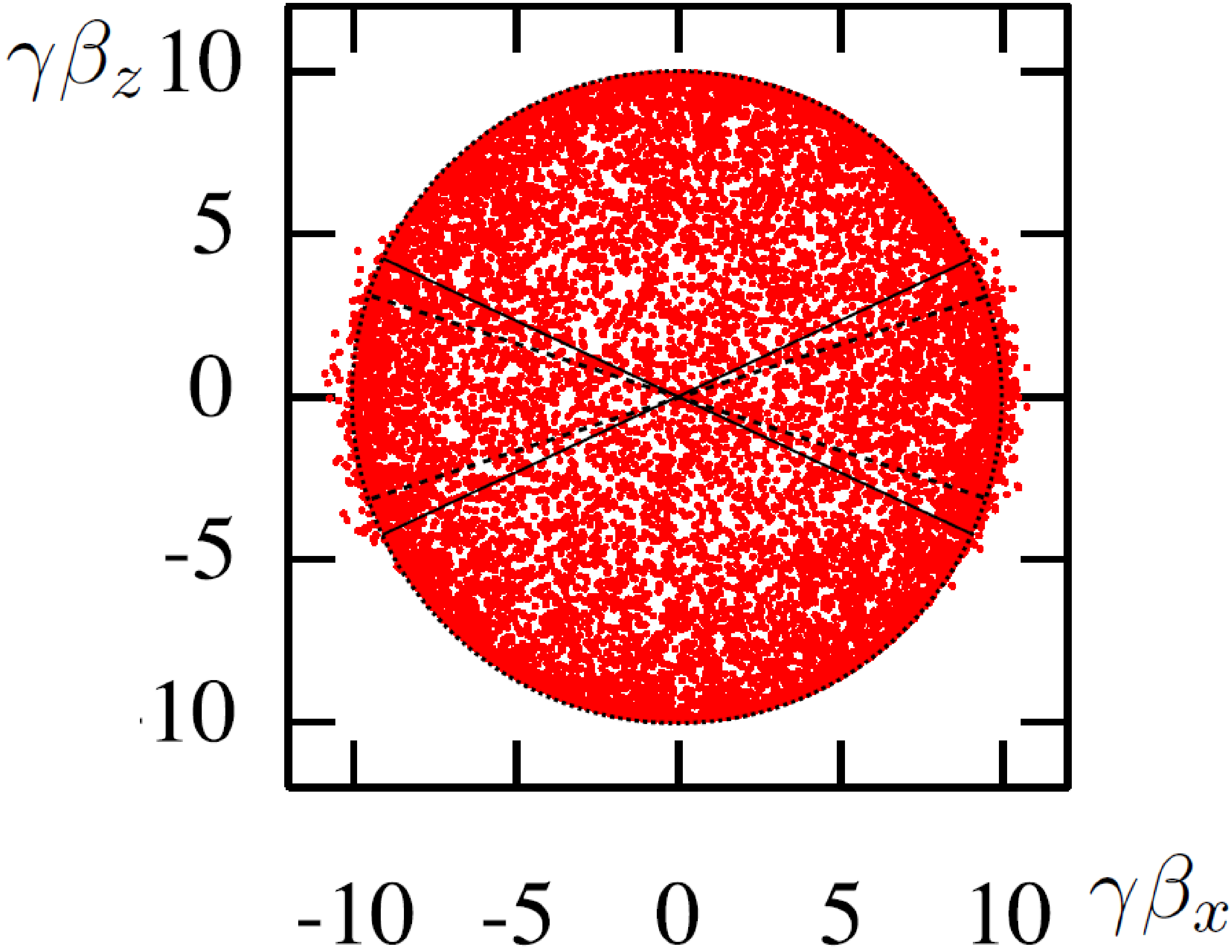


Teraki & Asano in prep.

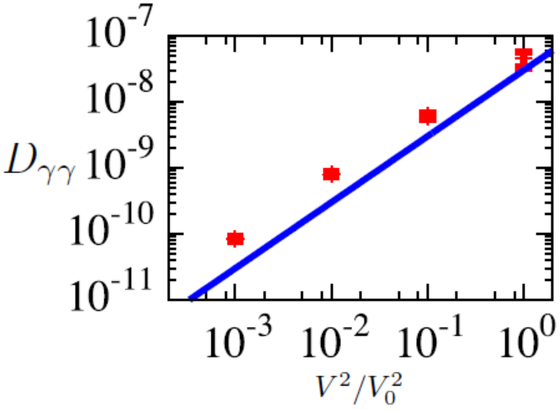
$\alpha = 10, k = 10 - 20$, **10 mode**
Kolmogorov

Enrgy diffusion

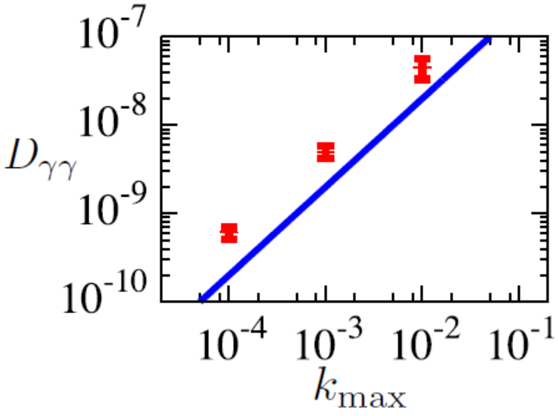
Energy gain by the TTD resonance



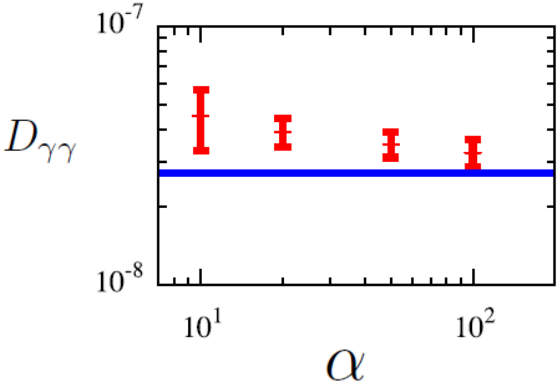
Dependence on Parameters



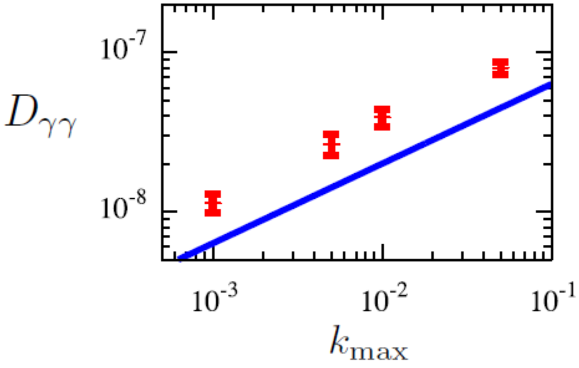
(a) Energy density dependence



(b) Maximum wavenumber dependence: $k_{\min}/k_{\max}$ is fixed.

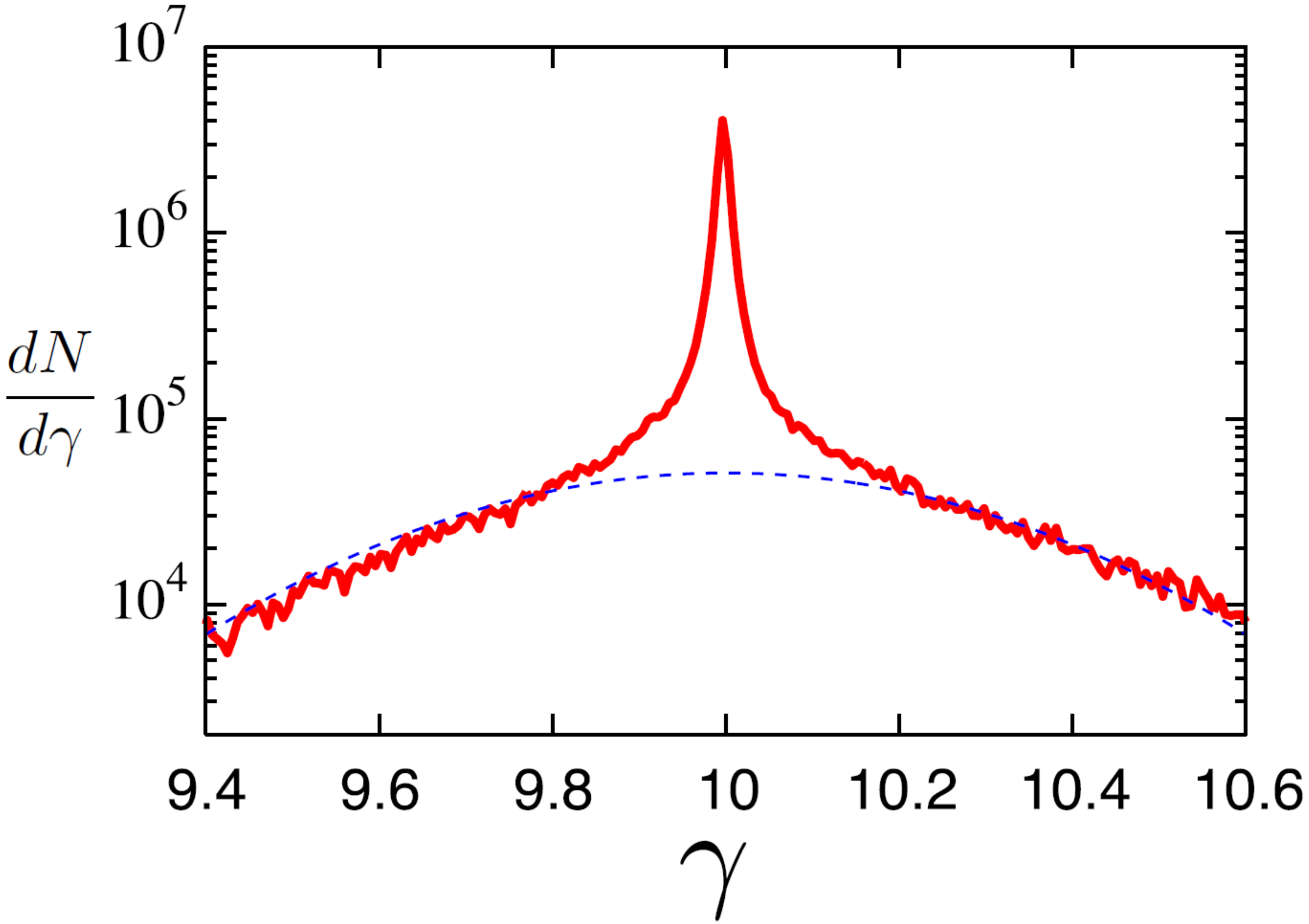


(c) α dependence



(d) Maximum wavenumber dependence: $k_{\min}$ is fixed

Energy Diffusion



Significant fraction of particles diffuses in the energy space.

Diffusion coefficient

$$\delta B(k) \equiv \sqrt{k_n P_{\mathrm{B}}(k_n)}$$

Diffusion by mirror force

$$\frac{\Delta p}{\delta t} \sim \frac{p_\perp v_\perp}{2B}\nabla_{\|}|B_{\|}| \sim p v_\perp k_{n,\|}\frac{\delta B(k_n)}{B_0}$$

$$\delta t \sim \frac{1}{v_{\|} k_{n,\|}}$$

$$\Delta\mu \sim \frac{\Delta p}{p} \sim \frac{v_\perp}{v_{\|}}\frac{\delta B(k_n)}{B_0}$$

$$D_{\mu\mu} = \frac{(\Delta\mu)^2}{\delta t} \sim \sum_n v_\perp^2 k_{\|}^2 \left(\frac{k_n P_B(k_n)}{B_0^2}\right) \cdot \delta t$$

$$\Delta\gamma \sim \frac{\mathcal{V}_{\mathrm{ph}}}{c}\gamma\Delta\mu$$

$$D_{\gamma\gamma} \sim \sum_n \frac{(\Delta\gamma)^2}{\delta t} \sim \sum_n \gamma^2 \left(\frac{\mathcal{V}_{\mathrm{ph}}}{c}\right)^2 c k_n \left(\frac{k_n P_B(k_n)}{B_0^2}\right)$$

$$D_{\gamma\gamma,\mathrm{fast}} \sim \gamma^2 \left(\frac{\mathcal{V}_{\mathrm{ph}}}{c}\right)^2 c k_{\mathrm{max}} \left(\frac{k_{\mathrm{max}} P_B(k_{\mathrm{max}})}{B_0^2}\right)$$

$$P_B \propto k^{-\nu}$$

$$D_{\gamma\gamma,\mathrm{fast}} \sim \gamma^2 \left(\frac{V}{c}\right)^2 c k_{\mathrm{max}}(\nu - 1)\left(\frac{k_{\mathrm{max}}}{k_{\mathrm{min}}}\right)^{1-\nu} \quad \epsilon_{\mathrm{res,fast}}$$

Hard Sphere-like diffusion

$$D_{\gamma\gamma,\mathrm{fast}} \sim \gamma^2 \left(\frac{V}{c}\right)^2 ck_{\mathrm{max}}(\nu-1)\left(\frac{k_{\mathrm{max}}}{k_{\mathrm{min}}}\right)^{1-\nu} \epsilon_{\mathrm{res,fast}}$$

$v_F = 0.1c, \delta V = 10^{-3}$(**weak turb.**)