



# spectral curvature in TeV blazars: physical insight on stochastic acceleration

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Half a Century of Blazars and Beyond

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#### LP SPECTRAL DISTRIBUTION OF HBLs



#### acceleration signature in the Es-vs-b trend

Tramacere+2007,2009



#### acceleration signature in the E<sub>S</sub>-vs-L<sub>s</sub> trend



The log-parabola origin: physical insight

## log-parabola is not a "new" model...

#### **KARDASHEV 1962**

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N. S. KARDASHEV

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At first, for simplicity, we consider the effect of each process viewed separately on the energy spectrum, and then the simultaneous effect of two or more processes.

Spectra of Isolated Processes

1. Random and Systematic Acceleration. The kinetic equation is

$$\frac{\partial N}{\partial t} = \alpha_1(t) \frac{\partial}{\partial E} \left( E^2 \frac{\partial N}{\partial E} \right) - \alpha_2(t) \frac{\partial}{\partial E} (EN) .$$

Let the energy distribution be specified, at each instant of time  $t_0$ , by the  $\delta$ -function in the neighborhood of energy  $E_0$ :

and

$$N(E, 0) = N_0 \delta(E - E_0) \qquad \qquad \overset{E_{\max}}{\underset{0}{\int}} k$$

$$\int_{0}^{\infty} N(E, 0) dE = N_0. \qquad \qquad \overset{E_{\max}}{\underset{0}{\int}} k$$

Then, utilizing the techniques developed, e.g., in [13],



# log-parabola is not a "new" model...

$$\frac{\partial n(\gamma,t)}{\partial t} = \frac{\partial}{\partial \gamma} \left\{ - \left[ S(\gamma,t) + D_A(\gamma,t) \right] n(\gamma,t) + D_p(\gamma,t) \frac{\partial n(\gamma,t)}{\partial \gamma} \right\} - \frac{n(\gamma,t)}{T_{esc}(\gamma)} + Q(\gamma,t)$$

analytical solution for:  $D_p \sim \gamma q, q=2$ 

"hard-sphere" case

Melrose 1968,

$$n(\gamma, t) = \frac{N_0}{\gamma \sqrt{4\pi D_{p0} t}} \exp\left\{-\frac{[\ln(\gamma/\gamma_0) - (A_{p0} - D_{p0})t]^2}{4D_{p0} t}\right\}$$

# The origin of the log-parabolic shape: statistical derivation



systematic



Log-Parabolic representation



$$\log(n(\gamma)) \propto \frac{(\log \gamma - \mu)^2}{2\sigma_{\gamma}^2} \propto r \; [\log(\gamma) - \mu]^2$$

#### Tramacere+2011

#### statistical approach

$$n(\gamma) = \frac{N_0}{\gamma \sigma_{\gamma} \sqrt{2\pi}} \exp\left[\frac{-\left(\ln(\gamma/\gamma_0) - n_s \left[\ln\bar{\varepsilon} - \frac{1}{2}(\sigma_{\varepsilon}/\bar{\varepsilon})^2\right]\right)^2}{2n_s (\sigma_{\varepsilon}/\bar{\varepsilon})^2}\right].$$

#### diffusion equation approach

$$n(\gamma, t) = \frac{N_0}{\gamma \sqrt{4\pi D_{p0} t}} \exp\left\{-\frac{[\ln(\gamma/\gamma_0) - (A_{p0} - D_{p0})t]^2}{4D_{p0} t}\right\}$$

$$\mathbf{r} \propto \frac{1}{D_{p0}t} \rightarrow \left( D_{p0} \propto \left( \frac{\sigma_{\varepsilon}}{\overline{\varepsilon}} \right)^2 \right)$$

# The curvature *r* is inversely proportional to $t => n_s$ and $D_p => \sigma_{\varepsilon}$

log-parabolic shape natural consequence of dispersion



$$\log(n(\gamma)) \propto \frac{(\log \gamma - \mu)^2}{2\sigma_{\gamma}^2} \propto r \ [\log(\gamma) - \mu]^2$$

#### Physical insight of the curvature:

#### A self-consistent approach

acc+cooling

#### injection term

$$L_{inj} = \frac{4}{3}\pi R^3 \int \gamma_{inj} m_e c^2 Q(\gamma_{inj}, t) d\gamma_{inj} \quad (erg/s)$$



#### set-up of the accelerator



#### spectral trends

single flare



### IC cooling and equilibrium



 $U_{ph}$  (R= 1x10<sup>13</sup> cm) >>  $U_{ph}$  (R= 1x10<sup>15</sup> cm)

IC prevents higher energies in more compact accelerators (if all the parameters are the same) **Impact on rapid TeV variability!** 

 curvature decreasing trend-> acceleration is dominating
 pure log-parabolic shape -> acceleration is dominating system far from equilibrium

•exp cut-off shape-> system ~@equilibrium

•the equilibrium energy can change if B and t\_acc are unchanged, depending on R, a smaller value of R implies a larger IC cooling, hence less energetic particles

## effect of the turbulence index q



## effect of the turbulence index q

B=1.0 G, t<sub>D0</sub>=10<sup>3</sup>, R=5x10<sup>15</sup> cm



## b distributions and q



#### **Continuous injection**



#### Fermi I+Fermi II Mrk 421 2006

LP+PL spectra Synch index~[1.6-1.7]=>s~[2.2-2.4]

r~0.7-0.8<<req~6



Lemoine, Pelletier 2003







#### spectral trends

#### multiple flares and population trends

## $E_s$ - $b_s$ X-ray trend and $\gamma$ -ray predictions



•data span 13 years, both flaring and quiescent states

- •We are able to reproduce these long-term behaviours, by changing the value of only one parameter  $(D_p)$
- •for q=2, curvature values imply distribution far from the equilibrium (b~[1.0-0.7])
- •More data needed at GeV/TeV, curvature seems to be cooling-dominated
- •Similar trend observed in GRBs (Massaro & Grindlay 2001)

$L_{\text{inj}} (E_s - b_s \text{ trend})$	) (erg s <sup>-1</sup> )	$  5 \times 10^{39}$
$L_{\text{inj}}$ (E <sub>s</sub> -L <sub>s</sub> trend	) (erg $s^{-1}$ )	$5 \times 10^{38}, 5 \times 10^{39}$
q		2
$t_A$	(s)	$1.2 \times 10^{3}$
$t_{D_0} = 1/D_{P0}$	(s)	$[1.5 \times 10^4, 1.5 \times 10^5]$
T <sub>inj</sub>	(s)	104
$T_{\rm esc}$	(R/c)	2.0

#### $E_s$ - $L_s$ X-ray trend and $\gamma$ -ray predictions



• the  $E_s-S_s$  ( $E_s-L_s$ ) relation follows naturally from that between  $E_s$  and  $b_s$ 

•the low  $L_{inj}$  objets (Mrk 501 vs Mrk 421) reach a larger  $E_S$ , compatibly with larger  $\gamma_{eq}$ 

- Mrk 421 MAGIC data on 2006 match very well the Synchrotron prediction with simultaneous X-ray data
- •the average index of the trend  $L_s \propto E_S^{\alpha}$  with  $\alpha \sim 0.6$ , is compatible with the data, and with a scenario in which a typical constant energy  $(L_{ini} \times t_{ini})$  is injected for any flare (jet-feeding problem), whilst the peak dynamic is ruled by the turbulence in the magnetic field.

# backup slides

#### self-consistent approach: acc+cooling





•R~  $10^{13}$ - $10^{15}$  cm • $\delta$ B/B<<1 , B~[0.01-1.0] G • $\beta_A \sim 0.1$ -0.5 • $\lambda_{max}$ <R => ~  $10^{[9-15]}$  cm • $\rho_g$ < $\lambda_{max}$  =>  $\gamma_{max} \sim 10^{7.5}$ 



#### Flare: acc.-dominated-vs-equil.,R= 10<sup>15</sup> cm, q=2



mono energetic inj., t<sub>inj</sub><<t<sub>acc</sub>, t<sub>inj</sub><<t<sub>sim</sub>
we measure r@peak as a function of the time
two phase: acceleration-dominated, equilibrium
equil. distribution:

•f=1 for q=2 and S, full TH, or full KN
•equil. curv.: r~2.5, (r<sub>3p</sub>~6.0) for TH or full KN
•equil. curv.: r~0.6, (r<sub>3p</sub>~4.0) for TH-KN

$$n(\gamma) \propto \gamma^2 \exp\left[\frac{-1}{f(q,\dot{\gamma})} \left(\frac{\gamma}{\gamma_{eq}}\right)^{f(q,\dot{\gamma})}\right]$$



# BH $R \le c \Delta t / (1 + z)$ R~R<sub>g</sub> • M<sub>BH</sub> disk/jet feeding



**\**max

ρg

R

Jet

 $R \le c \Delta t \delta / (1+z)$ 



### *E<sub>s</sub>-b<sub>s</sub>* X-ray trend and $\gamma$ -ray predictions



- •data span 13 years, both flaring and quiescent states
- •We are able to reproduce these long-term behaviours, by changing the value of only one parameter (q)
- •curvature values imply distribution far from the equilibrium (b~[0.7-1.0])
- •More data needed at GeV/TeV, curvature seems to be cooling-dominated

$L_{\rm inj}$ (E <sub>s</sub> -b <sub>s</sub> tren	nd) (erg $s^{-1}$ )	$5 \times 10^{39}$
$L_{\text{inj}}$ ( $E_s$ – $L_s$ trend) (erg s <sup>-1</sup> )		$5 \times 10^{38}, 5 \times 10^{39}$
q		[3/2, 2]
$t_A$	(s)	$1.2 \times 10^{3}$
$t_{D_0} = 1/D_{P0}$	(s)	$[1.5 \times 10^4, 1.5 \times 10^5]$
$T_{\rm inj}$	(s)	10 <sup>4</sup>
$T_{\rm esc}$	(R/c)	2.0

#### HBLs case



#### acceleration signature in the Es-vs-Ls trend



## **SEDs** evolution



#### Strong cooling



•Full bands curvature related to EED broadness, acceleration signature

•High energy band, dominated by cooling, moving towards the equilibrium



## Moving Ep above 30 keV

Low cooling

#### Strong cooling

3.0



## Effect o B on SEDs



## Rapid Variability





#### acceleration signature in the Es-vs-Ls trend







#### $D_p$ -driven trends $t_{D=}[1.5x10^4-1.5x10^5]$ , $L_{inj}$ =const.





## effect of $\lambda_{max}$ , $\lambda_{coher}$

