The Cusp and other lessons from NA48/2

Gino Isidori
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- How large can direct CPV in $K^{\pm} \to (3\pi)^{\pm}$ be?
- The $K \to 3\pi$ cusp
- The most important lessons from NA48/2

How large can direct CPV in $K^{\pm} \to (3\pi)^{\pm}$ be?

- After many years spent investigating CP violation in the neutral kaon sector, *Italo's dream* was finding a an evidence of matter-antimatter asymmetry with charged kaons.

- On general grounds, if CP violation is mediated by $\Delta S=1$ interactions (as predicted in the SM), we should observe a difference in partial decay widths (or kinematical distributions) of K^+ and K^- decays.

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- On general grounds, if CP violation is mediated by $\Delta S=1$ interactions (as predicted in the SM), we should observe a difference in partial decay widths (or kinematical distributions) of K^+ and K^- decays.

- The most promising candidate are the slope asymmetries in $K \to 3\pi$:

$$|A(K\to 3\pi)| \propto 1+gY+jX+hY^2+kX^2+\dots \qquad Y=(s_3-s_0)/M_\pi^2 \quad X=(s_1-s_2)/M_\pi^2$$

$$(\Delta g)_+ = \frac{g_{++-}-g_{--+}}{g_{++-}+g_{--+}} \neq 0 \quad \longleftrightarrow \quad \text{(direct) CPV}$$

- How large these asymmetries can be, both within & beyond the SM, is a problem that kept some of us busy for quite some time...

How large can direct CPV in $K^{\pm} \to (3\pi)^{\pm}$ be?

1989

Prediction of Direct {CP} Violation for $K \to 3\pi$ Decays in Chiral Perturbation Theory

A.A. Belkov, A.V. Lanyov (Serpukhov, IHEP), G. Bohm (DESY, Zeuthen), D. Ebert (Dubna, JINR)

Published in **Phys.Lett. B232 (1989) 118-122**

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CP violation in K+- ---> 3 pi decays and lattice QCD B factors
Gino Isidori, Luciano Maiani, Alessandra Pugliese (Rome U. & INFN, Rome). Dec 1991
Published in **Nucl.Phys. B381 (1992) 522-543**

My first two publications... !

→ we proved Δg can be at most ~ few×10^{-5} in the SM

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1999 **Direct CP violation in K ---> 3 pi decays induced by SUSY chromomagnetic penguins**
G. D'Ambrosio (INFN, Naples & Naples U.), G. Isidori (Frascati), G. Martinelli (Orsay, LAL). Nov 1999.
Published in **Phys.Lett. B480 (2000) 164-170**

→ $\Delta g \sim 10^{-4}$ possible beyond the SM (*with some fine-tuning in ε'*)

How large can direct CPV in $K^{\pm} \to (3\pi)^{\pm}$ be?

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN/SPSC 2000-003
CERN/SPSC/P253 add.3
January 25, 2000

ADDENDUM III
(to Proposal P253/CERN/SPSC)
for a Precision Measurement of Charged Kaon Decay Parameters with an Extended NA48 Setup

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Italo's dream was not too much influenced by our theoretical work.

He was still convinced it was important to look on real data (*ask Nature, not theoreticians...*) how much K+ and K- decay distributions can differ.

He was indeed one of the main proponents of a beautiful experiment (NA48/2) addressing this question...

How large can direct CPV in $K^{\pm} \rightarrow (3\pi)^{\pm}$ be?

… that despite didn't observed CPV violation in $K \rightarrow 3\pi$,

did produced very interesting results, among which the fascinating (*and totally unexpected...*) discovery of the cusp effect:

Top-cited papers by NA48/2:

1. **Observation of a cusp-like structure in the pi0 pi0 invariant mass distribution from K+- ---> pi+- pi0 pi0 decay and determination of the pi pi scattering lengths** (165)
   NA48/2 Collaboration (J.R. Batley (Cambridge U.) *et al.*). Nov 2005. 16 pp.
   Published in **Phys.Lett. B633 (2006) 173-182**
   DOI: 10.1016/j.physletb.2005.11.087
   e-Print: **hep-ex/0511056 | PDF**
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   CERN Document Server; ADS Abstract Service
   Detailed record - Cited by 165 records 100+

2. **Search for direct CP violating charge asymmetries in K+- ---> pi+- pi+ pi- and K+- ---> pi+- pi0 pi0 decays** (152)
   NA48/2 Collaboration (J.R. Batley (Cambridge U.) *et al.*). Jun 2007. 28 pp.
   Published in **Eur.Phys.J. C52 (2007) 875-891**
   CERN-PH-EP-2007-021
   DOI: 10.1140/epjc/s10052-007-0456-7
   e-Print: **arXiv:0707.0697 [hep-ex] | PDF**
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   CERN Document Server; ADS Abstract Service
   Detailed record - Cited by 152 records 100+

3. **New high statistics measurement of K(e4) decay form factors and pi pi scattering phase shifts** (134)
   NA48/2 Collaboration (J.R. Batley (Cambridge U.) *et al.*). Oct 2007. 24 pp.
   Published in **Eur.Phys.J. C54 (2008) 411-423**
   CERN-PH-EP-2007-035
   DOI: 10.1140/epjc/s10052-008-0547-0
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
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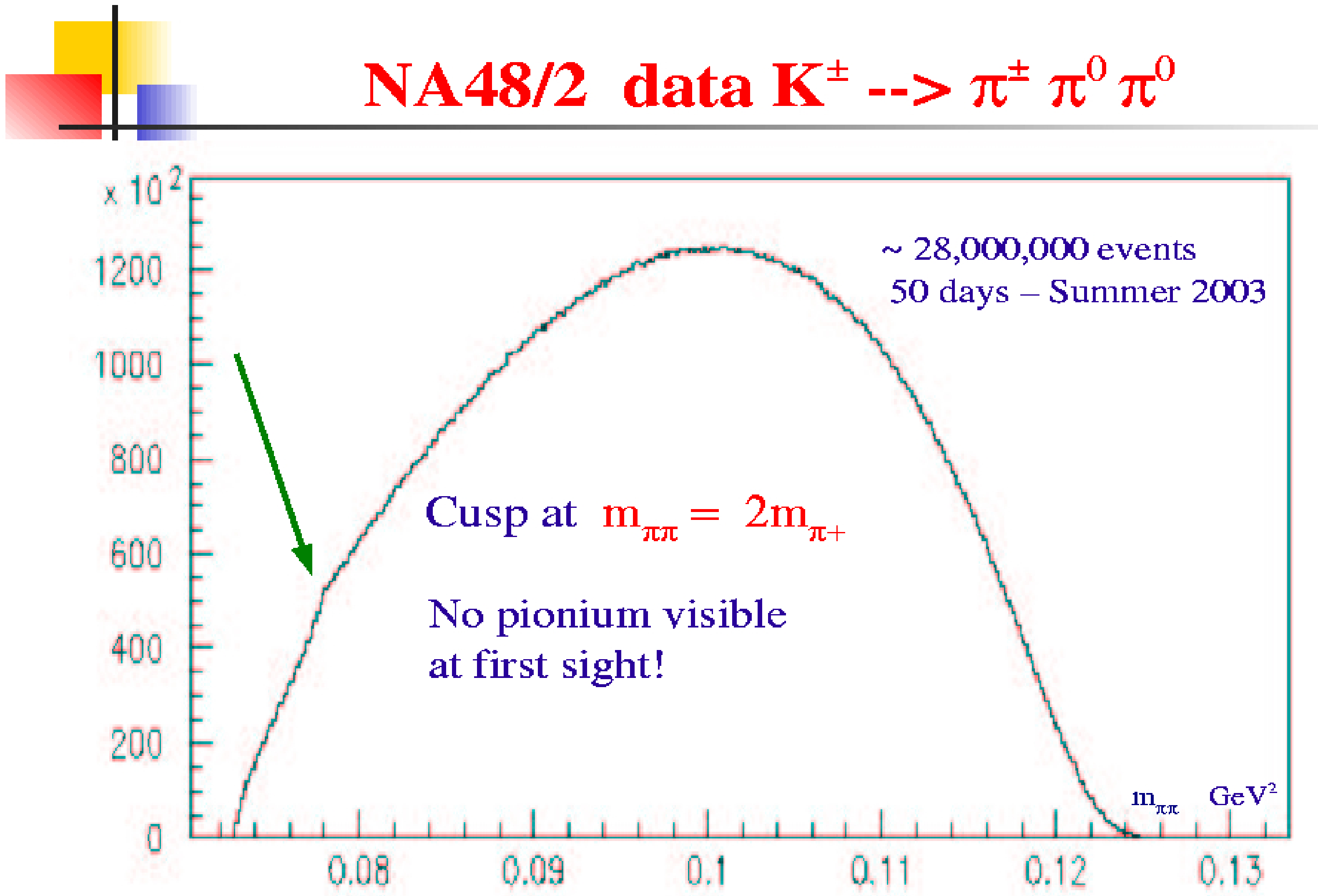
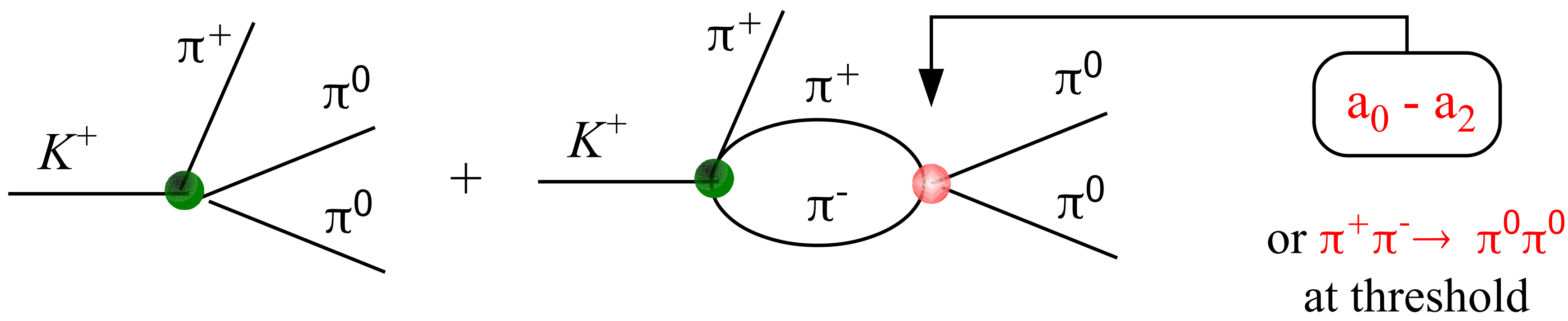
The $K \to 3\pi$ cusp



I. Mannelli, CERN March '05

The $K \rightarrow 3\pi$ cusp

The origin of this discontinuity is due to a re-scattering effect, as pointed out by Nicola Cabibbo [*a "young" member of NA48/2 at that time...*] as soon as he looked at the data:



The decay amplitude is an analytic function of the (neutral) di-pion invariant mass $s = (M_{\pi^0\pi^0})^2$, but the existence of a real intermediate state implies a discontinuity across the real axis for $s > s_0 = (2m_{\pi^+})^2$

$$T(s+i\varepsilon) - T(s-i\varepsilon) = i\,\rho_{\pi\pi}(s)\, T_{K\rightarrow 3\pi}(s)\, T_{\pi\pi\rightarrow\pi\pi}(s)\, \Theta(s-s_0)$$

$\rightarrow$ $v_{\pi+\pi-}(s) \sim (s-s_0)^{1/2}$

$\pi+\pi-$ phase-space

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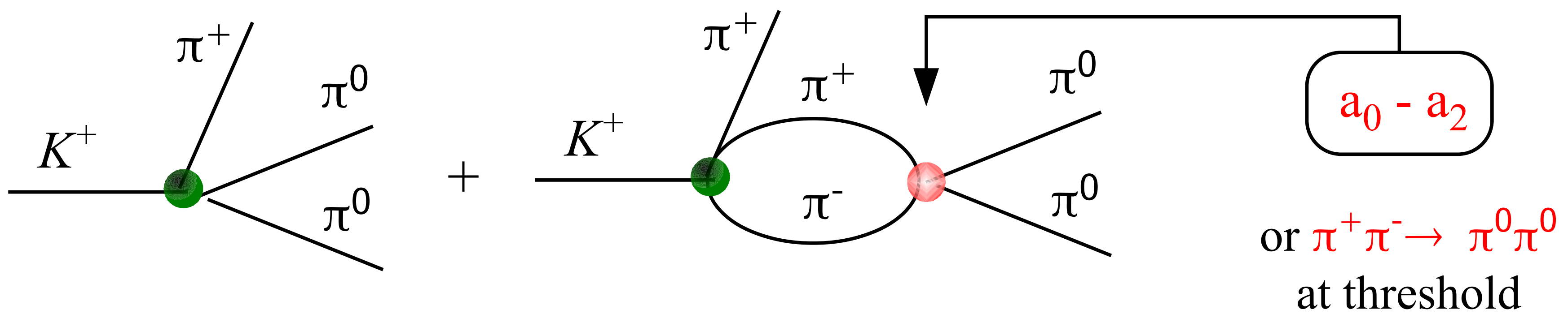

$s > s_0$ $\quad T(s) = A(s) + B(s)\,(s-s_0)^{1/2}$

Analytic continuation below the threshold

$s < s_0$ $\quad T(s) = A(s) + i\,B(s)\,(s_0-s)^{1/2}$

A(s) & B(s) regular around s_0

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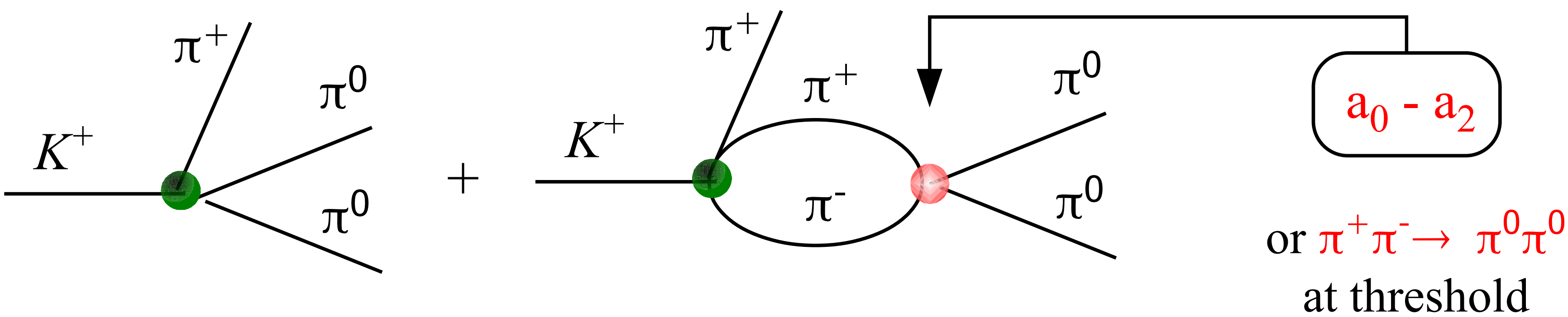

$s > s_0$ $\quad T(s) = A(s) + B(s)\,(s-s_0)^{1/2}$

$$|T(s)|^2 = [\, \mathrm{Re}A(s) + \mathrm{Re}\,B(s)\,(s-s_0)^{1/2}\,]^2 + [\, \mathrm{Im}A(s) + \mathrm{Im}\,B(s)\,(s-s_0)^{1/2}\,]^2$$

$s < s_0$ $\quad T(s) = A(s) + i\,B(s)\,(s_0-s)^{1/2}$

$$|T(s)|^2 = [\, \mathrm{Re}A(s) - \mathrm{Im}\,B(s)\,(s_0-s)^{1/2}\,]^2 + [\, \mathrm{Im}A(s) + \mathrm{Re}\,B(s)\,(s_0-s)^{1/2}\,]^2$$

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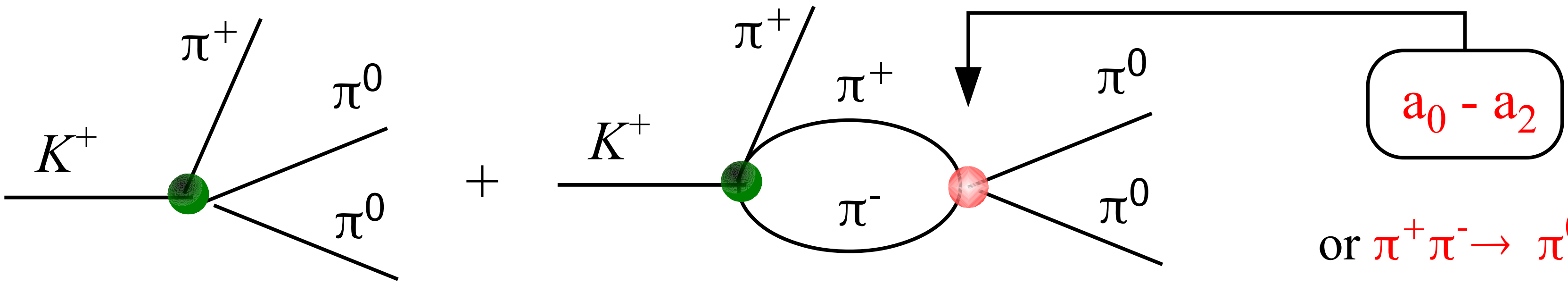


or $\pi^+\pi^- \rightarrow \pi^0\pi^0$ at threshold

$s > s_0$ $\quad T(s) = A(s) + B(s)\,(s-s_0)^{1/2}$

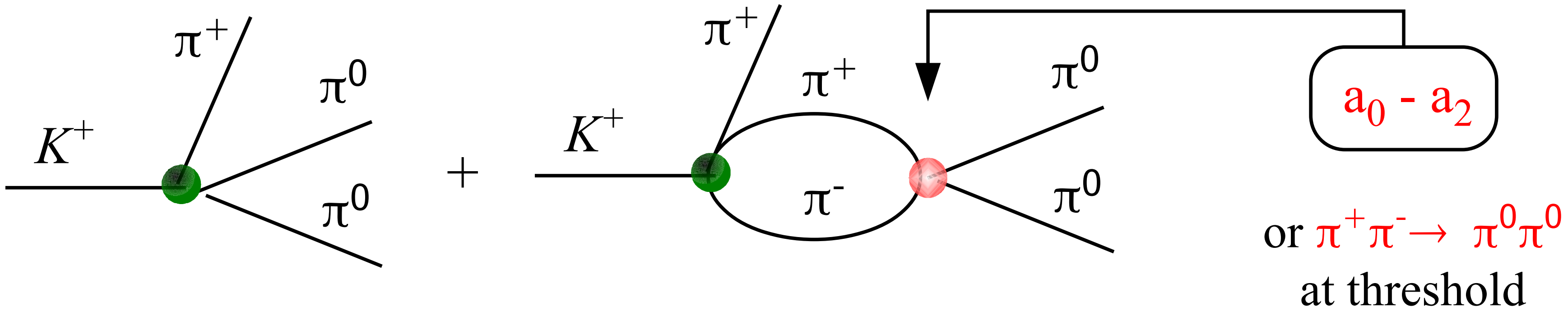
$$\rightarrow |T(s)|^2 = [\,\mathrm{Re}A(s) + \cancel{\mathrm{Re}\,B(s)\,(s-s_0)^{1/2}}\,]^2 + [\,\cancel{\mathrm{Im}A(s)} + \mathrm{Im}\,B(s)\,(s-s_0)^{1/2}\,]^2$$

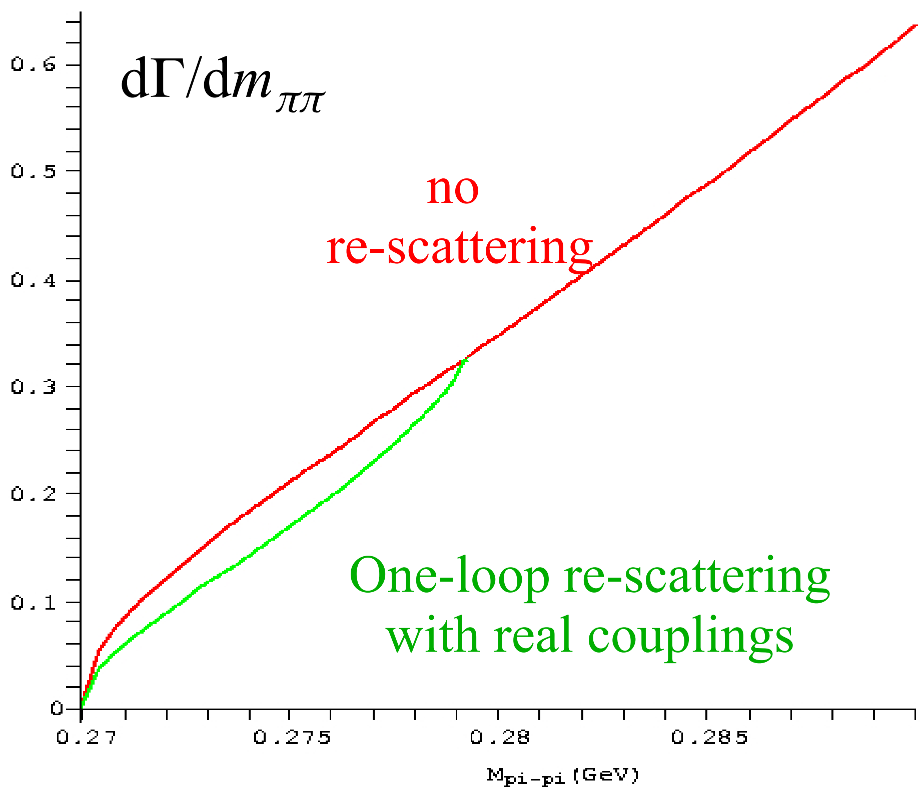
$s < s_0$ $\quad T(s) = A(s) + i\,B(s)\,(s_0-s)^{1/2}$

$$\rightarrow |T(s)|^2 = [\,\mathrm{Re}A(s) - \mathrm{Im}\,B(s)\,(s_0-s)^{1/2}\,]^2 + [\,\cancel{\mathrm{Im}A(s) + \mathrm{Re}\,B(s)\,(s_0-s)^{1/2}}\,]^2$$

in the limit where A(s) = real & B(s) = imaginary (real one-loop couplings)
we have a square-root behavior below the threshold

The $K \to 3\pi$ cusp





It then become clear this effect [*the Cabibbo-Mannelli effect*] could be a new way to measure $\pi\pi$ scattering lengths

Provided we were able to describe it in detail, also beyond the one-loop level

The $K \to 3\pi$ cusp

Why are $\pi\pi$ scattering lengths interesting?

- Very accurate (unique) prediction by means of Chiral Perturbation Theory:

$$a_0\, m_\pi = 0.220 \pm 0.005 \qquad a_2\, m_\pi = -0.0445 \pm 0.0010$$
$$(a_0 - a_2)\, m_\pi = 0.265 \pm 0.004$$

1.5% relative error !

Weinberg '79
⋮
Colangelo *et al.* '00-'01

- Their smallness (and their accurate prediction) is a direct consequence of the pseudo-Goldstone boson nature of the pions

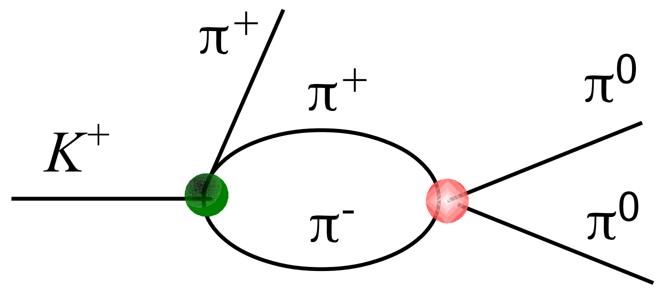
- The only way to modify these predictions is to modify the basic assumptions of CHPT → different values can be obtained assuming a more complicated structure for the QCD vacuum (in particular assuming m_π^2 is not linear in m_q)

$$m_\pi^2 = (m_u + m_d)\, \frac{\langle 0 |\, \bar{q}_L q_R \,| 0 \rangle}{F_\pi^2} + O(m_q^2)$$

The measure of the a_I is a direct probe of the QCD vacuum !

The $K \to 3\pi$ cusp

Toward a precise prediction of the cusp effect (I):



A full calculation of $K \to 3\pi$ within CHPT is not very useful:

- slow convergence of the chiral expansion
- too many free parameters

...but we don't need to compute the full decay amplitude !

- possible to perform a systematic expansion in powers of the a_I of the amplitudes which determine the coefficient of the singularity

Ad hoc construction which maximize the available experimental info on $K \to 3\pi$ and use only:

- Unitarity & analyticity
- Smallness of the a_I
- Smallness of $v_{\pi\pi} = (s-s_0)^{1/2}$

Cabibbo & G.I., JHEP '05

$$T(s) = A(s) + B(s)\,(s-s_0)^{1/2}$$

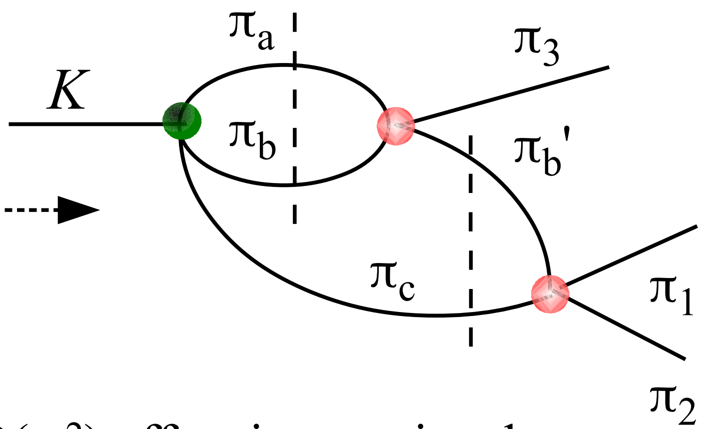
A(s) & B(s) regular around s_0

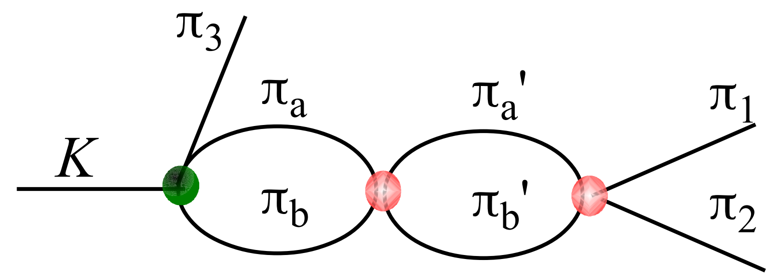
$$T(s) = A(s) + B(s)\,(s-s_0)^{1/2}$$

$\mathrm{Re}A(s) = O(1)$ exp. data
$\mathrm{Im}A(s) = O(a_I)$ one-loop

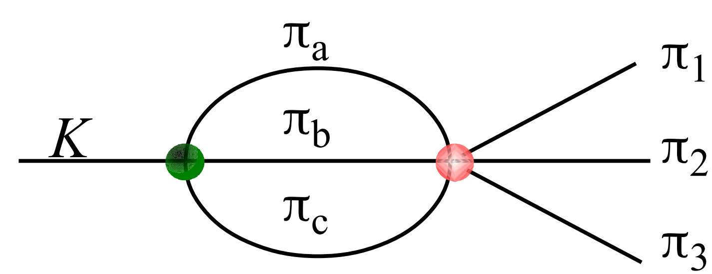
$\mathrm{Im}B(s) = O(a_I)$ one-loop
$\mathrm{Re}B(s) = O(a_I^2)$ two-loop

relevant 2-loop topologies:



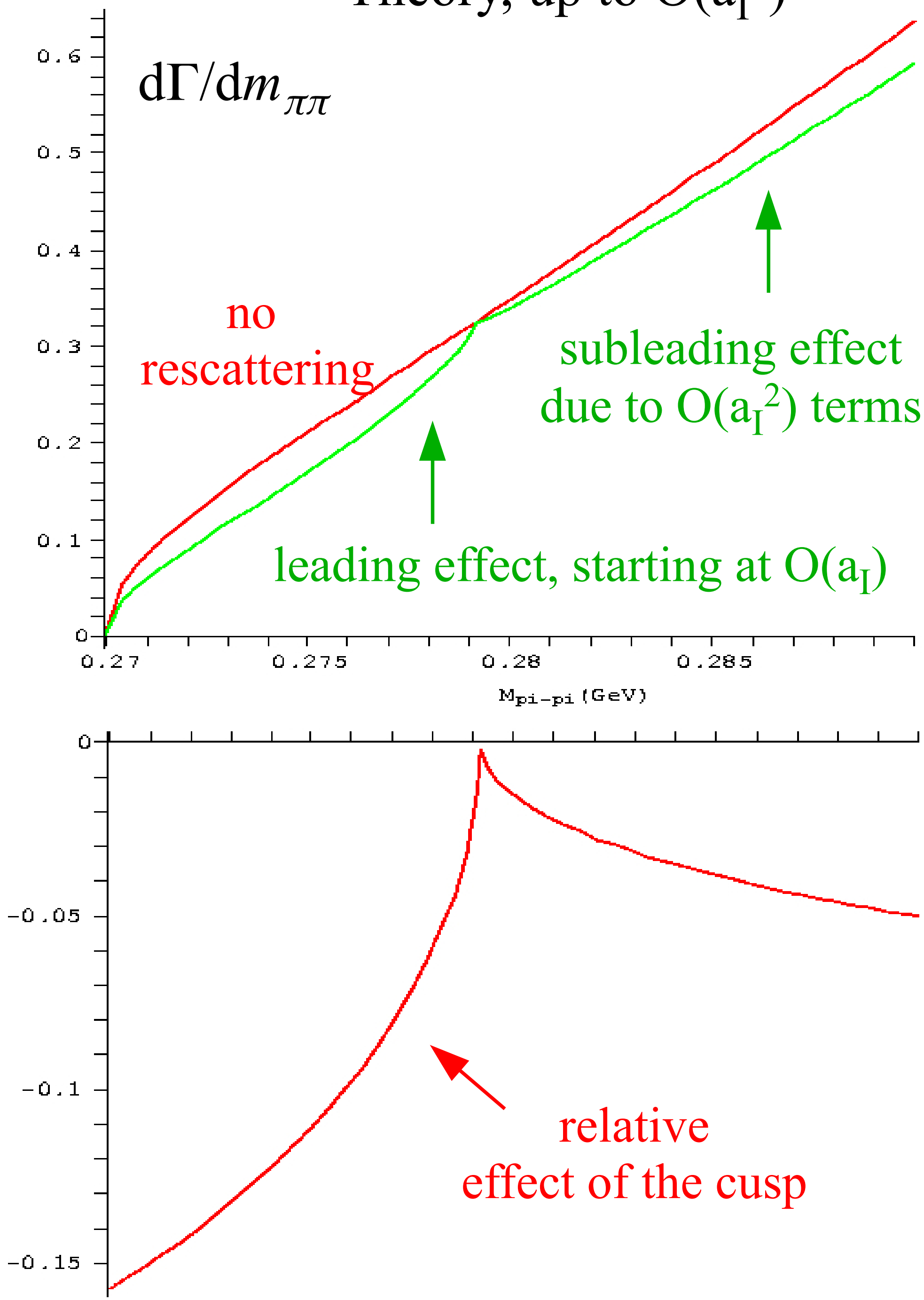


The largest $O(a_I^2)$ effect is associated to this double cut (new phase due to pions with "large" relative velocity)

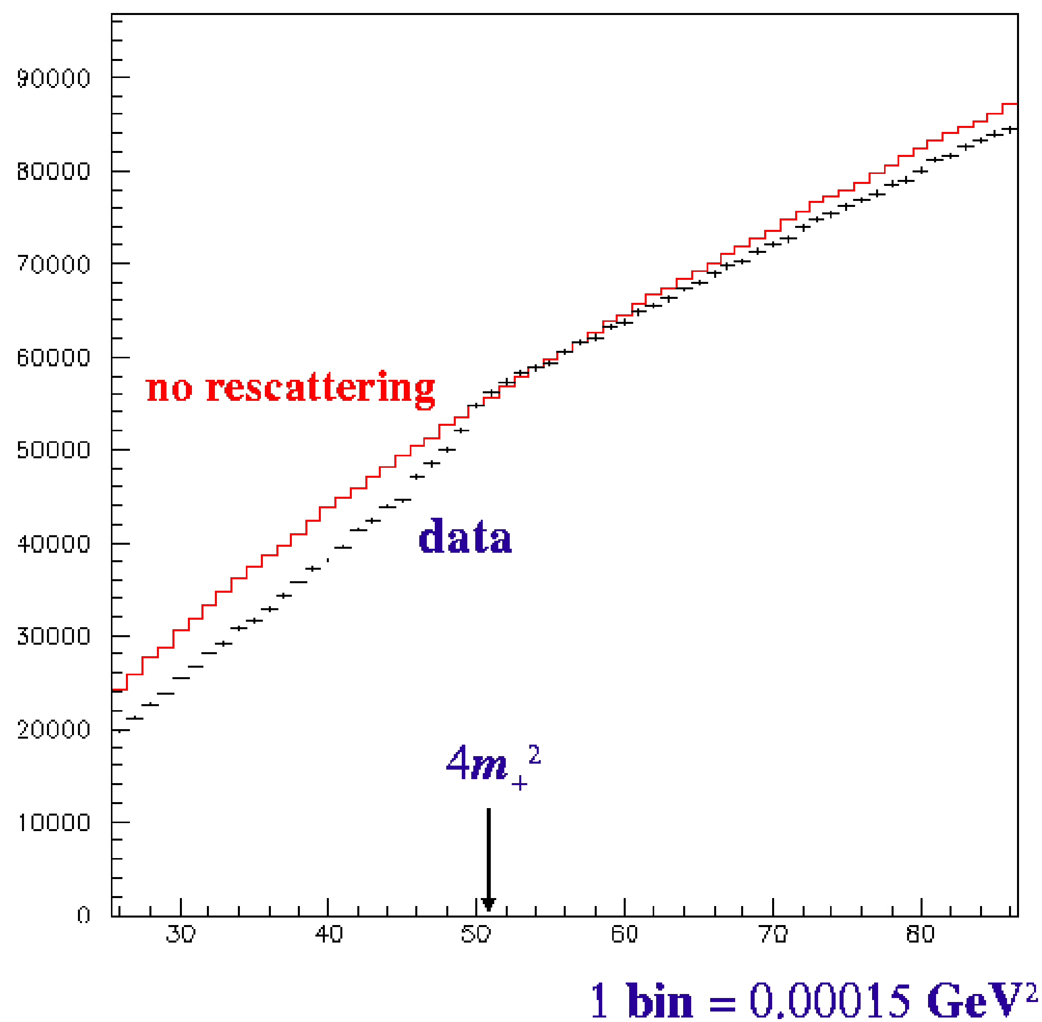


N.B.: at $O(a_I^2)$ the parameterization depends in a different way from (a_0-a_2) & a_2 : ➡ possible to determine both combinations !

Theory, up to $O(a_I^2)$



NA48 data



The $K \rightarrow 3\pi$ cusp

Toward a precise prediction of the cusp effect (II):

- Our analysis of the cusp effect was done in an effective theory where m_{π^+}-m_{π^0} (*and soft photons*) are the only source of isospin breaking ⇒ a more accurate treatment require the systematic inclusions of e.m. corrections beyond bremsstrahlung

Bissegger, Fuhrer, Gasser, Kubis, Russetsky '09

- E.m. effects turns out to produce corrections which can effectively be re-absorbed into a shift of the a_I except very close to the $\pi^+\pi^-$ threshold, where they are responsible for the formation of a bound state: pionium [*this is one of the reasons why a consistent inclusion of e.m. effect took quite some time...*]

Probability of pionium formation:

$$\frac{\Gamma(K^+ \rightarrow \pi^+ + A_{\pi\pi})}{\Gamma(K^+ \rightarrow \pi^+ \pi^+ \pi^-)} \approx 7.4 \times 10^{-6}$$

Silagadze '94

Life-time of the pionium:

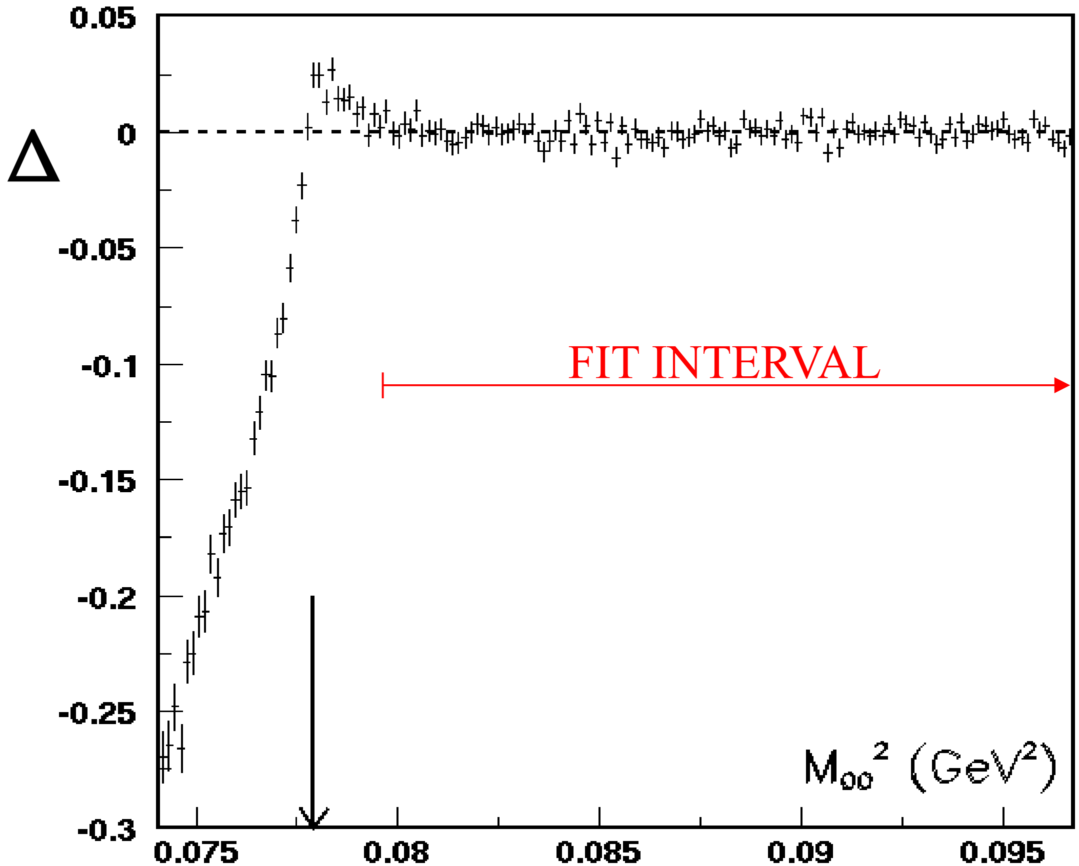
$$\Gamma(A_{\pi\pi}) \approx 3 \times 10^{-15}\,\mathrm{s}$$

Gasser *et al.* '01

▸ *The $K \to 3\pi$ cusp*: a closer look to NA48/2 data [*courtesy of Luigi di Lella*]

$\Delta \equiv$ (data – fit) /data versus M_{oo}^2

Without the re-scattering effect....



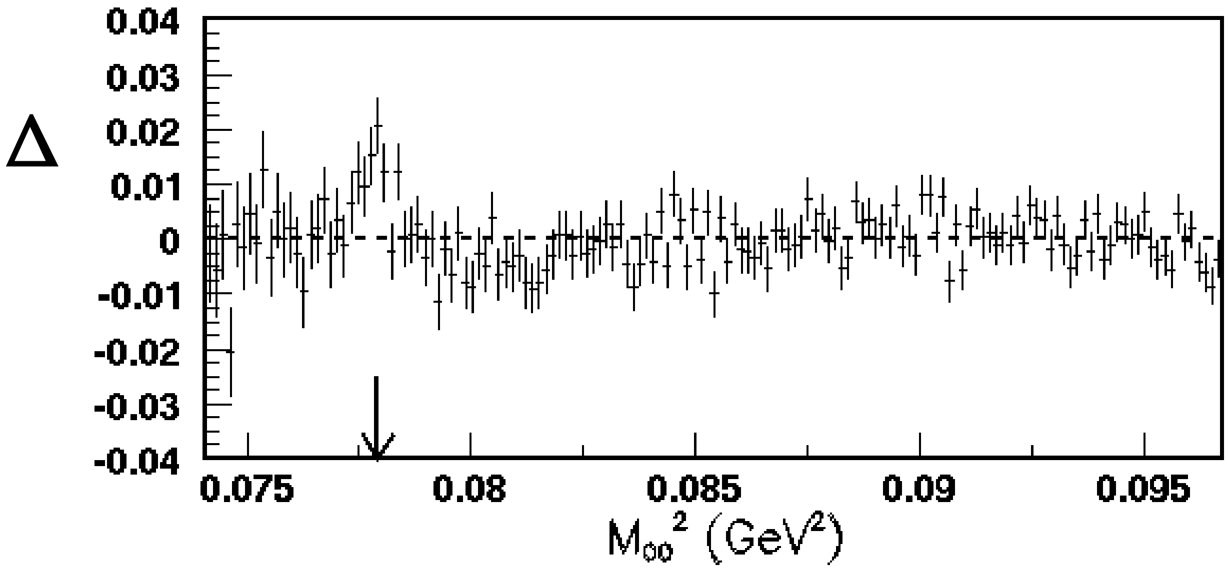
$\chi^2 =$ **13574 / 148 d.o.f.** $\Rightarrow$ **120 / 110 d.o.f.**

▶ *The $K \to 3\pi$ cusp*: a closer look to NA48/2 data [*courtesy of Luigi di Lella*]

$\Delta \equiv$ (data – fit) /data versus M_{oo}^2

One-loop parameterization [Cabibbo, P.R.L. '04]

One additional parameter with respect to the PDG fit: $(a_0-a_2)m_\pi$



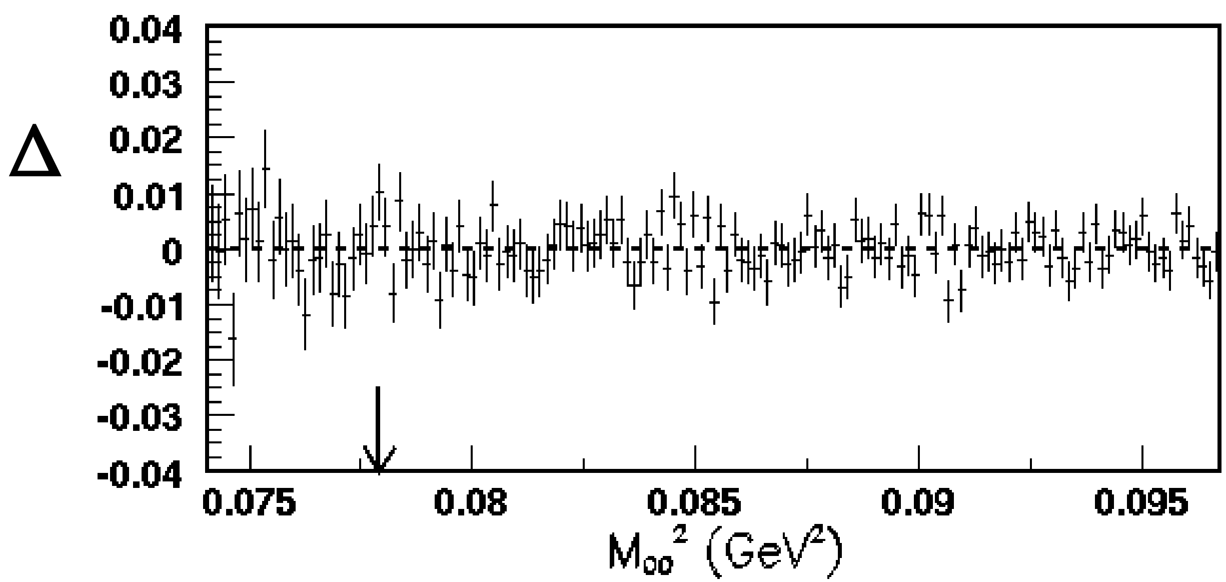
$\chi^2 = 217 \ / \ 147$ d.o.f.

▶ *The $K \rightarrow 3\pi$ cusp*: a closer look to NA48/2 data [*courtesy of Luigi di Lella*]

$\Delta \equiv$ (data – fit) /data versus M_{oo}^2

Two-loop parameterization [Cabibbo & G.I., JHEP '05]

Two additional parameters with respect to the PDG fit: $(a_0-a_2)m_\pi$ & a_2m_π



$\chi^2 =$ 156 / 146 d.o.f.

▶ *The $K \to 3\pi$ cusp*: a closer look to NA48/2 data [*courtesy of Luigi di Lella*]

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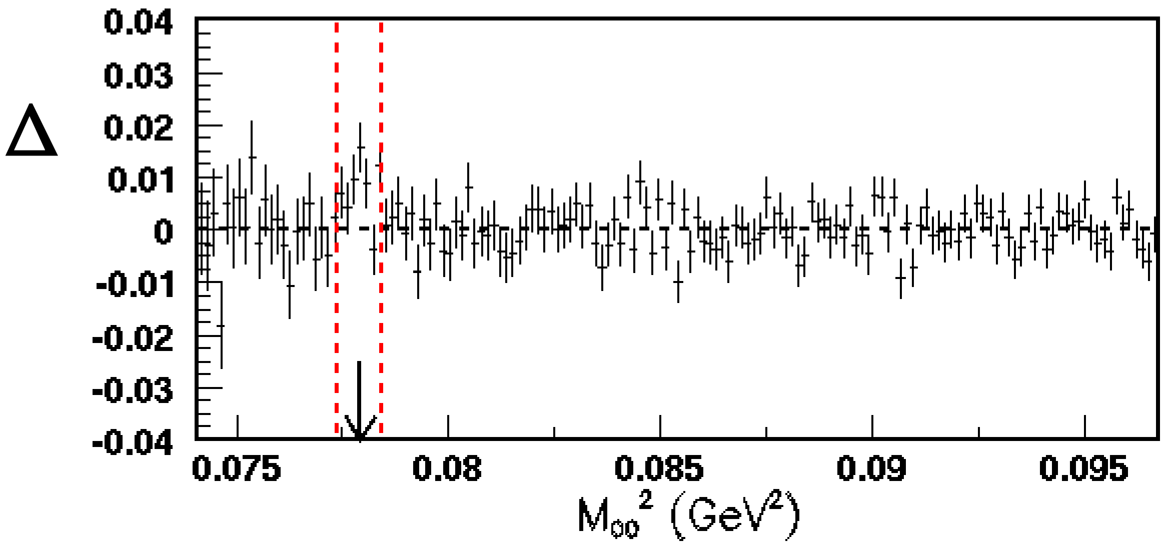
Fit excluding 7 bins centered around $M_{00} = 2m_{\pi^+}$



Excess of events in excluded bins ⇒ evidence for pionium

Statistical significance ~ 2.5 σ
[agreement with th. expectations]

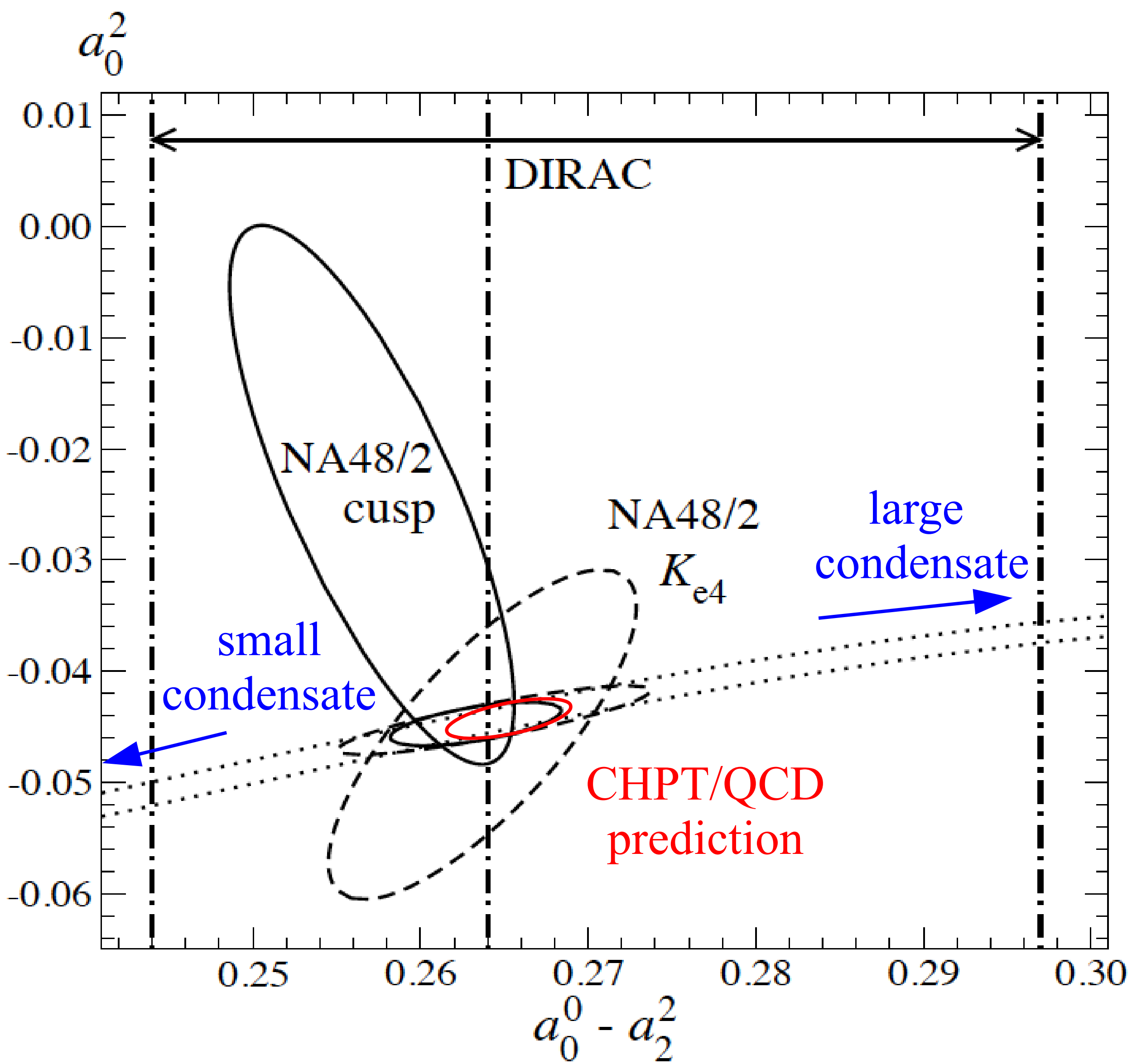
χ^2 = 141 / 139 d.o.f. ⇒

$$a_0^0 - a_0^2 = 0.2571 \pm 0.0048_{\rm stat} \pm 0.0025_{\rm syst} \pm 0.0014_{\rm ext}$$
$$a_0^2 = -0.024 \pm 0.013_{\rm stat} \pm 0.009_{\rm syst} \pm 0.002_{\rm ext}$$

Values corrected for e.m. effects [Bissiger *et al.* '09]

The $K \to 3\pi$ cusp

Final impact of NA48/2 in the determination of $\pi\pi$ scattering lengths:



Bastian Kubis '09
Cusp effects in meson decays

- *The most important lessons from NA48/2*
  [*at least from my perspective...*]:

  - The importance of "having a vision", possibly challenging current beliefs, and able to inspire others [*"Italo's dream"...*]

  - The importance of a close collaboration between theory and experiments [*The "Cabibbo-Mannelli" effect...*]

  - The most interesting results are the unexpected ones... [*The cusp effect was true discovery !*]