The 3D structure of nucleons

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The exploration of the 3-dimensional structure of the nucleon, both in momentum and in configuration space, is one of the major issues in high energy hadron physics.

Information on the 3-dimensional structure of the nucleon is embedded in the Transverse Momentum Dependent distribution and fragmentation functions (TMDs).

In a very simple phenomenological approach, hadronic cross sections and spin asymmetries are generated, within a QCD factorization framework, as convolutions of distribution and (or) fragmentation functions with elementary cross sections.

This simple approach can successfully describe a wide range of experimental data.
Intrinsic Transverse Momentum

We cannot learn about the spin structure of the nucleon without taking into account the transverse motion of the partons inside it.

Transverse motion is usually integrated over, but there are important spin-$k_\perp$ correlations which should not be neglected.

Several theoretical and experimental evidences for transverse motion of partons within nucleons, and of hadrons within fragmentation jets.
Where can we learn about the 3D structure of matter?
Where can we learn about the 3D structure of matter?
Experimental data for TMD studies

Unpolarized and Polarized SIDIS scattering

\[ \sigma_{\text{SIDIS}} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z) \]

Allows the extraction of TMD distribution and fragmentation functions
**Experimental data for TMD studies**

Unpolarized and Polarized Drell-Yan scattering

\[
\sigma_{\text{Drell-Yan}} = f_q(x, q^2) \otimes f_{\bar{q}}(x, q^2) \otimes \hat{\sigma} \otimes q \rightarrow q\bar{q}
\]

Allows extraction of distribution functions

Unpolarized and Polarized SIDIS scattering

\[
\sigma_{\text{SIDIS}} = f_p(x) \otimes \hat{\sigma} \otimes D_{h_1h_2}(z)
\]

\[
\sigma_{h_1 h_2} \propto D(1) \otimes D(2) \otimes \hat{\sigma}
\]

Allows extraction of distribution and fragmentation functions

Polarized Drell-Yan scattering

\[
e^+ e^- \rightarrow h_1 h_2 X
\]

Polarized SIDIS scattering

\[
e^+ e^- \rightarrow h_1 h_2 X
\]

Allows extraction of fragmentation functions
From the theory point of view ...

<table>
<thead>
<tr>
<th>Perturbative QCD</th>
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<td>Parton model</td>
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<tr>
<td>Factorization theorems</td>
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</table>

TMD factorization holds at large $Q^2$ and $P_T \approx k_\perp \approx \lambda_{QCD}$

Two scales: $P_T \ll Q^2$

(Collins, Soper, Ji, Ma, Yuan, Qiu, Vogelsang, Collins, Metz)

\[
\frac{d \sigma^{\ell(S_t)+p(S)\rightarrow \ell'+h+X}}{d x_B d Q^2 d z_H d^2 P_T d \phi_S} = \rho_{\lambda_{\ell_1}\lambda_{\ell_2}}^{q/p,S} \otimes \rho_{\lambda_{q_1},\lambda_{q_2}}^{q/p,S} f_{q/p,S}(x, k_\perp) \otimes M_{\lambda_\ell,\lambda_q;\lambda_{\ell'},\lambda_{q'}} \hat{M}^{*}_{\lambda_{\ell'},\lambda_{q'};\lambda_{\ell},\lambda_{q}} \otimes D_{\lambda_{q},\lambda_{q'}}^h \lambda'(z, p_\perp)
\]

TMD-PDF | hard scattering | TMD-FF
**Phenomenology**

- **THEORY**
  - Perturbative QCD
  - Factorization theorems
  - Resummation
  - ...  

- **PHENOMENOLOGY**
  **Mission**: devise simple, flexible and efficient models to link THEORY with EXPERIMENTS

- **EXPERIMENTS**
  - Drell-Yan scattering
  - Di-hadron production from $\text{e}^{+}\text{e}^{-}$ scattering
  - DIS and SIDIS processes
  - Inclusive single particle production from hadronic scattering
**Phenomenology**

The blind men and the elephant
from H. Avakian

Experiments → Blind men
Several different experiments measuring the same observable, with limited coverage

Phenomenology → “where everything comes together nicely”
Combine different sources of information to get the whole picture
How can we learn about the 3D structure of matter?
The mechanism which describes how quarks and gluons are bound into hadrons is embedded in the parton distribution and fragmentation functions (PDFs and FFs), the so-called “soft parts” of hadronic scattering processes. These are non-perturbative objects which connect the ideal world of pointlike and massless particles (pQCD) to our much more complex real world, made of nucleons, nuclei and atoms.

**PDFs versus TMDs**
Collinear parton distribution functions

Unpolarized distribution functions

\[ q = q_+ + q_- \]
\[ g = g_+ + g_- \]

Helicity distribution functions

\[ \Delta q = q_+ - q_- \]
\[ \Delta g = g_+ - g_- \]

Transversity distribution functions

\[ \Delta_T q = q_+ - q_- \]
TMD distribution and fragmentation functions

Distribution

$\frac{N}{\text{twist 2}}$

$f_1^q(x, k_T^2)$

$h_{1T}^q(x, k_T^2)$

$h_{1L}^q(x, k_T^2)$

$h_{1T}^{\perp}(x, k_T^2)$

$h_{1L}^{\perp}(x, k_T^2)$

$g_1^q(x, k_T^2)$

$g_{1L}^q(x, k_T^2)$

Fragmentation

$D_1^q(z, p_T^2)$

$H_1^{\perp}(z, p_T^2)$

Correlations between spin and transverse momentum
The Sivers function is related to the probability of finding an unpolarized quark inside a transversely polarized proton. It embeds the correlation between the proton spin and the quark transverse momentum. The Sivers function is $T$-odd.

The Sivers function is particularly interesting, as it provides information on the partonic orbital angular momentum.
The Collins function

\[ D_{h/q,s_q}(z, p_\perp) = D_{h/q}(z, p_\perp) + \frac{1}{2} \Delta^N D_{h/q}^{\uparrow}(z, p_\perp) s_q \cdot (\hat{p}_q \times \hat{p}_\perp) \]

\[ = D_{h/q}(z, p_\perp) + \frac{p_\perp}{z M_h} H_{1,q}^{\perp}(z, p_\perp) s_q \cdot (\hat{p}_q \times \hat{p}_\perp) \]

The Collins function is related to the probability that a transversely polarized struck quark will fragment into a spinless hadron.

The Collins function embeds the correlation between the fragmenting quark spin and the transverse momentum of the produced hadron.

The Collins function is chirally odd.
Polarized TMDs are best studied in polarized processes, most commonly they are extracted from spin or azimuthal asymmetries.

However, to compute these asymmetries in a reliable way, we must be able to reproduce the unpolarized cross sections in the best possible way, over the largest possible range in $q_T$. 
Unpolarized TMDs ...
... where it all begins
Calculating a cross section which describes a hadronic process over the whole $q_T$ range is a highly non-trivial task.

Let's consider Drell Yan processes (for historical reasons).

Fixed order calculations cannot describe DY data at small $q_T$:

At Born Level the cross section is vanishing
At order $\alpha_s$ the cross section is divergent...

\[ q_T \rightarrow 0 \]

\[ \frac{1}{\sigma_0} \frac{d\sigma}{dq_T^2} = \frac{2C_F}{2\pi q_T^2} \alpha_s \ln \left( \frac{M^2}{q_T^2} - \frac{3}{2} \right) \]
Naive TMD approach

\[
\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp\left(-P_T^2/\langle P_T^2 \rangle\right)}{\pi \langle P_T^2 \rangle}
\]

- Considering the same DY process at different energies:

Each data set is Gaussian but with a different width
Drell-Yan phenomenology

Does the $q_T$ distribution behave like a Gaussian?

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp\left(-\frac{P_T^2}{\langle P_T^2 \rangle}\right)}{\pi\langle P_T^2 \rangle}$$

Clearly this is not a Gaussian tail!
Drell-Yan phenomenology

Fixed order calculations cannot describe correctly DY cross sections at small $q_T$

DY cross sections do not show a Gaussian behaviour at large $q_T$
Fixed order calculations cannot describe correctly DY/SIDIS data at small $q_T$.

$$\frac{1}{\sigma_0} \frac{d\sigma}{dq_T^2} = \frac{2C_F}{2\pi q_T^2} \alpha_s \ln \left( \frac{M^2}{q_T^2} - \frac{3}{2} \right)$$

These divergencies are taken care of by TMD evolution/resummation.

The cross section is written in $b_T$ space:

$$\frac{1}{\sigma_0} \frac{d\sigma}{dQ^2 dy dq_T^2} = \int \frac{d^2 b_T e^{i q_T \cdot b_T}}{(2\pi)^2} \sum_j e_j^2 W_j(x_1, x_2, b_T, Q) + Y(x_1, x_2, q_T, Q)$$
The W term is designed to work well at low and moderate $q_T$, when $q_T \ll Q$. (Actually, W is devised to work down to $q_T \sim 0$, however collinear-factorization works up to $q_T > M$; therefore, TMD-factorization and collinear-factorization can be simultaneously applied only when $q_T > M$).

The W term becomes unphysical when $q_T \geq Q$, where it becomes negative (and large).

The Y term corrects for the misbehaviour of W as $q_T$ gets larger, providing a consistent (and positive) $q_T$ differential cross section.

The Y term should provide an effective smooth transition to large $q_T$, where fixed order perturbative calculations are expected to work.
Resummation / TMD factorization

Example: the CSS resummation scheme:

\[
W_j(x_1, x_2, b_T, Q) = \exp \left[ S_j(b_T, Q) \right] \sum_{i,k} C_{ji} \otimes f_i(x_1, \frac{C_1^2}{b_T^2}) \quad C_{jk} \otimes f_k(x_2, \frac{C_1^2}{b_T^2})
\]

\[
S_j(b_T, Q) = - \int_{\frac{C_1^2}{b_T^2}}^{Q^2} \frac{d\kappa^2}{\kappa^2} \left[ A_j(\alpha_s(\kappa)) \ln \left( \frac{Q^2}{\kappa^2} \right) + B_j(\alpha_s(\kappa)) \right]
\]

At large \( b_T \) the scale \( \mu \) becomes too small!

Non-trivially connected to the physical region: \( Q^2 \gg q_T^2 \simeq \Lambda_{QCD}^2 \)

- All TMD evolution schemes require a model to deal with the non-perturbative region
- Working in \( b_T \) space makes phenomenological analyses more difficult, as we lose intuition and direct connection with “real world experience”. (Experimental data are in \( q_T \) space).
This is a perturbative scheme. All the scales must be frozen when reaching the non-perturbative region:

\[ b_T \rightarrow b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \]

Then we define a non-perturbative function for large \( b_T \):

\[
\frac{W_j(x_1, x_2, b_T, Q)}{W_j(x_1, x_2, b_*, Q)} = F_{NP}(x_1, x_2, b_T, Q)
\]

Perturbatively calculable, but process dependent

\[
W_j(x_1, x_2, b_T, Q) = \sum_{i,k} \exp[S_j(b_*, Q)] \left[ C_{ji} \otimes f_i(x_1, \mu_b) \right] \left[ C_{jk} \otimes f_k(x_2, \mu_b) \right] F_{NP}(x_1, x_2, b_T, Q)
\]

\[ C_1 = 2 \exp(-\gamma_E) \]

Non-perturbative, must be inferred from experiment, but universal

For this scheme to work, 4 distinct kinematic regions have to be identified.

They should be large enough and well separated.

### TMD regions

<table>
<thead>
<tr>
<th>TMD evolution</th>
<th>Matching region (Y factor)</th>
<th>Fixed Order collinear QCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_T \sim \lambda_{QCD}$</td>
<td>$q_T \ll Q$</td>
<td>$q_T \sim Q$</td>
</tr>
<tr>
<td>$q_T \geq Q$</td>
<td>Soft gluon radiation</td>
<td>Hard gluon emission</td>
</tr>
</tbody>
</table>
To perform phenomenological studies we need a non perturbative function.

\[ F_{NP}(x_1, x_2, b_T, Q) \]

Davies-Webber-Stirling (DWS) \[
\exp \left[ -g_1 - g_2 \ln \left( \frac{Q}{2Q_0} \right) \right] b^2;
\]

Ladinsky-Yuan (LY) \[
\exp \left[ -g_1 - g_2 \ln \left( \frac{Q}{2Q_0} \right) \right] b^2 - \left[ g_1 g_3 \ln(100x_1 x_2) \right] b;
\]

Brock-Landry-Nadolsky-Yuan (BLNY) \[
\exp \left[ -g_1 - g_2 \ln \left( \frac{Q}{2Q_0} \right) - g_1 g_3 \ln(100x_1 x_2) \right] b^2
\]

CSS for DY processes

Nadolsky et al.* analyzed successfully low energy DY data and $Z_0$ production data using different parametrizations.

\[ b_{max} = 0.5 \text{ GeV}^{-1} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DWS-G fit</th>
<th>LY-G fit</th>
<th>BLNY fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_1$</td>
<td>0.016</td>
<td>0.02</td>
<td>0.21</td>
</tr>
<tr>
<td>$g_2$</td>
<td>0.54</td>
<td>0.55</td>
<td>0.68</td>
</tr>
<tr>
<td>$g_3$</td>
<td>0.00</td>
<td>-1.50</td>
<td>-0.60</td>
</tr>
</tbody>
</table>

CDF Z Run-0

$N_{fit}$ (fixed) 1.00 1.00 1.00

R209

$N_{fit}$ 1.02 1.01 0.86

E605

$N_{fit}$ 1.15 1.07 1.00

E288

$N_{fit}$ 1.23 1.28 1.19

DØ Z Run-1

$N_{fit}$ 1.01 1.01 1.00

CDF Z Run-1

$N_{fit}$ 0.89 0.90 0.89

$\chi^2$ 416 407 176

$\chi^2$/DOF 3.47 3.42 1.48

SIDIS processes

\[ \ell + p \rightarrow \ell' + h + X \]
As mentioned above

- fixed order pQCD calculation fail to describe the SIDIS cross sections at small $q_T$,
- the cross section tail at large $q_T$ is clearly non-Gaussian.

**Need resummation of large logs and matching perturbative to non-perturbative contributions**


Anselmino, Boglione, Gonzalez, Melis, Prokudin, JHEP 1404 (2014) 005

**Naive TMD approach**


Simple phenomenological ansatz can reproduce low $q_T$ data

$$f_{q/p}(x, k_T) = f(x) \frac{e^{-k_T^2/\langle k_T^2 \rangle}}{\pi \langle k_T^2 \rangle}$$

$$D_{h/q}(z, p_T) = D_{h/q}(z) \frac{e^{-p_T^2/\langle p_T^2 \rangle}}{\pi \langle p_T^2 \rangle}$$

$$F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

$$\langle P_T^2 \rangle = \langle p_T^2 \rangle + z_h^2 \langle k_T^2 \rangle$$

\[\begin{align*}
&\langle k_T^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2 \\
&\langle p_T^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2 \\
&\chi^2_{dof} = 1.69
\end{align*}\]
Naive TMD approach


\[ F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle} \]

\[ \langle P_T^2 \rangle = \langle p_{\perp}^2 \rangle + z_h^2 \langle k_{\perp}^2 \rangle \]

\[ \langle k_{\perp}^2 \rangle = 0.60 \pm 0.14 \text{ GeV}^2 \]

\[ \langle p_{\perp}^2 \rangle = 0.20 \pm 0.02 \text{ GeV}^2 \]

\[ \chi^2_{\text{dof}} = 3.42 \]

Fit over 6000 data points with 2 free parameters

\[ N_y = A + B y \]

“The point-to-point systematic uncertainty in the measured multiplicities as a function of \( p_{\perp}^2 \) is estimated to be 5% of the measured value. The systematic uncertainty in the overall normalization of the \( p_{\perp}^2 \)-integrated multiplicities depends on \( z \) and \( y \) and can be as large as 40%.”

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities: flavour dependence

A. Signori, A. Bacchetta, M. Radici, G. Schnell, JHEP 1311 (2013) 194

proton target

\[ \chi^2 / \text{d.o.f.} = 1.63 \pm 0.12 \]

\[ \text{no flavor dep.} \]

\[ 1.72 \pm 0.11 \]

5/7 parameters

Much more complex

parametrization of \( x \) and \( z \) dependence

\[ \pi^- \]

\[ 1.80 \pm 0.27 \]

\[ 1.83 \pm 0.25 \]

\[ \pi^+ \]

\[ 2.64 \pm 0.21 \]

\[ 2.89 \pm 0.23 \]

\[ K^- \]

\[ 0.78 \pm 0.15 \]

\[ 0.87 \pm 0.16 \]

\[ K^+ \]

\[ 0.46 \pm 0.07 \]

\[ 0.43 \pm 0.07 \]
\[ F_{UU} = \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle} \]

\[ \langle P_T^2 \rangle = \langle p_{\perp}^2 \rangle + z_h^2 \langle k_{\perp}^2 \rangle \]

\[ \langle k_{\perp}^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2 \]
\[ \langle p_{\perp}^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2 \]

\[ \chi^2_{\text{dof}} = 1.69 \]

All four bins have been overlapped in the same panel

Hard to decouple the \( Q^2 \) dependence from HERMES data alone

Anselmino et al. JHEP 1404 (2014) 005
HERMES and COMPASS multiplicities cover the same range in $Q^2$ ...

\[
\langle k^2 \rangle = g_1 + g_2 \ln \left( \frac{Q^2}{Q_0^2} \right) + g_3 \ln (10 \, e \, x) \]
\[
\langle p^2 \rangle = g'_1 + z^2 g'_2 \ln \left( \frac{Q^2}{Q_0^2} \right) \]
\[
\langle P^2_T \rangle = g'_1 + z^2 \left[ g_1 + g_2 \ln \left( \frac{Q^2}{Q_0^2} \right) + g_3 \ln (10 \, e \, x) \right] \]

- HERMES multiplicities show no sensitivity to these parameters
- COMPASS fitting is much more involved. After correcting for normalization, we find that the total $\chi^2$ decreases from 3.42 to 2.69.
To ensure momentum conservation, write the cross section in the Fourier conjugate space

\[
\delta^2(q_T - k_{1T} - k_{2T} - \ldots - k_{nT} + \ldots) = \int \frac{d^2b_T}{(2\pi)^2} e^{-i b_T \cdot (q_T - k_{1T} - k_{2T} - \ldots - k_{nT} + \ldots)}
\]

\[
\frac{1}{\sigma_0} \frac{d\sigma}{dQ^2 dy dq_T^2} = \left[ \int \frac{d^2b_T e^{i q_T \cdot b_T}}{(2\pi)^2} X_{div}(b_T) \right] + Y_{reg}(q_T)
\]

\[X_{div}(b_T) \rightarrow W(b_T) = \exp[S(b_T)] \times (\text{PDFs and Hard coefficients})\]

\[
\frac{d\sigma^{total}}{dx \, dy \, dz \, dq_T^2} = \pi \sigma_0^{DIS} \int \frac{d^2b_T e^{i q_T \cdot b_T}}{(2\pi)^2} W^{SIDIS}(x, z, b_T, Q) + Y^{SIDIS}(x, z, q_T, Q)
\]

Resummed part

Regular part
Fit of HERMES and COMPASS data

Attempting “Resummation” in SIDIS ...

\[ \chi^2_{\text{HERMES}} = 1.32 \]

\[ \chi^2_{\text{COMPASS}} = 1.12 \]

Y-term is neglected

\[ N = 2 \] (One overall normalization parameter is required)

\[ g_1 \sim 0.5 \] (too large compared to the value extracted from DY data)

\[ g_2 \sim 0.5 \]

\[ g_3 \sim -0.03 \]
Global fits

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori, JHEP06 (2017) 081

Extraction of partonic transverse momentum distributions from semi-inclusive deep-inelastic scattering, Drell-Yan and Z-boson production

Alessandro Bacchetta, Filippo Delcarro, Cristian Pisano, Marco Radici, Andrea Signori

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Abstract: We present an extraction of unpolarized partonic transverse momentum distributions (TMDs) from a simultaneous fit of available data measured in semi-inclusive deep-inelastic scattering, Drell-Yan and Z-boson production. To connect data at different scales, we use TMD evolution at next-to-leading logarithmic accuracy. The analysis is restricted to the low-transverse-momentum region, with no matching to fixed-order calculations at high transverse momentum. We introduce specific choices to deal with TMD evolution at low scales, of the order of 1 GeV$^2$. This could be considered as a first attempt at a global fit of TMDs.

$\chi^2_{\text{tot}} = 1.55$

- Y-term is neglected
- Sum of two Gaussian $k_t$ distributions is introduced
Global fits

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori, JHEP06 (2017) 081

Although the shape in transverse momentum space is well described, normalization is very problematic.

$\chi^2_{\text{tot}} = 1.55$

- Y-term is neglected
- Sum of two Gaussian $k_T$ distributions is introduced
What's going on ???
For this scheme to work, 4 distinct kinematic regions have to be identified

They should be large enough and well separated

\[ q_T \sim \lambda_{QCD} \]
\[ q_T \ll Q \]
\[ q_T \sim Q \]
\[ q_T \geq Q \]

Intrinsic \( q_T \)  
Soft gluon radiation  
Hard gluon emission
For this scheme to work, 4 distinct kinematic regions have to be identified

- They should be large and well separated

**TMD evolution**

- Intrinsic $q_T \sim \lambda_{QCD}$
- Matching region ($Y$ factor)
- Fixed order QCD: $q_T < Q$, $q_T \sim Q$, $q_T \geq Q$
The Y factor is very large (even at low $q_T$)

However, it could be affected by large theoretical uncertainties

Boglione, Gonzalez, Melis, Prokudin, JHEP 02 (2015) 095

The Y factor cannot be neglected !!!

New prescription for Y factor, $b^*$ and W


\[
\frac{d\sigma^{NLO}}{dx \, dy \, dz \, dq_T^2} = \frac{d\sigma^{ASY}}{dx \, dy \, dz \, dq_T^2} + Y
\]

\[
\sigma^{ASY} = Q^2/q_T^2 \left[ A \, \text{Ln}(Q^2/q_T^2) + B + C \right]
\]
Other issues related to TMD regions ...

- TMD regions are defined in terms of $q_T$ and not in terms of $P_T$.

- $q_T = P_T / z$

- $q_T = Q/4$

- $q_T = Q$
The rather large size of the K-factor can be understood as a consequence of the opening of a new dominant ('leading-order') channel, and not to the 'genuine' increase in the partonic cross section [...] The dominance of the new channel is due to the size of the gluon distribution at small $x_B$ and to the fact that the H1 selection cuts highlight the kinematical region dominated by the $\gamma + g \rightarrow g + q + \bar{q}$ partonic process. In particular, without the experimental cuts for the final state hadrons, the $gg$ component represents less than 25% of the total NLO contribution at small $x_B$.


How can we address the normalization problem ???

- K factor depends on $p_T$
- Kinematics cuts can affect the size of K factors ... up to a factor 10!
  Stringent cuts on the pion production angle in H1 data suppresses LO and NLO contributions in a different way
Large transverse momentum behaviour in SIDIS

There are large discrepancies between data and fixed order calculations. They seem to be generated by collinear PDFs and FFs.
Summary

- Naive TMD Models can describe HERMES and COMPASS data at low transverse momentum.

- Similarly to DY, the $Q^2$ dependence is not clearly visible in the shape of the spectrum.

- TMD resummation is difficult:
  - no information on unpolarized TMD fragmentation functions
  - global fitting is affected by normalization issues
  - Y-term is not included
  - the non-perturbative behaviour seems to be dominant
  - difficult to work in $b_T$ space where we lose phenomenological intuition

- Fixed order calculation fail to reproduce the correct behaviour of the cross section at large transverse momentum.

- Need an extra effort to devise theories/models/prescriptions which simultaneously explain experimental data from different experiments, over a wide range of transverse momentum values.

- Need new, high-precision experimental data to be able to perform solid and realistic phenomenological analyses of TMD physics (EIC !)
Extracting the unpolarized TMD fragmentation function from $e^+e^-$ data
**e^+e^- scattering processes**

- Recent data on Collins azimuthal asymmetries from BELLE, BaBar and BES III
- No modern data available (yet) on unpolarized cross sections

TASSO, MARKII data available for unpolarized e^+e^- → h X

- $p_T$ distributions
- Different energies
- Integrated over $z$
- No charge separation

**TMD evolution**

**Big limitation**

Direct observation of the hadron momentum component, transverse to the fragmenting parent parton (jet axis)
Recent data on Collins azimuthal asymmetries from BELLE, BaBar and BES III

No modern data available (yet) on unpolarized cross sections

TASSO, MARKII data available for unpolarized $e^+e^- \rightarrow h \ X$

- $p_T$ distributions
- Different energies
- Integrated over $z$
- No charge separation

Direct observation of the hadron momentum component, transverse to the fragmenting parent parton (jet axis)
Modeling the cross section

Assuming factorization ...

\[
\frac{d\sigma^h}{dz \, d^2p_\perp} \bigg|_{\text{model}} = \frac{4\pi \alpha^2}{3s} \sum_q e_q^2 \, D_q^h(z, p_\perp; Q^2)
\]

Use this ...

To get information about that ...

To leading order

\[
= \frac{4\pi \alpha^2}{3s} \sum_q e_q^2 \, D_q^h(z, Q^2) \, h(p_\perp)
\]

Issues to investigate:
- Appropriate functional form for \( g_{j/p} \)
- Scale evolution induced by \( g_k \)

\[
\tilde{D}_{h/q}(z, b_\perp; Q) = \sum_i \left[ \left( \tilde{C}_{j/q} \otimes \frac{d_{h/j}}{z^2} \right) e^{\Gamma_D(Q)} \right] \exp \left\{ g_{j/p}(x, b_\perp) + g_K(b_\perp) \log \left( \frac{Q}{Q_0} \right) \right\}
\]
Modeling the $p_{\perp}$ dependence

Fit of TASSO data, using **gaussian** $p_{\perp}$ dependence

$$\frac{1}{\sigma_0} \frac{d\sigma}{dp_{\perp}} \rightarrow \pi p_{\perp}N \left[ \int dz \frac{\sum c_i^2 P_i(z; Q^2)}{\sum c_i^2} h(p_{\perp}) + \delta Q \right]$$

1) **α** extracted separately for each $\sqrt{s}$ value

The lack of information about $z$ hinders a full TMD extraction of the FF.

Fit of TASSO data, using **power law** $p_{\perp}$ dependence

$$h(p_{\perp}) = 2(\alpha - 1)M^2 (\alpha - 1) \frac{1}{(p_{\perp}^2 + M^2)^{\alpha}}$$

2) $\langle p_{\perp}^2 \rangle = 2g_1 + 2g_2 z^2 \log \left( \frac{Q}{2Q_0} \right)$ Tension between the shape of the peak and that of the tale

Modeling the $p_{\perp}$ dependence

Fit of TASSO data, using **gaussian** $p_{\perp}$ dependence

$$\frac{1}{d\sigma/dp_{\perp}} \rightarrow \pi p_{\perp} N \left[ \int dz \frac{\sum c_i^2 e^{-\alpha_i(z; Q^2)}}{\sum_i c_i^2} \right] h(p_{\perp}) + \delta Q$$

Normalization problematic

Fit of TASSO data, using **power law** $p_{\perp}$ dependence

$$h(p_{\perp}) = 2(\alpha - 1)M^2(\alpha-1) \frac{1}{(p_{\perp}^2 + M^2)^{\alpha}}$$

$$\alpha = \alpha_0 + \tilde{\alpha} \log \left( \frac{Q}{Q_0} \right)$$

The lack of information about $z$ hinders a full TMD extraction of the FF.

\[ \langle p_{\perp}^2 \rangle = 2g_1 + 2g_2 z^2 \log \left( \frac{Q}{2Q_0} \right) \]

Tension between the shape of the peak and that of the tale.

MARKII

\[ \sqrt{s} = 29 \text{ GeV} \]

TASSO fit estimate

TASSO

PLUTO
Interpreting our results ...

**MODEL**

\[ h(p_{\perp}) = 2(\alpha - 1)M^2(\alpha-1) \frac{1}{(p^2_{\perp} + M^2)^{\alpha}} \]


**TMD scheme**

(under the assumption that integration over \( z \) does not alter the structural form of the non perturbative exponential)

\[ F^{-1} \left\{ \frac{d\sigma^h}{d^2p_{\perp}} \right\} \propto \exp \left\{ \tilde{g}(b_{\perp}) \log \left( \frac{Q}{Q_0} \right) \right\} \]

\[ b_{\perp}^{\alpha_0} \exp \left\{ \tilde{g}(b_{\perp}) \log \left( \frac{Q}{Q_0} \right) \right\} \propto b_{\perp}^\alpha \]

Logarithmic behavior of alpha may be interpreted as a consequence of the Log in the definition of the TMD FF.

\[ \alpha = \alpha_0 + \tilde{\alpha} \log \left( \frac{Q}{Q_0} \right) \]

\[ g_K(b_{\perp}) \xrightarrow{\text{large } b_{\perp}} \tilde{\alpha} \log(\nu b_{\perp}) \]
Interpreting our results ...

**MODEL**

\[ h(p_\perp) = 2(\alpha - 1)M^2 (\alpha - 1) \frac{1}{(p_\perp^2 + M^2)^\alpha} \]

**TMD scheme**

(under the assumption that integration over z does not alter the structural form of the non perturbative exponential)

**There are caveats on this interpretation:**

- it is consistent with theoretical expectations but it is not unique.

- Lack of information on z-dependence of the TMD FF in the TASSO and MARK II measurements (and possible correlations between Q and z of different origin) hinders a more solid conclusion about TMD evolution effects in these data sets.

Logarithmic behavior of alpha may be interpreted as a consequence of the Log in the definition of the TMD FF.

\[ \alpha = \alpha_0 + \tilde{\alpha} \log \left( \frac{Q}{Q_0} \right) \]

\[ g_K(b_\perp) \xrightarrow{\text{large } b_\perp} \tilde{\alpha} \log(\nu b_\perp) \]
Extracting the Sivers function from SIDIS data
Sivers effect: COMPASS vs. HERMES


Apparently … some tension between COMPASS and HERMES data

However, COMPASS and HERMES span different ranges in $Q^2$ and have different $< Q^2 >$.

Kinematics effects
Possible signal of TMD evolution?
Signal of some tension between independent fit solutions for COMPASS and HERMES data

Start by using a very simple model only 5 parameters and no Q evolution

Use only u flavour and only π+ data

**New extraction of the Sivers function**

Signal of some tension between independent fit solutions for COMPASS and HERMES data


Start by using a very simple model only 5 parameters and no Q evolution

Use only u flavour and only $\pi^+$ data

![Combined fit](image)
New extraction of the Sivers function

Tension relaxes when the asymmetry is computed using the appropriate unpolarized widths for each data set

**Reference Fit**

**COMPASS-2015** (COMPASS widths no-evolution)

**HERMES-2009** (HERMES widths no-evolution)

**SIMULTANEOUS FIT OF**
HERMES-2009 (HERMES widths - no evolution)
COMPASS-2015 (COMPASS widths – no evolution)
New extraction of the Sivers function

Sivers widths: HERMES vs. COMPASS

If we use different unpolarized widths for HERMES and COMPASS data, do we have to use different Sivers widths as well?

Allowing for different Sivers widths for each experiments, does not improve the quality of the fit, and the extracted values are very similar.

Uncertainty bands – Sivers first moment

Reference Fit (no-evolution)

Sivers First moment u-contribution

Study of Low-x Uncertainties (include $\alpha_u$ and $\alpha_d$ in the parametrization of the Sivers function)

Sivers First moment d-contribution

Uncertainty bands – Sivers Asymmetries


Reference Fit (no-evolution)

Uncertainty bands from the reference fit (light-blue) become artificially small at small x.

Study of Low-x Uncertainties
(include $\alpha_u$ and $\alpha_d$ in the parametrization of the Sivers function)

Alpha fit gives better estimates of uncertainties at small x (gray bands).
Uncertainty bands – Sivers Asymmetries


Reference Fit
(no-evolution)

Uncertainty bands from the reference fit (light-blue)

Study of Low-x Uncertainties
(include $\alpha_u$ and $\alpha_d$ in the parametrization of the Sivers function)

$\alpha$-fit gives better estimates of uncertainties at large-x as well (gray bands).

COMPASS (2017)

$P_T \rightarrow 1h^+X$

$\langle Q^2 \rangle < 3.6 GeV^2$

$\langle Q^2 \rangle < 5.5 GeV^2$

$\langle Q^2 \rangle < 12.9 GeV^2$

$\langle Q^2 \rangle < 36.8 GeV^2$

$\langle Q^2 \rangle = 1.95 GeV^2$

$\langle Q^2 \rangle = 4.97 GeV^2$

$\langle Q^2 \rangle = 9.42 GeV^2$

$\langle Q^2 \rangle = 26.5 GeV^2$