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The LHCb observation [1] was further supported by another two articles by the same group [2,3]:

The pentaquark as a compact five quark state

- Using group theory techniques we found that the compact pentaquark states belong to an SU(3) flavour octet.
- The masses of the octet pentaquark states were calculated by means of a Gürsey-Radicati mass formula extension.
- The partial decay widths were calculated by means of an effective Lagrangian:

\[ \mathcal{L}^{3/2} = i \bar{P} \mu \left[ \frac{g_1}{2M_N} \Gamma^{-} N \right] \eta^{\mu \nu} - i \bar{P} \mu \left[ \frac{ig_2}{(2M_N)^2} \Gamma^{-} \partial_{\nu} N + \frac{ig_3}{(2M_N)^2} \Gamma^{-} N \partial_{\nu} \right] \eta^{\mu \nu} + \text{H.c.} \]
Strong decay widths

Example:

\[ P^{0+}(4377) \rightarrow J/\psi + P \]

Thus, the higher partial wave terms proportional to \( \frac{p}{M_N} \) and \( \frac{p}{M_N}^3 \) can be neglected.

We can consider only the first term of the Effective Lagrangian.

\[
\mathcal{L}^{3/2^-}_{PNJ/\psi} = i \bar{P}_\mu \left[ \frac{g_1}{2M_N} \Gamma^{-} N \psi^{\mu\nu} - i \bar{P}_\mu \left[ \frac{ig_2}{(2M_N)^2} \Gamma^{-} \partial_\nu N \right. \\
+ \left. \frac{ig_3}{(2M_N)^2} \Gamma^{-} N \partial_\nu \right] \psi^{\mu\nu} + \text{H.c.} \right]
\]

the momenta of the final states in the pentaquark decays into \( J/\psi + P \) are fairly small compared with the nucleon mass [3].

Strong decay widths

Example:

$P^{0+}(4377) \rightarrow J/\psi + P$

$$
\mathcal{L}_{PNJ/\psi}^{3/2-} = i\not{P}_\mu \left[ \frac{g_1}{2M_N} \Gamma^\nu N \right] \gamma^{\mu\nu} - i\not{P}_\mu \left[ \frac{ig_2}{(2M_N)^2} \Gamma^\nu N \partial^\nu \right] + \frac{ig_3}{(2M_N)^2} \Gamma^{-N} \partial^\nu \right] \psi^{\mu\nu} + \text{H.c.,}
$$

$$
\Gamma(P^{0+} \rightarrow NJ/\psi) = \frac{g_{NJ/\psi}^2}{(2\pi)^3} \frac{P_N}{M_{P^{0+}}} \left( E_N + M_N \right) \left[ 2E_N(E_N - M_N) + (M_{P^{0+}} - M_N)^2 + 2M_{J/\psi}^2 \right],
$$

$P^{0+}$ field

$J/\psi$ field

Proton field
Strong decay widths

Example:

\[ P^{0+}(4377) \rightarrow J/\psi + P \]

The Pentaquark masses were predicted by the GR mass formula extension, **BUT** what about the coupling constant \( g_1 \)？

Observe that:

\[ \bar{g}_{\Delta J/\psi} = \frac{g_1}{2M_\Delta}, \quad \bar{g}_{\Sigma J/\psi} = \frac{g_1}{2M_\Sigma} \quad \text{and} \quad \bar{g}_{\Xi J/\psi} = \frac{g_1}{2M_\Xi} \]

so we only need to know \( g_1 \) to calculate the partial width for all the channels.
Since the pentaquark states have been observed in the $J/\psi P$ channel, it is natural to expect that they can be produced in $J/\psi P$ photoproduction process. Wang et al. [3] calculated the cross section of the pentaquark states in $J/\psi P$ photoproduction and compared it with the available experimental data. The coupling between $J/\psi P$ and the two pentaquark states is extracted by assuming that the decay width of each pentaquark state $J/\psi P$ is 5% of the total width. In conclusion, they found that to be consistent also with the available photoproduction data, the branching ratio for both the pentaquark states needs to be $BR[P^{0+}(4377) \to J/\psi + P] \leq 0.05$.

In conclusion, they found that to be consistent also with the available photoproduction data, the branching ratio for both the pentaquark states needs to be $BR[P^0+(4377) \to J/\psi + P ] \leq 0.05$.

The total width of the lightest pentaquark state is known from LHCb analysis ($\Gamma_{tot} = 205$ MeV).

We used the upper limit found by Wang.
Now we are able to calculate the partial width for each decay channel!

\[ \Gamma_{N J/\Psi} = B(P^+ \rightarrow J/\Psi p) \Gamma_{tot} = 10.25 \text{ MeV}, \]

<table>
<thead>
<tr>
<th>Initial state ( P^{I0} )</th>
<th>Channel ( \Lambda J/\Psi )</th>
<th>Partial width [MeV]</th>
<th>( 0.78 \Gamma_{N J/\Psi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P^{I-}, P^{10}, P^{1+} )</td>
<td>( \Sigma J/\Psi )</td>
<td>0.71 ( \Gamma_{N J/\Psi} )</td>
<td>0.82 ( \Gamma_{N J/\Psi} )</td>
</tr>
<tr>
<td>( P^{2-}, P^{20} )</td>
<td>( \Xi J/\Psi )</td>
<td>0.62 ( \Gamma_{N J/\Psi} )</td>
<td>0.74 ( \Gamma_{N J/\Psi} )</td>
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</table>

<table>
<thead>
<tr>
<th>Initial state ( P^{I0} )</th>
<th>Channel ( \Lambda J/\Psi )</th>
<th>Partial width [MeV]</th>
<th>7.94</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P^{I-}, P^{10}, P^{1+} )</td>
<td>( \Sigma J/\Psi )</td>
<td>7.21</td>
<td></td>
</tr>
<tr>
<td>( P^{2-}, P^{20} )</td>
<td>( \Xi J/\Psi )</td>
<td>6.35</td>
<td></td>
</tr>
</tbody>
</table>
Coupled channel between the meson-baryon states

- Near the thresholds, resonances are expected to have an exotic structure, like the hadronic molecules.

- The observed pentaquarks are found to be just below the $\bar{D}^* \Sigma_c$ ($P_c^+(4380)$) and the $\bar{D}^* \Sigma_c^*$ ($P_c^+(4450)$) thresholds. Moreover, the $\bar{D}^* \Lambda_c$ threshold is only 25 MeV below the $\bar{D} \Sigma_c$ threshold. For this reason, the $\bar{D} \Lambda_c$, $\bar{D}^* \Lambda_c$ channels are not irrelevant in the hidden-charm meson-baryon molecules.

In Phys.Rev. D96 (2017) no.1, 014018 E. Santopinto e Y. Yamaguchi considered the coupled channel systems of $\bar{D} \Lambda_c$, $\bar{D}^* \Lambda_c$, $\bar{D} \Sigma_c$, $\bar{D} \Sigma_c^*$, $\bar{D}^* \Sigma_c$ and $\bar{D}^* \Sigma_c^*$ to predict the bound and the resonant states in the hidden-charm sector. The binding interaction between the meson and the baryon is given by the One Meson Exchange Potential (OMEP).
Coupled channel between the meson-baryon states

In particular the bound and resonant states with $J^P = \frac{3^+}{2}, \frac{3^-}{2}, \frac{5^+}{2}$ and $\frac{5^-}{2}$ with isospin $I = \frac{1}{2}$ are studied by solving the coupled channel Schrödinger equations.

- Free parameter of the model: the cut-off parameter $\Lambda$;
- $\Lambda$ is fixed to reproduce the heaviest resonant state $P_c^+(4450)$.
Coupled channel between the meson-baryon states

results

<table>
<thead>
<tr>
<th>$\Lambda$ [MeV]</th>
<th>1300</th>
<th>1400</th>
<th>1500</th>
<th>1600</th>
<th>1700</th>
<th>1800</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J^P = 3/2^-$</td>
<td>4236.9 – i0.8</td>
<td>4136.0</td>
<td>4006.3</td>
<td>3848.2</td>
<td>3660.0</td>
<td>3438.26</td>
</tr>
<tr>
<td></td>
<td>4381.3 – i11.4</td>
<td>4307.9 – i18.8</td>
<td>4242.6 – i1.4</td>
<td>4150.1</td>
<td>4035.2</td>
<td>3897.3</td>
</tr>
<tr>
<td></td>
<td>4368.5 – i64.9</td>
<td>4348.7 – i21.1</td>
<td>4312.7 – i16.0</td>
<td>4261.0 – i7.0</td>
<td>4187.7 – i0.9</td>
<td>4092.5</td>
</tr>
<tr>
<td>$J^P = 3/2^+$</td>
<td>4223.0 – i97.9</td>
<td>4206.7 – i41.2</td>
<td>4169.3 – i5.3</td>
<td>4104.2</td>
<td>3996.7</td>
<td>3855.8</td>
</tr>
<tr>
<td></td>
<td>4363.3 – i57.0</td>
<td>4339.7 – i26.8</td>
<td>4311.8 – i6.6</td>
<td>4268.5 – i1.3</td>
<td>4193.2 – i0.1</td>
<td>4091.6</td>
</tr>
<tr>
<td>$J^P = 5/2^-$</td>
<td>—</td>
<td>4428.6 – i89.1</td>
<td>4391.7 – i88.8</td>
<td>4338.2 – i56.2</td>
<td>4286.8 – i27.3</td>
<td>4228.3 – i7.4</td>
</tr>
<tr>
<td>$J^P = 5/2^+$</td>
<td>—</td>
<td>—</td>
<td>4368.0 – i9.2</td>
<td>4305.8 – i1.9</td>
<td>4222.7 – i1.4</td>
<td>4111.1</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>4398.5 – i15.0</td>
<td>4357.8 – i8.2</td>
<td></td>
</tr>
</tbody>
</table>

Good agreement for the mass and quantum numbers of the lightest pentaquark $P_c^+(4380)$

The masses and widths of the two observed pentaquark states; BE AWARE: the mass of the lightest one is a prediction, while the mass of the heaviest is fitted to fix the cut-off parameter $\Lambda$.
In the current problem of pentaquark $P_c$, there are two competing sets of channels: the meson-baryon (MB) channels and the five-quark channels [1].

Motivation

Can a couple channel between the MB channels and the core contribution describe in a realistic way the pentaquark states?

Coupled channel between the meson-baryon states and the five quark states [1]

- hidden-charm pentaquarks as $\bar{D} \Lambda_c, \bar{D}^* \Lambda_c, \bar{D} \Sigma_c, \bar{D}^* \Sigma_c, \bar{D} \Sigma^*_c$ and $\bar{D}^* \Sigma^*_c$, molecules coupled to the five-quark states.

- For the first time some predictions for the hidden bottom pentaquarks as $\bar{B} \Lambda_b, \bar{B}^* \Lambda_b, \bar{B} \Sigma_b, \bar{B}^* \Sigma_b, \bar{D} \Sigma^*_b$ and $\bar{B}^* \Sigma^*_b$ molecules coupled to the five-quark states are provided.

- In particular, by solving the coupled channel Schrödinger equation, we study the bound and resonant hidden-charm and hidden-bottom pentaquark states for $J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$ and isospin $I = \frac{1}{2}$.

The meson-baryon channels describe the dynamics at long distances, while the five-quark part describes the dynamics at short distances (of the order of 1 fm or less).

The Model

Kinetic energy and OPEP of the Meson-Baryon system

Proportional to the spectroscopic factors $S_i^\alpha$:

$$V_{ij}^{5q} = -f \sum_\alpha S_i^\alpha S_j^\alpha e^{-Ax^2}$$

Kinetic energy and harmonic oscillator potential of the five quark states.
The figure shows the bound and resonant state energies of the hidden-charm molecules (solid lines) with various coupling constants. Dot-dashed lines are $\bar{D}\Lambda_c$ and $\bar{D}^*\Lambda_c$ threshold. Dashed lines are the $\bar{D}\Sigma_c$, $\bar{D}^*\Sigma_c$, $\bar{D}\Sigma_c^*$, and $\bar{D}^*\Sigma_c^*$ thresholds.
You can see the dependence of the obtained energy spectra for $J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$ on the coupling constant of the model $\frac{f}{f_0}$. This coupling constant is a free parameter proportional to the coupling strength between the Meson-Baryon and the compact five-quark channels. The filled circle in figures shows the starting point where the state is found.

The lowest threshold $\bar{D} \Lambda_c$ is at 4153, 46 MeV and the state whose energy is lower than the threshold is a bound state.
When the five-quark potential is turned-off, i.e. when $\frac{f}{f_0} = 0$, there are no resonant states and no bound states for any value of quantum numbers $J^P$. In the hidden-charm sector the OPEP alone is not strong enough to produce bound and resonant $P_c$ states.
Many $\bar{B}\Lambda_B$ and $\bar{B}^*\Lambda_B$ bound states appear.

Some $\bar{B}\Lambda_B$ bound states are produced even without introducing the five-quark potential ($\frac{f}{f_0} = 0$)!

Dot-dashed lines are the $\bar{B}\Lambda_B$ and $\bar{B}^*\Lambda_B$ thresholds. Dashed lines are the $\bar{B}\Sigma_B, \bar{B}\Sigma^*_B, \bar{B}^*\Sigma_B$ and $\bar{B}^*\Sigma^*_B$ thresholds.
Dot-dashed lines are the $\bar{B} \Lambda_B$ and $\bar{B}^* \Lambda_B$ thresholds. Dashed lines are the $\bar{B} \Sigma_B, \bar{B} \Sigma^*_B$, $\bar{B}^* \Sigma_B$ and $\bar{B}^* \Sigma^*_B$ thresholds.

Hot topic: first results for the hidden-bottom sector

As a matter of fact, we have found that, unlike the charm-sector, in which the five-quark potential is needed to produce bound states, in the bottom sector the OPEP alone provides sufficiently strong attraction to generate several bound and resonant states.
Moreover, many states appear when the 5q potential is switched on.

As a consequence, the hidden-bottom pentaquarks are more likely to form rather than the hidden-charm pentaquarks.

The hidden-bottom sector is the more interesting environment to search the pentaquark states.
In the hidden bottom sector, the kinetic energy of the meson-baryon system is suppressed with respect to the charm sector due to the higher reduced mass of the system.

Why?

Why so more likely to find bound and resonant states in the bottom sector?

In the hidden-bottom sector, the OPEP is strong enough to produce states due to the mixing effect enhanced by the small mass splitting between $B, B^*$ and $\Sigma_B, \Sigma_B^*$. 
As a result of our calculation we find that in the charm sector, one needs the five-quark potential in addition to the pion exchange potential in order to produce bound and resonant states, whereas, in the bottom sector, the pion exchange interaction alone is strong enough to produce states.

It emerges that the hidden-bottom pentaquarks are more likely to form than their hidden-charm counterparts; for this reason, we suggest that the experimentalists should look for pentaquark states in the bottom sector.
Back up slides
The BOUND AND RESONANT STATES are obtained by solving the coupled-channel Schrödinger equation with the One Pion Exchange and the five-quark potentials. This is a very hard task! Powerful numerical method are needed in order to solve the coupled channel problem.