Quark-gluon correlations in the twist-3 TMD using light-front wave functions

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3DSPIN MAPPING THE PROTON IN 3D

European Research Council

Cross-section

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Soft hadronic term in the cross-section

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$$\Phi(x, \boldsymbol{k}_\perp) = \frac{1}{2}\left(f_1(x, \boldsymbol{k}_\perp)\gamma^- + ... + \frac{M}{P^+} e(x, \boldsymbol{k}_\perp) + \frac{\not{k}_\perp}{P^+} f^\perp(x, \boldsymbol{k}_\perp) + ... \right)$$

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Quark-quark correlator

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Twist >2: parton's correlation

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Twist-3, T-even, unpolarized

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$$\psi_\pm = \mathcal{P}_\pm \psi = \frac{1}{2}\gamma^\mp \gamma^\pm \psi$$

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$$k^+ \geq 0$$

$$\mathcal{O}_{\rm tw3} = -\frac{i}{2}\int_0^{\xi^-} d\zeta^- \overline{\psi}_+(0)\sigma^{j+}\Big[\mathcal{W}_1(0^-,\mathbf{0}_\perp;\zeta^-,\boldsymbol{\xi}_\perp)\overrightarrow{D}_{\perp,j}(\zeta^-,\boldsymbol{\xi}_\perp) + \overleftarrow{D}^\dagger_{\perp,j}(0)\mathcal{W}_1(0^-,\mathbf{0}_\perp;\zeta^-,\boldsymbol{\xi}_\perp)\Big]\psi_+(0^+,\zeta^-,\boldsymbol{\xi}_\perp)$$

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Proton matrix element and F.T.

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Proton matrix element and F.T.

$$e_{tw3} = -\frac{P^+}{M}\frac{g_s}{2}\int \frac{d\xi^- d\boldsymbol{\xi}_\perp}{2(2\pi)^3} e^{ik^+\xi^- - i\boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp}$$
$$\times \Bigg(\int_{0^-}^{\xi^-} d\zeta^- \int_{\infty^-}^{\zeta^-} d\eta^- \langle P|\overline{\psi}(0)\mathcal{W}_1(0^-,\mathbf{0}_\perp;\eta^-,\boldsymbol{\xi}_\perp)G^+{}_j(0^+,\eta^-,\boldsymbol{\xi}_\perp)$$
$$\times \sigma^{j+}\mathcal{W}_1(\eta^-,\boldsymbol{\xi}_\perp;\zeta^-,\boldsymbol{\xi}_\perp)\psi(0^+,\zeta^-,\boldsymbol{\xi}_\perp)\rangle|P$$
$$+ \int_{0^-}^{\xi^-} d\zeta^- \int_{0^-}^{\infty^-} d\eta^- \langle P|\overline{\psi}(0)\mathcal{W}_1(0^-,\mathbf{0}_\perp;\eta^-,\mathbf{0}_\perp)G^+{}_j(0^+,\eta^-,\mathbf{0}_\perp)$$
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Decomposition

$$e(x,\boldsymbol{k}_\perp) = \frac{\delta(x)}{M}\int \frac{d\boldsymbol{\xi}_\perp}{2(2\pi)^2} e^{-i\boldsymbol{\xi}_\perp\cdot\boldsymbol{k}_\perp}\,\langle P|\overline{\psi}(0)\psi(\xi)|P\rangle\,\Big|_{\substack{\xi^+=0\\ \xi_-=0}}$$

$$+\,\widetilde{e}(x,\boldsymbol{k}_\perp) + \frac{m}{xM} f_1(x,\boldsymbol{k}_\perp)$$

$$+\int_{-1}^{1} dy\,\left(\widetilde{e}(y,\boldsymbol{k}_\perp) + \frac{m}{xM} f_1(y,\boldsymbol{k}_\perp)\right)$$

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Pure twist-3 Twist-2

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Efremov and Schweitzer JHEP 0308(2003)006

Decomposition

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“Tilde” Terms

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Trace of the **quark-gluon-quark** correlator

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In light-cone gauge the tilde terms $\propto \mathcal{F}.\mathcal{T}.\, \langle P|\overline{\psi}(0)\Gamma^i A_{\perp,i}\psi(\xi)|P\rangle$

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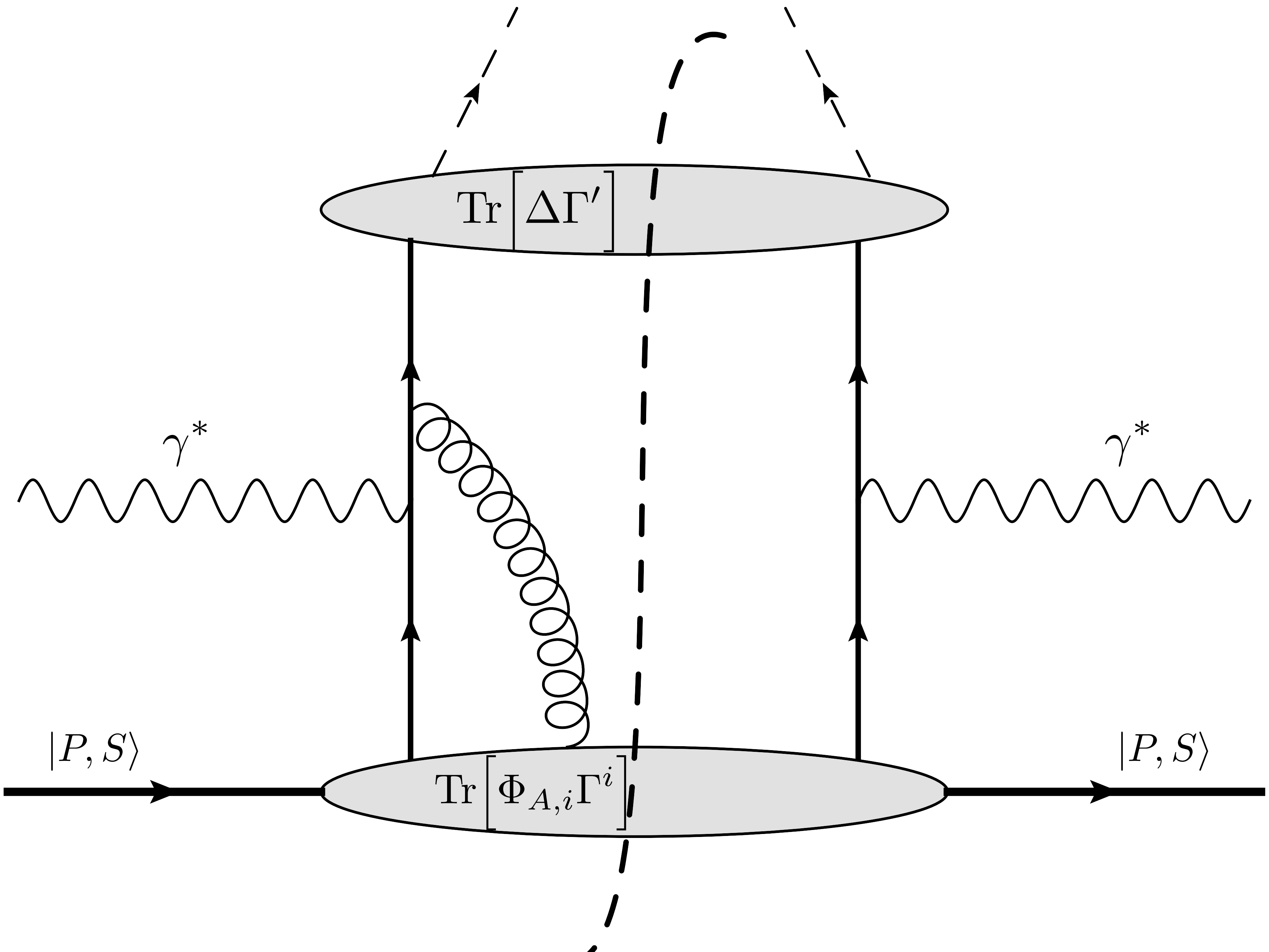

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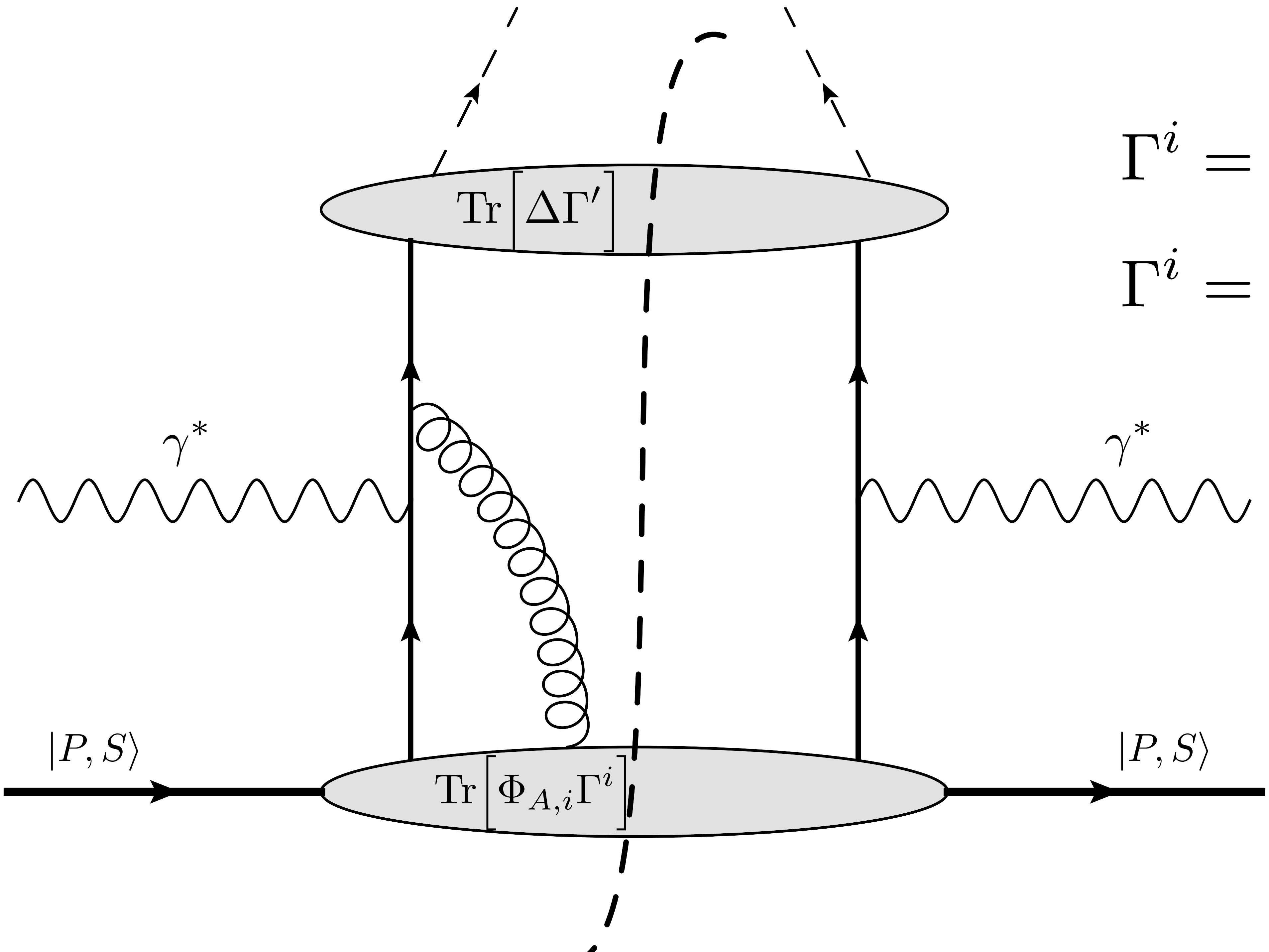

$$\Gamma^i = \sigma^{i+}$$

$$\Gamma^i = \left(g_\perp^{ji} + i\epsilon_\perp^{ji}\gamma^5\right)\gamma^+$$

Light-Front Wave Functions

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Light-front Fock expansion of the proton state

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$$|P, \Lambda\rangle = \Psi^{\Lambda}_{3q}|3q\rangle + \Psi^{\Lambda}_{3q+g}|3q+g\rangle + \cdots$$

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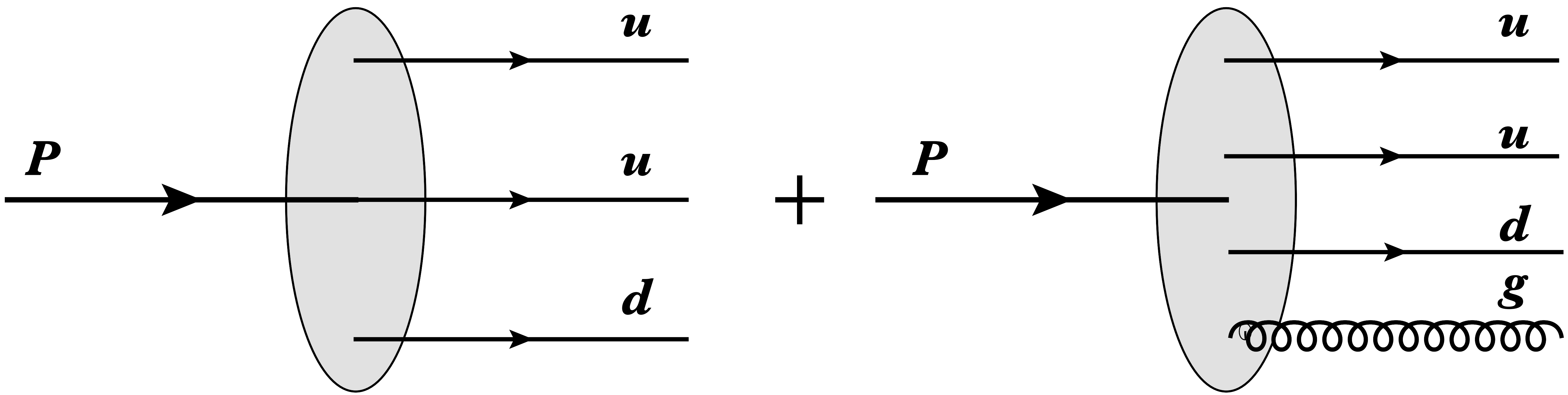

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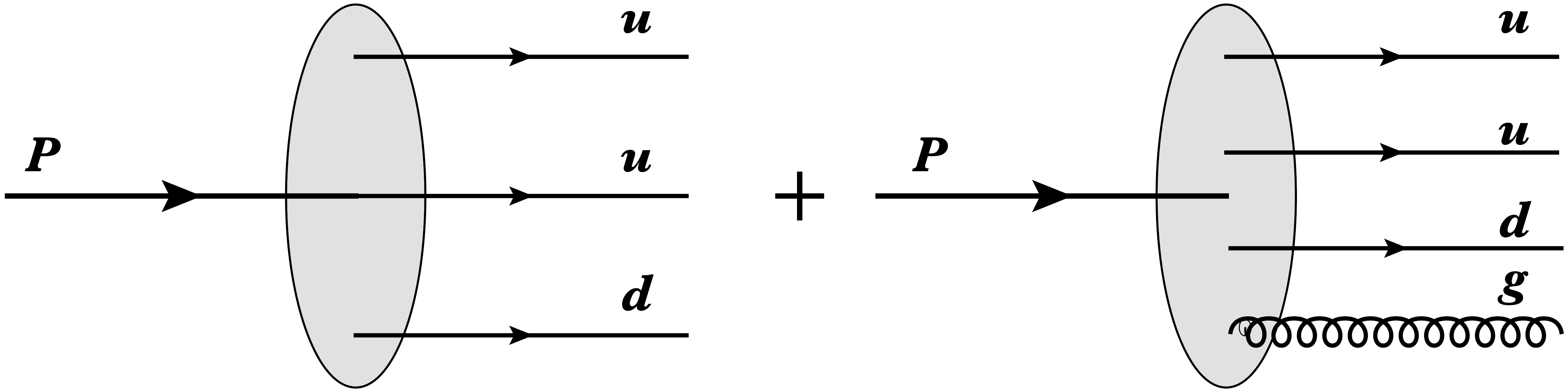

Brodsky,Pauli,Phys.Rept.301(1998)299

Light-Front Wave Amplitudes

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6 independent LFWAs for 3q

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$$\psi^1 \;,\; \psi^2 \quad L_z = 0 \qquad \psi^5 \quad L_z = -1$$

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Ji,Ma,Yuan NPB652(2003)383

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3 independent (& leading) LFWAs for L_z=0 of 3q+g

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$$\Psi^{\downarrow}(1,2,3,4) \qquad \Psi^{1,\uparrow}(1,2,3,4) \qquad \Psi^{2,\uparrow}(1,2,3,4)$$

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Ji,Ma,Yuan NPB652(2003)383

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Opposite polarization
of the proton and the gluon

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Ji,Ma,Yuan NPB652(2003)383

3 independent (& leading) LFWAs for L_z=0 of 3q+g

$$\Psi^{\downarrow}(1,2,3,4) \qquad \Psi^{1,\uparrow}(1,2,3,4) \qquad \Psi^{2,\uparrow}(1,2,3,4)$$

$\Psi^{\downarrow}(1,2,3,4)$ → Opposite polarization of the proton and the gluon

$\Psi^{1,\uparrow}(1,2,3,4)$, $\Psi^{2,\uparrow}(1,2,3,4)$ → Same polarization of the proton and the gluon

Light-Front Wave Amplitudes

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$\psi^1 \;,\; \psi^2 \quad L_z = 0 \qquad \psi^5 \quad L_z = -1$

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Ji,Ma,Yuan NPB652(2003)383

3 independent (& leading) LFWAs for L_z=0 of 3q+g

$\Psi^{\downarrow}(1,2,3,4) \qquad \Psi^{1,\uparrow}(1,2,3,4) \qquad \Psi^{2,\uparrow}(1,2,3,4)$

$\Psi^{\downarrow}$: Opposite polarization of the proton and the gluon

$\Psi^{1,\uparrow}$, $\Psi^{2,\uparrow}$: Same polarization of the proton and the gluon

Braun,et al.PRD83(2011)094023

Overlap Representation

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"tilde" contribution in terms of LFWFs

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$$\tilde{e} \propto \int \Psi^L_{3q+g}(1,2,3,4) \left(\Psi^L_{3q}\right)^* (1,2,3+4)$$

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"tilde" contribution in terms of LFWFs

$$\tilde{e} \propto \int \Psi^{L}_{3q+g}(1,2,3,4)\left(\Psi^{L}_{3q}\right)^{*}(1,2,3+4)$$

$$f^{\perp} \propto \int \Psi^{L}_{3q+g_{\uparrow/\downarrow}}(1,2,3,4)\left(\Psi^{L\pm1}_{3q}\right)^{*}(1,2,3+4)$$

Distribution Amplitudes

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$$\langle 0|\mathcal{O}(z_1, ..., z_n)|P\rangle \propto \int [dx]_n e^{-iP^+ \sum x_i z_i} \mathcal{DA}(x_1, ..., x_n)$$

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$\downarrow$

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↓

6 independent DAs for 3q operator

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126 independent DAs for 3q+g

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↓

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Also for DAs
twist expansion

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Also for DAs
twist expansion

$$\mathcal{DA} \longleftrightarrow \text{LFWA}$$

The model

The model

Up to now only L=0 LFWFs

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$$\Psi_N^{L=0} = \phi(x_1, ..., x_N)\Omega_N \left(\{x_i\}, \{\boldsymbol{k}_{\perp,i}\}, \{a_i\}\right)$$

The model

Up to now only *L*=0 LFWFs

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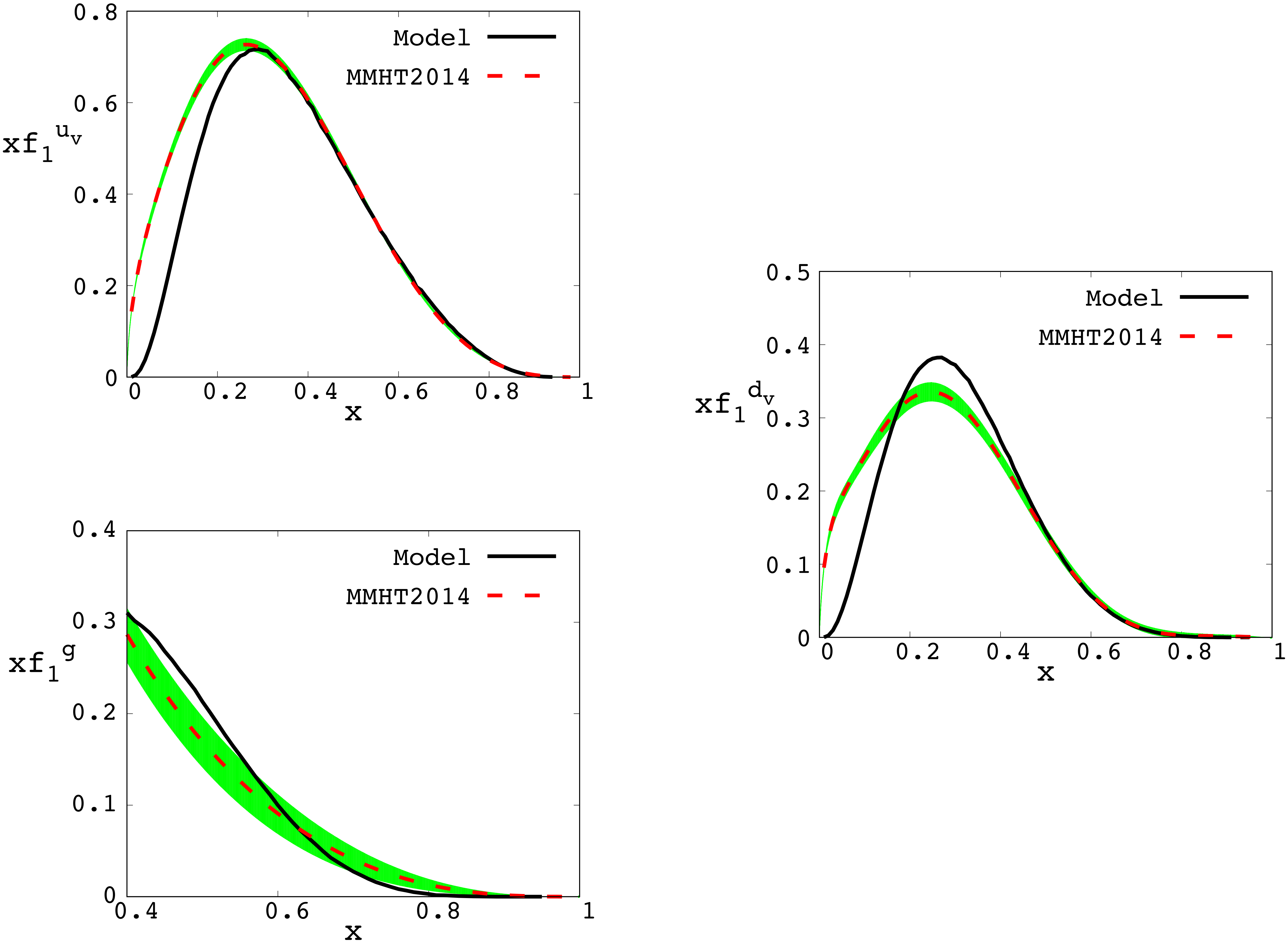
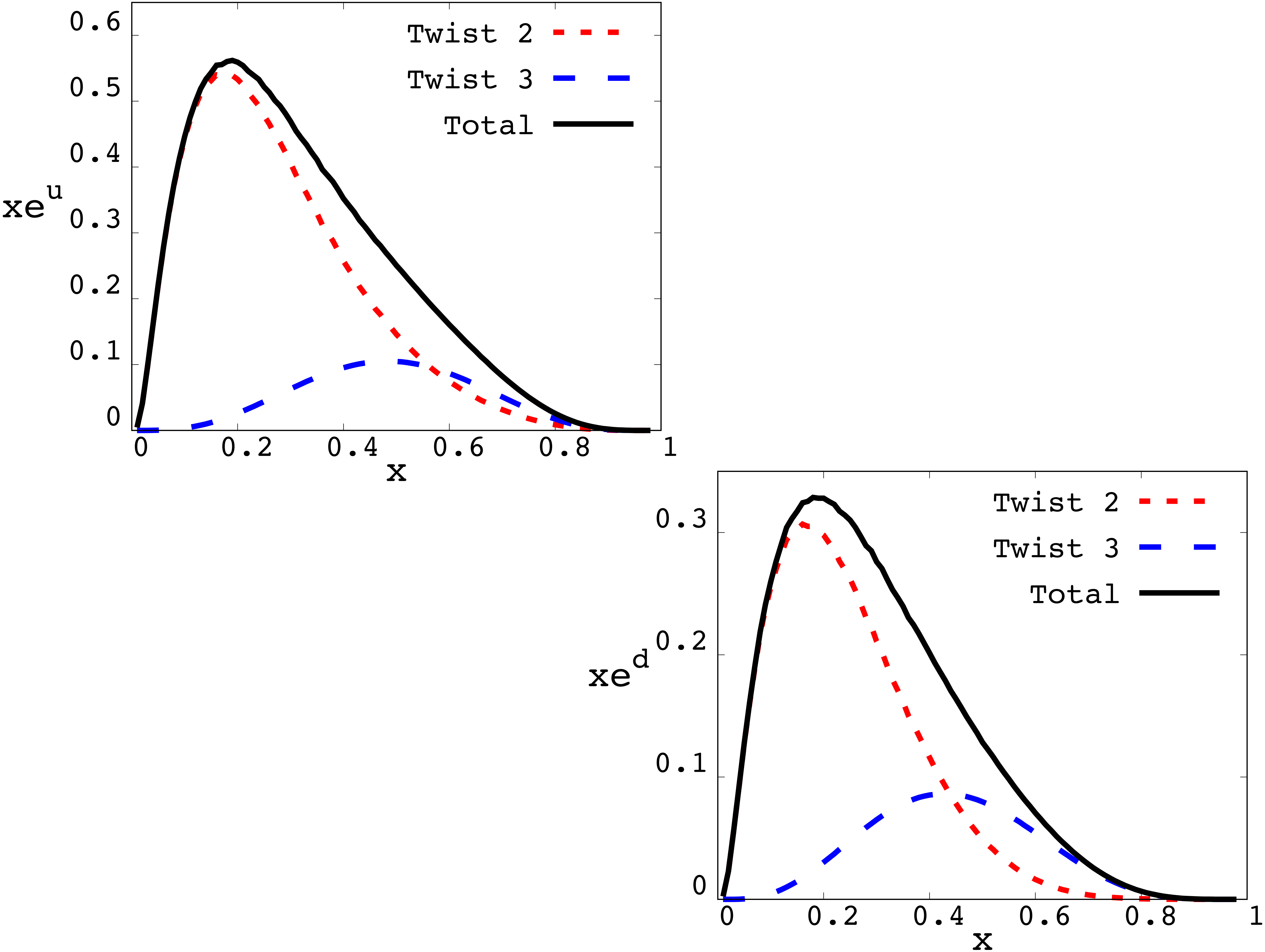
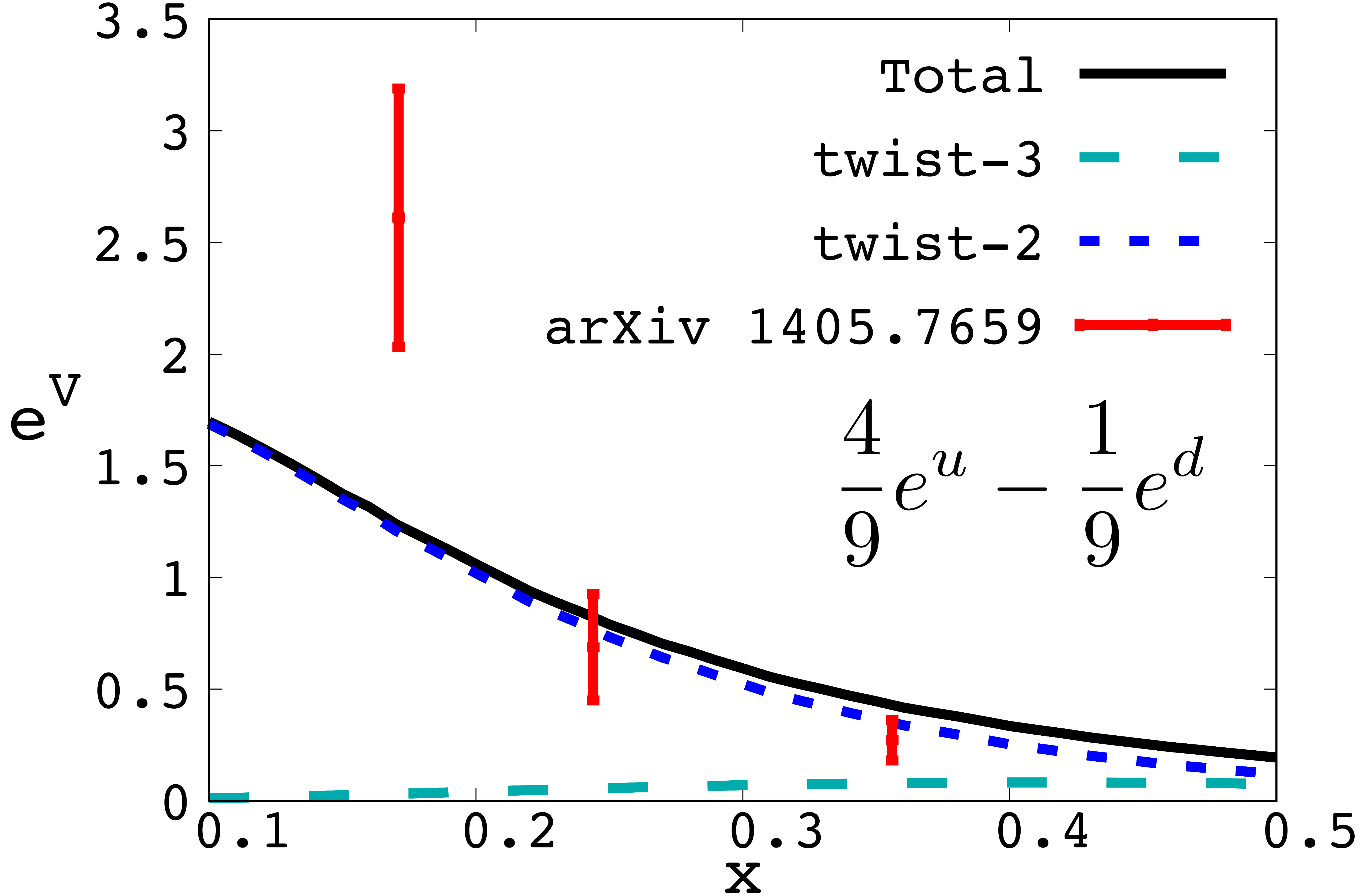
$L \neq 0$ LFWFs are related to higher-twist DAs

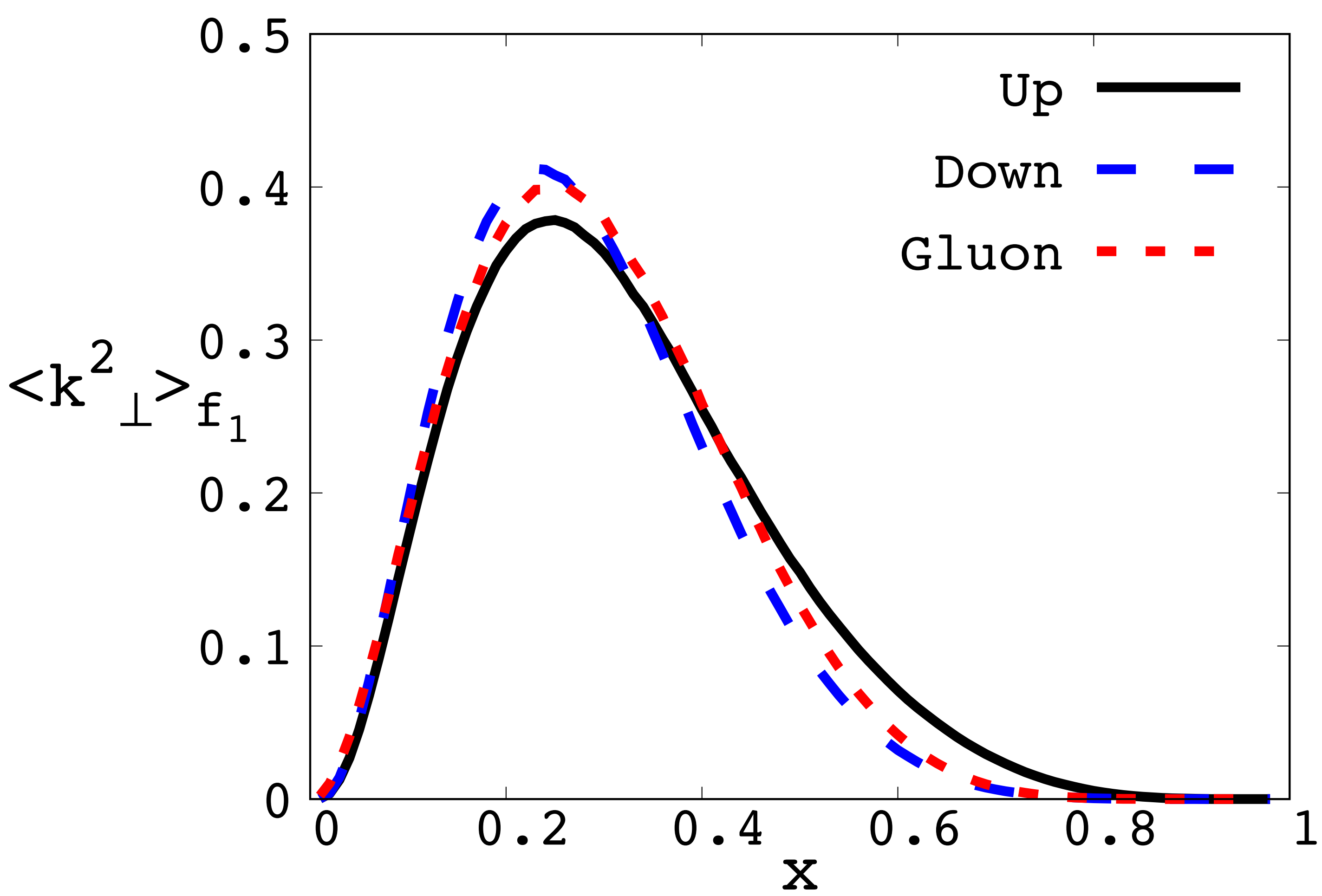


Model
MMHT2014
$xf_1^{u_v}$
x
Model
MMHT2014
$xf_1^{d_v}$
x
Model
MMHT2014
xf_1^{g}
x

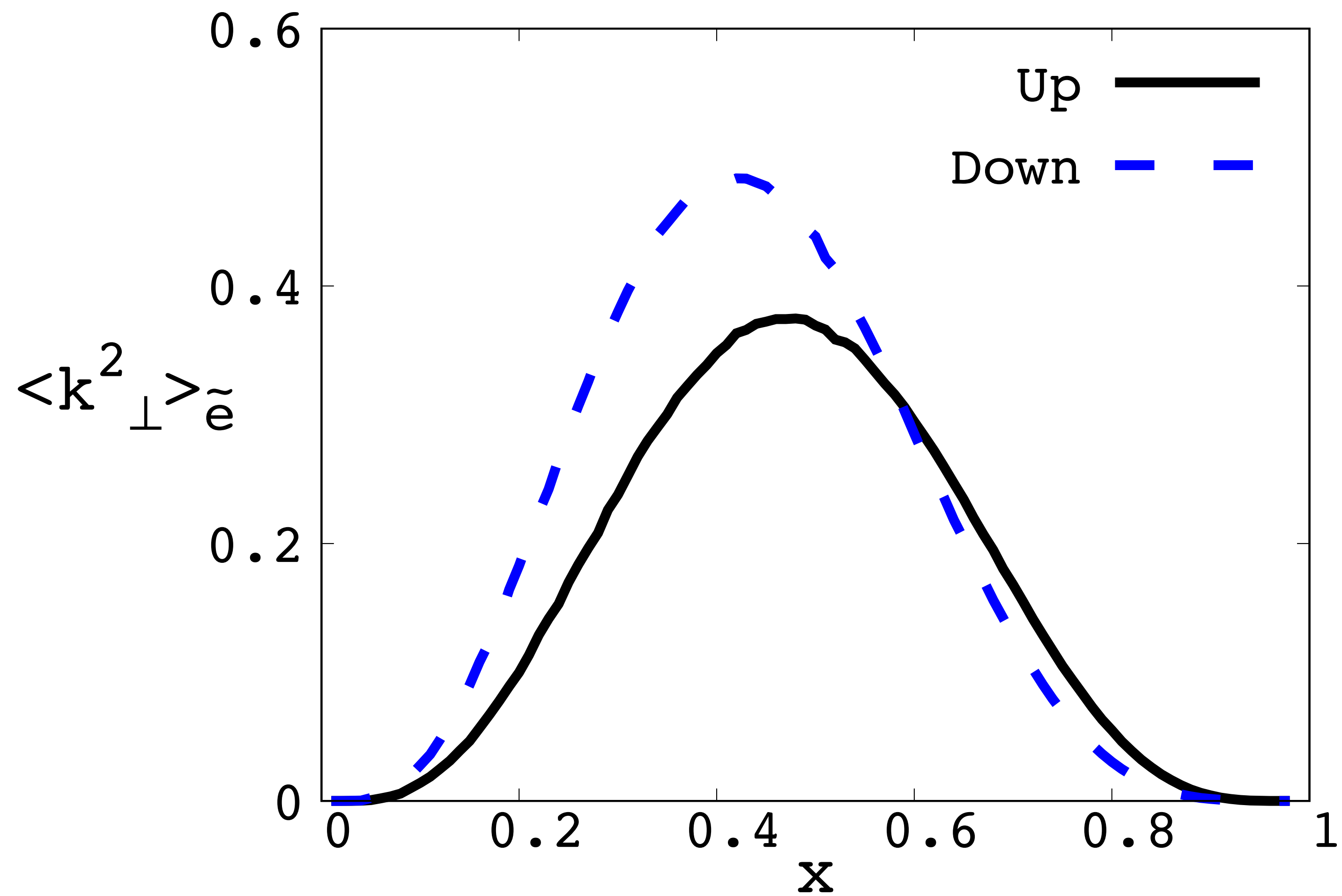

Twist 2
Twist 3
Total
xe^u
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0
0.2
0.4
0.6
0.8
1
Twist 2
Twist 3
Total
xe^d
x
0
0.1
0.2
0.3
0
0.2
0.4
0.6
0.8
1

Valence Distribution





Up
Down
Gluon
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
x
$<k^2_\perp>_{f_1}$



Up
Down
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
x
$<k^2_\perp>_{\tilde{e}}$

Why?

Why?

Parton's correlation!

Why?

Parton's correlation!

Constituent gluon!

Why?

Parton's correlation!

Constituent gluon!

Future prospects?

Why?

Parton's correlation!

Constituent gluon!

Future prospects?

OAM for 3q state

Why?

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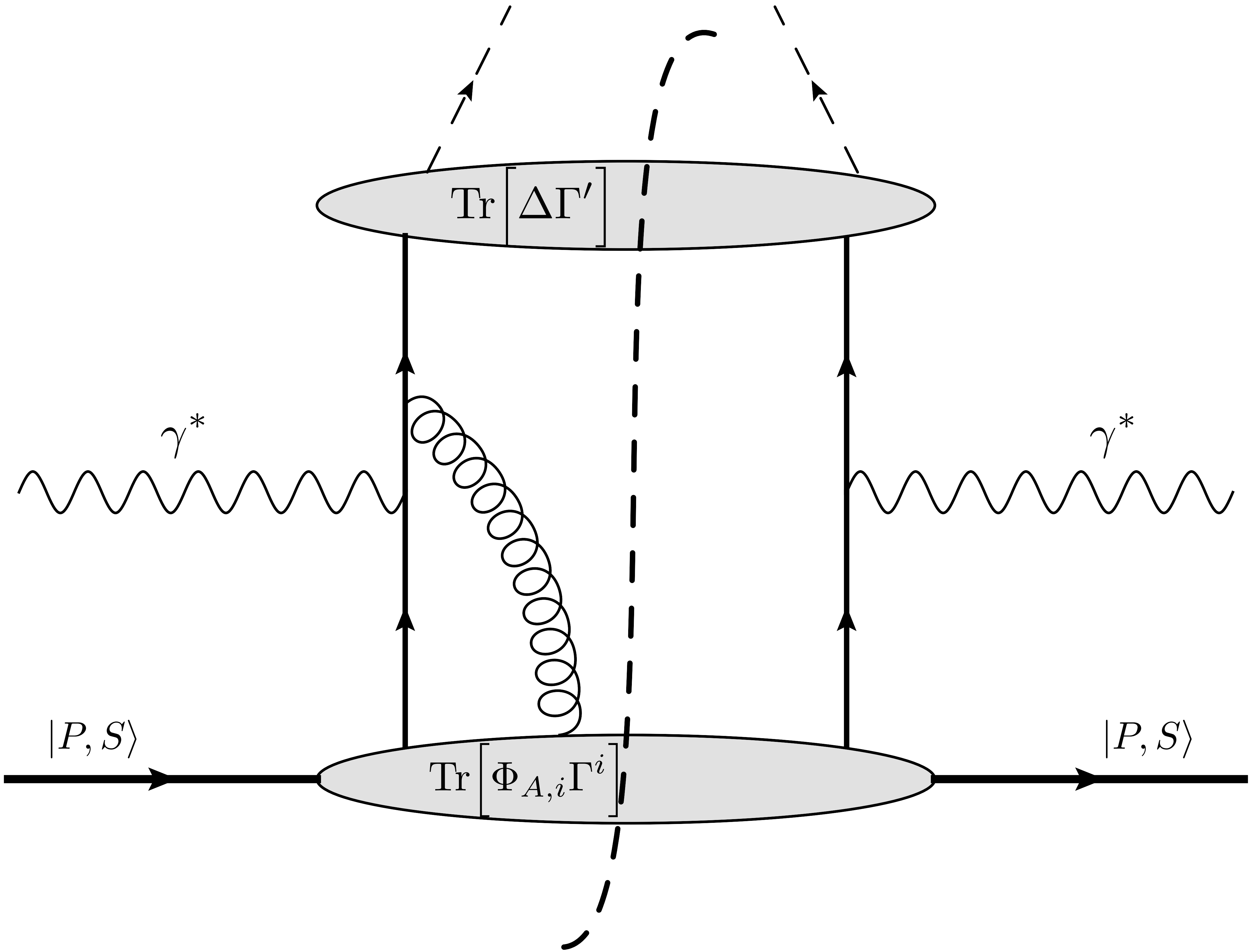
OAM for 3q state

Wider # of PDF/TMD

BACKUP

SIDIS



$\mathrm{Tr}\left[\Delta\Gamma'\right]$
γ^*
γ^*
$|P,S\rangle$
$\mathrm{Tr}\left[\Phi_{A,i}\Gamma^i\right]$
$|P,S\rangle$

Model

The overlap results are independent from a specific model

LFWAs ⟷ Distribution Amplitudes

$$\Psi^{(0)}(1,2,3) = \frac{1}{4\sqrt{6}} \phi(x_1, x_2, x_3) \Omega_3(1,2,3;a_3)$$

Leading-twist 3q distribution amplitude

Gaussian in $\boldsymbol{k}_\perp$

$$\Omega_N = \frac{\left(16\pi^2 a_N^2\right)^{N-1}}{\prod_{i=1}^{N} x_i} exp\left\{ - a_N^2 \sum_{i=1}^{N} \frac{k_{\perp,i}^2 + m_i^2}{x_i} \right\}$$

Model

For the 3q+g state:

$$\Psi^{\downarrow}(1,2,3,4) = \frac{1}{\sqrt{2x_4}}\, \phi^{\downarrow}(x_1,x_2,x_3,x_4)\, \Omega_4(1,2,3,4;a_{\downarrow})$$

$$\Psi^{1\uparrow}(1,2,3,4) = \frac{1}{\sqrt{2x_4}}\, \psi(x_1,x_2,x_3,x_4)\, \Omega_4(1,2,3,4;a_{\uparrow})$$

$$\Psi^{2\uparrow}(1,2,3,4) = \frac{1}{\sqrt{2x_4}}\, \theta(x_1,x_2,x_3,x_4)\, \Omega_4(1,2,3,4;a_{\uparrow})$$

$$a_{\downarrow} = a_{\uparrow} = a_4$$

Related to the three leading-twist 3q+g distribution amplitudes

Remark: only states with zero OAM

Model

$$\phi = 120 f_N x_1 x_2 x_3 \left(1 + A(x_1 - x_3) + B(x_1 + x_3 - 2x_2)\right)$$

$$\phi^{\downarrow} = -\frac{M}{96 g_s}\frac{8!}{2}\lambda_1^g x_1 x_2 x_3 x_4^2$$

$$\psi = -\frac{M}{96 g_s}\frac{8!}{4}\left(\lambda_2^g + \lambda_3^g\right) x_1 x_2 x_3 x_4^2$$

$$\theta = -\frac{M}{96 g_s}\frac{8!}{4}\left(\lambda_2^g - \lambda_3^g\right) x_1 x_2 x_3 x_4^2$$

PARAMETERS

$$f_N, \quad A, \quad B, \quad a_3,$$

$$\lambda_1, \quad \lambda_2, \quad \lambda_2, \quad \frac{a_4}{a_3}$$

$$m_q, \quad m_g,$$

$$F_{\mathrm{UU}}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\left[\ldots - \frac{\boldsymbol{P}_h \cdot \boldsymbol{k}_\perp}{|\boldsymbol{P}_h|M}\left(xf^{\perp}D_1 + \frac{M_h}{M}h_1^{\perp}\frac{\tilde{H}}{z}\right)\right]$$

$$F_{\mathrm{LU}}^{\sin\phi_h} = \frac{2M}{Q}\mathcal{C}\left[\ldots - \frac{\boldsymbol{P}_h \cdot \boldsymbol{p}_\perp}{|\boldsymbol{P}_h|M}\left(xeH_1^{\perp} + \frac{M_h}{M}f_1\frac{\tilde{G}^{\perp}}{z}\right)\right]$$