NUCLEAR PHYSICS AROUND THE UNITARITY LIMIT

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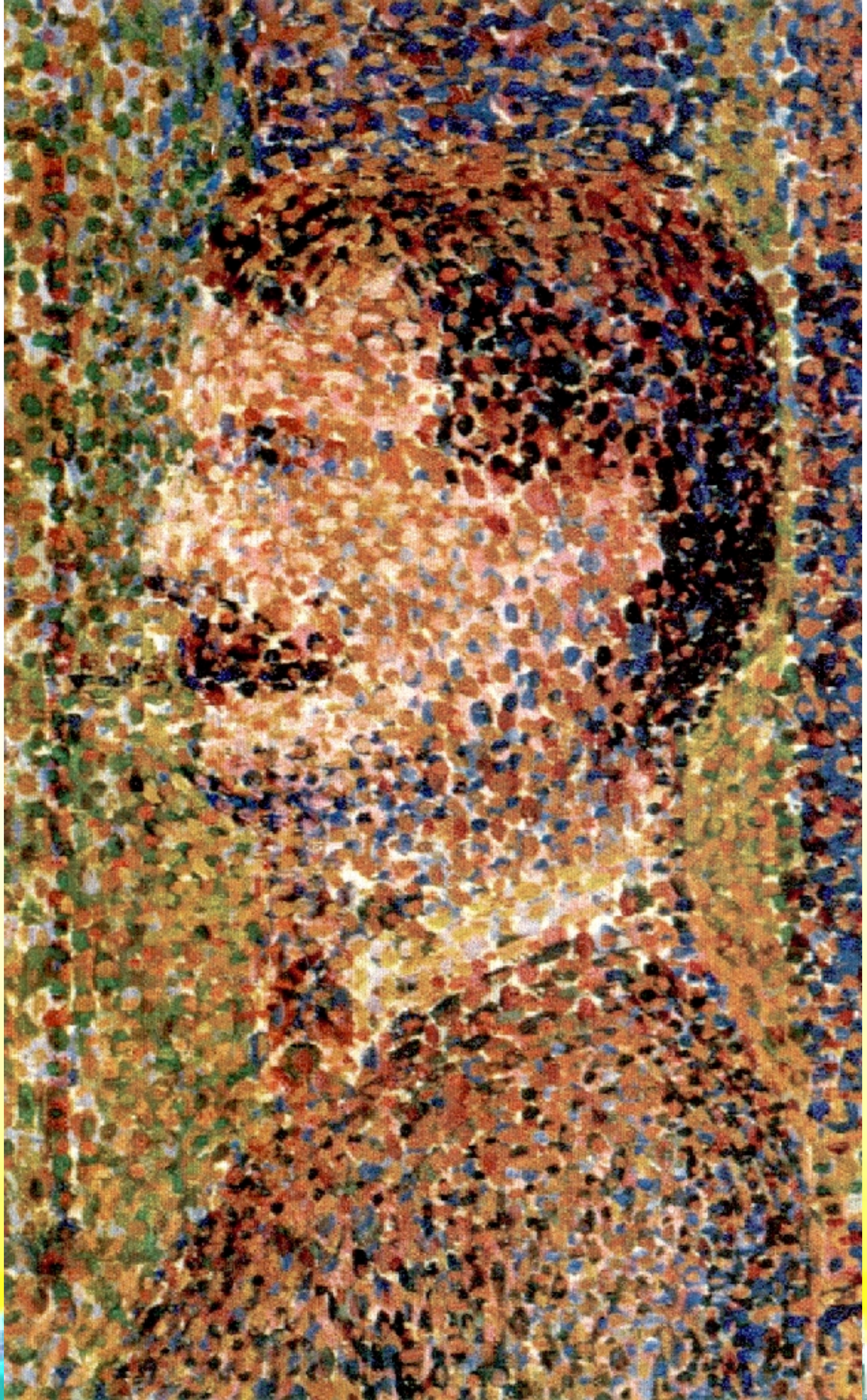
El Greco,
1586-90

"We must sow the seed, not hoard it."
Santo Domingo (1170-1221)



Seurat,
La Parade
(detail)

Outline

- What is essential?
- Effective Field Theory
- Unitarity: light nuclei
- Unitarity: bosonic clusters
- Unitarity: matter
- Conclusion

with

S. König,
H.W. Grießhammer
& H.-W. Hammer

S. Gandolfi,
J. Carlson,
& S.A. Vitiello

S. Lyu,
& B. Long

What is essential in nuclear physics?

Traditional approach

1) Describe NN precisely to some high energy
2) Append 3N forces as needed

→ Lost in details:

e.g., chiral potentials at NLO, N^{2}LO, N^{3}LO, N^{4}LO, …

(apparently no nuclear matter saturation at LO!)

Here

1) Describe NN and 3N approximately
2) Treat all other interactions in *perturbation theory*

cf. atomic systems in QED

“simplicity emerging
from complexity”

Expansion
around unitarity

König, Grießhammer,
Hammer + v.K. ’15 ’16
König ’16
v.K. ’17

$A=2$

$$T_2(k \ll R^{-1}) = \frac{4\pi}{m}\left(a_2^{-1} + ik - \frac{r_2}{2}k^2 + \frac{P_2}{4}k^4 + \ldots\right)^{-1} + (l>0)$$

scattering length

effective range

shape parameter

unitarity limit $a_2^{-1} \approx \sqrt{mB_2} \to 0$

$$\underbrace{\frac{r_2}{2} \sim R, \quad \frac{P_2}{4} \sim R^3, \quad \ldots}$$ typically

nucleons

$R^{-1} \sim m_\pi$

${}^1S_0$

$$\left|a_{2,I=1,I_3=0}m_\pi\right|^{-1} \simeq 0.06$$

$$\left|a_{2,I=1,I_3=+1}m_\pi\right|^{-1} - \left|a_{2,I=1,I_3=0}m_\pi\right|^{-1} \simeq 0.12$$

$$\left|a_{2,I=1,I_3=-1}m_\pi\right|^{-1} - \left|a_{2,I=1,I_3=0}m_\pi\right|^{-1} \simeq 0.02$$

${}^3S_1$ $\left|a_{2,I=0}m_\pi\right|^{-1} \simeq 0.26$

$$T_2\left(\left|a_2^{-1}\right| \ll k \ll R^{-1}\right) = \frac{4\pi}{m}(ik)^{-1}\left(1 + \mathcal{O}\left(\frac{1}{ka_2}, kR\right)\right)$$

unitarity window

no parameter!

"universality"

$A>2$

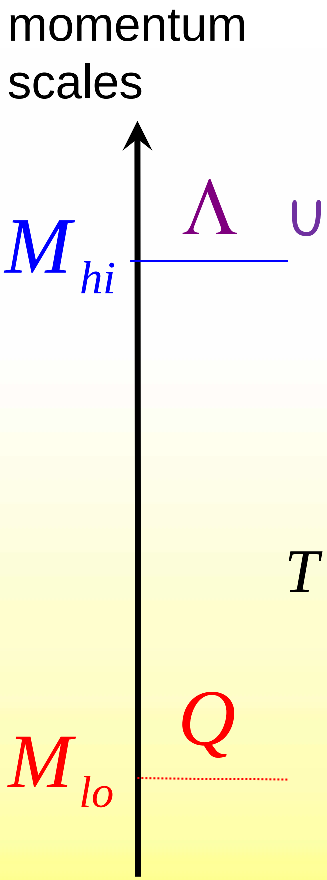
binding momenta

$$Q_A/m_\pi \sim \left(\frac{2m_N}{m_\pi^2}\frac{B_A}{A}\right)^{1/2} \sim 0.7$$

in unitarity window!

Effective Field Theory

most general action with symmetries



arbitrary UV regulator

model independence

i.e., consistent with QCD!

$$\frac{\partial T}{\partial \Lambda} = 0$$

renormalization-group invariance

"low-energy constants"

$$T(Q \sim M_{lo} \ll M_{hi}) \propto \sum_{\nu=0}^{\infty} \left[\frac{Q}{M_{hi}}\right]^{\nu} F^{(\nu)}\underbrace{\left(\frac{Q}{M_{lo}}, \frac{Q}{\Lambda}; \overbrace{\gamma_i^{(\nu)}\left(\frac{\Lambda}{M_{lo}}, \frac{M_{lo}}{M_{hi}}\right)}\right)}$$

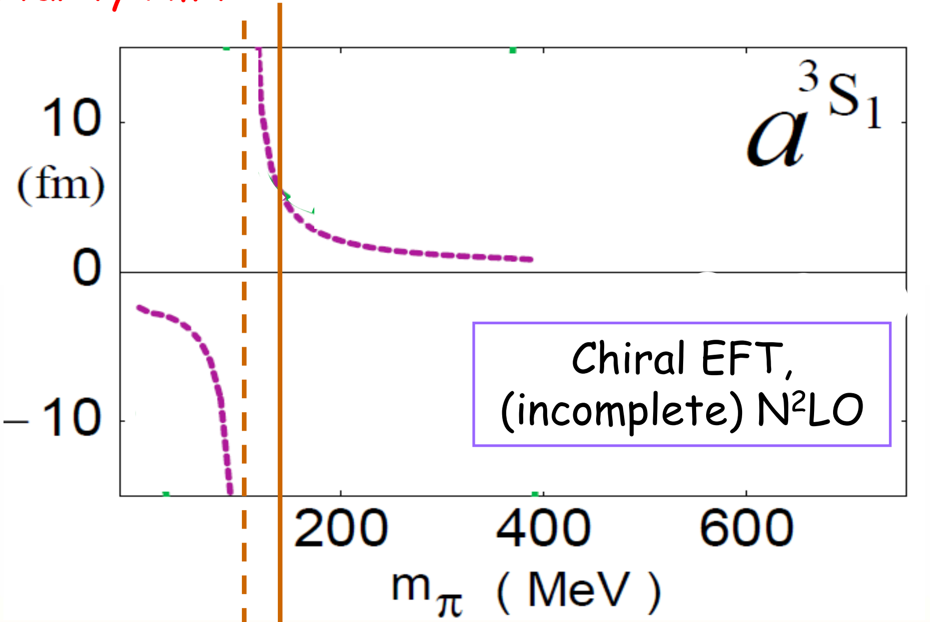
controlled expansion

$N^{\nu}LO$

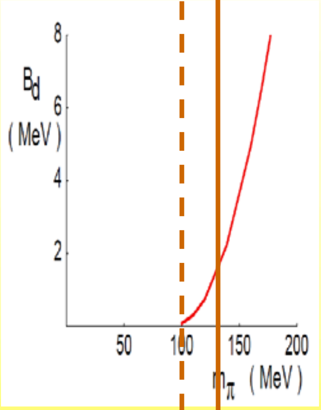
non-analytic functions,
from solution of dynamical eq.
(*e.g.* Lippmann-Schwinger)

The quantum generalization of the multipole expansion

unitarity limit



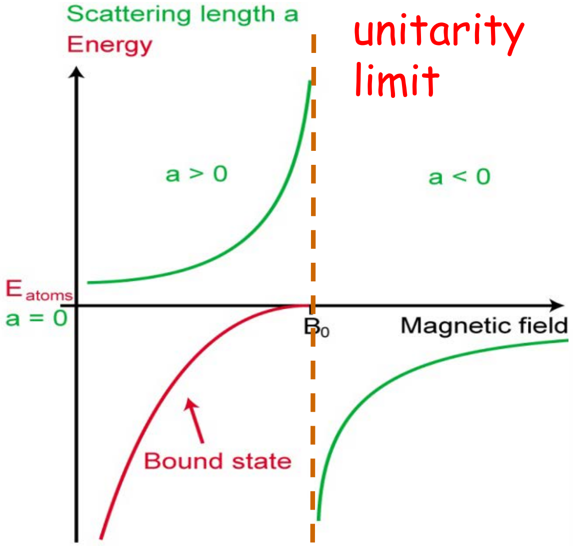
$m_\pi^*\left(M_{QCD}\right)$ $m_\pi \simeq 140\,\mathrm{MeV}$

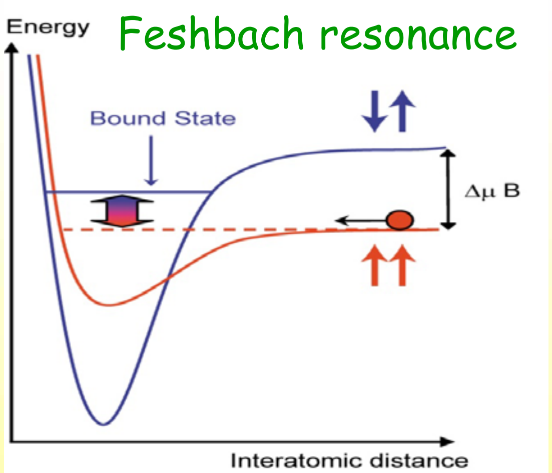


Beane, Bedaque, Savage + v.K. '02
Beane + Savage '03'04
Epelbaum, Glöckle + Meißner '03
Braaten + Hammer '03
...

$$a_2 \gg m_\pi^{-1} \sim r_2$$

[can do nuclear physics at unphysical quark masses as well, but that's another talk]





MIT webpage

or "accidentally", *e.g.* ^{4}He atoms

$$a_2 \gg l_{\mathrm{vdW}} \sim r_2$$

Pionless EFT

$Q \sim M_{lo} \ll M_{hi} \sim m_\pi$

- d.o.f.: nucleons
- symmetries: ~~Lorentz~~, ~~P~~, ~~T~~, ~~B~~, SU(3)$_c$, U(1)$_{em}$
  (trivial)

$$S_{EFT} = \int \frac{dt}{2m_N} \int d^3r \left\{ \psi^+ \left(2im_N \frac{\partial}{\partial t} + \vec{\nabla}^2 \right) \psi \right.$$

projector on isospin *I*

$$- 4\pi \sum_{I=0,1} C_{0I} \; \psi^+ \psi^+ \boxed{P_I} \psi \psi$$

$$- \frac{(4\pi)^2}{3} D_0 \; \psi^+ \psi^+ \psi^+ \; \psi \psi \psi$$

$$\left. + \; \ldots \right\}$$

more derivatives,
more bodies,
isospin violation,
...

most general action

Universality:
first orders
apply also to
neutral atoms

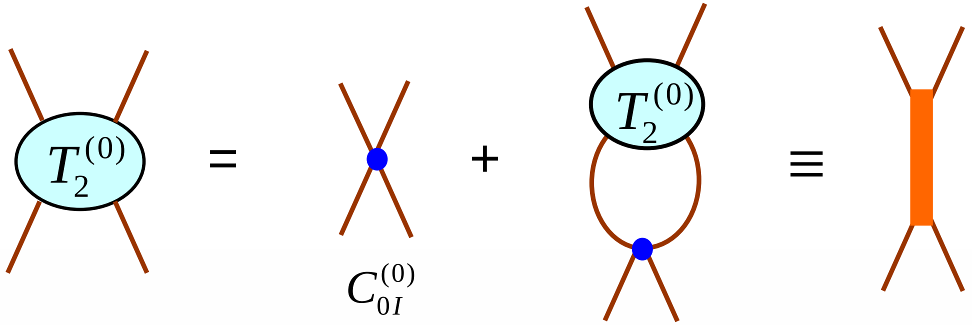
$m_\pi \to 1/l_{\mathrm{vdW}}$ *where* $V(r) = -\frac{l_{\mathrm{vdW}}^4}{2mr^6} + \ldots$

Bedaque, Hammer
\+ v.K. '99'00
Bedaque, Braaten
\+ Hammer '01
...

v.K. '97'98
Kaplan, Savage + Wise '98
Chen, Rupak + Savage '99
...

$A = 2$

LO



singular potential! $V_2^{(0)} = \frac{4\pi}{m_N} C_{0I}^{(0)}\, \delta^{(3)}(\vec{r}) \to \frac{4\pi}{m_N} C_{0I}^{(0)}(\Lambda)\, \delta_\Lambda^{(3)}(\vec{r})$

$\cancel{C_{0I}^{(0)}(\Lambda) = C_{0I}^{(0)} \Rightarrow B_d \propto \Lambda^2 / m_N}$

renormalization: $C_{0I}^{(0)}(\Lambda) = -\frac{1}{\theta_0 \Lambda} \Leftrightarrow B_d^{(0)} = 0$

regulator-dependent number → θ_0

scale invariance

$$\left.\begin{array}{ll} x \to \alpha x & \Lambda \to \alpha^{-1}\Lambda \\ \frac{t}{m} \to \alpha^2 \frac{t}{m} & \psi \to \alpha^{-3/2}\psi \end{array}\right\} \quad S_{EFT}^{(0)} \to S_{EFT}^{(0)}$$

Mehen, Stewart + Wise '00
König, Grießhammer, Hammer + v.K. '16
v.K. '17

NLO

- scattering lengths
- effective ranges
- Coulomb

in distorted-wave Born approximation

etc.

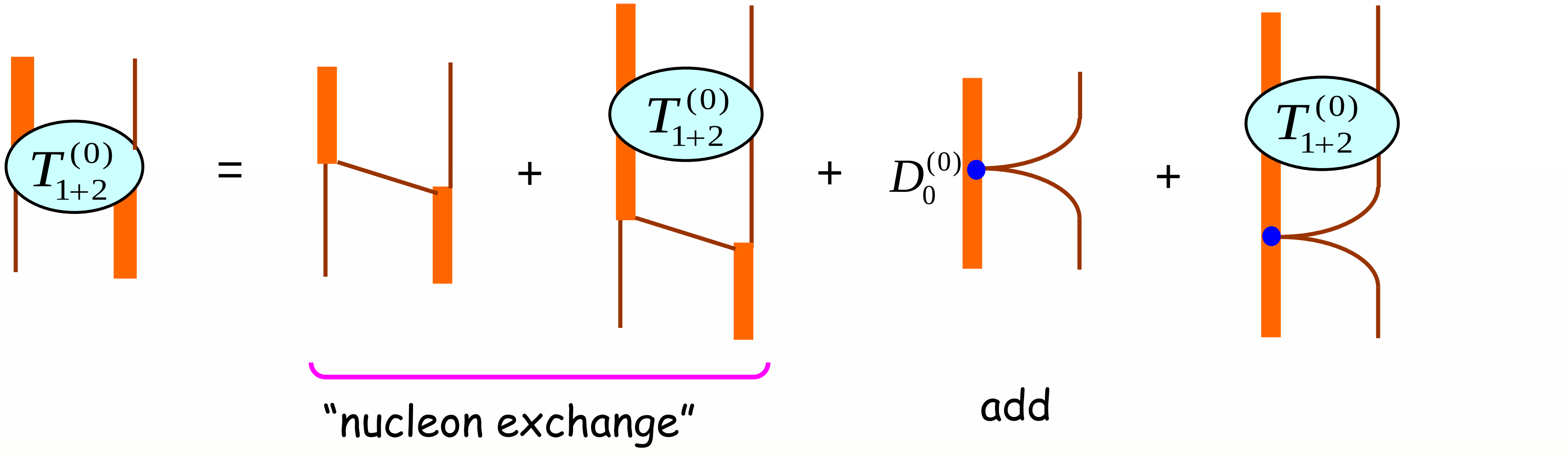
scale invariance
broken explicitly

e.g., $B_d^{(2)} = 1\big/\left(m_N a_{2,I=0}^2\right)$

Bedaque, Hammer + v.K. '99 '00
...

LO

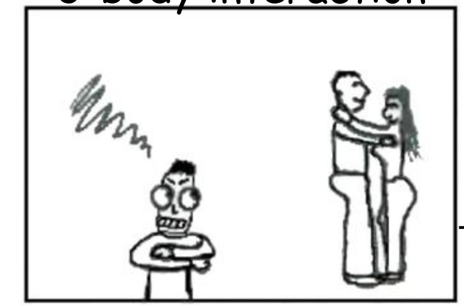
$A=3$



Thomas '35 $\Rightarrow \cancel{B_t \propto \Lambda^2/m_N}$

$$V_3^{(0)} = \frac{(4\pi)^2}{m_N} D_0^{(0)}(\Lambda)\, \delta_\Lambda^{(3)}(\vec{r}_{12})\, \delta_\Lambda^{(3)}(\vec{r}_{23})$$

3-body interaction



Wikipedia*

renormalization:

$$D_0^{(0)}(\Lambda) \propto \frac{1}{\Lambda^4} \frac{\sin\left(s_0 \ln(\Lambda/\Lambda_*) + \arctan s_0^{-1}\right)}{\sin\left(s_0 \ln(\Lambda/\Lambda_*) - \arctan s_0^{-1}\right)}$$

$s_0 = 1.00624...$

dimensionful parameter
(dimensional transmutation)

$\Lambda_* \simeq 49\ \text{MeV} \Leftrightarrow B_t = 8.48\ \text{MeV}$

discrete
scale
invariance

$$x \to \alpha_n x \qquad \Lambda \to \alpha_n^{-1}\Lambda$$

$$\frac{t}{m} \to \alpha_n^2 \frac{t}{m} \qquad \psi \to \alpha_n^{-3/2}\psi$$

$$\Bigg\} \quad S_{EFT}^{(0)} \to S_{EFT}^{(0)}$$

$$\alpha_n = \exp(n\pi/s_0) = (22.7)^n \equiv f^{\,n}$$

$$n = \cancel{\ldots, -2, -1,}\, 0, 1, 2, \ldots$$

ground state

$$\updownarrow$$

Efimov states

$$B_{3n}^{(0)} = \exp(-2n\pi/s_0)\, k_*^2/m_N \lesssim M_{hi}^2/m_N$$

$$k_* \propto \Lambda_*$$

Efimov '71
...

Similar for bosons

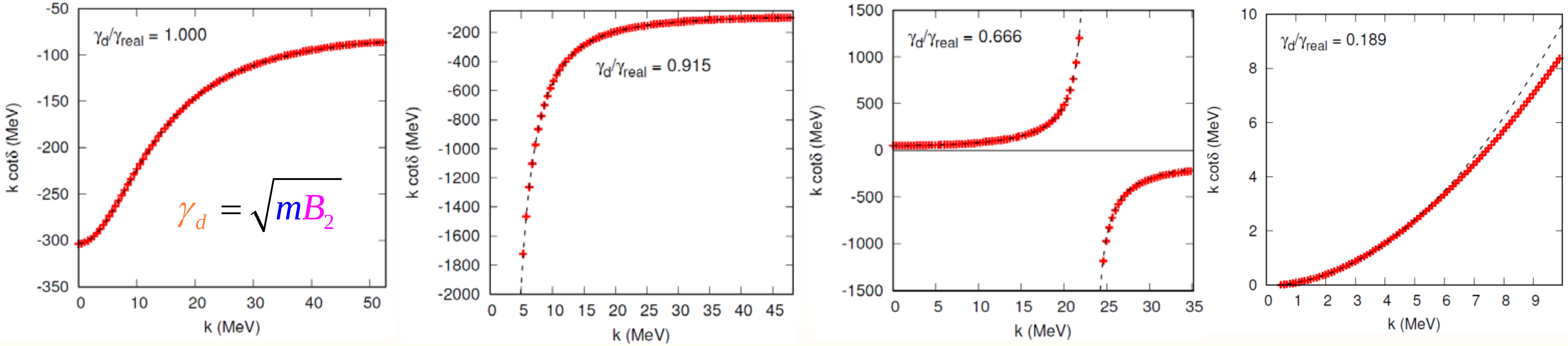
Bedaque, Hammer + v.K. '99 '00
...

Rupak, Vaghani,
Higa + v.K. '18

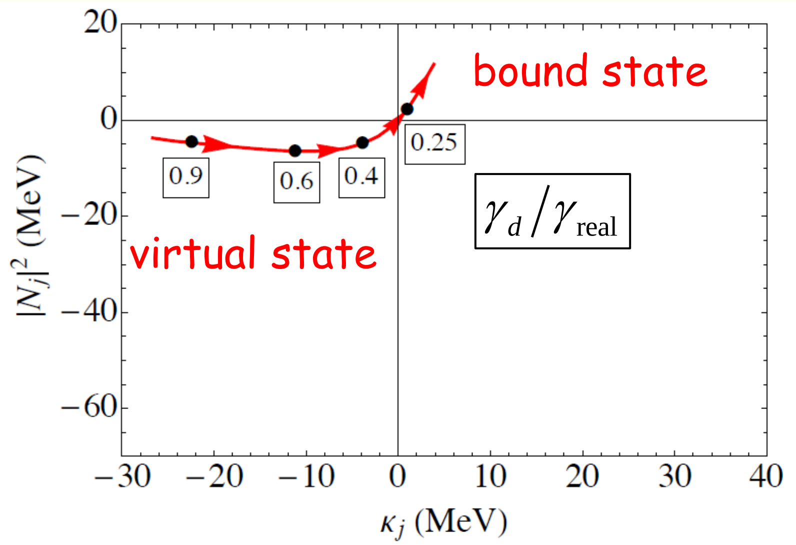
The first excited state of triton

LO

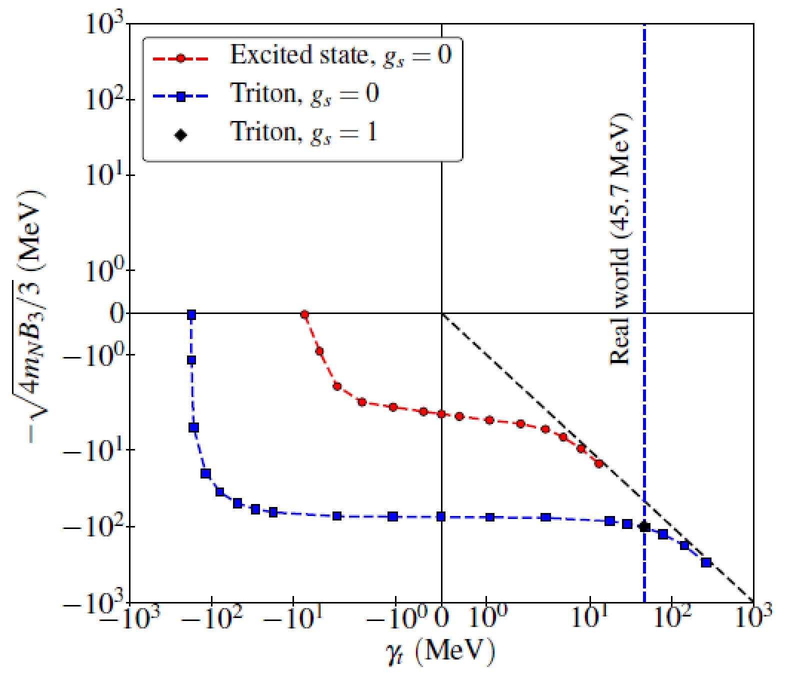
nd scattering below deuteron break-up → unitarity



normalization



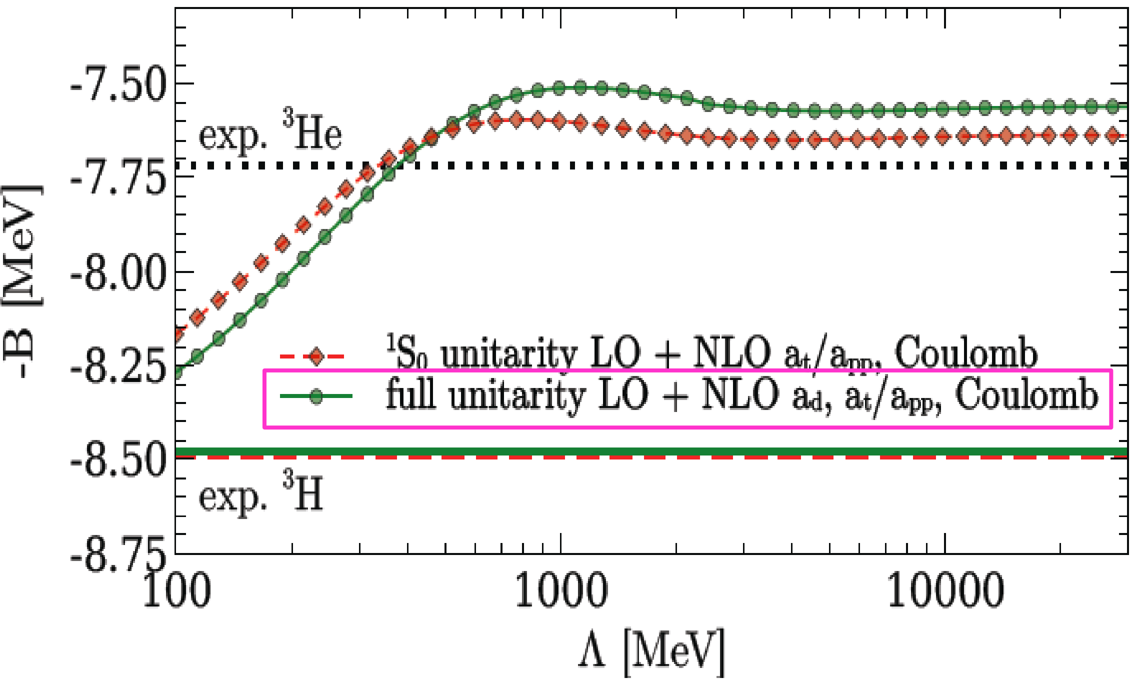
binding momentum



König, Grießhammer, Hammer + v.K. '16

NLO

no new three-body force



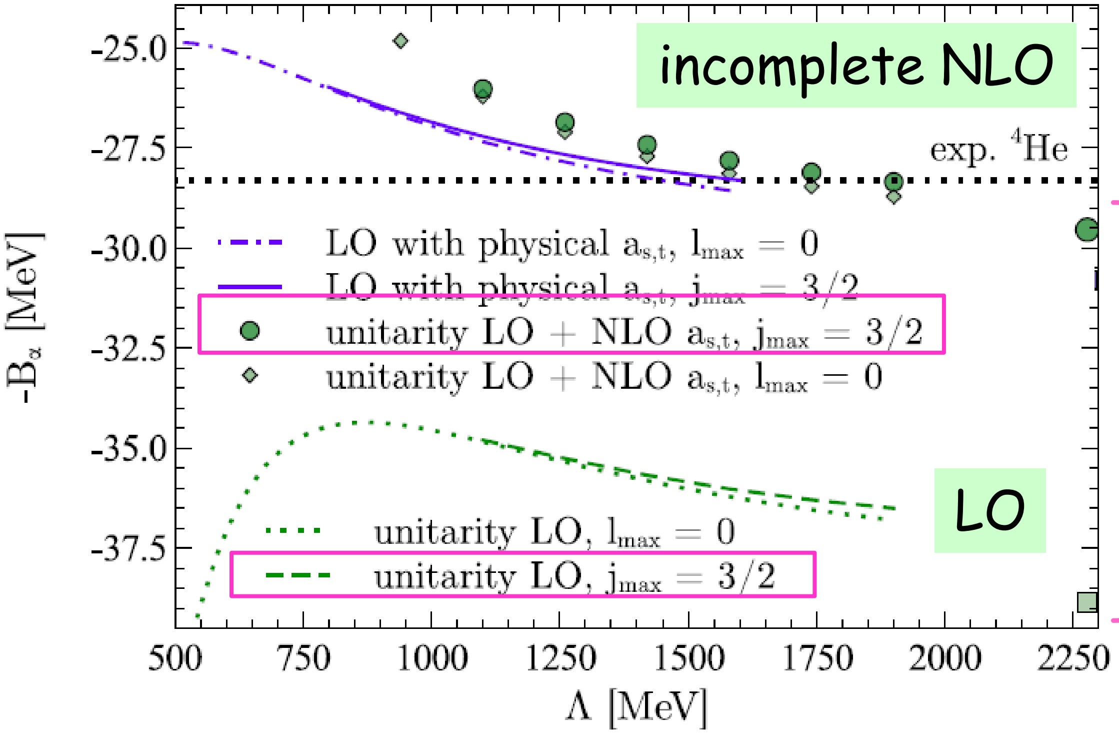
$$B_h^{(1)} - B_t \simeq -(0.92 \pm 0.18)\ \text{MeV}$$

vs.

$$-0.764\ \text{MeV (exp)}$$

(Fadeev-Yakubovski)

König, Grießhammer,
Hammer + v.K. '16



no ranges

no Coulomb

$A = 4$

infinite-cutoff extrapolation

⇒ no 4BF up to this order

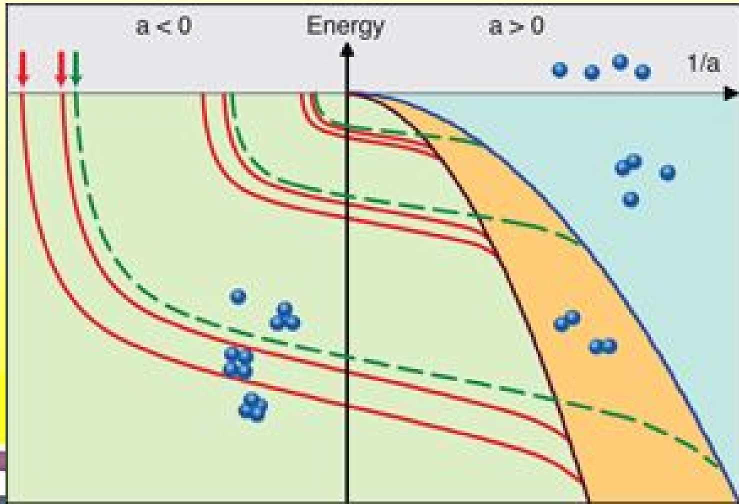
Hammer, Meißner + Platter '05
Stetcu, Barrett + v.K. '07
...

$$B_\alpha^{(1,\mathrm{inc})} \simeq (29.5 \pm 8.7)\ \mathrm{MeV} \quad \text{vs.} \quad 28.4\ \mathrm{MeV\ (exp)}$$

$$B_{\alpha*}^{(0)} - B_t \approx 0 \qquad -0.3\ \mathrm{MeV\ (exp)}$$

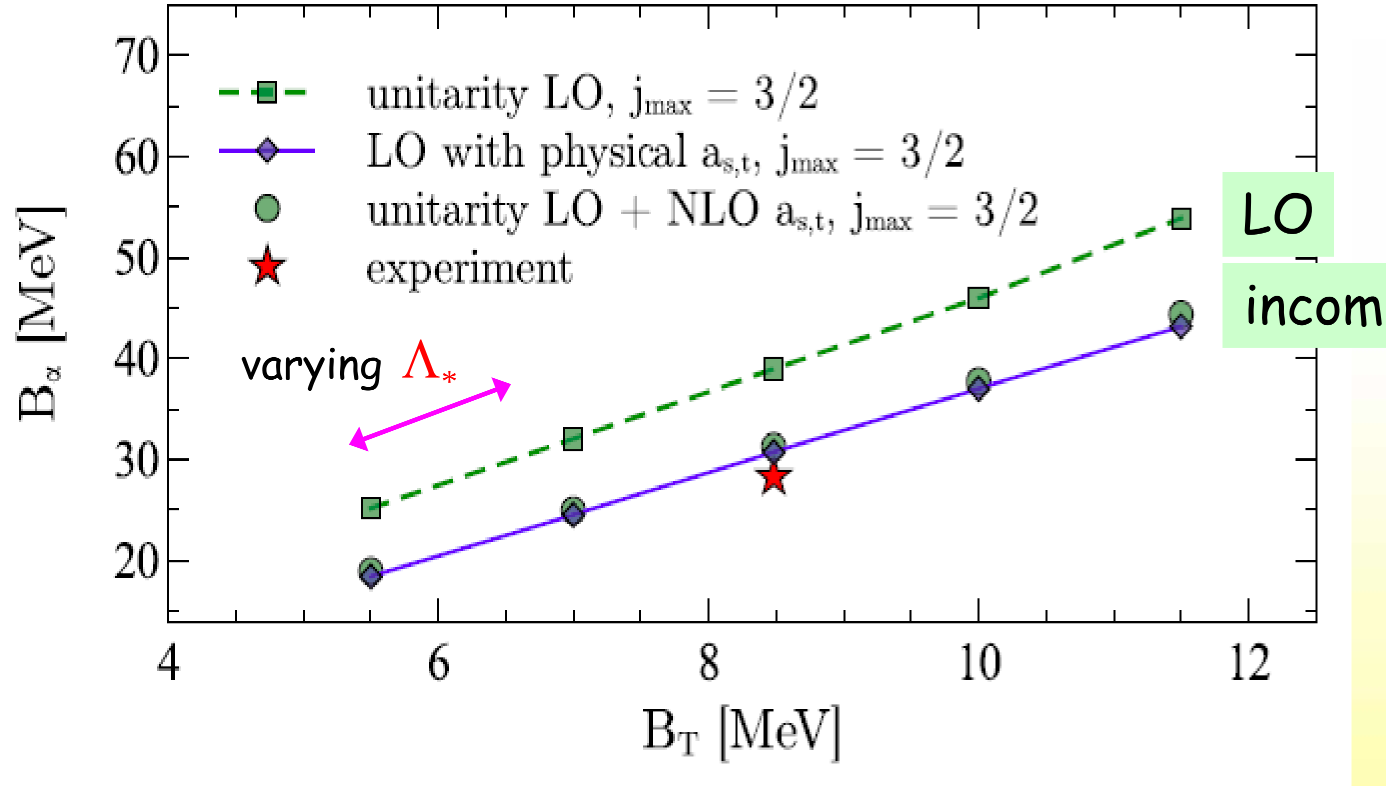
Hammer + Platter '07
...

Similar for bosons



König, Grießhammer,
Hammer + v.K. '16

Tjon line

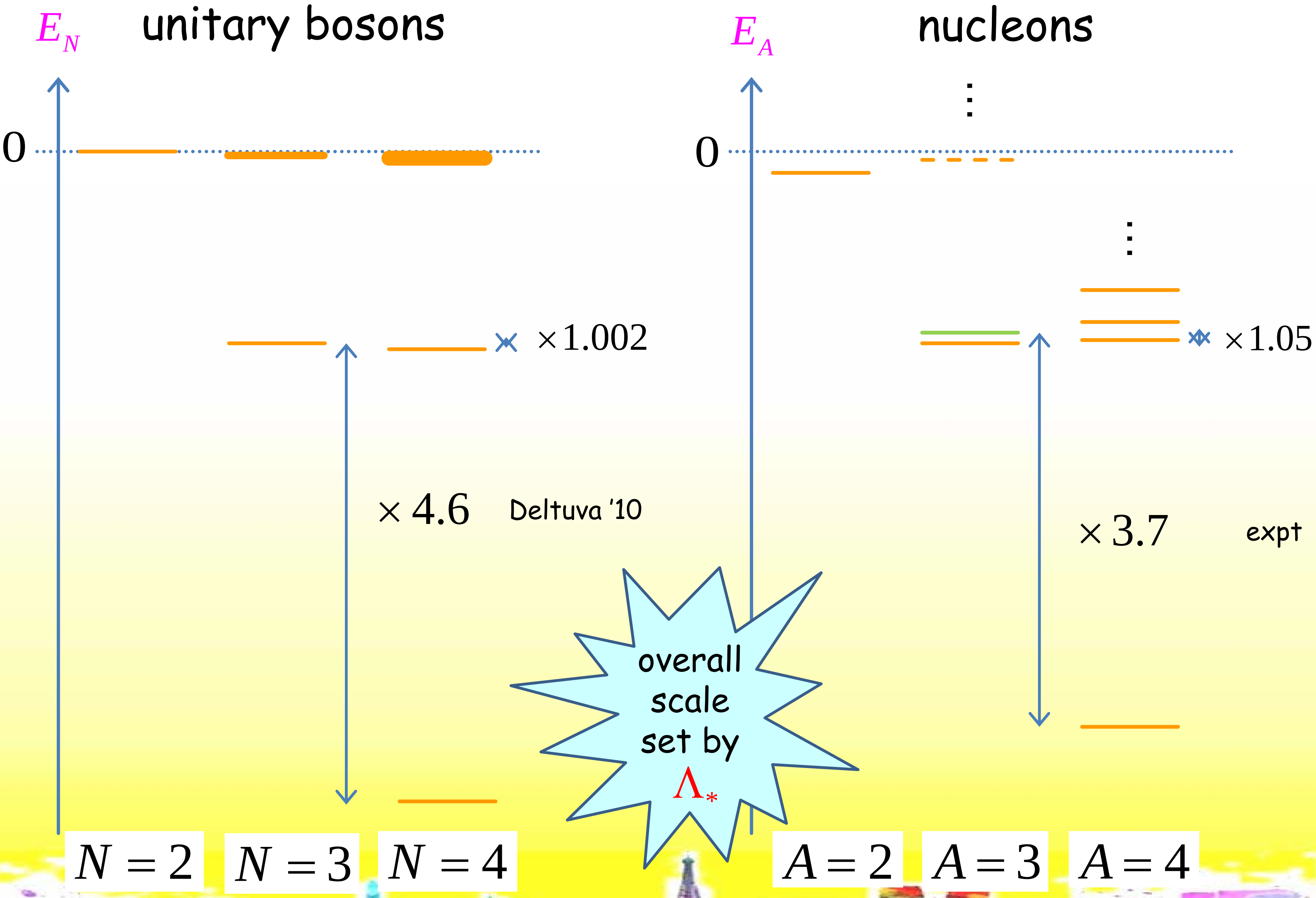


LO

incomplete NLO

perturbatively close to experimental point

Schematically



overall scale set by Λ_*

Unitary bosons

$N \geq 4$

No more-body forces

Bazak, Eliyahu + v.K. '16

LO

discrete

scale

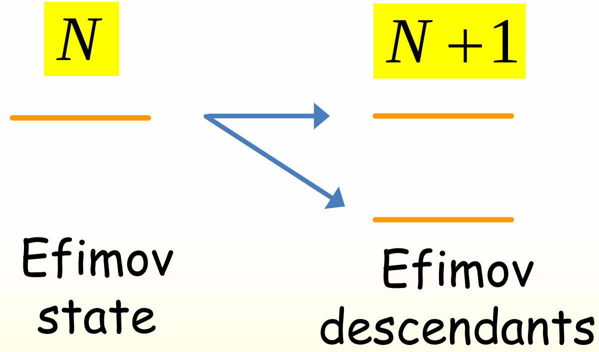
invariance

doubling

Hammer + Platter '07

von Stecher '10'11

Gattobigio, Kievsky + Viviani '11'12



N

$N+1$

Efimov state

Efimov descendants

Towers upon towers!

Ground states

single scale →

$$\frac{B_N^{(0)}(\Lambda_*)}{N} = \boxed{\kappa_N} \frac{B_3(\Lambda_*)}{3}$$

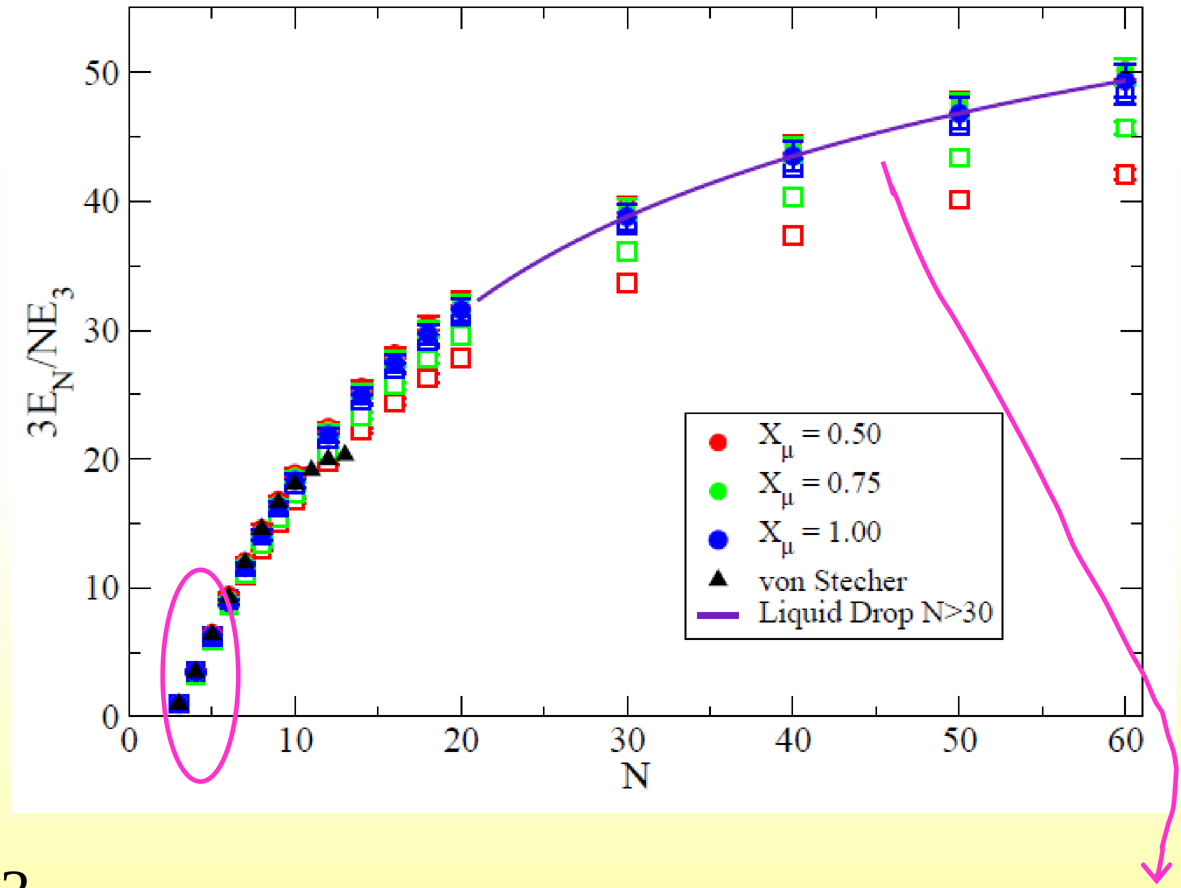
universal numbers

$$\left\{ \begin{array}{l} \kappa_2 \equiv 0 \\ \kappa_3 \equiv 1 \\ \kappa_4 \simeq 3.5 \\ \kappa_{N \geq 5} \simeq ? \end{array} \right.$$

(Variational and Diffusion Monte Carlo)

Gandolfi, Carlson,
Vitiello + vK '17

LO



$$\Lambda_2/Q_3 \simeq 26-38$$

$$\Lambda_3/\Lambda_2 = 0.5-1$$

saturation!

cf. Piatecki + Krauth '14
Comparin + Krauth '16

$$\kappa_N \approx \frac{3}{N}\left(N-2\right)^2$$

Bazak, Eliyahu
+ v.K. '16

$$\kappa_N = \kappa_\infty\left[1+\eta N^{-1/3}+\mathcal{O}\left(N^{-2/3}\right)\right]$$

$$\kappa_\infty = 90\pm10 \qquad \eta = -1.7\pm0.3$$

$A \geq 4$

Nucleons around unitarity

discrete
scale
invariance

doubling?

Ground states

single scale

LO

$$\frac{B_A^{(0)}(\Lambda_*)}{A} = \kappa_A \frac{B_3(\Lambda_*)}{3}$$

$$\left\{ \begin{array}{ll} \kappa_2 \equiv 0 & \\ \kappa_3 \equiv 1 & \\ \kappa_4 \simeq 3.5 & \text{same as for bosons} \\ \kappa_{A\geq 5} \simeq ? & \text{should grow slower than bosons} \end{array} \right.$$

same
saturation mechanism
as for bosons?

Equation of State at Unitarity

$k_\rho \equiv \left(6\pi^2 \rho / \boxed{g}\right)^{1/3}$ degeneracy

two-state fermions

scale invariance ⇒

$$\lim_{N\to\infty} \frac{E_N}{N} = \frac{3k_\rho^2}{10m}\left(\xi + \dots\right)$$

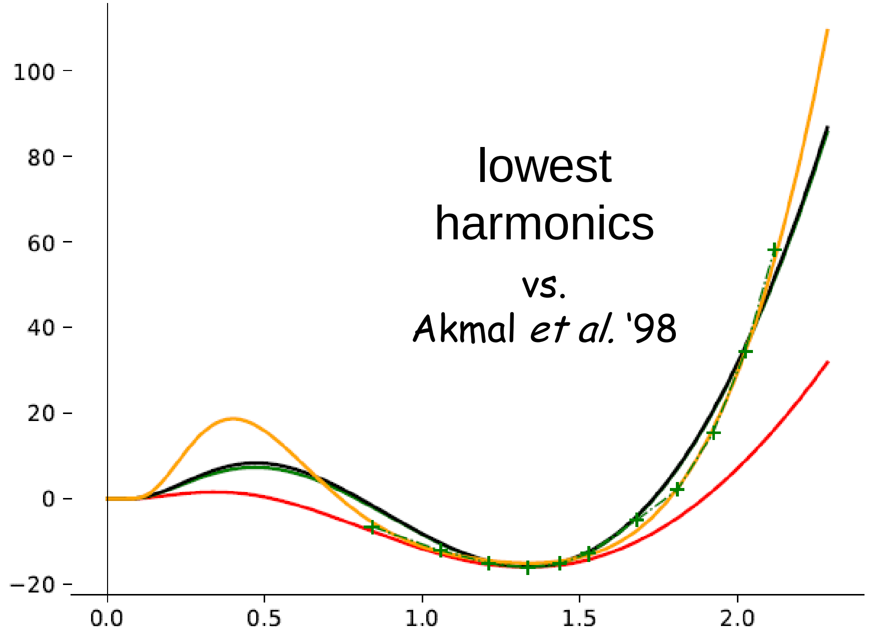
$$\xi \simeq 0.4$$

Bertsch '99
Carlson *et al.* '03
...

bosons and nucleons

discrete scale invariance ⇒

$$\lim_{N\to\infty} \frac{E_N}{N} = \frac{3k_\rho^2}{10m}\mathfrak{G}^{(0)}\left[s_0 \ln\left(k_\rho / k_*\right)\right] + \dots$$



lowest harmonics
vs.
Akmal *et al.* '98

$$\mathfrak{G}^{(0)}\left[\eta + n\pi\right] = \mathfrak{G}^{(0)}\left[\eta\right]$$

$$\equiv \gamma_0 - \sum_{l=1}^{\infty}\left[\gamma_l \cos(2l(\eta-\eta_0)) + \gamma_l' \sin(2l(\eta-\eta_0))\right]$$

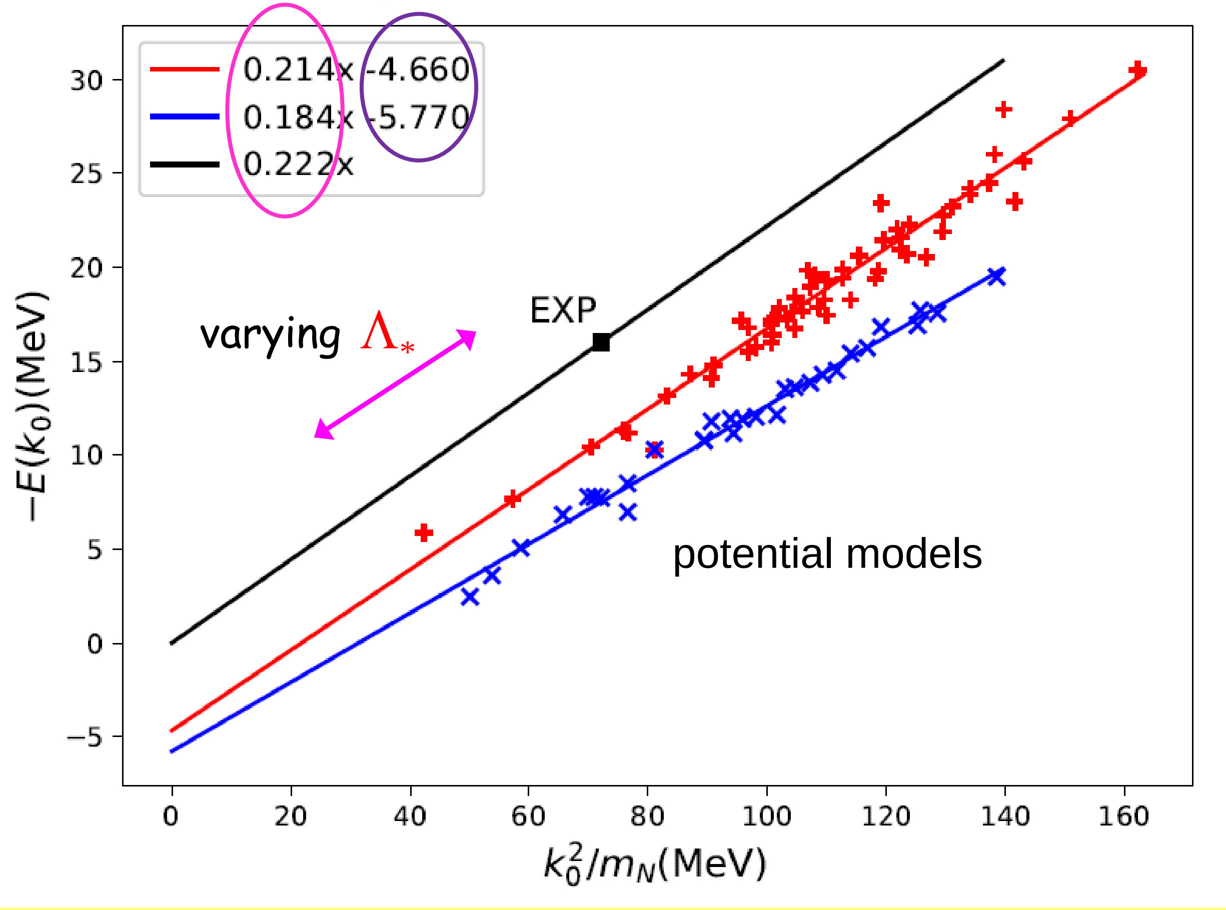
v.K. '17
Lyu, Long + v.K., in preparation

$$\left(\lim_{N\to\infty}\frac{E_N}{N}\right)_{\text{equil}} = \frac{3}{10}\mathfrak{G}^{(0)}\left[\eta_0\right]\frac{k_0^2}{m_N}+\dots \qquad \mathfrak{G}^{(0)\prime}\left[\eta_0\right]\simeq-\frac{2}{s_0}\mathfrak{G}^{(0)}\left[\eta_0\right]$$

v.K. '17
Lyu, Long + v.K.,
in preparation

Coester line

similar to
Delfino *et al.* '06



Approximate discrete scale invariance consistent with *ab initio* calculations

Conclusion

- Systems near unitarity can be described model-independently by Pionless EFT
  - properties given by essentially **one** parameter Λ_*
  - details obtained in perturbation theory

cf. parallel results from a phenomenological perspective:
Kievsky *et al.*
Grasso *et al.*

- Light nuclei in a sweet spot:
  dilute enough to be insensitive to interaction details,
  dense enough for approx discrete scale invariance

- A mechanism for nuclear saturation?

more nucleons	Gezerlis *et al.*, in progress
nuclear matter	Lyu *et al.*, in progress
higher orders	Bazak *et al.*, in progress