

# Achievements and challenges in understanding nucleon-deuteron reactions

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LENPIC collaboration

**LENPIC** (Low Energy Nuclear Physics International Collaboration): to understand nuclear structure and reactions with chiral forces

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Theoretical description of 3N continuum (elastic nucleon-deuteron scattering and the deuteron breakup reaction) requires solution of the following Faddeev-type equation for  $T|\phi_d\rangle$  state:

$$T|\phi_d\rangle = tP|\phi_d\rangle + (1 + tG_0)V_4^{(1)}(1 + P)|\phi_d\rangle \\ + tPG_0T|\phi_d\rangle + (1 + tG_0)V_4^{(1)}(1 + P)G_0T|\phi_d\rangle,$$

where  $|\phi_d\rangle \equiv |\varphi_d\rangle|\vec{q}_0\rangle$  is composed of the deuteron internal wave function and the state of the relative nucleon-deuteron motion.

The transition amplitude for elastic scattering is given by:

$$\langle\phi_d'|U|\phi_d\rangle = \langle\phi_d'|PG_0^{-1}|\phi_d\rangle + \langle\phi_d'|V_4^{(1)}(1 + P)|\phi_d\rangle \\ + \langle\phi_d'|PT|\phi_d\rangle + \langle\phi_d'|V_4^{(1)}(1 + P)G_0T|\phi_d\rangle$$

for breakup :

$$\langle\phi_0|U_0|\phi_d\rangle = \langle\phi_0|(1 + P)T|\phi_d\rangle$$

# Results with standard (AV18,CDBonn, Nijm1 and Nijm2) NN potentials and 3NF's (TM and Urbana IX) : good description of Nd elastic scattering data up to ~30 MeV with NN potentials only

Few-Body Systems 16, 127–142 (1994)

Few-Body Systems  
© Springer-Verlag 1994  
Printed in Austria



Physics Reports 274 (1996) 107–285

PHYSICS REPORTS

## Resolution of Discrepancy Between Backward Angle Cross-Section Data for Neutron-Deuteron Elastic Scattering

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## The three-nucleon continuum: achievements, challenges and applications

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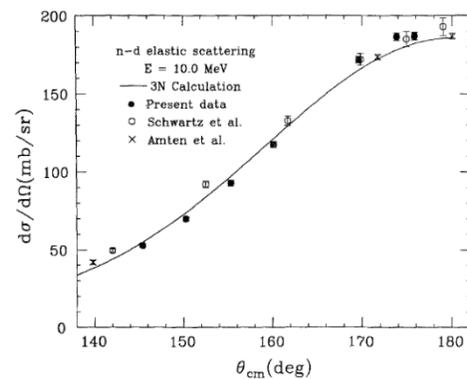


Fig. 13. Backward-angle  $d\sigma/d\Omega$  for  $n$ - $d$  elastic scattering at 10 MeV. The data and curve are the same as in Fig. 2 with the addition of the present data

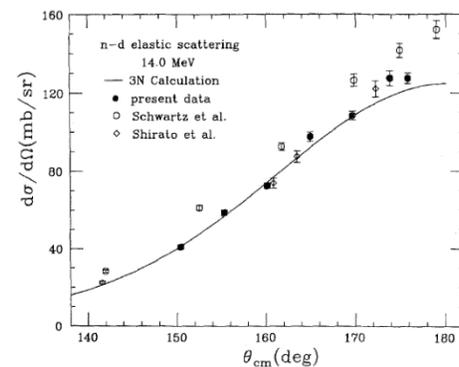


Fig. 14. Backward-angle  $d\sigma/d\Omega$  for  $n$ - $d$  elastic scattering at 14 MeV. The data and curve are the same as in Fig. 3 with the addition of the present data and the exclusion of the data of Brillmann et al. [13], which do not extend to backward angles

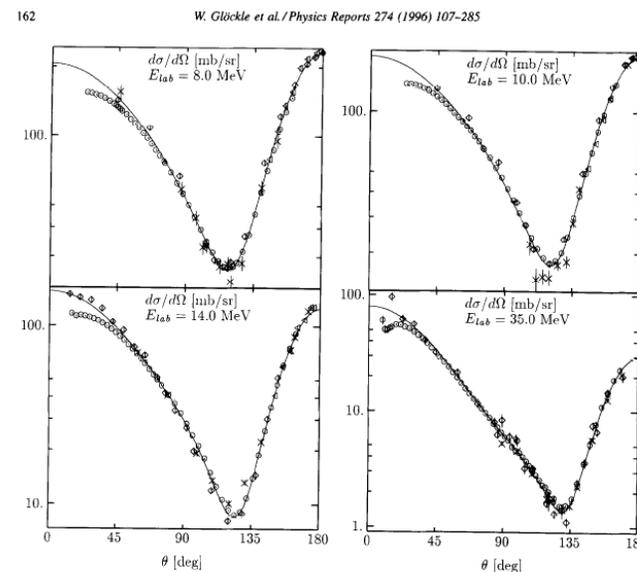


Fig. 7. Angular distributions for elastic Nd scattering. Comparison of pd and nd data. 8 MeV: nd data (○) [431], (●) [252], (×) [436], (◊) [234]; pd data (○) [416]. 10 MeV: nd data (○) (10.3 MeV) [431], (×) [24], (◊) [234]; pd data (○) [416]. 14 MeV: nd data (×) [440], (◊) [234], (◊) [47]; pd data (○) [416]. 35 MeV: nd data (×) [62], (◊) (36 MeV) [411]; pd data (○) [71]. The solid curve is the AV18 NN force prediction for 8, 10 and 14 MeV and the Nijm I NN force prediction for 35 MeV.

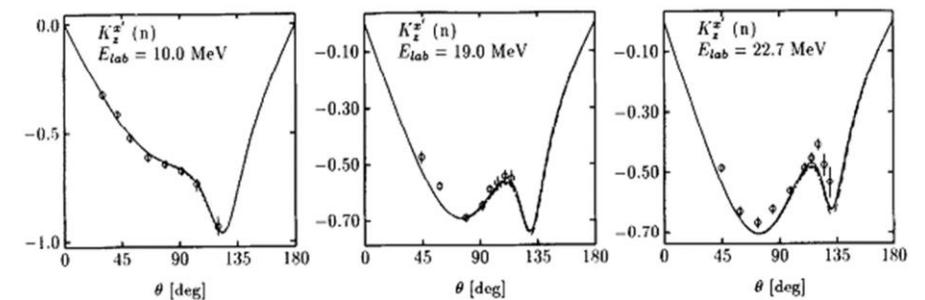


Fig. 22. The nucleon to nucleon spin transfer coefficient  $K_z^x(n)$  for elastic Nd scattering. Comparison of data with NN force predictions. 10 MeV: pd data (○) [457]. 19 MeV: pd data (○) [474]. 22.7 MeV: pd data (○) [300,301,186]. The NN force predictions are (—) Nijm I, (---) Nijm II, (- - - -) Nijm 93, and (· · · · ·) AV18.

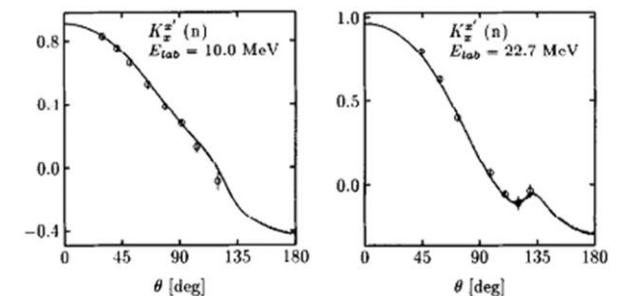
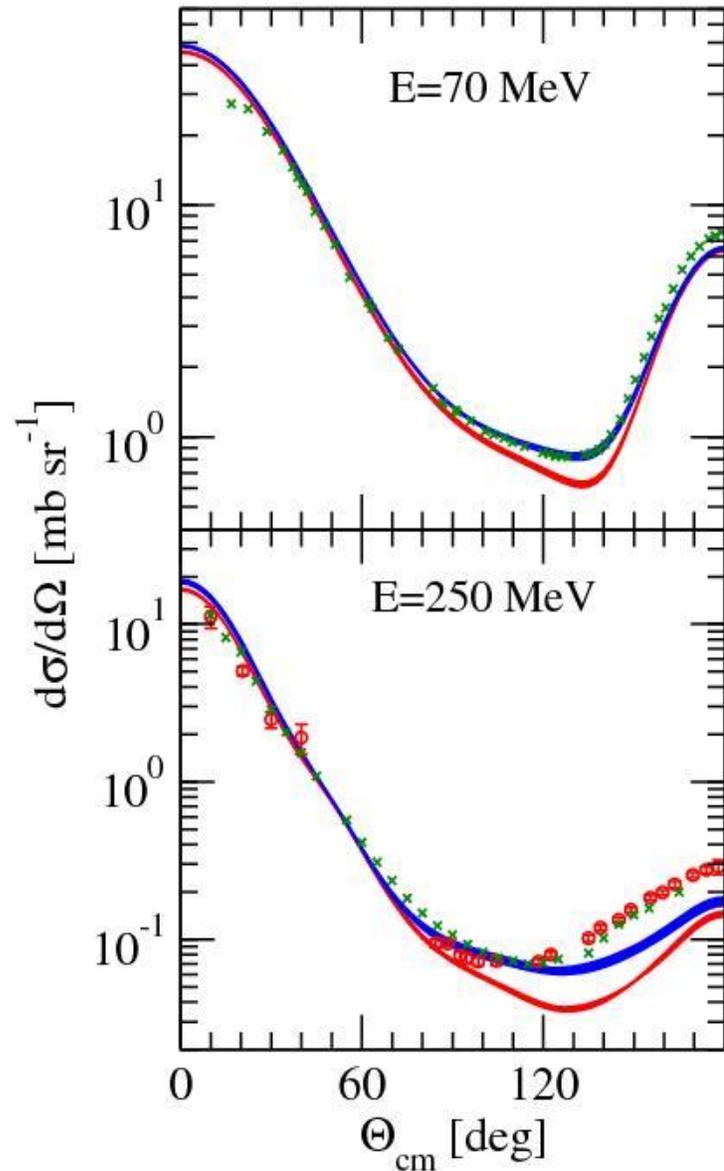


Fig. 23. The nucleon to nucleon spin transfer coefficient  $K_z^x(n)$  for elastic Nd scattering. Comparison of data with NN force predictions. 10 MeV: pd data (○) [457]. 22.7 MeV: pd data (○) [300,301,186]. The NN force predictions are (—) Nijm I, (---) Nijm II, (- - - -) Nijm 93, and (· · · · ·) AV18.

# Higher energy discrepancies

## Elastic scattering $d(p,p)d$

- NN only (AV18, CD Bonn, Nijm1, Nijm2)
- NN+3NF TM99



data 70: K.Sekiguchi et al., PR C65, 034003 (2002)

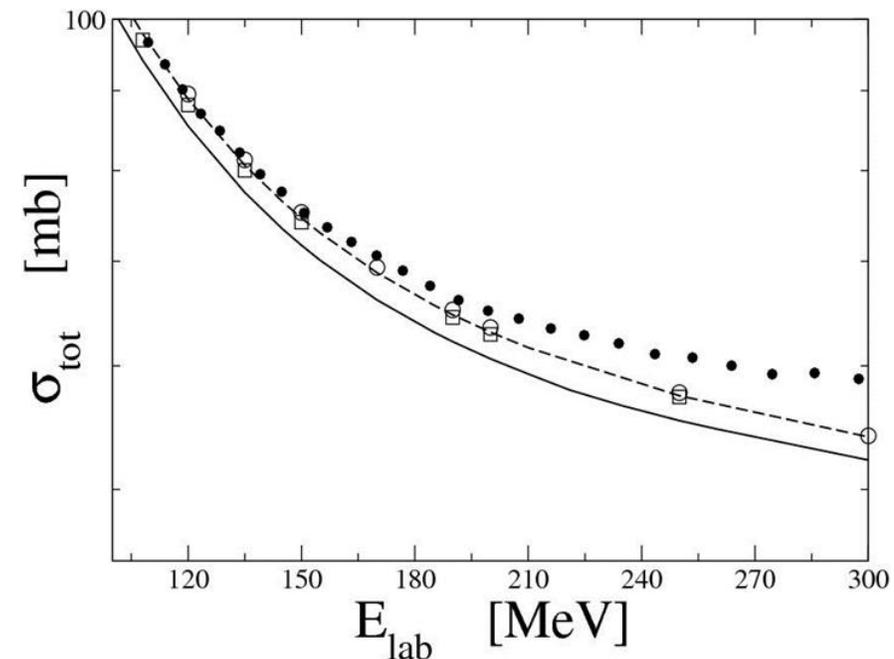
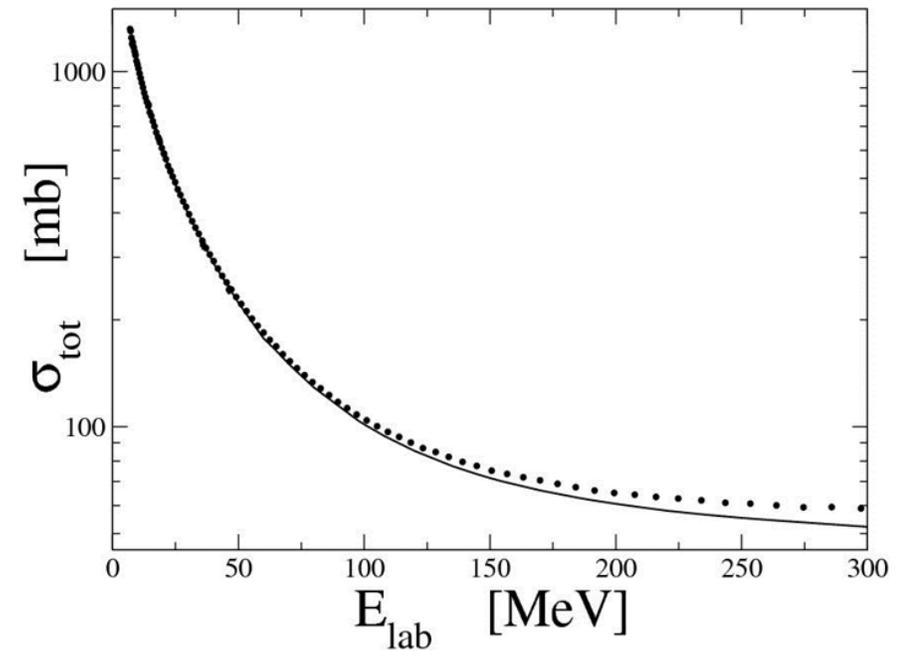
data 250:

x nd – Y.Maeda et al., PR C76, 014004 (2007)

o pd – K.Hatanaka et al., PR C 66, 044002 (2002)

**Total nd cross section: (W.P.Abfalterer et al. PRL 81(1998)57)**

- up to ~ 50 MeV good agreement with predictions based on 2N forces
- adding 3NF provides explanation of the disagreement up to ~ 150 MeV
- at even larger energies a clear disagreement which increases with energy



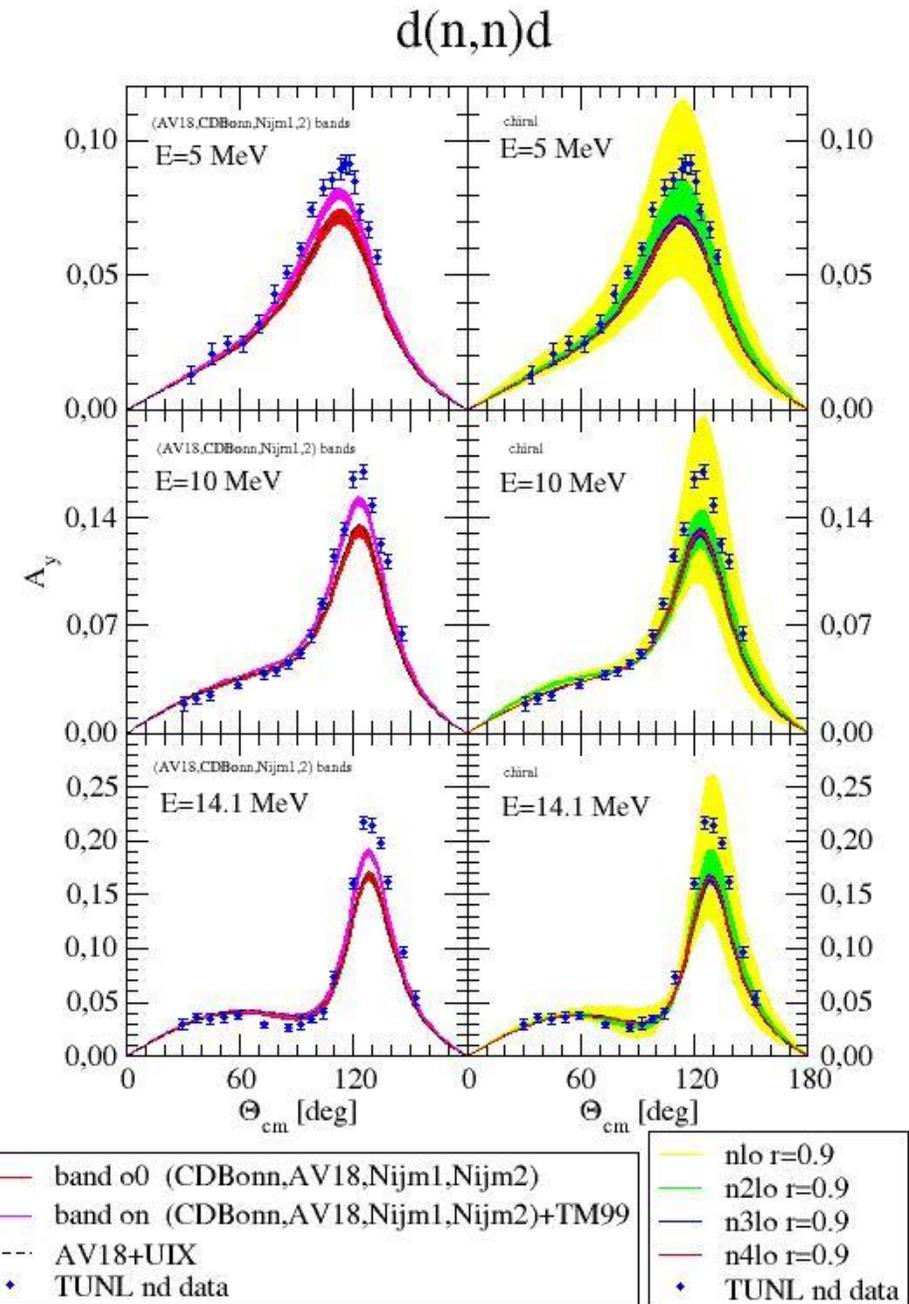
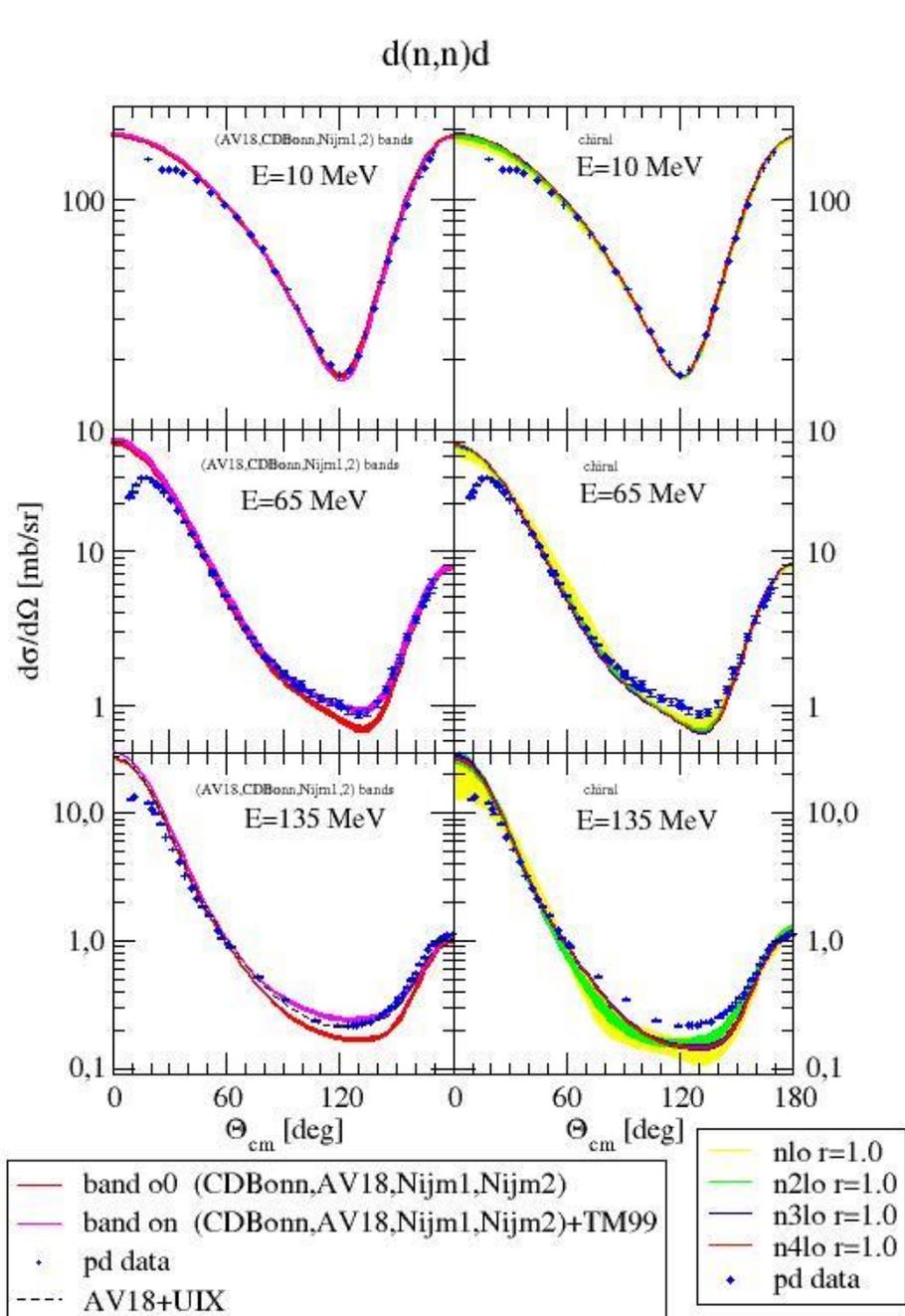
- relativistic effects are **not responsible** for large discrepancies in elastic Nd scattering and in the nd total cross section at higher energies
- those discrepancies must come from **neglect of short-range 3NF components**, which become active at higher energies
- such short-range 3NF's appear in a meson-exchange picture from e.g.  **$\pi$ - $\rho$**  and  **$\rho$ - $\rho$**  exchanges. In  $\chi$ PT a number of short-range 3NF components appear in N<sup>2</sup>LO and N<sup>3</sup>LO orders of chiral expansion

**Challenge:** application of NN and 3NF's derived consistently in the framework of chiral perturbation theory

## Few remarks on chiral forces:

- In order to reproduce properly 2N data up to about 250 MeV N3LO order of chiral expansion is required
  - About few years ago: NN interaction up to N3LO, 3NF derived up to N4LO
  - nonlocal momentum space regularization has been applied:  
 $V \rightarrow f(p', \Lambda) V(p', p) f(p, \Lambda)$  with  $f(p, \Lambda) = e^{-p^6/\Lambda^6}$   
 $V^4 \rightarrow f(p', q', \Lambda) V^4(p', q', p', q) f(p, q, \Lambda)$  with  $f(p, q, \Lambda) = e^{-(p^2+0.75q^2)^3/\Lambda^6}$   
what leads to finite cut-off artefacts (problems when applied to higher energy Nd scattering)
- New, improved chiral force, presented by Bochum-Bonn group in 2014:
  - E. Epelbaum, H. Krebs, U.-G. Meißner, Eur. Phys. J. A51 (2015) 3,26 – up to N3LO
  - E. Epelbaum, H. Krebs, U.-G. Meißner arXiv:1412.4623 [nucl-th] – up to N4LO
  - Local regularization in the coordinate space  $V_{lr}(\mathbf{r}) \rightarrow V_{lr}(\mathbf{r})f(\mathbf{r})$  with  $f(r) = (1 - e^{-r^2/R^2})^n$
  - $R=0.8-1.2$  fm what corresponds to  $\Lambda=330-500$  MeV
  - Such regularization preserves long-range OPE and TPE physics
  - All LECs in the long-range part are taken from pion-nucleon scattering without fine tuning
  - Very good description of the deuteron properties, phase shifts etc.

- NN developed up to N4LO: E.Epelbaum et al. arXiv:1412.4623 [nucl-th]
- Novel way of quantifying the theoretical uncertainties due to the truncation of the chiral expansion: E.Epelbaum et al. arXiv:1412.0142 [nucl-th]



- Theoretical uncertainty grows with energy and decreases with increasing order: one thus expects precise predictions starting from N3LO
- For many observables the results at N2LO and higher orders differ from data well outside the range of quantified observables, thus providing a clear evidence for missing three-nucleon forces

# Chiral 3N potential in N<sup>2</sup>LO order:

E. Epelbaum, Prog. Part. Nucl. Phys. 57, 654 (2006)

$$V_{123} = V_{2\pi}^{(3)} + V_{1\pi,cont}^{(3)} + V_{cont}^{(3)}$$

$$V_{2\pi}^{(3)} = \sum_{i \neq j \neq k} \frac{1}{2} \left( \frac{g_A}{2F_\pi} \right)^2 \frac{(\vec{\sigma}_i \circ \vec{q}_i)(\vec{\sigma}_j \circ \vec{q}_j)}{(\vec{q}_i^2 + M_\pi^2)(\vec{q}_j^2 + M_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta$$

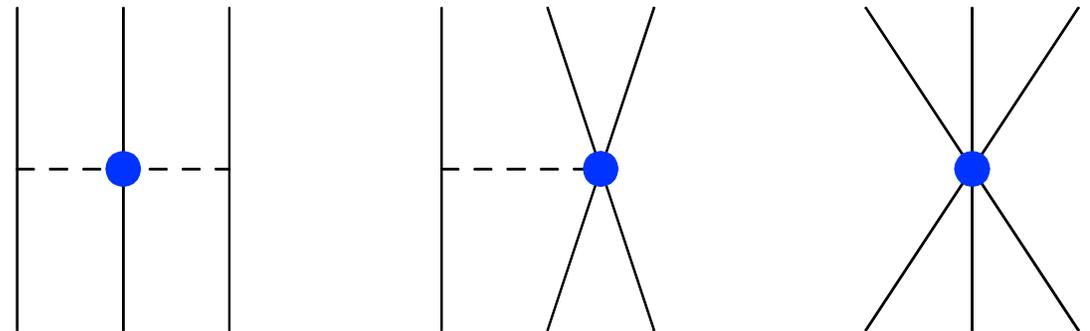
$$\vec{q}_i \equiv \vec{p}_i' - \vec{p}_i$$

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta} \left[ -\frac{4c_1 M_\pi^2}{F_\pi^2} + \frac{2c_3}{F_\pi^2} \vec{q}_i \circ \vec{q}_j \right] + \sum_\gamma \frac{c_4}{F_\pi^2} \varepsilon^{\alpha\beta\gamma} \tau_k^\gamma \vec{\sigma}_k \circ [\vec{q}_i \times \vec{q}_j]$$

$$V_{1\pi,cont}^{(3)} = - \sum_{i \neq j \neq k} \frac{g_A}{8F_\pi^2} D \frac{\vec{\sigma}_j \circ \vec{q}_j}{\vec{q}_j^2 + M_\pi^2} (\vec{\tau}_i \circ \vec{\tau}_j) (\vec{\sigma}_i \circ \vec{q}_j)$$

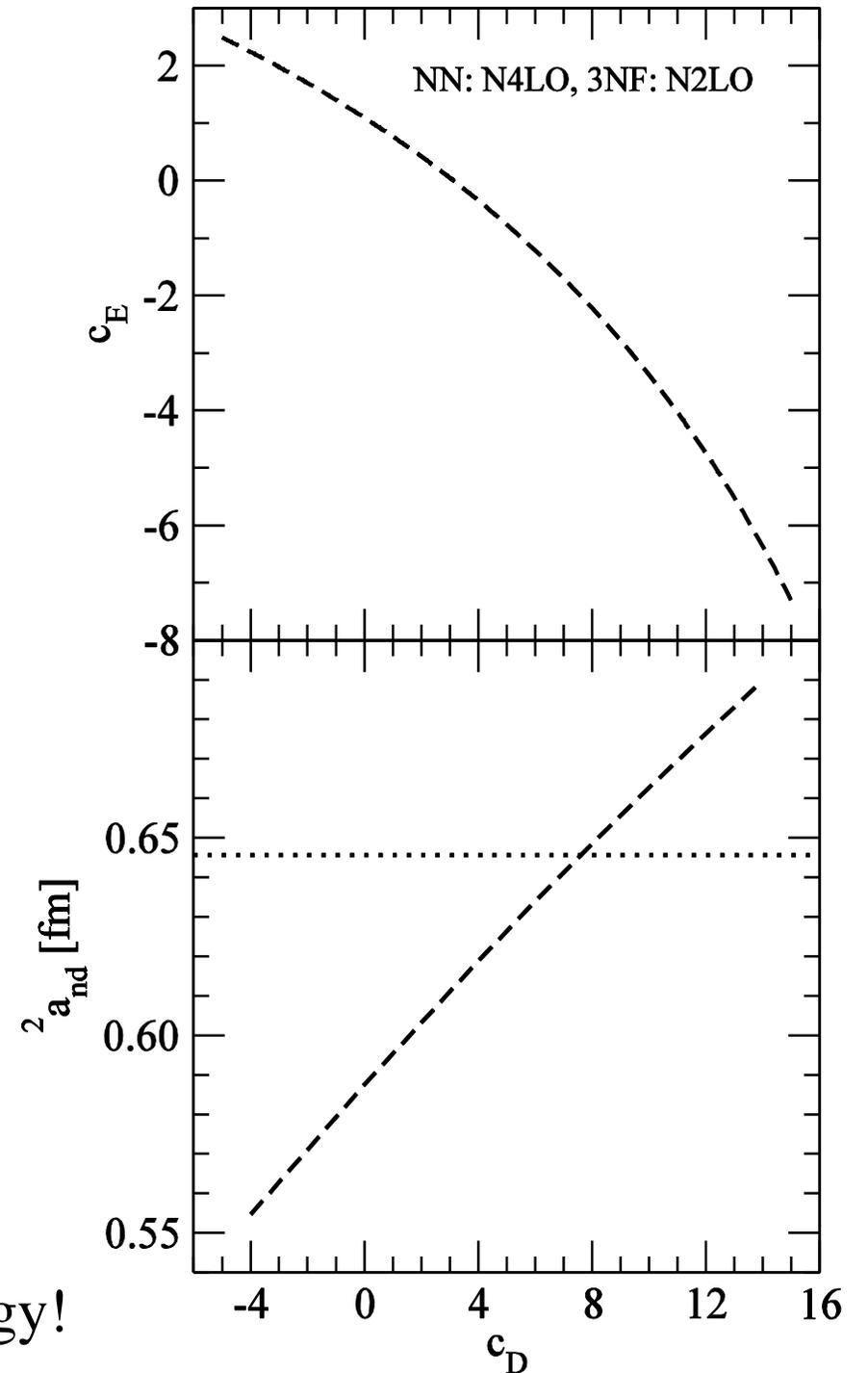
$$V_{cont}^{(3)} = \frac{1}{2} \sum_{j \neq k} E (\vec{\tau}_j \circ \vec{\tau}_k)$$

Two free parameters : D i E



A correlation function for the strengths  $c_D$  and  $c_E$  of the N2LO 3NF contact terms (as a NN potential N4LO version is taken) – these pairs of  $c_D$ - $c_E$  provide a proper  ${}^3\text{H}$  binding energy

Doublet nd scattering length  ${}^2a_{\text{nd}}$ .  
The experimental value  ${}^2a_{\text{nd}}^{\text{exp}}=0.645(7)$  fm.

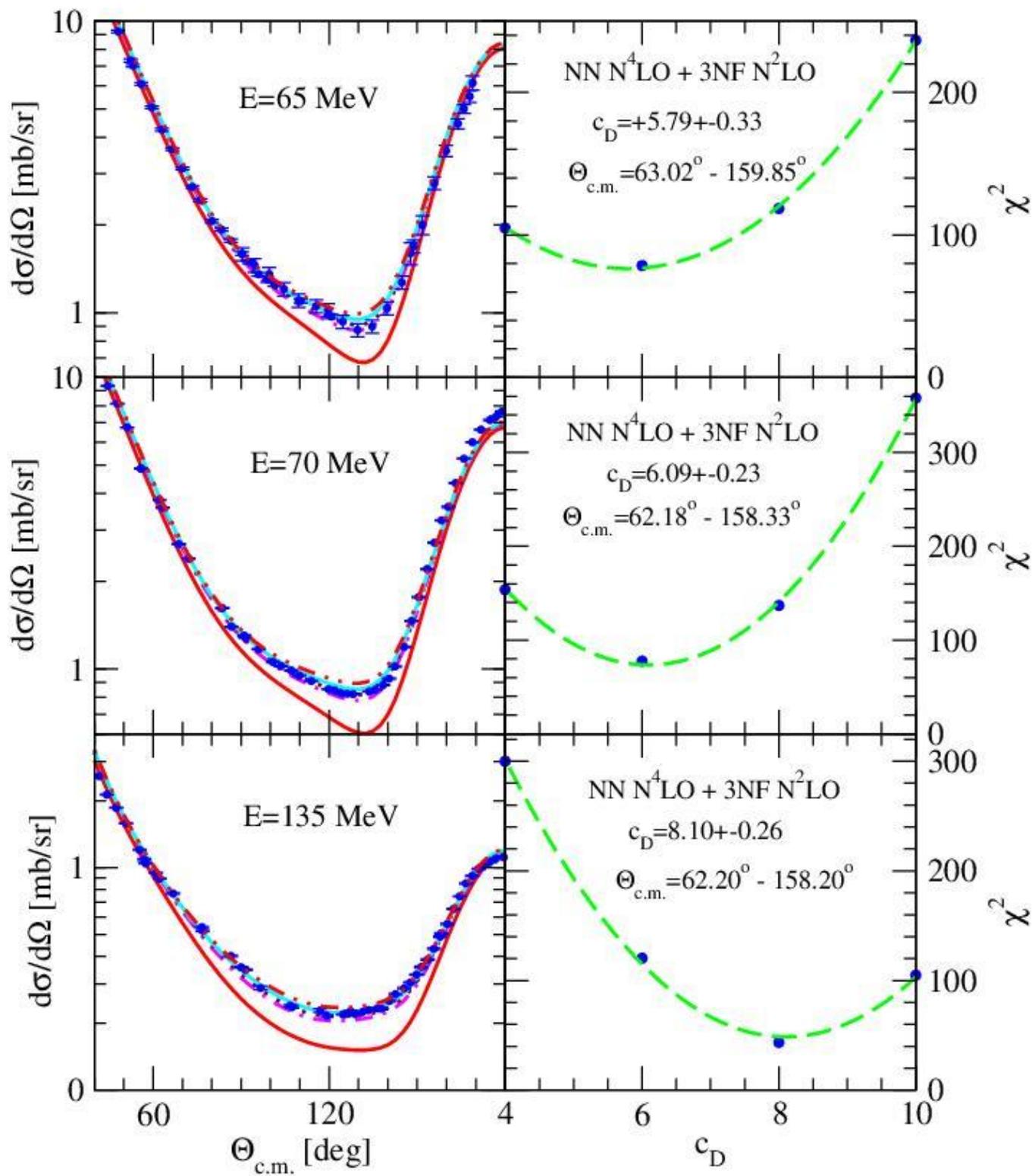


There is a correlation between  $a_{\text{nn}}$  and  ${}^3\text{H}$  binding energy!

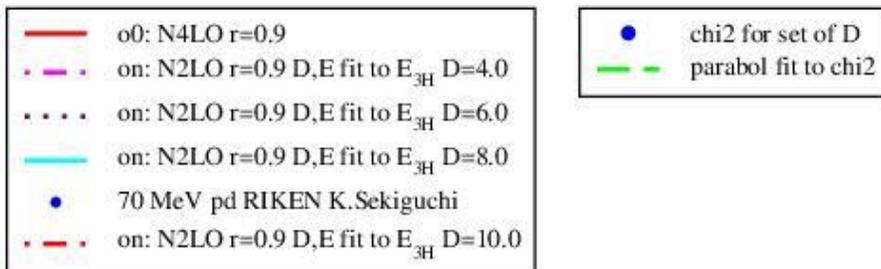
$$D = \frac{c_D}{F_\pi^2 \Lambda_\chi}$$

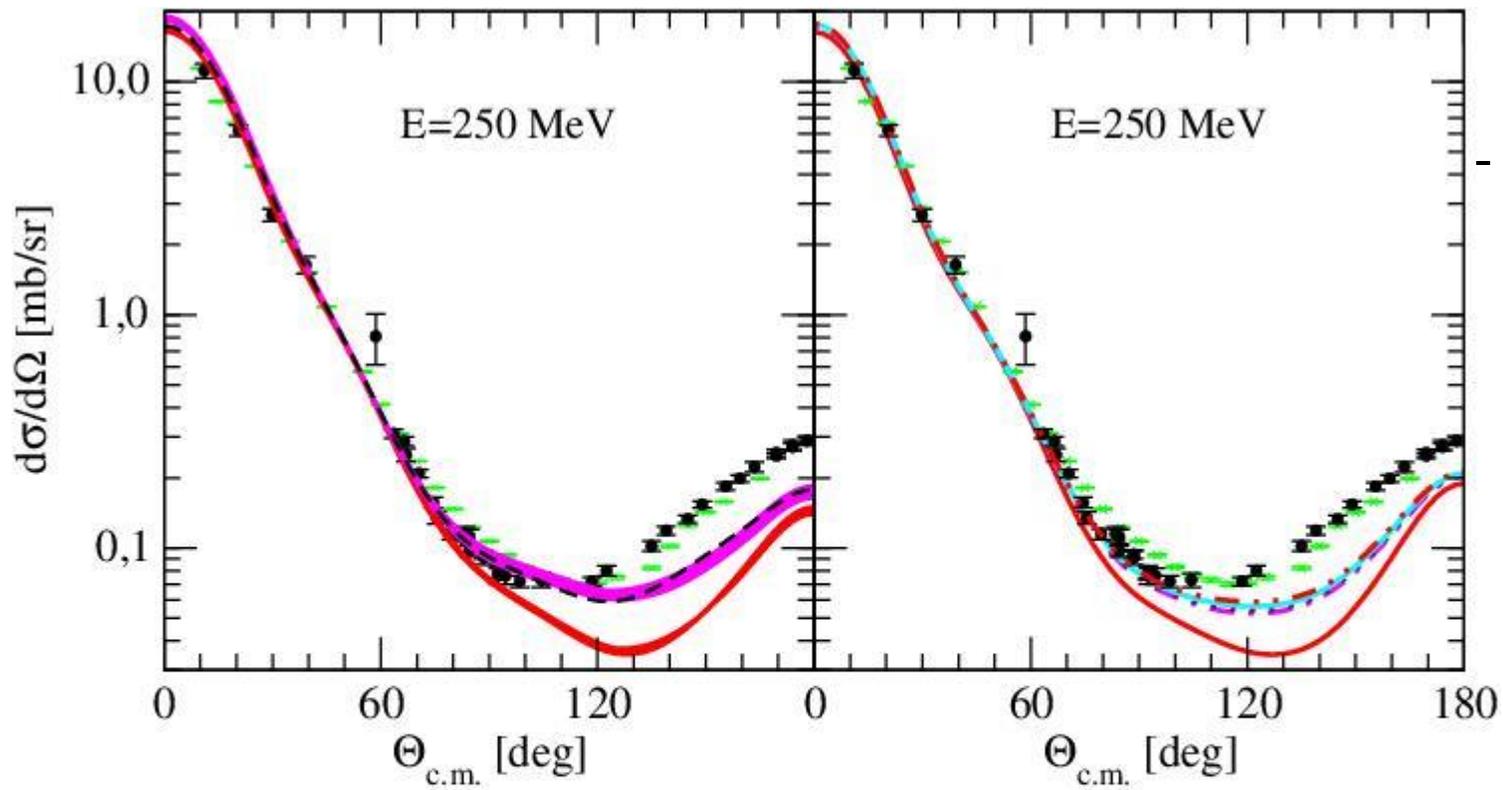
$$E = \frac{c_E}{F_\pi^4 \Lambda_\chi}$$

$\Lambda_\chi = 700\text{MeV}$  - chiral symmetry breaking scale

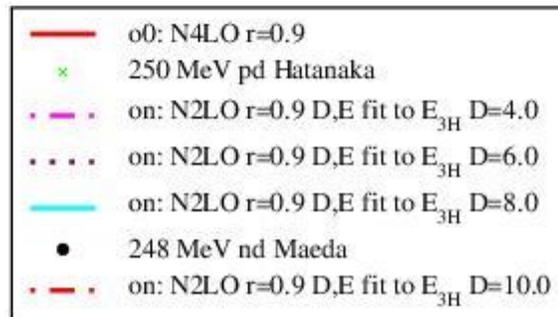
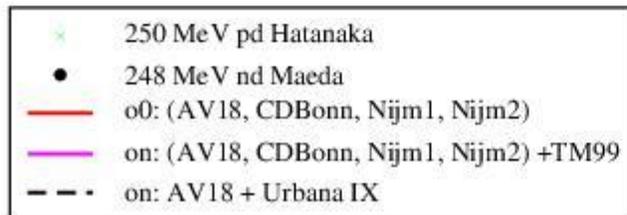


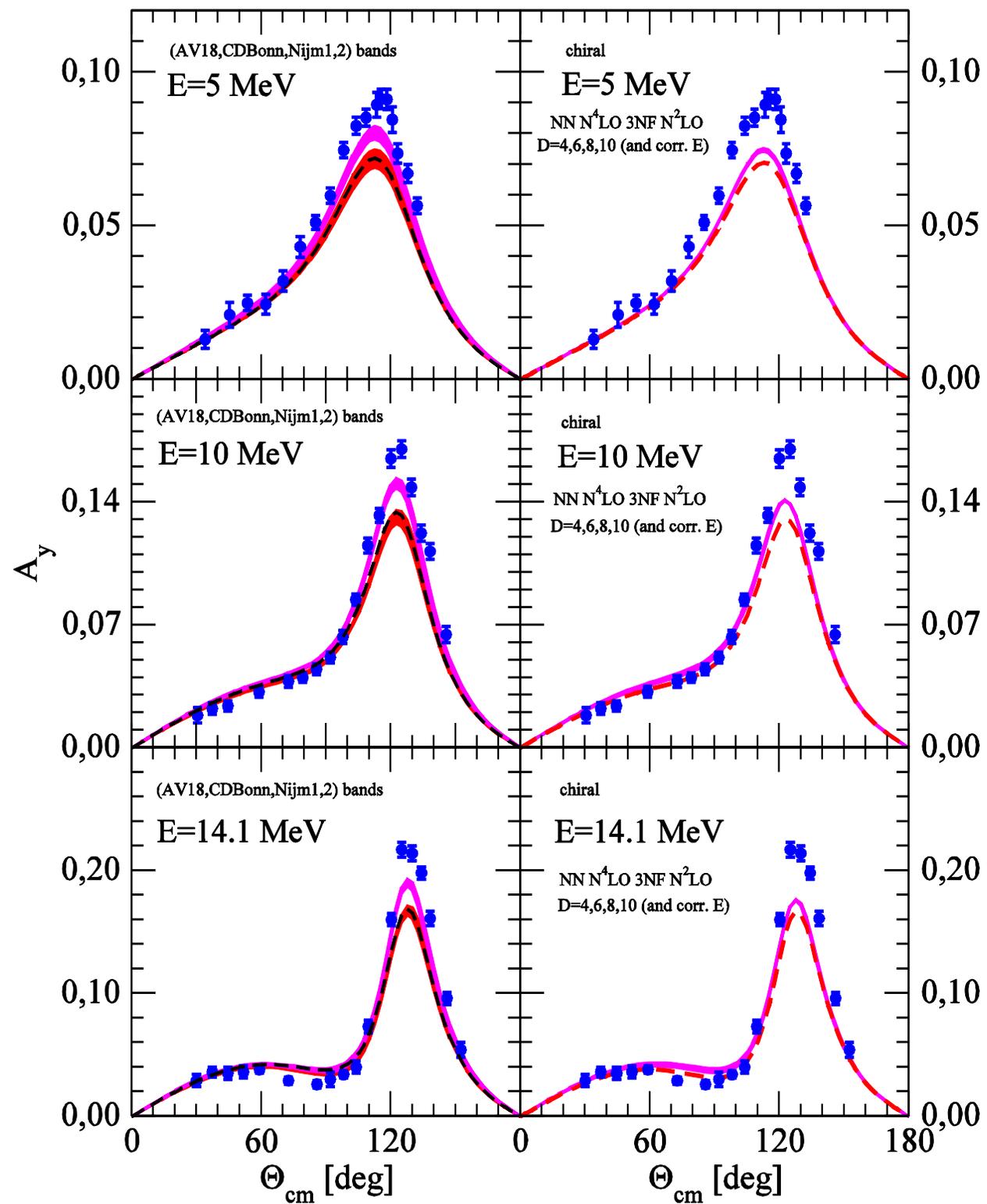
- Determination of  $c_D$  ( $c_E$ ) by  $\chi^2$ -fit to elastic Nd scattering cross section data
- The values of  $c_D$  found at 65 and 70 MeV are compatible and agree within error bars
- At 135 MeV the different value of  $c_D$  reflects growing importance from higher chiral order contributions





- Similarity of standard and chiral predictions:  
 can be traced back to the fact, that the basic  
 mechanism underlying the standard and  
 chiral N2LO 3NF's is the  $2\pi$ -exchange  
 mechanism

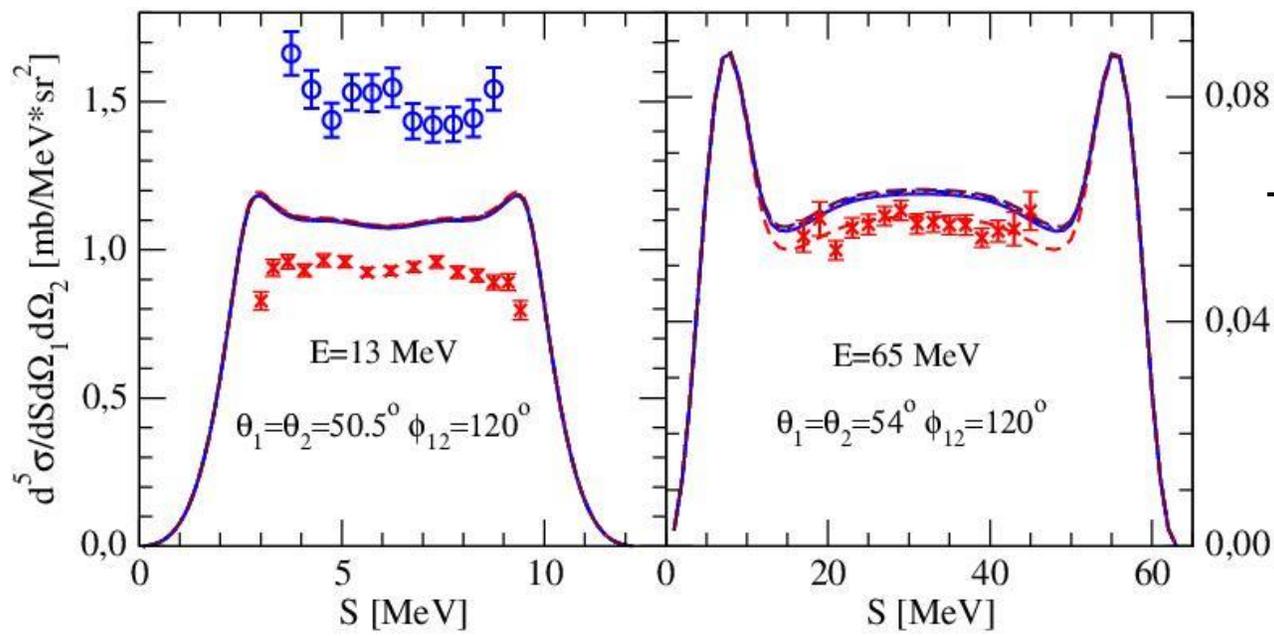




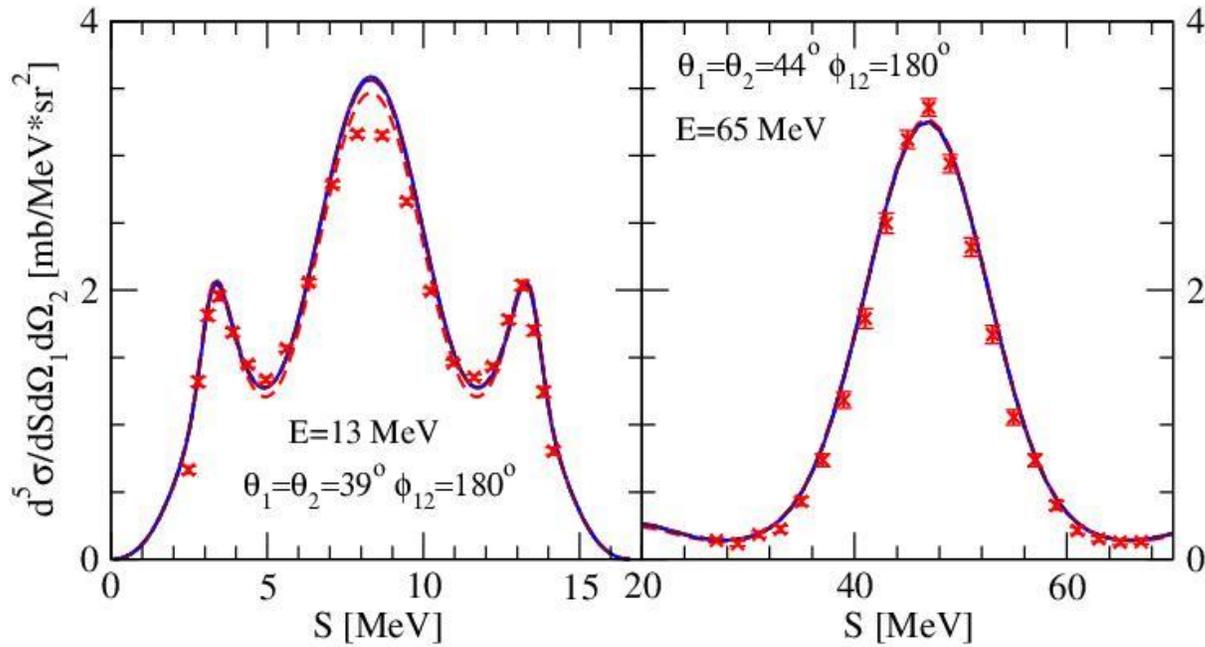
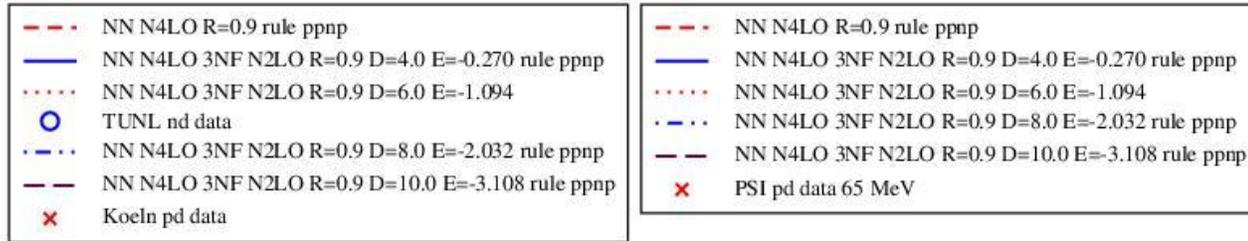
- small effects of N2LO 3N force – twice smaller than effects of the TM99 3NF
- effects are practically independent from  $c_D$  ( $c_E$ ) values used (from the correlation line)
- will 3N force explain  $A_y$  puzzle (N3LO 3NF) ?
- alternative: wrong low energy NN phase-shifts (in P-waves) ?

— o0 (CDB,AV18,Nijm1,Nijm2)  
 — on (CDB,AV18,Nijm1,Nijm2)+TM99  
 - - - AV18+UIX  
 • TUNL nd data

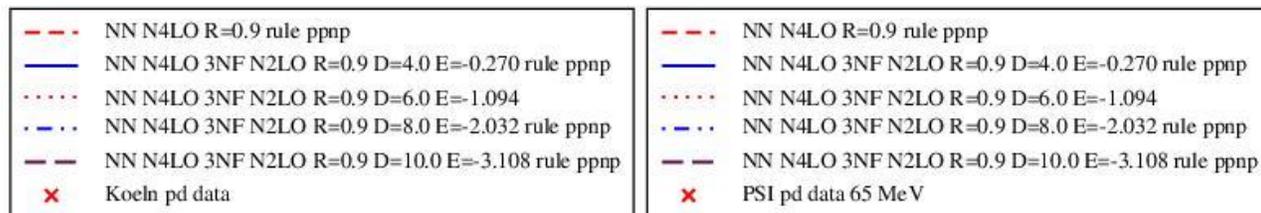
— NN n4lo +3NF n2lo R=0.9 fm, D-E corr(D=4,6,8,10)  
 - - - n4lo r=0.9 fm  
 • TUNL nd data

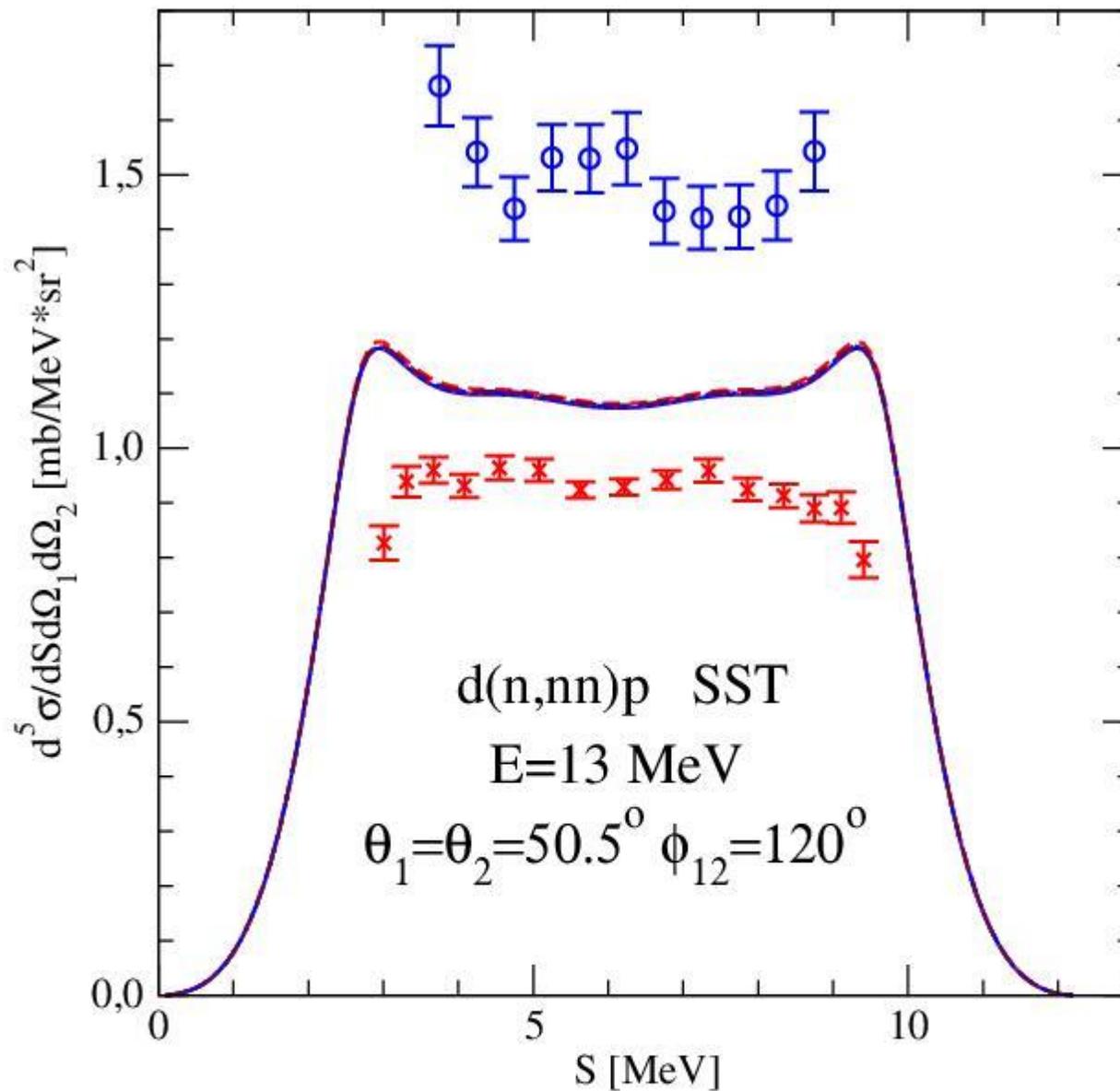


- no 3NF effects at 13 MeV and small effects at 65 MeV

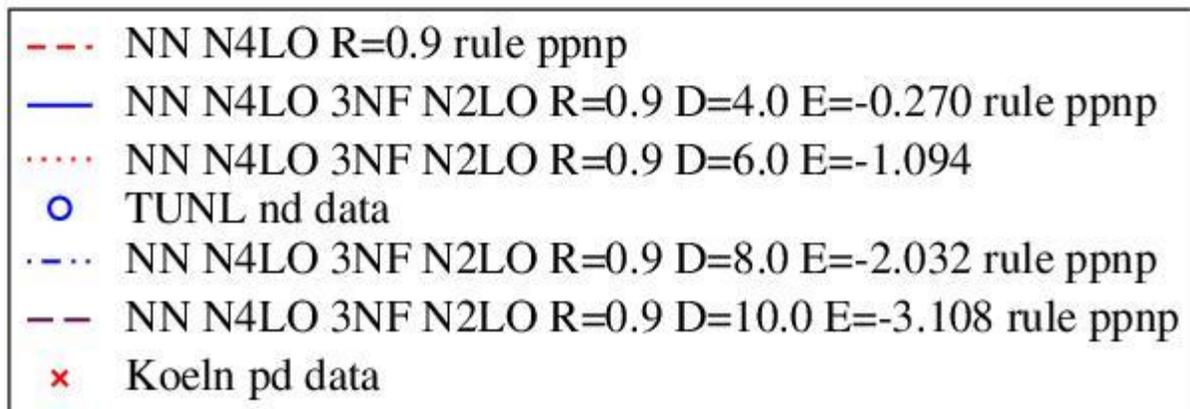


- QFS practically insensitive to action of the 3NF



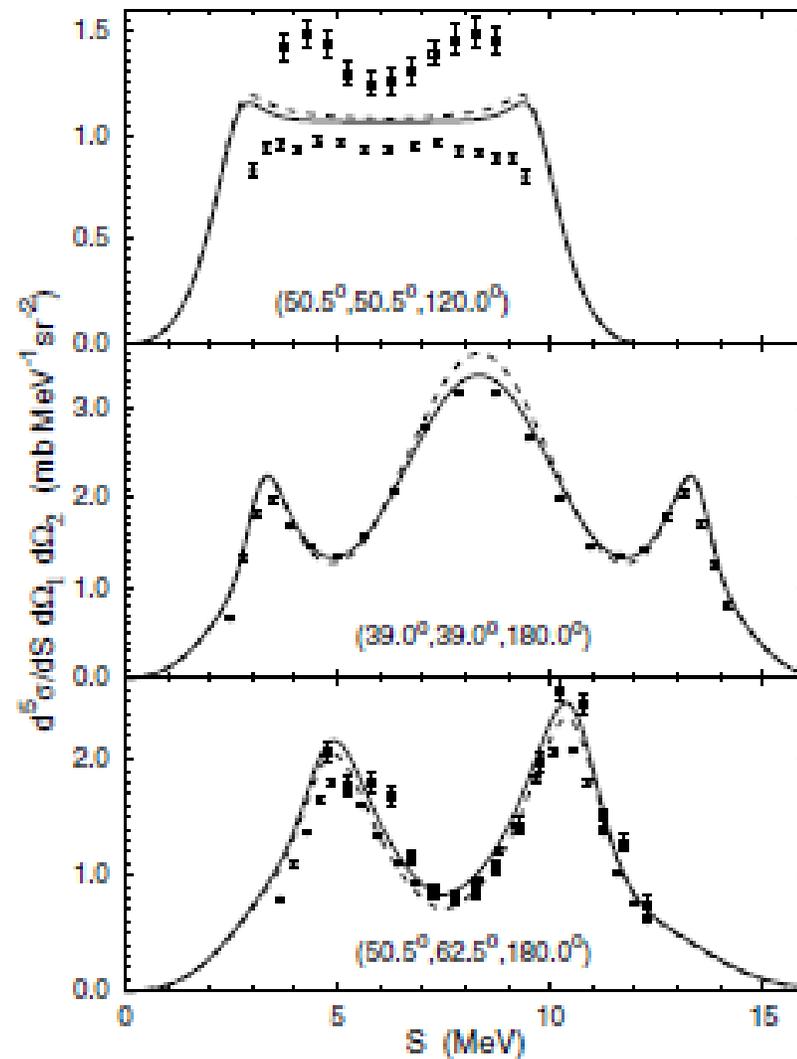


- pd data different from nd data and both are different from theory
- No 3NF effects for that configuration
- three independent measurements (TUNL, Erlangen, Bochum) support nd data – even change from nn to np coincidences was checked
- independent measurements (Koeln, Fukuoka) support pd data – careful checking was made for configurations around SST



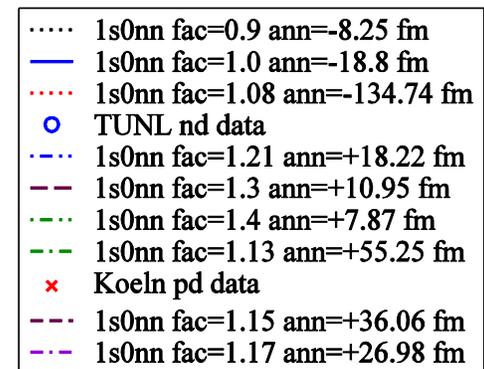
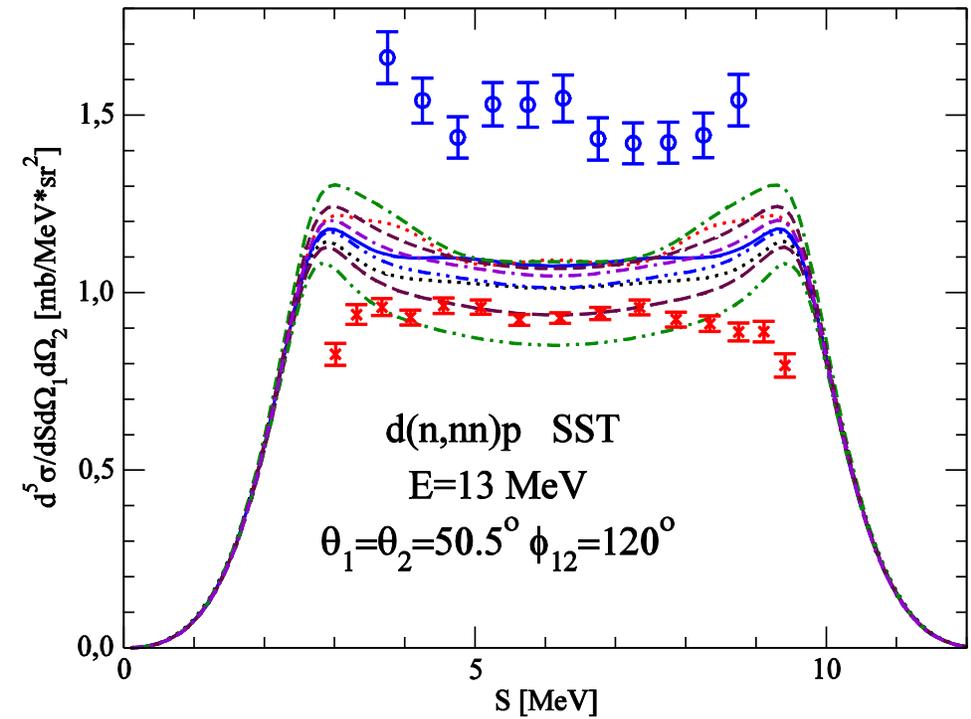
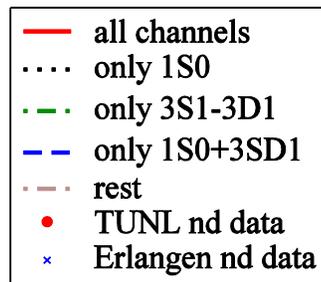
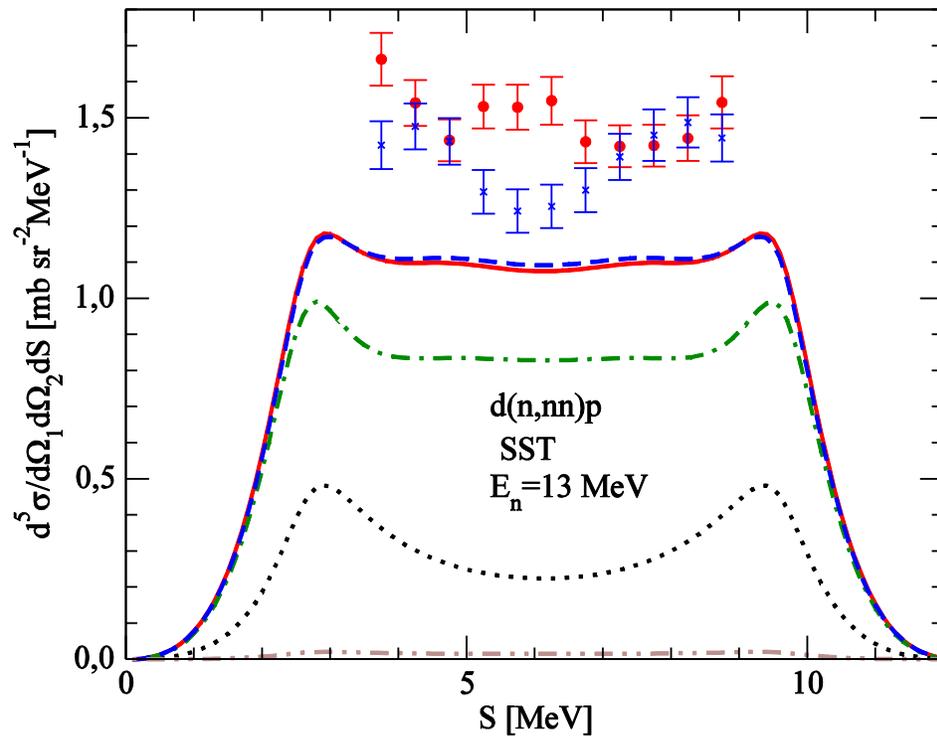
## Calculation of Proton-Deuteron Breakup Reactions including the Coulomb Interaction between the Two Protons

A. Deltuva,<sup>1,\*</sup> A. C. Fonseca,<sup>1</sup> and P. U. Sauer<sup>2</sup>



- practically no pp Coulomb force effects for low energy space star configuration !

FIG. 2. Differential cross section for  $pd$  breakup at 13 MeV proton lab energy for space star, quasi-free scattering, and col-linear configurations (from top to bottom). Results for CD Bonn +  $\Delta$  potential including the Coulomb interaction (solid curves) are compared to results without Coulomb (dashed curves). The experimental  $pd$  data (circles) are from Ref. [18] and  $nd$  data (squares) are from Ref. [19].



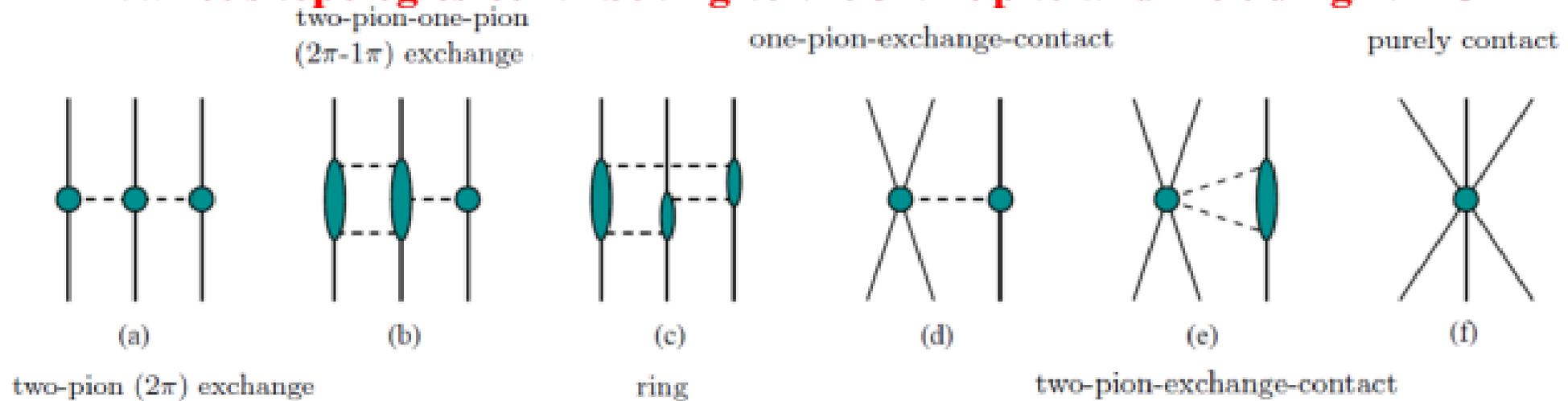
- only  $^1S_0$  and  $^3S_1$  contribute at this energy
- is something wrong with  $^1S_0$  (nn or pp) at low energies ?

- making  $^1S_0$  nn stronger (positive scattering length - bound  $^1S_0$  nn state) could explain pd data (but not nd data !)

## Summary:

- Nd elastic scattering and deuteron breakup reaction reveal large sensitivity to underlying nuclear forces --> good tools to test nuclear Hamiltonian
- call for consistency between 2N and 3N forces: chiral perturbation theory approach
- semilocal coordinate-space regularized (SCS) chiral forces support (semi)phenomenological forces predictions
- SCS NN+N2LO 3NF provide smaller 3NF effects for low-energy analyzing power than TM99 3NF – N2LO 3NF unable to explain  $A_y$  puzzle
- low-energy breakup space-star configuration reveals large theoretical cross-section discrepancies both to nd and pd data
- Big challenge: application of chiral N<sup>3</sup>LO three-nucleon forces to 3N continuum and, together with consistent chiral electroweak currents, to reactions induced on 3N bound states by electroweak external probes

## Various topologies contributing to the 3NF up to and including N<sup>4</sup>LO



□ N<sup>2</sup>LO: (a) + (d) + (f) (E.Epelbaum et al., PR C66, 064001 (2002))

□ N<sup>3</sup>LO: (a) + (b) + (c) + (d) + (e) + (f) + rel

V.Bernard et al., PR C77, 064004 (2008) - long range contributions (a), (b), (c)

V.Bernard et al., PR C84, 054001 (2011) - short range terms (e)

and leading relativistic corrections

**N<sup>3</sup>LO contributions do not involve any unknown low energy constants !**

The full N<sup>3</sup>LO 3NF depends on two parameters  $c_D$  and  $c_E$  coming with (d) and (f) terms, respectively.

They are adjusted to two chosen 3N observables.

□ N<sup>4</sup>LO (longest range contributions): (a) + (b) + (c) + (d) + (e) + (f) (H.Krebs et al., arXiv:1203.0067)