Achievements and challenges in understanding nucleon-deuteron reactions

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LENPIC (Low Energy Nuclear Physics International Collaboration): to understand nuclear structure and reactions with chiral forces
Theoretical description of 3N continuum (elastic nucleon-deuteron scattering and the deuteron breakup reaction) requires solution of the following Faddeev-type equation for $T|\phi_d\rangle$ state:

$$T|\phi_d\rangle = tP|\phi_d\rangle + (1 + tG_0)V_4^{(1)}(1 + P)|\phi_d\rangle$$

$$+ tPG_0T|\phi_d\rangle + (1 + tG_0)V_4^{(1)}(1 + P)G_0T|\phi_d\rangle,$$

where $|\phi_d\rangle \equiv |\varphi_d\rangle|\bar{q}_0\rangle$ is composed of the deuteron internal wave function and the state of the relative nucleon-deuteron motion.

The transition amplitude for elastic scattering is given by:

$$\langle \phi_d'|U|\phi_d\rangle = \langle \phi_d'|PG_0^{-1}|\phi_d\rangle + \langle \phi_d'|V_4^{(1)}(1 + P)|\phi_d\rangle$$

$$+ \langle \phi_d'|PT|\phi_d\rangle + \langle \phi_d'|V_4^{(1)}(1 + P)G_0T|\phi_d\rangle$$

for breakup:

$$\langle \phi_0|U_0|\phi_d\rangle = \langle \phi_0|(1 + P)T|\phi_d\rangle$$

Transition amplitude for the deuteron breakup is given by:
Results with standard (AV18, CD-Bonn, Nijm1 and Nijm2) NN potentials and 3NF’s (TM and Urbana IX): good description of Nd elastic scattering data up to ~30 MeV with NN potentials only.
Higher energy discrepancies

Elastic scattering d(p,p)d

- NN only (AV18, CD Bonn, Nijm1, Nijm2)
- NN+3NF TM99

Total nd cross section: (W.P.Abfalterer et al. PRL 81(1998)57)
- up to ~ 50 MeV good agreement with predictions based on 2N forces
- adding 3NF provides explanation of the disagreement up to ~ 150 MeV
- at even larger energies a clear disagreement which increases with energy

Data 70: K.Sekiguchi et al., PR C65, 034003 (2002)

Data 250:
- x nd – Y.Maeda et al., PR C76, 014004 (2007)
relativistic effects are **not responsible** for large discrepancies in elastic Nd scattering and in the nd total cross section at higher energies

those discrepancies must come from **neglection** of short-range 3NF components, which become active at higher energies

such short-range 3NF’s appear in a meson-exchange picture from e.g. \(\pi-\rho\) and \(\rho-\rho\) exchanges. In \(\chi\)PT a number of short-range 3NF components appear in \(N^2\)LO and \(N^3\)LO orders of chiral expansion

**Challenge:** application of NN and 3NF’s derived consistently in the framework of chiral perturbation theory
Few remarks on chiral forces:

- In order to reproduce properly 2N data up to about 250 MeV N3LO order of chiral expansion is required
  - About few years ago: NN interaction up to N3LO, 3NF derived up to N4LO
  - nonlocal momentum space regularization has been applied:
    \[ V \rightarrow f(p',\Lambda) \, V(p',p) \, f(p,\Lambda) \, \text{with} \, \quad f(p,\Lambda) = e^{-p^2/\Lambda^2} \]
    \[ V^4 \rightarrow f(p',q',\Lambda) \, V^4(p',q',p',q) \, f(p,q,\Lambda) \, \text{with} \, \quad f(p,q,\Lambda) = e^{-(p^2+0.75q^2)^3/\Lambda^6} \]
    what leads to finite cut-off artefacts (problems when applied to higher energy Nd scattering)

- New, improved chiral force, presented by Bochum-Bonn group in 2014:
  - Local regularization in the coordinate space \( V_{lr}(r) \rightarrow V_{lr}(r)f(r) \) with \( f(r) = (1-e^{-r^2/R^2})^n \)
  - \( R=0.8–1.2 \, \text{fm} \) what corresponds to \( \Lambda=330-500 \, \text{MeV} \)
  - Such regularization preserves long-range OPE and TPE physics
  - All LECs in the long-range part are taken from pion-nucleon scattering without fine tuning
  - Very good description of the deuteron properties, phase shifts etc.
• NN developed up to N4LO: E.Epelbaum et al. arXiv:1412.4623 [nucl-th]
• Novel way of quantifying the theoretical uncertainties due to the truncation of the chiral expansion: E.Epelbaum et al. arXiv:1412.0142 [nucl-th]

• Theoretical uncertainty grows with energy and decreases with increasing order: one thus expects precise predictions starting from N3LO
• For many observables the results at N2LO and higher orders differ from data well outside the range of quantified observables, thus providing a clear evidence for missing three-nucleon forces
Chiral 3N potential in $N^2$LO order:


$$V_{123} = V_{2\pi}^{(3)} + V_{1\pi,\text{cont}}^{(3)} + V_{\text{cont}}^{(3)}$$

$$V_{2\pi}^{(3)} = \sum_{i \neq j \neq k} \frac{1}{2} \left( \frac{g_A}{2F_\pi} \right)^2 \frac{\left( \vec{\sigma}_i \circ \vec{q}_i \right) \left( \vec{\sigma}_j \circ \vec{q}_j \right)}{\left( \vec{q}_i^2 + M_\pi^2 \right)\left( \vec{q}_j^2 + M_\pi^2 \right)} \frac{}{} \frac{}{F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta}$$

$$\bar{q}_i \equiv \vec{p}_i^l - \vec{p}_i$$

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta} \left[ \frac{4c_1 M_\pi^2}{F_\pi^2} + \frac{2c_3}{F_\pi^2} \vec{q}_i \circ \vec{q}_j \right] + \sum_{\gamma} \frac{c_4}{F_\pi^2} \epsilon^{\alpha\beta\gamma} \tau_k^\gamma \vec{\sigma}_k \circ \left[ \vec{q}_i \times \vec{q}_j \right]$$

$$V_{1\pi,\text{cont}}^{(3)} = - \sum_{i \neq j \neq k} \frac{g_A}{8F_\pi^2} D \frac{\vec{\sigma}_j \circ \vec{q}_j}{\vec{q}_j^2 + M_\pi^2} \left( \vec{\tau}_i \circ \vec{\tau}_j \right) \left( \vec{\sigma}_i \circ \vec{q}_j \right)$$

$$V_{\text{cont}}^{(3)} = \frac{1}{2} \sum_{j \neq k} E \left( \vec{\tau}_j \circ \vec{\tau}_k \right)$$

Two free parameters: D i E
A correlation function for the strengths $c_D$ and $c_E$ of the N2LO 3NF contact terms (as a NN potential N4LO version is taken) – these pairs of $c_D$-$c_E$ provide a proper $^3\text{H}$ binding energy.

Doublet nd scattering length $^2a_{\text{nd}}$.
The experimental value $^2a_{\text{nd}}^{\text{exp}}=0.645(7)$ fm.

There is a correlation between $a_{\text{nn}}$ and $^3\text{H}$ binding energy!

$$D = \frac{c_D}{F_\pi^2 \Lambda_\chi} \quad E = \frac{c_E}{F_\pi^4 \Lambda_\chi} \quad \Lambda_\chi = 700\text{MeV} - \text{chiral symmetry breaking scale}$$
Determination of $c_D$ ($c_E$) by $\chi^2$-fit to elastic Nd scattering cross section data

The values of $c_D$ found at 65 and 70 MeV are compatible and agree within error bars.

At 135 MeV the different value of $c_D$ reflects growing importance from higher chiral order contributions.
Similarity of standard and chiral predictions: can be traced back to the fact, that the basic mechanism underlying the standard and chiral N2LO 3NF’s is the $2\pi$-exchange mechanism.
- small effects of N2LO 3N force – twice smaller than effects of the TM99 3NF
- effects are practically independent from $c_D$ ($c_E$) values used (from the correlation line)
- will 3N force explain $A_y$ puzzle (N3LO 3NF)?
- alternative: wrong low energy NN phase-shifts (in P-waves)?
- no 3NF effects at 13 MeV and small effects at 65 MeV

- QFS practically insensitive to action of the 3NF
- pd data different from nd data and both are different from theory

- No 3NF effects for that configuration

- three independent measurements (TUNL, Erlangen, Bochum) support nd data – even change from nn to np coincidences was checked

- independent measurements (Koeln, Fukuoka) support pd data – careful checking was made for configurations around SST
Calculation of Proton-Deuteron Breakup Reactions including the Coulomb Interaction between the Two Protons

A. Deltuva, A. C. Fonseca, and P. U. Sauer

- practically no pp Coulomb force effects for low energy space star configuration!

FIG. 2. Differential cross section for \( pd \) breakup at 13 MeV proton lab energy for space star, quasi-free scattering, and collinear configurations (from top to bottom). Results for CD Bonn +\( \Delta \) potential including the Coulomb interaction (solid curves) are compared to results without Coulomb (dashed curves). The experimental \( pd \) data (circles) are from Ref. [18] and \( nd \) data (squares) are from Ref. [19].
- only $^1S_0$ and $^3S_1$ contribute at this energy

- is something wrong with $^1S_0$ (nn or pp) at low energies?

- making $^1S_0$ nn stronger (positive scattering length - bound $^1S_0$ nn state) could explain pd data (but not nd data !)
Summary:

- Nd elastic scattering and deuteron breakup reaction reveal large sensitivity to underlying nuclear forces — good tools to test nuclear Hamiltonian
- Call for consistency between 2N and 3N forces: chiral perturbation theory approach
- Semilocal coordinate-space regularized (SCS) chiral forces support (semi)phenomenological forces predictions
- SCS NN+N2LO 3NF provide smaller 3NF effects for low-energy analyzing power than TM99 3NF — N2LO 3NF unable to explain Ay puzzle
- Low-energy breakup space-star configuration reveals large theoretical cross-section discrepancies both to nd and pd data
- Big challenge: application of chiral N^3LO three-nucleon forces to 3N continuum and, together with consistent chiral electroweak currents, to reactions induced on 3N bound states by electroweak external probes
Various topologies contributing to the 3NF up to and including N^4LO

- N^2LO: (a) + (d) + (f) (E.Epelbaum et al., PR C66, 064001 (2002))

- N^3LO: (a) + (b) + (c) + (d) + (e) + (f) + rel
  V.Bernard et al., PR C77, 064004 (2008) - long range contributions (a), (b), (c)
  V.Bernard et al., PR C84, 054001 (2011) - short range terms (e)
  and leading relativistic corrections

N^3LO contributions do not involve any unknown low energy constants!

The full N^3LO 3NF depends on two parameters c_D and c_E coming with (d) and (f) terms, respectively. They are adjusted to two chosen 3N observables.

- N^4LO (longest range contributions): (a) + (b) + (c) + (d) + (e) + (f) (H.Krebs et al., arXiv:1203.0067)