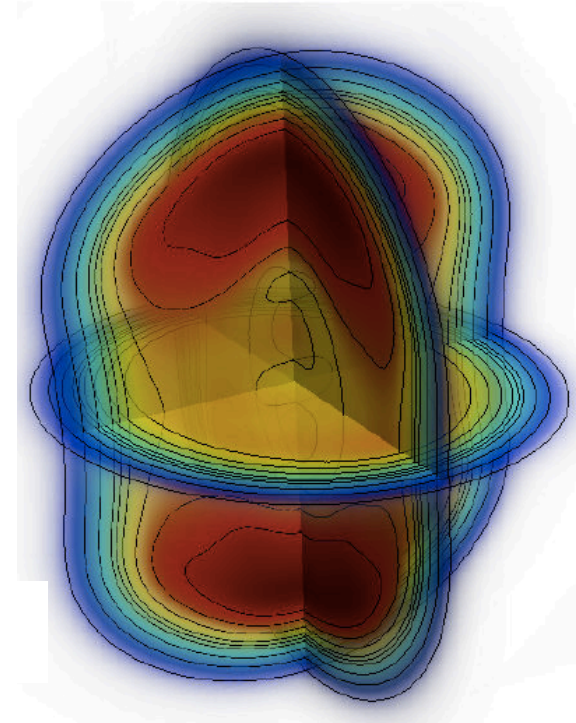
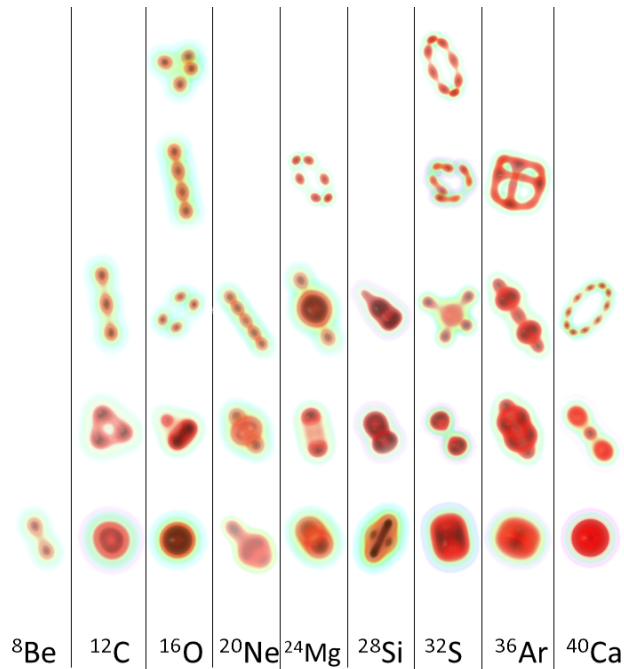


# Investigation of nuclear cluster phenomenology with the relativistic EDF approach



2018 European Nuclear Physics Conference  
2-7 September 2018, Bologna

# EDF method & clusters

- EDF: many-body system mapped into the **one-body density** and its powers, gradient

$$\rho_0(\mathbf{r}) = \rho_0(\mathbf{r}, \mathbf{r}) = \sum_{\sigma\tau} \rho(\mathbf{r}\sigma\tau; \mathbf{r}\sigma\tau)$$

$$\rho_1(\mathbf{r}) = \rho_1(\mathbf{r}, \mathbf{r}) = \sum_{\sigma\tau} \rho(\mathbf{r}\sigma\tau; \mathbf{r}\sigma\tau) \tau$$

$$\mathbf{s}_0(\mathbf{r}) = \mathbf{s}_0(\mathbf{r}, \mathbf{r}) = \sum_{\sigma\sigma'\tau} \rho(\mathbf{r}\sigma\tau; \mathbf{r}\sigma'\tau) \boldsymbol{\sigma}_{\sigma'\sigma}$$

$$\mathbf{s}_1(\mathbf{r}) = \mathbf{s}_1(\mathbf{r}, \mathbf{r}) = \sum_{\sigma\sigma'\tau} \rho(\mathbf{r}\sigma\tau; \mathbf{r}\sigma'\tau) \boldsymbol{\sigma}_{\sigma'\sigma} \tau$$

$$\mathbf{j}_T(\mathbf{r}) = \frac{i}{2} (\nabla' - \nabla) \rho_T(\mathbf{r}, \mathbf{r}') \Big|_{\mathbf{r}=\mathbf{r}'}$$

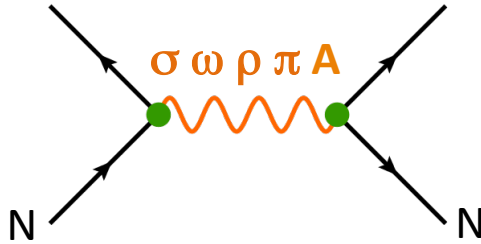
$$\mathcal{J}_T(\mathbf{r}) = \frac{i}{2} (\nabla' - \nabla) \otimes \mathbf{s}_T(\mathbf{r}, \mathbf{r}') \Big|_{\mathbf{r}=\mathbf{r}'}$$

$$\tau_T(\mathbf{r}) = \nabla \cdot \nabla' \rho_T(\mathbf{r}, \mathbf{r}') \Big|_{\mathbf{r}=\mathbf{r}'}$$

$$\mathbf{T}_T(\mathbf{r}) = \nabla \cdot \nabla' \mathbf{s}_T(\mathbf{r}, \mathbf{r}') \Big|_{\mathbf{r}=\mathbf{r}'}$$

- Most general** antisymmetrised product of nucleonic wavefunctions
- Not any a priori assumption** on the nucleons' wave function
- Correlations** beyond the mean-field effectively included by the EDF
- Investigate nuclear structure on the **whole nuclear chart**
- Relativistic**: the depth of the central potential is **consistently predicted**

# Relativistic EDF in nuclei



$$\mathcal{L}_{int} = -g_{\sigma}(\rho_v)\bar{\psi}\sigma\psi - g_{\omega}(\rho_v)\bar{\psi}\gamma_{\mu}\omega^{\mu}\psi - g_{\rho}(\rho_v)\bar{\psi}\gamma_{\mu}\vec{\rho}^{\mu}\cdot\vec{\tau}\psi - \frac{f_{\pi}(\rho_v)}{m_{\pi}}\bar{\psi}\gamma_5\gamma_{\mu}\partial^{\mu}\vec{\pi}\cdot\vec{\tau}\psi - e\bar{\psi}\gamma_{\mu}A^{\mu}\psi$$

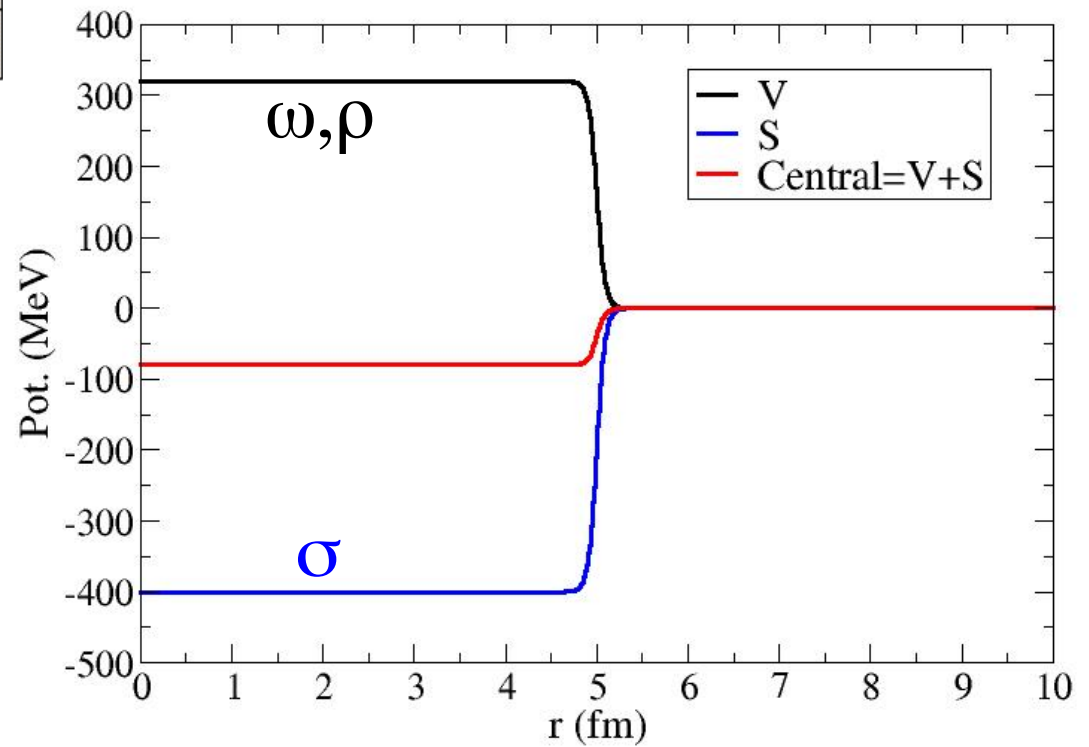
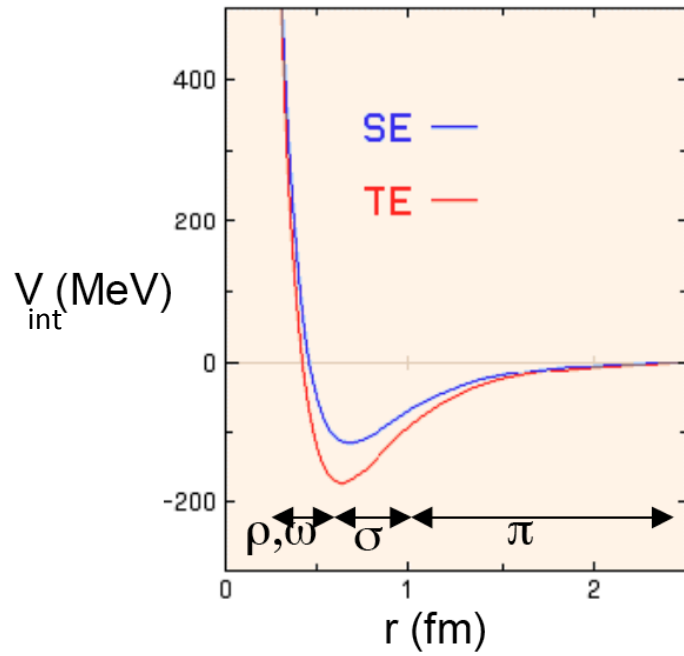
EDF  $[\rho ; \sigma, \omega, \rho, \pi, A]$

$$\left\{ p \frac{1}{2\tilde{M}(r)} p + W(r) + V_{ls}(r) l.s \right\} \varphi_i = \varepsilon_i^{NR} \varphi_i$$

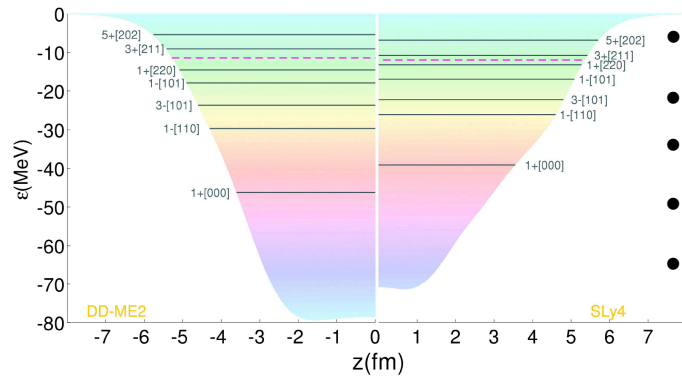
$$W(r) = [V + S](r)$$

$$V_{ls}(r) = \frac{1}{2\tilde{M}^2(r)} \frac{1}{r} \frac{d}{dr} (V - S)$$

# V and S potentials



# Origins of nuclear clustering



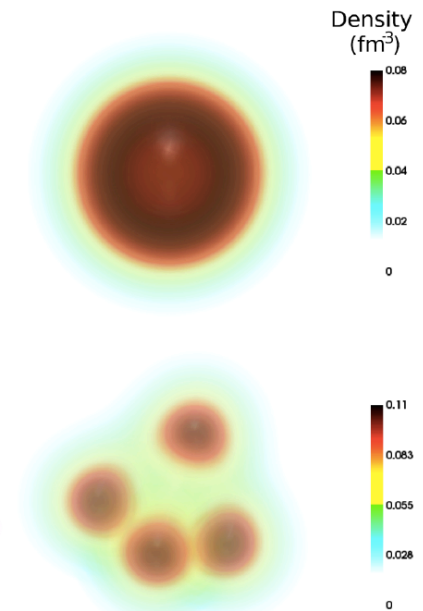
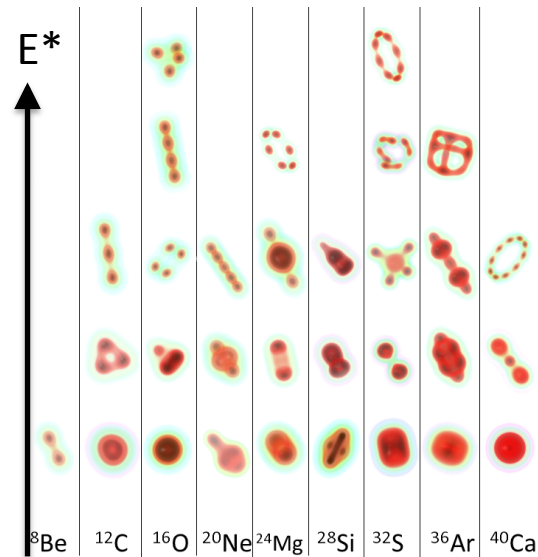
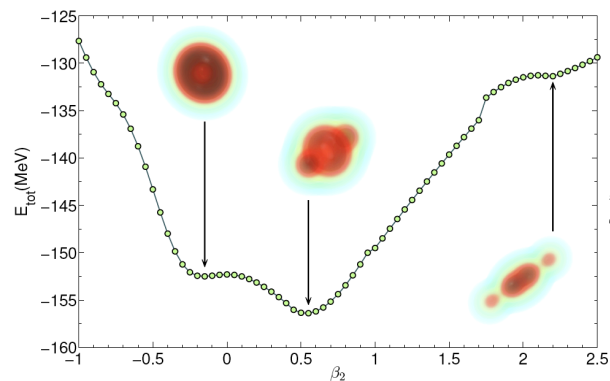
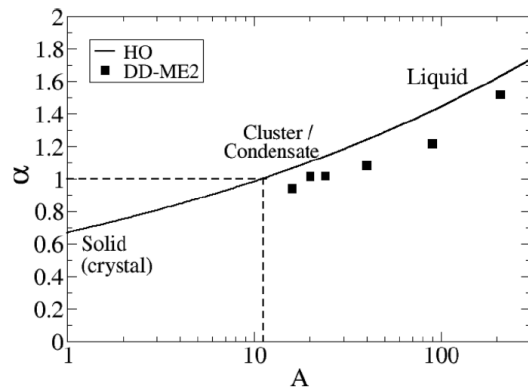
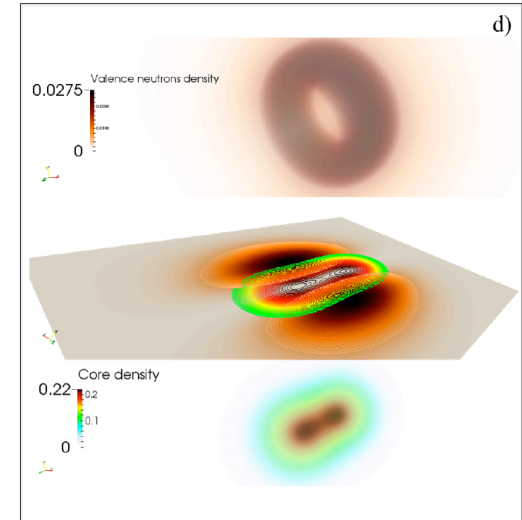
- Depth of the confining potential
- Heavy vs. Light nuclei
- Deformation / excitation energy
- Density
- Neutron excess

J.-P. Ebran, E. Khan, T. Niksic, D. Vretenar, Nature 487(2012)341

PRC87(2013)044307

PRC90(2014)054329

PRC89(2014)031303(R)



# What's new ?

**1) Quantitative results:** comparison with exp. rotational bands in  $^{20}\text{Ne}$  and  $^{12}\text{C}$

*P. Marevic , J.-P. Ebran, E. Khan, T. Niksic , and D. Vretenar, PRC 97, 024334 (2018)*

**2) More qualitative results :** localisation over the nuclear chart

*J.-P. Ebran, E. Khan, **R.-D. Lasserri**, and D. Vretenar. PRC 97, 061301(R) (2018)*

**3) Improvement of the method :** effect of pairing and quarteting on clustering

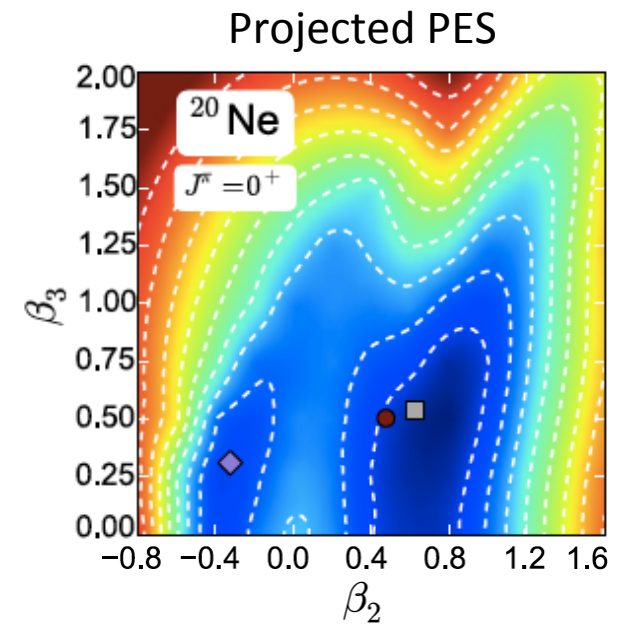
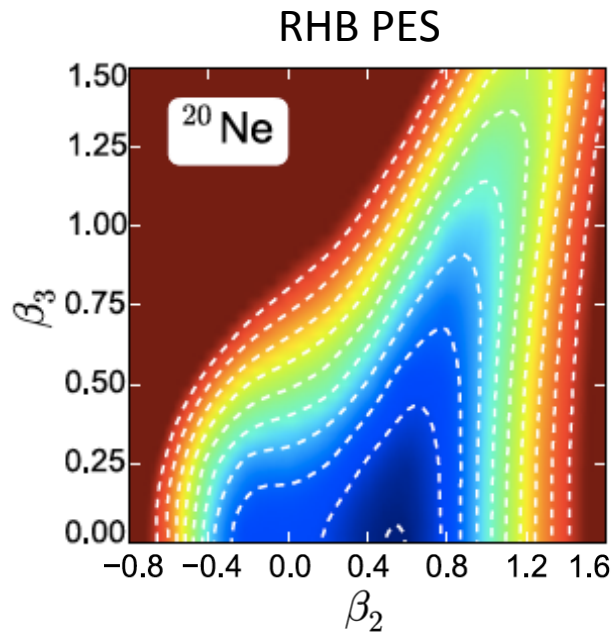
***R.-D. Lasserri**, J.-P. Ebran, E. Khan, and N. Sandulescu, PRC 98, 014310 (2018)*

# Comparison with exp. on $^{20}\text{Ne}$

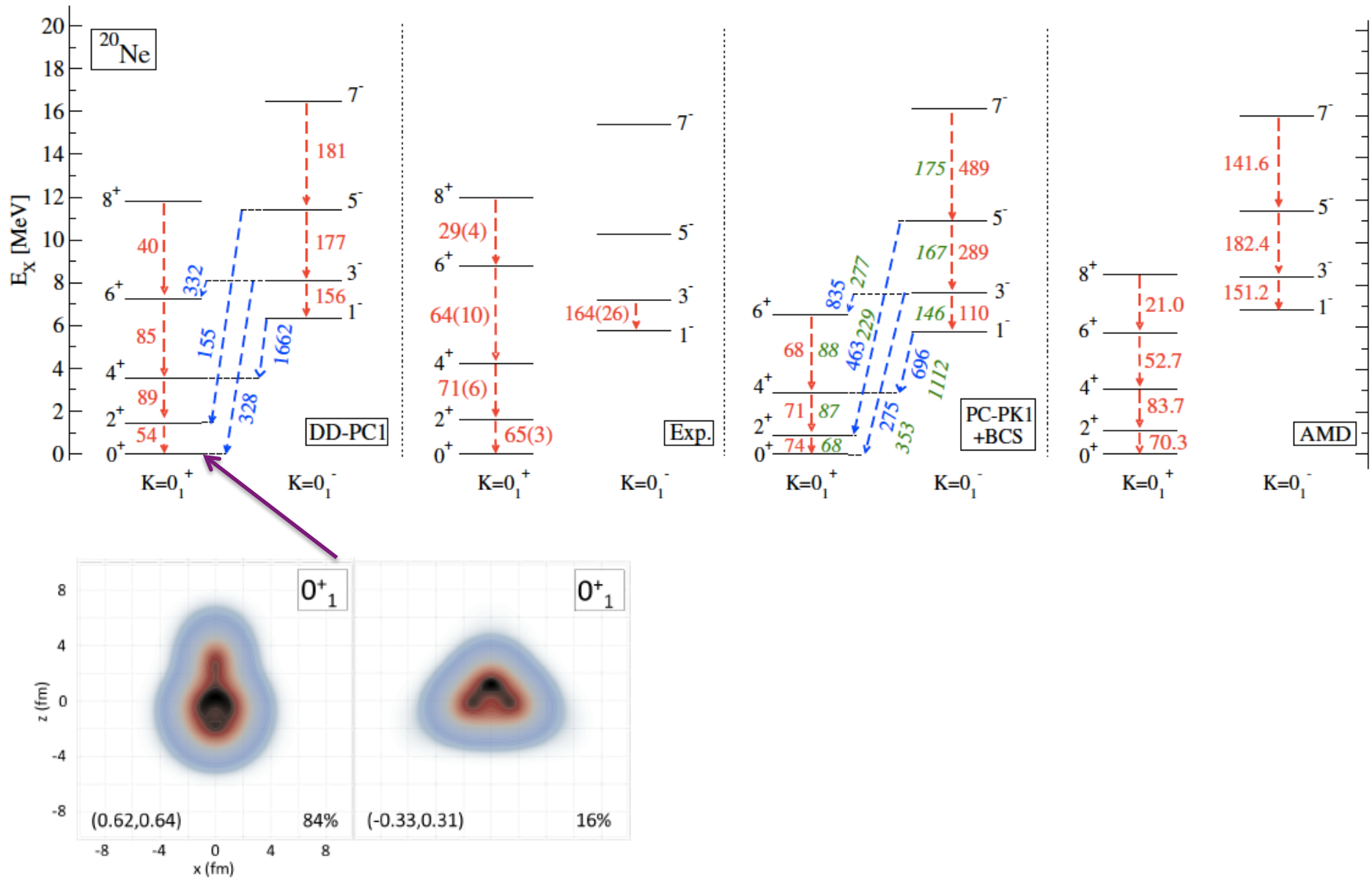
- GCM on top of axially symmetric /reflection asymmetric RHB (DD-PC1) :

$$|JM\pi; \alpha\rangle = \sum_j \sum_K f_\alpha^{JK\pi}(q_j) \hat{P}_{MK}^J \hat{P}^\pi |\phi(q_j)\rangle$$

- Angular momentum, parity and particle number projections



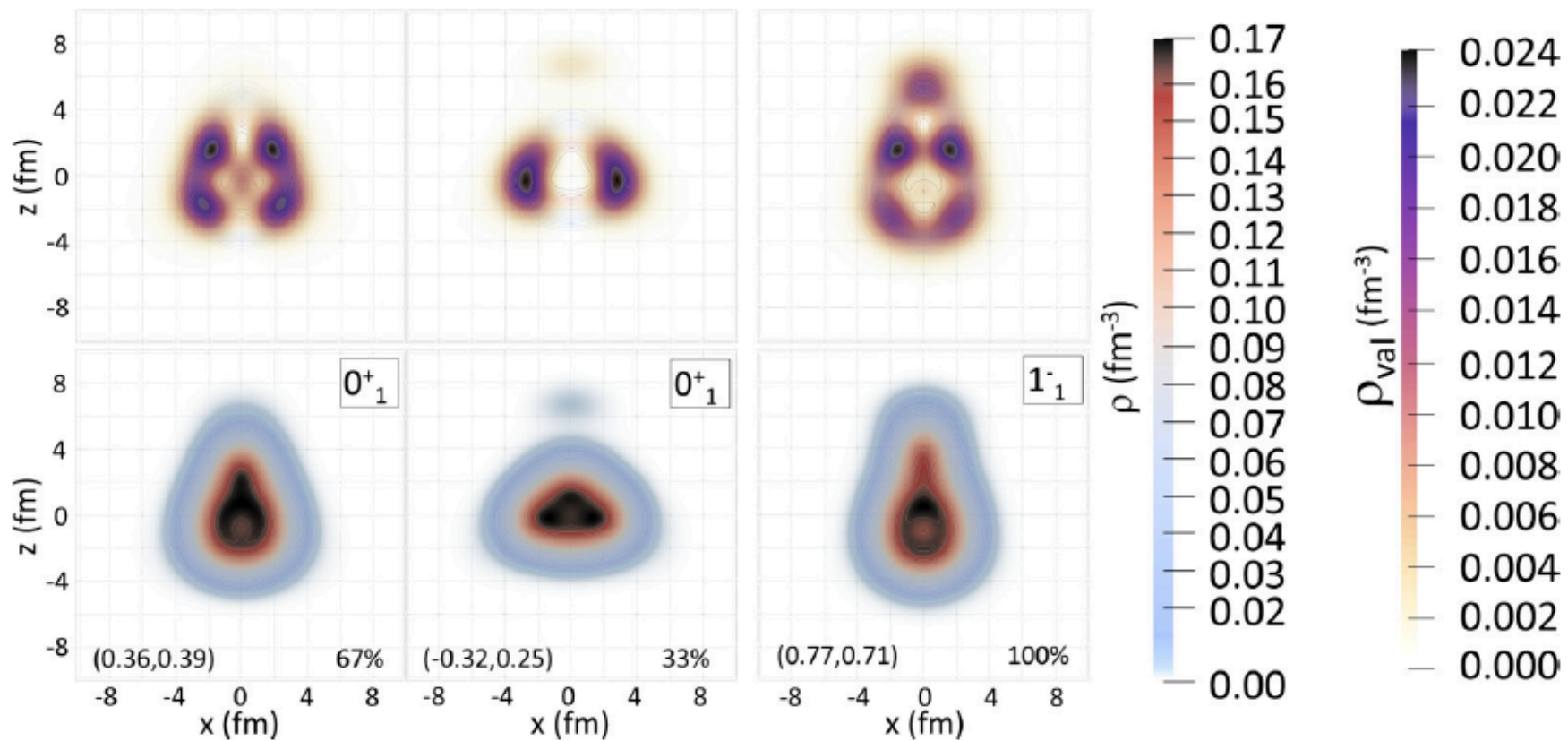
# Comparison with the data



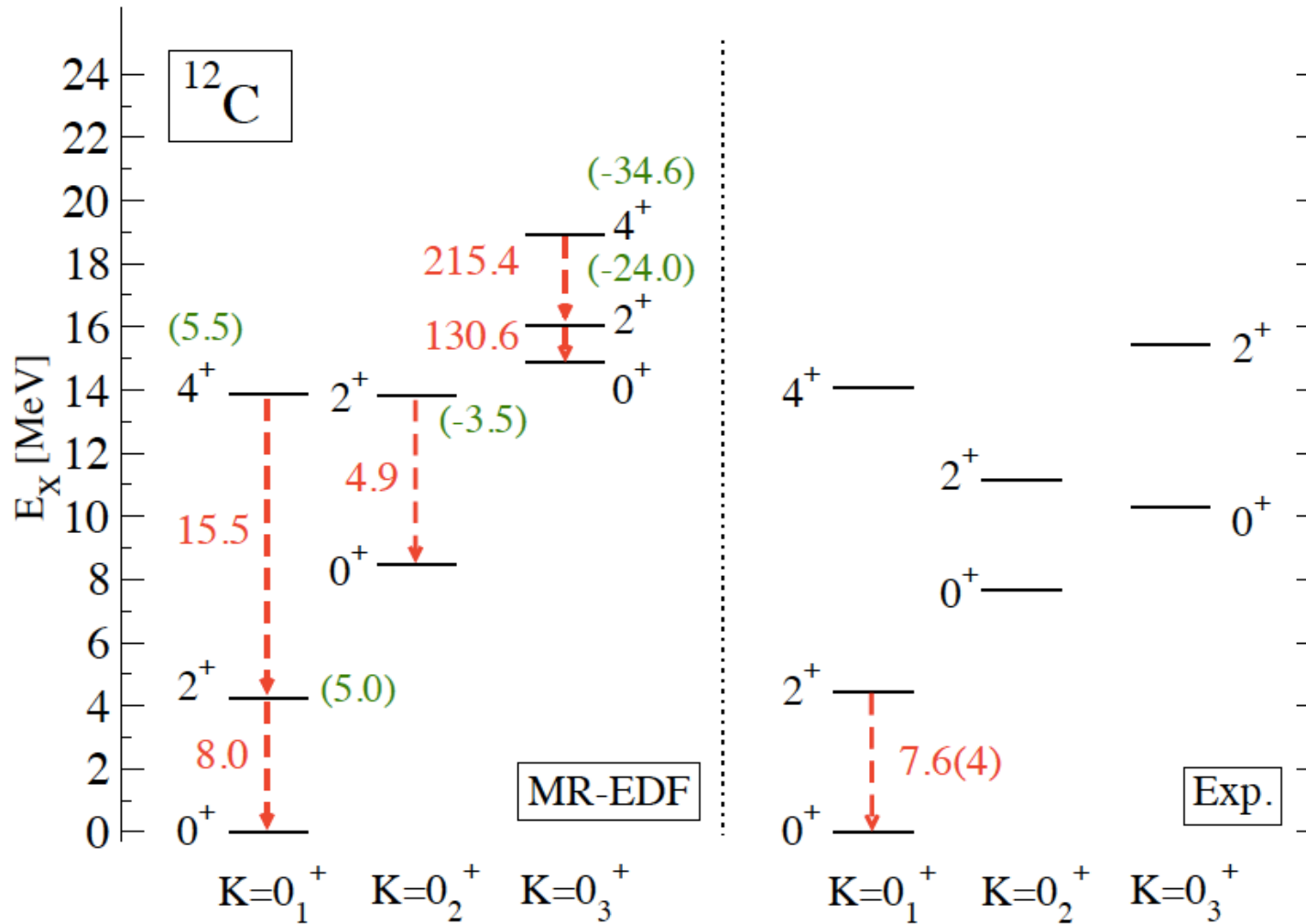


# Analysis of the densities

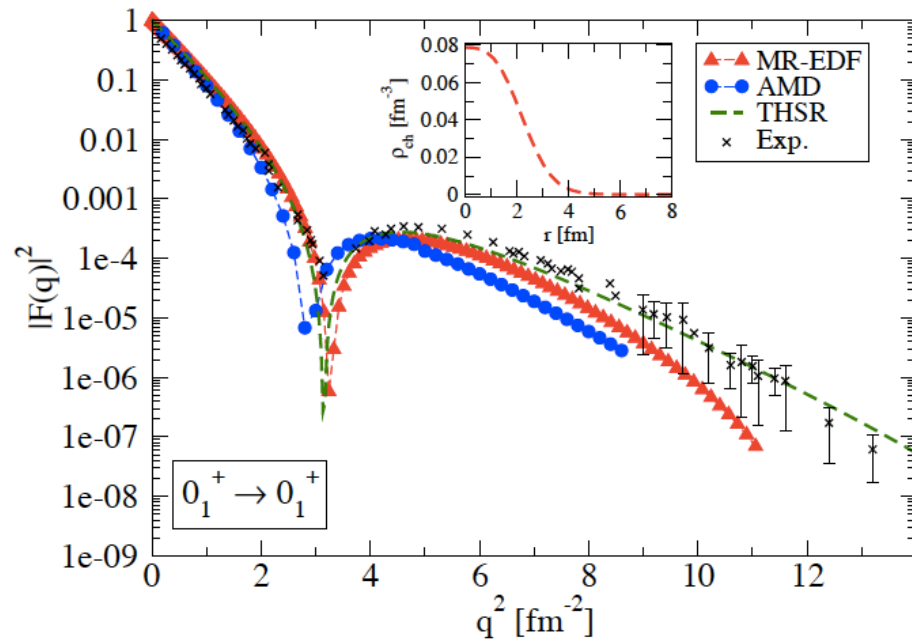
$^{24}\text{Ne}$



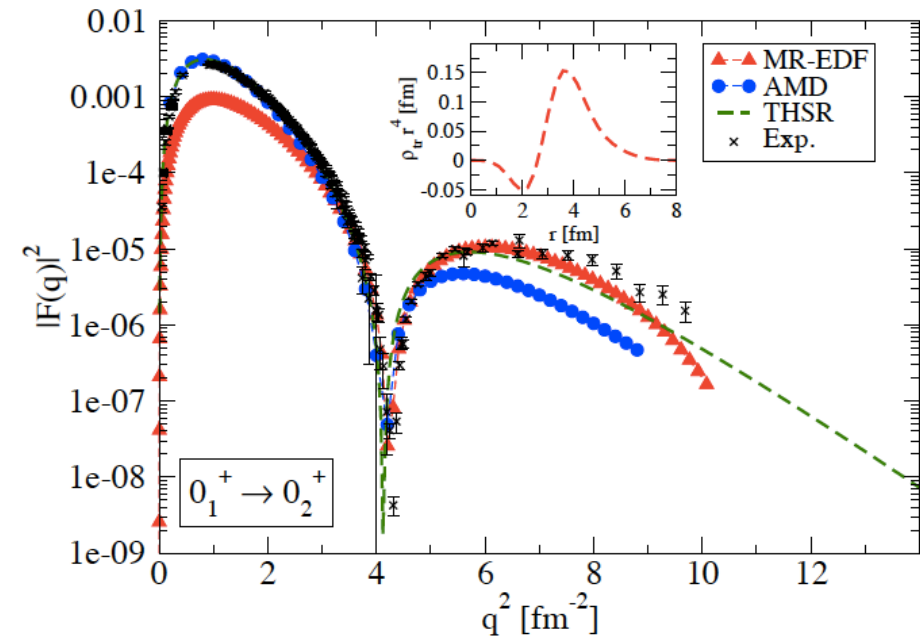
# Comparison with exp. on $^{12}\text{C}$



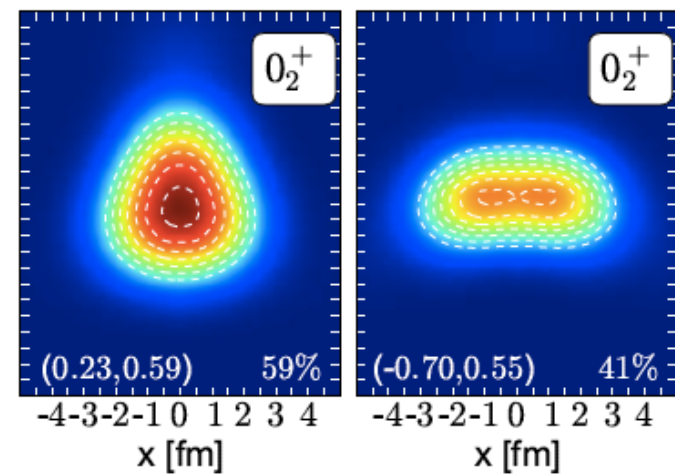
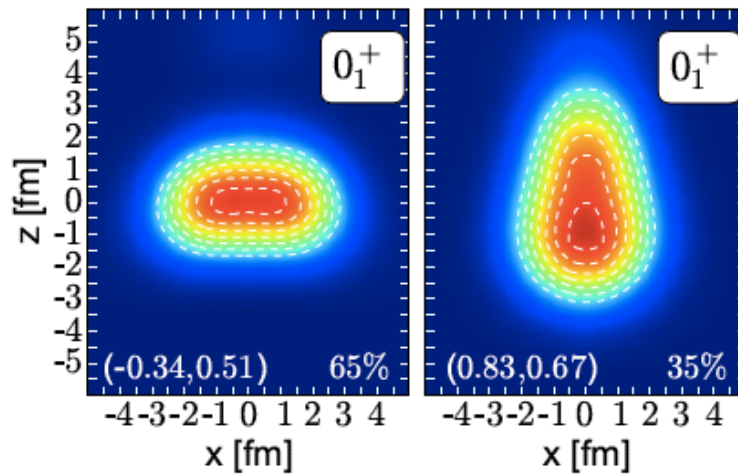
# Comparison with exp. on $^{12}\text{C}$



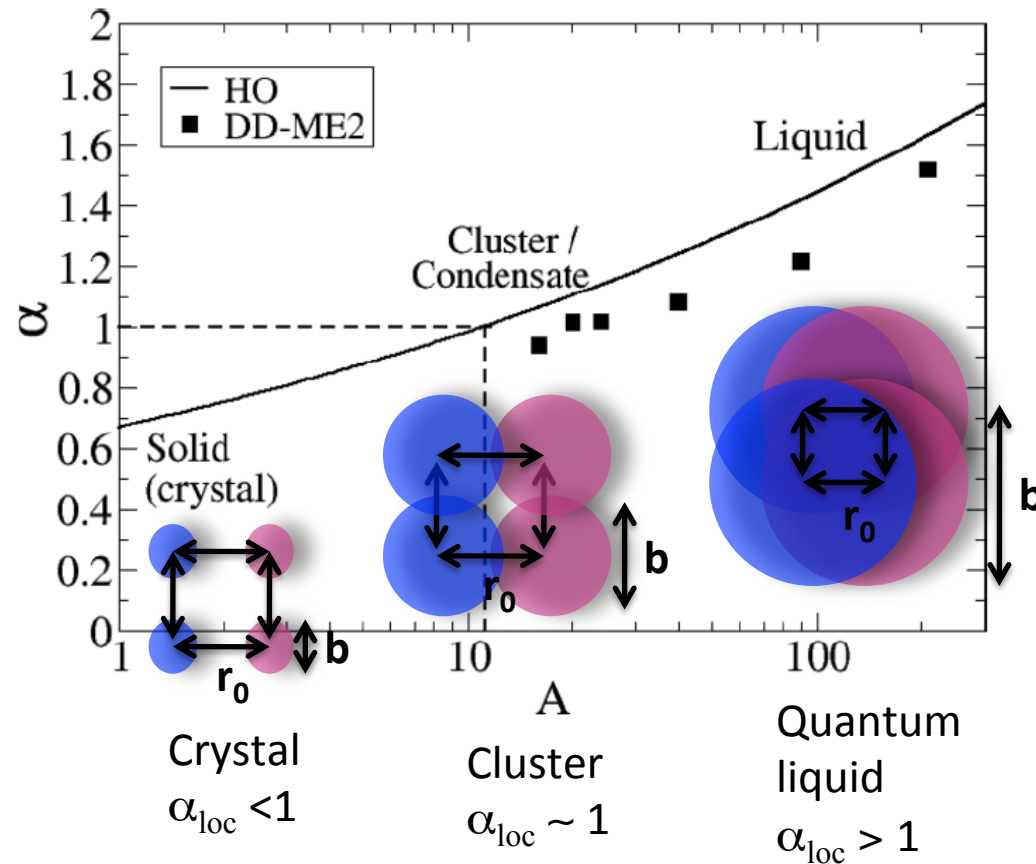
g.s.



Hoyle



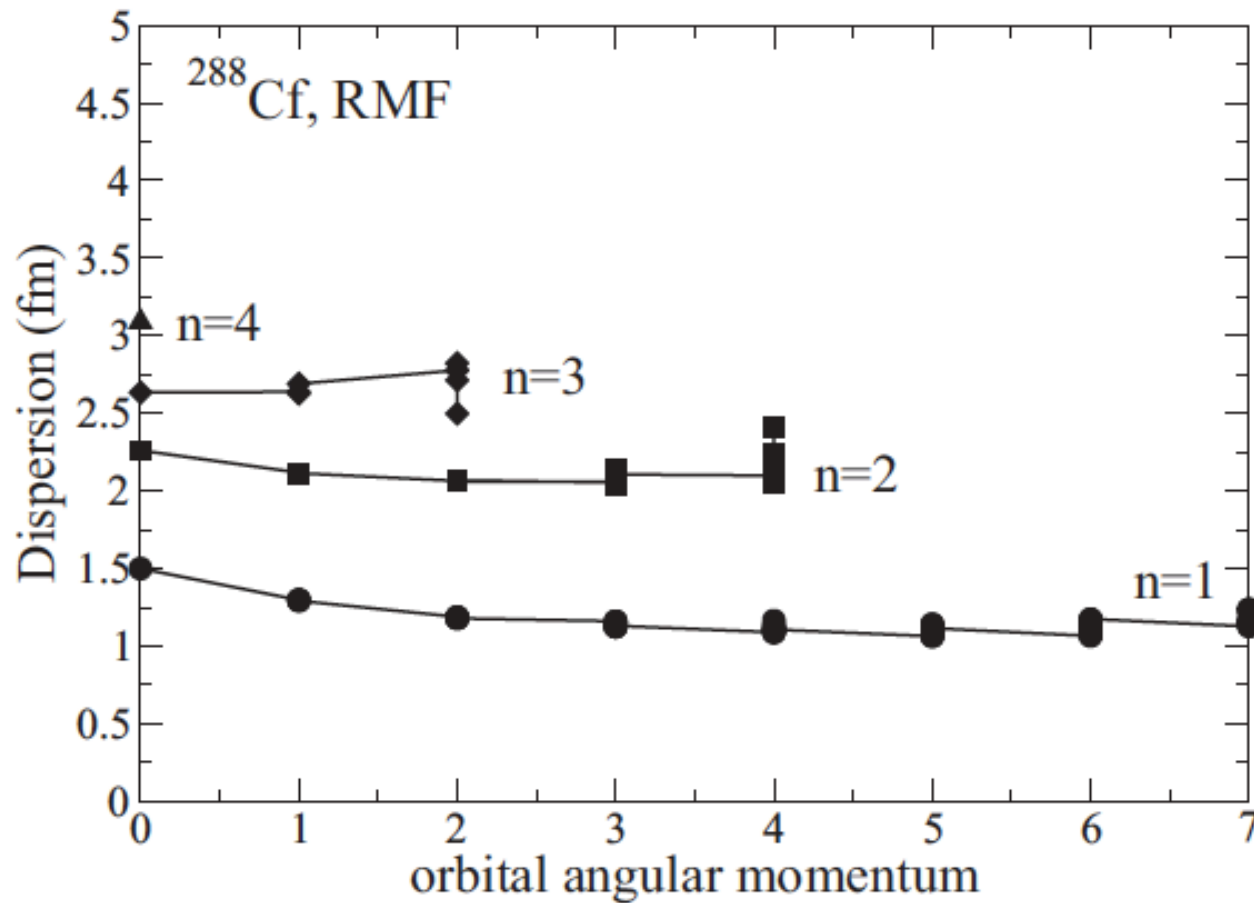
# Localisation



$$\alpha_{loc} = \frac{2\Delta r}{r_0} \simeq \frac{b}{r_0} = \frac{\sqrt{\hbar} A^{1/6}}{(2mV_0 r_0^2)^{1/4}}$$

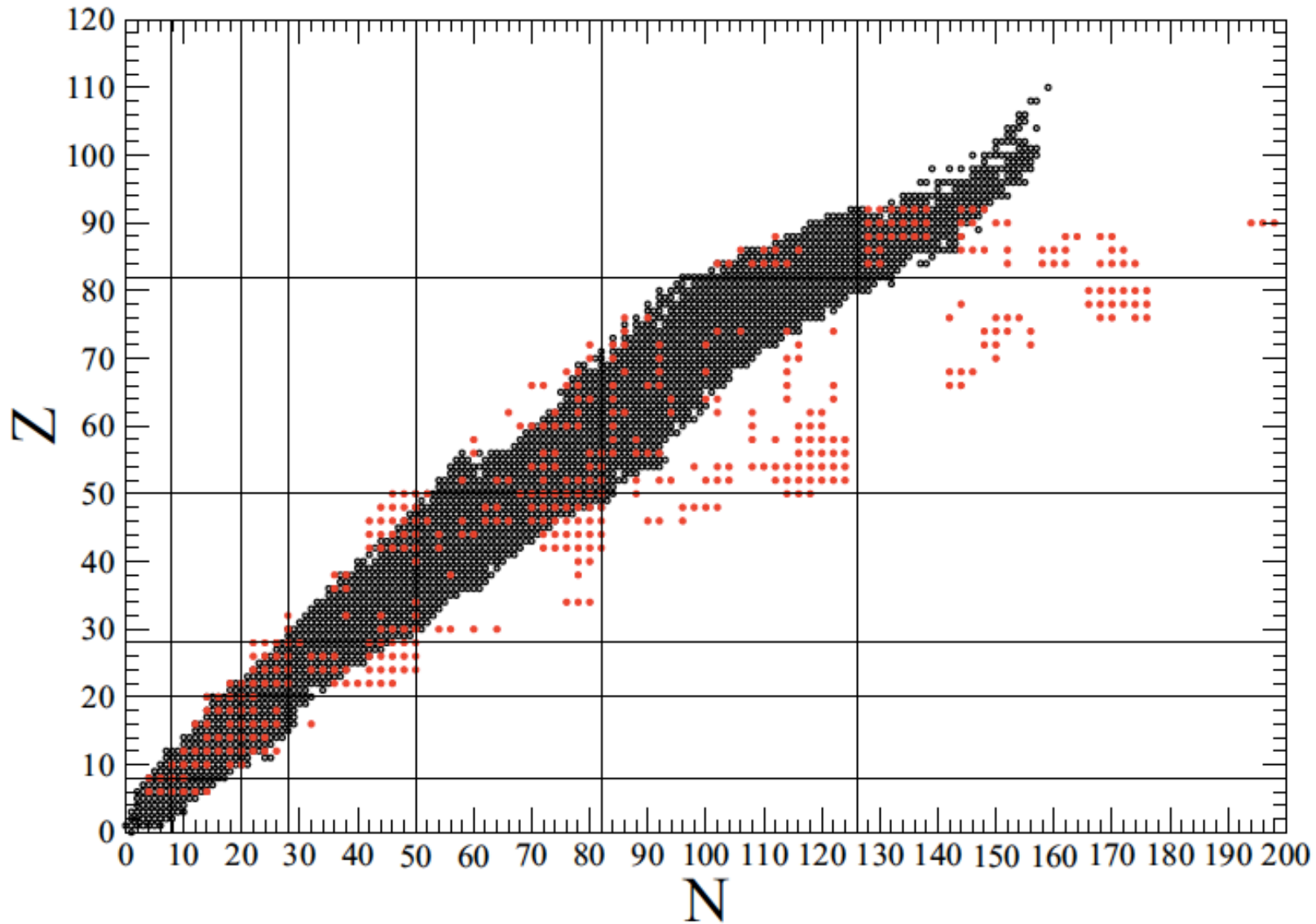
Single particle state dependence of the localisation ?

# Dispersion in nuclei: a striking pattern



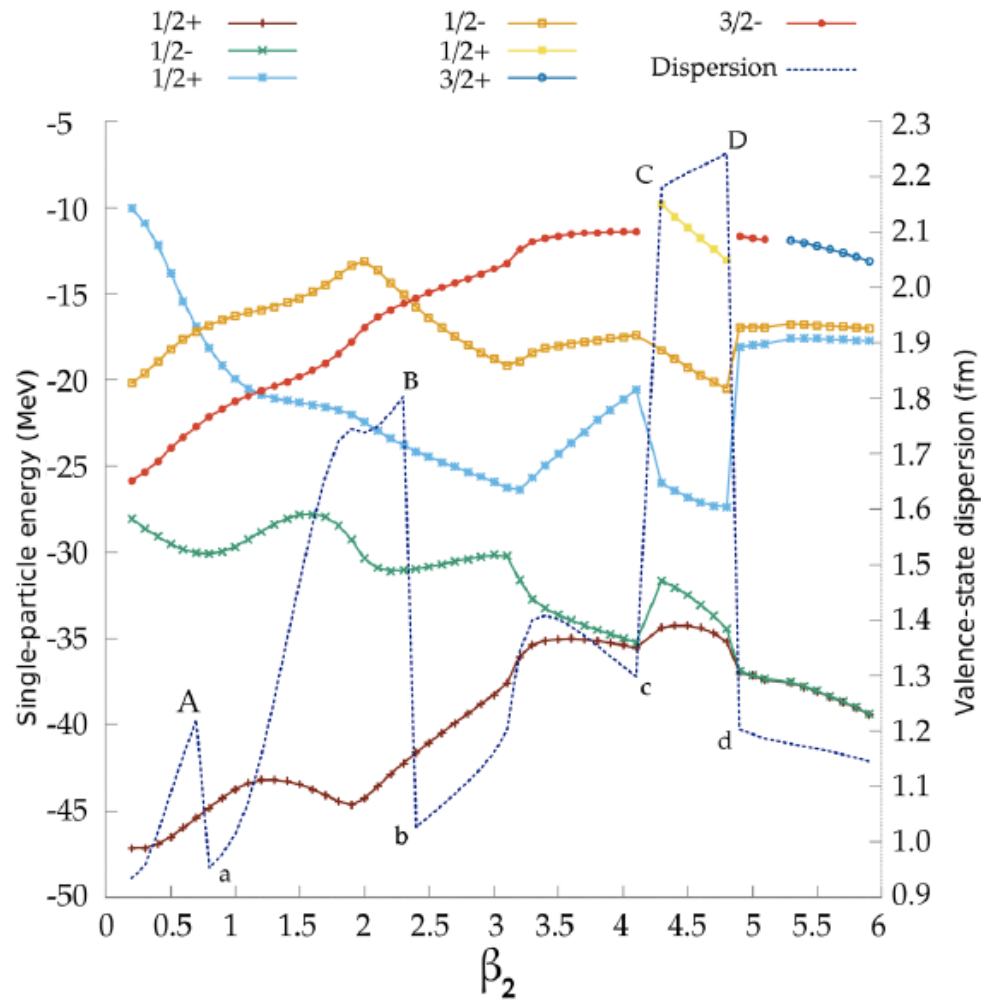
$$\alpha_{\text{loc}} = \frac{2\Delta r}{r_0} \simeq \frac{b}{r_0} \sqrt{2n-1} = \frac{\sqrt{\hbar(2n-1)}}{(2mV_0r_0^2)^{1/4}} A^{1/6}$$

# $\alpha$ -valence localisation over the nuclear chart

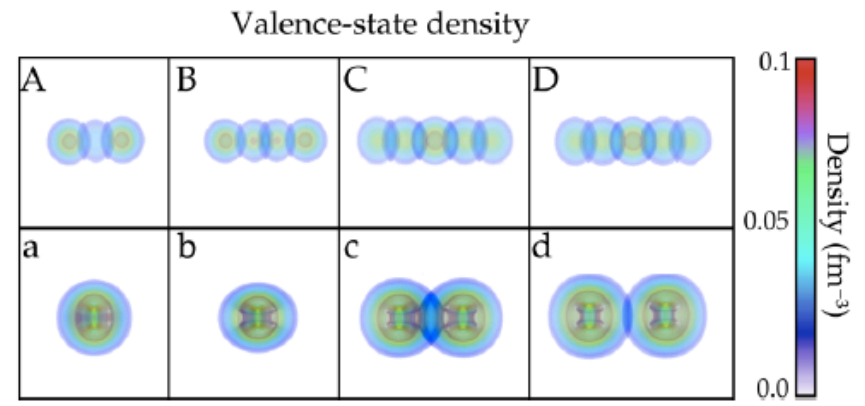


Axially symmetric RHB DD-ME2 calc.

# Dispersion and s.p. states

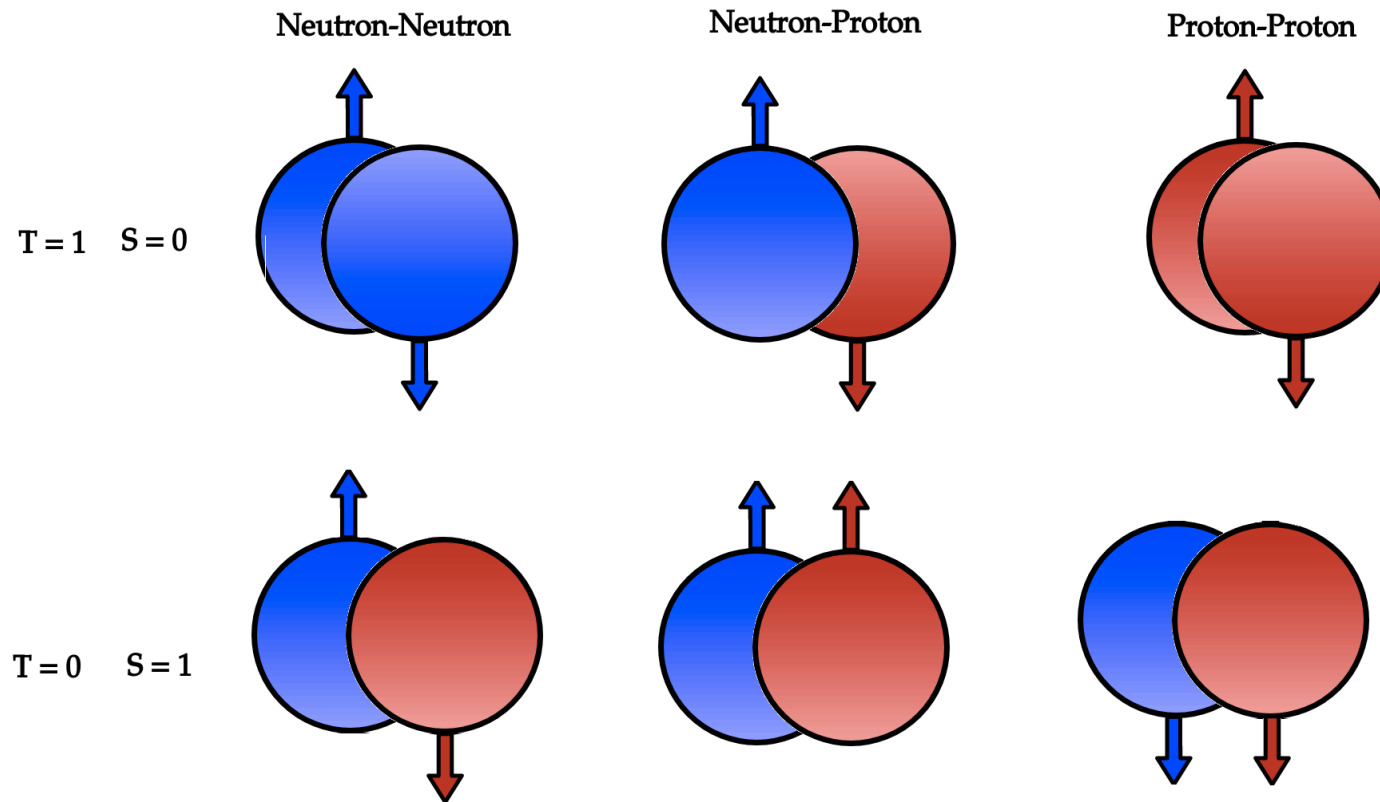


$^{20}\text{Ne}$



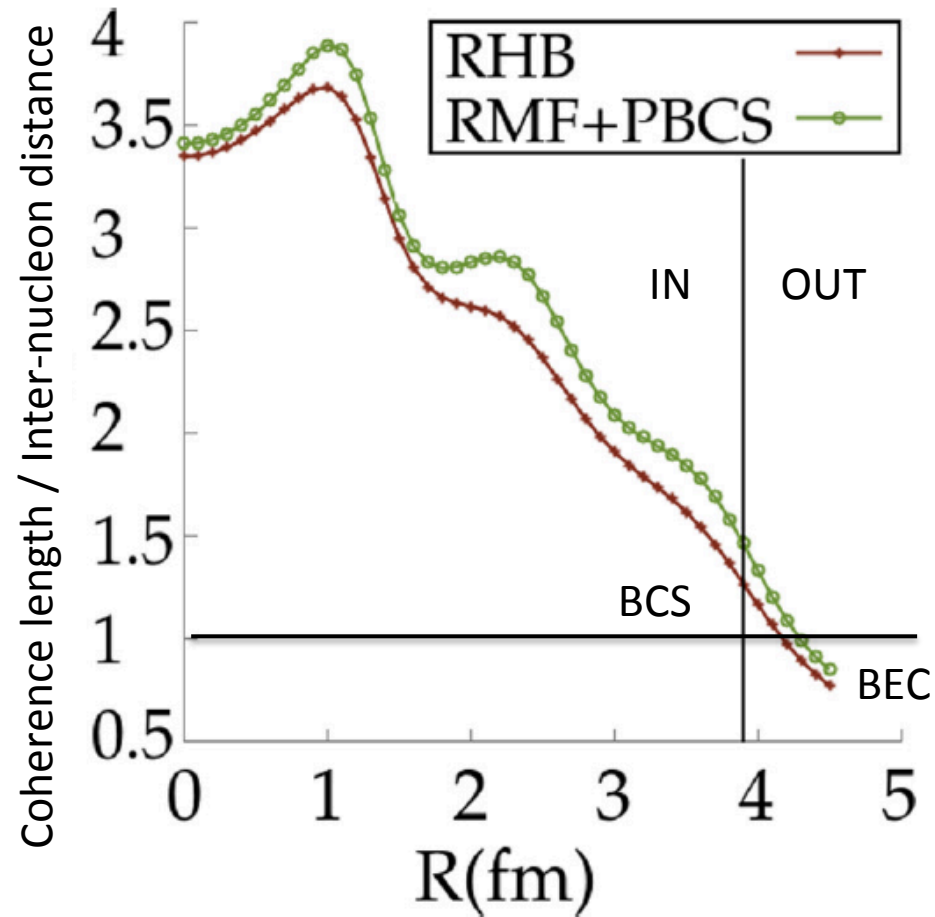
# Impact of pairing and quarteting on clustering

Quarteting = projected BCS in the nn, pp and np channels

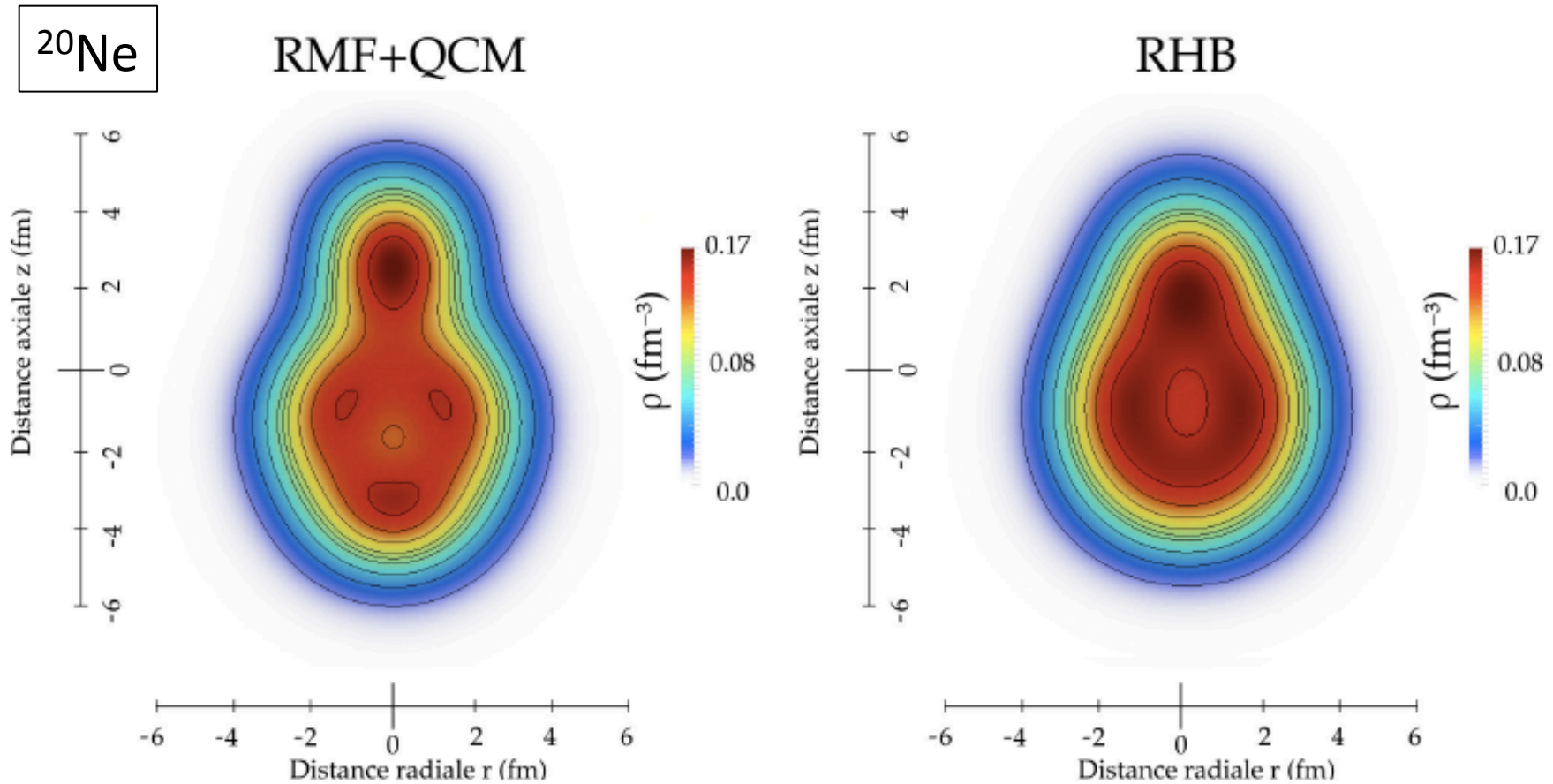




# Pairing coherence length vs. inter-nucleon distance

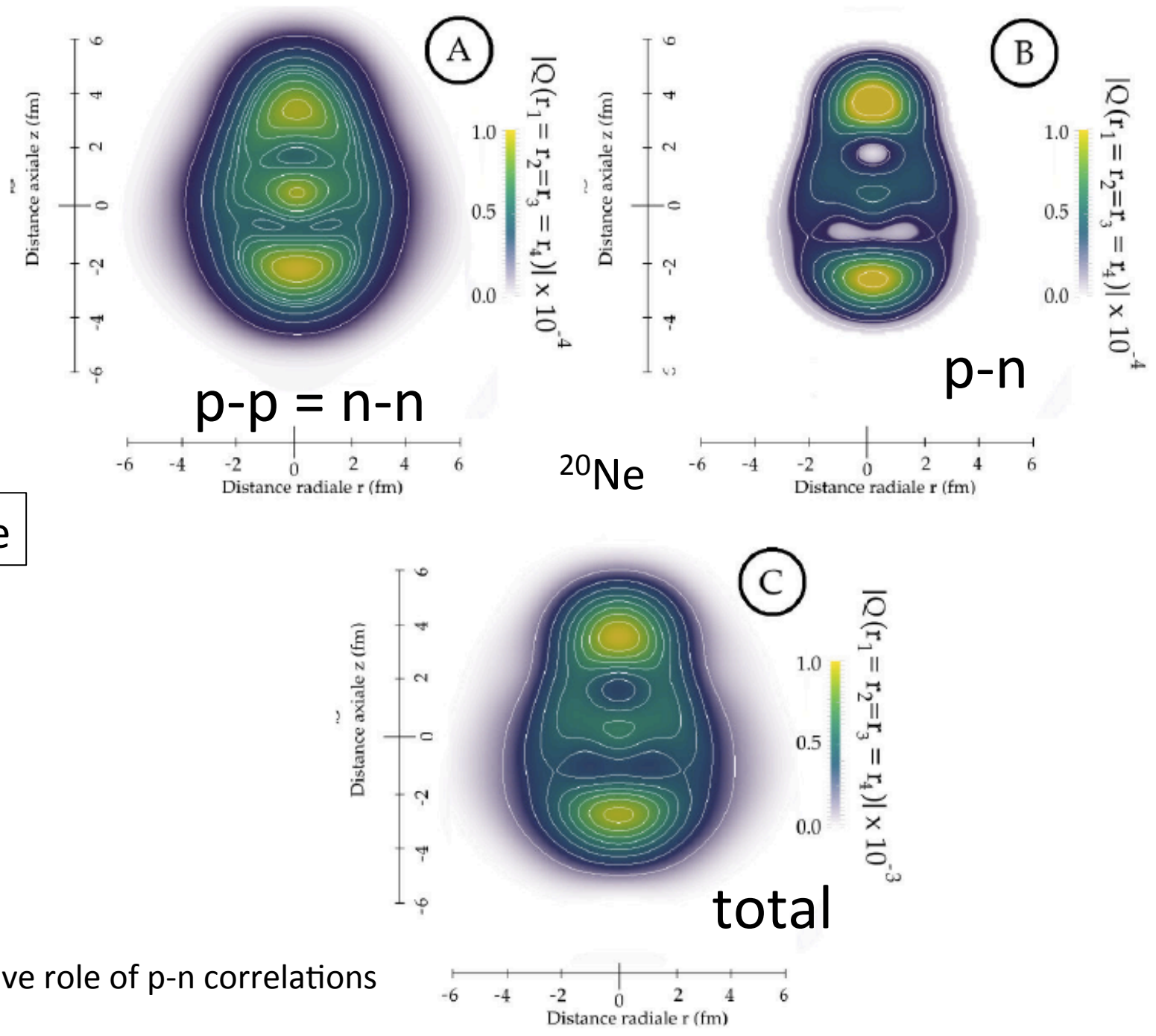


# Effect of pairing and quarteting on clustering



- Pairing acts against clustering (smearing of the density)
- Quarteting is compatible with clustering

$^{20}\text{Ne}$



Decisive role of p-n correlations

# Conclusions and outlooks

- Nuclear clustering described by relativistic EDF both on qualitative and quantitative grounds
  - Clustering persist with quarteting
  - General and microscopic view and predictive power on nuclear clustering
- 
- More comparison with data
  - Clustering in heavy nuclei
  - Relating clustering structure and alpha (cluster) radioactivity
  - Density dependence and the Mott transition