Isospin-symmetry breaking in nuclear structure

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CNRS/IN2P3 – Université de Bordeaux, France

European Nuclear Physics Conference, Bologna, Italy,
September 3–7, 2018

Plan of the talk

Isospin-symmetry breaking (ISB) in nuclear structure

I Introduction
- Experimental evidence
- ISB in the *NN* interaction and light nuclei

II Many-body approach (Shell Model) with effective charge-dependent interactions
- Phenomenological versus microscopic description
- Isobaric multiplet splitting

III Isospin-forbidden transitions and isospin mixing (β-delayed proton emission)

IV ISB correction to superallowed $0^+ \rightarrow 0^+$ β-decay

V Applications to astrophysical issues

I. Introduction

Isospin symmetry

$$\psi_p(\vec{r}) = \psi(\vec{r}) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \ \psi_n(\vec{r}) = \psi(\vec{r}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad [\hat{t}_i, \hat{t}_j] = i\epsilon_{ijk}\hat{t}_k\,, \quad [\hat{\mathbf{t}}^2, \hat{t}_i] = 0$$

Experimental evidence on isospin-symmetry breaking (ISB)

- Splittings of isobaric multiplets
- Isospin-forbidden processes (isospin-forbidden Fermi β-decay, $E1$-transitions in $N = Z$ nuclei, isospin-forbidden nucleon(s) emission, etc)

Importance of an accurate description of the isospin-symmetry breaking

- Structure and decay modes of proton-rich nuclei and nuclei along $N = Z$ line
- ISB corrections to superallowed Fermi $0^+ \rightarrow 0^+$ β-decay for the tests of the Standard Model (CVC test and unitarity of the CKM matrix)
- Astrophysics applications (masses or proton-rich nuclei and p-capture reactions)

Isospin-symmetry breaking

Sources of the isospin-symmetry breaking in nuclei:

- Coulomb interaction between protons (V_{pp}^{Coul})
- $M_p - M_n \approx 1.3$ MeV and charge-dependent forces of nuclear origin

Experimental evidence on charge-dependent *NN* forces

- *NN* scattering length in the $^1S_0$ channel

$$a_{nn} - a_{pp} = 1.65 \pm 0.60 \text{ fm} \qquad \textit{Charge-symmetry breaking (CSB)}$$

$$\frac{1}{2}(a_{nn} + a_{pp}) - a_{np} = 5.6 \pm 0.6 \text{ fm} \qquad \textit{Charge-independence breaking (CIB)}$$

- ^{3}H and ^{3}He binding energy difference

$$\Delta BE_{exp} \approx 764 \text{ keV} \quad \Delta BE_{th}^{Coul} \approx (680 \pm 30) \text{ keV}$$

- Nolen-Schiffer anomaly: the Coulomb interaction cannot reproduce observed difference of binding energies of mirror nuclei

  J.A. Nolen, J.P. Schiffer, ARNS 19, 471 (1969)

Charge-dependent *NN* forces

Classification of two-nucleon forces (*E.M. Henley, G.A. Miller, 1979*):

- Class I: $V_I = \alpha + \beta \hat{\mathbf{t}}(1) \cdot \hat{\mathbf{t}}(2)$
- Class II: $V_{II} = \alpha \hat{t}_3(1) \hat{t}_3(2)$
- Class III: $V_{III} = \alpha(\hat{t}_3(1) + \hat{t}_3(2))$
- Class IV: $V_{IV} = \alpha(\hat{t}_3(1) - \hat{t}_3(2)) + \beta[\hat{\mathbf{t}}(1) \times \hat{\mathbf{t}}(2)]_3$

$$V_I > V_{II} > V_{III} > V_{IV}$$

Theory of ISB *NN* forces

- Meson-exchange models

  G.A. Miller et al, Phys. Rep. 194, 1 (1990); R. Machleidt, PRC64, 024001 (2001), . . .

- χ EFT (ISB *NN* and 3*N* forces)

  U. van Kolck, J.L. Friar, E. Epelbaum, U.-G. Meissner, . . .
  see for review E. Epelbaum et al, RMP81, 1773 (2009)

Light nuclei

GFMC with CD interactions (AV18 + IL7) for ^{3}H - ^{3}He, $A = 7, 8$, isospin-mixing in ^{8}Be

e.g., R.B. Wiringa, S. Pastore, S.C.Pieper, G.A. Miller, PRC88, 044333 (2013)

II. Many-body models with INC Hamiltonians

- Shell model (from 60's ...)

  W.E. Ormand, B.A. Brown et al 1985 –; S. Nakamura, K. Muto, T. Oda (1994); A.P. Zuker, S.M. Lenzi, M.A. Bentley et al, 2001 – 2018; K. Kaneko et al, 2010 – 2018; Y.H. Lam, N. Smirnova, E. Caurier, 2013 – 2018.
- No-core shell model

  E. Caurier, P. Navratil et al, 2002, ...
- Gamow shell-model, SMEC

  N. Michel, W. Nazarewicz, M. Ploszajczak, PRC82 (2010), ...
- HF + Tamm-Dankoff, RPA, HTDA

  I. Hamamoto, H. Sagawa, N. V. Giai, J. Dobaczewski, T. Suzuki, G. Colo, J. Le Bloas, N. Auerbach et al, 1993 – 2018
- Relativistic RPA

  H. Liang, N. V. Giai, J. Meng, 2009 –
- J-projected and T-projected DFT with ISB terms

  W. Satula, J. Dobaczewski et al, 2009 – 2018, P. Baczyk et al, PLB778, 178 (2018)
- VAP technique on the HFB basis

  A. Petrovici et al, 2008 – 2018
- Isovector giant monopole resonance

  G. Colo et al (1993); N. Auerbach, Phys. Rep. 98, 273 (1983); PRC (2009)
- ...

Isospin-symmetry breaking in the shell model

Isospin-invariant shell model: $[\hat{H}, \hat{\mathbf{T}}^2] = [\hat{H}, \hat{T}_i] = 0$

$$\hat{H}\Psi_{TT_z} \equiv (\hat{H}_0 + \hat{V}_0)\Psi_{TT_z} = E_T\Psi_{TT_z}, \quad \hat{H}_0\Phi_{TT_z} = E_0\Phi_{TT_z}$$

$$\Psi_{TT_z} = \sum_k C_{Tk}\Phi_{TT_z k} \quad \Rightarrow \quad \langle\Phi_k|\hat{H}|\Phi_l\rangle \quad \Rightarrow \quad \{E_T, C_{Tk}\}$$

Isospin-nonconserving (INC) term from *classes II - III* forces only

$$\hat{V}_{INC} = \hat{V}_C + \hat{V}_{CD} = \sum_{k=0,1,2} \hat{V}^{(k)}$$

$$\begin{aligned} \hat{V}^{(0)} &= (v_{pp} + v_{nn} + v_{np}^{T=1})/3 \\ \hat{V}^{(1)} &= v_{pp} - v_{nn} \\ \hat{V}^{(2)} &= (v_{pp} + v_{nn})/2 - v_{np}^{T=1} \end{aligned} \qquad \epsilon_i = \epsilon_i^p - \epsilon_i^n$$

In first order perturbation theory:

$$\langle\Psi_{TT_z}|\hat{V}_{INC}|\Psi_{TT_z}\rangle = E^{(0)}(\alpha, T) + E^{(1)}(\alpha, T)T_z + E^{(2)}(\alpha, T)\left[3T_z^2 - T(T+1)\right]$$

Isobaric-multiplet mass equation – IMME (*E.P. Wigner, 1957*):

$$M(\alpha, T, T_z) = a(\alpha, T) + b(\alpha, T)T_z + c(\alpha, T)T_z^2$$

Shell-model with charge-dependent Hamiltonians

$$\hat{H}_{INC}\Psi(\alpha_p,\alpha_n) \equiv (\hat{H}_0 + \hat{V}_0 + \hat{V}_{INC})\Psi(\alpha_p,\alpha_n) = E\Psi(\alpha_p,\alpha_n)$$

How to construct charge-dependent Hamiltonian $\hat{H}_{INC}$?

- **Phenomenological** effective charge-dependent part (V_{INC}) is added to a well-known isospin-invariant Hamiltonian

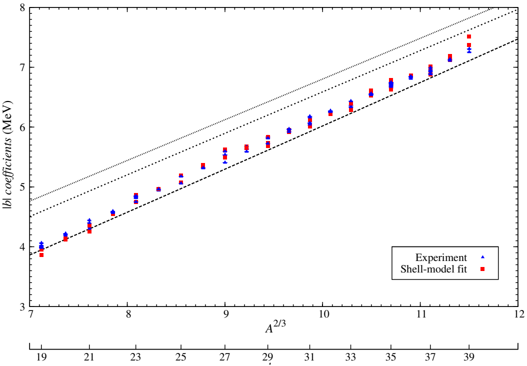
  W.E. Ormand, B.A. Brown, NPA490, 1 (1989)
  S. Nakamura, K. Muto, T. Oda, NPA575, 1 (1994)
  Y.H. Lam, N.A. Smirnova, E. Caurier, PRC87 (2013)
  A.P. Zuker, S.M. Lenzi, et al, PRL89 (2002)
  K. Kaneko, Y. Sun, T. Mizusaki, S. Tazaki, PRL110, 172505 (2013), . . .

- **Microscopic** effective charge-dependent interaction from NN ($+3N$) force

  J.D. Holt, J. Menendez, A. Schwenk, PRL110, 022502 (2013)
  W.E. Ormand, B.A. Brown, M. Hjorth-Jensen, PRC96, 024323 (2017)
  N.A. Smirnova, B.R. Barrett et al, in preparation

b coefficients in the *sd* shell ($v_{pp} - v_{nn}$)

USDA/USDB (*Brown, Richter, 2006*)
plus V_C, V_{CD}, $\tilde{\epsilon}_i \rightarrow$ 5 parameters *Y.H. Lam, Smirnova, Caurier, PRC87 (2013)*



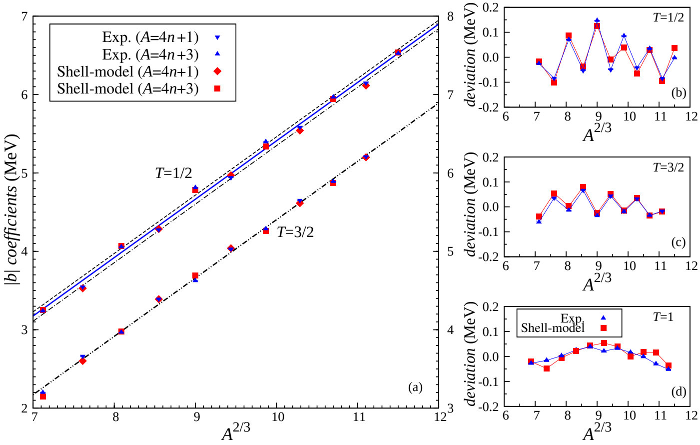
Comparison with uniformly charged sphere and *Möller, Nix, ADNDT39 (1988)*

- 81 data points ($T = 1/2, 1, 3/2, 2$): rms $\approx$ 32 keV
- experimental IMME database: *Y.H. Lam et al* **ADNDT99**, *680 (2013)*

Staggering of the *sd*-shell *b*-coefficients

Theory: *Y.H. Lam, N. Smirnova, E. Caurier, 2013*

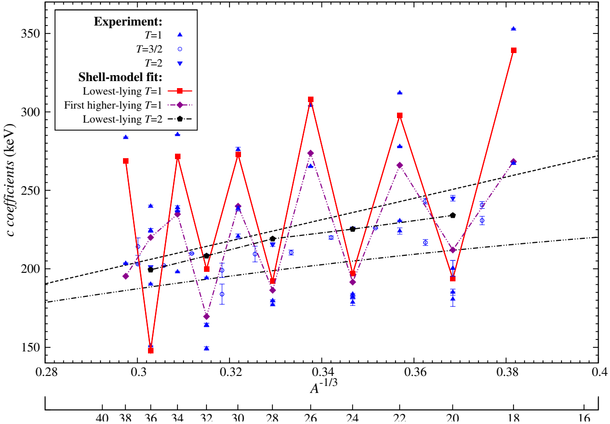
Data: *Y.H. Lam et al, ADNDT99, 2013; M. MacCormick, G. Audi, NPA925, 2014*



c-coefficients in the sd shell ($v_{pp} + v_{nn} - 2v_{pn}$)

Theory (shell model): *Y.H. Lam, N. Smirnova, E. Caurier, PRC87 (2013)*

Data: *Y.H. Lam et al, ADNDT99, (2013); M. MacCormick, G. Audi, NPA925, (2014)*



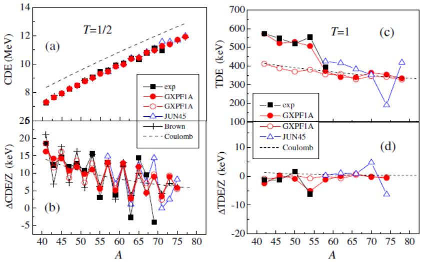
- 2-parameter fit on 51 data points ($T = 1, 3/2, 2$): rms $\approx$ 10 keV
- Black dashed line: $c = \frac{3e^2}{5r_0} A^{-1/3}$; double-dot-dashed line: *Möller, Nix, ADNDT39, 1988*

Mirror and triplet displacement energies in the *pf* and $pf_{5/2}g_{9/2}$ shell-model space

K. Kaneko, Y. Sun, T. Mizusaki, S. Tazaki, PRL110, 172505 (2013)

Details

- GXPF1A and JUN45 (charge-independent)
- Theoretical ISPE
- $\hat{V}_C$ (Coulomb)
- $\hat{V}_{INC}$: $J = 0$ nn, pp, np matrix elements in $f_{7/2}$
- *b* and *c* coefficients as a function of *A*



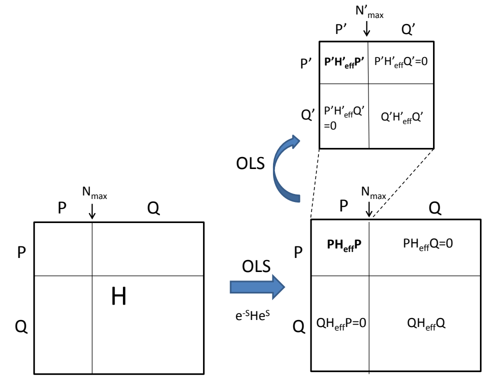
Applications for astrophysics

Prediction of masses of heavy nuclei along $N = Z$ and mapping of the proton drip-line

W.E. Ormand, PRC53, 214 (1996); PRC55, 2407 (1997); J. Cole, PRC54 (1996), . . .

Ab-initio effective interaction from the NCSM

$$H = \sum_{i<j=1}^{A} \frac{(\vec{p}_i - \vec{p}_j)^2}{2Am} + \sum_{i<j=1}^{A} V_{ij}^{NN}$$



Flow

- NCSM for ^{18}F at $N_{\max} = 4$ with V_{eff} (OLS)
- $\tilde{H}'_{eff}$ for ^{18}F at $N'_{\max} = 0$ (OLS)
- NCSM for ^{16}O (core energy)
- NCSM ^{17}O, ^{17}F (one-body terms)
- Single-particle energies and $T = 0, 1$ TBMEs in *sd* shell

S. Okubo, Progr. Theor. Phys. 12 (1954); K. Suzuki, S.Y.Lee, Prog. Theor. Phys. 68 (1980)

E. Dikmen, A.F. Lisetskiy, B.R. Barrett et al, PRC91, 064301 (2015)

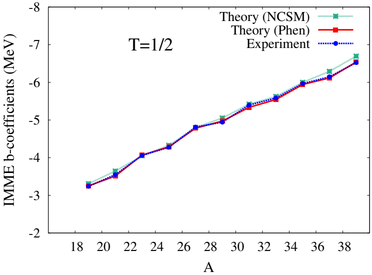
Ab-initio effective interaction from the NCSM

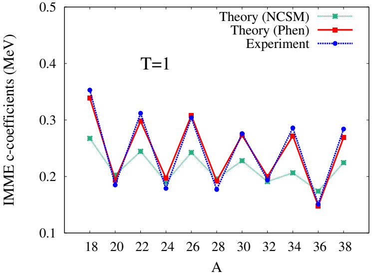
Present results obtained from the NCSM

- Daejeon16 *NN* potential (based on SRG-evolved chiral N3LO)

  A.M. Shirokov, I.J. Shin, Y. Kim, P. Maris, M. Sosonkina, J.P. Vary, PLB761, 81 (2016)

- $N_{max} = 4$ and $\hbar\Omega$=14 MeV
- ^{18}O, ^{18}F, ^{18}Ne $\Rightarrow$ pp,nn, pn ($T = 0, 1$) TBMEs in *sd* shell (charge-dependent)





- *b*-coefficients: rms (NCSM) $\approx$ 86 keV, rms (phen) $\approx$ 30 keV
- *c*-coefficients: rms (NCSM) $\approx$ 51 keV, rms (phen) $\approx$ 11 keV

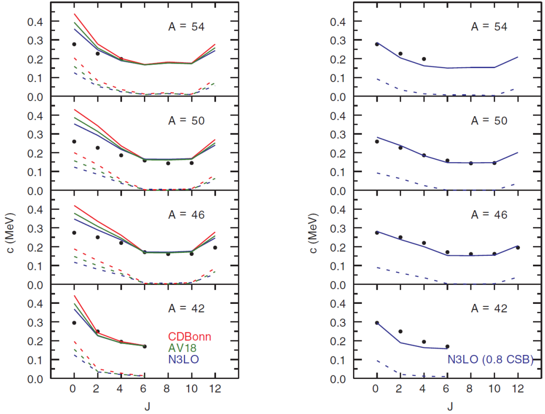
N. Smirnova, B.R. Barrett, Y. Kim et al, in preparation

Microscopic effective CD forces in the *pf* shell

W.E. Ormand, B.A. Brown, M. Hjorth-Jensen, PRC96, 024323 (2017)

Details

- CD-Bonn, AV18, N^{3}LO *NN* potentials (with CSB and CIB forces)
- *G*-matrix
- V_{eff} for *pf* shell obtained within many-body perturbation theory (to 3rd order)
- Coulomb interaction is added (except for AV18)
- *c* coefficients as a function of *J*



Outcome

Overprediction of *c* coefficients !
Similar conclusions from MEDs in $A = 54$ studies: *A. Gadea et al, PRL97, 152501 (2006)*

Mirror and triplet energy differences (MED and TED)

MEDs and TEDs as a function of J

$$MED(J) = E^*_{J,T_z=T} - E^*_{J,T_z=-T}$$

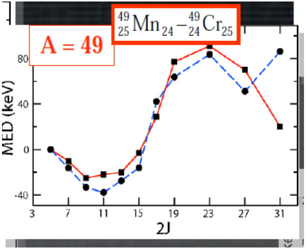
$$TED(J) = E^*_{J,T_z=T} + E^*_{J,T_z=-T} - 2E^*_{J,T_z=0} \quad (T=1)$$

Detailed information on change with J

- pair alignment
- nuclear radii (deformation)
- V_{INC}: $J = 0$ (isotensor) and $J = 2$ (isovector) matrix elements

A.P. Zuker, S.M. Lenzi, M.A. Bentley et al; S.M Lenzi, M.A. Bentley, PPNP59, 497 (2007)

K.Kaneko et al, PRC89, 031302 (2014); . . .



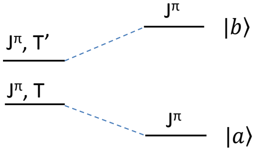
Courtesy of S. Lenzi

Neutron skin from MED in ^{23}Na-^{23}Mg

$$\Delta r_{\nu\pi} = \sqrt{\langle r_\nu^2 \rangle} - \sqrt{\langle r_\pi^2 \rangle} = \frac{\zeta(N-Z)}{A} e^{g/Z}$$

A. Boso et al, PRL121, 032502 (2018)

III. Isospin impurities and mixing matrix element



$$|a\rangle = \sqrt{1-\alpha^2}|T\rangle + \alpha|T'\rangle$$
$$|b\rangle = \sqrt{1-\alpha^2}|T'\rangle - \alpha|T\rangle$$

$$\alpha^2 \sim \frac{\langle T|V_{INC}|T'\rangle^2}{(\Delta E)^2}$$

- Shell-model predictions of α^2 are hampered by an imprecise ΔE knowledge.
  $\Rightarrow$ Isospin-mixing matrix element!
- $|\langle T|V_{INC}|T'\rangle|$ varies from 0 to about $\sim$ 150 keV.
- Contributions from both Coulomb and INC terms
  S. Nakamura, K. Muto, T. Oda, NPA575, 1 (1994)
- Complications due to many-level mixing

Isospin-forbidden processes and isospin mixing

Experimental signatures

- **Fermi β-decay**: $(J^\pi, T) \to (J^\pi, T \pm 1)$,
  Fermi β-decay between non-analogue 0^+ states: $(0^\pi, T) \to (0^\pi, T \pm 1)$
  depletion/splitting of the Fermi strength $\Rightarrow$ *Model-independent way*

  E. Hagberg et al, PRL73, 396 (1994); P. Schuurmans et al, NPA672, 89 (2000); N.Severijns et al, PRC71, 064310 (2005); . . .

- Electromagnetic probes ($E1$ transitions in $N = Z$ nuclei, mirror transitions, . . .)

  E. Farnea et al, PLB551, 56 (2003); A. Lisetskiy et al, PRL89, 012502 (2002); N.S. Pattabiraman et al, PRC78, 024301 (2008); A. Corsi et al, PRC84, 041304 (2011); S. Ceruti, F. Camera, A. Bracco et al, PRL115, 222502 (2015)

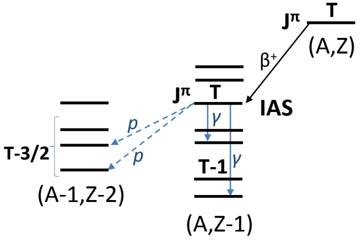
- Isospin-forbidden particle (p, $2p$, α, . . .) emission

  B. Blank, M.J.G. Borge, PPNP60, 403 (2008)
  N. Smirnova, B. Blank et al, PRC93, 044305 (2016); PRC95, 054301 (2017)

- Search for higher order terms in T_z in the IMME

  A. Signoracci, B.A. Brown, PRC84 (2011); A.T. Gallant et al, PRL113, 082501 (2014); . . .

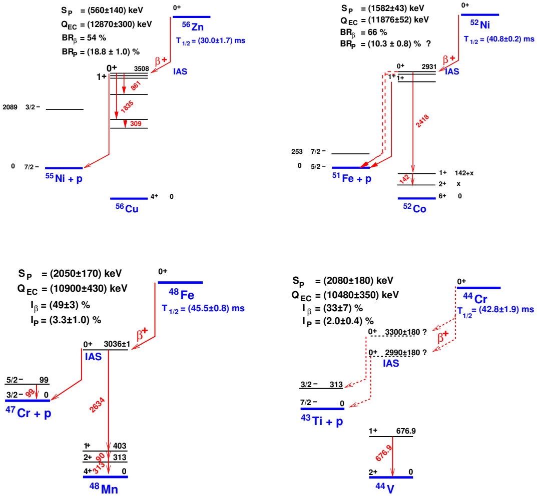
β-delayed $p\gamma$-emission and isospin-mixing



$$I_p(IAS)/I_\gamma(IAS)$$

Can we use this data to extract the amount of the isospin-mixing in the IAS ?

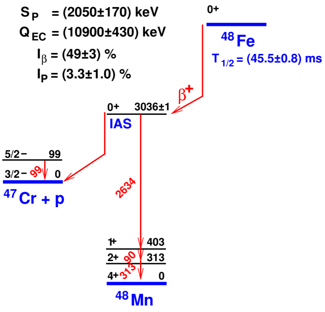
β-delayed $p\gamma$ emission: experimental examples



C. Dossat et al, NPA792, 18 (2007); B. Blank, M. Borge, PPNP60 (2008)

S. Orrigo et al, PRL112 (2014); S. Orrigo et al, PRC93 (2016); X. Xu et al, PRL117 (2016), . . .

β-delayed $p\gamma$-emission and isospin-mixing



$$\frac{I_p^{IAS}}{I_\gamma^{IAS}} = \frac{\Gamma_p^{IAS}}{\Gamma_\gamma^{IAS}}$$

$$I_\gamma^{IAS} = I_\beta^{IAS} - I_p^{IAS}$$

$$\Gamma_p^{IAS} = \theta^2 \, \Gamma_{sp}^{IAS}$$

$$\Rightarrow \quad \theta_{exp}^2 = \frac{\Gamma_\gamma^{IAS}}{\Gamma_{sp}^{IAS}} \times \frac{I_p^{IAS}}{I_\gamma^{IAS}}$$

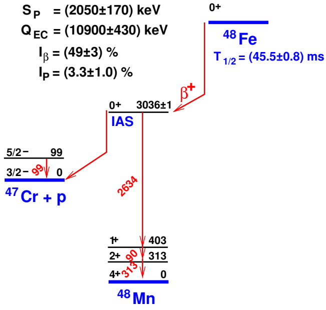
- Shell model: Γ_γ^{IAS}
- WS potential: Γ_{sp}^{IAS}

If a two-level mixing is valid:

$$\Rightarrow \quad \alpha_{exp}^2 = \frac{\Gamma_\gamma^{IAS}}{\theta_{\mathrm{T}-1}^2 \, \Gamma_{sp}^{IAS}} \times \frac{I_p^{IAS}}{I_\gamma^{IAS}}$$

N. Smirnova, B. Blank, B.A. Brown, W.A.Richter et al, PRC95, 054301 (2017)

Decay of ^{48}Fe



	Exp	cd-GXPF1A	cd-KB3G
$T_{1/2}$ (ms)	45.5(8)	45.2(5)	47.4(6)
E^{IAS} (keV)	3036(1)	3039	2921
Γ_γ (eV)		0.56	0.52
B_p (%)	14.4(7)	13.5	13.5

0^+, $T = 2$, IAS, is the lowest 0^+ state in ^{48}Mn.

$$\theta^2_{exp} = 3.9(7) \times 10^{-3}$$
$$\theta^2_{\mathrm{T}=1} = 0.19$$
$$\alpha^2_{exp} = 2.1(4)\%$$

$$\Gamma^{th}_{sp} = 15(2)\,\mathrm{eV}$$

N. Smirnova, B. Blank, B. A. Brown et al, PRC95, 054301 (2017).

IV. Superallowed $0^+ \to 0^+$ beta decay

Absolute *Ft* value

$$Ft^{0^+\to 0^+} \equiv ft^{0^+\to 0^+}(1+\delta_R')(1+\delta_{NS}-\delta_C) = \frac{K}{|M_F^0|^2 G_V^2 (1+\Delta_R)}$$

14 best known emitters (*ft*-value known with a precision $\lesssim 0.4\%$):

^{10}C, ^{14}O, ^{22}Mg, ^{26m}Al, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{42}Sc, ^{46}V, ^{50}Mn, ^{54}Co, ^{62}Ga, ^{74}Rb

J.C. Hardy, I.S. Towner, PRC91, 025501 (2015)

Radiative corrections

$$\Delta_R^V = (2.361 \pm 0.038)\%$$
$$\delta_R' \sim (1.50 \pm \sim 0.12)\%$$
$$|\delta_{NS}| \lesssim 0.3\%$$

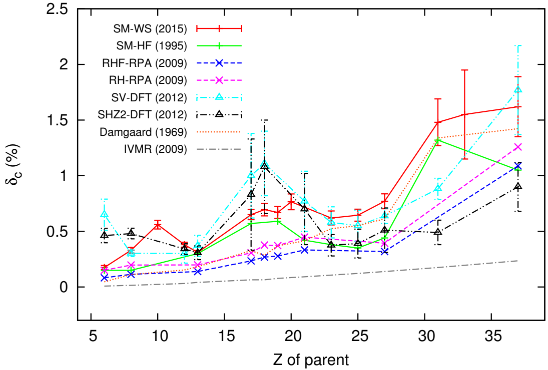
A. Sirlin, W.J. Marciano,
W. Jaus, G. Rasche

Nuclear-structure correction

$$|M_F|^2 = |M_F^0|^2(1-\delta_C)$$
$$|M_F^0|^2 = T(T+1) - T_{zi}T_{zf}$$
$$\delta_C \approx 0.1 - 2.0\%$$

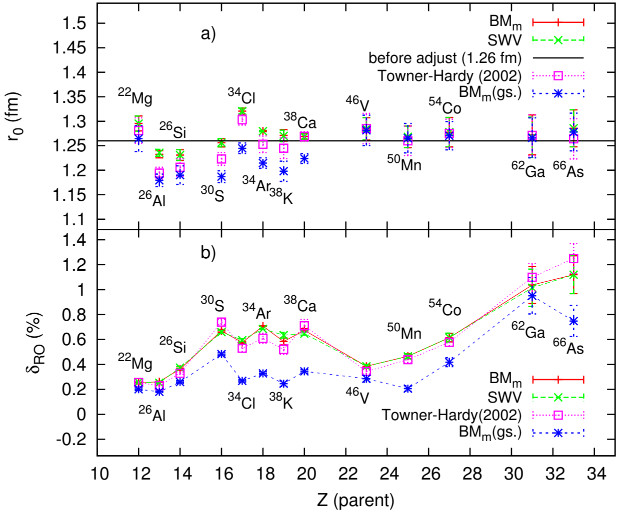
δ_C — large ambiguities from various theoretical models

Present status of δ_C from various models



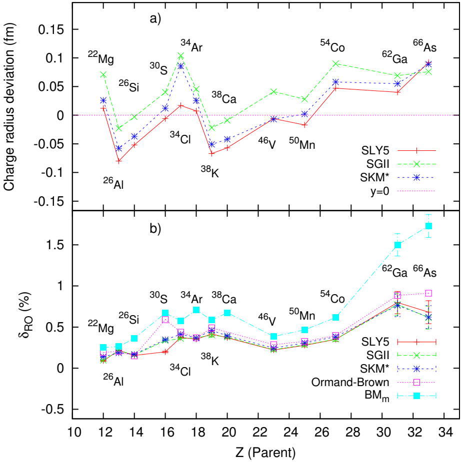
- Damgaard Model (*J. Damgaard*)
- Shell Model (INC) + WS (*I.S. Towner, J.C. Hardy*)
- Shell Model (INC) + HF (*W.E. Ormand, B.A. Brown*)
- JT-projected DFT (*W. Satula et al*)
- RHF-RPA and RH-RPA (*R. Liang et al*)
- Isovector Giant Monopole Resonance (*N. Auerbach*)

Results δ_{RO} from WS potential



L. Xayavong, N. Smirnova, PRC97, 024324 (2018).

HF potential adjusted



L. Xayavong, N.Smirnova, M. Bender, K. Bennaceur, Acta Phys. Pol. B10, 285 (2017).

V. Radiative p-capture reactions in stellar environment

Explosive hydrogen burning and proton capture reactions on $A > 20$ nuclei:

- Classical O-Ne novae (0.1-0.4 GK) $\Rightarrow$ nucleosynthesis near $N = Z$ line ($A \lesssim 40$)
- X-ray bursts ($\lesssim 2$ GK) $\Rightarrow$ nucleosynthesis up to proton drip-line ($A \lesssim 100$)
  rp-process

Proton capture reactions on radioactive $A > 20$ nuclei

$Q \lesssim 5$ MeV $\Rightarrow$ contribution of isolated resonances

$$N_A \langle \sigma v \rangle_r = 1.540 \times 10^{11} \; (\mu T_9)^{-3/2} \; \omega\gamma \; \exp\left(\frac{-11.605\, E_r}{T_9} \right) \; \mathrm{cm^3\, s^{-1}\, mol^{-1}}$$

$$\omega\gamma = \frac{2J_f + 1}{2(2J_i + 1)} \frac{\Gamma_p \Gamma_\gamma}{\Gamma_p + \Gamma_\gamma}$$

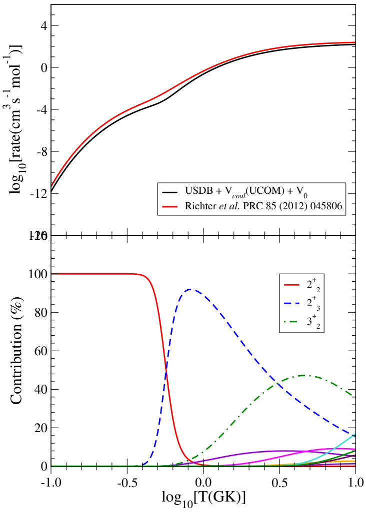
- Not enough experimental data $\Rightarrow$ Estimations: from mirror systems, when available
- Theoretical input (shell model): E_r, Γ_γ and Γ_p

H. Herndl et al, ***PRC52****, 1078 (1995; W. A. Richter, B. A. Brown, A. Signoracci, M. Wiescher,* ***PRC83****, 065803 (2011); . . .*

$^{35}Ar(p,\gamma)^{36}K$ reaction rate

Y. H. Lam et al (in preparation)

E_r, $\omega\gamma$ in ^{36}K are evaluated via *c*-coefficients V_{INC}



Summary and Perspectives

- **Phenomenological** INC shell-model Hamiltonian: robust description
- Importance of the Thomas-Ehrman shift!

  $^{24}Al(p,\gamma)^{25}Si$

  B. Longfellow et al, PRC97, 054307 (2018)
- **Microscopic description**: in progress
- Experimental data on ISB phenomena are welcome.

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