THEORY OF ALPHA-PARTICLE CONDENSATION IN NUCLEAR SYSTEMS
A nuclear Quantum Phase Transition

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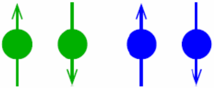
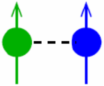
Clusters important aspect and richness of nuclear systems due to 4 Fermions :

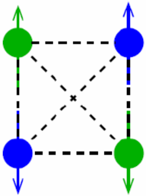


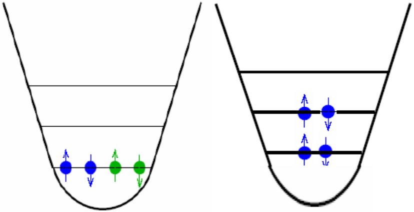
Dimer :



$\frac{E}{A} = 1$ MeV

Quartet :



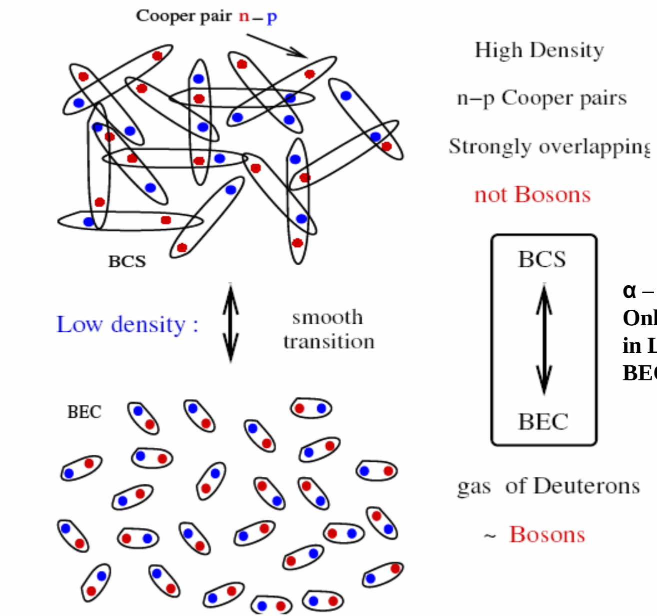


$\frac{E}{A} = 7$ MeV, $E^{*} = 20$ MeV

Proposal :

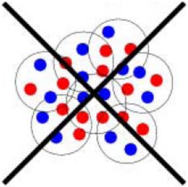
Trapping of 4 different species of Fermionic atoms.

Bi-Excitons

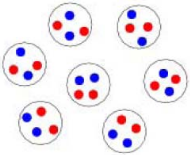


α – Particles
Only Exist
in Low Density
BEC Phase

Quartetting



No BCS phase (dense phase) of α-particles possible!



Bose-Einstein-Condensation of α-particles (dilute)

Finite nuclei ?

Exact ^{8}Be :

Density : $\frac{\rho_0}{3}$

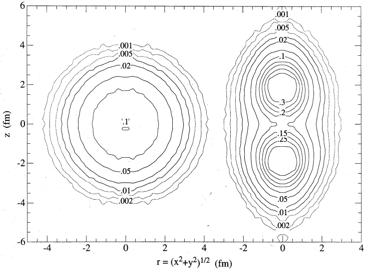
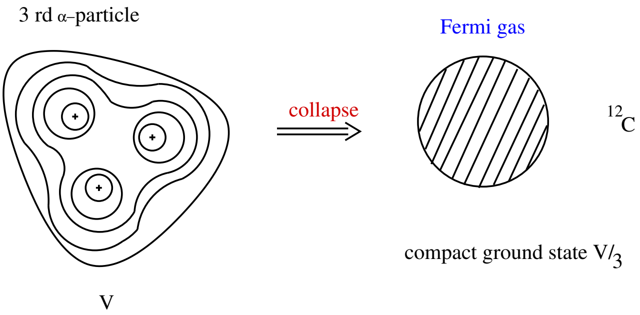
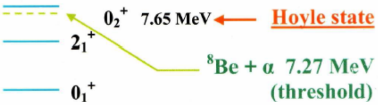


Fig. 15 (Wiringa, et al.)



Does a dilute 3α $^{12}C^*$ state exist ?
Similar to $^8Be + \alpha$?



The
Fred Hoyle
story

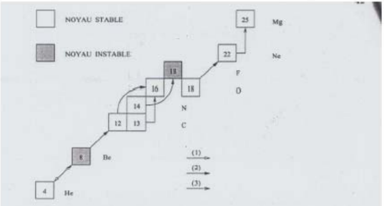
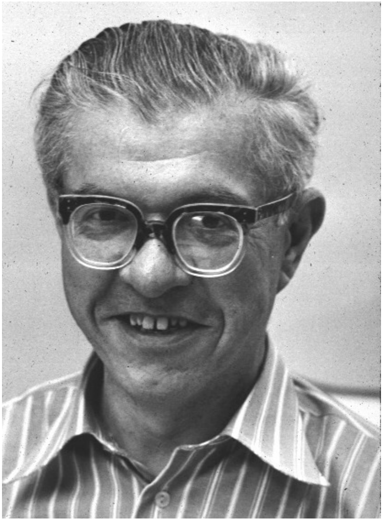


Figure 26. Réactions principales impliquées dans la combustion non explosive de He. Les flèches de type (1), (2) et (3) se rapportent respectivement à des réactions (γ, α), (α, γ), et (α, n)

At $T = 10^8 K$ helium burning
thermal equilibrium

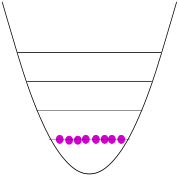
$$\alpha + \alpha + \alpha \;\rightarrow\; ^8Be \;\rightarrow\; \alpha \;\rightarrow\; ^{12}C^* \quad O_2^+$$

O_2^+ : dilute 3α state hypothesis !

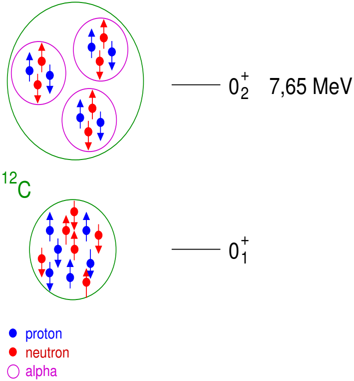


Fred Hoyle

Bosons

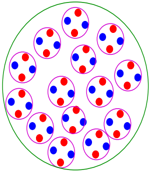


Back to nuclei



many α's

$\rightarrow$ condensate



strong cluster phenomena in lighter nuclei

Theoretical Description

Ideal Bose condensate : $\left|0\right\rangle = b_0^\dagger b_0^\dagger \cdots b_0^\dagger \left|vac\right\rangle$

α-particle condensate : $\left|\Phi_{\alpha C}\right\rangle = C_\alpha^\dagger C_\alpha^\dagger \cdots C_\alpha^\dagger \left|vac\right\rangle$

In r-space :

$$\left\langle \vec{r}_1, \vec{r}_2, \cdots, \vec{r}_{4n} \middle| \Phi_{\alpha C} \right\rangle = \mathcal{A}\left\{\Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4)\Phi(\vec{r}_5, \vec{r}_6, \vec{r}_7, \vec{r}_8)\cdots\Phi(\vec{r}_{4n-3}, \vec{r}_{4n-2}, \vec{r}_{4n-1}, \vec{r}_{4n})\right\}$$

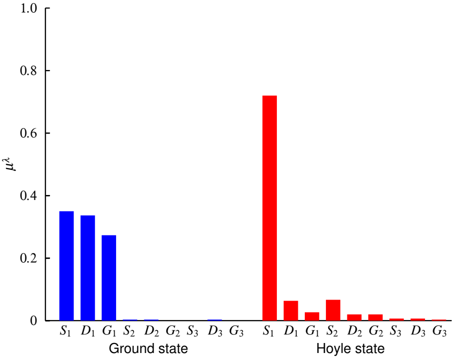
In comparison with pairing :

$$\left\langle \vec{r}_1, \vec{r}_2, \cdots \middle| \mathrm{BCS} \right\rangle = \mathcal{A}\left\{\Phi(\vec{r}_1, \vec{r}_2)\Phi(\vec{r}_3, \vec{r}_4)\cdots\right\}$$

Boson occupancies:

α-particle density matrix: $\rho_\alpha(\mathbf{R}, \mathbf{R}')$, **R**: c.o.m. of α

diagonalisation



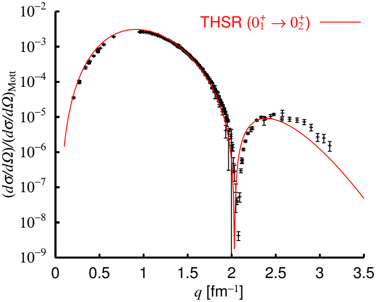
Comparison with experiment: inelastic form factor to Hoyle

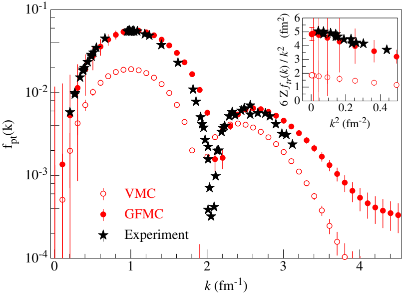
THSR and **GFMC**(Pieper et al.)

THSR

(Tohsaki, Horiuchi, Schuck, Roepke)

Vol_Hoyle
-------------- ~ 4 !!
Vol_12C

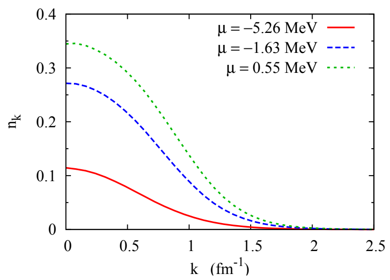




GFMC (Wiringa, Pieper et al., RMP 2017)

Quantum Phase Transition in infinite matter and 16O Lazauskas, Sogo et al., PRC 79, 051301

PHYSICAL REVIEW C 79, 051301(R) (2009)



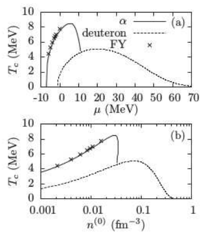
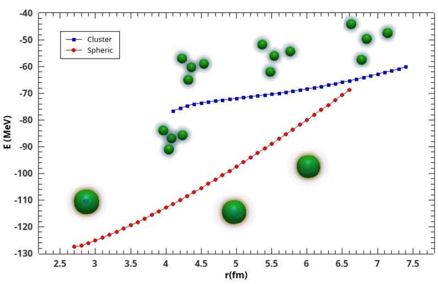
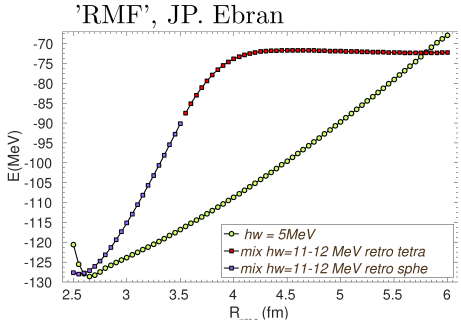


FIG. 2. Critical temperature of α and deuteron condensations as functions of (a) chemical potential and (b) density of free nucleon, derived from Eq. (4) for the α particle and Eq. (11) for the deuteron. Crosses ($\times$) correspond to calculations of Eq. (1) with the Malfliet-Tjon interaction (MT I-III) using the FY method.

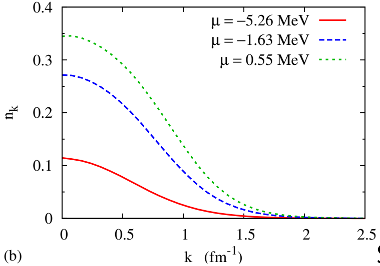
QPT in finite nuclei

Gogny force D1S, M. Girod; 'RMF', JP. Ebran

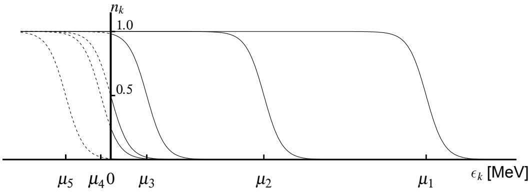




Fully self-consistent quartet alpha-order parameter $\langle c_1 c_2 c_3 c_4 \rangle \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4)$ at $T = 0$



Schematic view in case of BCS:



Alpha condensation stops at $\mu \simeq 0.55$ MeV

This corresponds to $\rho \sim \rho_0/5 \rightarrow$ Quantum Phase Transition!

Why does it stop?

Three-hole level densities:

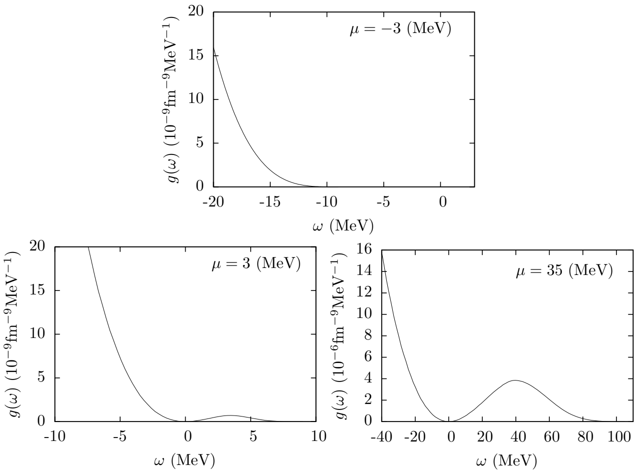


Figure: $3h$-level density for negative (top) and two positive (bottom) chemical potentials [?].

CONCLUSIONS

Hoyle state precursor of α particle condensation in nuclear matter

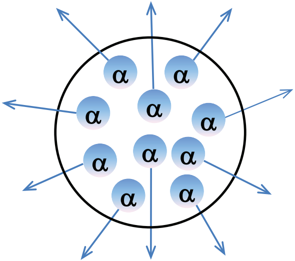
Alpha (quartet) condensation only exists at low density ($\rho < \rho_0/5$)

Alpha-Condensation is a Quantum Phase Transition with density as control parameter.

Stems from vanishing four particle level density at Fermi-energy

Alpha condensation predicted in heavier nuclei. E.g. ^{16}O: 6-th 0^+ state at 15.1 MeV

Dream: Coulomb explosion of ^{40}Ca



THANK YOU !