TOROIDAL MODE IN NUCLEI: FROM GIANT RESONANCE TO INDIVIDUAL STATES

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Joint Institute for Nuclear Research, Dubna, Moscow region, Russia

EuNPC, Bologna, Italy, 02-07.09.2018

Motivation-1

The giant toroidal dipole resonance (TDR) is interesting in many aspects:

- only known electric intrinsic vortical mode in nuclei
- measure of nuclear vorticity
- source of pygmy dipole resonance (PDR)
- constitutes low-energy part of ISGDR
- other areas: astrophysics, dark matter, metamaterials, …

Repko, P.-G. Reinhard, VON, J. Kvasil, PRC, 87, 024305 (2013).

P.-G. Reinhard, VON, A. Repko, J. Kvasil, Phys. Rev. C89, 024321 (2014)

However exper. observation and identification of TDR is complicated:

$$E(TDR) = 50 \div 70\ A^{-1/3} MeV \longleftrightarrow E(E2, T=0) = 64\ A^{-1/3} MeV$$

$$E(E1, T=1) = 81\ A^{-1/3} MeV$$

TDR is concealed by other modes located at the same energy region.

- available (α,α') experimental data on TDR in E1(T=0) can be disputed

A. Repko, J. Kvasil, VON, P.-G. Reinhard, EPJA, 53, 221 (2017)

Motivation-2

VON, A. Repko, J. Kvasil and P.-G. Reinhard, PRL 120, 182501 (2018)

Y. Kanada-En'yo and Y. Shikata, PRC 95, 064319 (2017).

Last studies show that in light nuclei with high prolate deformation there should exist individual low-energy toroidal states (TS) with $I^{\pi}K = 1^{-}1$.

Individual TS:

- is low-energy (lowest) $I^{\pi}K = 1^{-}1$ excitation which can be much easier observed and identified
- being found, can be used as the test for various reactions to probe vortical (toroidal) flow

- promising light nuclei ^{10}Be, ^{20}Ne, ^{24}Mg

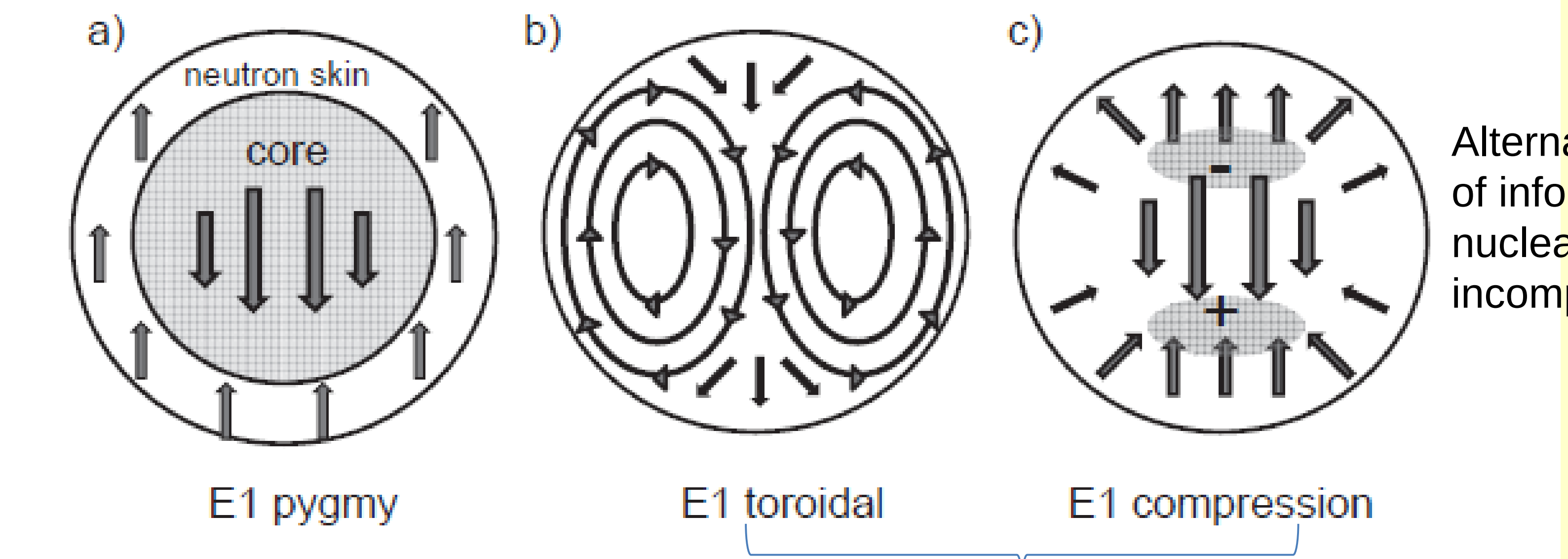
- N=Z nuclei are most promising since they do not have 1^{-} pygmy modes

^{24}Mg

VON, A. Repko, J. Kvasil and P.-G. Reinhard, PRL 120, 182501 (2018)

Exotic dipole resonances

R. Mohan et al (1971), | V.M. Dubovik (1975) S.F. Semenko (1981) | M.N. Harakeh (1977) S. Stringari (1982)



Alternative source of information on nuclear incompressibility

Dominate in E1(T=0) excitation channel
(due to suppression of dominant E1(T=1) motion)

irrotational | vortical | irrotational

$E = 50 \div 60\, A^{-1/3} MeV$ $E = 50 \div 70\, A^{-1/3} MeV$ $E = 132\, A^{-1/3} MeV$

- Different kinds of dipole oscillations with fixed c.m.

Reviews:

N. Paar et al, Rep. Prog. Phys. 70 691 (2007);
D. Savran et al, Prog. Part. Nucl. Phys. 70, 210 (2013)
VON, J. Kvasil, A. Repko,W. Kleinig, and P.-G. Reinhard, Phys. Atom. Nucl. 79, 842 (2016).

Origin of the toroidal operator

V.M. Dubovik and A.A. Cheshkov, Sov. J. Part. Nucl. v.5, 318 (1975).

Multipole electric operator (external field) :

$$\hat{M}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d\vec{r}\;\; j_\lambda(kr)\vec{Y}_{\lambda\lambda\mu}\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})\,]$$

$$j_\lambda(kr) = \frac{(kr)^\lambda}{(2\lambda+1)!!}[1-\frac{(kr)^2}{2(2\lambda+3)}+\ldots]$$

$$\hat{M}(Ek\lambda\mu) = \hat{M}(E\lambda\mu) + k\hat{M}_{tor}(E\lambda\mu)$$

Toroidal operator appears as the second order term in long-wave expansion of the electric operator

$$\hat{M}(E\lambda\mu) = \int d\vec{r}\,\rho(\vec{r})r^\lambda Y_{\lambda\mu}$$

standard electric operator
In long wave approximation

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC, 84, 034303 (2011)

Toroidal E1 operator:

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c}\int d\vec{r}\,[r^3+\frac{5}{3}r<r^2>_0]\,\vec{Y}_{11\mu}(\hat{\vec{r}})\cdot[\underbrace{\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})}_{\text{vortical flow}}]$$

Compression E1 operator:

$$\hat{M}_{com}(E1\mu) = -\frac{i}{10c}\int d\vec{r}\,[r^3-\frac{5}{3}r<r^2>_0]Y_{1\mu}\,[\underbrace{\vec{\nabla}\cdot\hat{\vec{j}}_{nuc}(\vec{r})}_{\text{irrotational flow}}]$$

$$\dot{\rho}+\vec{\nabla}\cdot\vec{j}_{nuc}=0$$

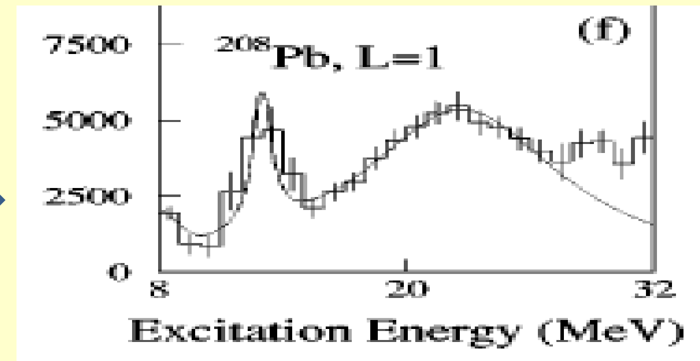
$$\hat{M}'_{com}(E1\mu) = \int d\vec{r}\,\hat{\rho}(\vec{r})\,[r^3-\frac{5}{3}r<r^2>_0]Y_{1\mu}$$

TDR and CDR constitute low- and high-energy ISGDR branches (?)

Familiar treatment → LE (toroidal) HE (compression)

Experiment: (α,α')

^{208}Pb
D.Y. Youngblood et al, 1977
H.P. Morsch et al, 1980
G.S. Adams et al, 1986
B.A. Devis et al, 1997
H.L. Clark et al, 2001
D.Y. Youngblood et al, 2004
M.Uchida et al, PRC 69, 051301(R) (2004)



There are also exp ISGDR data in

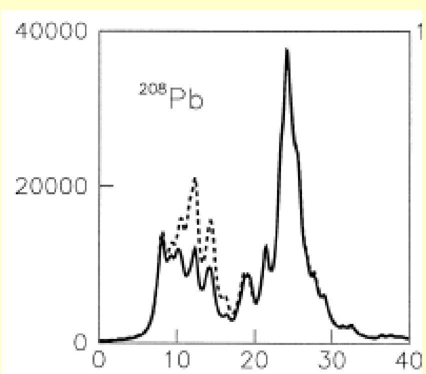
^{56}Fe, ^{58,60}Ni, ^{90}Zr, ^{116}Sn, ^{144}Sm, ...

Theory:

G. Colo et al, PLB 485, 362 (2000)
D. Vretenar et al, PRC, 65, 021301(R) (2002)
N. Paar, D. Vretenar, E. Kyan, G. Colo, Rep. Prog. Phys. 70 691 (2007);

TDR CDR

A. Repko, P.-G. Reinhard, V.O.N. and J. Kvasil,
PRC 87, 024305 (2013).

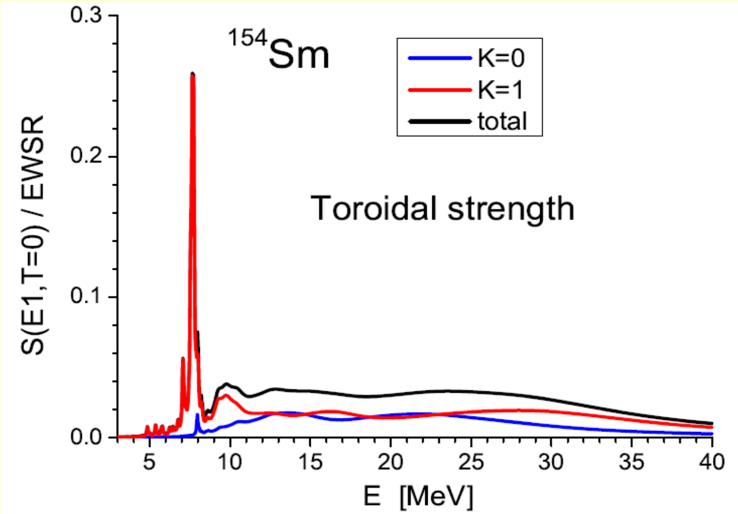


Perhaps Uchida observed at 10-17 MeV not TDR but mixed CDR/TDR . The main TDR peak should be at lower energy ~ 7-9 MeV. The observation of TDR in (a,a') can be disputed in general since (a,a') is determined by transition density while toroid depends on the vortical transition current. NEED IN NEW EXPERIMENTS!

Deformation features of TDR

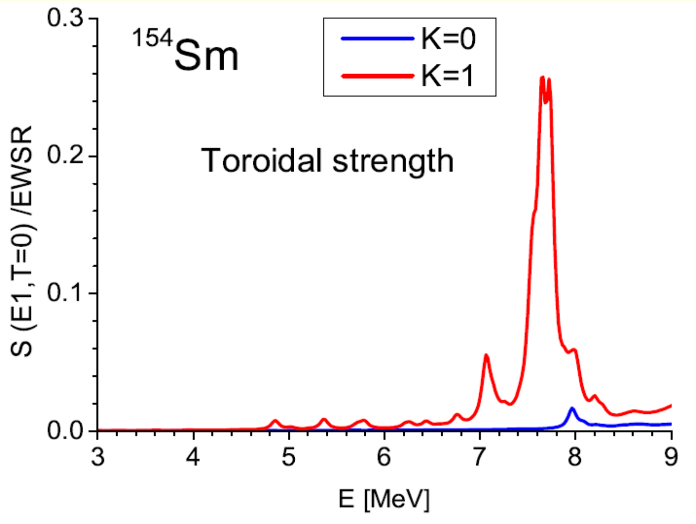
154Sm, SLy6 $\beta_2^{exp} = 0.339$

Energy-weighted strength functions

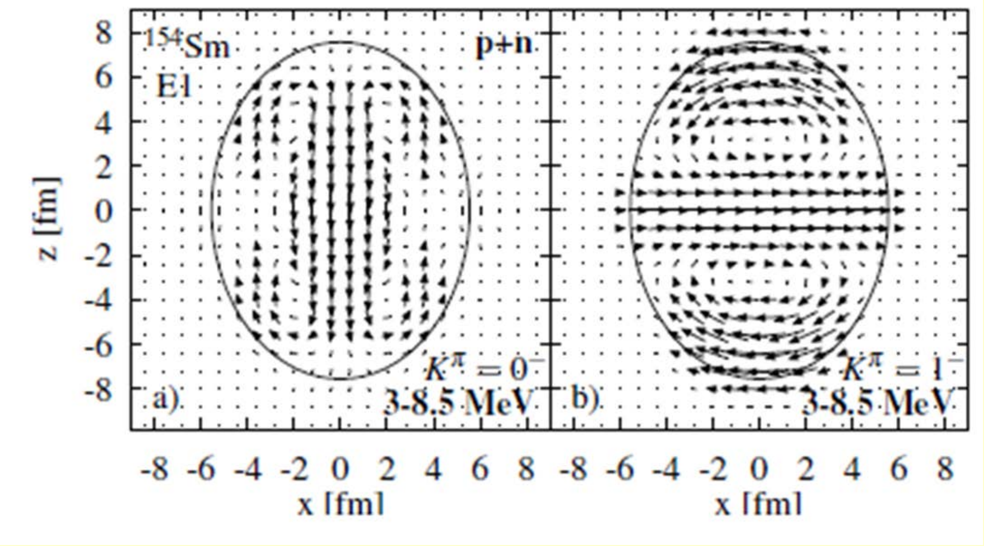


Skyrme QRPA, SLy6

A. Repko, J. Kvaail, VON,, P.-G. Reinhard, EPJA, 53, 221 (2017)



K=1 dominates !!!



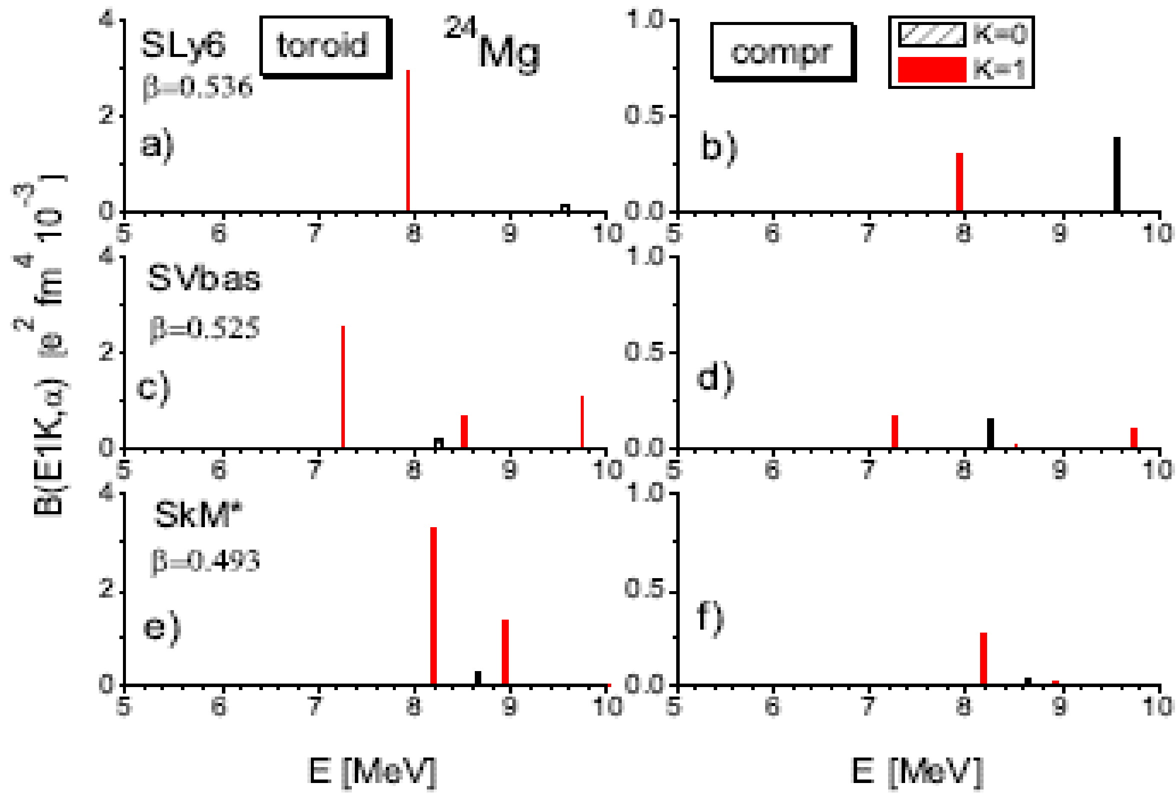
Similar results for other prolate nuclei.

K=1 dominance can in principle be used as TDR fingerprint in future experiments.

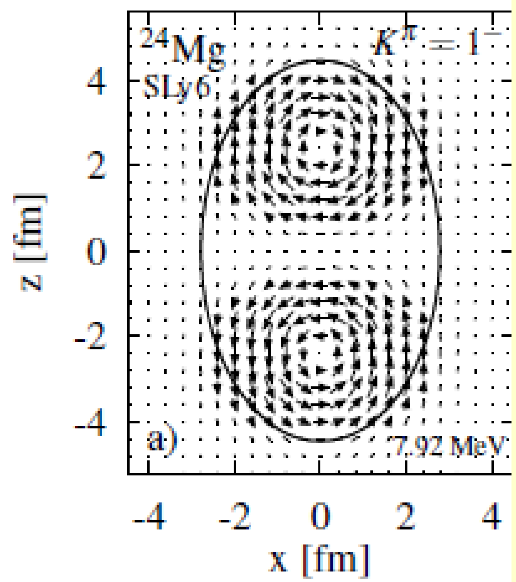
If prolate deformation is so important, then what we will get in nuclei with a huge axial deformation, like ^{24}Mg?

^{24}Mg $\beta_2^{exp} = 0.605$

VON, A. Repko, J. Kvasil, P.-G. Reinhard,
PRL 120, 182501 (2018)



QRPA results for
SLy6,
SVbas,
SkM*

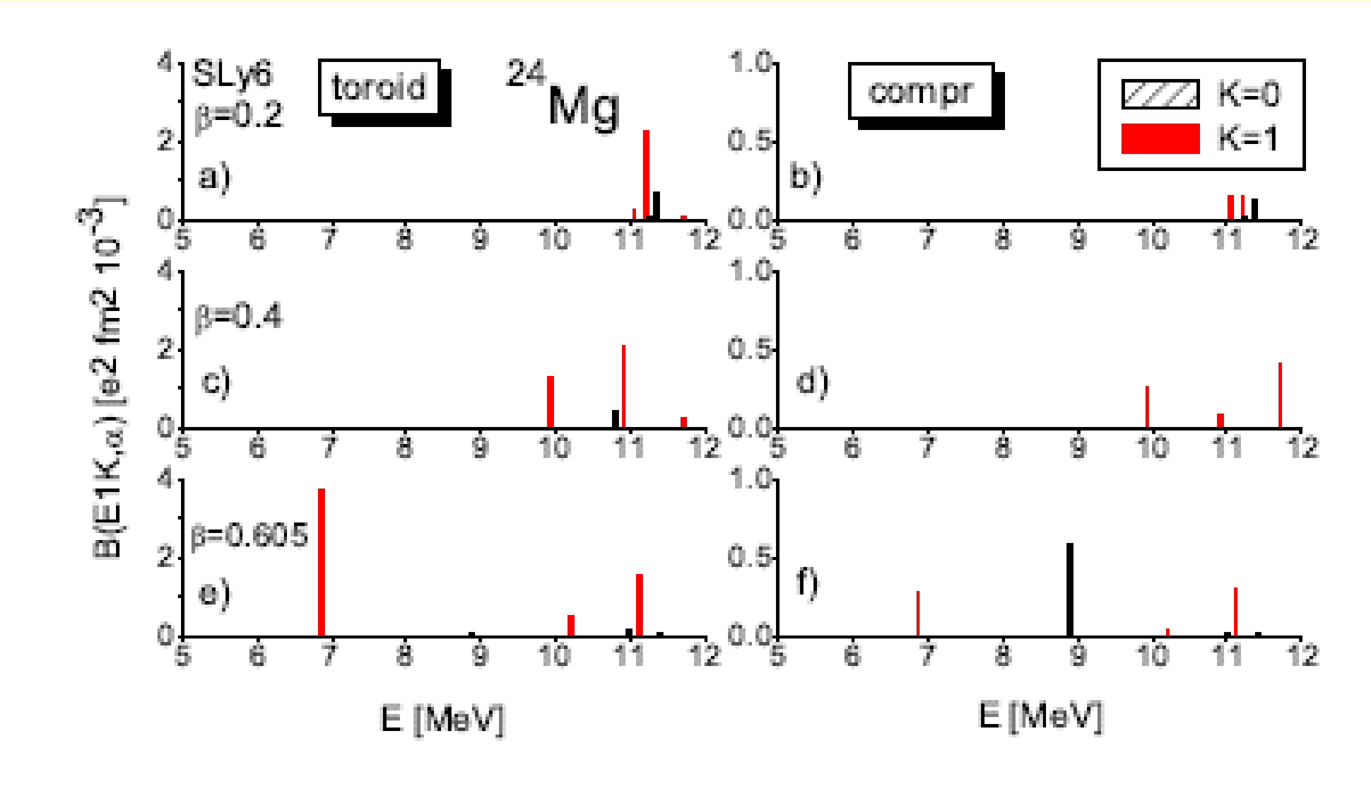


Persistence of the main result:
the **lowest** toroidal K=1 peak

The remarkable example of individual toroidal state!

Dependence on deformation

VON, A. Repko, J. Kvasil, P.-G. Reinhard, PRL 120, 182501 (2018)



TS becomes lowest due to of the large axial prolate deformation.

K=1 peak is:

- the lowest dipole state
- well separated from other states

To get individual lowest TS, two rigorous requirements should be held:

- huge prolate deformations
- sparse low-energy spectrum

This can be realized just in light deformed nuclei

PHYSICAL REVIEW C 95, 064319 (2017)

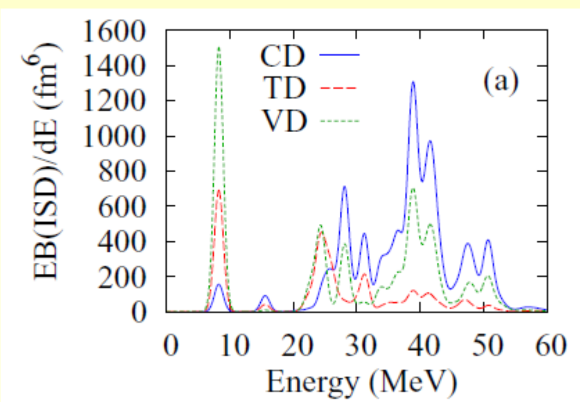
^{10}Be=^{8}He+2n

Toroidal, compressive, and $E1$ properties of low-energy dipole modes in ^{10}Be

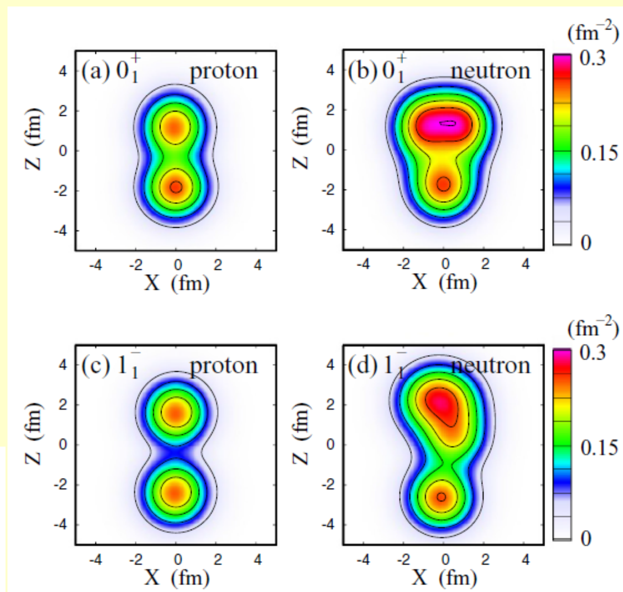
Yoshiko Kanada-En'yo and Yuki Shikata
Department of Physics, Kyoto University, Kyoto 606-8502, Japan

We studied dipole excitations in ^{10}Be based on an extended version of the antisymmetrized molecular dynamics, which can describe 1p-1h excitations and large amplitude cluster modes. Toroidal and compressive dipole operators are found to be good proves to separate the low-energy and high-energy parts of the isoscalar dipole excitations, respectively. Two low-energy 1^- states, the toroidal dominant 1_1^- state at $E \sim 8$ MeV and the $E1$ dominant 1_2^- state at $E \sim 16$ MeV, were obtained. By analysis of transition current densities, the 1_1^- state is understood as a toroidal dipole mode with exotic toroidal neutron flow caused by rotation of a deformed ^{6}He cluster, whereas the 1_2^- state is regarded as a neutron-skin oscillation mode, which are characterized by surface neutron flow with inner isoscalar flow caused by the surface neutron oscillation against the 2α core.

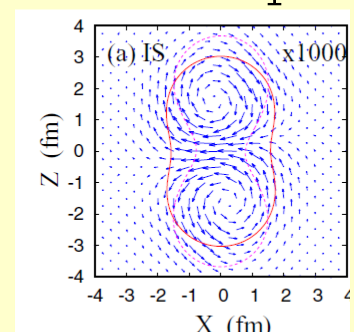
$I^{\pi}K = 1^{-}1_1$ lowest!



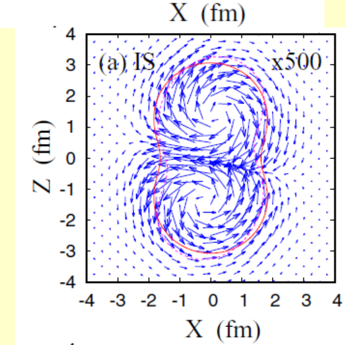
[21] J. Kvasil, V. O. Nesterenko, W. Kleinig, P.-G. Reinhard, and P. Vesely, Phys. Rev. C 84, 034303 (2011).
[22] A. Repko, P.-G. Reinhard, V. O. Nesterenko, and J. Kvasil, Phys. Rev. C 87, 024305 (2013).
[23] V. O. Nesterenko, J. Kvasil, A. Repko, W. Kleinig, and P.-G. Reinhard, Phys. At. Nucl. 79, 842 (2016).



$I^{\pi}K = 1^{-}1_1$

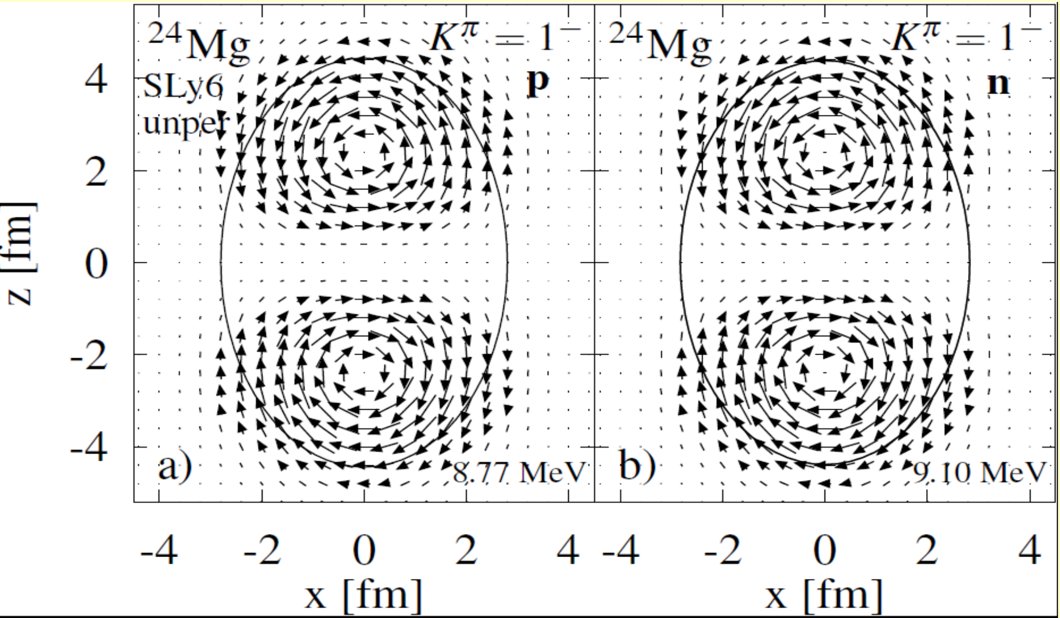


AMD



cluster GCM

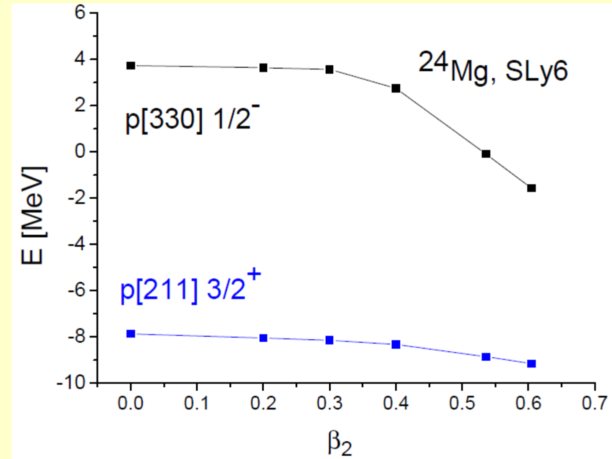
Toroidal flow: collective or 2qp origin?



$pp[211]\uparrow - [330]\uparrow$ (54%)

$nn[211]\uparrow - [330]\uparrow$ (39%)

Toroid is mainly 2qp (mean field) effect!



Explains the deformation effect in TS

The deformation-induced energy downshift is not universal.

Perhaps ^{10}Be, ^{24}Mg are unique nuclei where the toroidal mode is the lowest dipole state.

(α,α') is not good to observe vortical toroidal excitations.

What about (e,e')?

Here we also meet problem: impact of the magnetization current

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c}\int d\vec{r}\,[r^3 + \frac{5}{3} r < r^2 >_0]\,\vec{Y}_{11\mu}(\hat{r})\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})]$$

Nuclear current

$$\hat{\vec{j}}_{nuc}(\vec{r}) = \frac{e\hbar}{m}\sum_{q=n,p}(\hat{\vec{j}}^{q}_{con}(\vec{r}) + \hat{\vec{j}}^{q}_{mag}(\vec{r}))$$

$$\hat{\vec{j}}^{q}_{con}(\vec{r}) = -ie^{q}_{eff}\sum_{k\ni q}\left(\delta(\vec{r}-\vec{r}_k)\vec{\nabla}_k - \overleftarrow{\nabla}_k\delta(\vec{r}-\vec{r}_k)\right) \longrightarrow \text{toroidal flow}$$

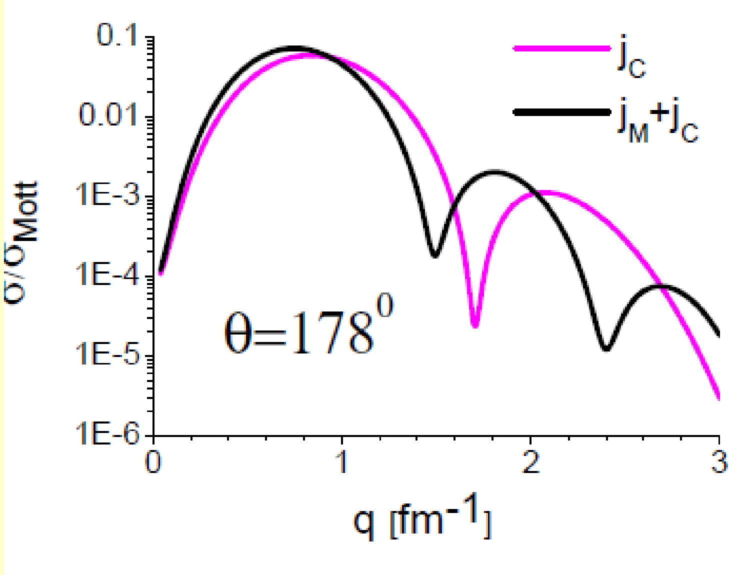
$$\hat{\vec{j}}^{q}_{mag}(\vec{r}) = \frac{g^{q}_{s}}{2}\gamma\sum_{k\ni q}\vec{\nabla}_k\times\hat{\vec{s}}_{qk}\delta(\vec{r}-\vec{r}_k), \qquad \gamma = 0.7$$

$$\mathrm{B(E1K,tor)}_{\mathrm{con}} = 0.63\,\mathrm{B(E1K,tor)}_{\mathrm{con+mag}}$$

$\hat{\vec{j}}^{q}_{mag}$ gives 37% of the toroidal response. Large effect !

What is the contribution of $\hat{\vec{j}}^{q}_{mag}$ to (e,e') for the toroidal state?

(e,e') in PWBA: role of magnetic nuclear current



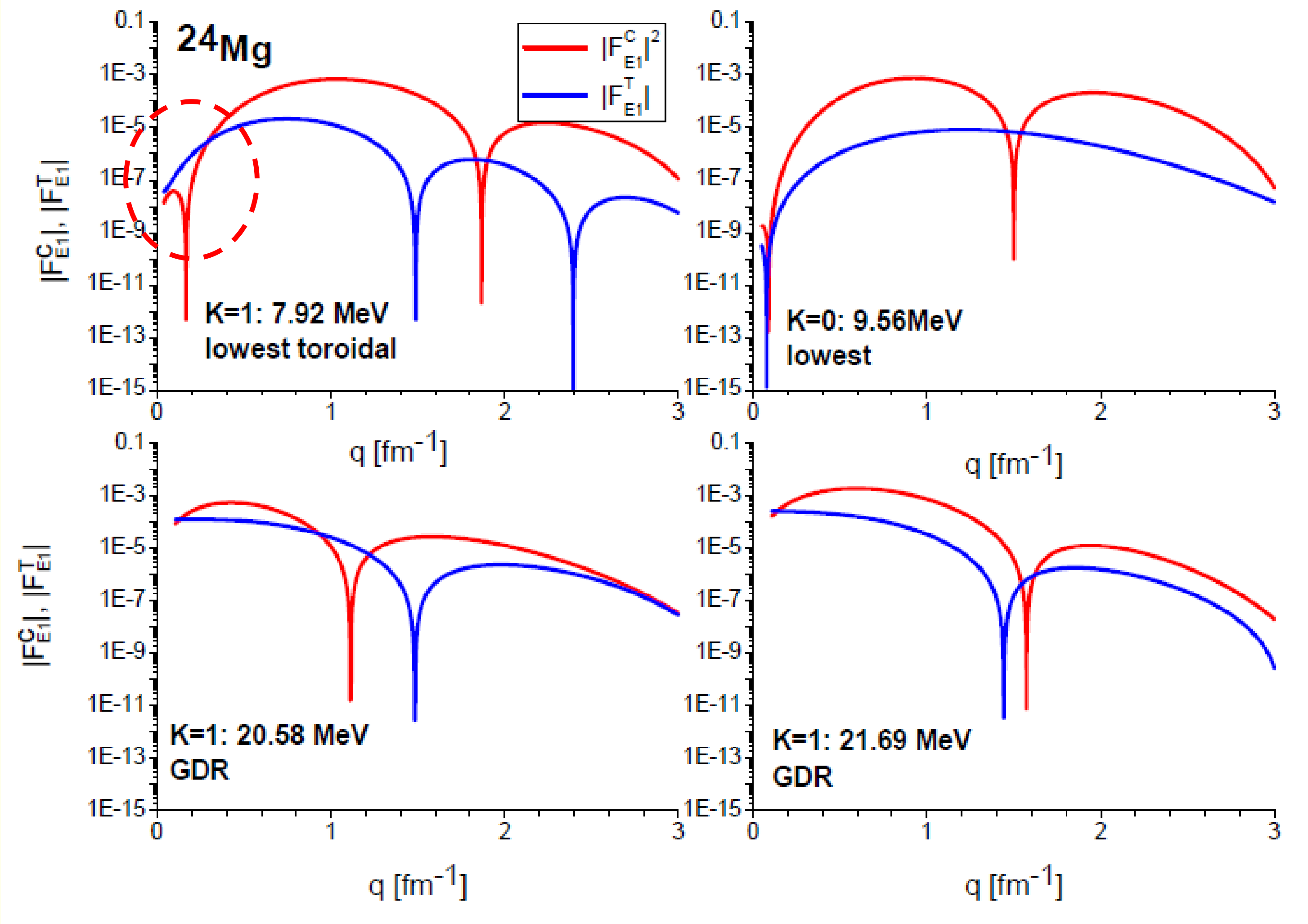
Impact of $\vec{j}_M$ current is:

- significant at $q > 1\ fm^{-1}$, makes toroidal effect unresolved
- small at low q

It will be difficult to observe toroid at $q > 1\ fm^{-1}$ where effect of $\vec{j}_M$ is large.

We should concentrate on low q where the influence of $\vec{j}_M$ is small.

(e,e'): Longitudinal $|F^C_{E1}|^2$ and transverse $|F^T_{E1}|^2$ form factors for different states in 24Mg



At low q, TS gives a specific diffraction minimum and $|F^T_{E1}|^2 > |F^C_{E1}|^2$

Possible signature of the toroid mode?

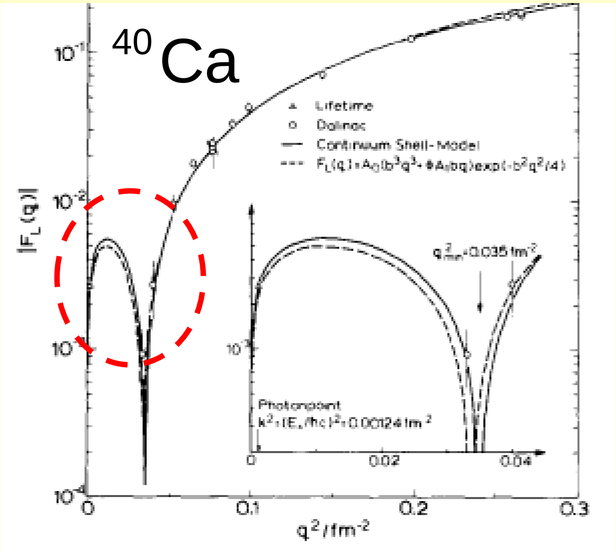
Interpretation of toroidal state (TS)

$$B(E1, T=1, 0^{+}0_{gs} \rightarrow 1^{-}1) = 2.52 \cdot 10^{-4} e^{2} fm^{2}$$ ← lowest T=0 state with a weak E1 transition

Low-energy dipole isoscalar states (LE-IDS) with isospin-forbidden E1 decay in 16O, 40Ca:

H. Miska et al, PLB, **59**, 441 (1975).
H.D . Graf et al, PLB, **72**, 179 (1977).
B. Castel, Y. Okuhara, and H. Sagawa, PRC, **42**, R1203 (1990).
P. Papakonstantinoua, V.Yu. Ponomarev, R. Roth and J. Wambach et al, EPJA , **47**, 14 (2011).

- LE-IDS has toroidal-like flow



TS also:
- is low-energy T=0 state with a weak E1 transition,
- has low-q minimum in the longitudinal form factor.

So perhaps E1(K=1) TS in 24Mg
is a realization of LE-IDS in strongly deformed nuclei

Conclusions

★ Toroidal dipole resonance (TDR) is interesting in many aspects but still has problems with experimental observation and identification.

★ We propose a new route in investigation of toroidal excitations: investigation of individual toroidal states (ITS) in light nuclei.

★ These states can:
- be much easier observed and identified,
- serve as new promising test cases for various nuclear reactions to probe vortical (toroidal) flow in nuclei.

★ Individual lowest toroidal state in highly deformed 24Mg.
- down-shifted by high prolate deformation,
- has large 2qp components, can correspond to observed 7.77-MeV and 8.44-MeV dipole states
- is significantly affected by $\vec{j}^{\,q}_{mag}$

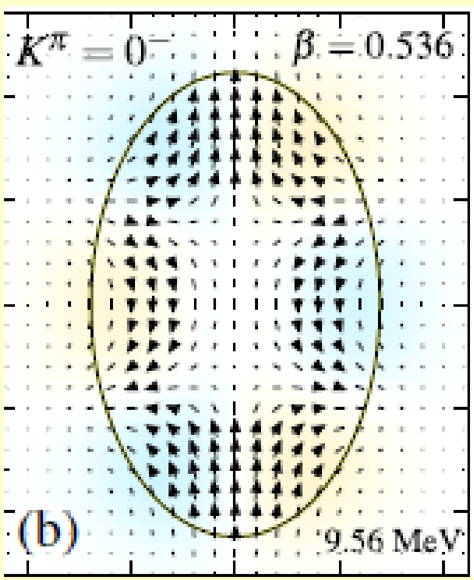
★ Signatures of ITS in nuclear reactions? (e,e'):
- $\vec{j}^{\,q}_{mag}$ contaminates toroidal contribution at $q > 1\ \mathrm{fm}^{-1}$
- specific diffraction minimum at low q
- TS as a realization of LE-IDM in deformed nuclei?

Thank you for attention!

Interpretation of compression state (CS)

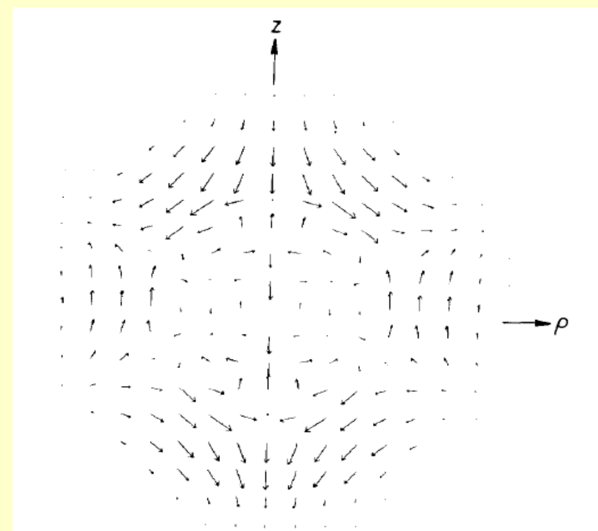
- Collective: $B(E30, T=0, 0^{+}0_{gs} \rightarrow 3^{-}0) = 17$ Wu

VON, A. Repko, J. Kvasil and P.-G. Reinhard, PRL 120, 182501 (2018)



CS with $K^{\pi}=0^{-}$ in ^{24}Mg

D. G. Raventhall and J. Wambach, NPA, 475, 468 (1987)



Octupole $I^{\pi}=3^{-}$ state in ^{208}Pb

So perhaps E1(K=0) CS is basically an octupole K=0 mode.

Comparison with the experiment

Spectrum of 24Mg

E_{level}(keV)	Jπ	$T_{1/2}$
0	0+	STABLE
1368.672 5	2+	1.33 ps 6
4122.889 12	4+	22 fs 2
4238.24 3	2+	41 fs 4
5235.12 4	3+	61 fs 7
6010.84 4	4+	49 fs 5
6432.30 11	0+	53 fs 8
7349.00 3	2+	6 fs 2
7555.04 15	1-	270 fs 55
7616.47 4	3-	890 fs 140
7747.51 9	1+	10 fs 3
7812.35 11	5+	20 fs 4
8114.2 20	6+	3.6 fs 10
8357.98 13	3-	56 fs 8
8437.31 15	1-	10 fs 2
8439.36 4	4+	3.8 fs 11
8654.53 15	2+	8.2 fs 21
8864.29 9	2-	4.4 fs 15
9003.34 9	2+	7.6 fs 14
9145.99 15	1-	
9284.22 14	2+	11 fs 2
9299.77 24		
9301.15 8	2+,3,4+	7 fs 2

QRPA calculations predicted individual toroidal 1^-(K=1) toroidal state at 7.2-8.2 MeV.

M. Kimura et al, PTP, 127, 287 (2012)

7.55 MeV $I^\pi = 1^-, 3^-, 5^-$ → K=0 (K-mixed)

8.44 MeV $I^\pi = 1^-, 2^-, 3^-$ → K=1

The predicted 1^-(K=1) toroidal state can match 7.55-MeV or 8.44-MeV dipole states.

(e,e’): PWBA cross section

H. Theissen, Springer Tacts in Mord. Phy., 65 , 1 (1972)
J. Heisenberg and H.P. Blok, Ann. Rev. Nucl. Part. Sci, 33, 569 (1983),

$$\sigma_{PWBA}(\theta,q)=\left[\frac{Ze^2}{E_i}\right]^2 f_{rec}\{v_c(\theta,E_i)|F_E^C(q)|^2+v_T(\theta,E_i)[|F_E^T(q)|^2+|F_M^T(q)|^2\}$$

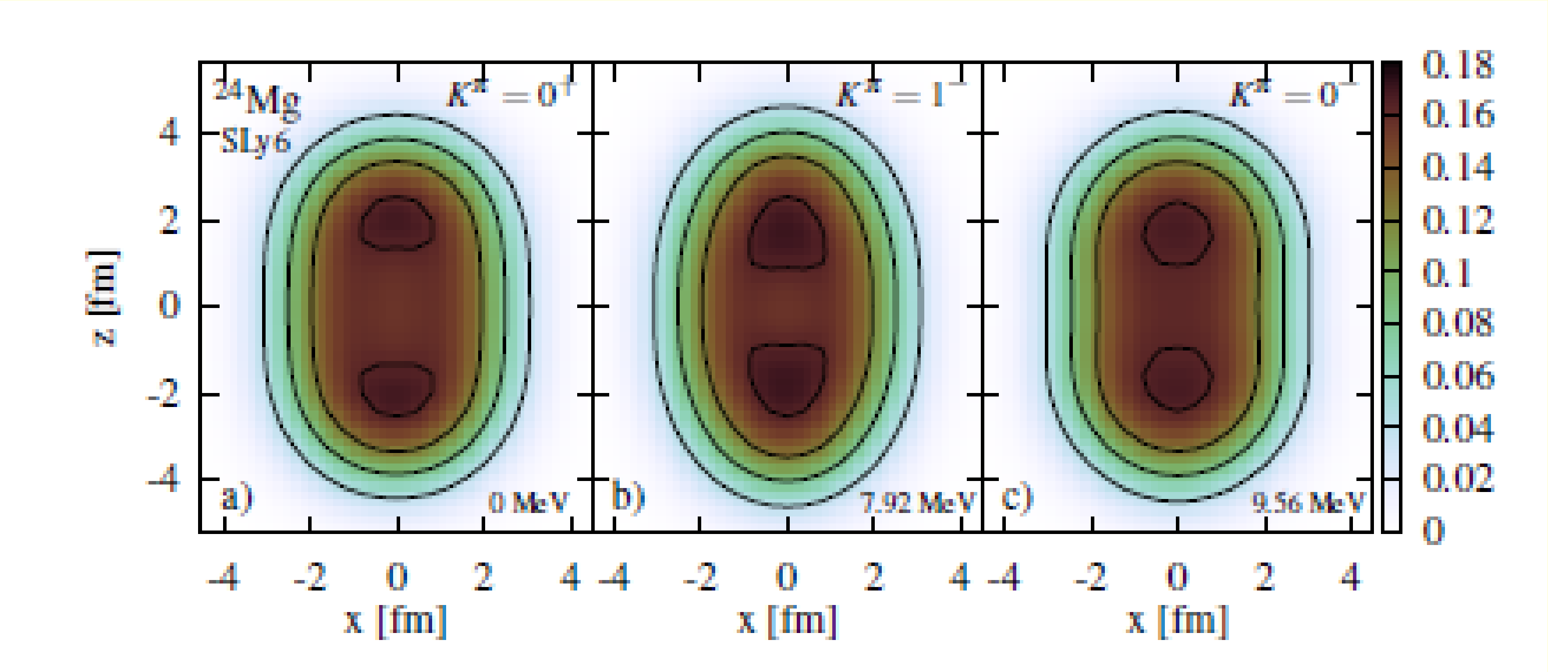
For $E_i >> E_v = E_i - E_f$, we get the familiar PWBA expression:

$$\sigma_{PWBA}(\theta,q)=\sigma_{Mott}(\theta,E_i)f_{rec}\{|F_E^C(q)|^2+(\frac{1}{2}+tg^2(\frac{\theta}{2})[|F_E^T(q)|^2+|F_M^T(q)|^2\}$$

For $I^\pi=1^-$ states, $F_M^T(q)=0$ but $\hat{\vec{j}}^{\,q}_{mag}$ contributes to $F_E^T(q)$

VON, A. Repko, J. Kvasil, P.-G. Reinhard, PRL 120, 182501 (2018)

Relation to cluster structure of 24Mg



Densities for the ground, toroidal and compression states

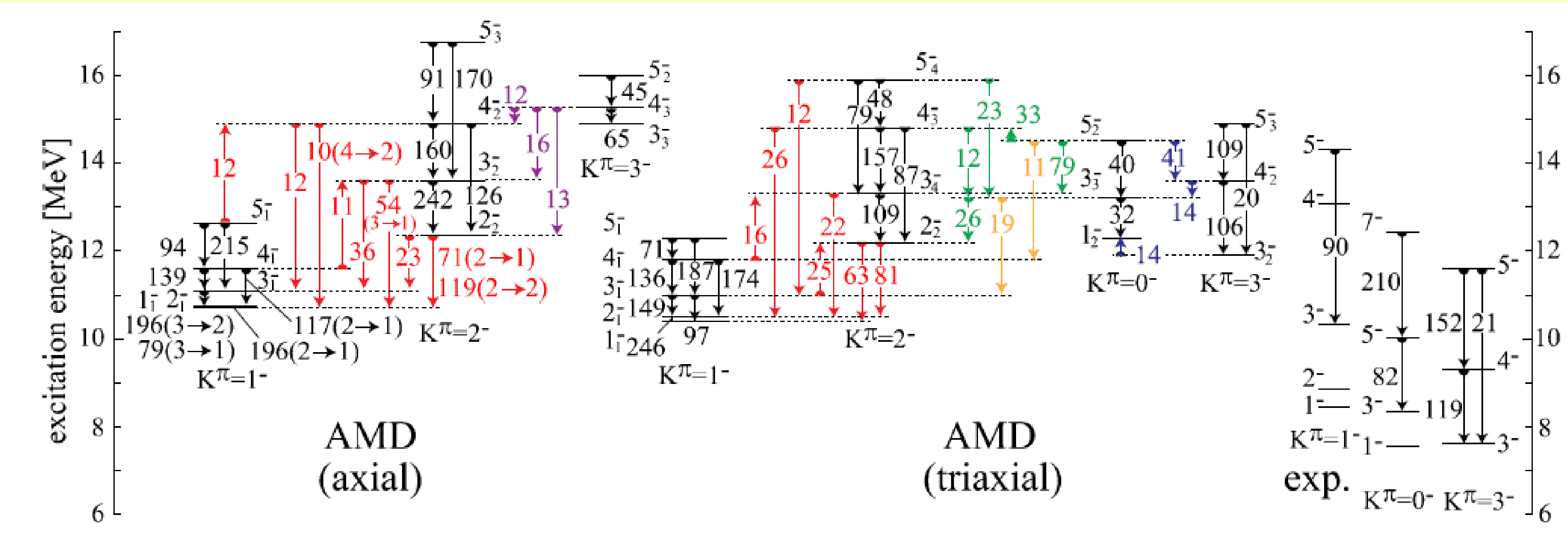
- excitation energies near α particle threshold S_{α}=9.3 MeV
- cluster structure in all three cases
- perhaps cluster structure favors the separation of vortex ad antivortex

Progress of Theoretical Physics, Vol. 127, No. 2, February 2012

Excitation and Structure Change of ^{24}Mg. I

— *Triaxial Deformed Mean Field in Low-Lying States* —

Masaaki KIMURA,[1] Ryosuke YOSHIDA[2] and Masahiro ISAKA[2]



Triaxiality affects $K^{\pi} = 2^{+}, 0^{-}, 3^{-}$ bands but not $K^{\pi} = 1^{-}$ band!

Spectrum of 24Mg

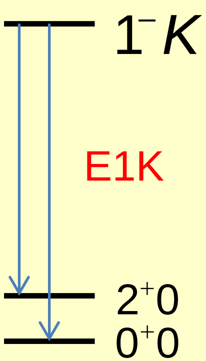
E_{level}(keV)	Jπ	$T_{1/2}$
0	0+	STABLE
1368.672 5	2+	1.33 ps 6
4122.889 12	4+	22 fs 2
4238.24 3	2+	41 fs 4
5235.12 4	3+	61 fs 7
6010.84 4	4+	49 fs 5
6432.30 11	0+	53 fs 8
7349.00 3	2+	6 fs 2
7555.04 15	1-	270 fs 55
7616.47 4	3-	890 fs 140
7747.51 9	1+	10 fs 3
7812.35 11	5+	20 fs 4
8114.2 20	6+	3.6 fs 10
8357.98 13	3-	56 fs 8
8437.31 15	1-	10 fs 2
8439.36 4	4+	3.8 fs 11
8654.53 15	2+	8.2 fs 21
8864.29 9	2-	4.4 fs 15
9003.34 9	2+	7.6 fs 14
9145.99 15	1-	
9284.22 14	2+	11 fs 2
9299.77 24		
9301.15 8	2+,3,4+	7 fs 2

Excitation levels [MeV]

theor	exper		
7.92	7.55	$I^\pi = 1^-, 3^-, 5^-$ →	K=0
	8.44	$I^\pi = 1^-, 2^-, 3^-$ →	K=1
9.56	9.15		

Alaga rules:

$$A_K = \frac{X(1^-K \to 0^+0)}{X(1^-K \to 2^+0)} = \left[\frac{\langle 1K1-K\,|\,00\rangle}{\langle 1K1-K\,|\,20\rangle}\right]^2$$



$A_0 = 0.5$

$A_1 = 2$

So K=1 branch can be discriminated

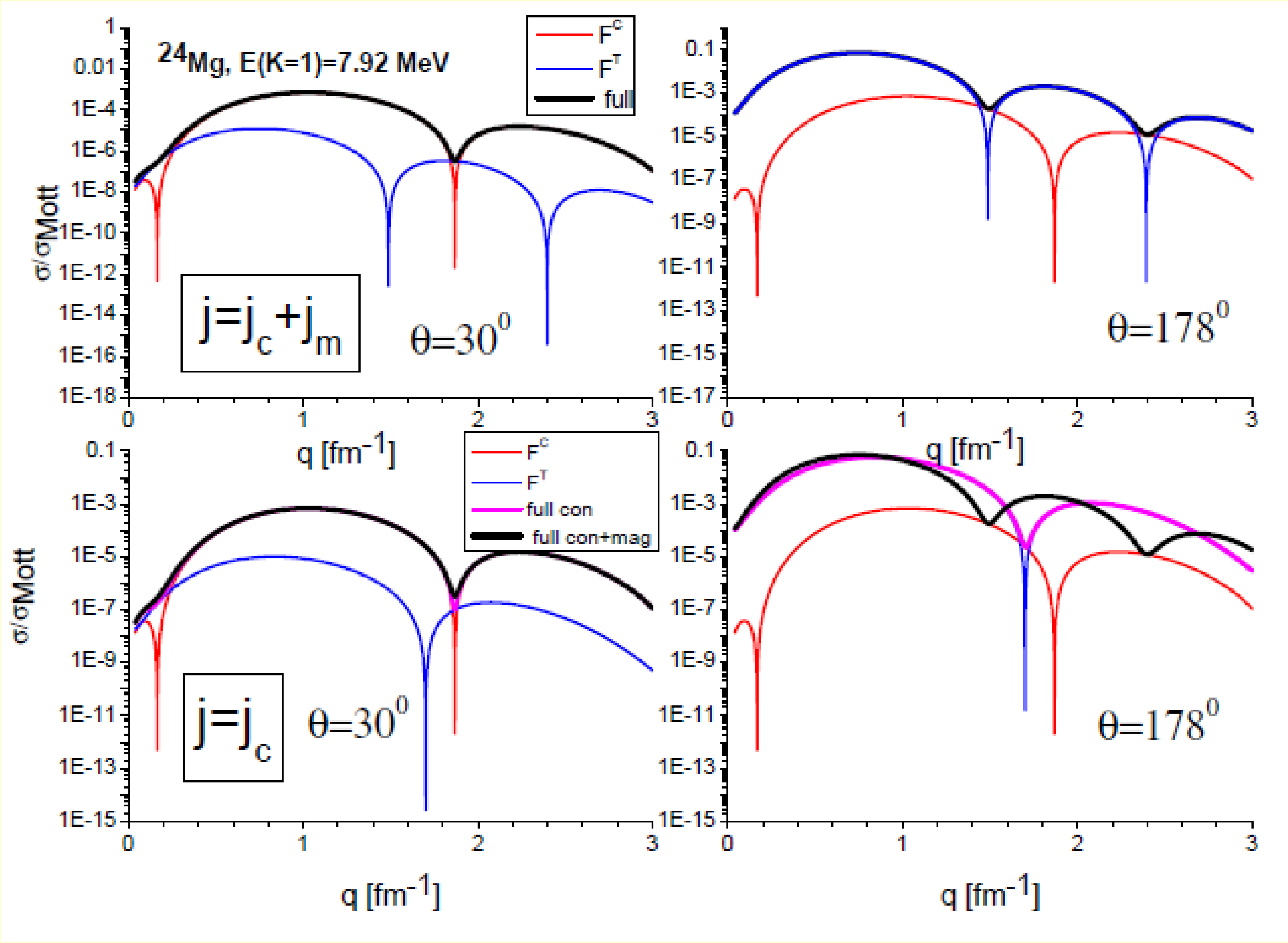
For 7.55-MeV state, $A^{exp}_{7.55} = 2.2 \to K = 1$

Perhaps the contradiction between band structure and Alaga rules can be explained by the strong triaxial effect for K=0 band. This band should be K-mixed.

M. Kimura et al, PTP, 127, 287 (2012)

Both 7.55-MeV and 8.44-MeV states can be candidates for TS.

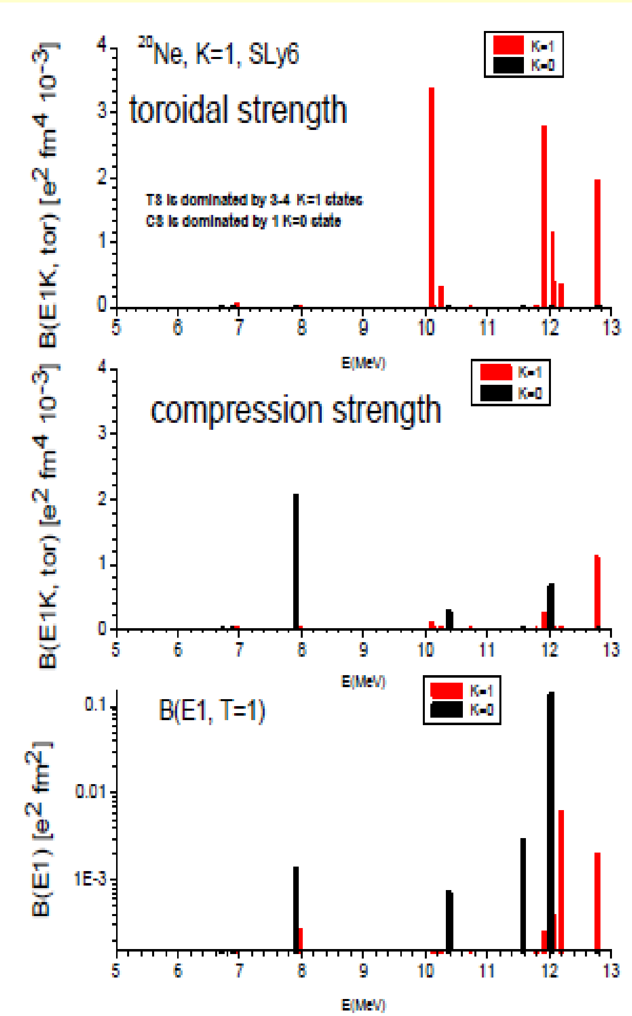
(e,e'): role of magnetic nuclear current



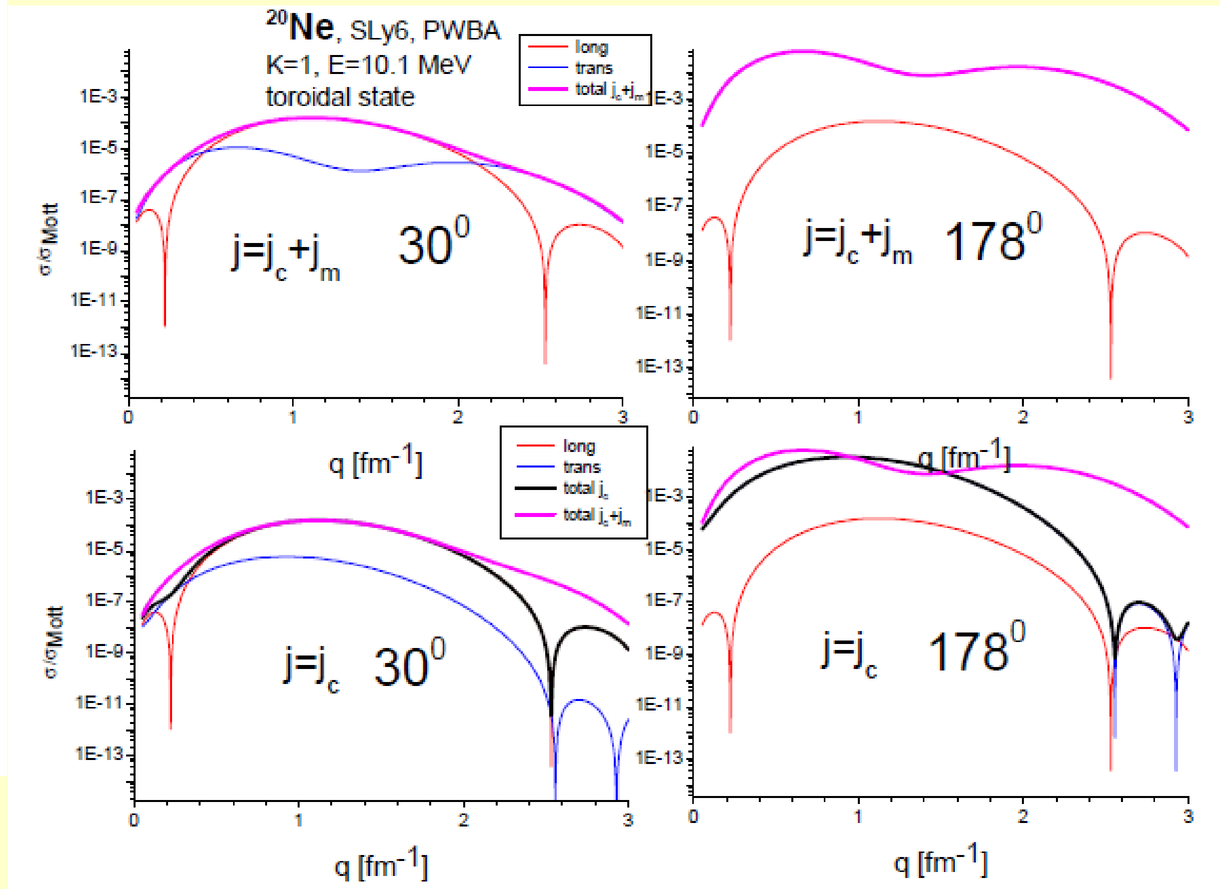
Impact of the magnetic nuclear current is:

- significant at $q > 1\ \mathrm{fm}^{-1}$, makes toroidal effect unresolved
- negligible at low q, makes reasonable the analysis of $|F^T_{E1}|^2 > |F^C_{E1}|^2$

20Ne



20Ne, K=1, SLy6
toroidal strength
TS is dominated by 3-4 K=1 states
CS is dominated by 1 K=0 state
K=1
K=0
B(E1K, tor) [e2 fm4 10-3]
E(MeV)
compression strength
B(E1K, tor) [e2 fm4 10-3]
B(E1, T=1)
B(E1) [e2 fm2]



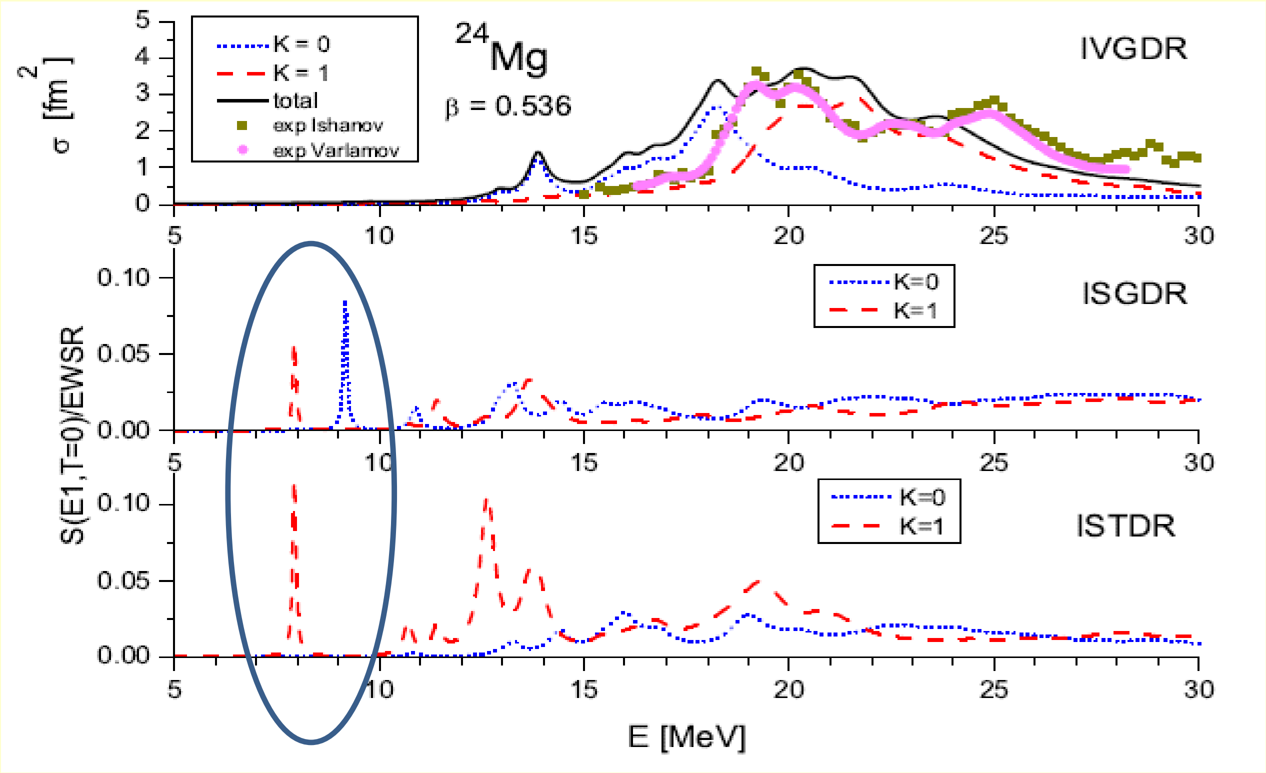
20Ne, SLy6, PWBA
K=1, E=10.1 MeV
toroidal state
long
trans
total jc+jm
total jc
σ/σMott
j=jc+jm 30⁰
j=jc+jm 178⁰
j=jc 30⁰
j=jc 178⁰
q [fm-1]

^{24}Mg $\beta_2^{exp} = 0.605$

VON, A. Repko, J. Kvasil and P.-G. Reinhard, PRL 120, 182501 (2018)

- extremely large prolate deformation
- rare low-energy spectrum
- cluster structure

→ Principle possibility for individual toroidal and compression states



QRPA, SLy6

Separate toroidal and compression dipole states!

Toroid: lowest by energy K=1 state!

(e,e’): PWBA cross section

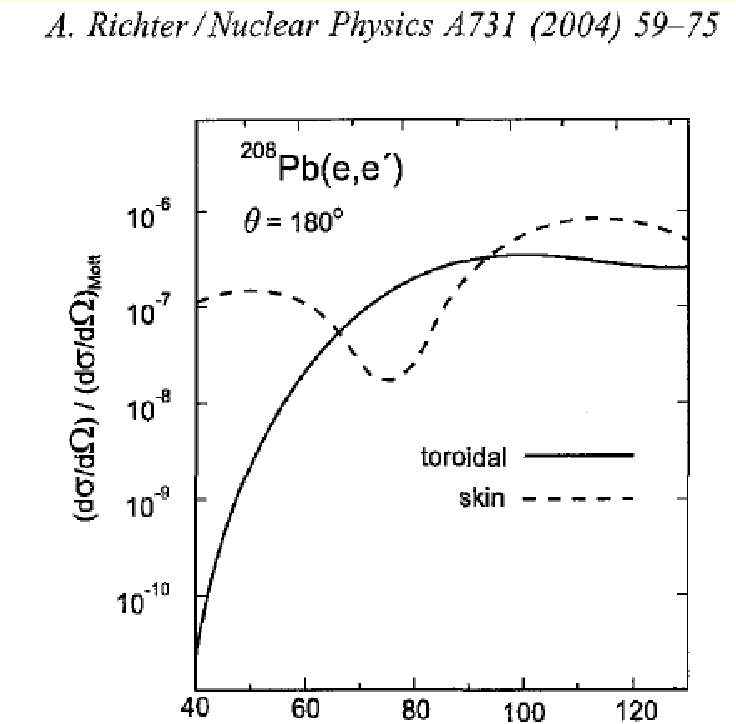
H. Theissen, Springer Tacts in Mord. Phy., 65 , 1 (1972)
J. Heisenberg and H.P. Blok, Ann. Rev. Nucl. Part. Sci, 33, 569 (1983),

$$\sigma_{PWBA}(\theta,q)=\left[\frac{Ze^2}{E_i}\right]^2 f_{rec}\{v_c(\theta,E_i)|F_E^C(q)|^2+v_T(\theta,E_i)[|F_E^T(q)|^2+|F_M^T(q)|^2\}$$

For $E_i >> E_v = E_i - E_f$, we get the familiar PWBA expression:

$$\sigma_{PWBA}(\theta,q)=\sigma_{Mott}(\theta,E_i)f_{rec}\{|F_E^C(q)|^2+(\frac{1}{2}+tg^2(\frac{\theta}{2})[|F_E^T(q)|^2+|F_M^T(q)|^2\}$$

For $I^\pi=1^-$ states, $F_M^T(q)=0$ but $\hat{\vec{j}}^q_{mag}$ contributes to $F_E^T(q)$



A. Richter, NPA, 731, 59 (2004).

Toroidal mode contributes to E1 transversal form-factor and perhaps can be discriminated in (e,e’) to back angles.

What we get for TS in 24Mg?

The model: self-consistent Skyrme QRPA (SLy6, SVbas, SkM*)

Mean field (SKYAX)

- 2D mesh in cylindrical coordinates
  - calculation box: >3R, mesh step 0.4 fm
  - s-p spectrum: ~2100 p+n sp levels up to +55 MeV
  - β_2=0.536(SLy6), β_2=0.525(SVbas), β_2=0.493(SkM*)

^{24}Mg

$\beta_2^{exp} = 0.605$

2qp basis (SLy6):

- 1900 (K=0) and 3600 (K=1) 2qp states until ~ 100 MeV
- EWSR(E1,T=1): 100%, EWSR(E1,T=0): 97%

Pairing:

- volume monopole pairing
- BCS with elements of HFB

2d QRPA (A. Repko):

A. Repko, J. Kvasil, V.O.N., and P.-G. Reinhard, arXiv:1510.01248[nucl-th]

- fully self-consistent
- matrix RPA, no separable ansatz,
- ph- and pp-channels, cmc
- parameterizations SLy6, SVbas, SkM*
- SLy6 as the main force (best description of equil. deformation and GDR)

Structure and spectroscopic properties of QRPA toroidal 7.92-MeV state: SLy6

Structure of $K^{\pi}=1^{-}_{1}$ toroidal state:

$pp[211]\uparrow-[330]\uparrow$ 54%

$nn[211]\uparrow-[330]\uparrow$ 39%

$F \rightarrow F+5$

$1d_{5/2} \rightarrow 1f_{7/2}$

Similar structure for SVbas and SkM* !

Δ=0.85 MeV - minimal collective shift

B(E3,T=0,$0^{+}0_{gs} \rightarrow 3^{-}1$)= 402 e^2 fm^6 (10.7 Wu) →

- holding asympt. E3(K=1) sel. rules
- strong octupole correlations, coupling of E1 and E3 modes

B(E1,T=1,$1^{-}1 \rightarrow 0^{+}0_{gs}$)=$2.52 \cdot 10^{-4} e^2 fm^2$

The state has a sizeable collectivity!

The most promising light prolate nuclei to search the toroidal states

^{10}Be	S_α=4.5 MeV	$E(1^-) = 5.96\ MeV$
^{20}Ne (β=0.72)	S_α=4.7 MeV	$E(1^-) = 5.79\ MeV$
^{22}Ne (β=0.57)	S_α=??? MeV	$E(1^-) = 6.69\ MeV$
^{24}Ne (β=0.41)	S_α=??? MeV	$E(1^-) = ???\ MeV$
^{24}Mg (β=0.61)	S_α=9.3 MeV	$E(1^-) = 7.92\ MeV$
^{26}Mg (β=0.48)	S_α=??? MeV	$E(1^-) = 7.06\ MeV$
^{28}Mg (β=0.48)	S_α=??? MeV	$E(1^-) = 5.19?\ MeV$
^{32}Mg (β=0.51)	S_α=??? MeV	$E(1^-) = 2.86?\ MeV$
^{34}Mg (β=0.55)	S_α=??? MeV	$E(1^-) = ???\ MeV$

Lowest dipole states lie close to S_α

Nuclei ^{10}Be, ^{20}Ne, ^{24}Mg are most relevant

TDR: points of interest

- The only known electric intrinsic vortical mode in nuclei:

  J. Kvasil, VON., W. Kleinig, P.in nucli-G. Reinhard, P. Vesely, **PRC** 84, 034303 (2011).

- Measure of nuclear dipole vorticity

  P.-G. Reinhard, VON, A. Repko, and J. Kvasil, **PRC** 89, 024321 (2014).

- TDR as a source of PDR

  A. Repko, P.-G. Reinhard, VON. and J. Kvasil, **PRC** 87, 024305 (2013).

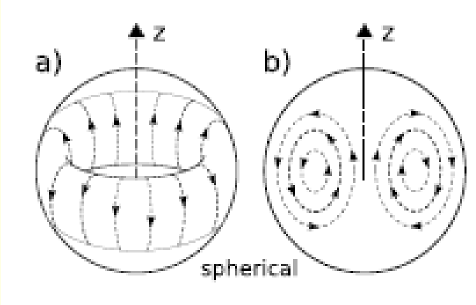
- TDR as spherical Hill's vortex (vortex ring)

  (well known in hydrodynamics of fluids in spherical confinement)

  M. J. M. Hill, Phil. Trans. Roy. Soc. London A 185, 213 (1894).

  The best known example of steady rotational solution of classical equations for inviscid incompressible fluid flow

  (small-amplitude oscillations, not spinning)

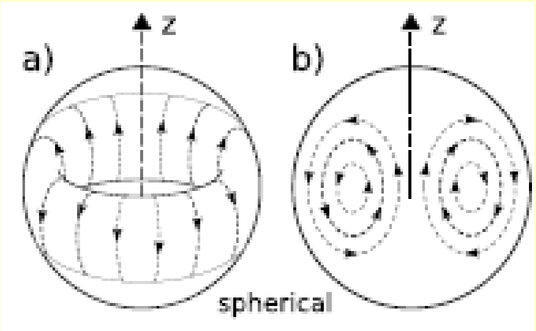


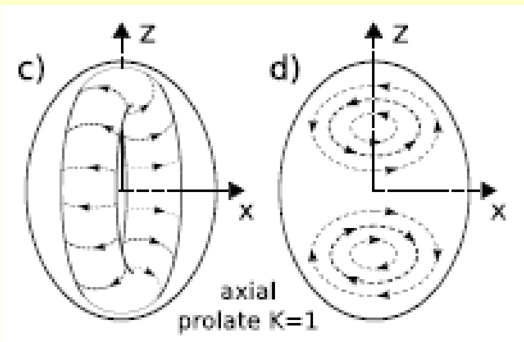
- Specific deformation features of TDR

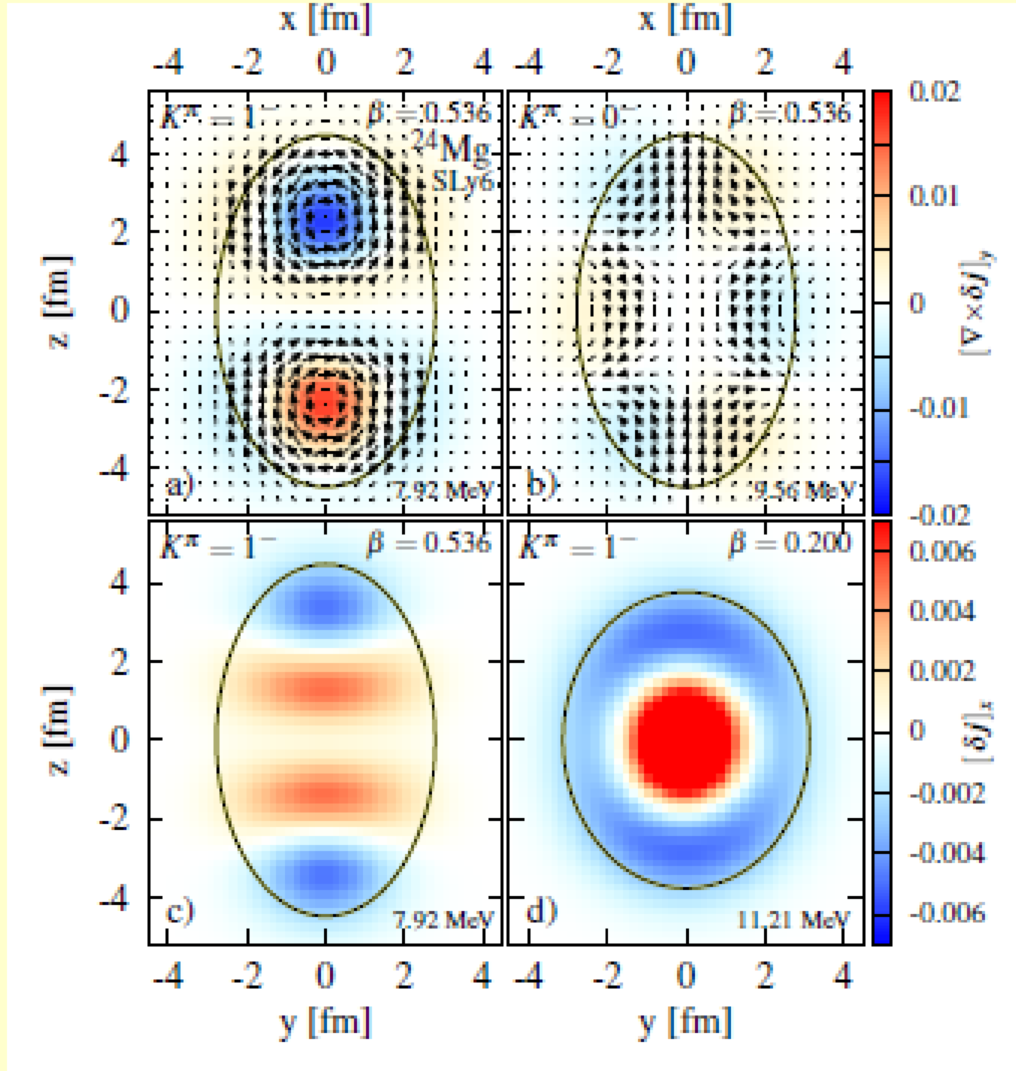
  J. Kvasil, A. Repko,VON, and P.-G. Reinhard, EPJA, 53, 221 (2017)

- Discussed in other areas (astrophysics, metamaterials, …)

Transformation of spherical Hill's vortex in highly deformed axial confinement



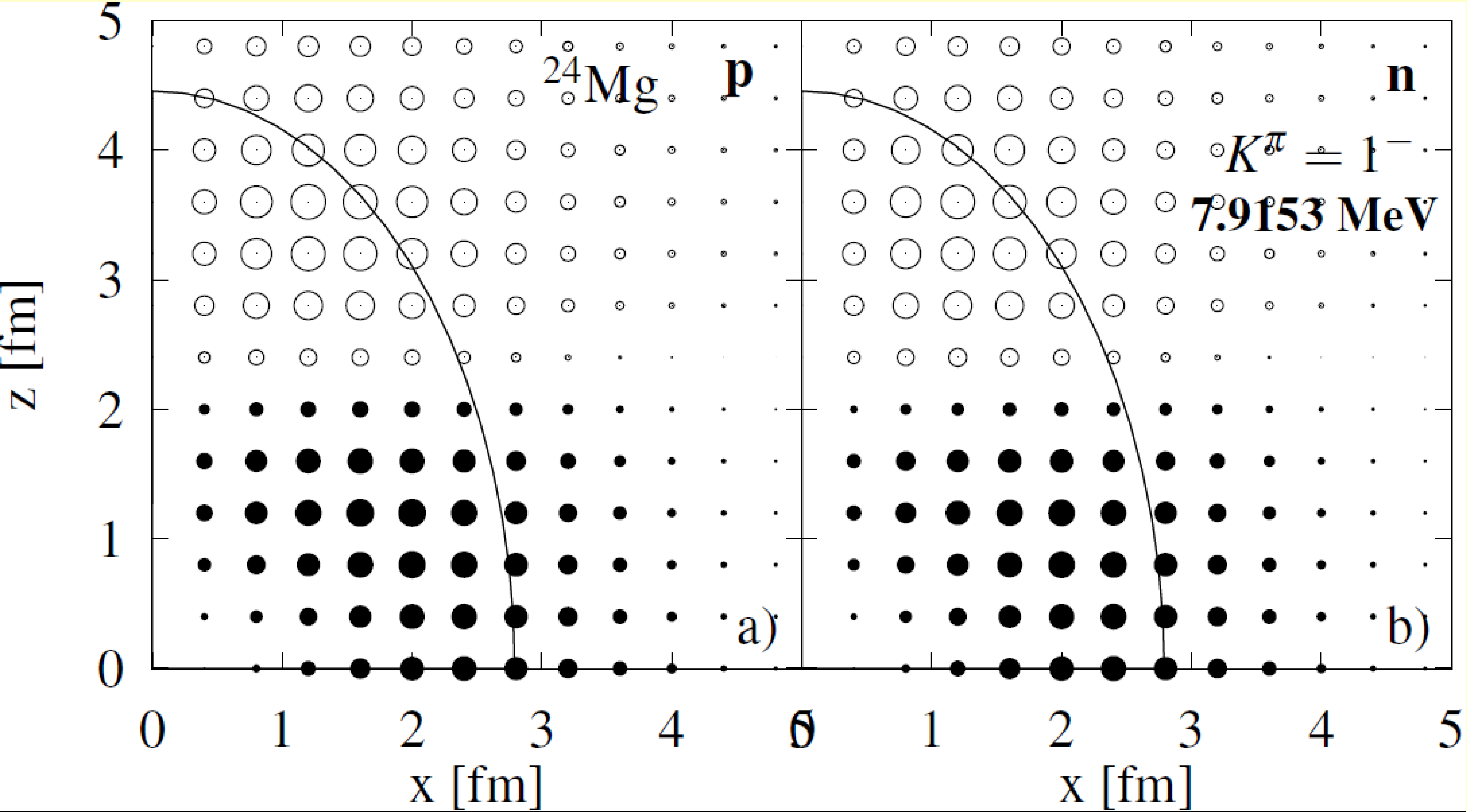




Spherical Hill's vortex:
the simplest vortex in
spherical confinement.
Well known in hydrodynamics.

In the confinement with a high axial prolate deformation Hill's vortex is converted to vortex-antivortex configuration!

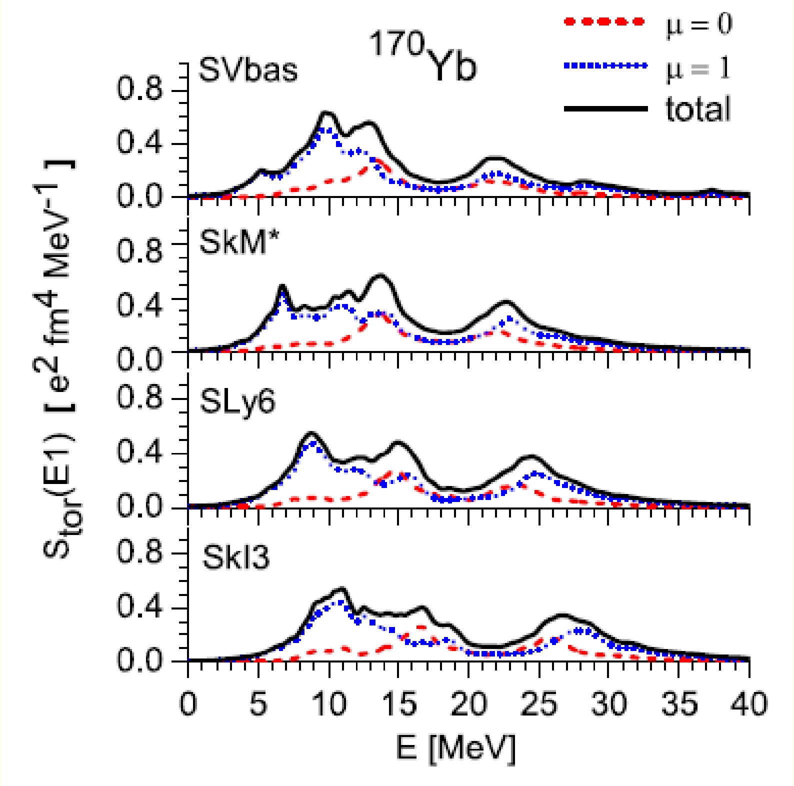
Proton and neutron transition densities in 24Mg



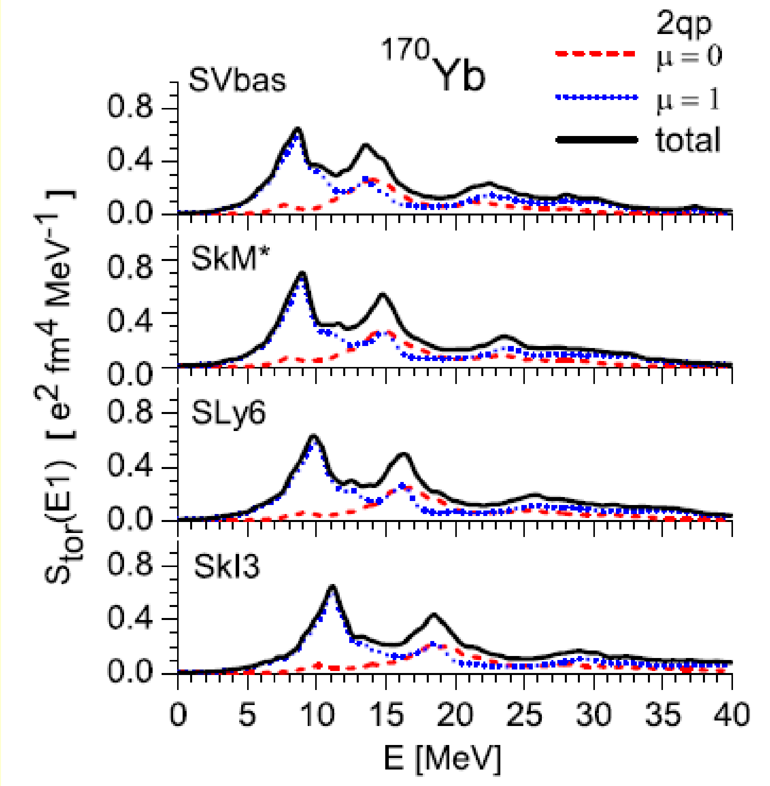
Deformation effects in the toroidal mode

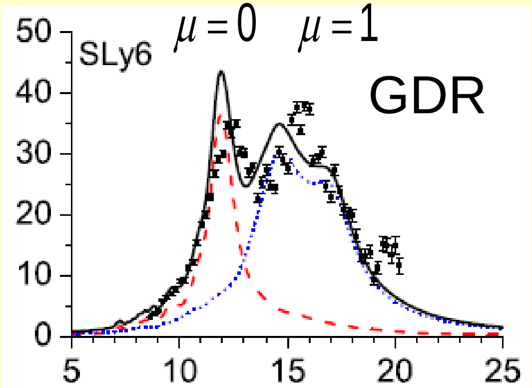
J. Kvasil, VON, W. Kleinig and P.-G. Reinhard, Phys. Scr. 89, 054023 (2014)

RPA



2qp





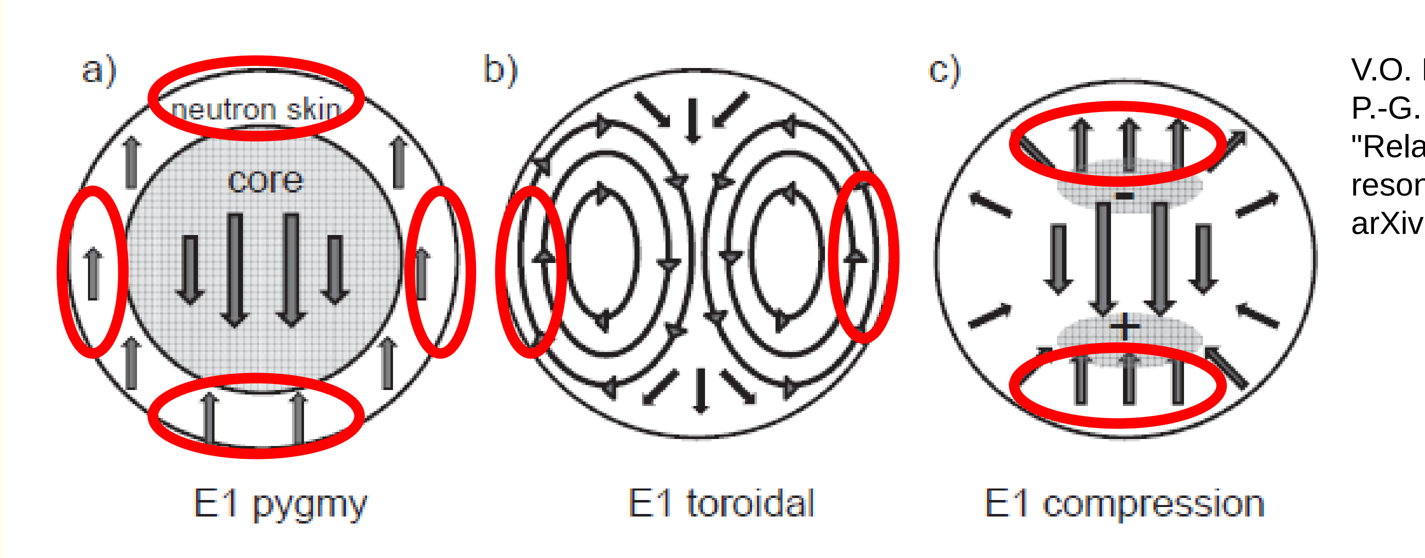
GDR: $E(\mu = 0) < E(\mu = 1)$

TM: $E(\mu = 0) > E(\mu = 1)$

Unusual sequence of $\mu = 0$ and $\mu = 1$ branches

Deformation (not resid. Interaction) effect

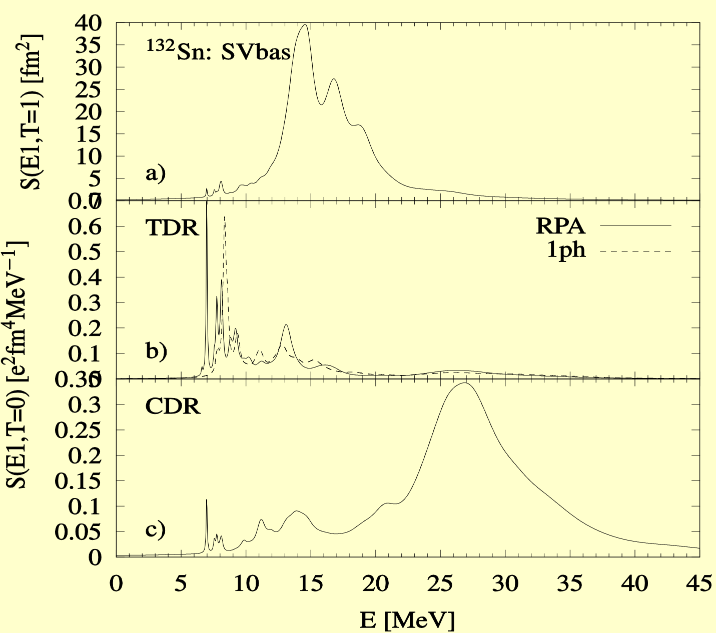
Should affect PDR properties



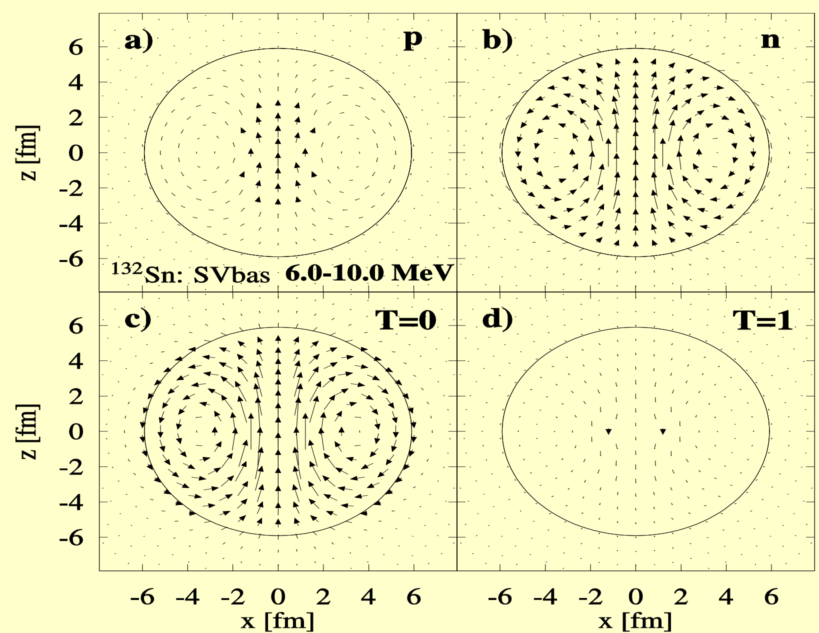
V.O. Nesterenko, A. Repko, P.-G. Reinhard, and J. Kvasil, "Relation of E1 pygmy and toroidal resonances", arXiv:1410.5634[nucl-th],

- PDR can be viewed as a local peripheral part of TDR and CDR

- Our calculations demonstrate the TDR motion in PDR energy region for other nuclei: Ni, Zr, Sn, …

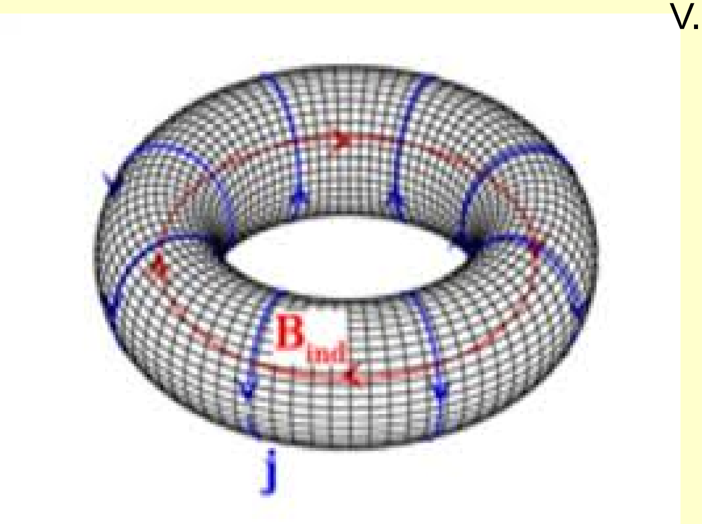


132Sn, SVbas, with PDR

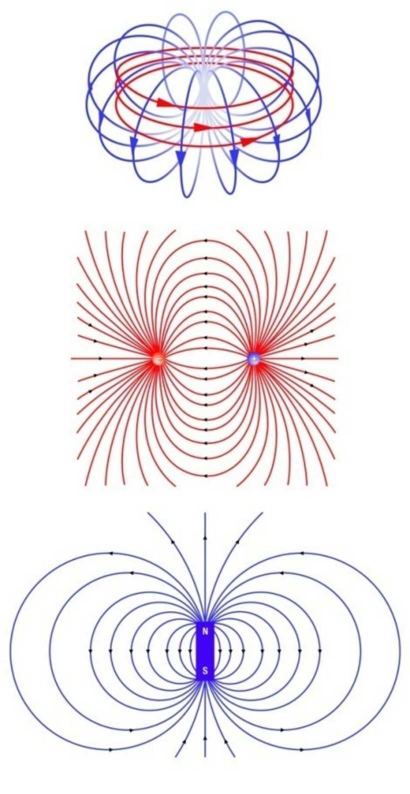


Toroidal moment

Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. 33, 1531 (1957)
V.M. Dubovik and L.A. Tosunyan, Part. Nucl., 14, 1193 (1983)



dipole moments



anapole

electric

magnetic

Zeldovich anapole: no electric and magnetic moments but the toroidal moment

$$\vec{T} = \frac{1}{10c}\int d\vec{r}\ [(\vec{j}\cdot\vec{r})\vec{r} - 2r^2\vec{j}]$$

$$T = \frac{\pi}{2c} jR_0^2 bn$$

Speculations with toroidal :

- Robert Scherrer and Chiu Man Ho (2013): attempt to explain dark matter by existence of Maiorana fermions with the anapole moment.

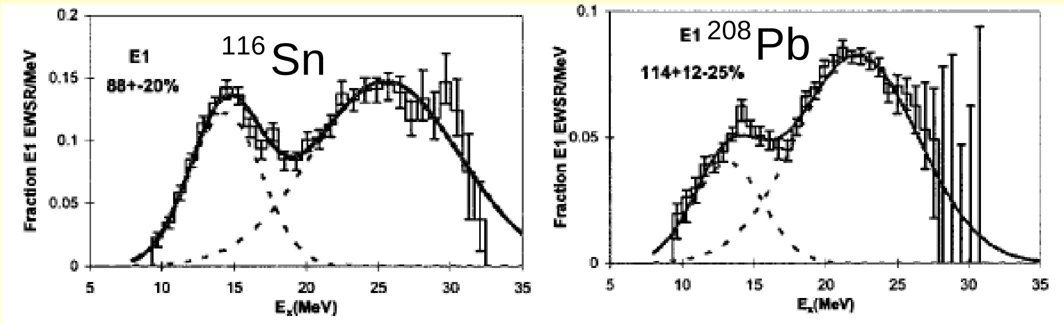
Available (α,α') experimental data

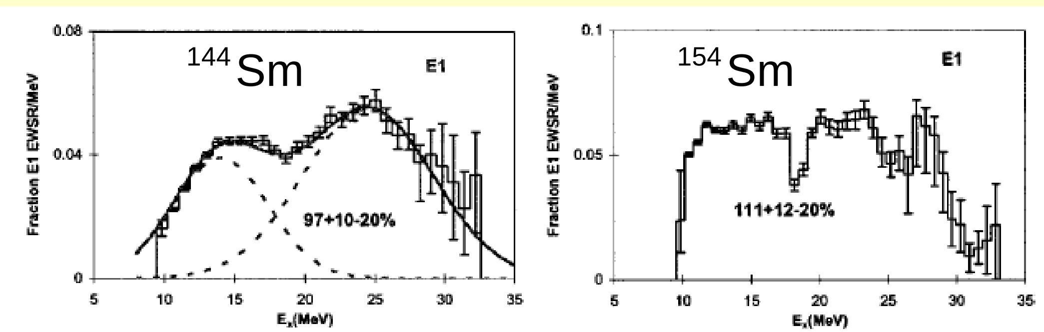
M.Uchida et al, PRC 69, 051301(R) (2004)

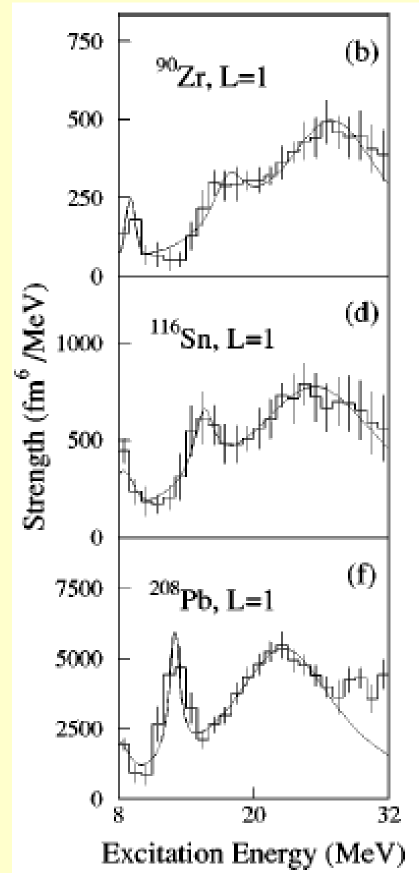
$^{90}Zr, ^{116}Sn, ^{144}Sm, ^{208}Pb$

D.H. Youngblood et al, PRC 69, 034315 (2004).

$^{116}Sn, ^{144}Sm, ^{154}Sm, ^{208}Pb$







-the only experiment for ISGDR in **deformed** nucleus
-**TDR is lost** since too high excitation energies (E>10 MeV) are considered

Perhaps TDR is observed at ~ 8 MeV

The familiar treatment of ISGDR experiments should be corrected:
- the low energy broad bump is not TDR but some TDR/CDR mixture
- the main peaked TDR is at a lower energy

Previous studies

D. VRETENAR, N. PAAR, P. RING, AND T. NIKŠIĆ PHYSICAL REVIEW C **65** 021301(R)

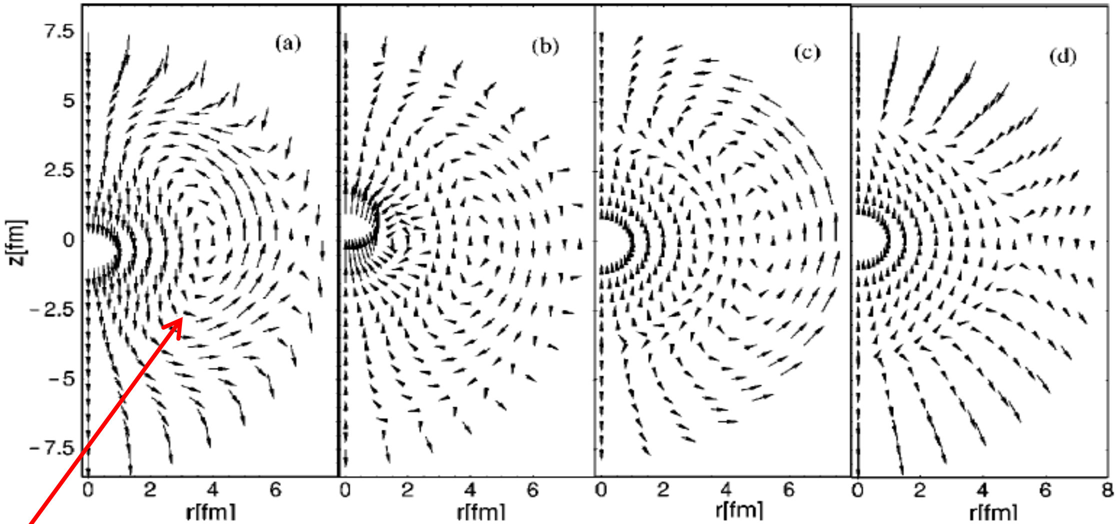


FIG. 3. Velocity distributions for the most pronounced dipole peaks in ^{116}Sn (see Fig. 2). The velocity fields correspond to the peaks at 8.82 MeV (a), 10.47 MeV (b), 17.11 MeV (c), and 30.97 MeV (d).

D. Vretenar et al,
relativistic mean field RPA

Toroidal-like flow in T=1 channel .

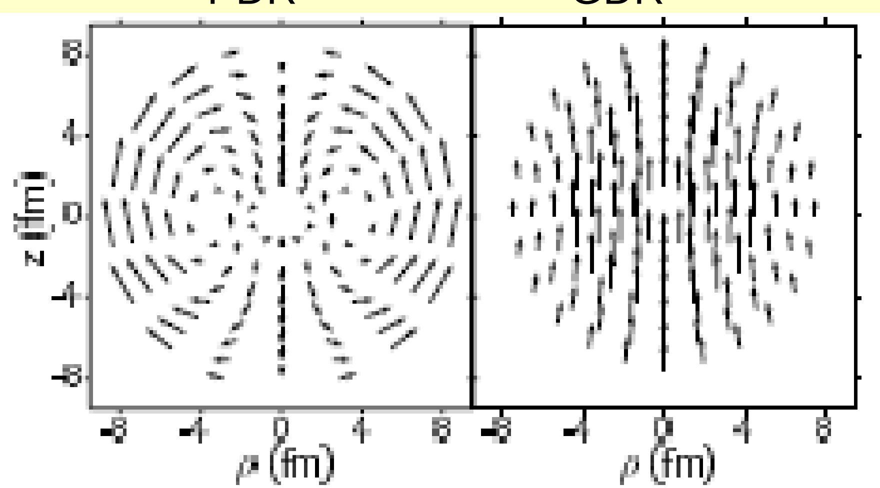
N.Ryezayeva et al, PRL 89, 272502 (2002).

QPM calculations taking into account complex configurations

Summed QPM velocity fields in 6.5-10.5 MeV region

$$\delta\vec{v} = \frac{N}{A}\delta\vec{v}_p - \frac{Z}{A}\delta$$

PDR GDR



However none of these studies has clamed the toroidal origin of PDR