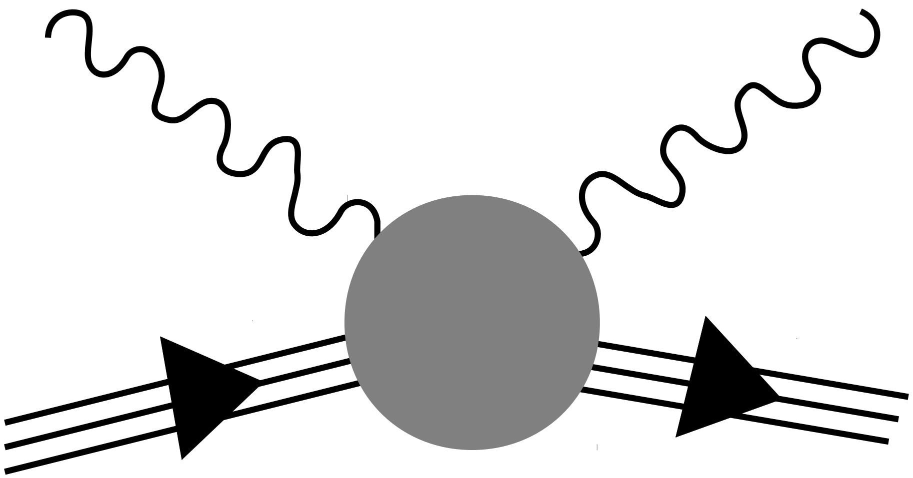
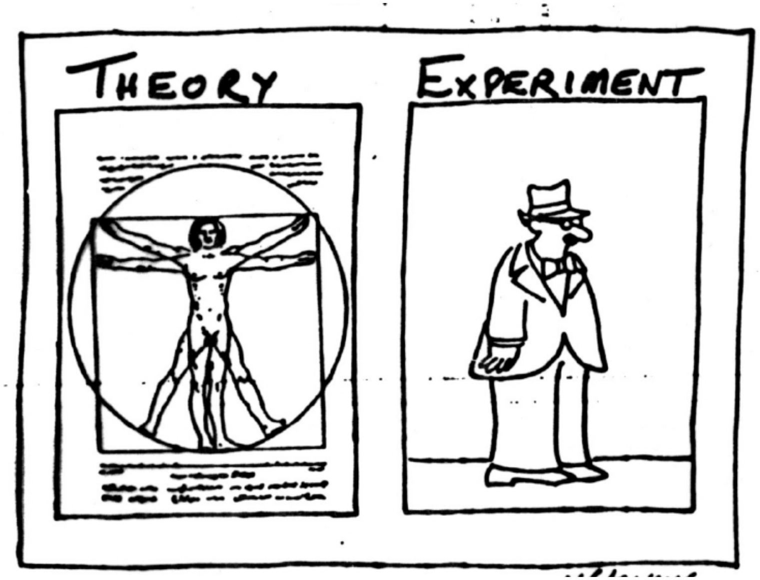
2018 European Nuclear Physics Conference (EuNPC)
4th September, Bologna





Scalar dipole dynamical polarizabilities from proton real Compton scattering data

UNIVERSITÀ DI PAVIA

INFN

Stefano Sconfietti in collaboration with ***Barbara Pasquini*** & ***Paolo Pedroni***
University of Pavia & INFN (Pavia)

Outline

WHAT?
Extraction of dipole dynamical polarizabilities (DDPs) from proton RCS data

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WHAT? (II)
Response of the nucleon to an external **dynamical** field (photon)

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Dispersion Relation approach + data analysis

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WHAT? (II)
Response of the nucleon to an external **dynamical** field (photon)

HOW?
Dispersion Relation approach + data analysis

WHY?
"...sure, it may give some practical results, but that's not why we do it" - R. P. Feynman

SOME STATISTICS

Complications

Gradient method to find the $\boldsymbol{\chi^2}$ minimum

VERY **high correlations** between parameters!

MINUIT WARNING IN HESSE
 ============== MATRIX FORCED POS-DEF BY ADDING
0.13727E-01 TO DIAGONAL.

VERY **low sensitivity** of the data to dynamical polarizabilities

NO WAY to find the “right” minimum and to define “right” errors on fit parameters

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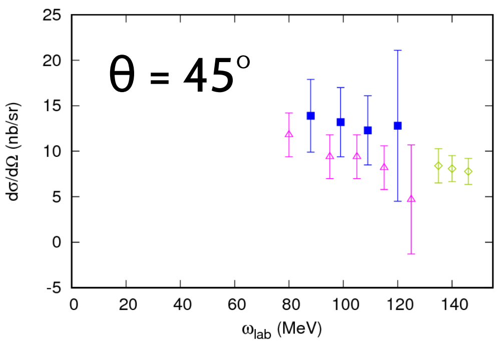
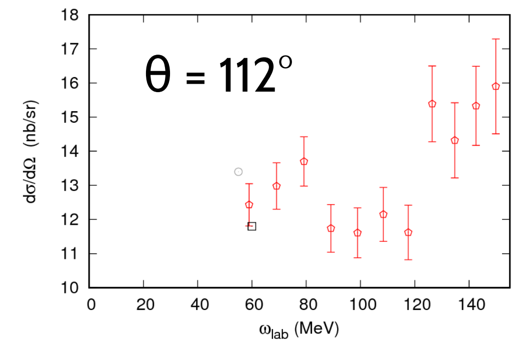
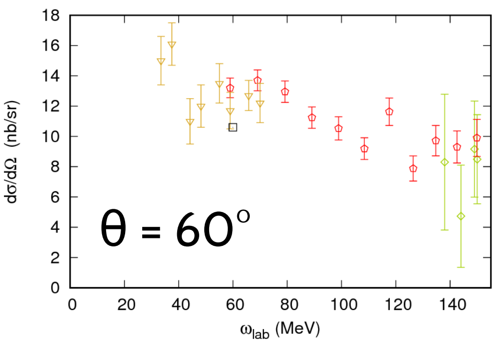
Combination of **SIMPLEX** method and **BOOTSTRAP** technique

The DATA set







Symbol	Set	Ref
	GOL 60	Goldansky et al.
	OdL 01	Olmos de León
	HAL 93	Hallin et al.
	HYM 59	Hyman et al.
	PUG 67	Pugh et al.
	FED 91	Federspiel et al.
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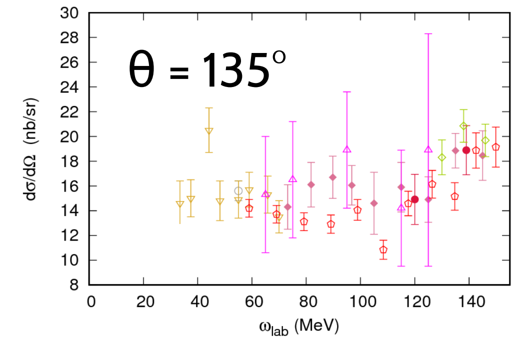
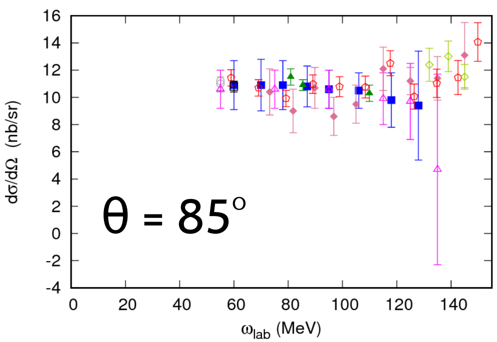
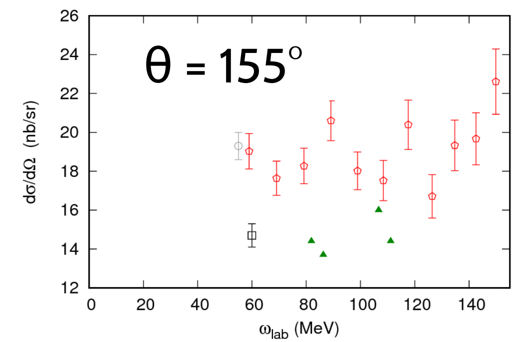
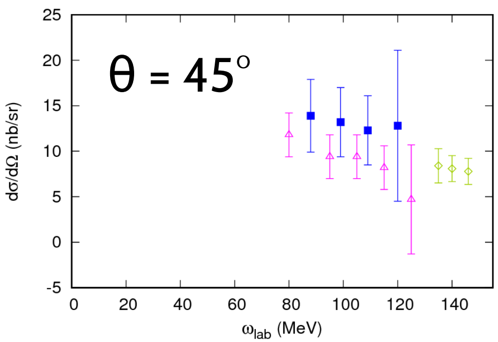


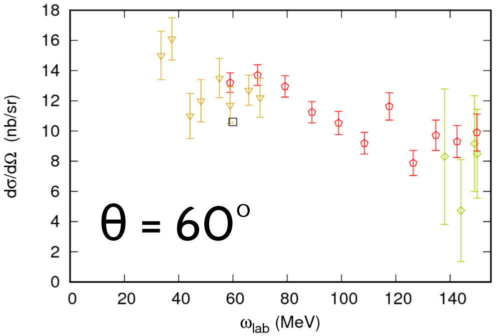


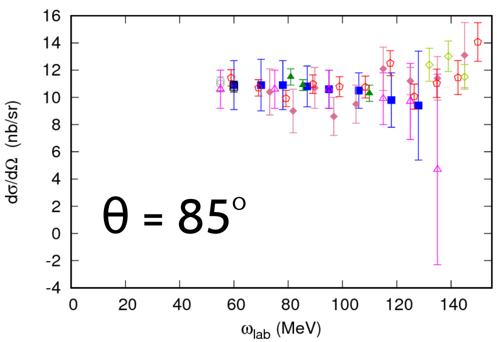


The DATA set

Half of the Spartans
that King Leonidas led to the Battle of Thermopylae...







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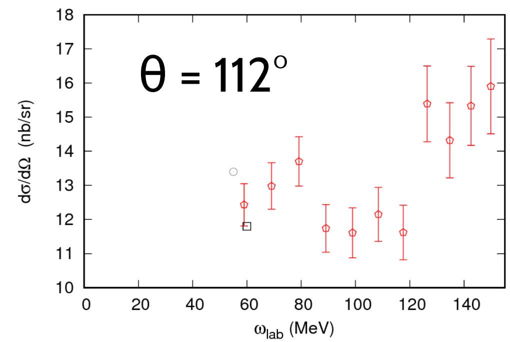

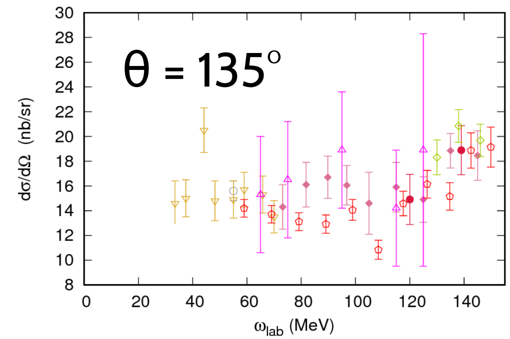

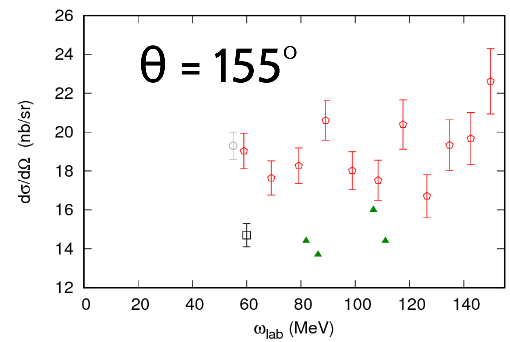

Parametric bootstrap sampling and systematics

$$S_{i,exp}^{boot} = S_{i,exp} \pm \gamma \, \sigma_{i,\mathrm{exp}}$$

Gaussian distributed

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How can we include systematical errors?

$$\chi^2_{mod} = \sum_{i=1}^{N_{tot}} \left[\frac{\mathcal{N}\mathcal{S}_{i,exp} - \mathcal{S}_{i,theory}}{\mathcal{N}\sigma_{i,exp}} \right]^2 + \left(\frac{\mathcal{N} - 1}{\sigma_{i,sys}} \right)^2$$

...one normalization factor per data set is needed!

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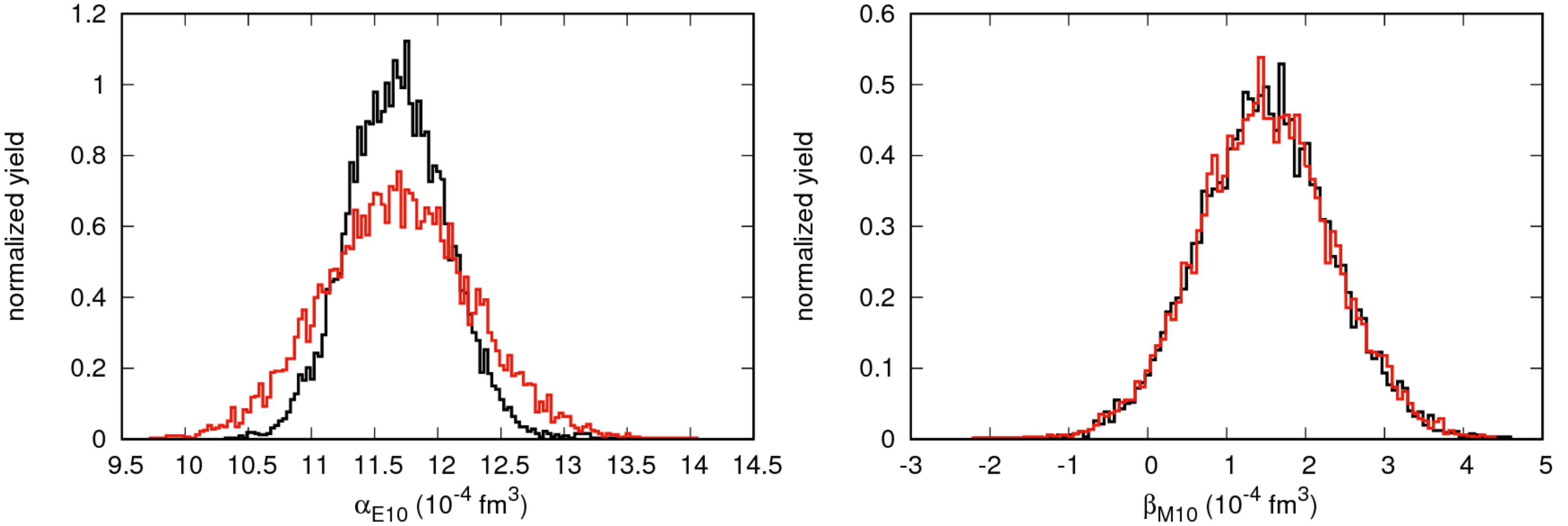
...one normalization factor per data set is needed!

$$S_{i,exp}^{boot} = \xi\left[S_{i,exp} \pm \gamma\,\sigma_{i,\mathrm{exp}}\right]$$

At every bootstrap cycle the systematical errors for each set can vary independently!

The effect of systematics (static & spin-independent polarizabilities)

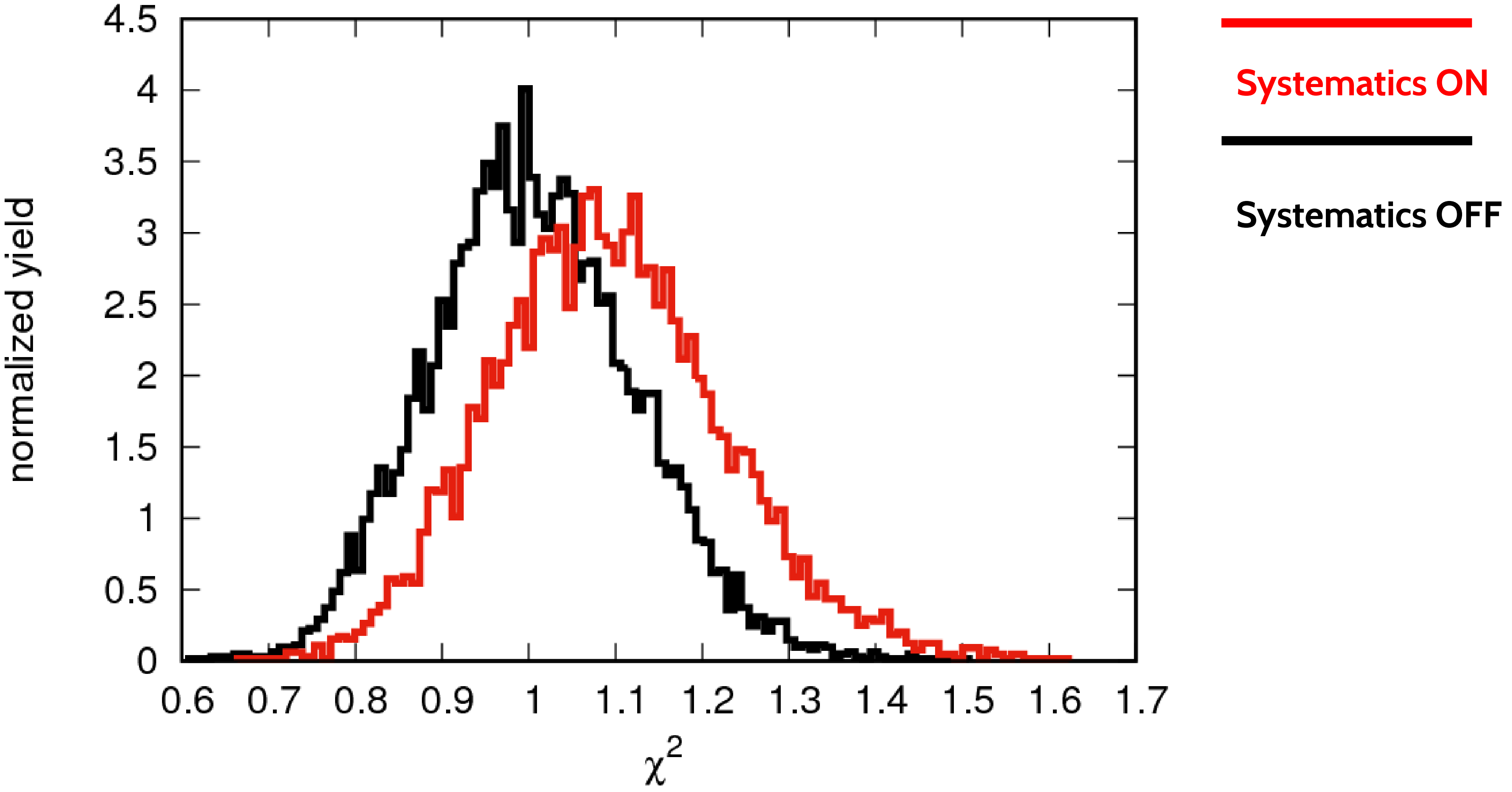


Systematics ON

Systematics OFF

Expected Gaussian shape + systematics enlarging

"χ^2" probability distribution in bootstrap framework (static pol.)



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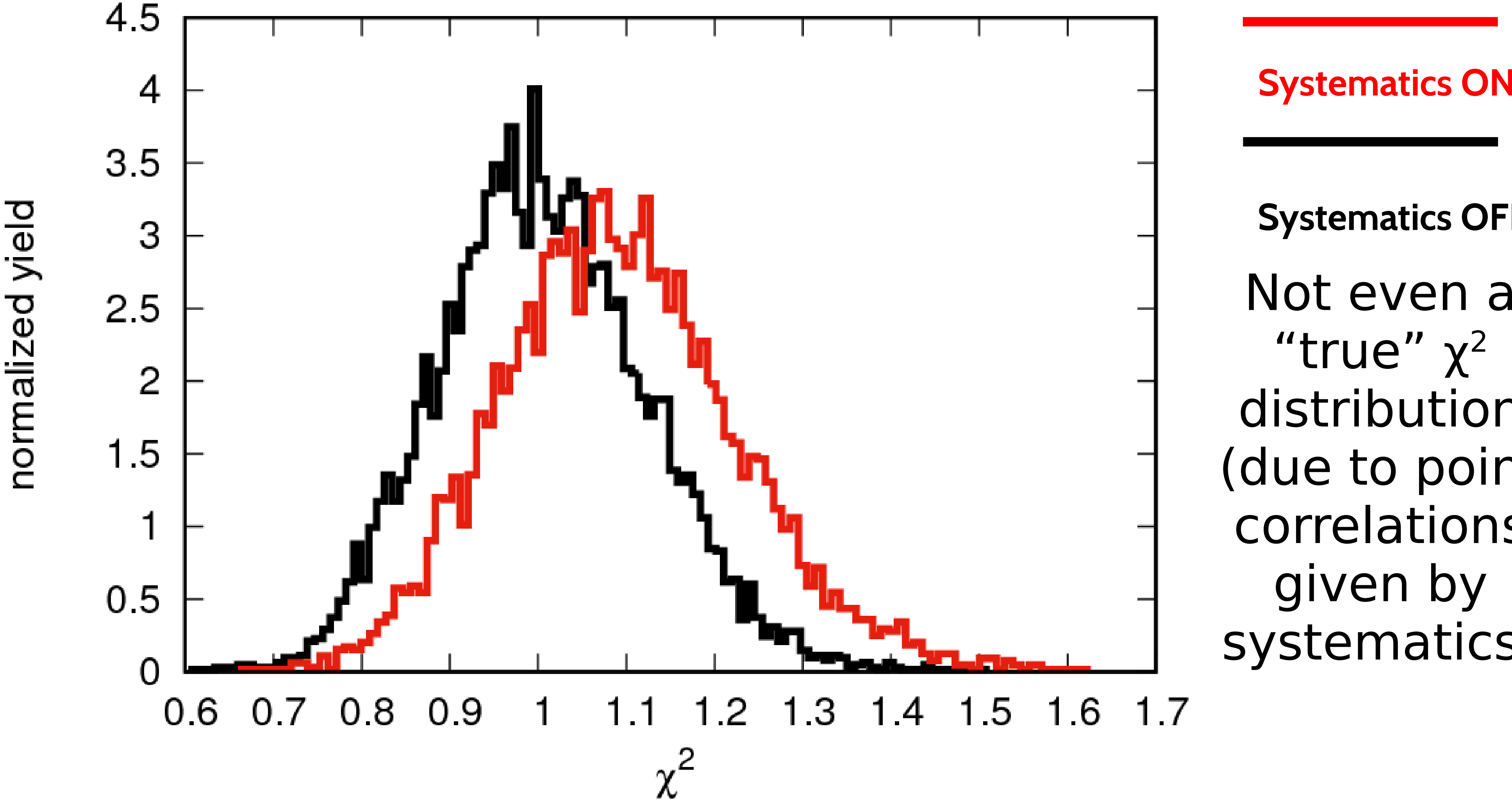
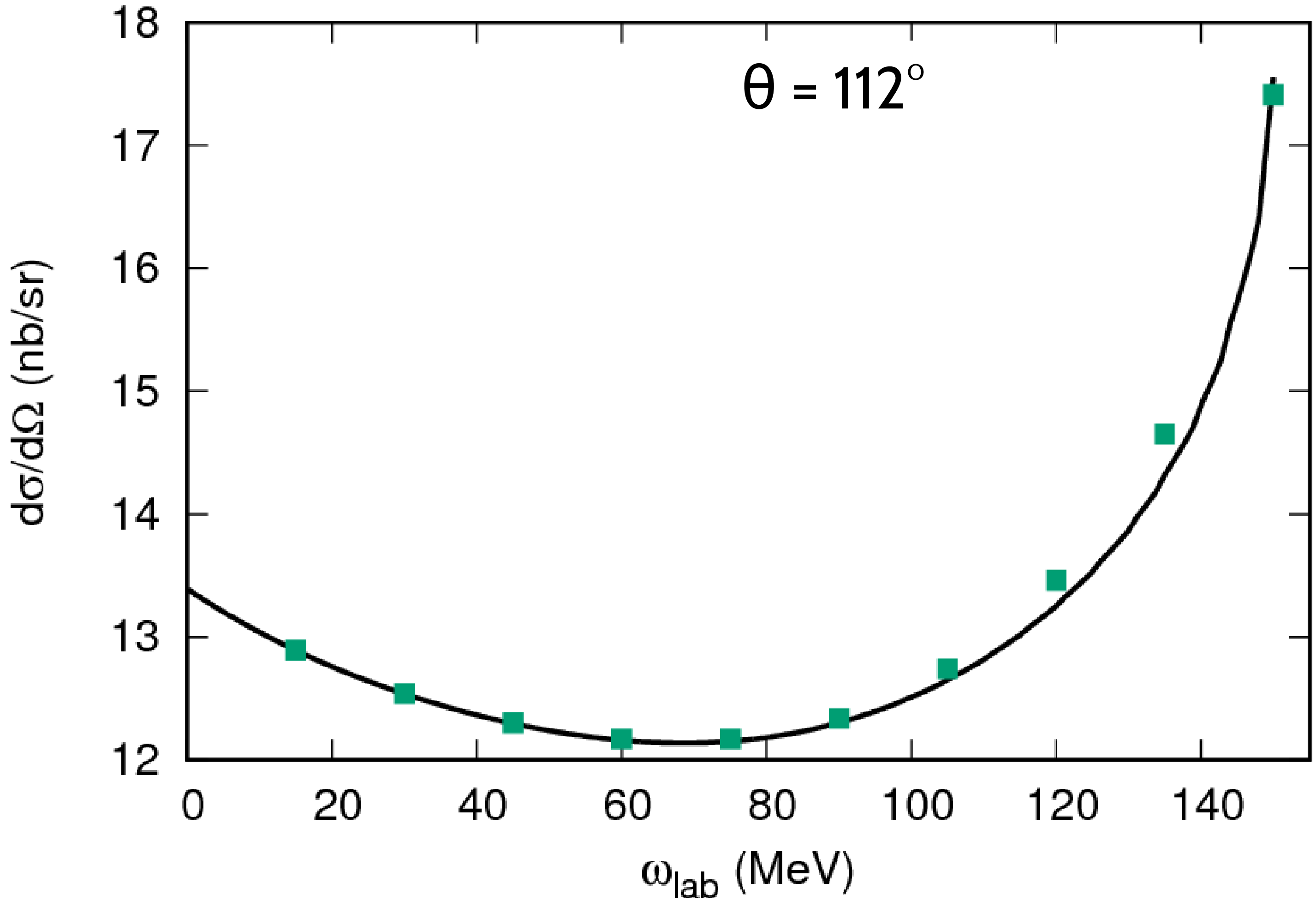


Not even a "true" χ^2 distribution (due to point correlations given by systematics)

DIPOLE
STATIC
POLARIZABILITIES

DRs VS (*LEX + multipoles*)

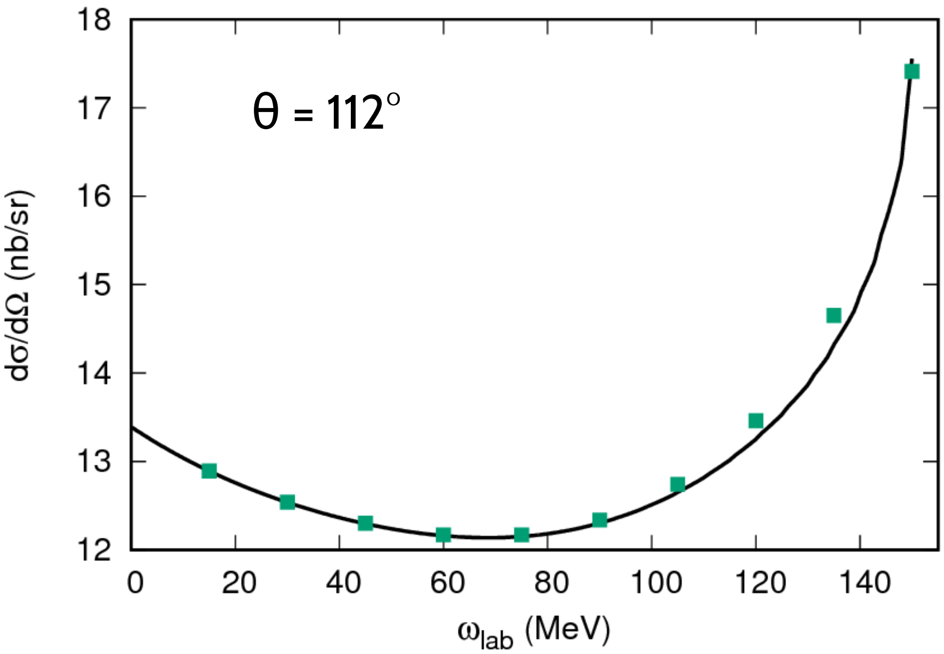


LEX + multipoles

DRs calculation

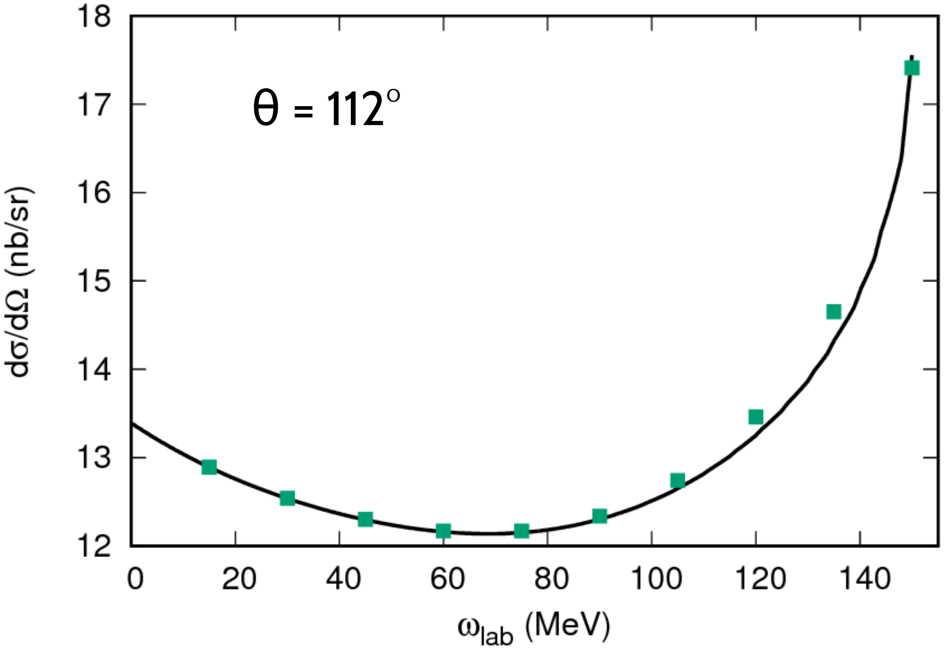
LEX: up to **ω^5**
multipoles: up to **l=3**

DRs VS (*LEX + multipoles*): fit



FIRST cross check:
comparison with
LEX + multipoles
and ***full DRs***

DRs VS (*LEX + multipoles*): fit



FIRST cross check:
comparison with
LEX + multipoles
and ***full DRs***

	α_{E1} $(10^{-4}\ \mathrm{fm}^3)$	β_{M1} $(10^{-4}\ \mathrm{fm}^3)$
full DRs	**11.9 ± 0.2**	**1.9 ± 0.2**
LEX + MULTIPOLES	**11.8 ± 0.2**	**2.0 ± 0.2**

B. Pasquini, P. Pedroni, S. Sconfietti, Phys. Rev. C 98, 015204 (2018)

Bootstrap VS gradient: systematics ON

	α_{E1}	β_{M1}
BOOTSTRAP	11.8 ± 0.2	2.0 ± 0.2
LEX + MULTIPOLES	11.8 ± 0.2	2.0 ± 0.2
BOOTSTRAP SYS ON	11.8 ± 0.3	2.0 ± 0.3

Bootstrap VS gradient: systematics ON

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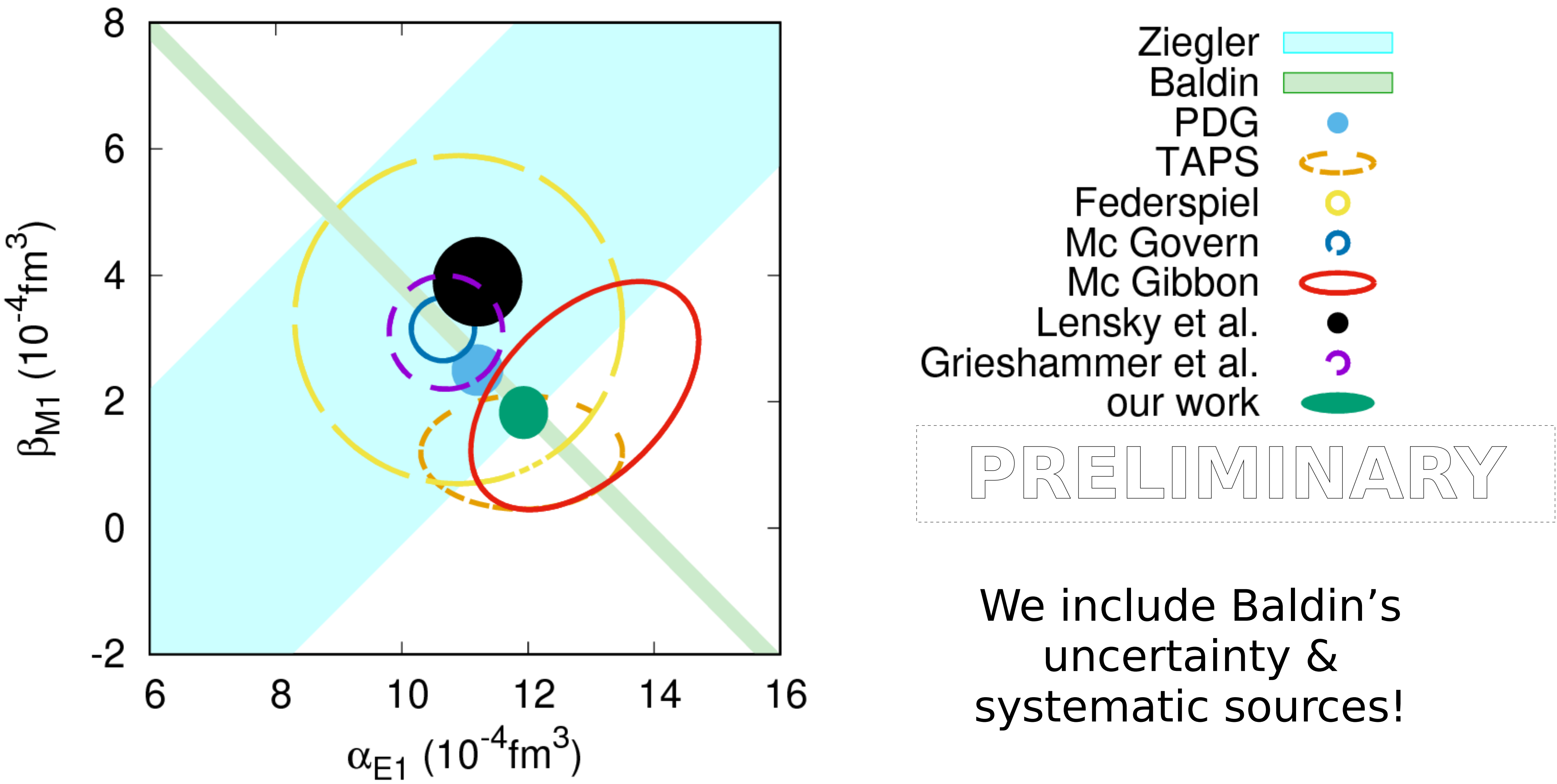
Systematical errors enlarge the error band of polarizabilities!

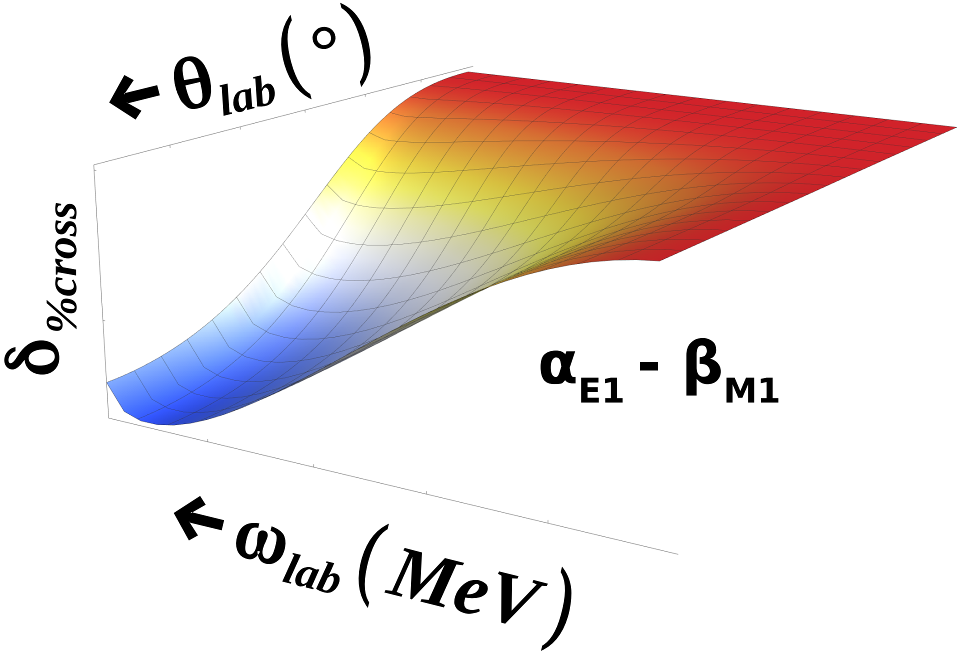
Summary plot

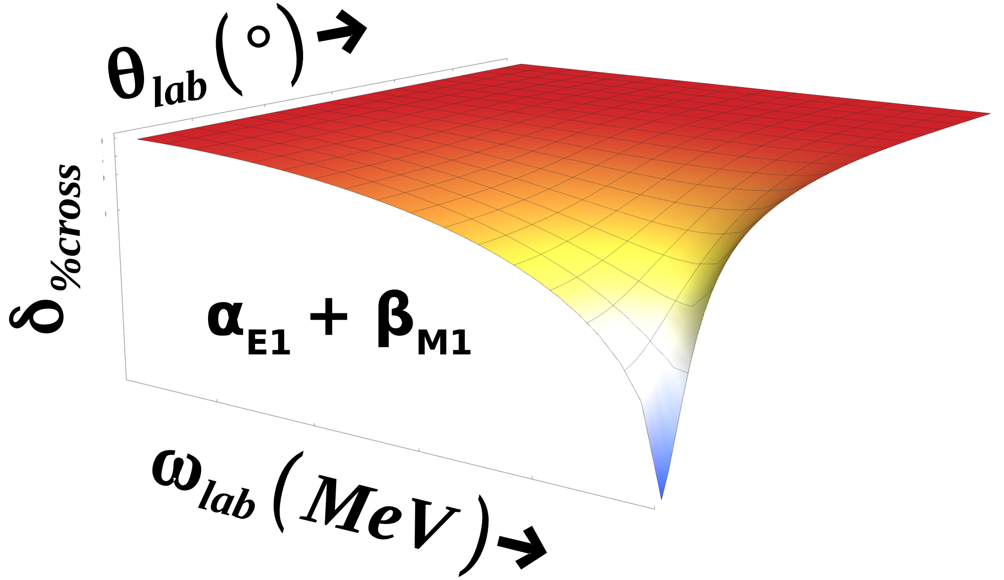


We include Baldin's uncertainty & systematic sources!

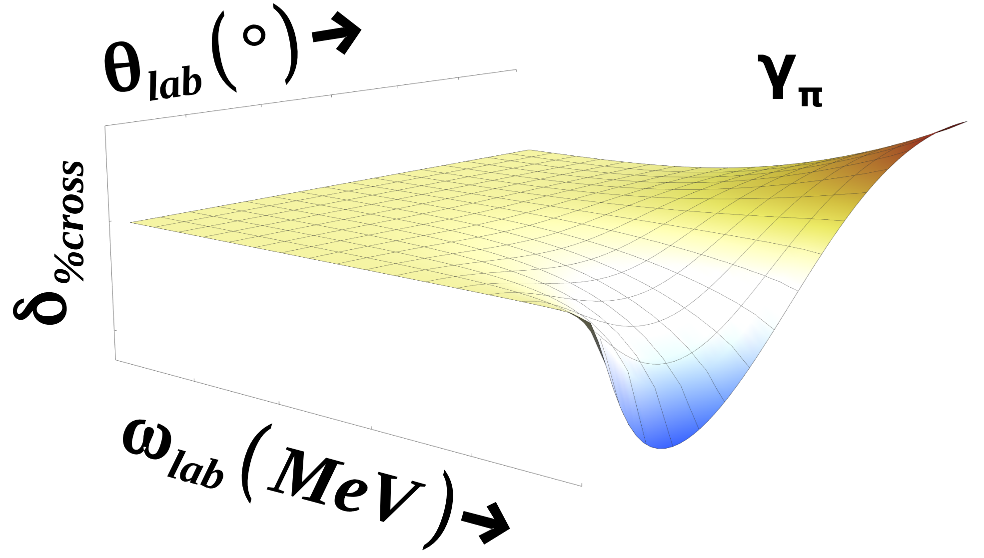
F. Hagelstein, R. Miskimen, V. Pascalutsa, Prog.Part.Nucl.Phys. 88 (2016) 29-97

Unpolarized differential cross section: sensitivity plots

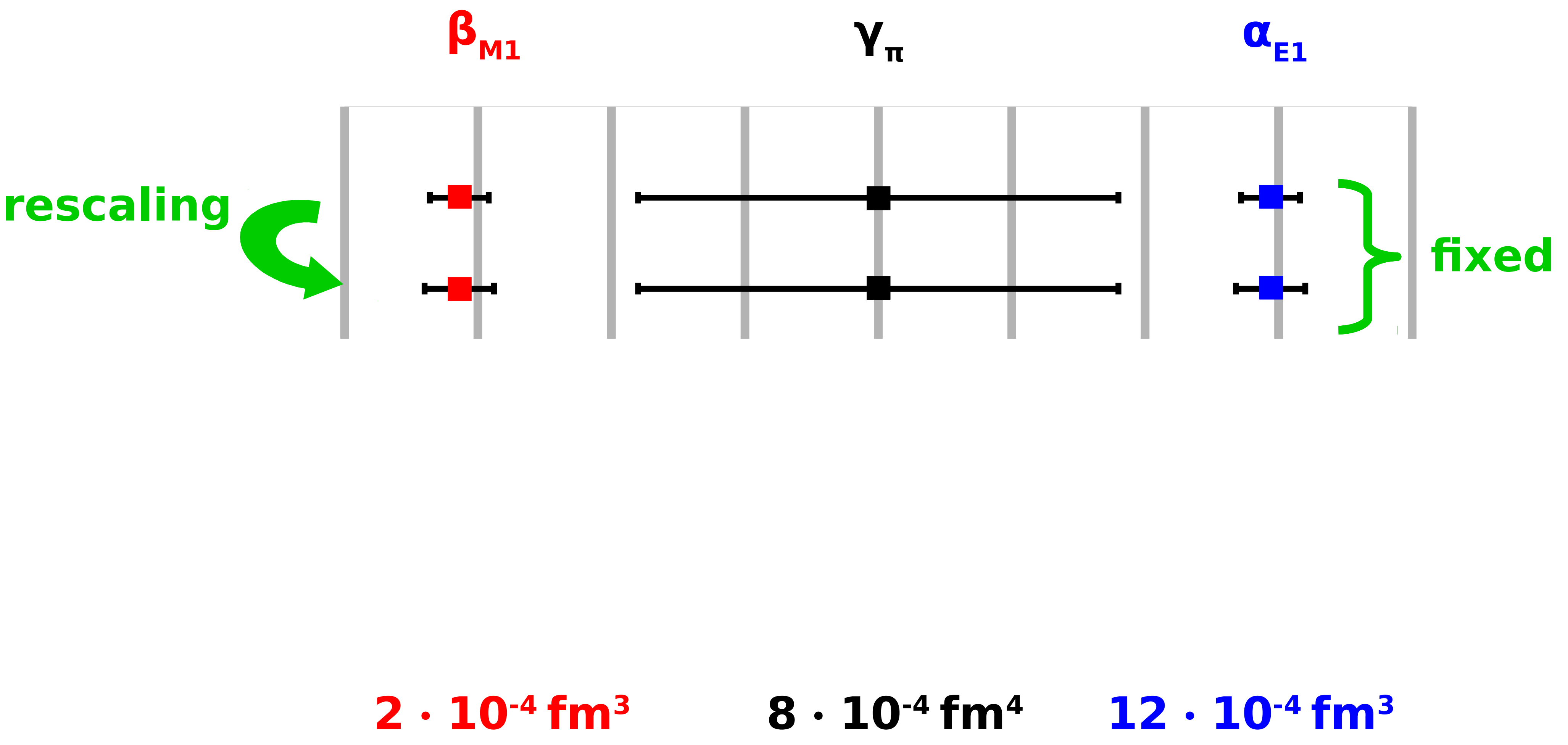




% variation of the observable when the particular polarizability is changed by a factor ± 10%



Static fit: γ_π free parameter



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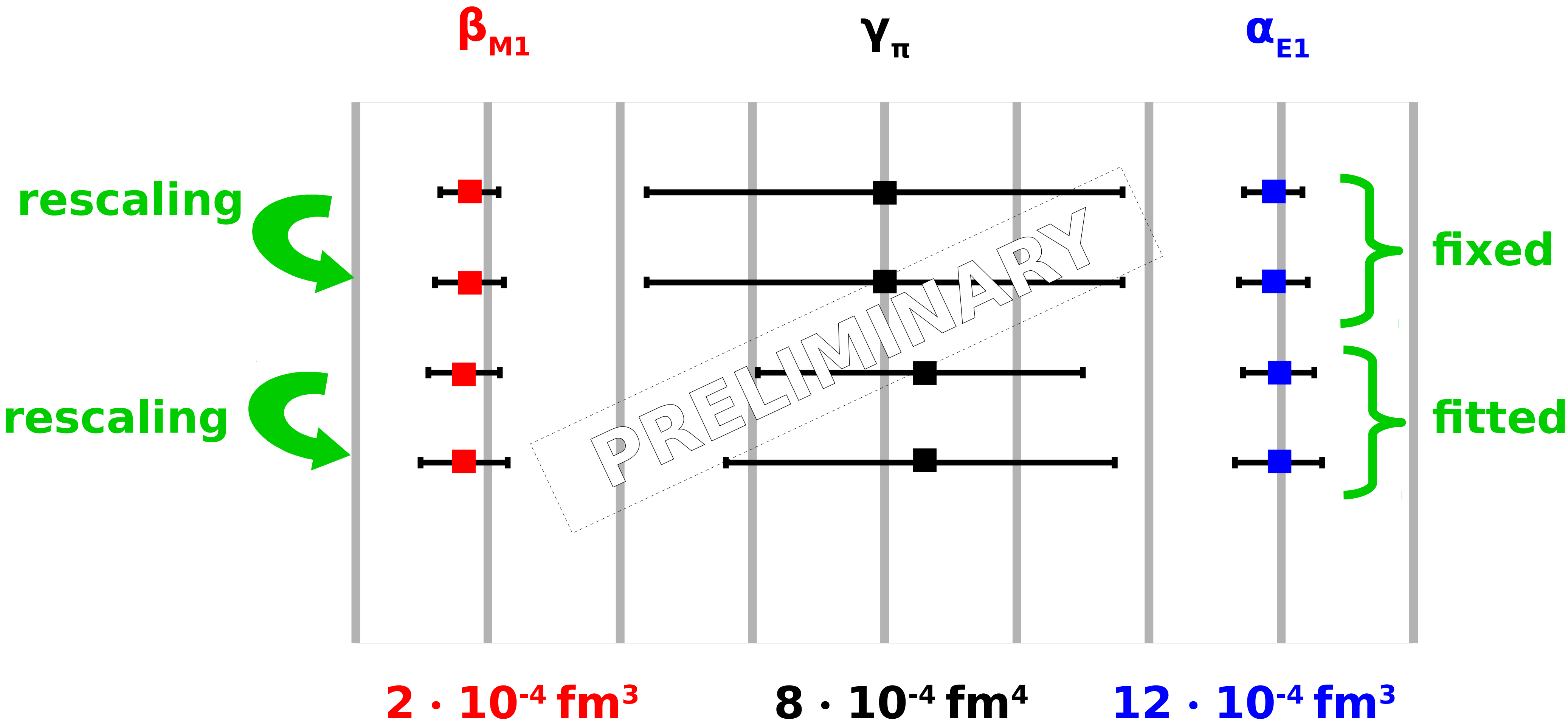

Central values and uncertainties are almost the same!

DIPOLE DYNAMICAL POLARIZABILITIES

Fit conditions

- ✓ Baldin’s sum rule
- ✓ Systematical errors ON
- ✓ FULL data set (150 data)
- ✓ TAPS data set (55 data)
- ✓ Errors on Baldin’s sum rule and γ_π included in the procedure

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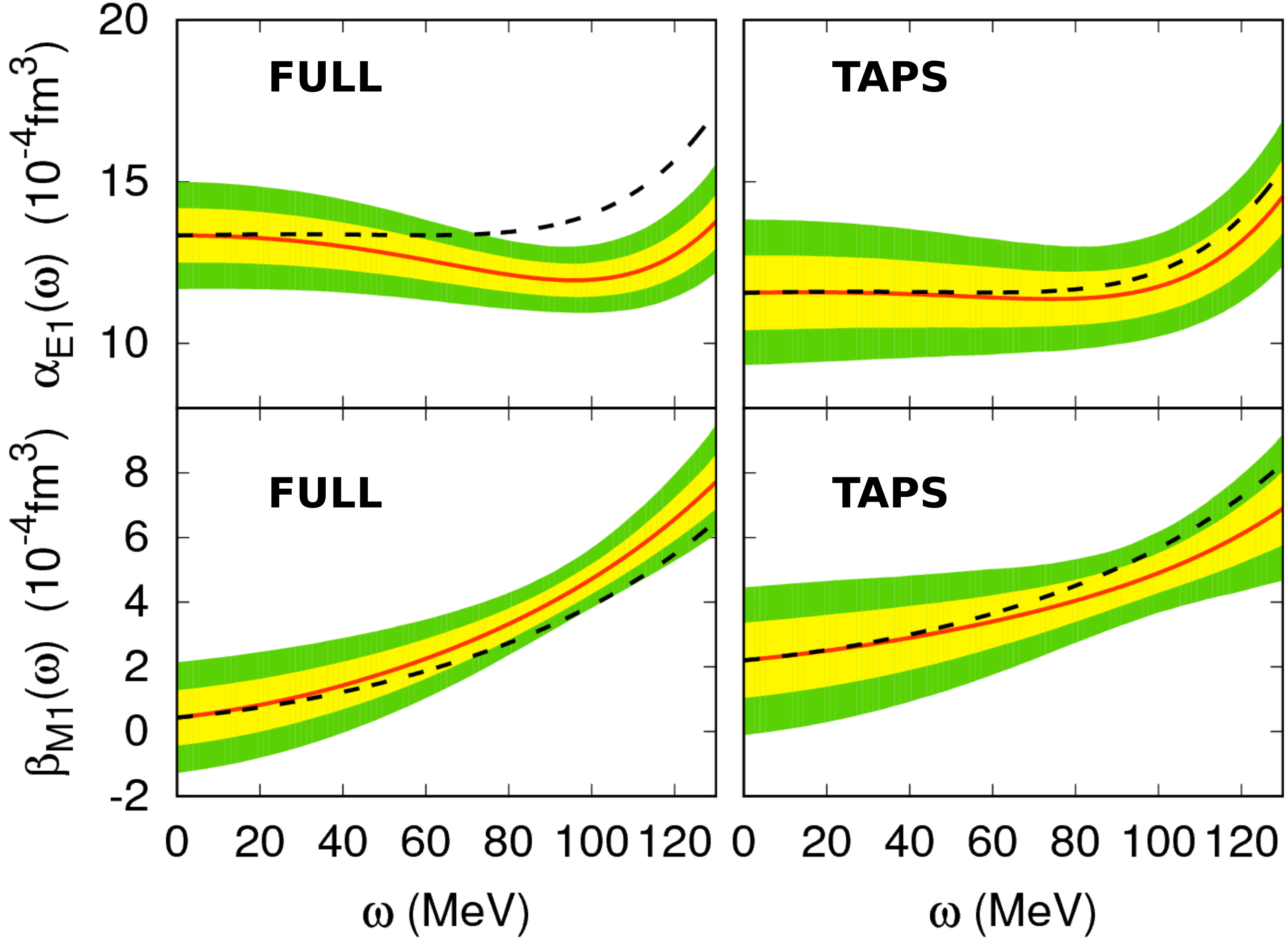
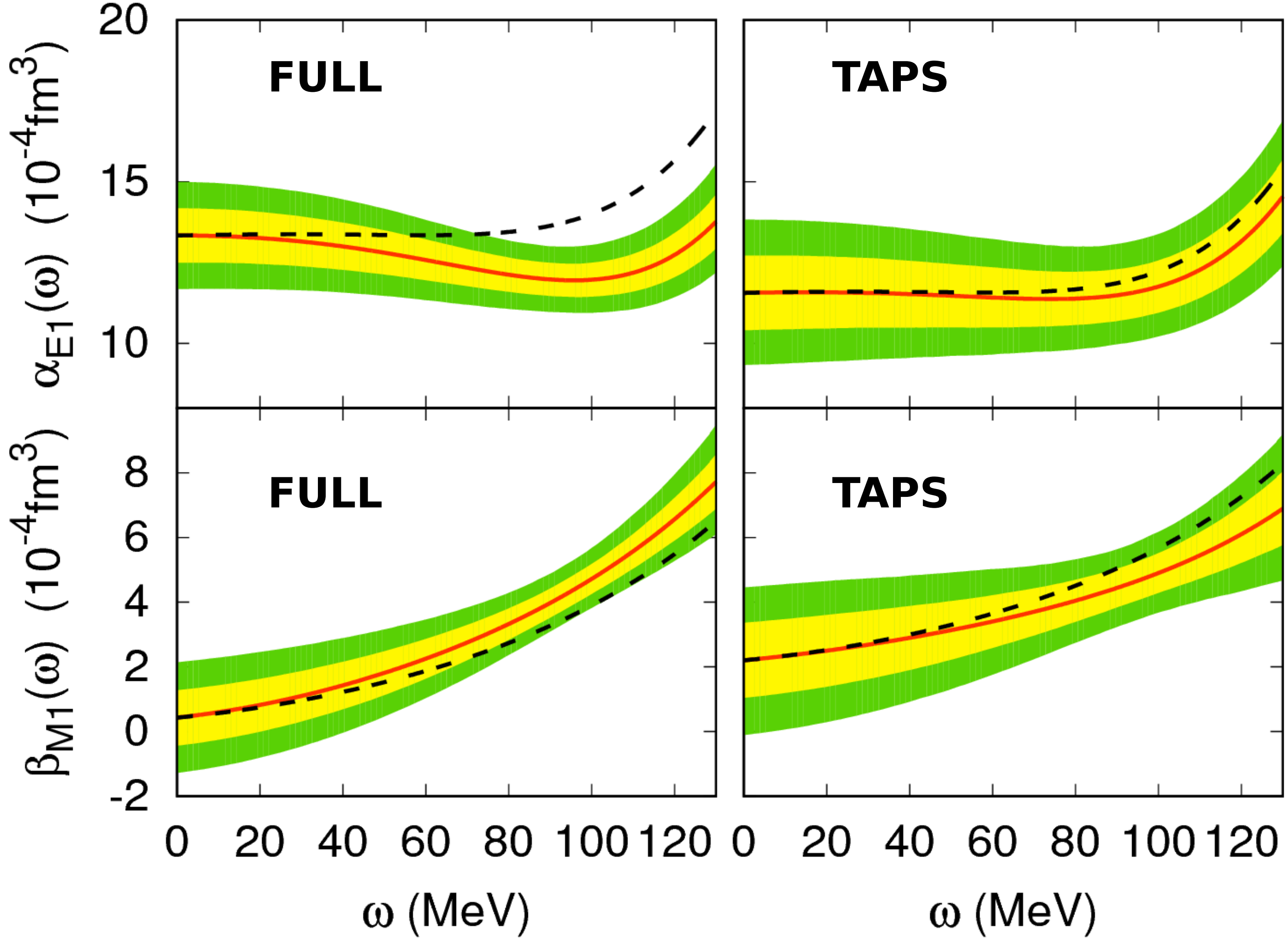
A. M. Baldin, Nucl. Phys. 18, 310 (1960)
V. Olmos de León, Eur. Phys. J. A 10, 207–215 (2001)

DDPs from the fit



DDPs from the fit



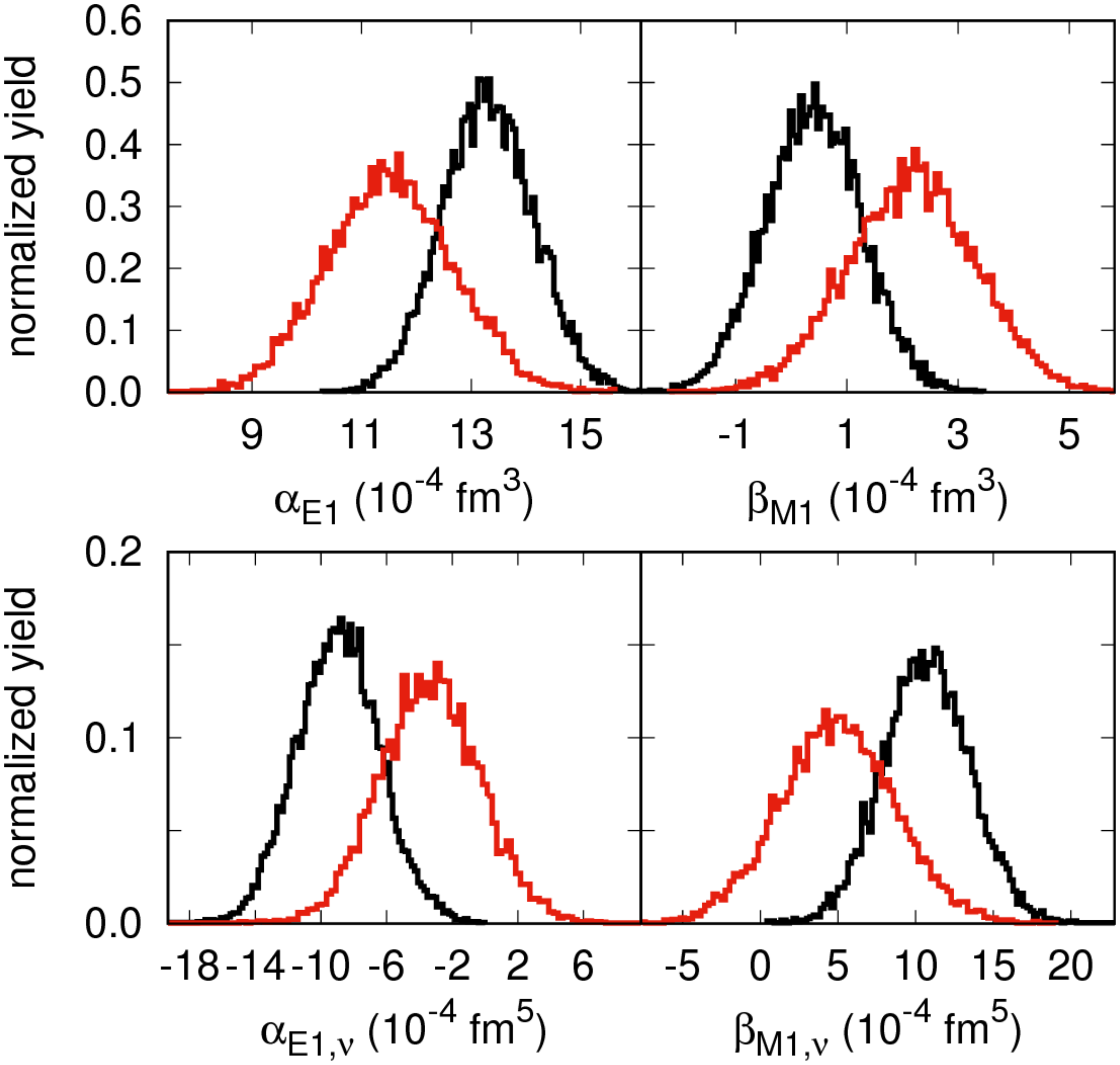
From FIT

DRs calculation

68% CL band

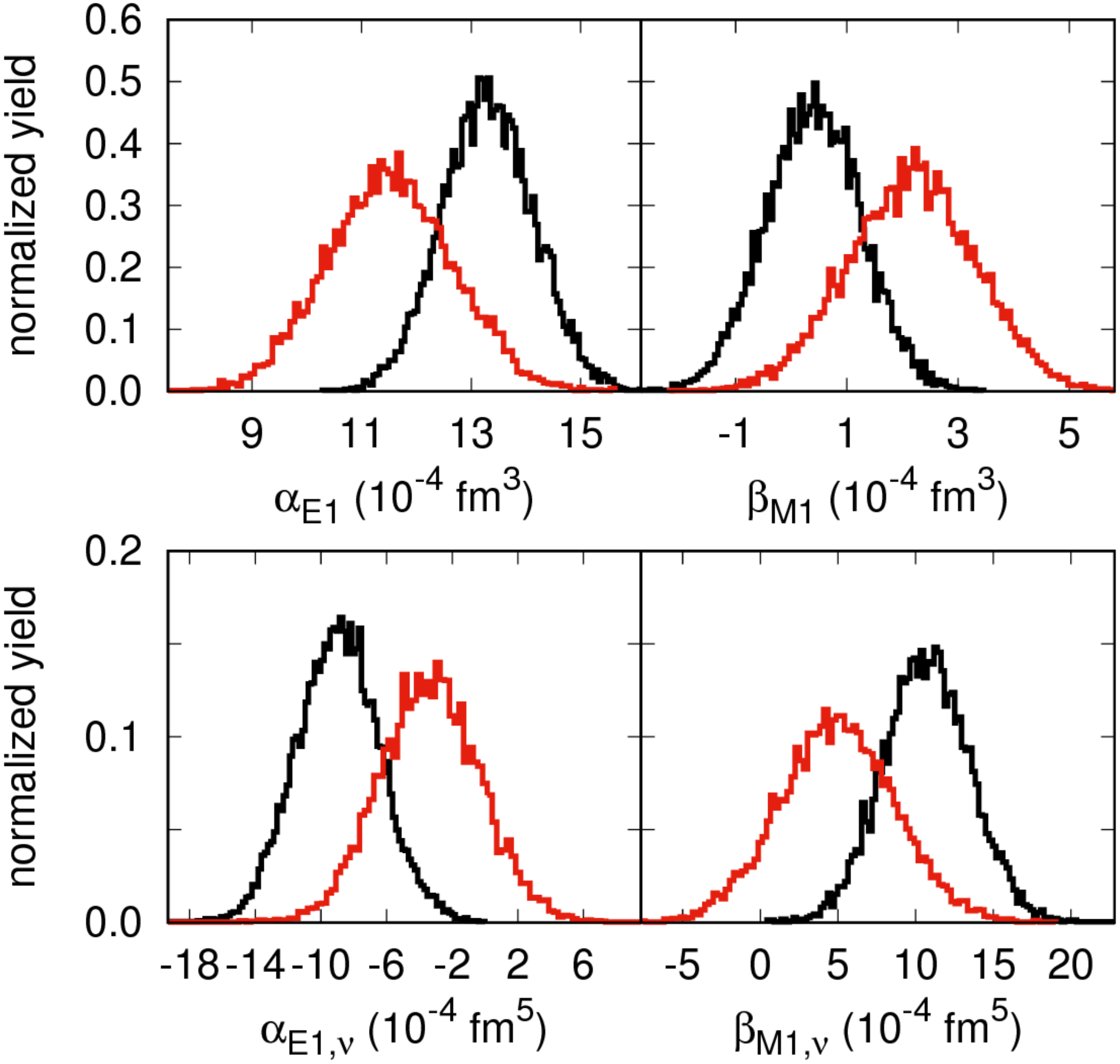
95 % CL band

DDPs from the fit: probability distributions



B. Pasquini, P. Pedroni, S. Sconfietti, Phys. Rev. C 98, 015204 (2018)

DDPs from the fit: probability distributions



TAPS data set

FULL data set

Probability distributions given by our technique (**not** assumed a priori)

Global results: numerical values

		FULL	TAPS
α_{E1}	$(10^{-4}\mathrm{fm}^3)$	13.3 ± 0.8	11.6 ± 1.1
$\alpha_{E1,\nu}$	$(10^{-4}\mathrm{fm}^5)$	-8.8 ± 2.5	-3.2 ± 3.1
β_{M1}	$(10^{-4}\mathrm{fm}^3)$	0.4 ∓ 0.9	2.2 ∓ 1.1
$\beta_{M1,\nu}$	$(10^{-4}\mathrm{fm}^5)$	10.8 ± 2.8	5.1 ± 3.7

Very STRONG dependence on data set (maybe due to different angular regions...)

B. Pasquini, P. Pedroni, S. Sconfietti, Phys. Rev. C 98, 015204 (2018)

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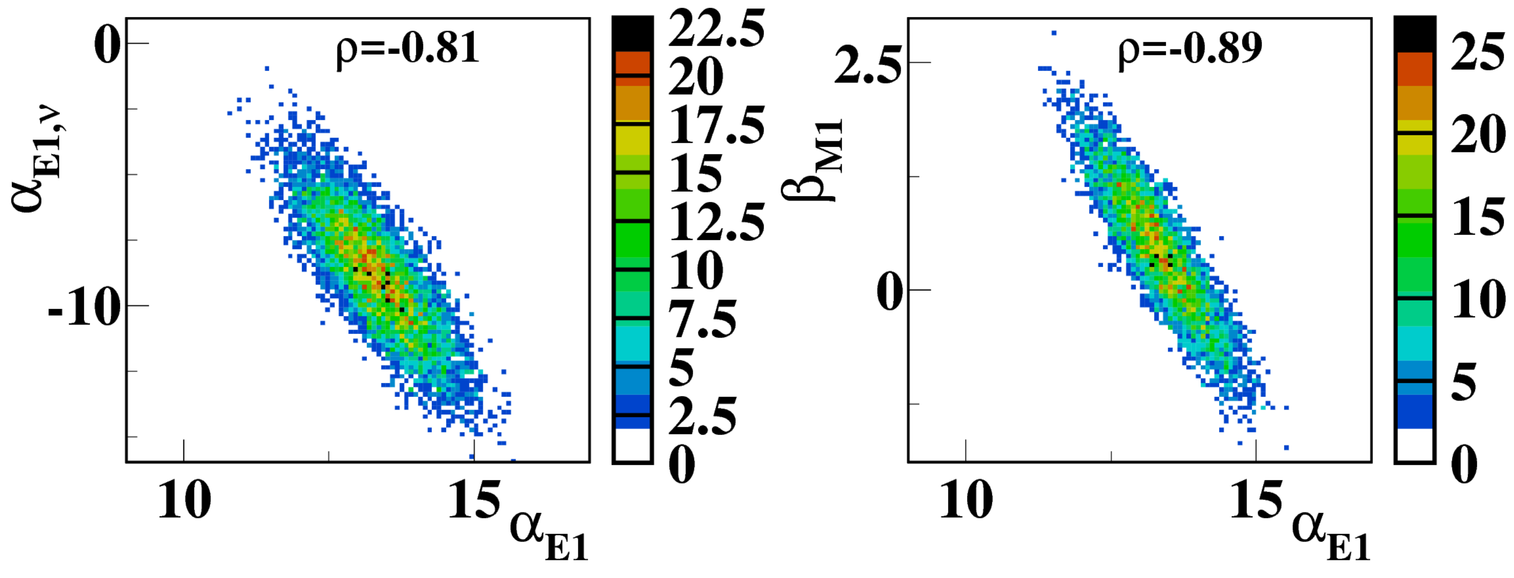
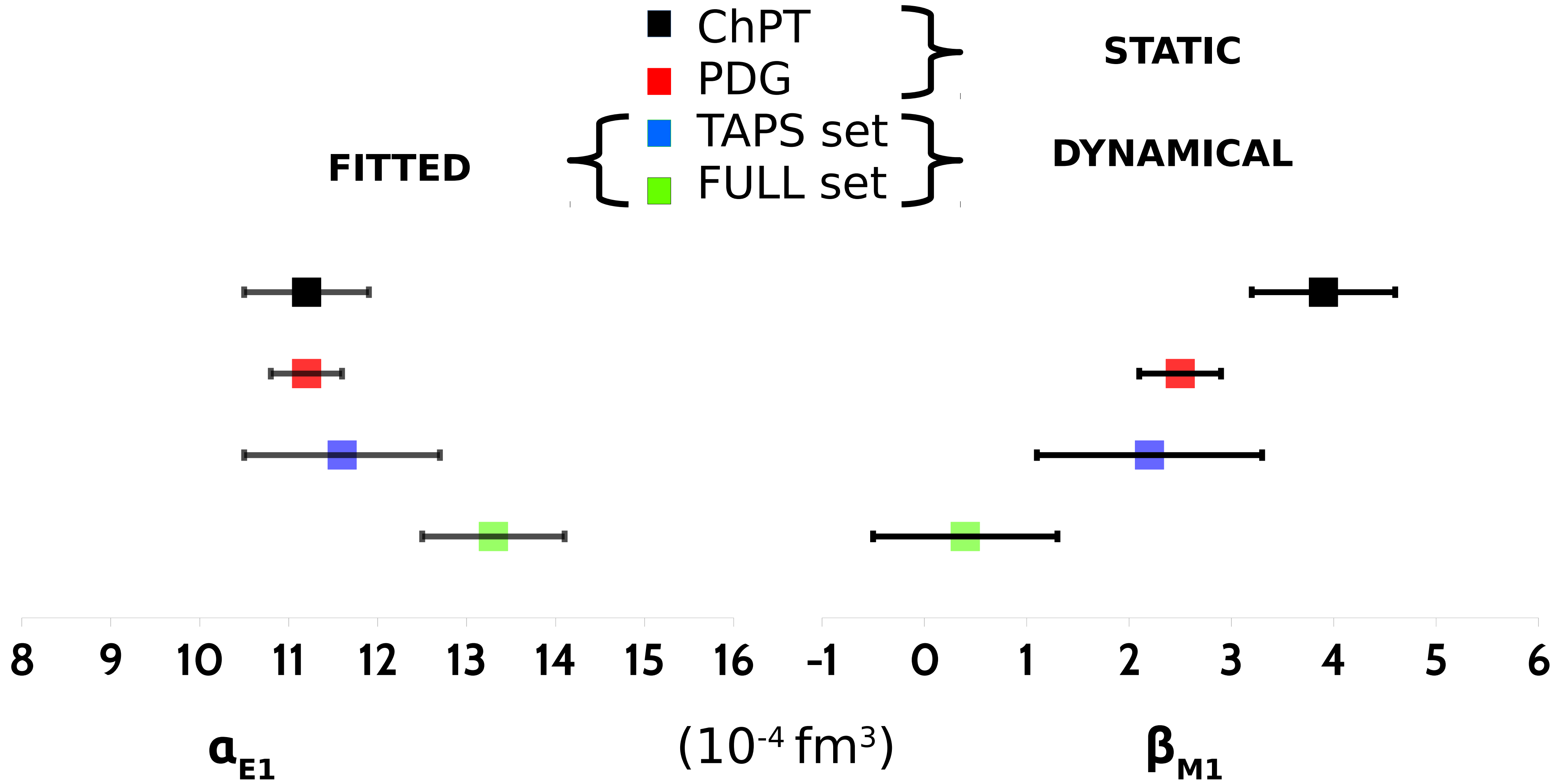
Very STRONG dependence on data set (maybe due to different angular regions...)

Very HIGH correlations among parameters

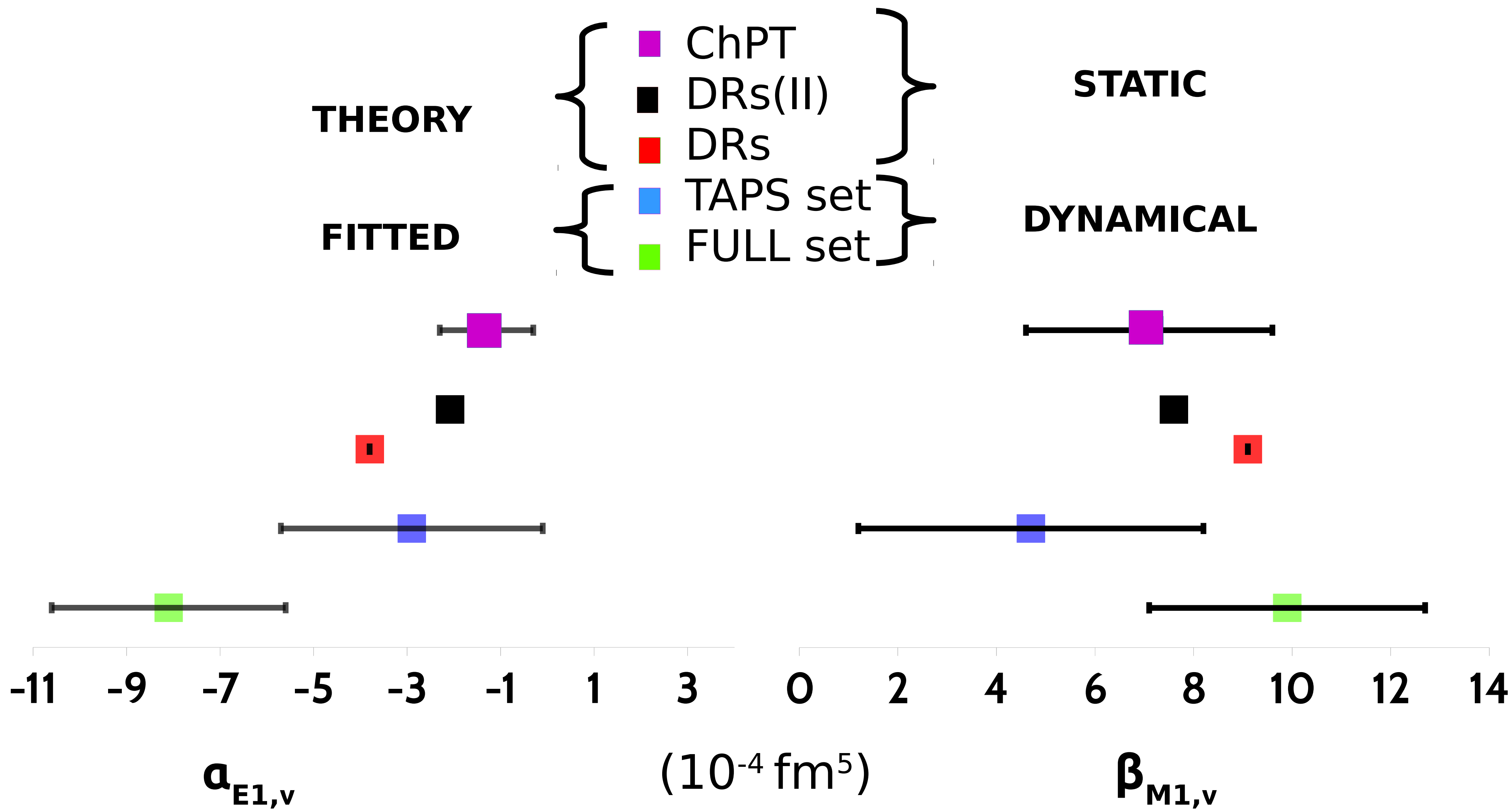


Global results: α_{E1} & β_{M1}



PDG: C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update
ChPt: V. Lensky, J. McGovern, and V. Pascalutsa, Eur. Phys. J. C75, 604 (2015)
B. Pasquini, P. Pedroni, S. Sconfietti, Phys. Rev. C 98, 015204 (2018)

Global results: $\alpha_{E1,\nu}$ & $\beta_{M1,\nu}$



DRs (I): B. R. Holstein, D. Drechsel, B. Pasquini, M. Vanderhaeghen. Phys.Rev. C61 (2000) 034316
DRs (II): B. Pasquini, D. Drechsel, M. Vanderhaeghen. Phys.Rev. C76 (2007) 015203
ChPt: V. Lensky, J. McGovern, and V. Pascalutsa, Eur. Phys. J. C75, 604 (2015)
B. Pasquini, P. Pedroni, S. Sconfietti, Phys. Rev. C 98, 015204 (2018)

Problems with the data set (what we could do)

Very strong dependence of polarizabilities on the specific data set!

Outliers → rescaling of all the statistic uncertainties by a factor

$$\sqrt{\chi^2}$$

O. Behnke, K. Kröninger, G. Schott and T. Schörner-Sadenius, *Data Analysis in High Energy Physics: A Practical Guide to Statistical Methods*, Wiley-VCH, 2013
N. Krupina, V. Lensky, V. Pascalutsa, Phys.Lett. B782 (2018) 34

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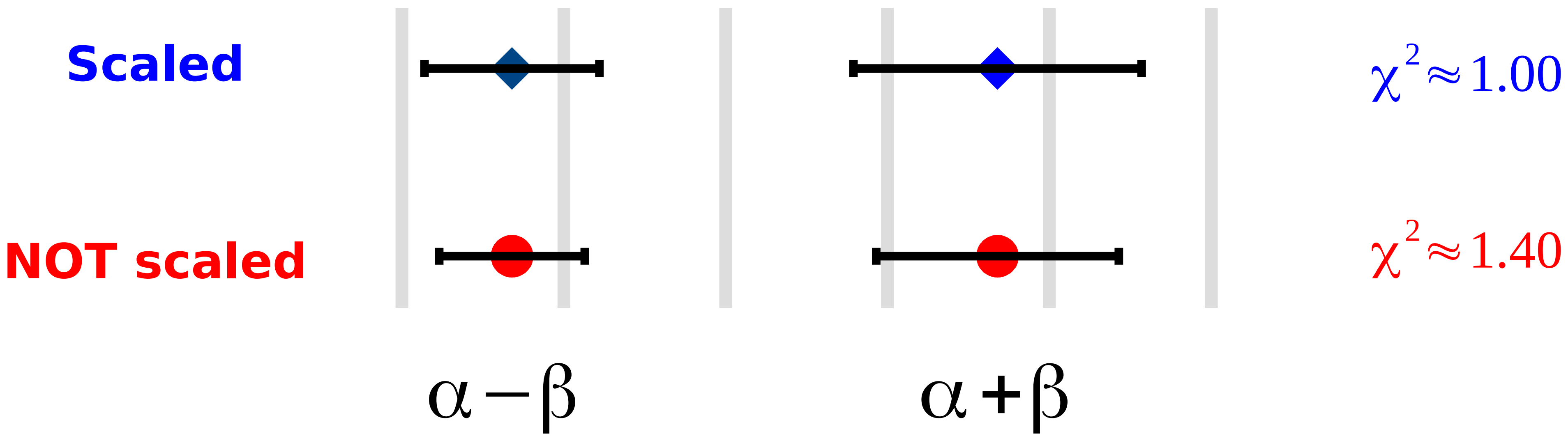

Effect: enlarging of errors on fitted parameters (~20%)

O. Behnke, K. Kröninger, G. Schott and T. Schörner-Sadenius, *Data Analysis in High Energy Physics: A Practical Guide to Statistical Methods*, Wiley-VCH, 2013
N. Krupina, V. Lensky, V. Pascalutsa, Phys.Lett. B782 (2018) 34

Conclusions & perspectives

Very useful and versatile technique for data analysis

Effect of systematic sources of uncertainties on the fitted parameters

Waiting for new data in order to reduce the uncertainties of the fitted parameters (MAMI)

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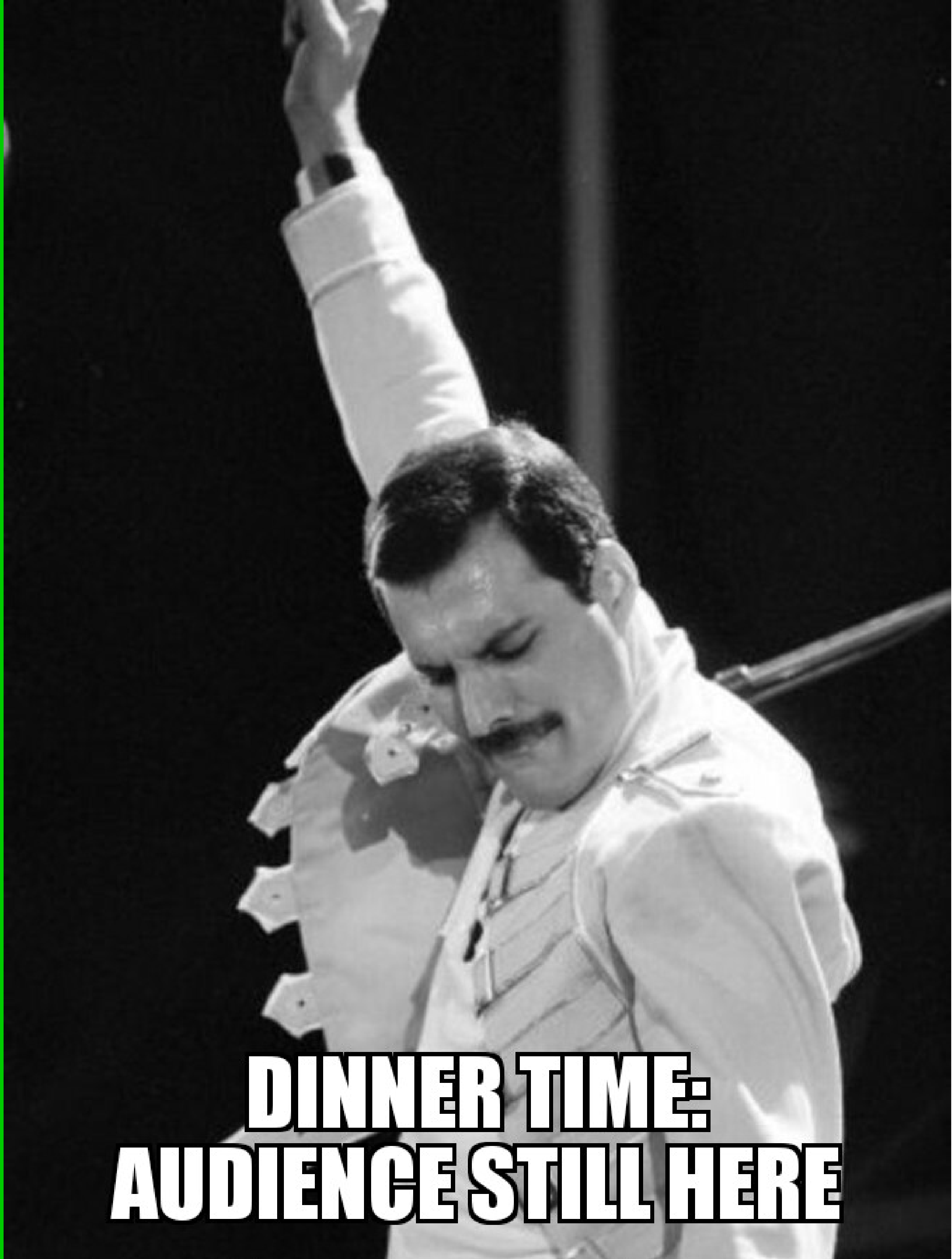
DDPs without LEX (double subtraction in DRs)

Fit of polarized observables in RCS with the same technique

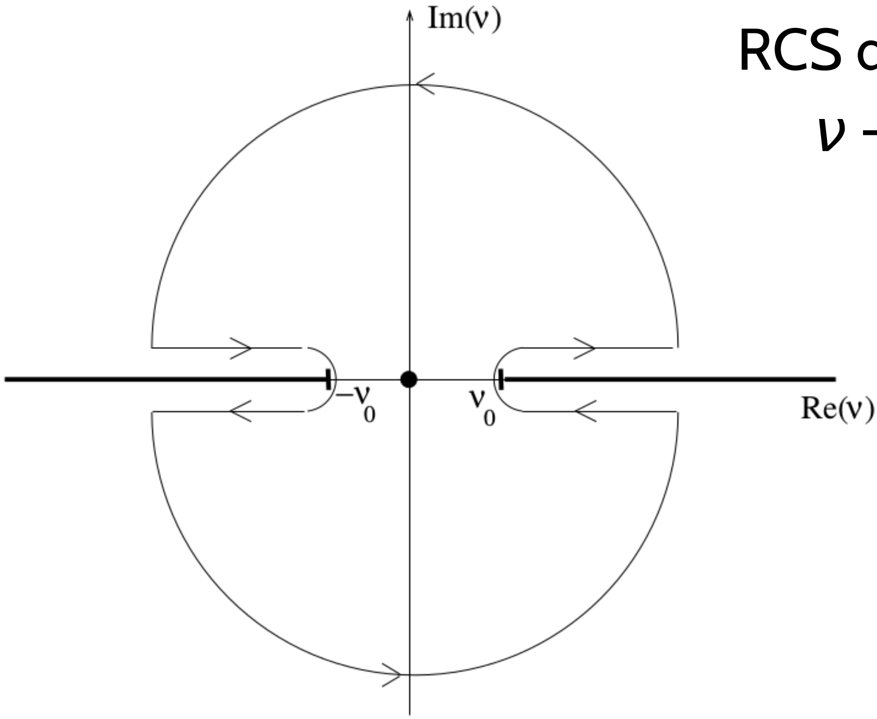
E. J. Downie et al., Proposal MAMI-A2/04-16 (2016)

Thank you!



Backup slides

Dispersion relations and RCS (I)



RCS differential cross section → 6 amplitudes A_i

ν → energy $\qquad$ t → transferred momentum

$$A_i(\nu,t) = A_i^B(\nu,t) + \int_{\nu_{thr}}^{\nu_{MAX}} \ldots + \int_{\cap} \ldots$$

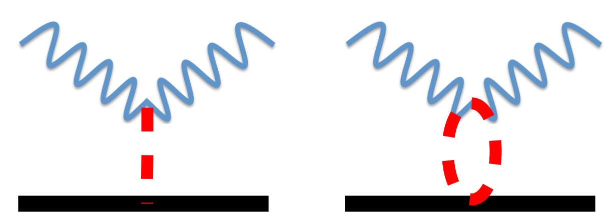
For i=3,…,6: “good” behavior

$$A_i(\nu,t) = A_i^B(\nu,t) + \int_{\nu_{thr}}^{\infty} \ldots + 0$$

For i=1,2: “bad” behavior

$$A_i(\nu,t) = A_i^B(\nu,t) + \int_{\nu_{thr}}^{\nu_{MAX}} \ldots + A_i^{AS}$$

Asymptotic contribution → meson exchange

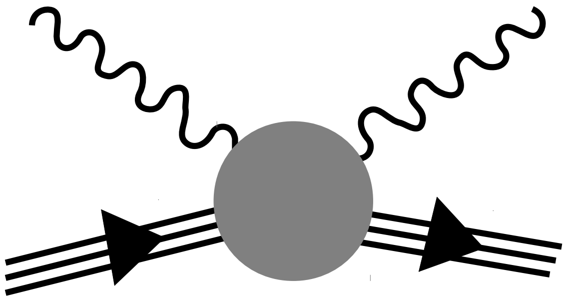


Contour behavior: that’s the problem! → faster convergence is needed…

SUBTRACTED DISPERSION RELATIONS

B. Pasquini , D. Drechsel M. Vanderhaeghen, Phys.Rev. C76 (2007) 015203
B. Pasquini, M. Vanderhaeghen, Ann.Rev.Nucl.Part.Sci. 68 (2018) 75-103

Dispersion relations and RCS (II)



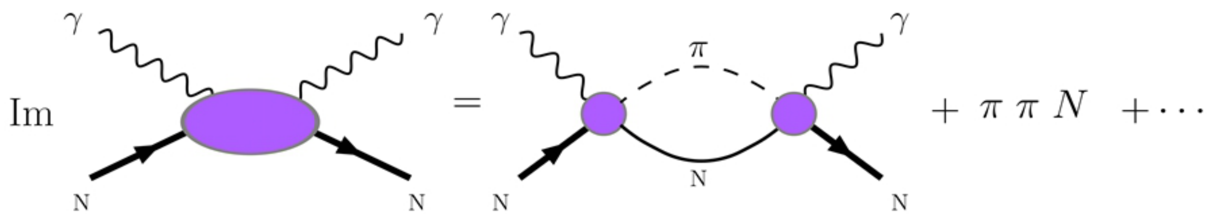
$A_i(0,0) = a_i$ → **Static polarizabilities**

$$A_i(\nu,t) = A_i^s(\nu,0) + A_i^t(0,t) + A_i(0,0)$$

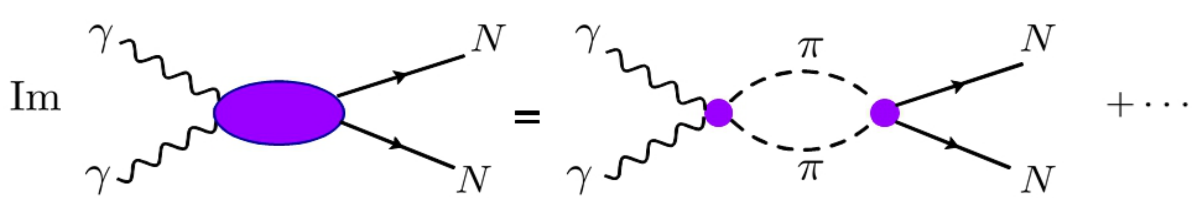
Subtracted Dispersion Relations (*s*-channel)

$$A_i^s(\nu,0) \;=\; \frac{2}{\pi}\,\nu^2\,\mathrm{P}\int_{\nu_{thr}}^{\infty}\,\mathrm{Im_s}\,A_i(\nu',t)\,\frac{d\nu'}{\nu'(\nu'^2-\nu^2)}$$

s-CHANNEL



t-CHANNEL



D. Drechsel, M. Gorchtein, B. Pasquini, M. Vanderhaeghen, Phys.Rev. C61 (1999) 015204
B. Pasquini , D. Drechsel M. Vanderhaeghen, Phys.Rev. C76 (2007) 015203
B. Pasquini, M. Vanderhaeghen, Ann.Rev.Nucl.Part.Sci. 68 (2018) 75-103

Multipoles expansion and DYNAMICAL polarizabilities

$$R_1 = \sum_{l\geq 1}\{[(l+1)f_{EE}^{l+} + l f_{EE}^{l-}](lP_l' + P_{l-1}'') - [(l+1)f_{MM}^{l+} + l f_{MM}^{l-}]P_l''\}$$

$$R_2 = \sum_{l\geq 1}\left\{[(l+1)f_{MM}^{l+} + l f_{MM}^{l-}](lP_l' + P_{l-1}'') - [(l+1)f_{EE}^{l+} + l f_{EE}^{l-}]P_l''\right\}$$

DYNAMICAL POLARIZABILITIES

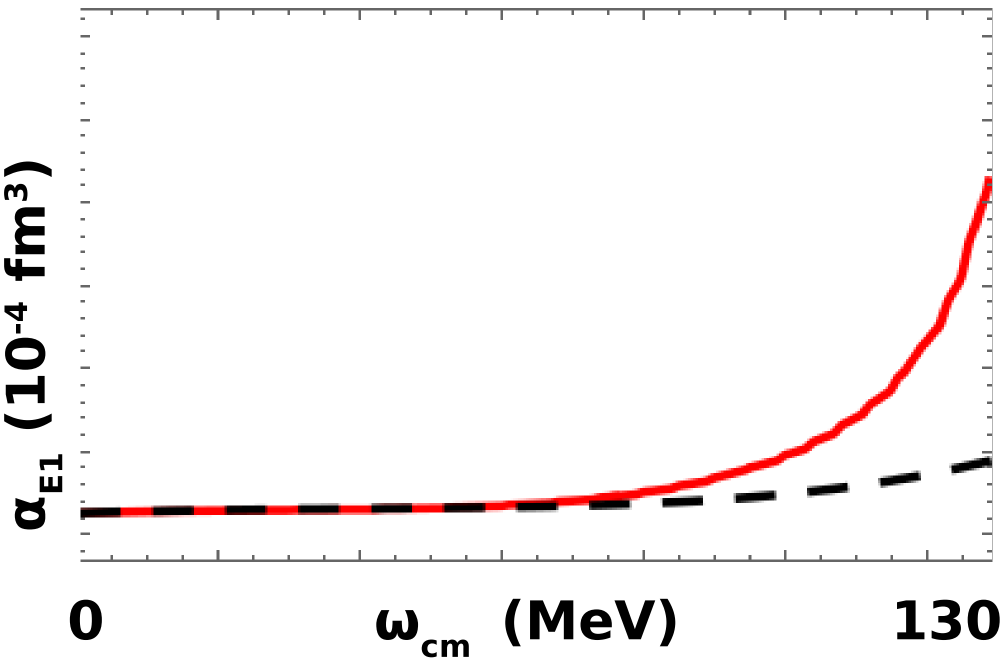
$$\alpha_{El} = a(l)\frac{(l+1)f_{EE}^{l+} + l f_{EE}^{l-}}{\omega^{2l}} \quad \beta_{Ml} = a(l)\frac{(l+1)f_{MM}^{l+} + l f_{MM}^{l-}}{\omega^{2l}}$$

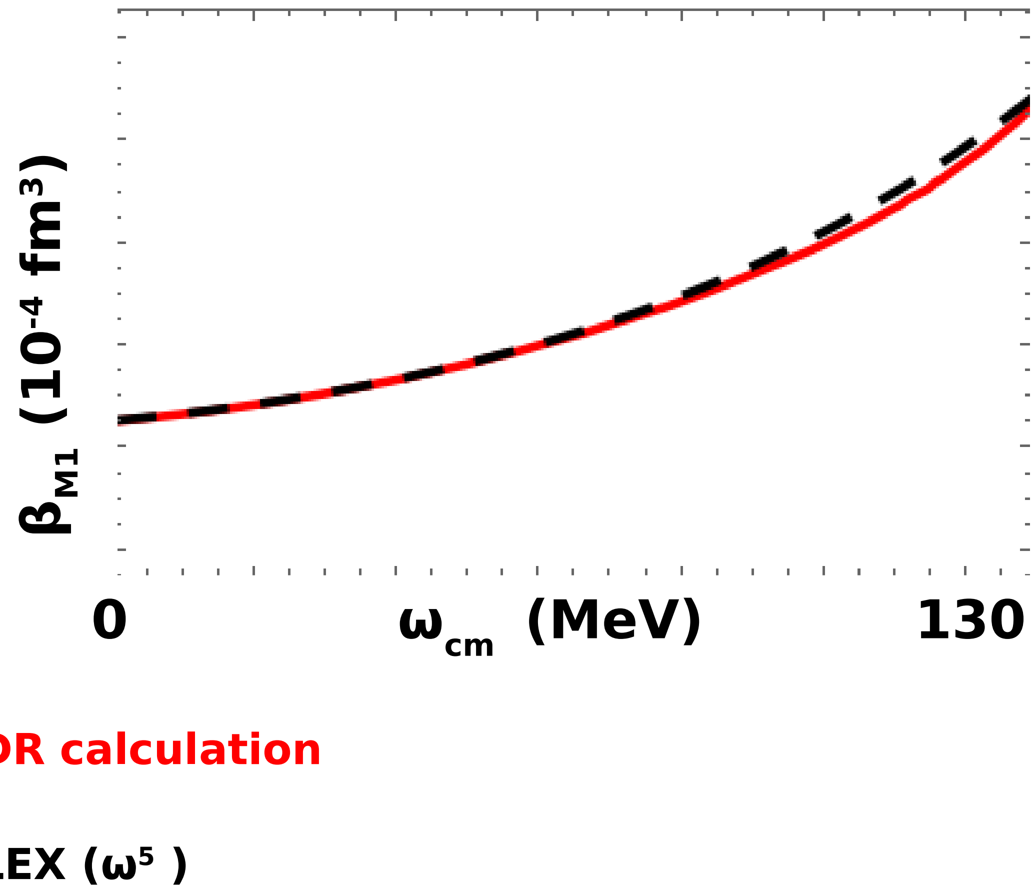
DIPOLE DYNAMICAL POLARIZABILITIES
(DDPs)

$\alpha_{E1}(\omega)$ $\beta_{M1}(\omega)$

D. Babusci, G. Giordano, A.I. L'vov, G. Matone, A.M. Nathan.Phys.Rev. C58 (1998) 1013-1041
V. Lensky, J. McGovern, and V. Pascalutsa, Eur. Phys. J. C75, 604 (2015)

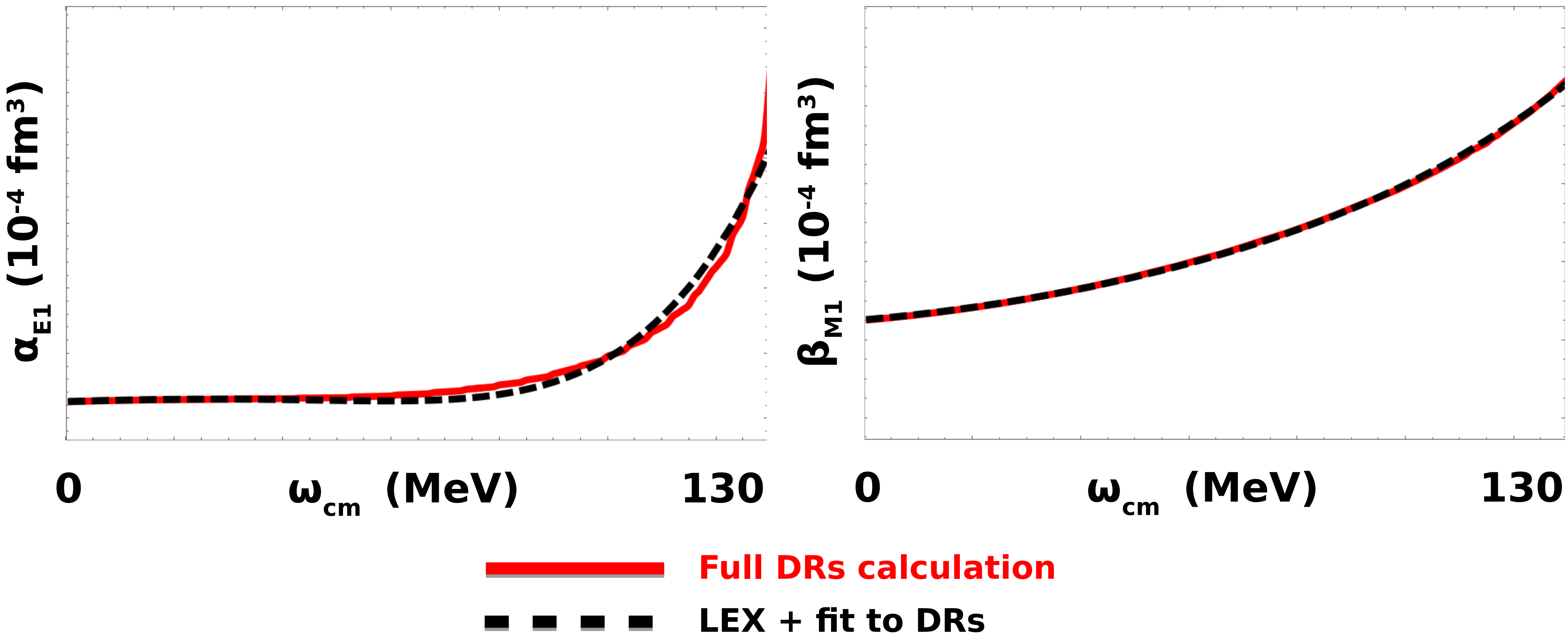
LEX is *very* low …





LEX + residual functions

$$DDP(\omega) = DDP_{LEX}(\omega) + f_R(\omega)$$



Some comments on the data set

Strong correlation between parameters

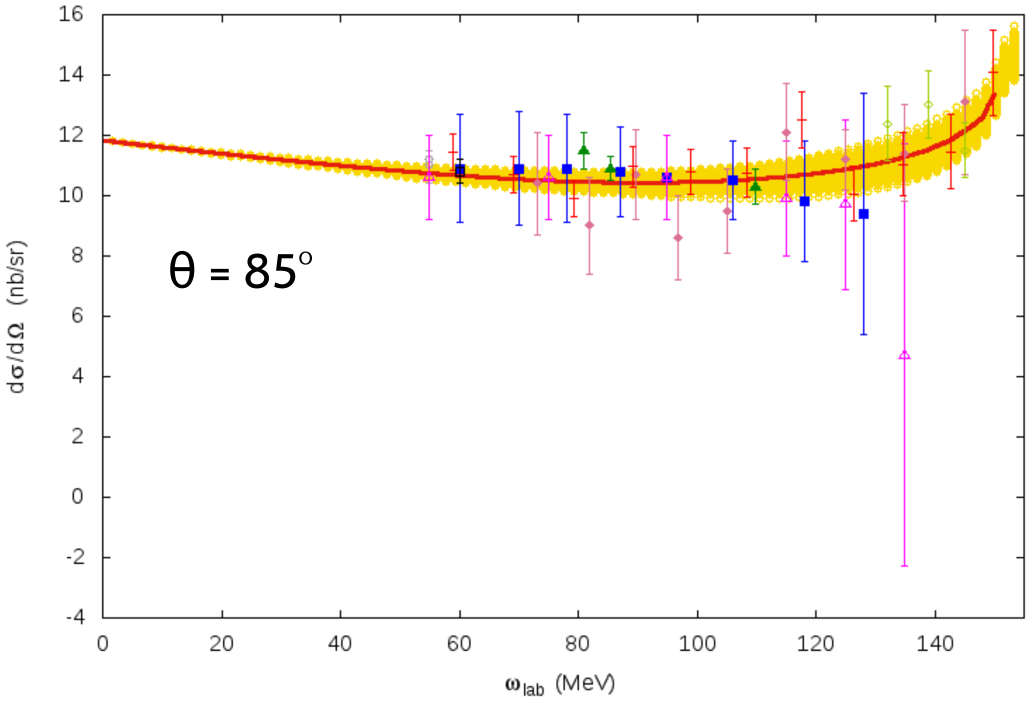
Reduction of parameters number thanks to sum rules

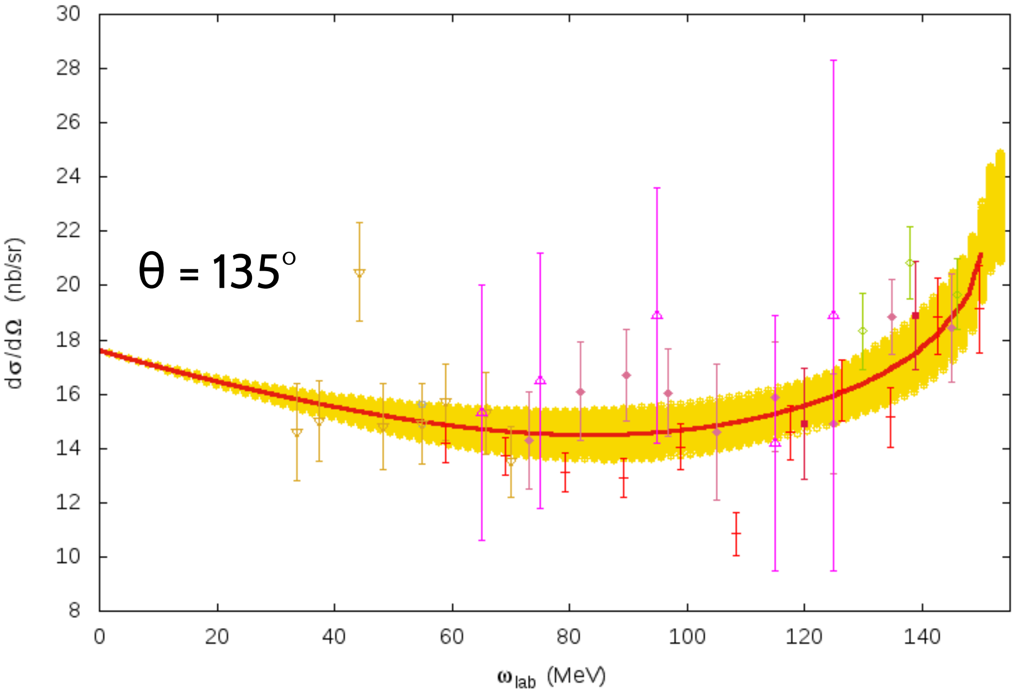
Identifications of the *outliers* (rescaling for statistic errors?)

The χ^2 is not the only *quality indicator* → no “definition” of data set

Waiting for new data (MAMI)

Differential cross section

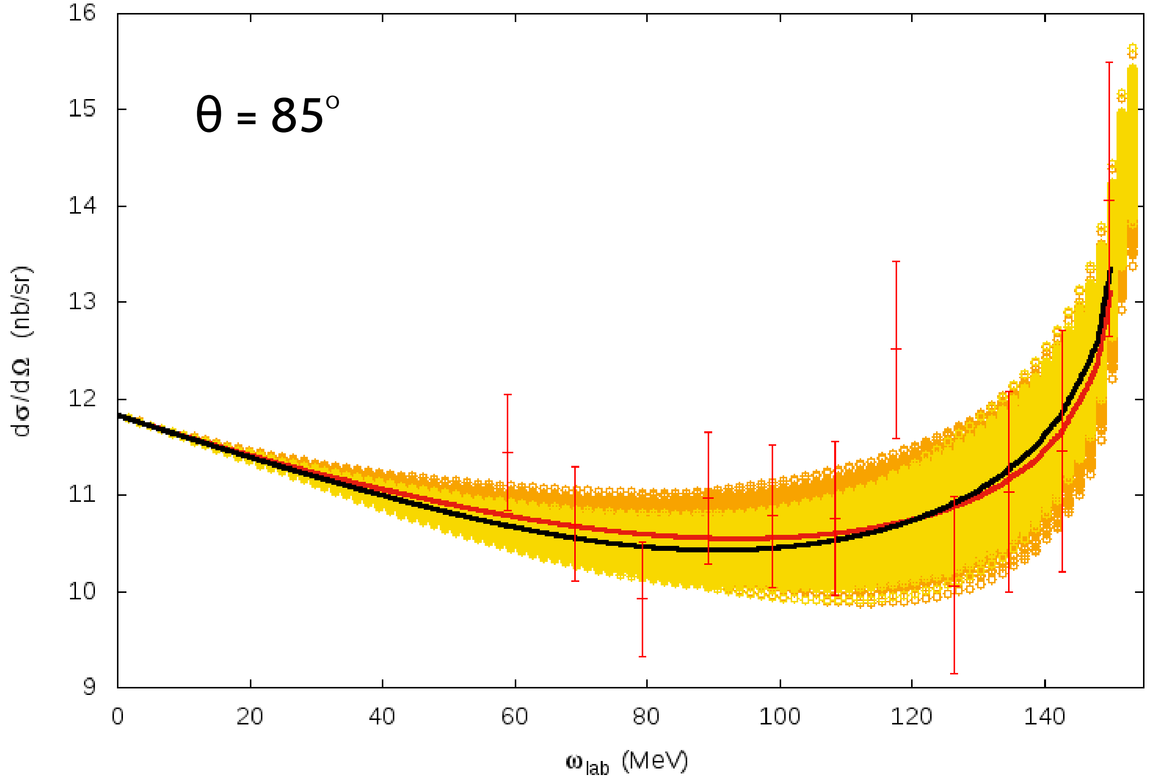




dσ/dΩ VS lab energy

100% error band from the bootstrap fit

TAPS vs FULL data set



TAPS

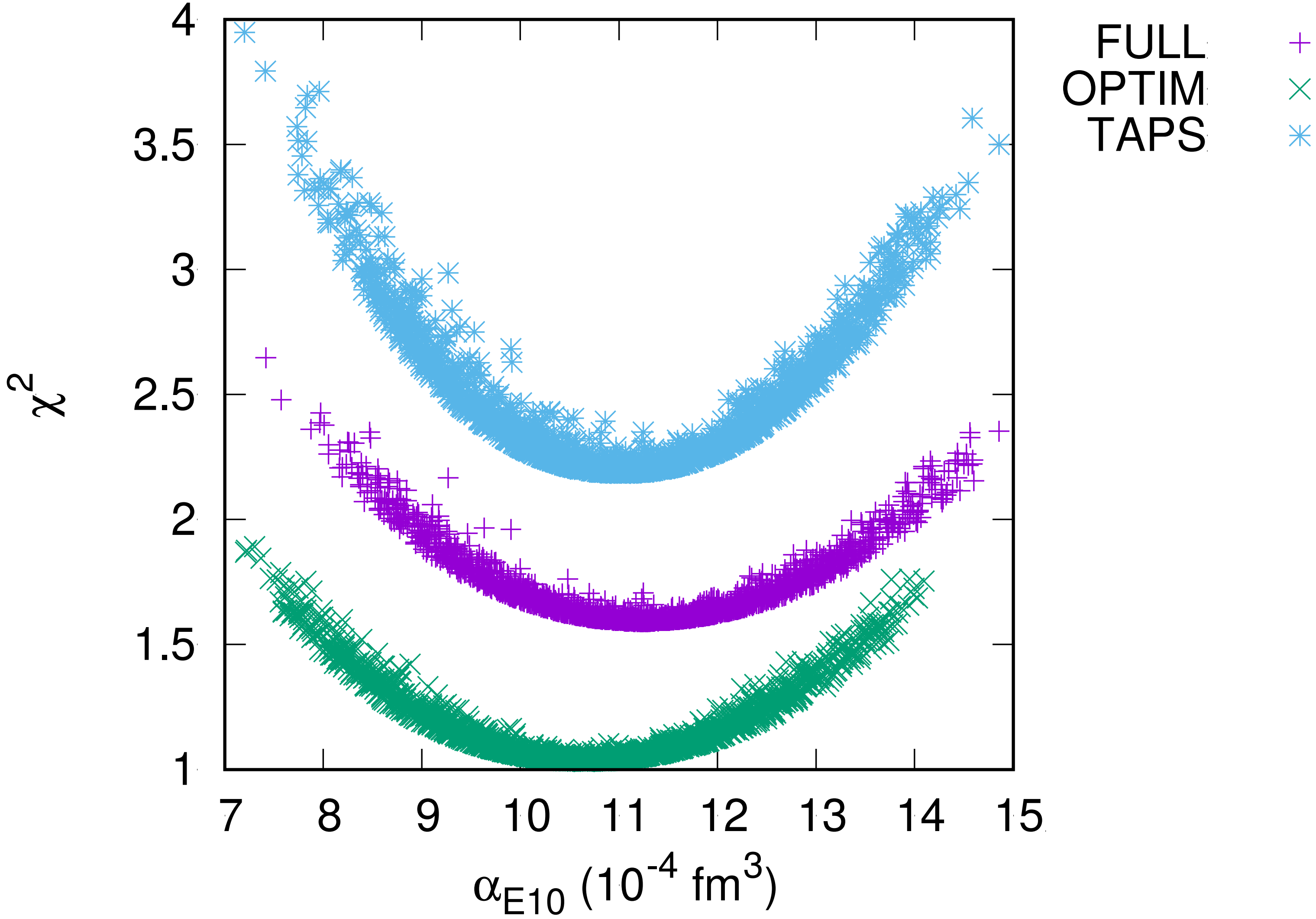
FULL

TAPS 100% error band

FULL 100% error band

VERY small difference both in calculation and in error band

χ^2 curvature close to its minimum



H. W. Grießhammer, J. A. McGovern, D. R. Phillips, and G. Feldman,Prog. Part. Nucl. Phys. 67, 841 (2012).