PROBING OF CHARMONIUM-LIKE EXOTICS WITH HADRON AND HEAVY ION COLLISIONS

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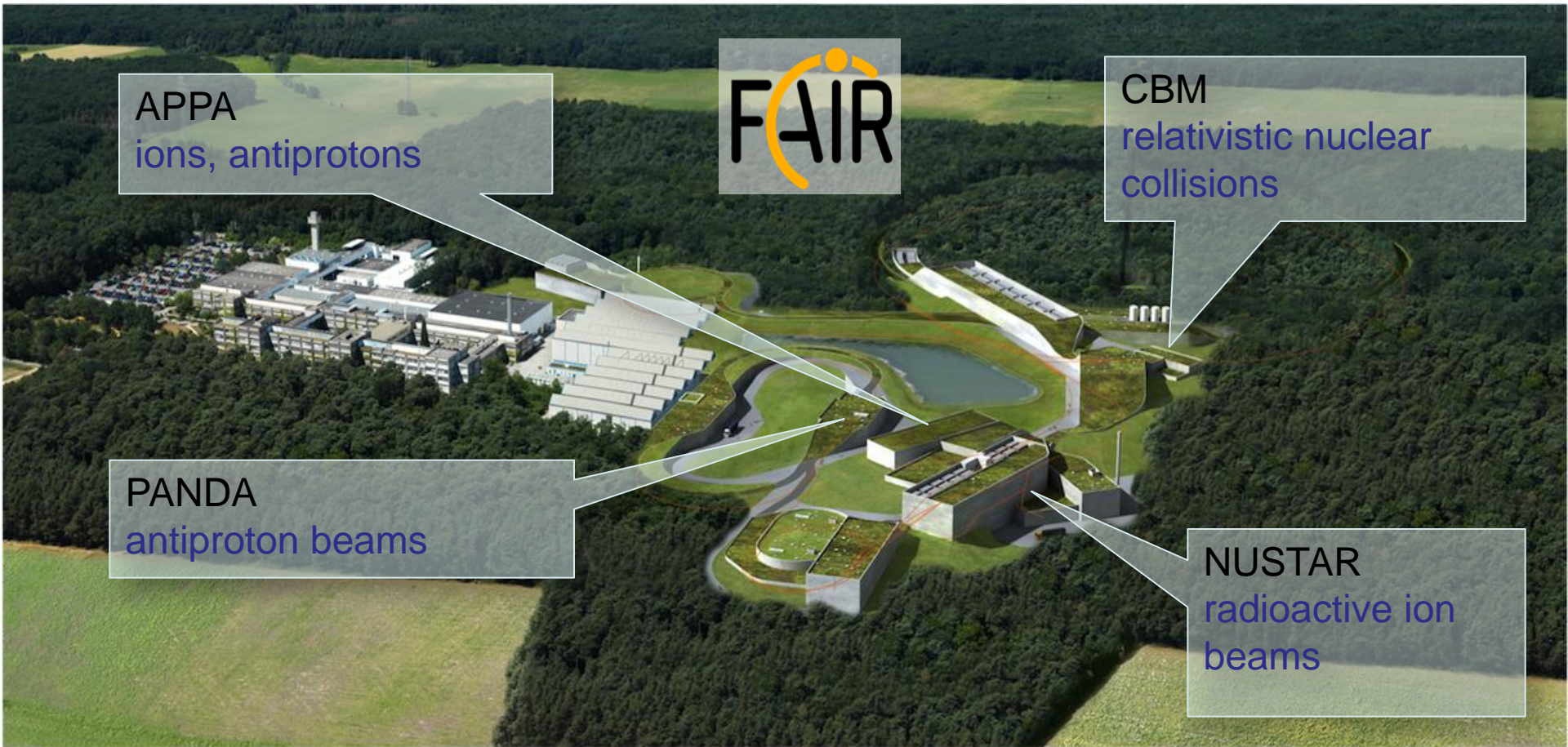
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Complex FAIR

APPA
ions, antiprotons

CBM
relativistic nuclear
collisions

PANDA
antiproton beams

NUSTAR
radioactive ion
beams

HESR: Storage_ring for $\bar{p}$
- Injection of $\bar{p}$ at 3.7 GeV/c
- Slow synchrotron (1.5-15 GeV/c)
- Luminosity up to L~ 2×10^{32} cm^{-2}s^{-1}
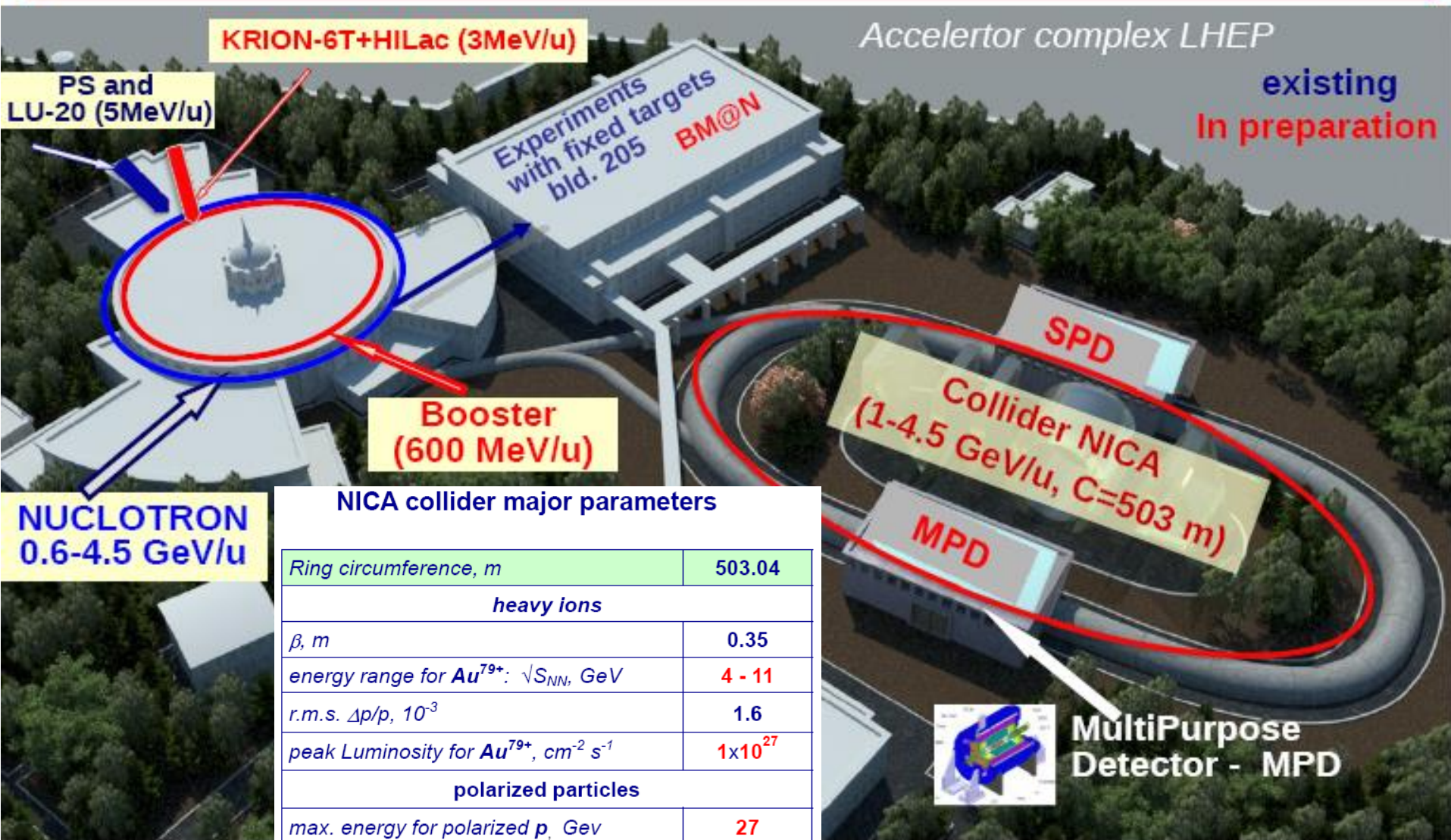- Beam cooling (stochastic & electron)

$\sqrt{s} \approx 5.5$ GeV

Antiproton production
- Proton Linac 70 MeV
- Accelerate p in SIS18 / 100
- Produce $\bar{p}$ on Cu target
- Collection in CR, fast cooling
- Accumulation in RESR
- Storage and usage in HESR

Complex NICA

Collider basic parameters: *beams: from* p *to Au;*
L~10^{27} cm^{-2} c^{-1} (Au), $\sqrt{s_{NN}}$ = 4-11 GeV; ~10^{32} cm^{-2} c^{-1} (p), $\sqrt{s}$ = 12-26 GeV;

KRION-6T+HILac (3MeV/u)

PS and LU-20 (5MeV/u)

Accelertor complex LHEP

Experiments with fixed targets bld. 205 BM@N

existing
In preparation

SPD

Collider NICA (1-4.5 GeV/u, C=503 m)

Booster (600 MeV/u)

MPD

NUCLOTRON 0.6-4.5 GeV/u

MultiPurpose Detector - MPD

### NICA collider major parameters

Ring circumference, m	503.04
heavy ions	
β, m	0.35
energy range for **Au^{79+}**: $\sqrt{S_{NN}}$, GeV	4 - 11
r.m.s. $\Delta p/p$, 10^{-3}	1.6
peak Luminosity for **Au^{79+}**, cm^{-2} s^{-1}	1x10^{27}
polarized particles	
max. energy for polarized **p**, Gev	27
peak Luminosity for **p**, cm^{-2} s^{-1}	1x10^{32}

Outline

- Physics case & motivation

- Conventional & exotic hadrons

- Review of recent experimental data

- Analysis & results

- Summary & perspectives

# WHY WE CONCENTRATE ON PHYSICS WITH ANTIPROTONS, PROTONS AND HEAVY IONS
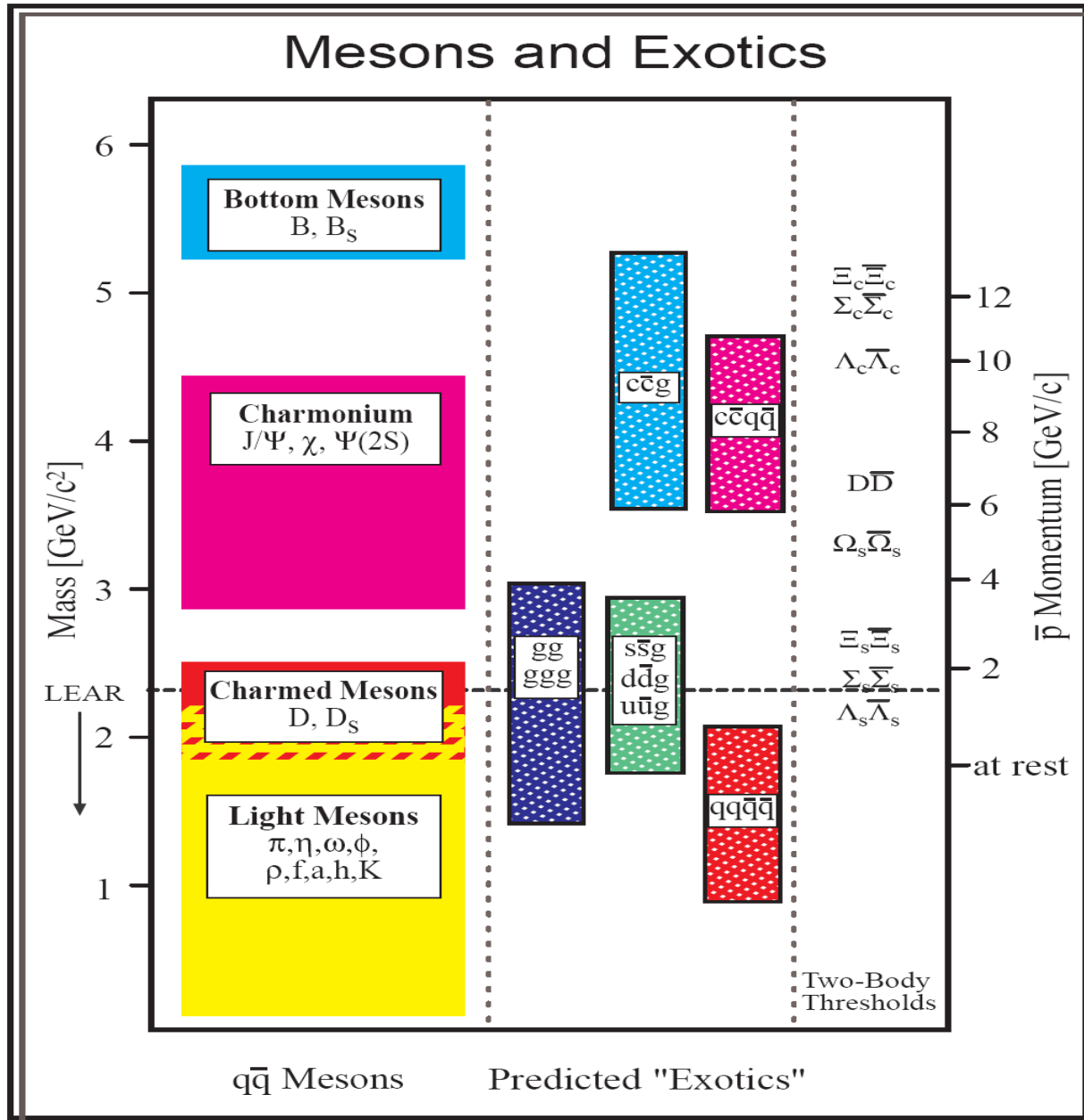
Expected masses of $\bar{q}q$-mesons, glueballs, hybrids and two-body production thresholds.

MOTIVATION

To look for different charmonium-like states (charmonium and exotics) in *pp\bar* annihilation, *pp* and *pA* collisions to obtain complementary results to the ones from *e+e-* interactions and *B*-meson decays

Motivation

- Predicted neutral charmonium states compared with found $c\bar{c}$ states, & both neutral & charged exotic candidates

- Based on Olsen [arXiv:1511.01589]

- Added 4 new J/ψφ states
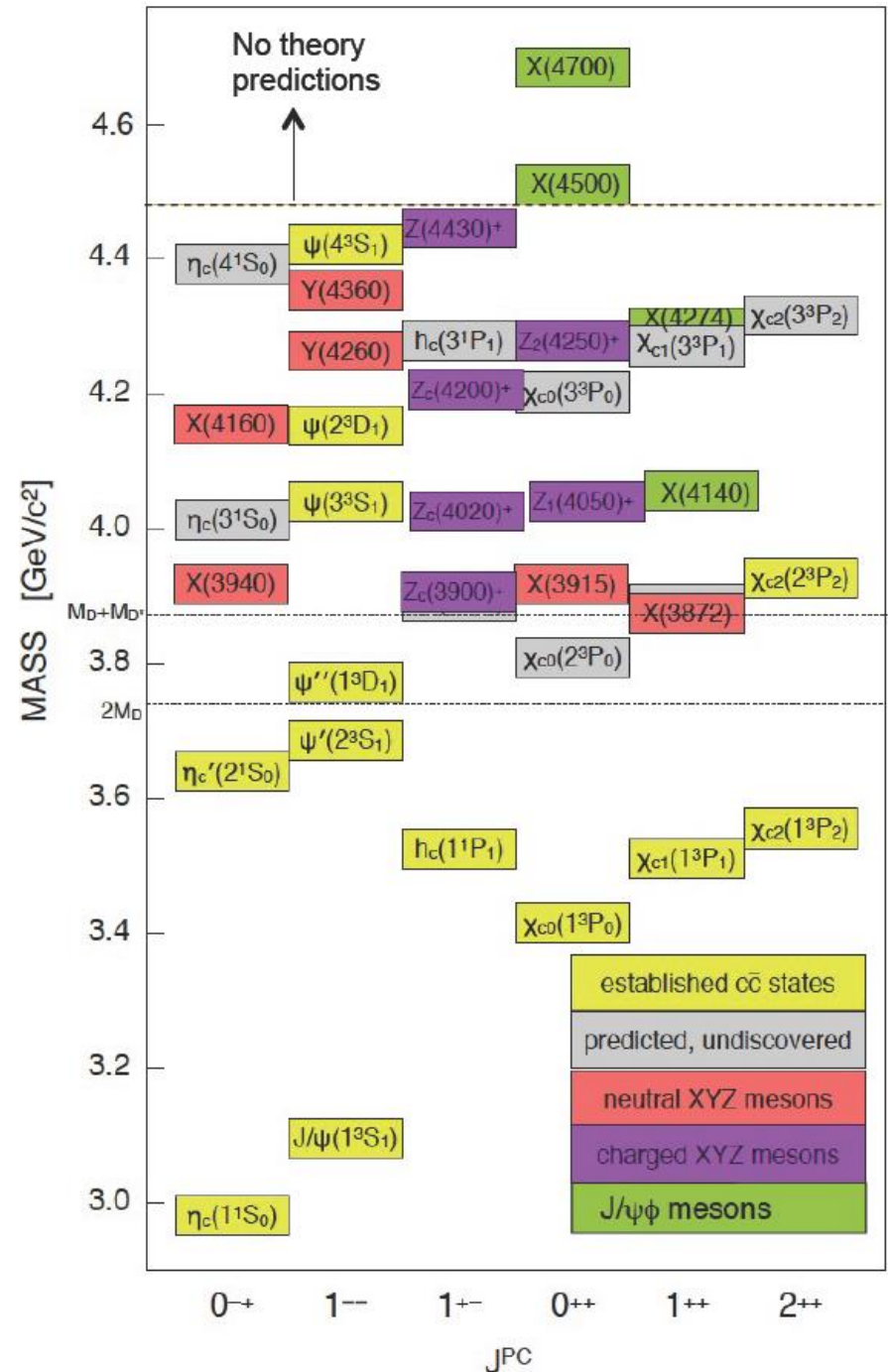
Charmonium-like states possess some well favored characteristics:

- is the simplest two-particle system consisting of quark & antiquark;

- is a compact bound system with small widths varying from several tens of keV to several tens of MeV compared to the light unflavored mesons and baryons

- charm quark c has a large mass (1.27 ± 0.07 GeV) compared to the masses of u, d & s (~ 0.1 GeV) quarks, that makes it plausible to attempt a description of the dynamical properties of charmonium-like system in terms of non-relativistic potential models and phenomenological models;

- quark motion velocities in charmonium-like systems are non-relativistic (the coupling constant, $\alpha_s \approx 0.3$ is not too large, and relativistic effects are manageable ($v^2/c^2 \approx 0.2$));

- the size of charmonium-like systems is of the order of less than 1 Fm ($R_{c\bar{c}} \sim \alpha_s \cdot m_q$) so that one of the main doctrines of QCD – asymptotic freedom is emerging;

<u>Therefore</u>:
♦ charmonium-like studies are promising for understanding the dynamics of quark interaction at small distances;
♦ charmonium-like spectroscopy represents itself a good testing ground for the theories of strong interactions:
- QCD in both perturbative and nonperturbative regimes
- QCD inspired potential models and phenomenological models
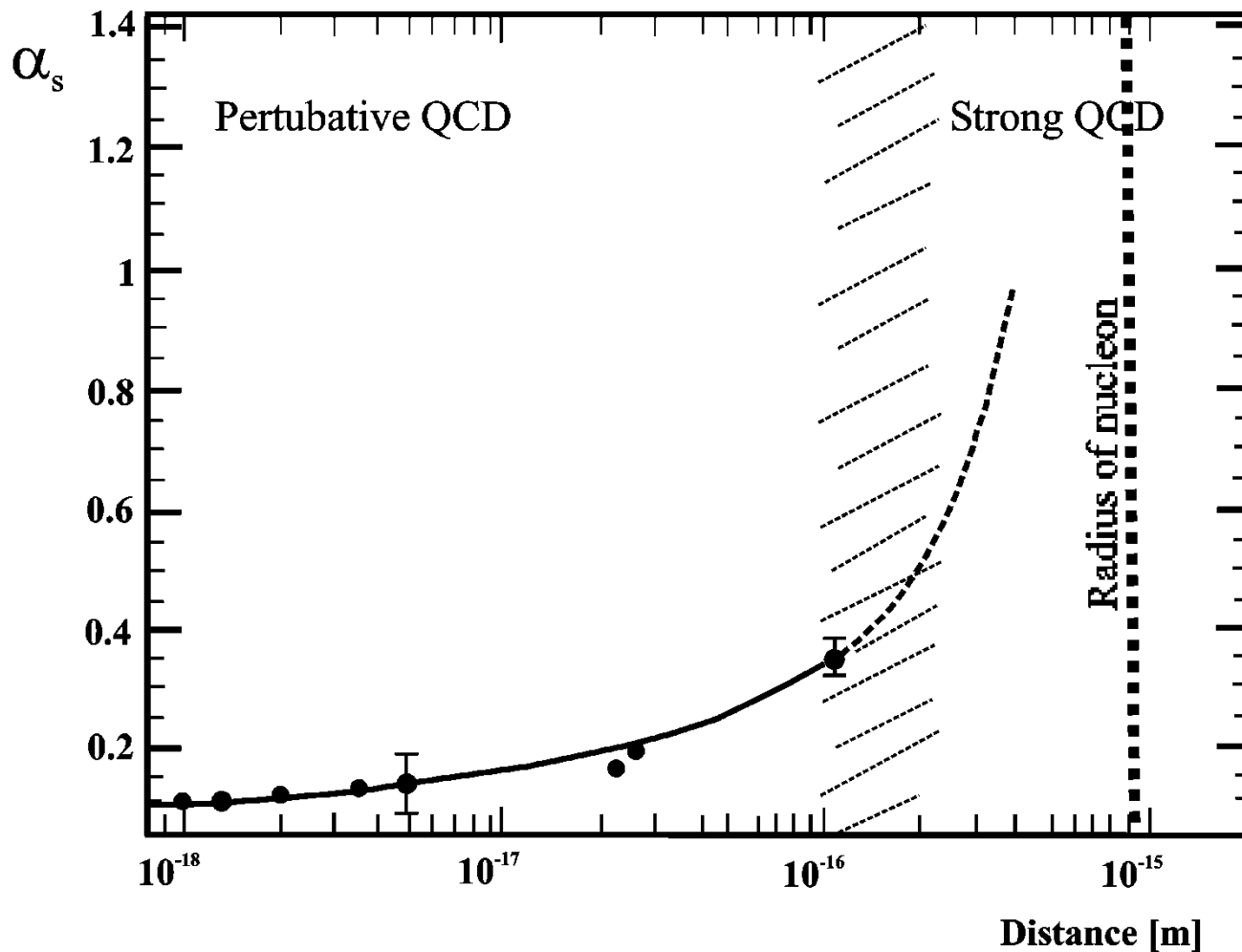
Coupling strength between two quarks as a function of their distance. For small distances ($\leq 10^{-16}$ m) the strengths α_s is ≈ 0.1, allowing a theoretical description by perturbative QCD. For distances comparable to the size of the nucleon, the strength becomes so large (strong QCD) that quarks can not be further separated: they remain confined within the nucleon and another theoretical approaches must be developed and applicable.

For charmonium (charmonium-like) states $\alpha_s \approx 0.3$ and $<v^2/c^2> \approx 0.2$.

The quark potential models have successfully described the charmonium spectrum, which generally assumes short-range coulomb interaction and long-range linear confining interaction plus spin dependent part coming from one gluon exchange. The zero-order potential is:

$$V_0^{(c\bar{c})}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + br + \frac{32\pi\alpha_s}{9m_c^2}\tilde{\delta}_\sigma(r)\vec{S}_c \cdot \vec{S}_{\bar{c}},$$

where $\tilde{\delta}_\sigma(r) = (\sigma/\sqrt{\pi})^3 e^{-\sigma^2 r^2}$ defines a gaussian-smeared hyperfine interaction.

Solution of equation with $H_0 = p^2/2m_c + V_0^{(c\bar{c})}(r)$ gives zero order charmonium wavefunctions.

T. Barnes, S. Godfrey, E. Swangon, Phys. Rev. D 72, 054026 (2005), hep-ph/ 0505002 & Ding G.J. et al., arXiv: 0708.3712 [hep-ph], 2008

The splitting between the multiplets is determined by taking the matrix element of the $V_{spin-dep}$ taken from one-gluon exchange Breit-Fermi-Hamiltonian between zero-order wave functions:

$$V_{spin-dep} = \frac{1}{m_c^2}\left[\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r}\right)\vec{L}\cdot\vec{S} + \frac{4\alpha_s}{r^3}T\right]$$

where α_s - coupling constant, b - string tension, σ - hyperfine interaction smear parameter.

Izmestev A. has shown *Nucl. Phys., V.52, N.6 (1990) & *Nucl. Phys., V.53, N.5 (1991)* that in the case of curved coordinate space with radius a (confinement radius) and dimension N at the dominant time component of the gluonic potential the quark-antiquark potential defines via Gauss equations. If space of physical system is compact (sphere S^3), the harmonic potential assures confinement: *Advances in Applied Clifford Algebras, V.8, N.2, p.235 - 270 (1998) .*
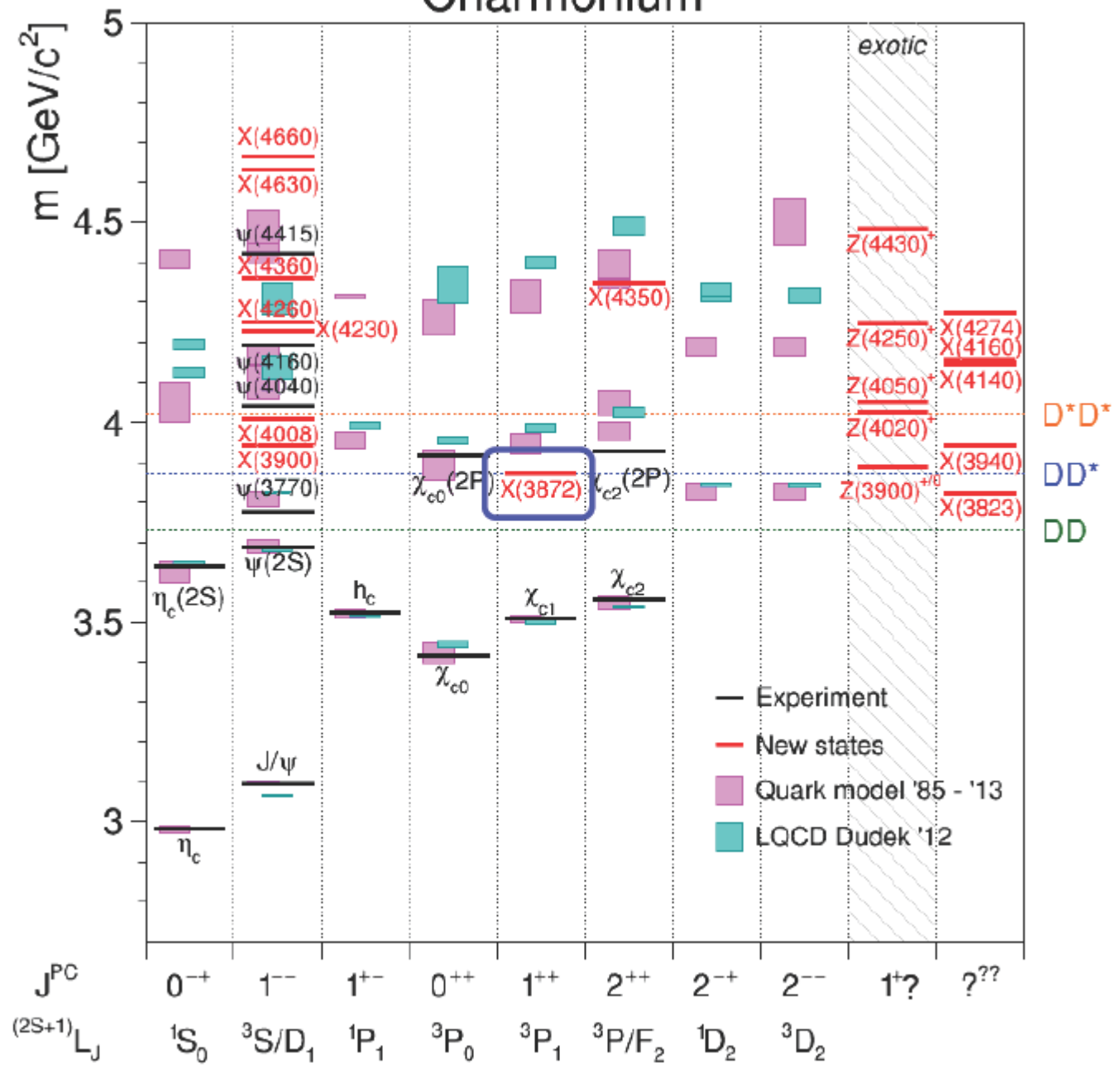
$$\Delta V_N(\vec{r}) = \text{const } G_N^{-1/2}(r)\delta(\vec{r}),$$

$$R(r) = \sin(r/a), \quad D(r) = r/a,$$

$$V_N(r) = V_0\int D(r)R^{1-N}(r)dr/r, \quad V_0 = \text{const} > 0.$$

$$V_3(r) = -V_0 \text{ctg}(r/a) + B, \quad V_0 > 0, \quad B > 0.$$

When cotangent argument in $V_3(r)$ is small: $r^2/a^2 << \pi^2$,

we get: $ctg(r/a) \approx a/r - r/3a$,

$$\begin{cases} V(r)|_{r\to 0} \sim 1/r \\ V(r)|_{r\to\infty} \sim kr \end{cases}$$

where $R(r)$, $D(r)$ and $G_N(r)$ are scaling factor, gauging and determinant of metric tensor $G_{\mu\nu}(r)$.

Charmonium

The $\overline{c}c$ system has been investigated in great detail first in e⁺e⁻-reactions, and afterwards on a restricted scale (E$_{\overline{p}}$ ≤ 9 GeV), but with high precision in $\overline{p}p$-annihilation (the experiments R704 at CERN and E760/E835 at Fermilab).

The number of unsolved questions related to charmonium has remained:

- singlet $^1D_2$ and triplet ^3D_J charmonium states are not determined yet;
- little is known about partial width of $^1D_2$ and ^3D_J charmonium states.
- higher laying singlet $^1S_0$, $^1P_1$ and triplet $^3S_1$, ^3P_J – charmonium states are poorly investigated;
- only few partial widths of ^3P_J-states are known (some of the measured decay widths don't fit theoretical schemes and additional experimental check or reconsideration of the corresponding theoretical models is needed, more data on different decay modes are desirable to clarify the situation);

AS RESULT :

- little is known on charmonium states above the the $D\overline{D}$ – threshold (S, P, D,....);
- many recently discovered states above $D\overline{D}$ - threshold (XYZ-states) expect their verification and explanation (their interpretation now is far from being obvious).

IN GENERAL ONE CAN IDENTIFY FOUR MAIN CLASSES OF CHARMONIUM DECAYS:

- decays into particle-antiparticle or $D\overline{D}$-pair: $\overline{c}c \rightarrow (\Psi, \eta_c, \chi_{cJ},..) \rightarrow \Sigma^0\overline{\Sigma}^0$, $\Lambda\overline{\Lambda}$, $\Sigma^0\overline{\Sigma}^0\pi$, $\Lambda\overline{\Lambda}\pi$;
- decays into light hadrons: $\overline{c}c \rightarrow (\Psi, \eta_c,..) \rightarrow \rho\pi$; $\overline{c}c \rightarrow \Psi \rightarrow \pi^+\pi^-$, $\overline{c}c \rightarrow \Psi \rightarrow \omega\pi^0, \eta\pi^0,...$;
- radiative decays: $\overline{c}c \rightarrow \gamma \eta_c, \gamma \chi_{cJ}, \gamma J/\Psi, \gamma \Psi', ...$;
- decays with J/Ψ, Ψ' and h_c in the final state: $\overline{c}c \rightarrow J/\Psi + X => \overline{c}c \rightarrow J/\Psi \pi^+\pi^-$, $\overline{c}c \rightarrow J/\Psi \pi^0\pi^0$; $\overline{c}c \rightarrow \Psi' + X => \overline{c}c \rightarrow \Psi' \pi^+\pi^-$, $\overline{c}c \rightarrow \Psi' \pi^0\pi^0$; $\overline{c}c \rightarrow h_c + X => \overline{c}c \rightarrow h_c \pi^+\pi^-$, $\overline{c}c \rightarrow h_c \pi^0\pi^0$.
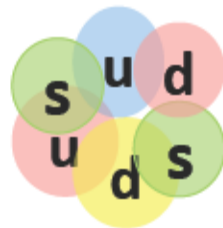
non-standard hadrons

non-qq̄ & non-qqq color-singlet combinations
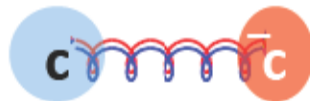

pentaquarks
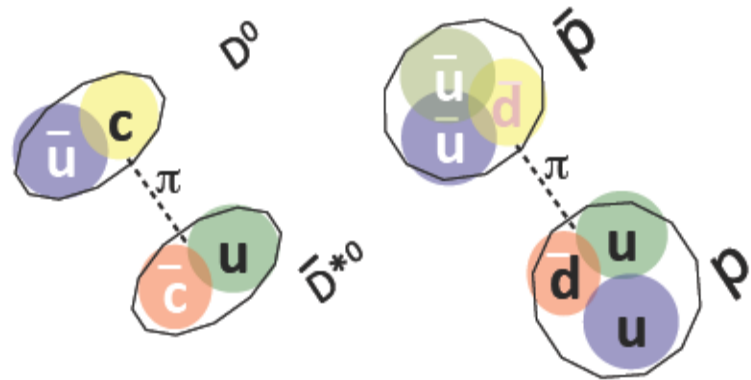
glueballs

H-dibaryon

diquark-diantiquarks

heptaquarks

hybrids

deusons

molecules

protonium

Multiquark states have been discussed since the 1st page of the quark model

A SCHEMATIC MODEL OF BARYONS AND MESONS *

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Where are they??

If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" [1-3], we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone [4]. Of course, with only strong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the F-spin by electromagnetism and the weak interactions determines the choice of isotopic spin and hypercharge directions.

Even if we consider the scattering amplitudes of strongly interacting particles on the mass shell only and treat the matrix elements of the weak, electromagnetic, and gravitational interactions by means

ber $n_t - n_{\bar{t}}$ would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin $\frac{1}{2}$ and $z = -1$, so that the four particles d^-, s^-, u^0 and b^0 exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" [6] q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q q q)$, $(q q q q \bar{q})$, etc., while mesons are made out of $(q \bar{q})$, $(q q \bar{q} \bar{q})$, etc. It is assuming that the lowest baryon configuration $(q q q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q \bar{q})$ similarly gives just **1** and **8**.

Two different kinds of experiments to study exotics:

- production experiment – $\overline{c}cg \to X + M$, where $M = \pi, \eta, \omega,\ldots$ (conventional states plus states with exotic quantum numbers)
- formation experiment (annihilation process) – $\overline{c}cg \to X \to M_1 M_2$ (conventional states plus states with non-exotic quantum numbers)

The low laying charmonium hybrid states:

	Gluon	
$(q\bar{q})_8$	1^- (TM)	1^+ (TE)
$^1S_0,\ 0^{-+}$	1^{++}	1^{--}
$^3S_1,\ 1^{--}$	0^{+-} ← exotic	0^{-+}
	1^{+-}	1^{-+} ← exotic
	2^{+-} ← exotic	2^{-+}

Charmonium-like exotics (hybrids, tetraquarks) predominantly decay via electromagnetic and hadronic transitions and into the open charm final states:

- $\overline{c}cg \to (\Psi, \chi_{cJ})$ + light mesons ($\eta, \eta', \omega, \varphi$) and $(\Psi, \chi_{cJ}) + \gamma$ - these modes supply small widths and significant branch fractions;
- $\overline{c}cg \to DD_J^*$. In this case *S-wave (L = 0) + P-wave (L = 1)* final states should dominate over decays to $D\overline{D}$ (are forbidden → *CP* violation) and partial width to should be very small.

The most interesting and promising decay channels of charmed hybrids have been, in particular, analyzed:

- $\overline{c}c \to \tilde{\eta}_{c0,1,2}\ (0^{-+}, 1^{-+}, 2^{-+})\ \eta \to \chi_{c0,1,2}\ (\eta, \pi\pi, \gamma;\ldots)$;
- $\overline{c}c \to \tilde{h}_{c0,1,2}(0^{+-}, 1^{+-}, 2^{+-})\ \eta \to \chi_{c0,1,2}\ (\eta, \pi\pi, \gamma;\ldots)$;
- $\overline{c}c \to \tilde{\Psi}\ (0^{--}, 1^{--}, 2^{--}) \to J/\Psi\ (\eta, \omega, \pi\pi, \gamma \ldots)$;
- $\overline{c}c \to \tilde{\eta}_{c0,1,2},\ \tilde{h}_{c0,1,2},\ \tilde{\chi}_{c1}\ (0^{-+}, 1^{-+}, 2^{-+}, 0^{+-}, 1^{+-}, 2^{+-}, 1^{++})\ \eta \to D\overline{D}_J^*\ (\eta, \gamma)$.

$J^{PC} = 0^- \to$ exotic!

According the constituent quark model tetraquark states are classified in terms of the diquark and diantiquark spin S_{cq}, $S_{\bar{c}\bar{q}}$, total spin of diquark-diantiquark system S, total angular momentum J, spacial parity P and charge conjugation C. The following states with definite quantum numbers J^{PC} are expected to exist:

- two states with $J=0$ and positive P-parity $J^{PC}=0^{++}$ i.e., $|\,0_{cq}, 0_{\bar{c}\bar{q}}\,; S=0, J=0\,\rangle$ and $|\,1_{cq}, 1_{\bar{c}\bar{q}}\,; S=0, J=0\,\rangle$;

- three states with $J=0$ and negative P-parity i.e., $|A\,\rangle = |\,1_{cq}, 0_{\bar{c}\bar{q}}\,; S=1, J=0\,\rangle$; $|B\,\rangle = |\,0_{cq}, 1_{\bar{c}\bar{q}}\,; S=1, J=0\,\rangle$; $|C\,\rangle = |\,1_{cq}, 1_{\bar{c}\bar{q}}\,; S=1, J=0\,\rangle$. State $|C\,\rangle$ is even under charge conjugation. Taking symmetric and antisymmetric combinations of states $|A\,\rangle$ and $|B\,\rangle$ we obtain a C-odd and C-even state respectively; therefore we have one state with $J^{PC}=0^{--}$ i.e., $|\,0^{--}\,\rangle = \frac{1}{\sqrt{2}}\big(|A\,\rangle + |B\,\rangle\big)$ and two states with $J^{PC}=0^{-+}$ i.e., $|\,0^{-+}\,\rangle_1 = \frac{1}{\sqrt{2}}\big(|A\,\rangle - |B\,\rangle\big)$; $|\,0^{-+}\,\rangle_2 = |\,C\,\rangle$.

- three states with $J=1$ and positive P-parity i.e., $|D\rangle = |\,1_{cq}, 0_{\bar{c}\bar{q}}\,; S=1, J=1\,\rangle$; $|E\rangle = |\,0_{cq}, 1_{\bar{c}\bar{q}}\,; S=1, J=1\,\rangle$; $|F\rangle = |\,1_{cq}, 1_{\bar{c}\bar{q}}\,; S=1, J=1\,\rangle$. State $|F\,\rangle$ is odd under charge conjugation. Operating $|D\rangle$ and $|E\rangle$ in the same way as for states $|A\,\rangle$ and $|B\,\rangle$ we obtain one state with $J^{PC}=1^{++}$ state i.e., $|\,1^{++}\,\rangle = \frac{1}{\sqrt{2}}\big(|D\,\rangle + |E\,\rangle\big)$ and two states with $J^{PC}=1^{+-}$ i.e., $|\,1^{+-}\,\rangle_1 = \frac{1}{\sqrt{2}}\big(|D\,\rangle - |E\,\rangle\big)$; $|\,1^{+-}\,\rangle_2 = |F\rangle$.

- one state with $J=2$ and positive P-parity $J^{PC}=2^{++}$ i.e., $|\,1_{cq}, 1_{\bar{c}\bar{q}}\,; S=1, J=2\,\rangle$.

• $\bar{p}p \to X \to J/\Psi\,\rho \to J/\Psi\,\pi\pi$; $\bar{p}p \to X \to J/\Psi\,\omega \to J/\Psi\,\pi\pi\pi$, $\bar{p}p \to X \to \chi_{cJ}\,\pi$ (decays into J/Ψ, Ψ', χ_{cJ} and light mesons);

• $\bar{p}p \to X \to D\bar{D}^* \to D\bar{D}\,\gamma$; $\bar{p}p \to X \to D\bar{D}^* \to D\bar{D}\,\eta$ (decays into $D\bar{D}^*$-pair).

Candidate exotic hadrons

Light quark sector

Charmonium-like

Charged charmonium-like

Hidden charmed pentaquarks

b-quark sector

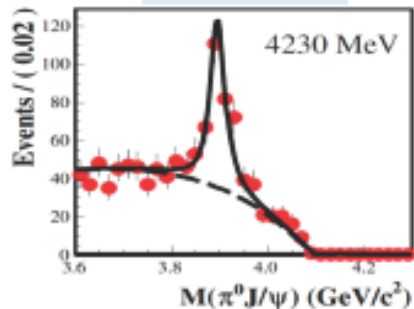| State	M (MeV)	Γ (MeV)	J^{PC}	Process (decay mode)	Experiment
$\pi_1(1400)$	1354 ± 25	330 ± 25	1^{-+}	$\pi^- p \to (\eta\pi^-)p$	MPS, Compass
				$p\bar{p} \to \pi^0(\pi^0\eta)$	Xtal Barrel
$X(1835)$	$135.7^{+5.0}_{-3.2}0$	99 ± 50	0^{-+}	$J/\psi \to \gamma(p\bar{p})$	BESII, CLEOc, BESIII
				$J/\psi \to \gamma(\pi^+\pi^-\eta')$	BESII, BESIII
$X(3872)$	3871.68 ± 0.17	<1.2	1^{++}	$B \to K + (J/\psi\,\pi^+\pi^-)$	Belle, BaBar, LHCb
				$p\bar{p} \to (J/\psi\,\pi^+\pi^-) + ...$	CDF, D0
				$B \to K + (J/\psi\,\pi^+\pi^-\pi^0)$	Belle, BaBar
				$B \to K + (D^0\bar{D}^0\pi^0)$	Belle, BaBar
				$B \to K + (J/\psi\,\gamma)$	BaBar, Belle, LHCb
				$B \to K + (\psi'\,\gamma)$	BaBar, Belle, LHCb
				$pp \to (J/\psi\,\pi^+\pi^-) + ...$	LHCb, CMS
$X(3915)$	3917.4 ± 2.7	28^{+10}_{-9}	0^{++}	$B \to K + (J/\psi\,\omega)$	Belle, BaBar
				$e^+e^- \to e^+e^- + (J/\psi\,\omega)$	Belle, BaBar
$\chi_{c2}(2P)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \to e^+e^- + (D\bar{D})$	Belle, BaBar
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$0(?)^{-(?)+}$	$e^+e^- \to J/\psi + (D^*\bar{D})$	Belle
				$e^+e^- \to J/\psi + (...)$	Belle
$G(3900)$	3943 ± 21	52 ± 11	1^{--}	$e^+e^- \to \gamma + (D\bar{D})$	BaBar, Belle
$Y(4008)$	4008^{+121}_{-49}	226 ± 97	1^{--}	$e^+e^- \to \gamma + (J/\psi\,\pi^+\pi^-)$	Belle
$Y(4140)$	$4146.5^{+6.4}_{-5.3}$	$83^{+30}_{-25}9$	1^{++}	$B \to K + (J/\psi\,\phi)$	CDF, CMS, LHCb
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$0(?)^{-(?)+}$	$e^+e^- \to J/\psi + (D^*\bar{D})$	Belle
$Y(4260)$	4263^{+8}_{-9}	95 ± 14	1^{--}	$e^+e^- \to \gamma + (J/\psi\,\pi^+\pi^-)$	BaBar, CLEO, Belle
				$e^+e^- \to (J/\psi\,\pi^+\pi^-)$	CLEO, BESIII
				$e^+e^- \to (J/\psi\,\pi^0\pi^0)$	CLEO, BESIII
$Y(4274)$	4273^{+19}_{-9}	56 ± 16	1^{++}	$B \to K + (J/\psi\,\phi)$	CDF, CMS, LHCb
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	$13.3^{+18.4}_{-10.0}$	$0/2^{++}$	$e^+e^- \to e^+e^- (J/\psi\,\phi)$	Belle
$Y(4360)$	4361 ± 13	74 ± 18	1^{--}	$e^+e^- \to \gamma + (\psi'\,\pi^+\pi^-)$	BaBar, Belle
$X(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^- \to \gamma(\Lambda_c^+\Lambda_c^-)$	Belle
$Y(4660)$	4664 ± 12	48 ± 15	1^{--}	$e^+e^- \to \gamma + (\psi'\,\pi^+\pi^-)$	Belle
$Z_c^+(3900)$	3890 ± 3	33 ± 10	1^{+-}	$Y(4260) \to \pi^- + (J/\psi\,\pi^+)$	BESIII, Belle
				$Y(4260) \to \pi^- + (D\bar{D}^*)^+$	BESIII
$Z_c^+(4020)$	4024 ± 2	10 ± 3	$1(?)^{+(?)-}$	$Y(4260) \to \pi^- + (h_c\,\pi^+)$	BESIII
				$Y(4260) \to \pi^- + (D^*\bar{D}^*)^+$	BESIII
$Z_1^+(4050)$	4051^{+24}_{-49}	82^{+51}_{-55}	$?^{?+}$	$B \to K + (\chi_{c1}\,\pi^+)$	Belle, BaBar
$Z^+(4200)$	4196^{+35}_{-32}	370^{+99}_{-149}	1^{+-}	$B \to K + (J/\psi\,\pi^+)$	Belle, LHCb
$Z_2^+(4250)$	4248^{+185}_{-45}	177^{+321}_{-72}	$?^{?+}$	$B \to K + (\chi_{c1}\,\pi^+)$	Belle, BaBar
$Z^+(4430)$	4477 ± 20	181 ± 31	1^{+-}	$B \to K + (\psi'\,\pi^+)$	Belle, LHCb
				$B \to K + (J\psi\,\pi^+)$	Belle
$P_c^+(4380)$	4380 ± 30	205 ± 88	$(3/2)^{-}$	$\Lambda_b^+ \to K + (J/\psi\,p)$	LHCb
$P_c^+(4450)$	4449.8 ± 3.0	39 ± 20	$(5/2)^{+}$	$\Lambda_b^+ \to K + (J/\psi\,p)$	LHCb
$Y_b(10890)$	10888.4 ± 3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \to (\Upsilon(nS)\,\pi^+\pi^-)$	Belle
$Z_b^+(10610)$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	"$\Upsilon(5S)$" $\to \pi^- + (\Upsilon(nS)\,\pi^+)$, $n=1,2,3$	Belle
				"$\Upsilon(5S)$" $\to \pi^- + (h_b(nP)\,\pi^+)$, $n=1,2$	Belle
				"$\Upsilon(5S)$" $\to \pi^- + (B\bar{B}^*)^+$, $n=1,2$	Belle
$Z_b^0(10610)$	10609 ± 6		1^{+-}	"$\Upsilon(5S)$" $\to \pi^0 + (\Upsilon(nS)\,\pi^0)$, $n=1,2,3$	Belle
$Z_b^+(10650)$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	"$\Upsilon(5S)$" $\to \pi^- + (\Upsilon(nS)\,\pi^+)$, $n=1,2,3$	Belle
				"$\Upsilon(5S)$" $\to \pi^- + (h_b(nP)\,\pi^+)$, $n=1,2$	Belle
				"$\Upsilon(5S)$" $\to \pi^- + (B^*\bar{B}^*)^+$, $n=1,2$	Belle

SUMMARY on Z_c from BES III

Are these states the same?!

Are these states the same?!

$e^+e^- \to \pi^{+(0)}\pi^{-(0)}J/\psi$

$Z_c(3900)^\pm$

$Z_c(3900)^0$
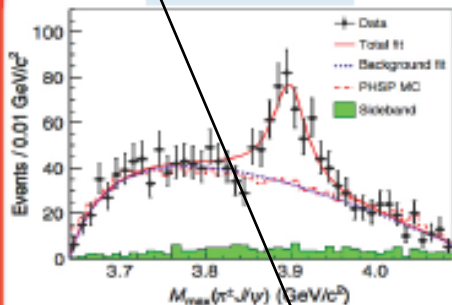
$e^+e^- \to (D\bar{D}^*)^{\pm(0)}\pi^{\mp(0)}$

$Z_c(3885)^\pm$
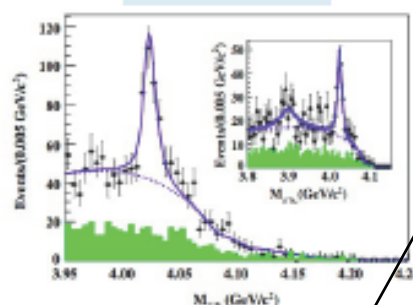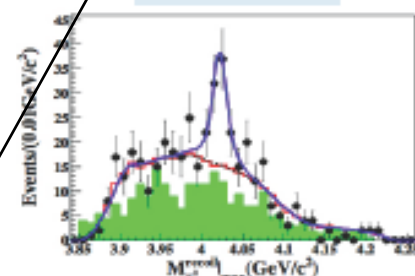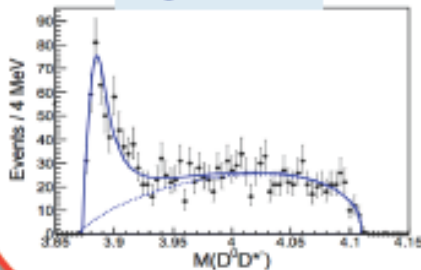
$Z_c(3885)^0$
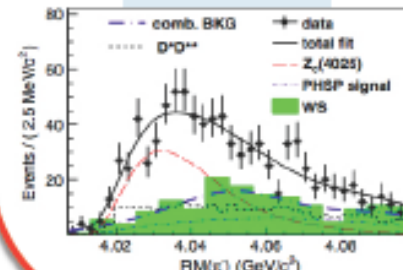
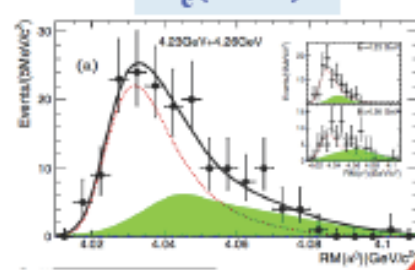$e^+e^- \to \pi^{+(0)}\pi^{-(0)}h_c$

$Z_c(4020)^\pm$

$Z_c(4020)^0$
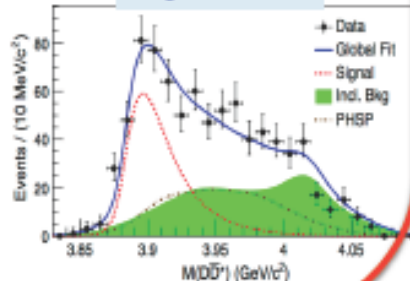
$e^+e^- \to (D^*\bar{D}^*)^{\pm(0)}\pi^{\mp(0)}$
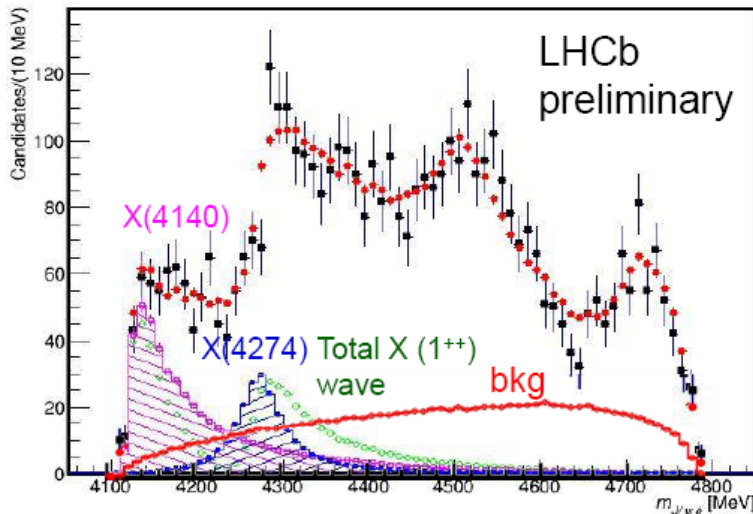
$Z_c(4025)^\pm$

$Z_c(4025)^0$

- Nature of these states? Isospin triplets?
- Different decay channels of the same states observed?
- Other decay modes?

The LHCb new resonances

In 2016 LHCb measured 4 new resonances with an amplitude analysis on $B^+ \to J/\Psi \phi K^+$ decay

- The $X(4140)$ 1^{++} state, Phys. Rev. Lett. 118, 022003 (2017), Phys. Rev. D 95, 012002 (2017)

  - Not seen by Belle, and BaBar

  - Seen by CDF and D0

  - The 1^{++} quantum numbers ruled out most of the multiquark models.

- The $X(4274)$ 1^{++}, Phys. Rev. Lett. 118, 022003 (2017), Phys. Rev. D 95, 012002 (2017)

  - Seen by CDF and CMS and Belle with a higher mass.

- The $X(4500)$ 0^{++} and $X(4700)$ 0^{++}, Phys. Rev. D 95, 012002 (2017)

X(1^{++})

Contri-bution	sign. or Ref.	M_0 MeV	Γ_0 MeV	Fit results F.F. %
All $X(1^+)$				16 ± 3 $^{+6}_{-2}$
$X(4140)$	8.4σ	4146.5 ± 4.5 $^{+4.6}_{-2.8}$	83 ± 21 $^{+21}_{14}$	13 ± 3.2 $^{+4.8}_{2.0}$
ave.		4146.9 ± 2.3	17.8 ± 6.8	
$X(4274)$	6.0σ	4273.3 ± 8.3 $^{+17.2}_{-3.6}$	56 ± 11 $^{+8}_{-11}$	7.1 ± 2.5 $^{+3.5}_{-2.4}$
CDF		4274.4 $^{+8.4}_{-6.7}\pm1.9$	32 $^{+22}_{-15}\pm8$	
CMS		$4313.8\pm5.3\pm7.3$	38 $^{+30}_{-15}\pm16$	

- ## Significant X(4140) 8.4σ,
  - mass consistent with the previous measurements, but the width substantially larger
  - $J^{PC}=1^{++}$ determined at 5.7σ including systematic errors

- ## Significant X(4274) 6.0σ,
  - Consistent with the unpublished CDF results. First significant claim for this structure.
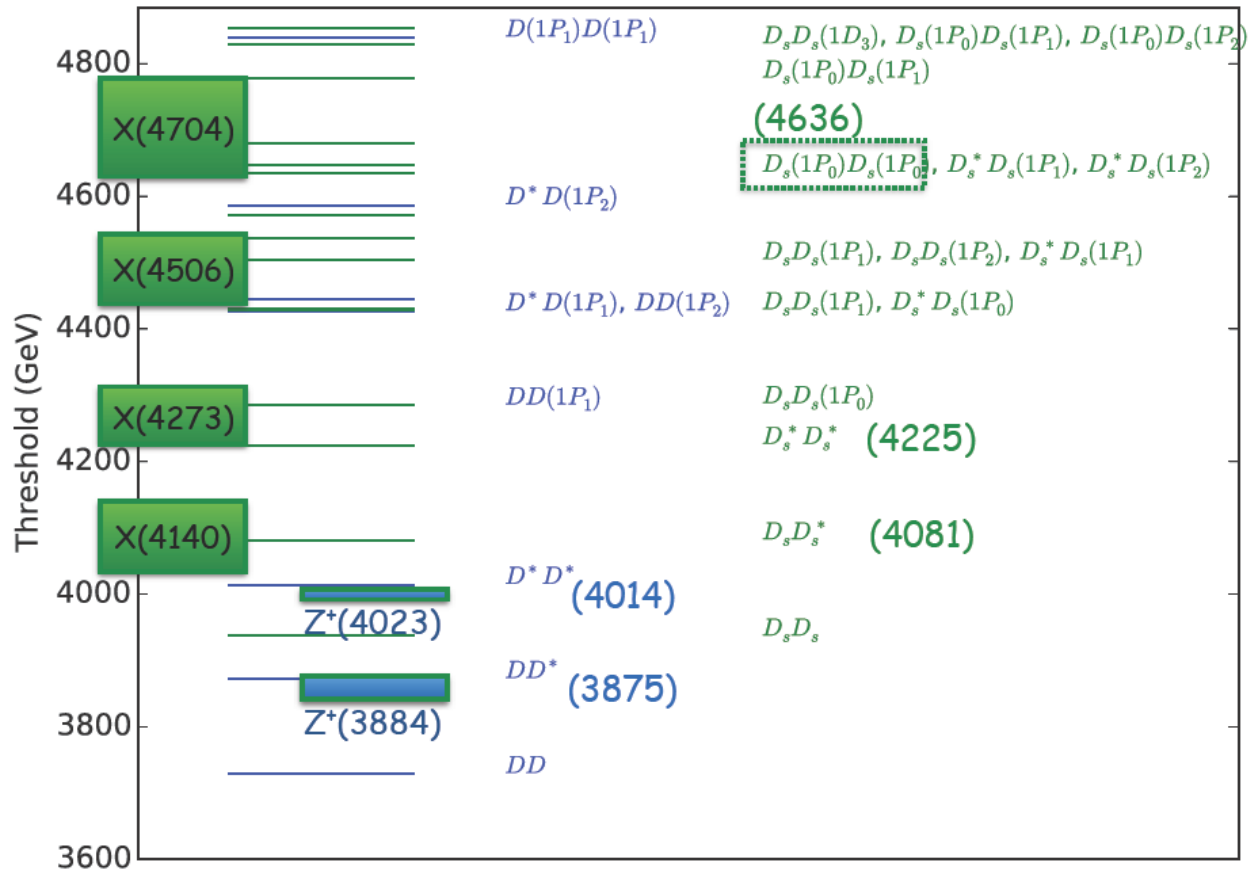 - $J^{PC}=1^{++}$ determined at 5.8σ including systematic errors

$X(0^{++})$
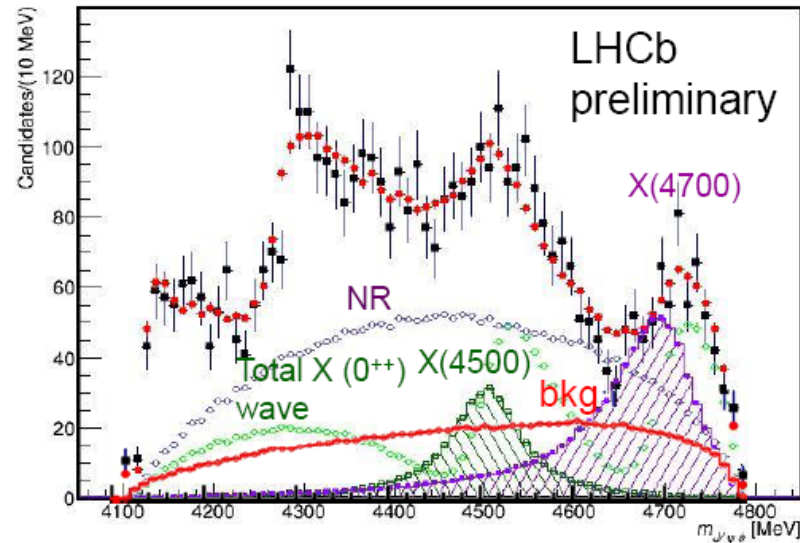
Contribution	sign.	Fit results		
		M_0 MeV	Γ_0 MeV	F.F. %
All $X(0^+)$				$28\pm 5\,^{+7}_{-7}$
$NR_{J/\psi\phi}$	6.4σ			$46\pm 11\,^{+11}_{-21}$
$X(4500)$	6.1σ	$4506\pm 11\,^{+12}_{-15}$	$92\pm 21\,^{+21}_{-20}$	$6.6\pm 2.4\,^{+3.5}_{-2.3}$
$X(4700)$	5.6σ	$4704\pm 10\,^{+14}_{-24}$	$120\pm 31\,^{+42}_{-33}$	$12\pm 5\,^{+9}_{-5}$

- Significant structures at higher masses, best described by two new 0^{++} resonances X(4500),X(4700):
  - Significances of 6.1σ, 5.6σ
  - $J^{PC}=0^{++}$ determined at 4.0σ, 4.5σ, respectively
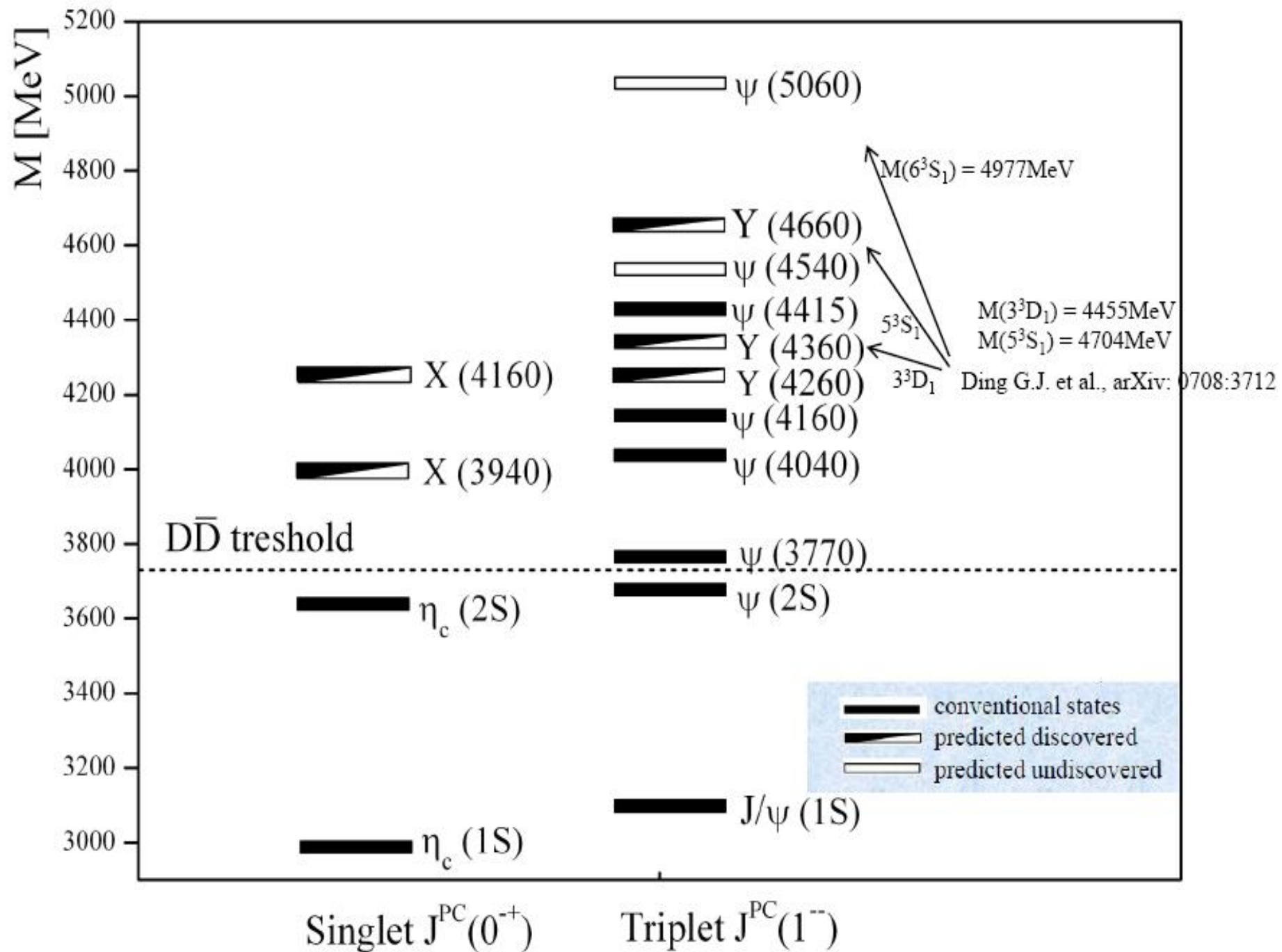
NEW STATES WITH ZERO STRANGENESS from LHCb

- strangeness zero states - charmonium ($\bar{c}s\bar{s}c$) structures
- SU(3) symmetry suggests new X_s states near the thresholds:
  D D_s*, D_s D*, D_s*D_s* : observable in B decays?   B -> X K: M_X < 4785 MeV

- No evidence in preliminary LQCD studies for ($\bar{c}s\bar{s}c$) tetraquark states.

M [MeV]

5200

5000 — ψ (5060)

M(6³S₁) = 4977MeV

4800

4600 — Y (4660)
— ψ (4540)

4400 — ψ (4415)
— Y (4360) 5³S₁
— Y (4260) 3³D₁

M(3³D₁) = 4455MeV
M(5³S₁) = 4704MeV
Ding G.J. et al., arXiv: 0708:3712

4200 — X (4160)
— ψ (4160)

4000 — X (3940)
— ψ (4040)

3800 DD̄ treshold
- - - - - - - - - - - - - - - - - - - -
— ψ (3770)

3600 — η_c (2S)
— ψ (2S)

3400
conventional states
predicted discovered
predicted undiscovered

3200

3000 — J/ψ (1S)

— η_c (1S)

Singlet J^PC(0^-+)   Triplet J^PC(1^--)

6 observed states can fit* into charmonium table
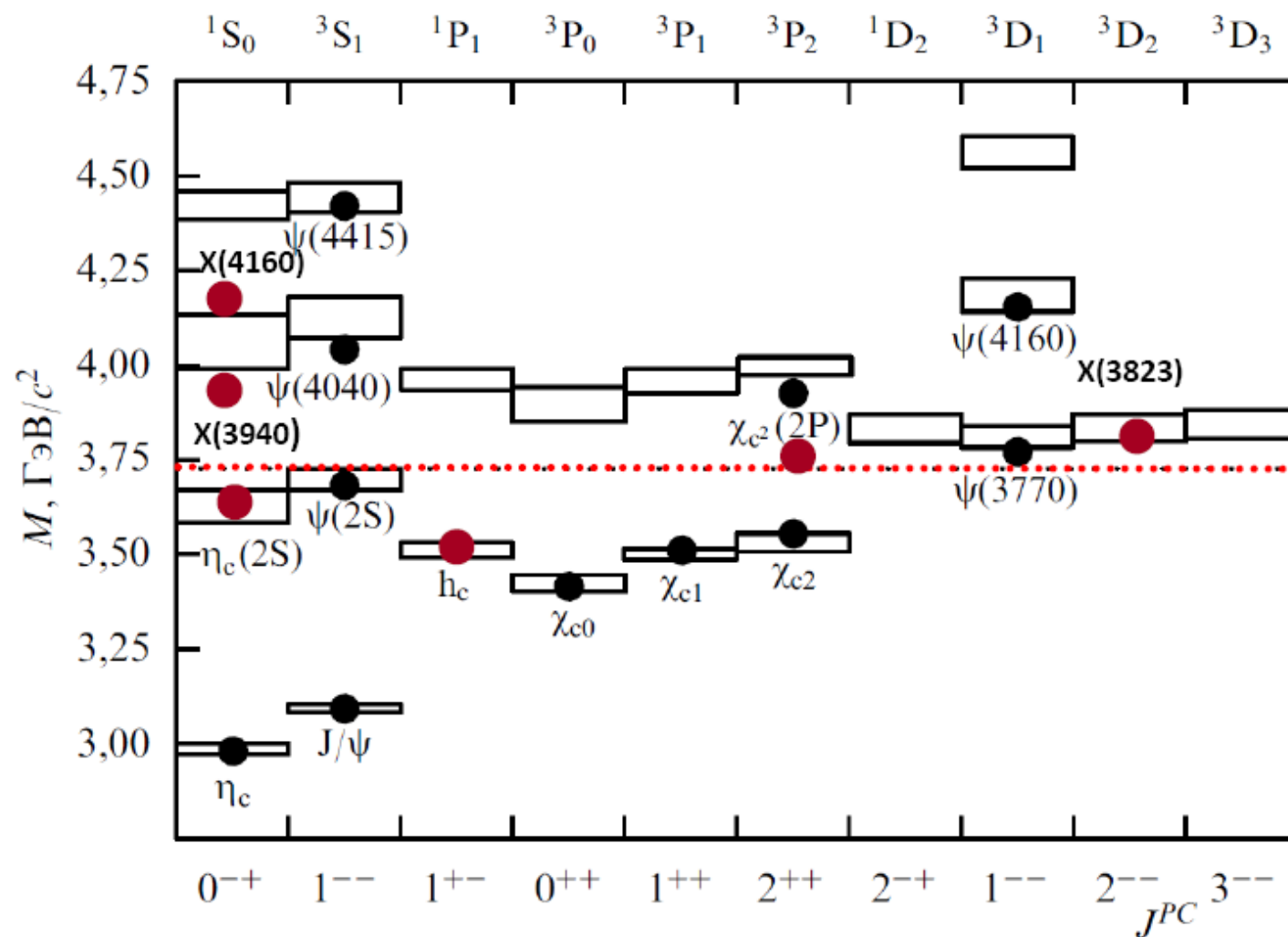
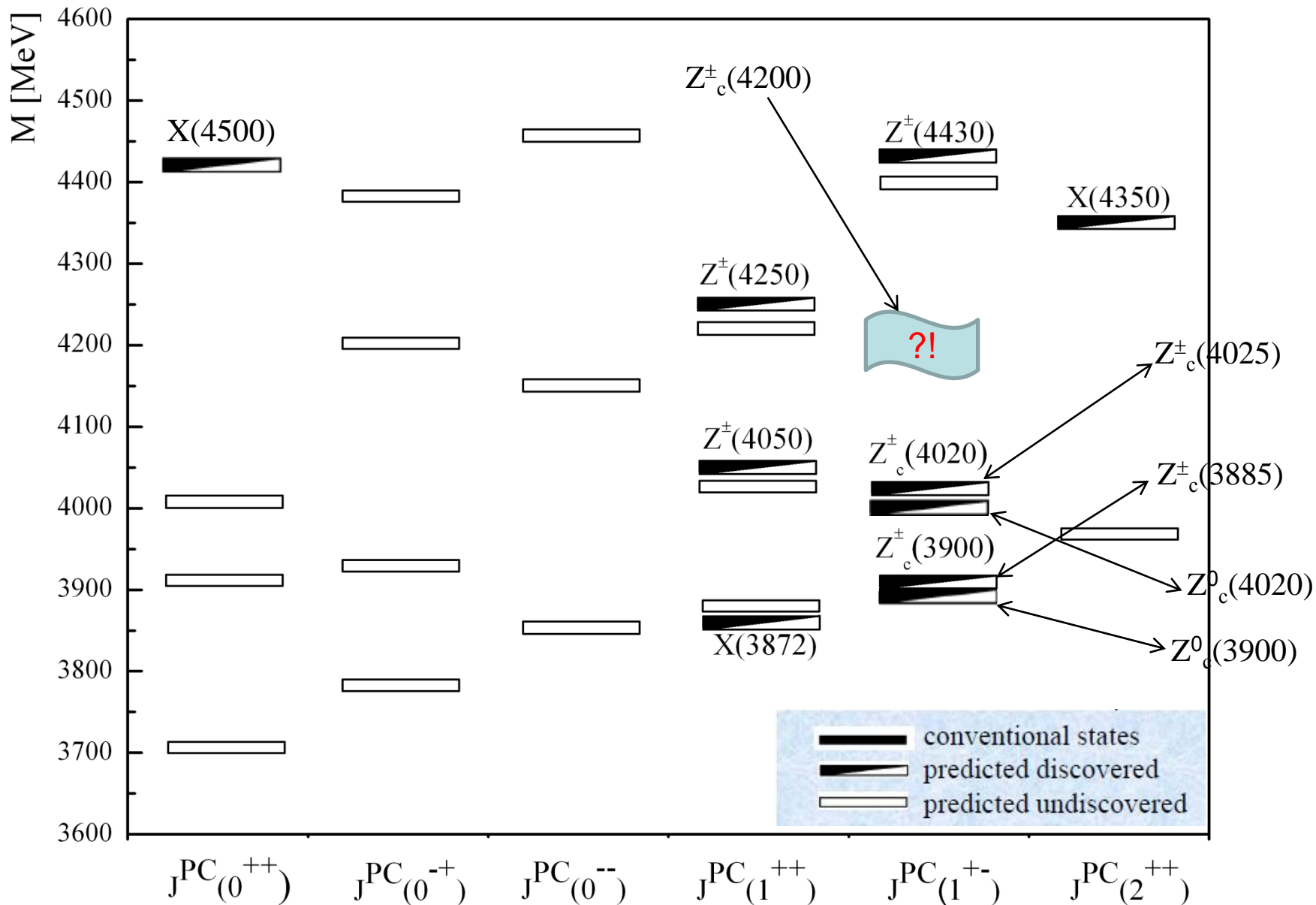* However, not easily: potential models need to be elaborated to describe new masses

What about others?

THE SPECTRUM OF TETRAQUARKS

What to look for

◉ Does the Z(4433) exist??

◉ Better to find charged X !

◉ Neutral partners of $Z(4433) \sim X(1^{+-}, 2S)$ should be close by few MeV and decaying to $\psi(2S)\, \pi/\eta$ or $\eta_c(2S)\, \rho/\omega$

◉ What about $X(1^{+-}, 1S)$? Look for any charged state at $\approx$ 3880 MeV (decaying to $\psi\pi$ or $\eta_c\rho$)

◉ Similarly one expects $X(1^{++}, 2S)$ states. Look at $M \sim 4200\text{-}4300$: $X(1^{++}, 2S) \rightarrow D^{(*)}D^{(*)}$

◉ Baryon-anti-baryon thresholds at hand (4572 MeV for $2M_{\Lambda c}$ and 4379 MeV for $M_{\Lambda c} + M_{\Sigma c}$). $X(2^{++}, 2S)$ might be over bb-threshold.

(L.Maiani, A.D.Polosa, V.Riquer, 0708.3997

TETRAQUARK STATES

There are indications of structures in $J/\psi\, \phi$ of the kind $[cs]_0 [\bar{c}\bar{s}]_1 + [cs]_1 [\bar{c}\bar{s}]_0$ — FROM LHCb.

SPECTRUM

$$\underset{4270}{\overline{\phantom{0^{++}}}}^{0^{++}} + K \qquad\qquad \overline{\phantom{1^{+-}}}^{1^{+-}} + K \qquad \underset{4270}{\overline{\phantom{2^{++}}}}^{2^{++}} + K$$

$$\underset{4140}{\overline{\phantom{1^{++}}}}^{1^{++}} \qquad \overline{\phantom{1^{+-}}}^{1^{+-}} - K$$

$$\overline{\phantom{0^{++}}}^{0^{++}} - 3K$$

and $4500\ 0^{++}$
$\qquad\quad 4700\ 0^{++}$

(RADIAL EXCITATIONS LIKE $Z(4430)$?)

PROBLEM: 4270 seems at the moment a 1^{++} !!

CALCULATION OF WIDTHS

The integral formalism (or in other words integral approach) is based on the possibility of appearance of the discrete quasi stationary states with finite width and positive values of energy in the barrier-type potential. This barrier is formed by the superposition of two type of potentials: short-range attractive potential $V_1(r)$ and long-distance repulsive potential $V_2(r)$.

Thus, the width of a quasi stationary state in the integral approach is defined by the following expression (integral formula):

$$\Gamma = 2\pi \left| \int_0^\infty \phi_L(r) V(r) F_L(r) r^2 dr \right|^2$$

$$(r < R): \int_0^R \left| \phi_L(r) \right|^2 dr = 1$$

where

where $F_L(r)$ – is the regular decision in the $V_2(r)$ potential, normalized on the energy delta-function; $\phi_{L(}r)$ – normalized wave function of the resonance state. This wave function transforms into irregular decision in the $V_2(r)$ potential far away from the internal turning point.

The integral can be estimated with the well known approximately methods: for example, the saddle-point technique or the other numerical method.

THE WIDTHS OF TRIPLET $^3S_1$ CHARMONIUM STATES
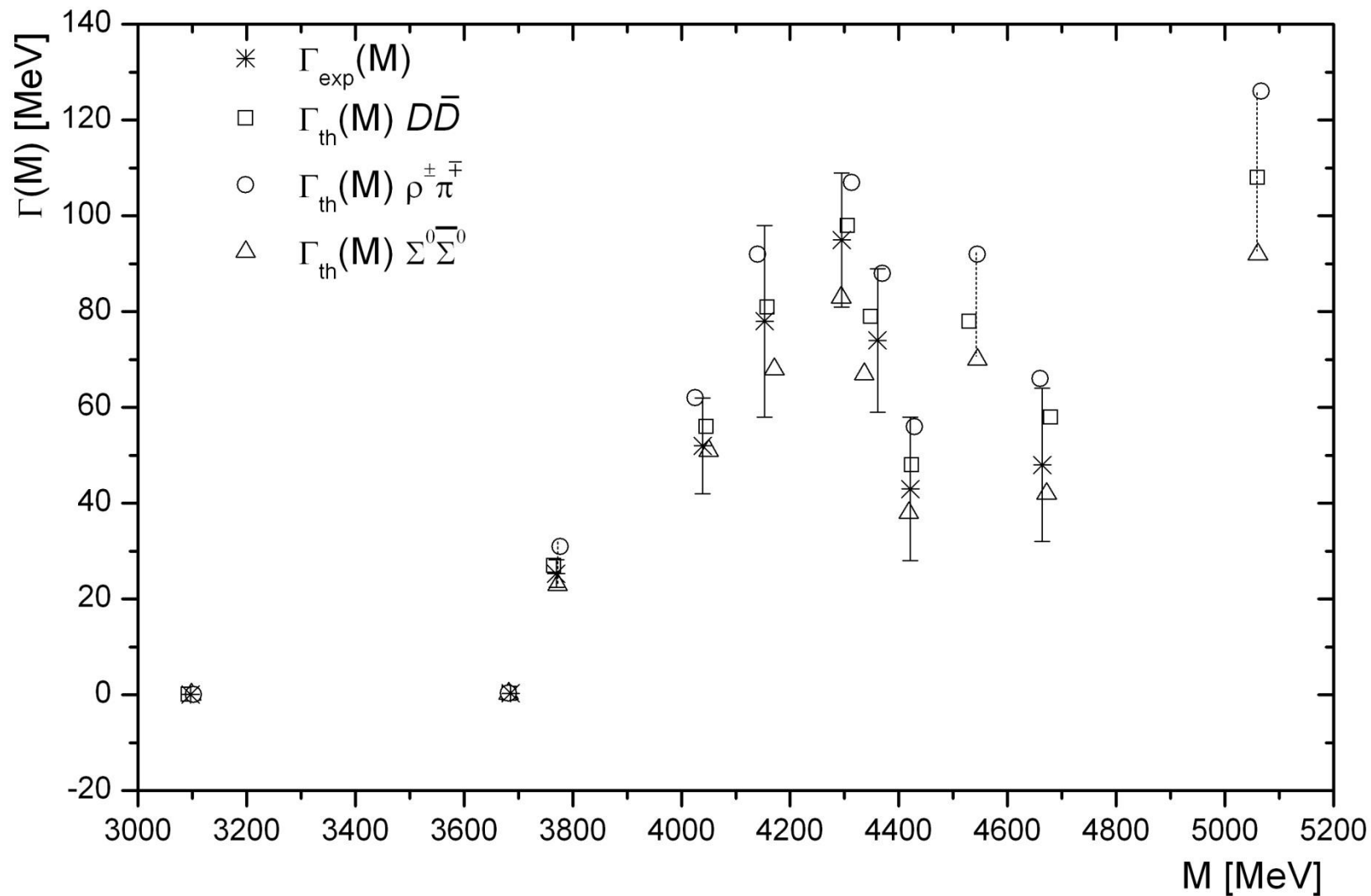
THE WIDTHS OF TETRAQUARKS WITH THE HIDDEN CHARM
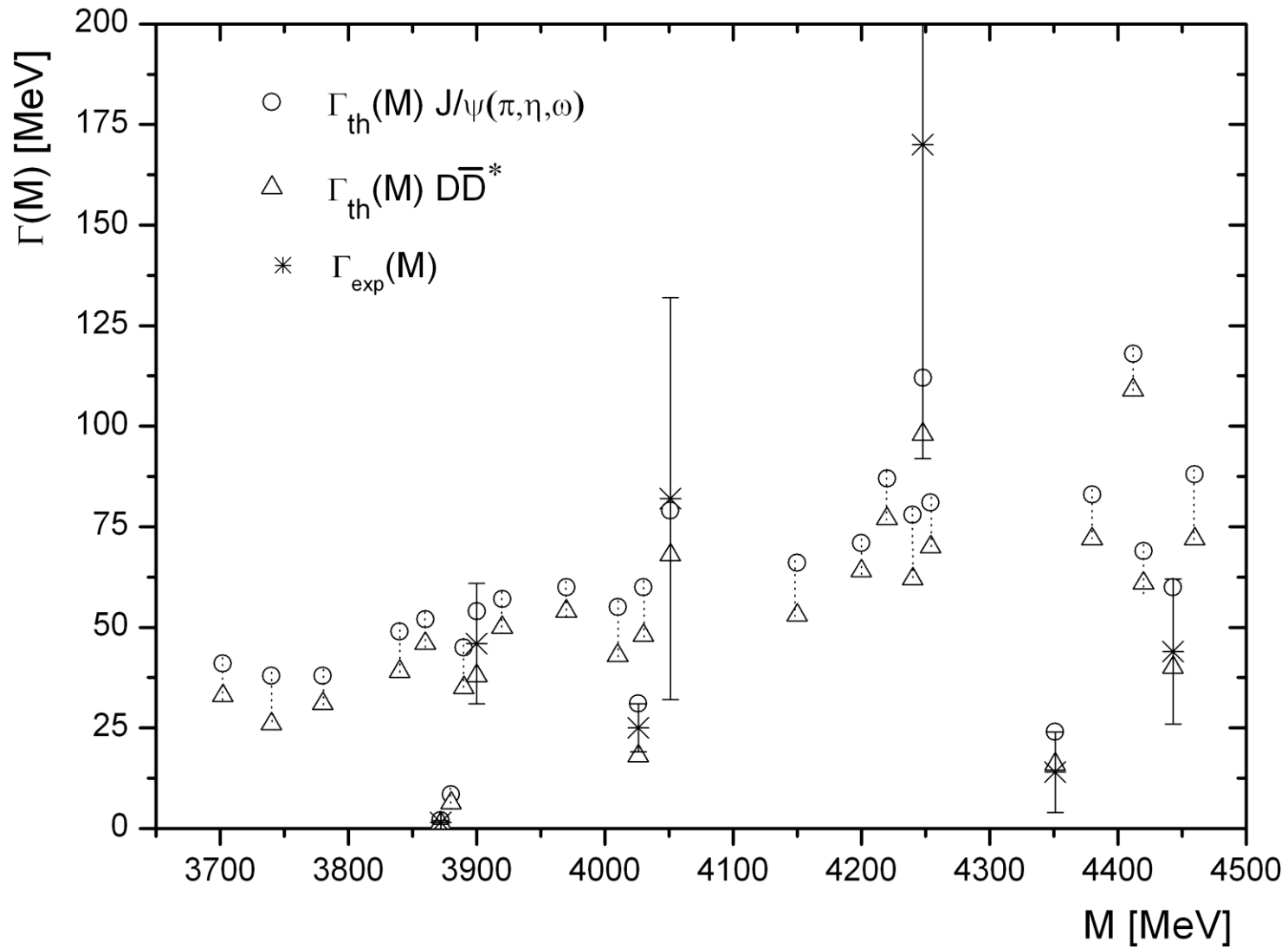
PHYSICS WITH *pp & pA* COLLISIONS:

• search for the bound states with gluonic degrees of freedom: glueballs and hybrids of the type $gg, ggg, \overline{Q}Qg, Q^3g$ in mass range from 1.3 to 5.0 GeV. Especially pay attention at the states $\overline{s}sg, \overline{c}cg$ in mass range from 1.8 – 5.0 GeV.

• charmonium-like states cc, i.e. $pp \rightarrow \overline{c}c\, pp;\ pp \rightarrow \overline{c}qcq'\, pp$ $(q, q' = u, d, s)$

• spectroscopy of heavy baryons with strangeness, charm and beauty:

$\Omega^0_c, \Xi_c, \Xi'_c, \Xi^+_{cc}, \Omega^+_{cc}, \Sigma^*_b, \Omega^-_b, \Xi^0_b, \Xi^-_b$.

$pp \rightarrow \Lambda_c X\,;\, pp \rightarrow \Lambda_c pX\,;\ pp \rightarrow \Lambda_c pD_s$ $pp \rightarrow \Lambda_b X,\, pp \rightarrow \Lambda_b pX;\ pp \rightarrow \Lambda_b pB_s$

• study of the hidden flavor component in nucleons and in light unflavored mesons such as $\eta, \eta', h, h', \omega, \varphi,\ f, f'$.

• search for exotic heavy quark resonances near the charm and bottom thresholds.

• *D*-meson spectroscopy and *D*-meson interactions: *D*-meson in pairs and rare *D*-meson decays to study the physics of electroweak processes to check the predictions of the Standard Model and the processes beyond it.

-*CP*-violation - Flavour mixing -Rare decays
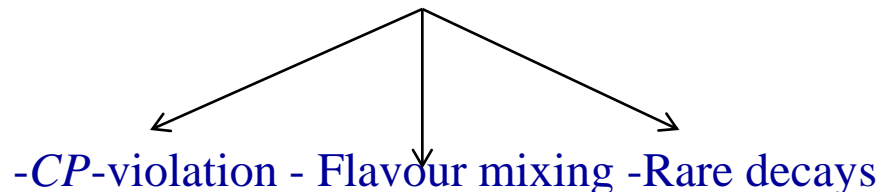
Running conditions

1. p+p at $\sqrt{s}$ = 25 GeV

2. Luminosity L = 10^{29} cm^{-2}c^{-1} - 10^{31} cm^{-2}c^{-1}
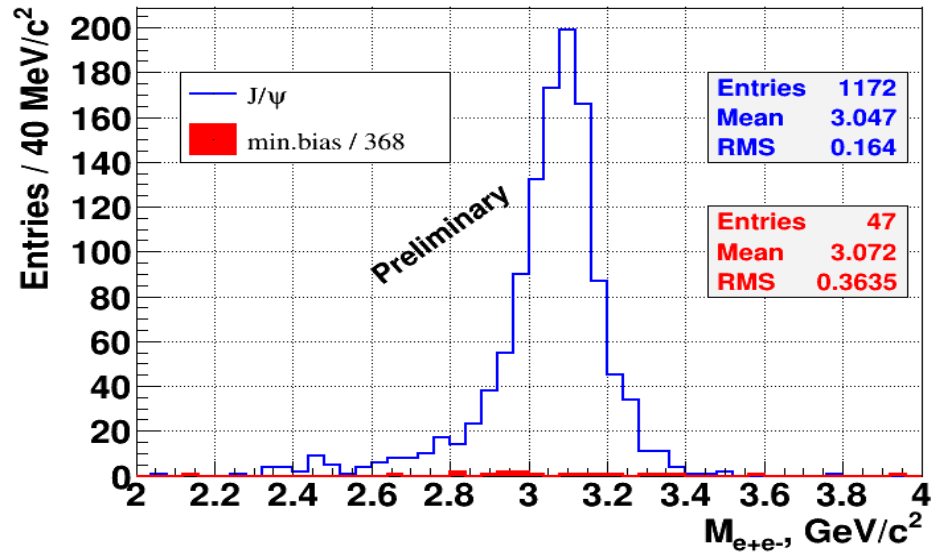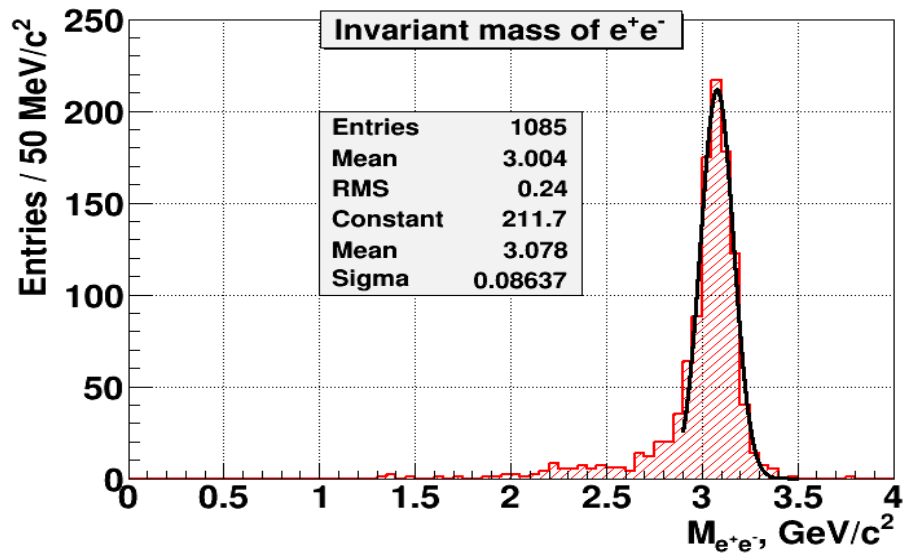
3. Running time 10 weeks:
$\qquad$ *integrated luminosity L_{int} = 604.8 nb^{-1} - 60.48 pb^{-1}*

Expectations for J/ψ

1. X-section $\sigma_{J/\psi}$ from Pythia8 108.7 nb

2. Statistics: $N_{J/\psi} = L_{int} \cdot \sigma_{J/\psi} \cdot Br_{J/\psi \to e+e-} \cdot Eff._{\Delta\eta=\pm1.5}$ =
604.8 $\cdot$ 108.7 $\cdot$ 0.06 $\cdot$ 0.8 = 3156

Invariant mass: e⁻ + e⁺

X(3872) state

1. X-section in Pythia8 for X(3872) is 4 nb (X(3872) $\equiv \psi$(3770) with mass 3.872 GeV)

2. Br (X3872$\rightarrow$J/ψ ρ^0) = 5.0%
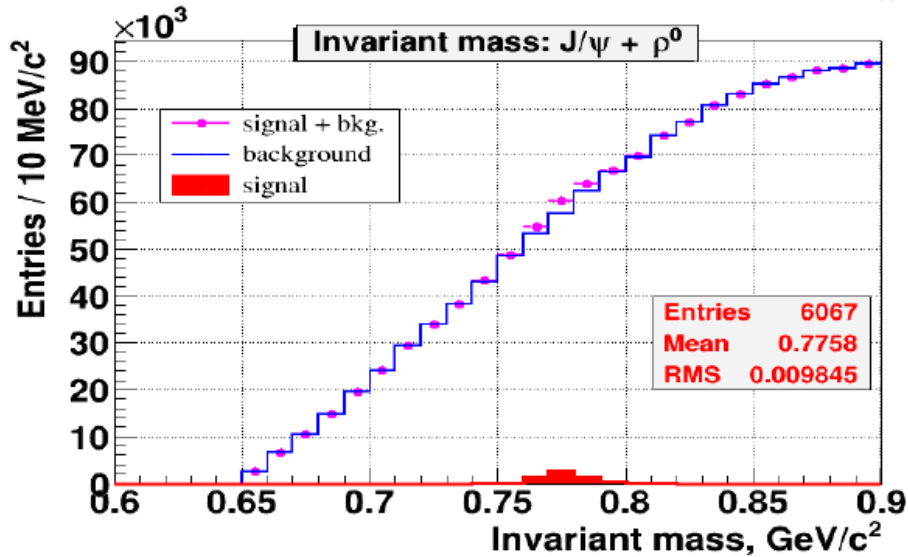   Br (X3872$\rightarrow$e+e- π+π–) = 0.3% $\rightarrow$ X-section = 12.2 pb
   1000 events at L = 10^{31} cm^{-2}s^{-1} : 95 days
   10^{32} cm^{-2}s^{-1} and 10 months: 31600 events

X(3872) → J/ψ + ρ⁰

Using mass combination: $M_{e+e-\pi+\pi-} - M_{e+e-}$
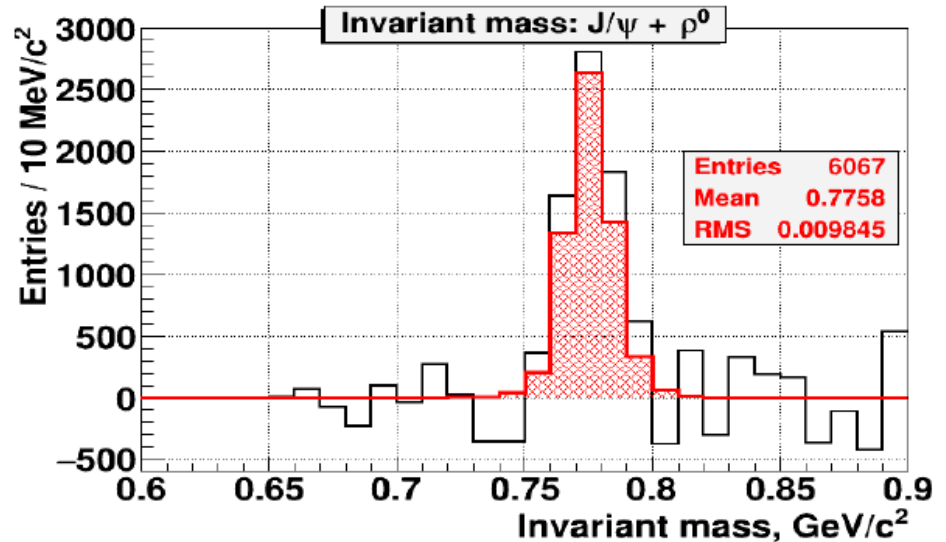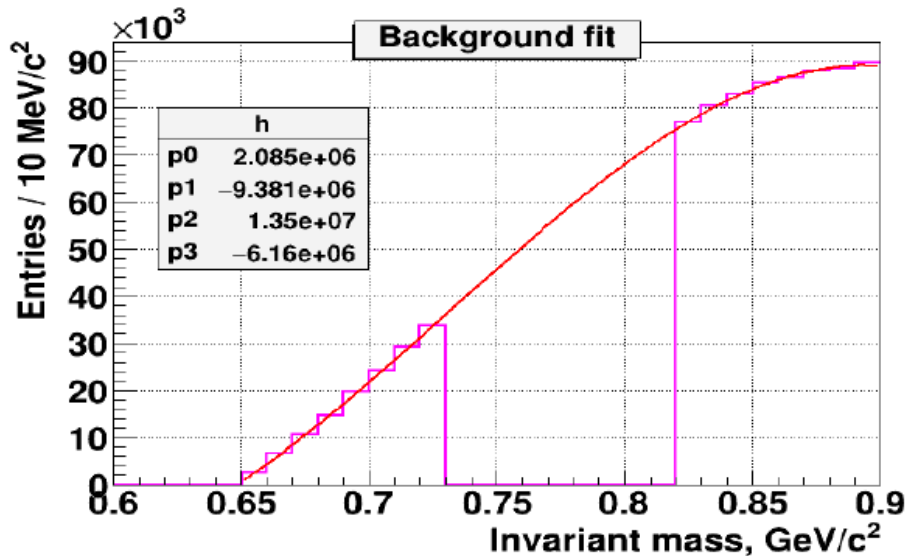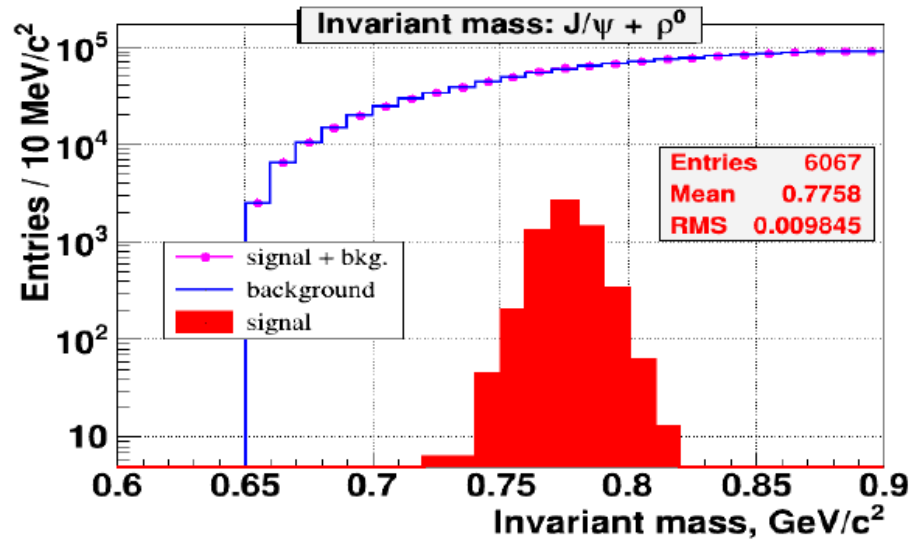
Probing the X(3872) meson structure with near-threshold pp and pA collisions at NICA

M.Yu. Barabanov[1], S.-K. Choi[2], S.L. Olsen[3†], A.S. Vodopyanov[1] and A.I. Zinchenko[1]

(1) *Joint Institute for Nuclear Research, Joliot-Curie 6 Dubna Moscow region Russia 141980*
(2) *Department of Physics, Gyeongsang National University, Jinju 660-701, Korea*
(3) *Center for Underground Physics, Institute for Basic Science, Daejeon 34074, Korea*

Pythia8 predictions for X(3872)

1. X-section of $\psi(3770)$ with m = 3.872 GeV at pp 12.5+6.5 GeV: 1.3 nb

*2. X-section at pCu: 1.3 * A (=63) = 81.9 nb*

3. Br (X(3872)$\rightarrow$J/ψ $\pi+\pi–$) = 5.00%
   Br (X(3872)$\rightarrow$D$^+$D$^-$) = 40.45%
   Br (X(3872)$\rightarrow$D^{0}D^{0}bar) = 54.55% $\Rightarrow$ D^{0}D^{0}barπ^0 = 35.29%*

4. Br (D+->K-$\pi+\pi+$) = 9.2%, Br (D0->K-$\pi+$) = 3.8%

*5. σ(pCu) * Br(J/ψ $\pi+\pi–$) * Br(e+e-) = 81.9 * 0.05 * 0.06 = 0.246 nb*
   *σ(pCu) * Br(D+D-) * Br(K$\pi\pi$)^2 = 81.9 * 0.4045 * 0.092 * 0.092 = 0.280 nb*
   *σ(pCu) * Br(D^{0}D^{0}barπ^0) * Br(Kπ)^2 = 81.9 * 0.3529 * 0.039 * 0.039 = 0.044 nb*

0.280 nb => L = 5.9 x 10^{29} (1000 events / 10 weeks)

Probably a mixture of DD̄* & a cc̄ "core"

Specific model by
Takizawa & Takeuchi, PTEP 9, 093D01

most of the time looks
like a D⁰D̄*⁰ molecule

~5%

C̄
C
produced via
this component

~90%
D̄*⁰
>7 fm
D⁰

"core"
state

$$\frac{1}{M_{X(3872)} - m_{D^0} - m_{D^{*0}} - \frac{q^2}{2\mu_0}}$$

~5%

D*⁻
≈1 fm
D⁺

C̄
C

$$\frac{1}{M_{X(3872)} - m_{D^+} - m_{D^{*-}} - \frac{q^2}{2\mu_0}}$$
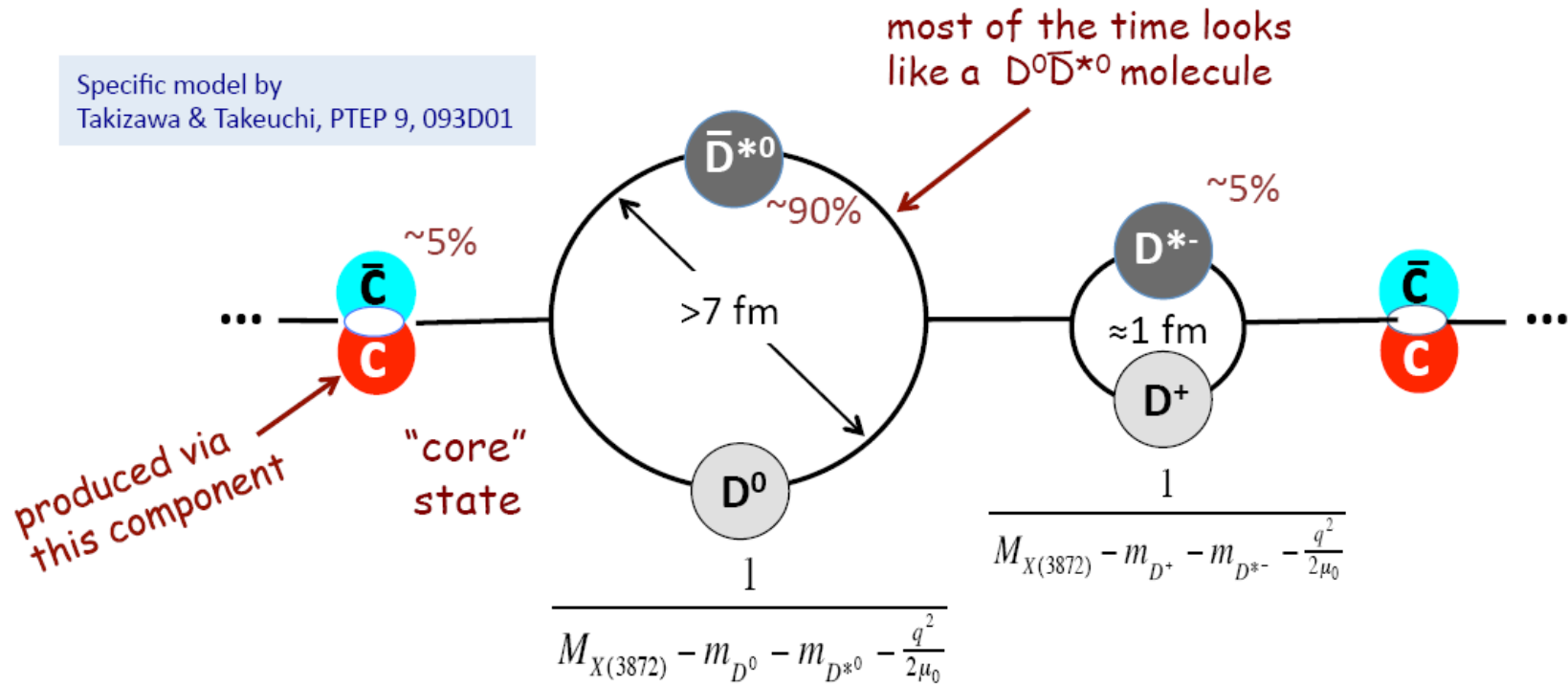
Near-threshold prod. via pp & pA
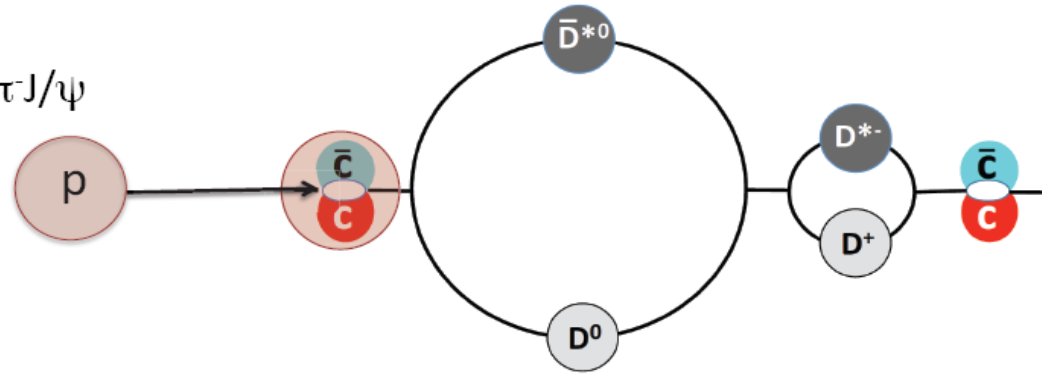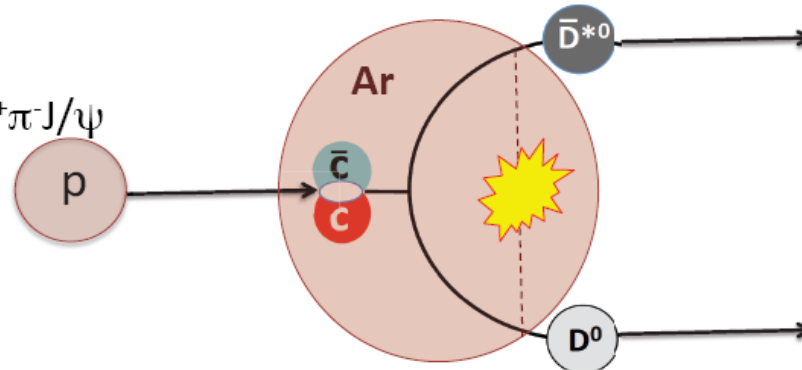
$$\sqrt{s_{pN}} \simeq 8 \text{ GeV}$$

Summary

◈ Many observed states remain puzzling and can not be explained for many years. This stimulates and motivates for new searches and ideas to obtain the nature of exotic states.

◈ A combined approach based on quarkonium potential model and confinement model has been proposed and applied to study charmonium and exotics.

◈ The most promising decay channels of charmonium-like states have been analyzed. Different charmonium-like states above DD\bar are expected to exist in the framework of the combined approach.

◈ Using the integral approach for the hadron resonance decay the widths of the expected states were calculated. They turn out to be relatively narrow of the order of several tens of MeV.

◈ Physics analysis for the antiproton-proton annihilation, proton-proton and proton-nuclear collisions is in progress nowadays. Preliminary results have been obtained.

◈ FAIR & NICA can provide important complimentary information and new discoveries. The necessity for further charmonium and exotics research has been demonstrated.

THANK YOU!

Charmonia measurements

Table 2

The ψ and ψ' cross sections in proton-induced interactions for $x_F > 0$ with the branching ratios to lepton pairs, $B(J/\psi \to \mu^+\mu^-) = (5.97 \pm 0.25)\%$ and $B(\psi' \to \mu^+\mu^-) = (0.77 \pm 0.17)\%$ [79], divided out. The ψ cross section into lepton pairs at $y = 0$, $B\,d\sigma(\psi)/dy|_{y=0}$ is also shown. All cross sections are per nucleon

| Ref.	A[a]	$\sqrt{s}$ (GeV)	$\sigma(\psi)$ (nb)	$B\,d\sigma(\psi)/dy|_{y=0}$ (nb)	$\sigma(\psi')$ (nb)	$\langle p_T^2 \rangle$ (GeV^2)
[33]	Be	6.1	$0.1^{+0.10}_{-0.05}$[b]	~ 0.01	—	—
[34]	p	6.7	0.31 ± 0.09[c]	0.055 ± 0.02	—	$0.62^{d,e}$
[1]	Be	7.3	$1^{+1.0}_{-0.5}$[b]	~ 0.1	—	$\sim 0.62^e$
[35]	p	8.6	1.2 ± 0.6	0.2 ± 0.1[f]	—	—
[36]	Be	11.5	11 ± 3[c]	1.2 ± 0.4	—	0.55 ± 0.09[c]
[37]	Be	16.8	—	5.6 ± 1.5	—	$\sim 1^e$
[38]	Be	16.8	69 ± 23	7.2 ± 2.5[g]	—	—
[9]	p	16.8	47 ± 10[h]	—	—	—
[9]	p	19.4	61 ± 11	4.1 ± 0.3	—	1.23 ± 0.05
[39][i]	C	20.5	147 ± 7	14.3 ± 1.5[g]	8.0 ± 4.5	—
[40][i]	C	20.5	95 ± 13	9.5 ± 1.0	12 ± 7	1.25 ± 0.10
[18]	Li	23.8	162 ± 22	—	16 ± 6	—
[41]	p	24.3	71.8 ± 9.3	6.2 ± 1.1	—	—
[42]	Be	27.4	110 ± 27[c]	8.9 ± 2.2	15 ± 8	0.91 ± 0.29[c]
[43]	p	30	—	9.1 ± 2.5	—	—
[44]	p	30.6	—	6.6 ± 1.8	—	—
[45]	Be	31.5	161 ± 35	8 ± 2	—	1.55 ± 0.11
[46]	p	52	—	7.5 ± 2.5	—	—
[47]	p	52	350 ± 160[c]	12 ± 5	—	1.2 ± 0.3
[48]	p	52	—	12.8 ± 3.2	—	—
[44]	p	52.7	—	11.0 ± 0.4	—	1.92 ± 0.15[e]
[43]	p	53	—	13.6 ± 3.1	—	—
[44]	p	62.4	—	10.2 ± 0.7	—	1.7 ± 0.2[e]
[43]	p	63	—	14.8 ± 3.3	—	—

Charmonia measurements

NA38

NA51

Table 4: J/ψ and ψ' absolute cross sections, in the dimuon channel, for the measured p-A reactions. Systematic uncertainties, not included, amount to 7 %.

	$\mathcal{L}$ (nb^{-1})	N^ψ	$B^\psi_{\mu\mu}\sigma^\psi$ (nb)	$B^{\psi'}_{\mu\mu}\sigma^{\psi'}$ (nb)
C	2232.5	15014 ± 140	55.8 ± 0.6	1.06 ± 0.07
Al	136.4	1851 ± 48	112.1 ± 2.8	1.52 ± 0.39
Cu (2)	63.0	2083 ± 51	267.8 ± 6.3	4.66 ± 0.31
Cu (10.1)	518.4	16522 ± 140	263.5 ± 2.4	4.58 ± 0.29
W (1.5)	25.4	1896 ± 48	606.1 ± 14.8	9.63 ± 0.77
W (5.6)	136.7	11533 ± 118	692.6 ± 7.4	11.00 ± 0.87

Target	H2	D2
N_ψ	301236 ± 601	312204 ± 630
$N_{\psi'}$	5705 ± 127	6219 ± 131
N_{DY}	1910 ± 44	2120 ± 46
$B\sigma_\psi$ (nb)	$5.50 \pm 0.01 \pm 0.36(0.06)$	$11.32 \pm 0.03 \pm 0.75(0.13)$
$B'\sigma_{\psi'}$ (nb)	$0.086 \pm 0.002 \pm 0.006(0.003)$	$0.188 \pm 0.004 \pm 0.015(0.006)$
σ_{DY} (pb)	$25.3 \pm 0.6 \pm 1.8(0.5)$	$55.0 \pm 1.2 \pm 3.9(1.2)$
$B'\sigma_{\psi'}/B\sigma_\psi(\%)$	$1.60 \pm 0.04 \pm 0.02$	$1.72 \pm 0.04 \pm 0.025$
$B\sigma_\psi/\sigma_{DY}$	$54.7 \pm 1.0 \pm 1.3$	$53.8 \pm 1.0 \pm 0.5$

Table 3: Numbers of J/ψ, ψ' and Drell-Yan events in the mass range [4.3–8.0] GeV/c^2 as well as the corresponding cross sections. B and B' are the branching ratios of the decay of J/ψ and ψ' resonances into two muons. Ratios of cross sections are also given. In the case of the ratio $B\sigma_\psi/\sigma_{DY}$, Drell-Yan pairs are taken in the mass range [2.9–4.5] GeV/c^2 in order to allow the comparison with other data from the NA38 experiment. Finally, the numbers given in parenthesis correspond to the fraction of systematic error which has to be taken into account in the comparison of the two targets.

X(3872) decay channels

strong $D\bar{D}^*$ coupling

$D^0\bar{D}^{*0}$

$\gamma J/\psi$
$\gamma\psi'$
$\pi^+\pi-J/\psi$
$\omega J/\psi$
$D^0\bar{D}^{*0}$
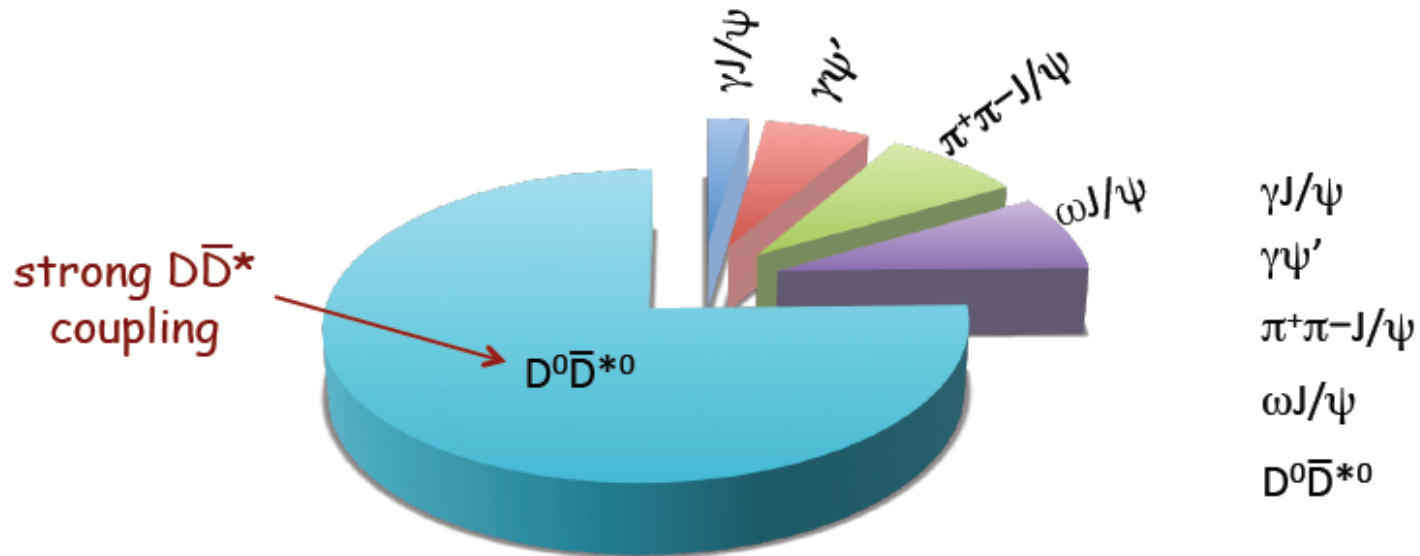
$\Gamma_{\text{"tot"}} \approx 15\ \Gamma(X(3872) \rightarrow \pi^+\pi-J/\psi)$

$\Gamma(X(3872) \rightarrow \pi^+\pi-J/\psi) < 80$ keV

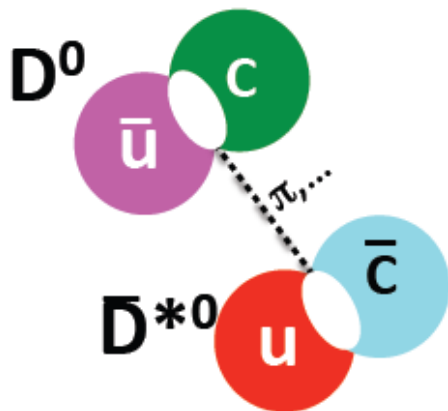$\Gamma(X(3872) \rightarrow p\bar{p}) < 0.002\Gamma(\pi^+\pi^- J/\psi) < 160 eV$

$BR(\bar{D}^{*0}D^0)\ /BR(J/\psi\pi^+\pi^-) = 9.9 \pm 3.2$

Models for the Y(3872)

D^0-$\bar{D}^{*0}$ molecule?

Lots of literature about this

D^0

$\bar{u}$ c

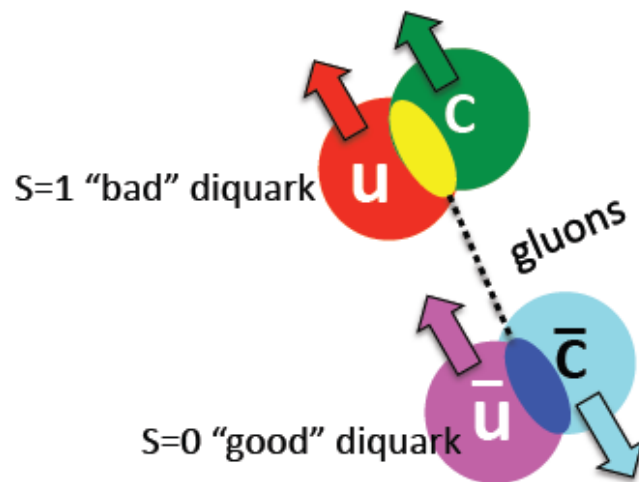$\pi, \ldots$

$\bar{D}^{*0}$

u $\bar{c}$

Impossible to produce such an
fragile extended object in prompt
high energy hadron colliders at
the rates reported by CDF & CMS

QCD diquark-diantiquark?

Maiani et al.   PRD 71, 014028 (2005)

S=1 "bad" diquark   u   c

gluons

u $\bar{c}$

S=0 "good" diquark $\bar{u}$

Predicts partner states (e.g.,
a nearby state with u→d) that
have yet be seen.

$SU(2)$ **group and quantization of momentum.**

The $SO(4)$ symmetry of the Schrodinger- Pauli problem, generated by the angular momentum

$$\vec{M} = [\vec{r} \times \vec{p}]$$

and the normalized Laplace-Runge-Lenz vector

$$\vec{A} = (-2mH)^{-1/2}(\frac{\vec{r}}{r} + (2m\alpha)^{-1}(\vec{M} \times \vec{p} - \vec{p} \times \vec{M})),$$

can be used as follows:

1) the eigenvalue problem is solved directly on making use of properties of $SU(2)$-group:

$$H = \frac{1}{2(((\vec{M} \pm \vec{A}), \vec{\sigma}) + 2\hbar)^2} \rightarrow -mc^2 \frac{\alpha^2}{2(n+1)^2}, \quad n = 0, 1, 2.$$

these eigenvalues (including degeneracy) are given by standard group theoretical arguments.

In this form $\vec{A}$ and $\vec{M}$ suffice to derive the bound states of the non-relativistic Coulomb problem.

In order to demonstrate this let us form two kinds of generators of the generators of $SO(4)$ group

$$\vec{M} = [\vec{r} \times \vec{p}], \quad \vec{N} = r_4\vec{p} - \vec{r}p_4,$$

by taking the following linear combinations

$$\vec{\mathcal{M}}_\pm = \frac{1}{2}(\vec{M} \pm \vec{N}).$$

The $SU(2)$ group generate the action on three-dimension sphere S^3. This action consists of the translation with whirling around the direction of translation.

The operator $\vec{N}$ on S^3 can be written as $\vec{N} = R\vec{p} + \vec{r}(\vec{r}\vec{p})/R$, where R is the radius of the sphere. Hamilton operator

$$H = \frac{2}{mR^2}\{\hbar + (\vec{\mathcal{M}}_\pm, \vec{\sigma})\}\{\hbar + (\vec{\mathcal{M}}_\pm, \vec{\sigma})\}$$

The spectrum of H:

$$H\Psi_n = \frac{\hbar^2}{2mR^2}(n+1)^2 \Psi_n, \quad n = 0, 1, 2, \ldots .$$

The discreteness of the energy spectrum is a consequence of the compactness of the group $SU(2)$, the space of which is the space of the solutions. When $R \to \infty$, the Hamiltonian tends to the Hamiltonian of Pauli equation. In this case

$$\mathcal{P}_\pm = (\vec{M} \pm \vec{N})/R \to \pm\vec{p},$$

The Dirac equation

$$2mH = \frac{(\vec{\sigma}\vec{\mathcal{M}} + 2\hbar)}{R}\frac{(\vec{\sigma}\vec{\mathcal{M}} + 2\hbar)}{R} \rightarrow$$

$$\rightarrow Det \begin{pmatrix} \frac{H_D}{c} - mc & \vec{\sigma}\vec{\mathcal{M}}_\pm + 2\hbar \\ \vec{\sigma}\vec{\mathcal{M}}_\pm + 2\hbar & \frac{H_D}{c} + mc \end{pmatrix} = 0 \rightarrow$$

$$\rightarrow \frac{H_D}{c}\Psi_\pm = (\vec{\alpha}\frac{\vec{\mathcal{M}}_\pm}{R} + \beta mc + \gamma_5\frac{2\hbar}{R})\Psi_\pm, \quad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.$$

$$(2.5)$$

The spectrum of this Hamiltonian is given by

$$\mathcal{E} = c\sqrt{m^2c^2 + \frac{\hbar^2(n+1)^2}{R^2}}, \quad n = 0, 1, 2, \dots .$$

From a physical point of view, it is clear that we deal with quantization of the momentum:

$$\mathcal{E} = c\sqrt{m^2c^2 + \mathcal{P}^2}, \quad \mathcal{P} = \frac{\hbar^2(n+1)^2}{R^2}, \quad n = 0, 1, 2, \dots .$$

Let us define the set of generators of $SO(4)$ group $\longrightarrow$ $\vec{M} = [\vec{r} \times \vec{p}]; \vec{N} = r_4 \vec{p} - \vec{r} p_4$

where $\vec{r}$ and $\vec{p}$ are coordinate and momentum operators, $\vec{M}$ is angular momentum operator.

Dilatation operator $\vec{N}$ defined on the sphere S^3 has the form $\longrightarrow$ $\vec{N} = R\vec{p} + \vec{r}(\vec{r}, \vec{p})/R$

The linear combinations of these orthonormal operators $\longrightarrow$ $\vec{\mu}_\pm = (\vec{M} \pm \vec{N})$

contribute two set of generators of the SU(2) group. Thus the $SU(2)$ group generates the action on a three-dimensional sphere S^3. This action consists of the translation with whirling around the direction of translation. We get a Hamiltonian:

$$H = \frac{1}{2mR^2}\{2\hbar + (\vec{\mu}_\pm, \vec{\sigma})\}\{2\hbar + (\vec{\mu}_\pm, \vec{\sigma})\}$$ where $\vec{\sigma}$ - spin operator, m - mass of the top.

When radius of the sphere: $R \to \infty$ $\longrightarrow$ $\vec{\mu}_\pm / R = (\vec{M} \pm \vec{N})/ R \to \pm\vec{p}$

the Hamiltonian tends to the Pauli operator for the free particle motion: $H = \frac{1}{2mR^2}\{2\hbar + (\vec{\mu}_\pm, \vec{\sigma})\}\{2\hbar + (\vec{\mu}_\pm, \vec{\sigma})\} \to \frac{1}{2m}(\vec{p}, \vec{\sigma})^2.$

The spectrum is:
$$H\Psi_n = \frac{\hbar^2}{2mR^2}(n+1)^2 \Psi_n, n = 0, 1, 2...$$

The wave function:
$$\Psi_n = |LSJM_J\rangle$$

was taken as eigenfunction of total momentum $\longrightarrow$ $\vec{J}^2 = ((\vec{\mu}_\pm + \vec{\sigma})/2)^2$ of the top.

* Advances in Applied Clifford Algebras, V.8, N.2, p.235-254 (1998) & V.8, N.2, p.255-270 (1998) .

In the framework of this approach in the relativistic case the Hamiltonian of a decaying resonance is defined with the equation ($R \rightarrow a+b$ is a binary decay channel):

$$H = \sqrt{m_a^2 + \frac{1}{R^2}\left((\vec{\mu}_\pm,\vec{\sigma})+2\hbar\right)^2} + \sqrt{m_b^2 + \frac{1}{R^2}\left((\vec{\mu}_\pm,\vec{\sigma})+2\hbar\right)^2}$$

were m_a and m_b are the masses of resonance decay products (particles a and b). The spectrum of the Hamiltonian is:

$$E = \sqrt{m_a^2 + \frac{\hbar^2(n+1)^2}{R^2}} + \sqrt{m_b^2 + \frac{\hbar^2(n+1)^2}{R^2}}, \quad n = 0, 1, 2...$$

Finally, the formula for resonance mass spectrum can be written in the following form (we used the system in which $\hbar = c = 1$):

$$E = M_{th} = \sqrt{m_a^2 + P_n^2} + \sqrt{m_b^2 + P_n^2} = \sqrt{m_a^2 + (nP_0)^2} + \sqrt{m_b^2 + (nP_0)^2} =$$

$$= \sqrt{m_a^2 + \left[\frac{n}{R_0}\right]^2} + \sqrt{m_b^2 + \left[\frac{n}{R_0}\right]^2}$$

where P_0 – is the basic momentum. The momentum of relative motion of decay products P_n (particles a and b in the center-of-mass system of decaying resonance) is quantized relatively P_0. R_0 is the parameter with dimension of the length conjugated to P_0.