Constraining the Symmetry Energy (Far) Above Saturation Density Using Elliptic Flow

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arXiv:1706.01300 [nucl-th]

ASYEOS-II collaboration meeting
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Transport Model

Quantum Molecular Dynamics (TuQMD):

previously applied to study:

- dilepton emission in HIC: K.Shekter, PRC 68, 014904 (2003); D. Cozma, PLB640,170 (2006); E.Santini PRC78,03410 (2008)
- EoS of symmetric nuclear matter: C. Fuchs, PRL 86, 1974 (2001); Z.Wang NPA 645, 177 (1999)
- In-medium effects and HIC dynamics: C. Fuchs, NPA 626,987 (1997); U. Maheswari NPA 628,669 (1998)

upgrades implemented in Bucharest:

- various parametrizations for the EoS: optical potential, symmetry energy PRC 88, 044912 (2013)
- various parametrizations for elastic cross-sections (also in medium ones) PLB 700, 139 (2011)
- threshold effects for baryon resonance & π meson emission/absorption PLB 753, 166 (2016)
- pion optical potential PRC 95, 014601 (2017)
- **planned**: threshold effects for reactions involving strangeness degrees of freedom

some recent upgrades (of interest for the current study):

- improved density profiles
- MST coalescence algorithm
- reduced Coulomb strength (factor 0.9)
- new Gogny type EoS parametrization

→ Softer value for L as compared to PRC88, 044912 (2013)

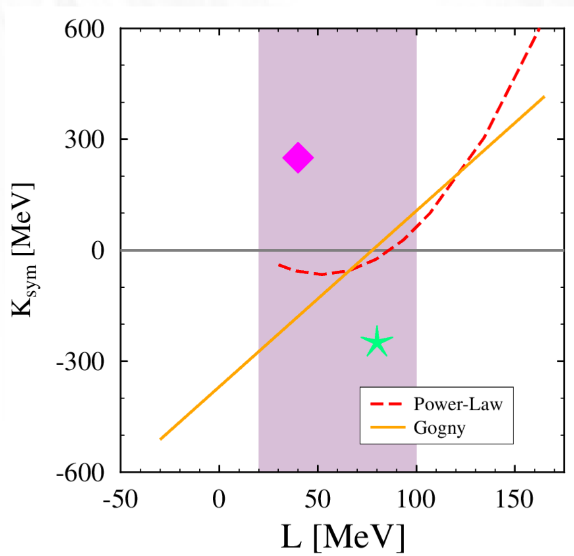
Isospin dependence of EoS

momentum dependent – generalization of the Gogny interaction:

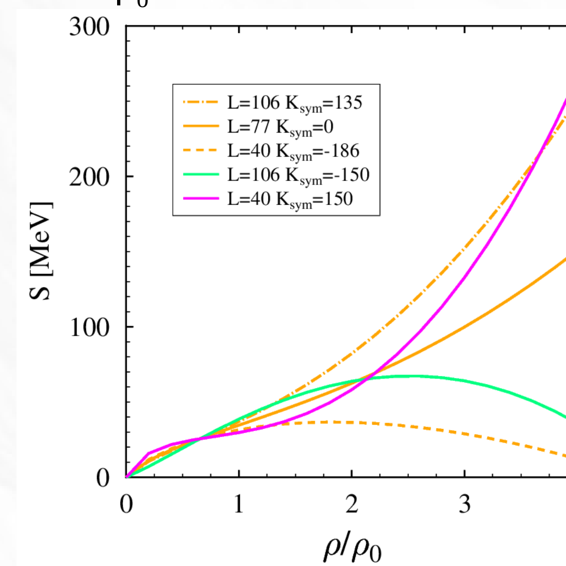
MDI potential — Das, Das Gupta, Gale, Li PRC67, 034611 (2003)

$$\frac{E}{N}(\rho,\beta,x)=\frac{1}{2}A_1 u+\frac{1}{2}A_2(x)u\beta^2+\frac{B}{\sigma+1}(1-x\beta^2)+\frac{1}{u\rho_0^2}\sum_{\tau,\tau'}C_{\tau\tau'}\int\int d^3p\,d^3p'\frac{f_\tau(p,p')f_{\tau'}(p,p')}{1+(\vec{p}-\vec{p}\,')^2/\Lambda^2}$$

$$A_2(x)=A_2^0+\frac{2xB}{\sigma+1}\bar{u}^{\sigma-1}\qquad u=\frac{\rho}{\rho_0}$$



$$S(\rho)=J+\frac{L}{3}\frac{\rho-\rho_0}{\rho_0}+\frac{K_{sym}}{18}\frac{(\rho-\rho_0)^2}{\rho_0^2}$$



$$\frac{E}{N}(\rho,\beta)=\bar{E}_s(\rho/\bar{\rho})^\gamma$$

momentum independent – power law

$$L=\frac{791}{80}\left(J-\frac{800\bar{E}_s}{791}\right)+\frac{49J}{20\bar{E}_s^2}\left(J-\frac{17\bar{E}_s}{14}\right)^2$$

$$K_{sym}=\frac{49J}{\bar{E}_s^2}\left(J-\frac{17\bar{E}_s}{14}\right)^2-\frac{9J}{4}$$

D. Blaschke et al. ArXiv:1604.08575

$$K_A\sim K_{vol}(1+cA^{-1/3})+K_\tau((N-Z)/A)^2+K_{Coul}Z^2A^{-4/3}$$

$$K_\tau=K_{sym}-6L-\frac{J_0}{K_0}L$$

results from experiment/theory

GMR: K_τ=-550±100 MeV T.Li et al. PRL 99, 162503 (2007)

-590±220 MeV J.Stone et al. PRC 89, 044316 (2014)

Higher Order Effects: -370±120 MeV L.W. Chen PRC 80, 014322 (2009)

The Gogny interaction

short + intermediate range interactions
zero range (dd) – needed to describe saturation properties

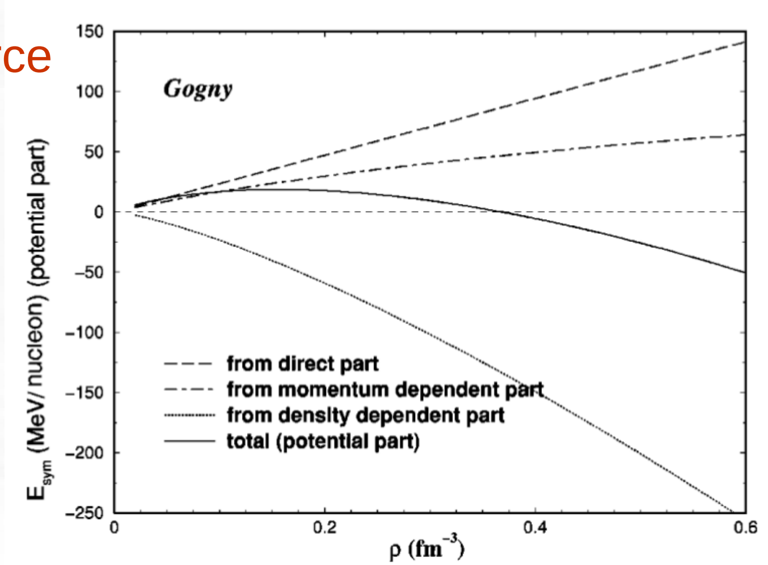
$$V(r)=\sum_i (W+BP_\sigma-HP_\tau-MP_\sigma P_\tau)_i e^{-\vec{r}^2/\mu_i^2}$$
$$+t_0(1+x_0P_\sigma)\rho^\alpha\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)\delta(\vec{r}_1-\vec{r}_2)$$
$$+iW_{LS}(\sigma_1+\sigma_2)(\overleftarrow{\nabla_1-\nabla_2})\delta(\vec{r}_1-\vec{r}_2)\overrightarrow{\nabla_1-\nabla_2}$$

3-body force

Decharge & Gogny, PRC 21, 1568 (1980)

total potential energy

$$V_T=\frac{1}{2}\sum_{p_1\sigma_1\tau_1p_2\sigma_2\tau_2}\langle\vec{p}_1,\sigma_1,\tau_1,\vec{p}_2,\sigma_2,\tau_2|v(r)$$
$$\times(|\vec{p}_1,\sigma_1,\tau_1,\vec{p}_2,\sigma_2,\tau_2\rangle-|\vec{p}_2,\sigma_2,\tau_2,\vec{p}_1,\sigma_1,\tau_1\rangle)$$



Das, Das Gupta, Gale, Li PRC67, 034611 (2003)

short-distance spin and spatial structure of 3n interaction
→ impacts S_0 and maximum mass of NS (L-S_0 -correlated linearly)

Gandolfi et al., PRC 85, 032801R (2012)
Steiner, Gandolfi, PRL 108, 081102 (2012)

New potential (MDI2)

momentum dependent potential **MDI2**

$$\frac{E}{N}(\rho,\beta,x,y)=\frac{1}{2}A_1 u+\frac{1}{2}A_2(x,y)u\beta^2+\frac{Bu^\sigma}{\sigma+1}(1-x\beta^2)+\frac{Du^2}{3}(1-y\beta^2)$$

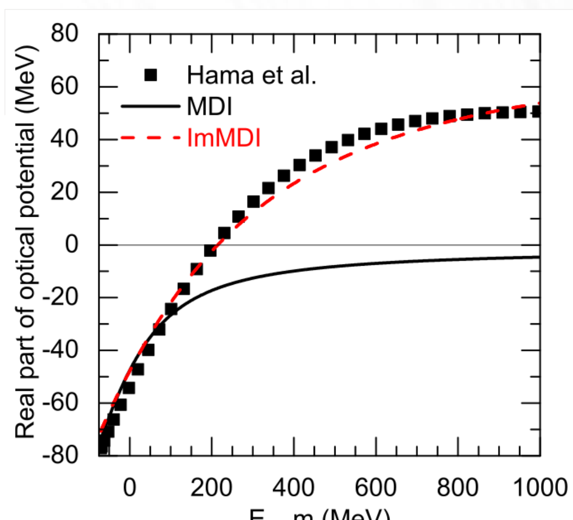
$$+\frac{1}{u\rho_0^2}\sum_{\tau,\tau'}C_{\tau\tau'}\int\int d^3p\,d^3p'\,\frac{f_\tau(p,p')f_{\tau'}(p,p')}{1+(\vec{p}-\vec{p}\,')^2/\Lambda^2}$$

$$A_2(x,y)=A_2^0+\frac{2xB}{\sigma+1}\bar{u}^{\sigma-1}+\frac{2yD}{3}\bar{u} \qquad u=\frac{\rho}{\rho_0}$$

Fit:
U_∞,K,J_0,m* -isoscalar
S(ũ),L,K_{sym},δm_{isv} -isovector

A_l^0, x, y $\qquad C_l - C_u$ $\qquad \delta^*_{n-p}$

momentum dependent part: similar with that of J. Xu et al. PRC 91, 014611 (2015)
(see also C. Hartnack, J. Aichelin PRC 49, 2801 (1994))
used previously to test model dependence: flow ratio PRC 88, 44912 (2013)
pion multiplicity ratio PLB 753, 166 (2016)
independent part: extra term (vary L vs. K_{sym} and also J_0 vs. K independently)



from J. Xu et al. PRC 91, 014611 (2015)

Input		Parameters	
ρ_0 [fm^{-3}]	0.16	Λ [MeV]	708.001
E_B [MeV]	-16.0	C_l [MeV]	-13.183
m_s^*/m	0.70	C_u [MeV]	-140.405
δ^*_{n-p} (ρ_0, β =0.5)	0.165	B [MeV]	137.305
K_0 [MeV]	245.0	σ	1.2516
J_0 [MeV]	-350.0	$\tilde{A}_l$ [MeV]	-130.495
$\tilde{\rho}$ [fm^{-3}]	0.10	$\tilde{A}_u$ [MeV]	-8.828
S($\tilde{\rho}$) [MeV]	25.4	D [MeV]	7.357

Coalescence Algorithm

Minimum spanning tree (MST) phase-space algorithm

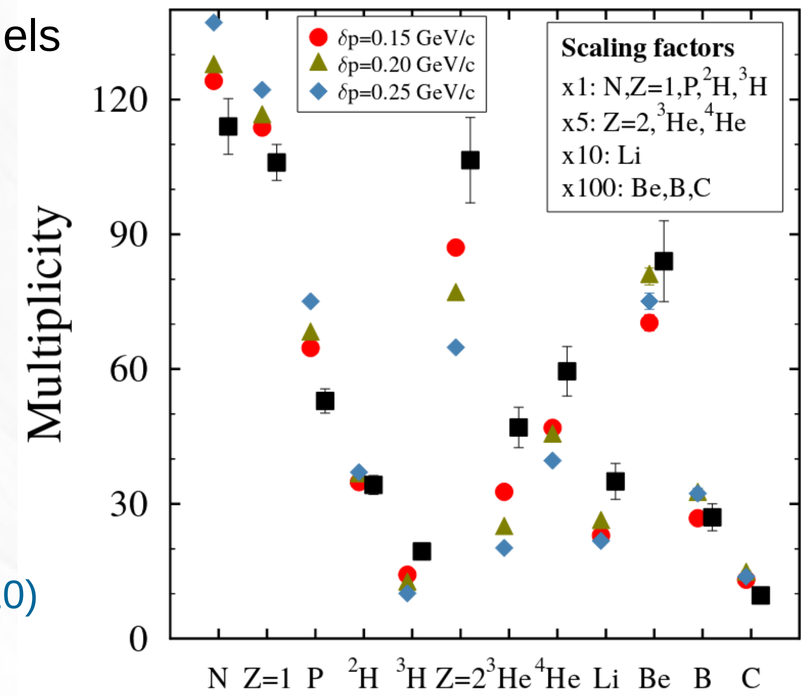
All clusters with A≤15, 23 additional A>15 (B,C,N,O)
Stable : lifetime > 1ms
Unstable : decay into stable using known decay channels
Ex: $^{4}Li \rightarrow p + {}^{3}He$
unknown/unphysical: evaporate n/p until a known nucleus is reached
Ex: $1p8n \rightarrow 6n + {}^{3}H$
All other clusters discarded

Multiplicities of p,n,^{2}H, ^{3}H, ^{3}He, ^{4}He, Li, Be, B, C **fitted** to FOPI exp data

$\delta p_n = \delta p_p = 0.2$ GeV/c fixed
δr_{pp}, δr_{pn}, δr_{nn} adjusted

W. Reisdorf et al, NPA 848, 366 (2010)

Au+Au @ 400 MeV/nucleon
b< 2.0 fm



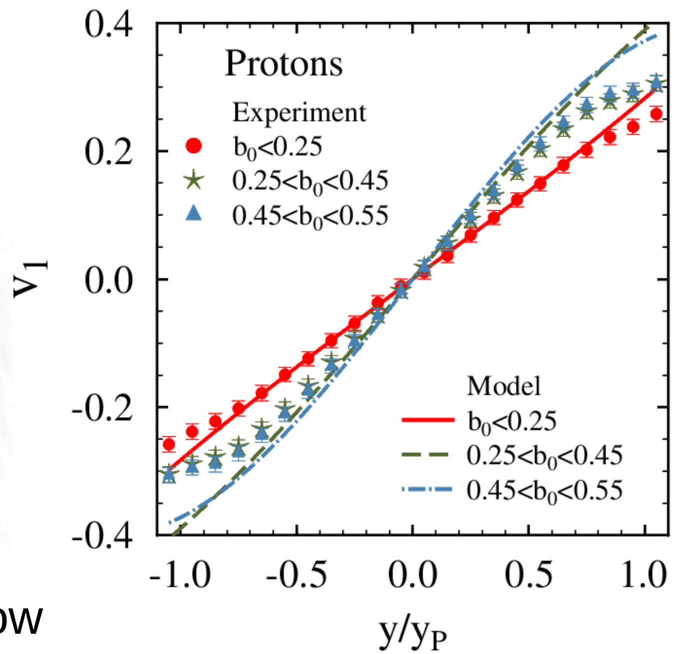
δp [GeV/c]	δr_{pp} [fm]	δr_{np} [fm]	δr_{nn} [fm]
0.15	6.25	6.83	5.77
0.20	2.75	4.25	4.25
0.25	1.75	2.99	2.75

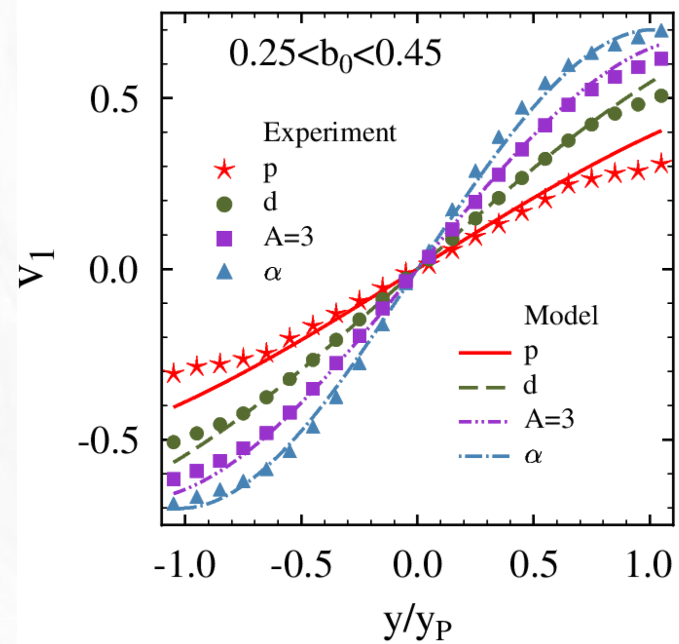
Comparison with FOPI data

W. Reisdorf, NPA 848, 366 (2010)

Transverse flow

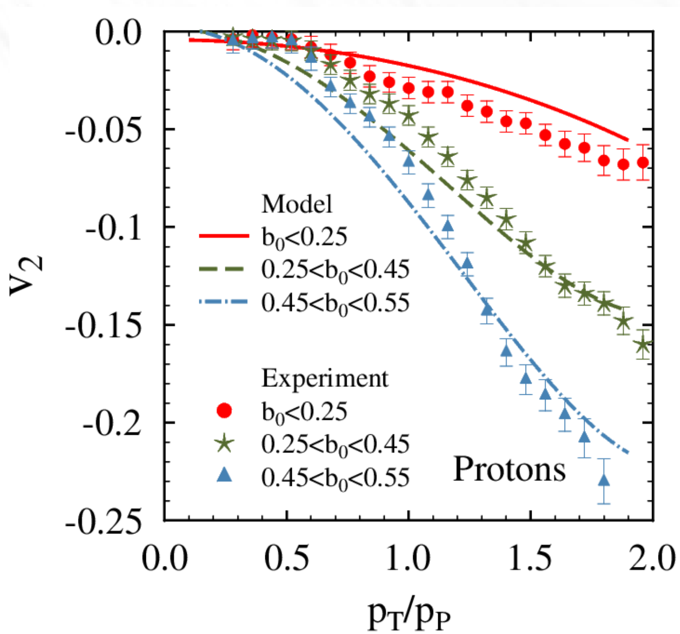
Au+Au @ 400 MeV/nucleon

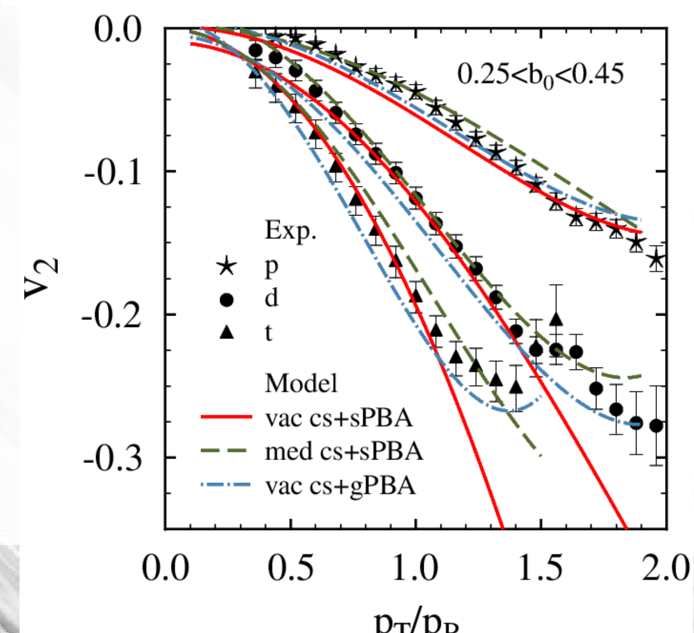




L=80 MeV
K_{sym}=0 MeV

Elliptic flow



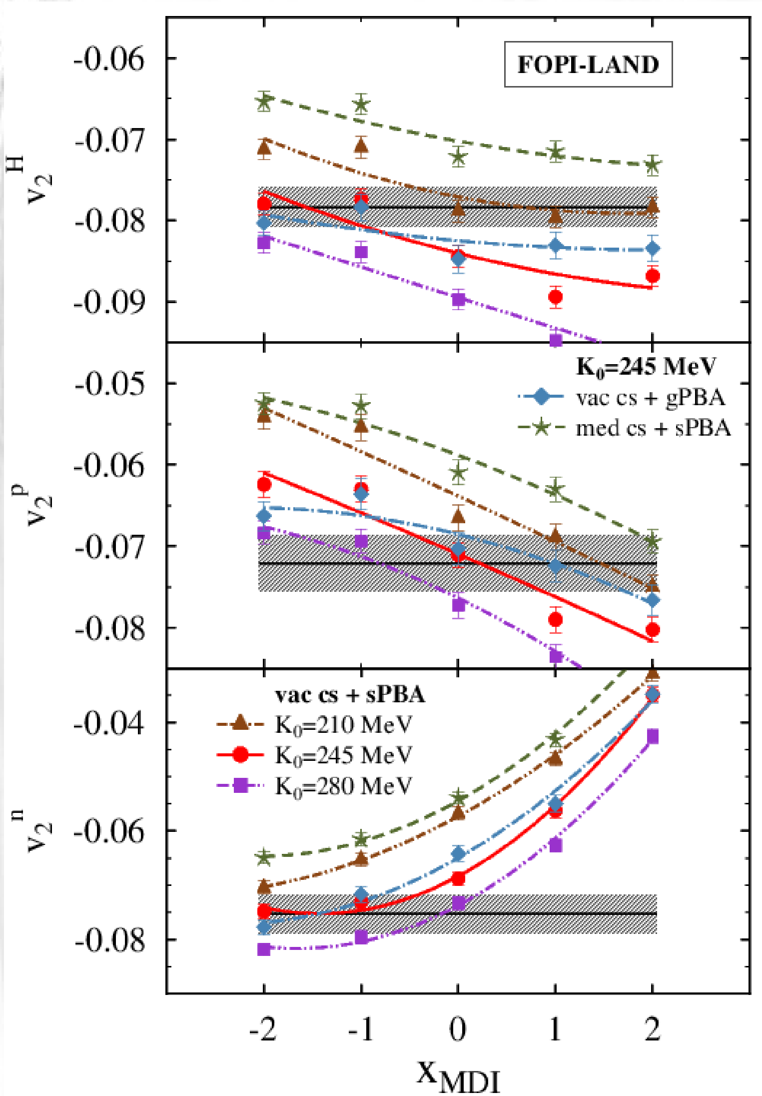


FOPI-LAND/ASYEOS data

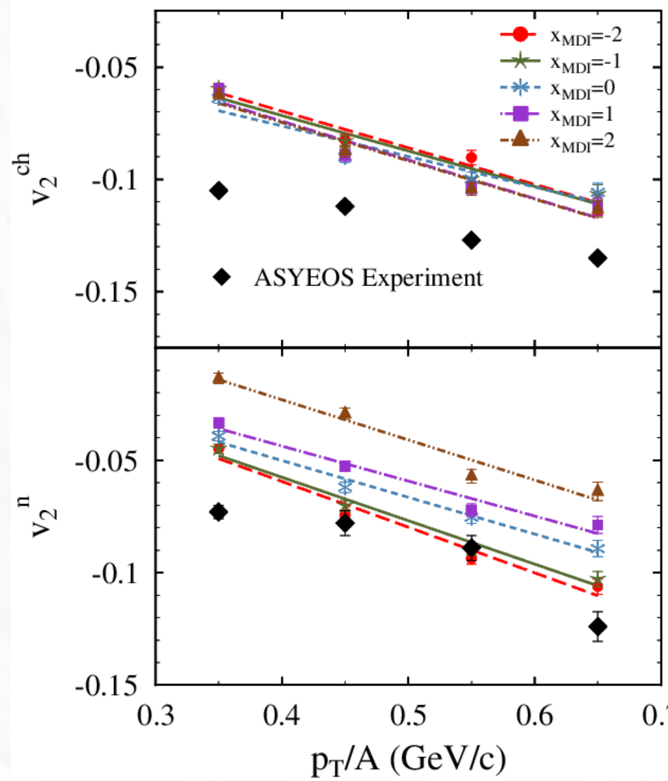
Au+Au @ 400 MeV/nucleon
b<7.5 fm

P. Russotto et al, PRC 94, 034608 (2016)

Y. Leifels et al, PRL 71, 963 (1993)



x_{MDI}	L [MeV]	K_{sym} [MeV
-2	151	349.0
-1	106	135.0
0	61	-81.0
1	15	-298.0
2	-31	-512.0



Systematical underestimation of v_2^{ch} (v_2^H)

$$\tilde{v}_2^H = \frac{M_p^{exp}\, v_2^p + M_d^{exp}\, v_2^d + M_t^{exp}\, v_2^t}{M_p^{exp} + M_d^{exp} + M_t^{exp}}$$

$$\tilde{v}_2^{ch} = \frac{M_p^{exp}\, v_2^p + \sum_{Z_i \geq 1, N_i \geq 1} M_{Z_i,N_i}^{exp}\, v_2^{Z_i,N_i}}{M_p^{exp} + \sum_{Z_i \geq 1, N_i \geq 1} M_{Z_i,N_i}^{exp}}$$

$$f_{corr}^H = \tilde{v}_2^H / v_2^H$$

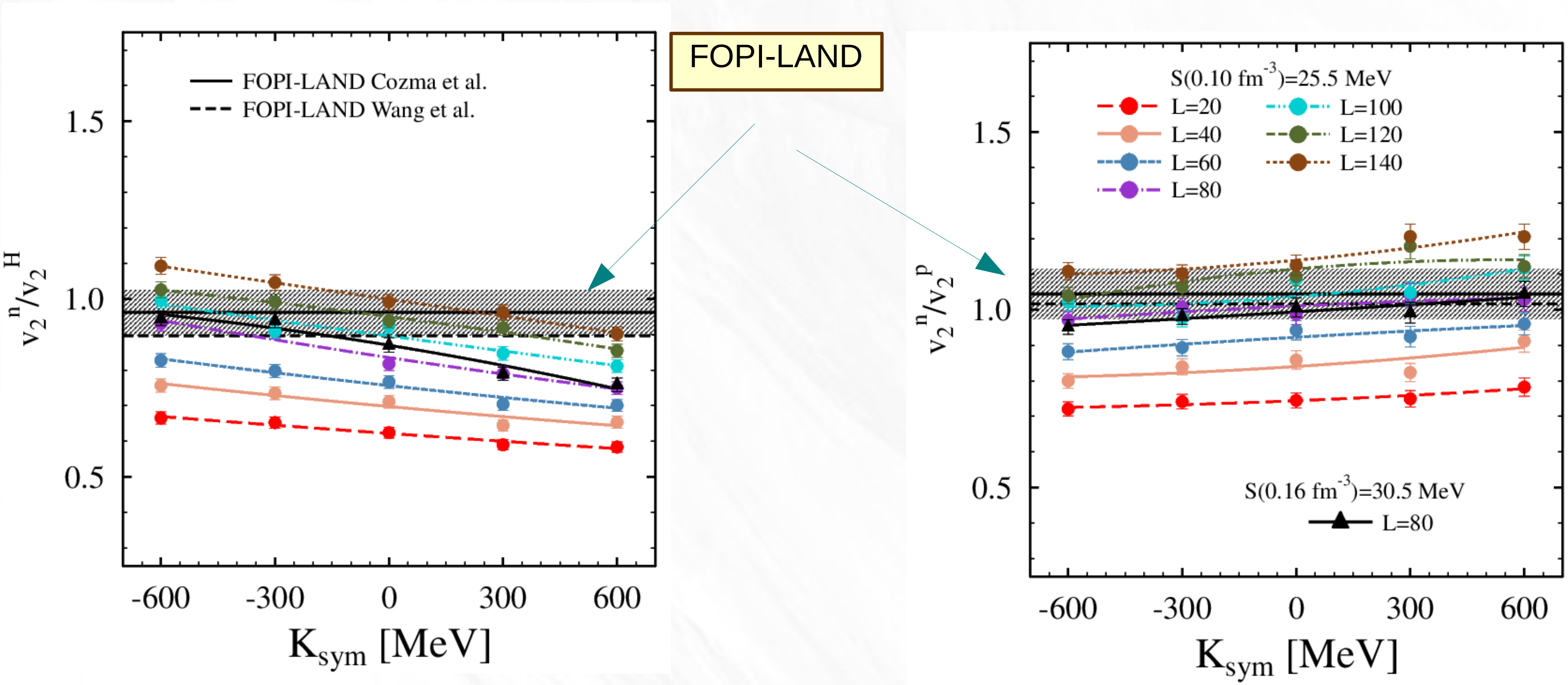
$$f_{corr}^{ch} = \tilde{v}_2^{ch} / v_2^{ch}$$

$$f_{corr}^H = 1.075 \pm 0.05$$

$$f_{corr}^{ch} = 1.10 \pm 0.05$$

Sensitivity to K_{sym} (and J)

3 dimensional parameter space: heavy-ion observables → 1 dim constraint
+nuclear structure → determine values



$S(0.10\,fm^{-3}) = 25.5\,MeV$

B.A. Brown, PRL111, 232502 (2013)

n/p and n/H (n/ch) elliptic flow ratios probe on average different densities

1.4-1.5 ρ_0 1.0-1.1 ρ_0

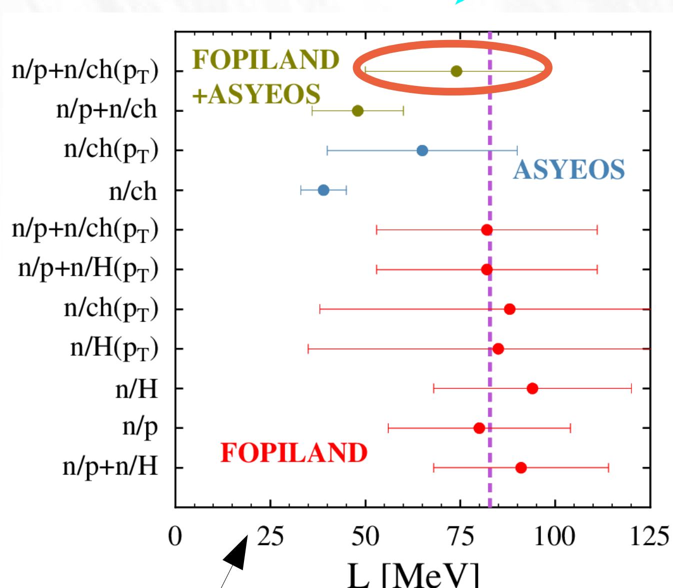
P. Russotto et al, PRC 94, 034608 (2016)

Constraints for L and K_{sym}

cMDI2

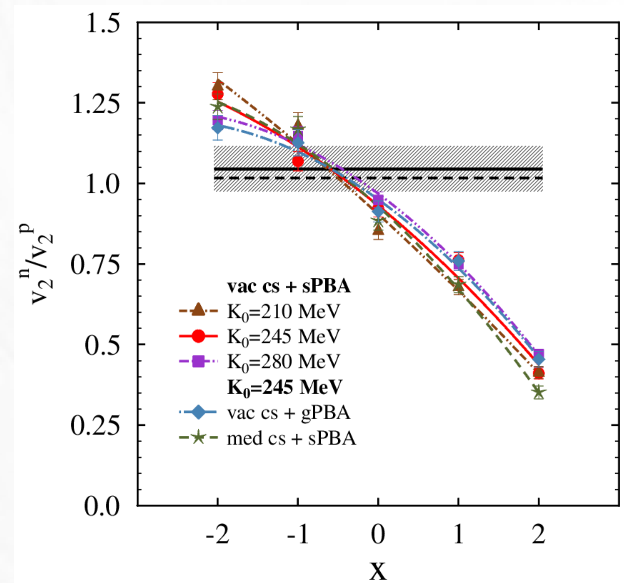
only experimental errors included
charged-particles flow uncorrected (mult ratios)

L	=	74 ±24 MeV
K_{sym}	=	-17 ± 114 MeV
J	=	35.1 ± 2.1 MeV

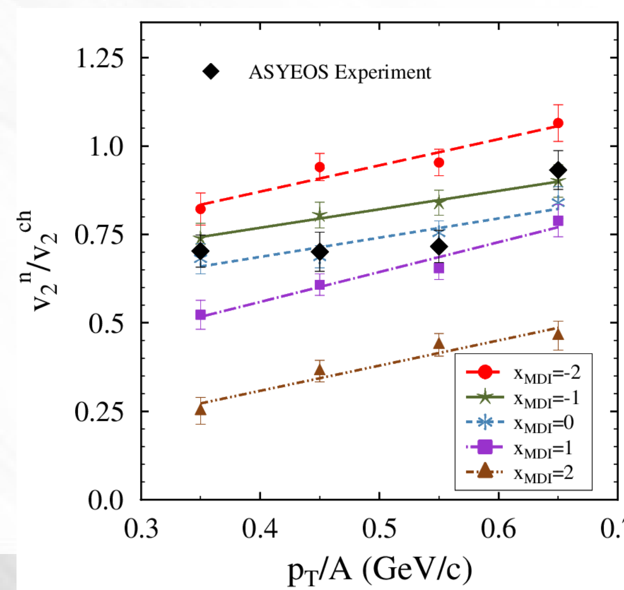


L and K_{sym} constrained as for MDI
vaccum Li-Machleidt elastic cs
K_0=245 MeV

FOPI-LAND



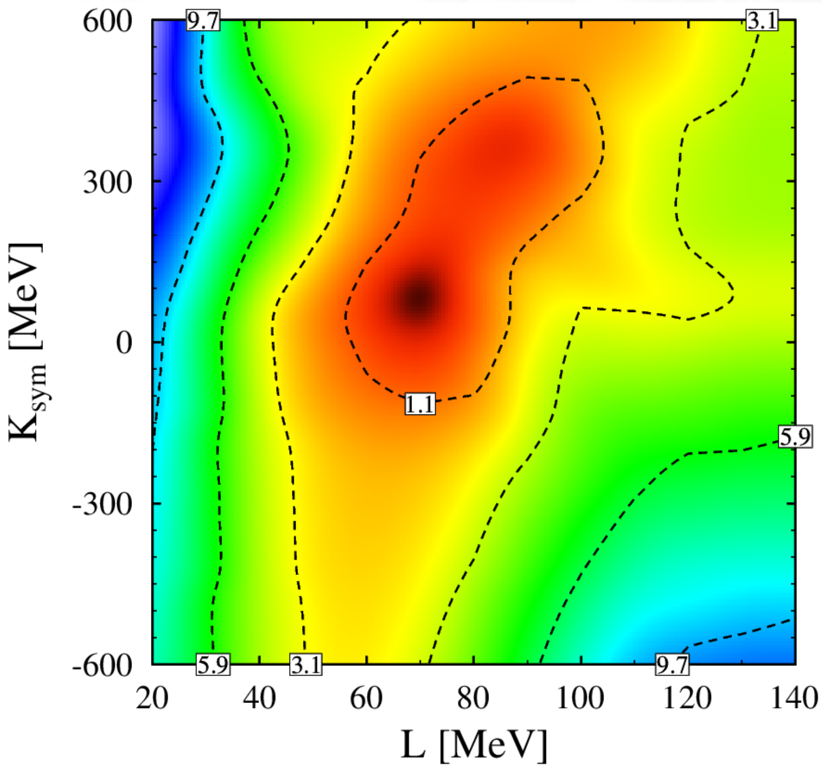
ASYEOS



Constraints for L and K_{sym}

MDI2

n/p + n/ch (p_T) elliptic flow ratios

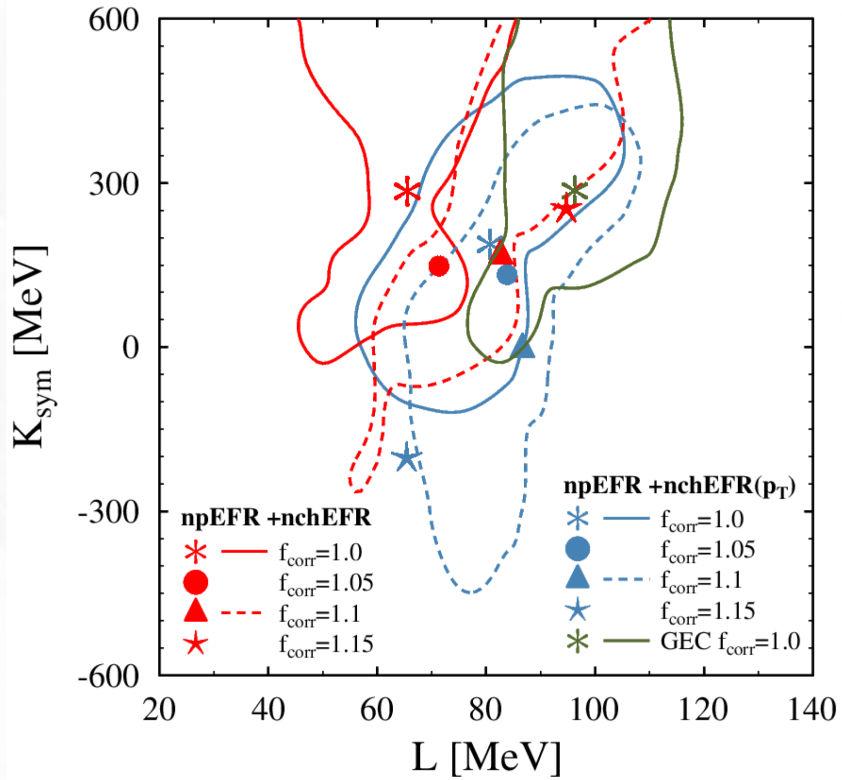


uncorrected n/p +n/ch(p_T)

L = 81 ±24 MeV
K_{sym} = 188 ± 307 MeV

uncorrected n/p +n/ch

L = 66 ±20 MeV
K_{sym} = 285 ± 315 MeV



combined corrected

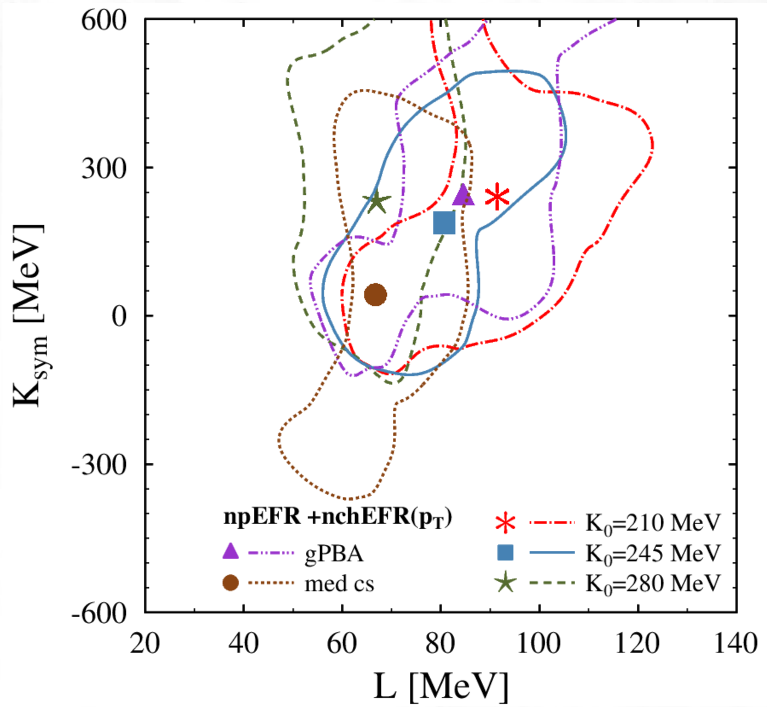
L = 85 ±22 MeV
K_{sym} = 188 ± 315 MeV

GEC – enforce total energy conservation in the collision term
- needed to explain π^-/π^+ multiplicity ratio

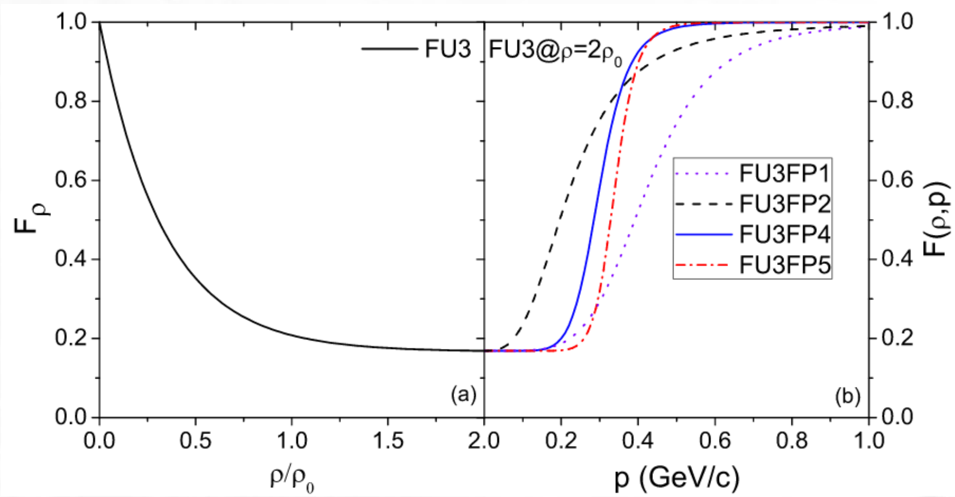
D.C. PLB 753, 166 (2016); PRC 95, 014601 (2017)

Model dependence (MDI2)

Isoscalar compressibility modulus



Empirical in-medium NN cross-sections



FU3FP4 Y. Wang et al., PRC 89, 034606 (2014)

Values for K_0

HIC: flow observables 210-230 MeV

Y.Wang et al., PRC 89, 034606 (2014);
A. Le Fevre et al. NPA 945, 112 (2016)

ISGMR: non-relativistic Skyrme/Gogny: 230-240 MeV
relativistic EDF: 250-270 MeV G. Colo et al., PRC 70, 024307 (2004)

Pauli blocking algorithm: gaussian Wigner one-nucleon distribution

$$P_{ij} = Erfc\left(\frac{|\boldsymbol{r}_i - \boldsymbol{r}_j|}{\sqrt{2L_N}}\right) Erfc\left(\frac{|\boldsymbol{p}_i - \boldsymbol{p}_j|}{4} \frac{\sqrt{2L_N}}{\hbar}\right)$$

Model dependence (MDI2)

Isovector n-p effective mass difference

N-A scattering

Δm*=(0.32±0.15)β C.Xu et al. PRC 82, 054607 (2015)

Δm*=(0.41±0.15)β X.H. Li et al. PRB 743, 408 (2015)

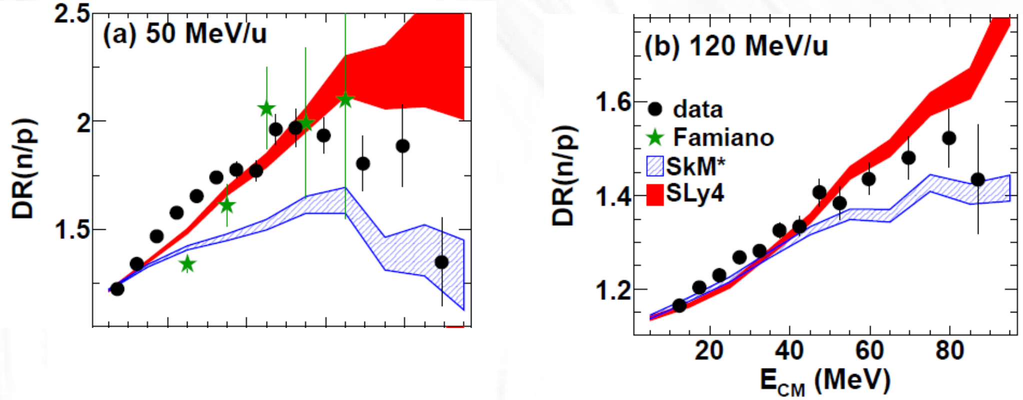
IVGDR,EDP, ISGQR

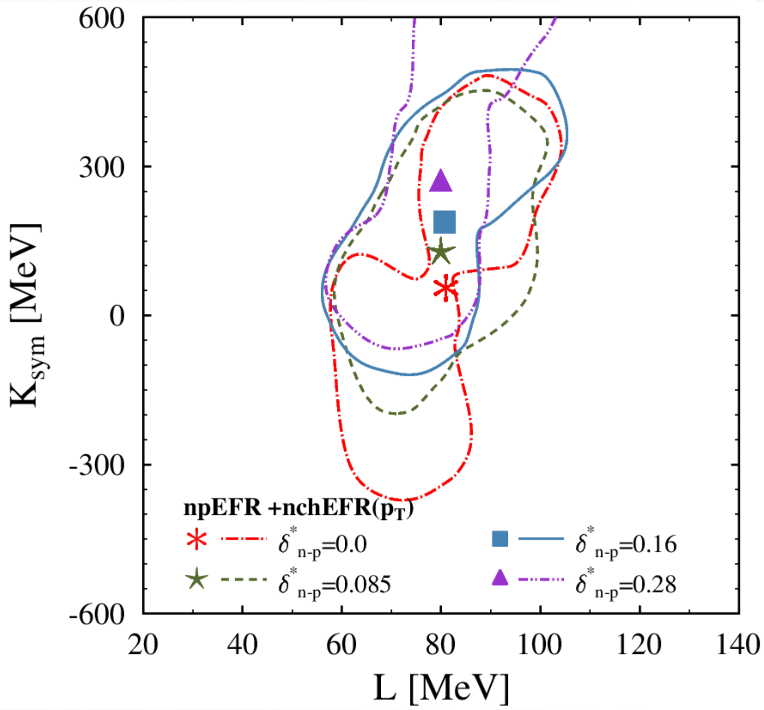
Δm*=(0.33±0.16)β Z.Zhang et al. PRC 93, 034335 (2016)

HICs: DR of n/p in Sn+Sn

Δm*<0.0

D.D.S.Coupland et al., PRC 94, 011601 (2016)





$$m_s^{*\,2}(\rho,p) = \frac{m^2 - 2Ep\frac{\partial V}{\partial p} + E^2\left(\frac{\partial V}{\partial p}\right)^2}{\left[1+\frac{E}{p}\frac{\partial V}{\partial p}\right]^2}$$

$$\delta m^*_{n-p}(\rho,\beta,p) \equiv \frac{m^*_n - m^*_p}{m} \approx \frac{\frac{E}{p}\left(1+\frac{p^2}{2m^2}\right)\left(\frac{\partial V^p}{\partial p} - \frac{\partial V^n}{\partial p}\right)}{1+\frac{E}{p}\left(\frac{\partial V^p}{\partial p} + \frac{\partial V^n}{\partial p}\right)}$$

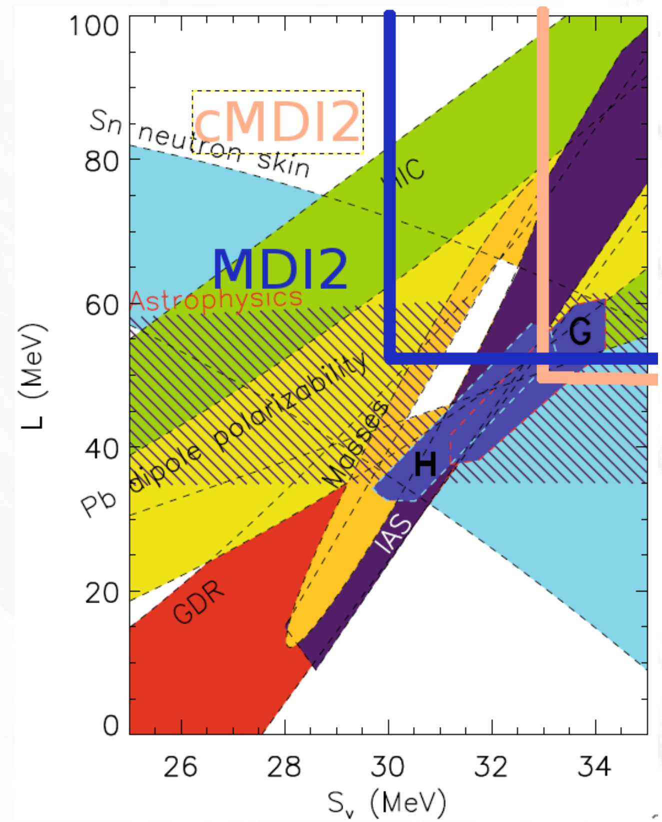
Constraints for L and K_{sym}

S_0= 36±3 MeV (cMDI2)
S_0= 36±6 MeV (MDI2)

- Free of systematical uncertainties (cMDI2)
  neutron-proton v_2^n/v_2^p

J.M.Lattimer & A.W. Steiner, EPJA 50, 40 (2014)

$$L = 84 \pm 30(\text{exp}) \pm 18(\text{th})\ \text{MeV}$$
$$K_{sym} = 30 \pm 142(\text{exp}) \pm 85(\text{th})\ \text{MeV}.$$



- Full MDI2 freedom

neutron-proton v_2^n/v_2^p +neutron-charged part. v_2^n/v_2^{ch}

$$L = 85 \pm 22(\text{exp}) \pm 20(\text{th}) \pm 12(\text{sys})\ \text{MeV}$$
$$K_{sym} = 96 \pm 315(\text{exp}) \pm 170(\text{th}) \pm 166(\text{sys})\ \text{MeV}$$

- Isovector compressibility: $K_\tau = K_{sym} - 6L - \frac{J_0}{K_0} L$

$$K_\tau = -354 \pm 228\ \text{MeV}(\text{cMDI2})$$
$$K_\tau = -290 \pm 421\ \text{MeV}(\text{MDI2}).$$

Literature:
ISGMR: -500±100 MeV
Gogny interaction: -370±100 MeV

Constraints for L and K_{sym}

- Free of systematical uncertainties (cMDI2) neutron-proton v_2^n/v_2^p

$$L = 84 \pm 30(\mathrm{exp}) \pm 18(\mathrm{th})\ \mathrm{MeV}$$
$$K_{sym} = 30 \pm 142(\mathrm{exp}) \pm 85(\mathrm{th})\ \mathrm{MeV}.$$

- Full MDI2 freedom

neutron-proton v_2^n/v_2^p +neutron-charged part. v_2^n/v_2^{ch}

$$L = 85 \pm 22(\mathrm{exp}) \pm 20(\mathrm{th}) \pm 12(\mathrm{sys})\ \mathrm{MeV}$$
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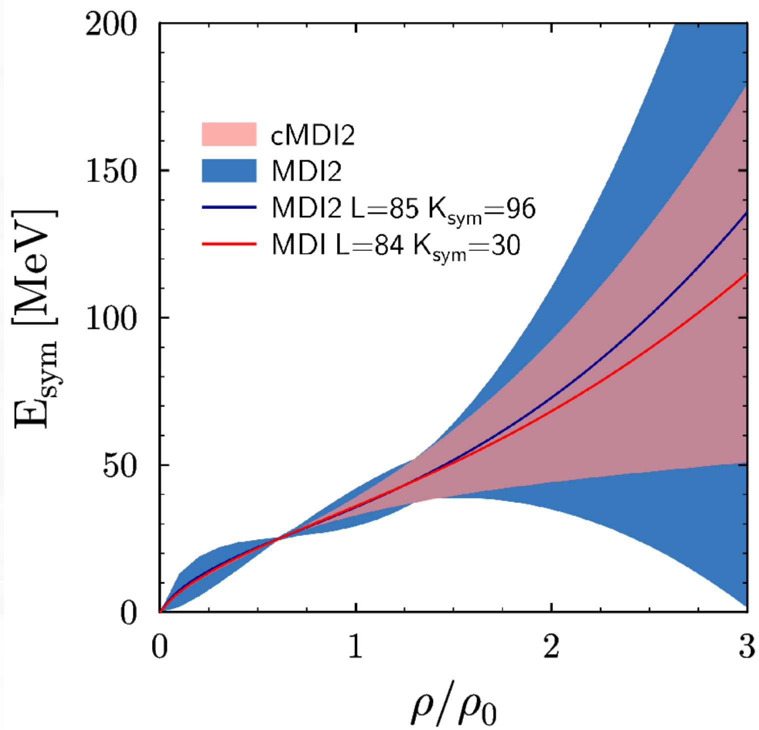

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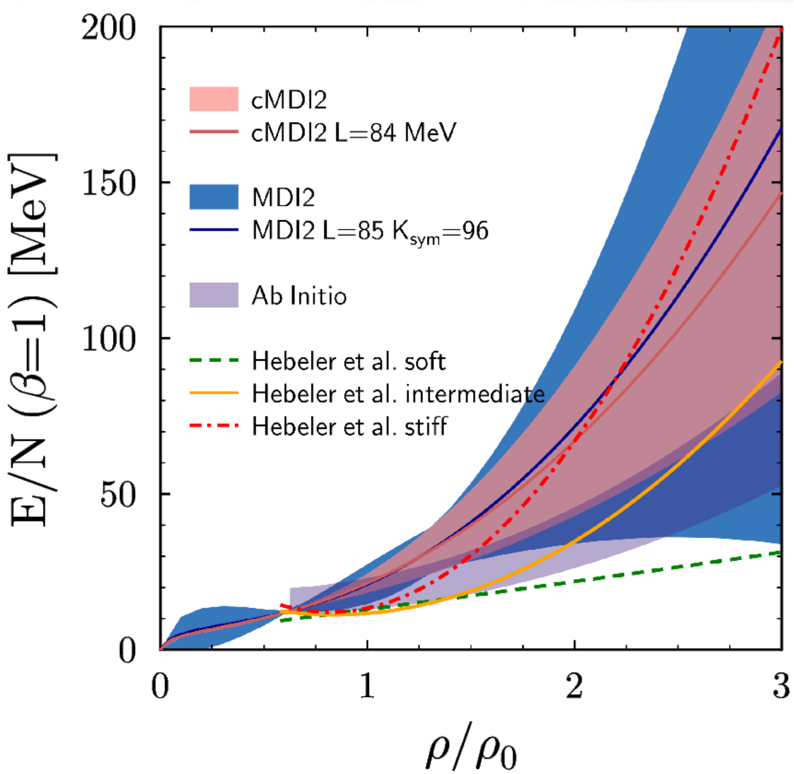
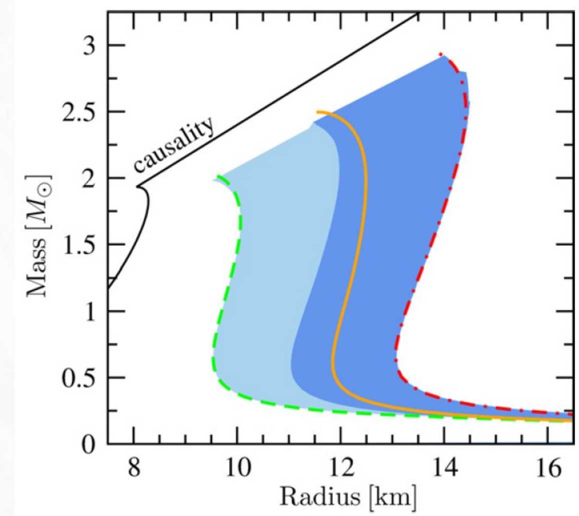
EFT Models & Astrophysics





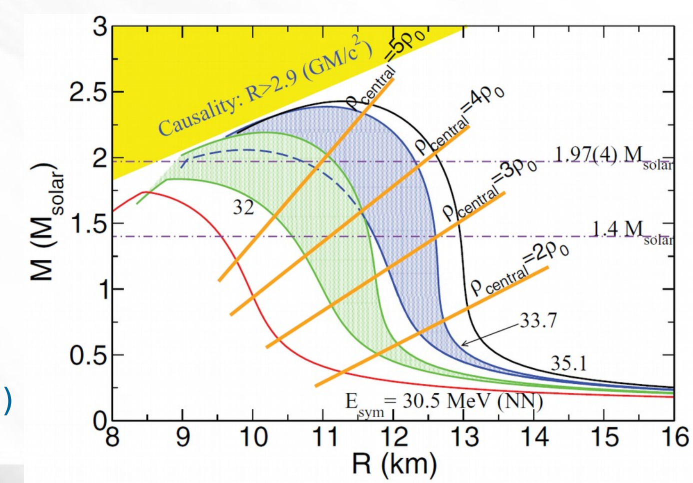
χEFT

Hebeler et al., PRL 105, 161102 (2010)
Hebeler et al., ApJ 773, 11 (2013)

QMC

Gandolfi et al, PRC 85, 032801R (2012)
Steiner et al, PRL 108, 081102 (2012)

- 2N+3N model → constrain neutron matter up to ρ_0
- extrapolation to high density ($2\rho_0$) uncertain
- high density – polytopes
- rule out many EoSs using measured NS masses



$M_{NS}=1.97\pm0.04\ M_\odot$ Demorest et al., Nature 467, 1081 (2010)

$M_{NS}=2.01\pm0.04\ M_\odot$ Antoniadis et al., Science 340, 6131 (2013)

$M_{PSR}=2.40\pm0.12\ M_\odot$ $M_{PSR}>1.66\ M_\odot$

vanKerkwijk et al, ApJ 728, 95 (2011)

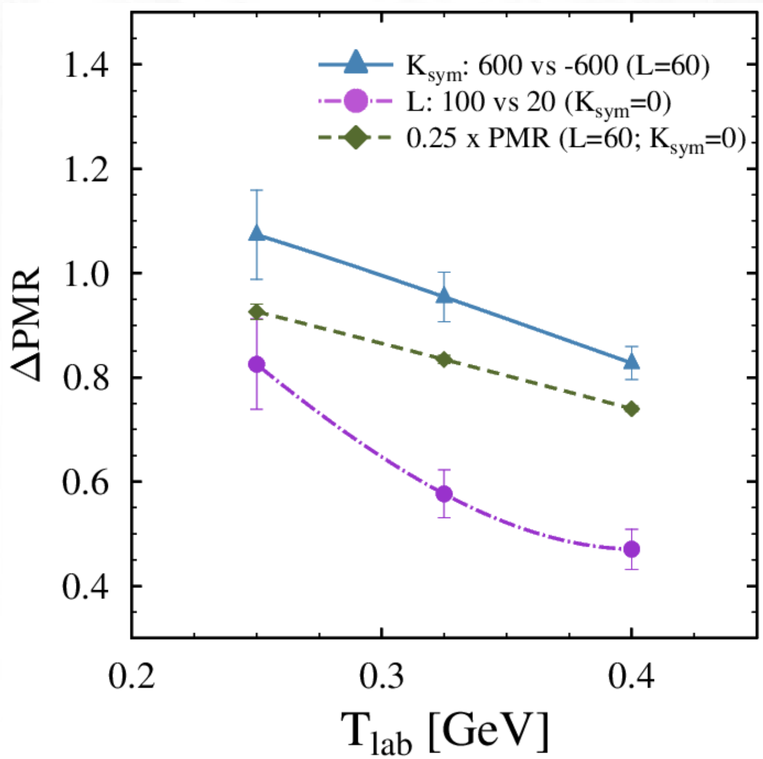
1) Fitted Δr, Δp
2) Extra curves
Δr=4.0 fm Δp=0.2 GeV/c
Δr=3.0 fm Δp=0.3 GeV/c

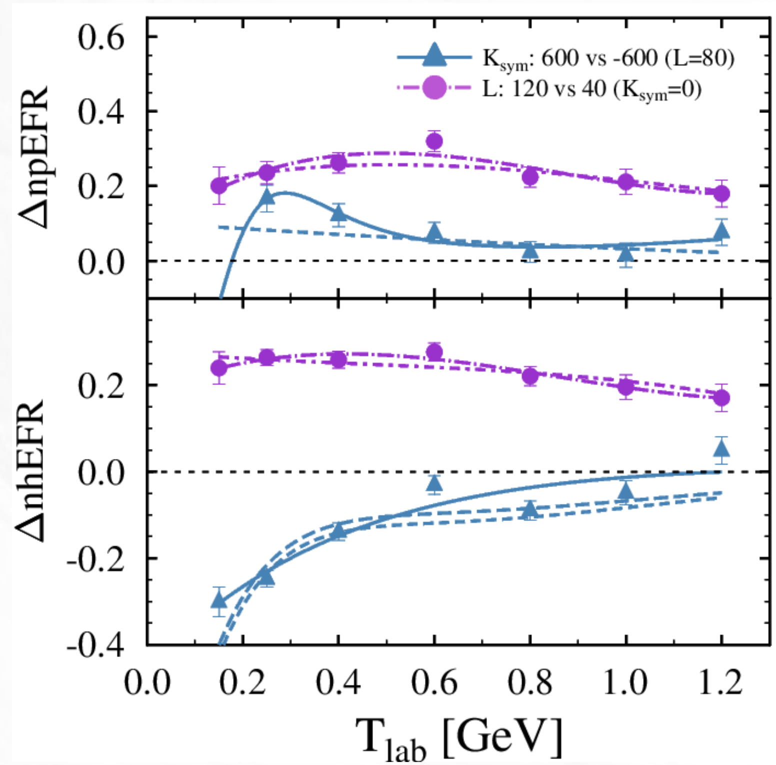
Perspectives

Pions

Au+Au HICs

Elliptic Flow





Experiment: SAMURAI-TPC, ASYEOS2

Theory:

Pion potential above saturation
(FOPI exp data for pion ellipic flow)

Multiplicities of light clusters
(coalescent invariant spectra, transport model with cluster d.o.f.)

Goal (6 years): σ(L)≈10 MeV σ(K_{sym})≈100 MeV

Summary / Conclusions

Improved Transport Model: improved density profiles of nuclei
MST coalescence model- not a satisfactory description of FOPI cluster multiplicities
-good description of elliptic flow parameters of individual fragment species
MDI2 potential – momentum dependent part in agreement with the empirical one (pA reactions)
- extra term to vary L and K_{sym} independently

Constraints for Symmetry Energy:
MDI2 - EFR sensitive to L & K_{sym}
- sensitivity to J suppressed
- systematic error due to underprdiction of ch/p multiplicit ratio

arXiv:1706.01300 [nucl-th]

$$L = 84 \pm 30(\text{exp}) \pm 18(\text{th})\ \text{MeV}$$
$$K_{sym} = 30 \pm 142(\text{exp}) \pm 85(\text{th})\ \text{MeV}$$

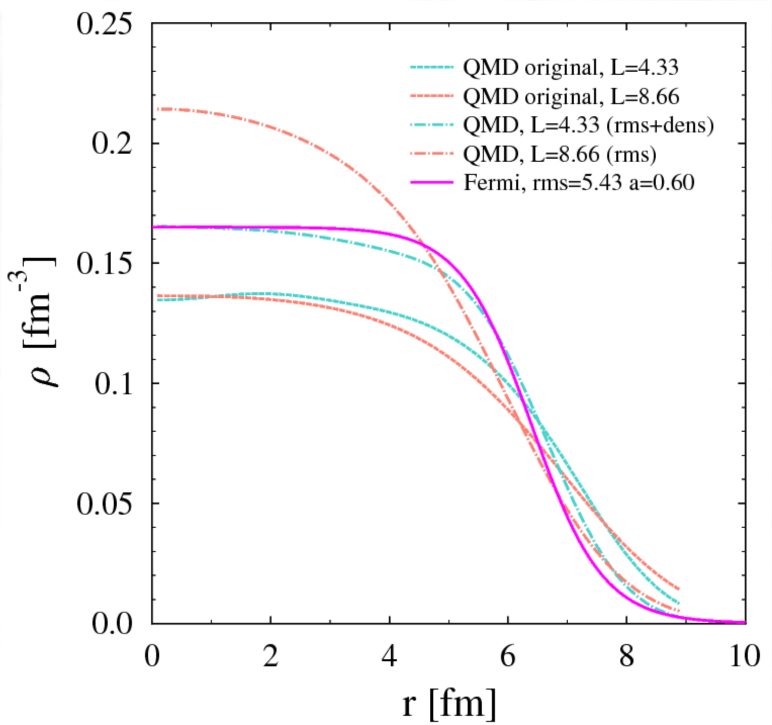
FOPI-LAND n/p

$$L = 85 \pm 22(\text{exp}) \pm 20(\text{th}) \pm 12(\text{sys})\ \text{MeV}$$
$$K_{sym} = 96 \pm 315(\text{exp}) \pm 170(\text{th}) \pm 166(\text{sys})\ \text{MeV}$$

FOPI-LAND n/p + ASYEOS n/ch

Perspectives: higher accuracy EFRnp needed (ASYEOS2 Coll)
Au+Au probe 250 MeV/nuc for K_{sym} & 600 MeV/nuc for L

Density Profiles; Coulomb Strength



$$\rho(\vec{r}) = \int d^3t \frac{1}{(0.5\pi L^2)^{3/2}} \exp[-(\vec{r}-\vec{t})^2/(0.5L^2)] f(\vec{t}, L^2)$$

$$h(r) \equiv r f(r) \qquad g(r) \equiv r\rho(r)$$

$$h(r,L) = g(r) + \sum_{n=1} \frac{(-1)^n}{n!8^n} (L^2)^n g^{(2n)}(r)$$

Stability of nuclei:
L^2=4.33 fm^2 – light nuclei
L^2=8.66 fm^2 – heavy nuclei

Static properties:
rms, skin – lower values for L^2

Compromise needs to be made:
cannot satisfy both

Coulomb strength:

- enforce agreement with Coulomb contribution to the empirical mass formula: scale strength by a factor of 0.9

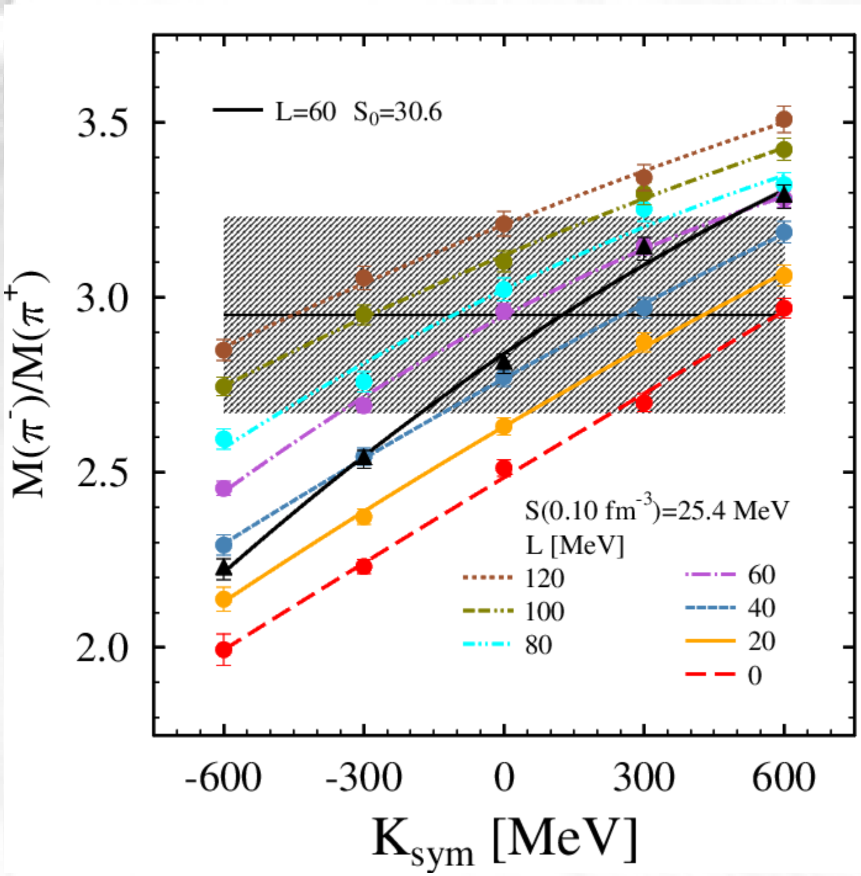
Liu, Wang, Deng, Wu, PRC 84, 014333 (2011).

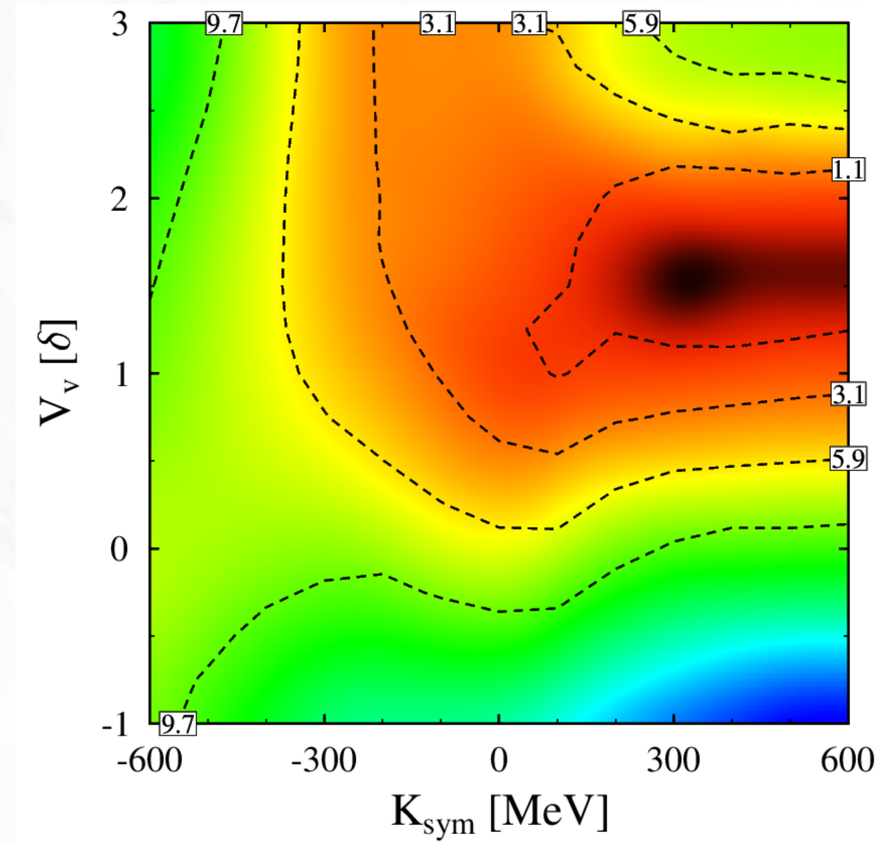
- needed for a closer reproduction of experimental ratio of average p_T of charged pions

Cozma, PRC 95, 014601 (2017)

Sensitivity to K_{sym}

L=60 MeV ≈ central value of elliptic flow constraint (same model)





probed density ≈ 1.7±0.1 ρ_0

Excluded combintations:

soft L + soft K_{sym}

stiff L + stiff K_{sym}

Elliptic flow constraint: D.C. arXiv:1706:01300

$$L = 85 \pm 22(\text{exp}) \pm 20(\text{th}) \pm 12(\text{sys})\ \text{MeV}$$
$$K_{sym} = 96 \pm 315(\text{exp}) \pm 170(\text{th}) \pm 166(\text{sys})\ \text{MeV}$$

Symmetry Energy

EoS: **Symmetric** **Asymmetric NM**

$$\frac{E(\rho)}{N}=\frac{E(\rho_0)}{N}+\frac{K_0}{18}\frac{(\rho-\rho_0)^2}{\rho_0^2}+\frac{J_0}{162}\frac{(\rho-\rho_0)^3}{\rho_0^3}$$

$$P=\rho^2\frac{\partial E(\rho)}{\partial\rho} \qquad K_0=9\frac{\partial P}{\partial\rho}$$

$$\frac{E(\rho,\beta)}{N}=\frac{E(\rho,\beta=0)}{N}+S(\rho)\beta^2 \qquad \beta=\frac{\rho_n-\rho_p}{\rho}$$

$$S(\rho)=S(\rho_0)+\frac{L}{3}\frac{\rho-\rho_0}{\rho_0}+\frac{K_{sym}}{18}\frac{(\rho-\rho_0)^2}{\rho_0^2}$$

Sources:

finite nuclei $\rho/\rho_0\leq 1$

heavy−ions $\rho/\rho_0\leq 3$

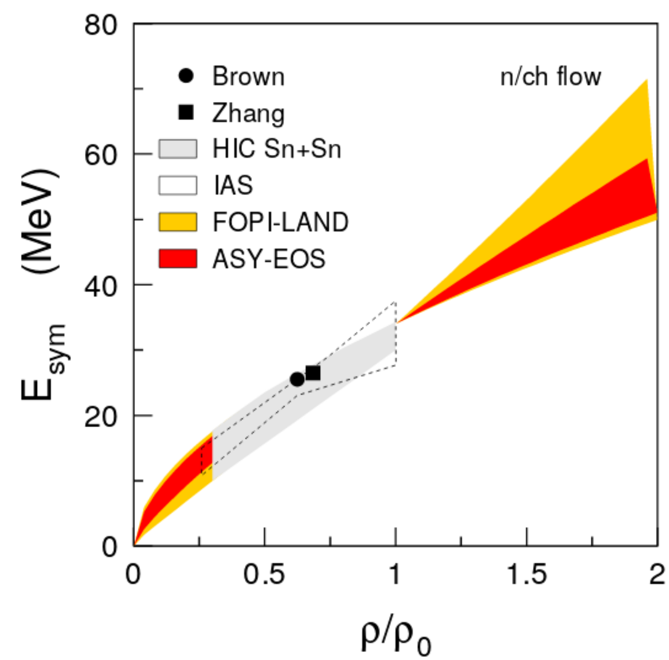
neutron stars $\rho/\rho_0\leq 10$

Flow observables:

$$\frac{dN}{d\phi}\sim 1+2v_1\cos\phi+2v_2\cos 2\phi$$

elliptic flow (v_2)

transverse flow (v_1)



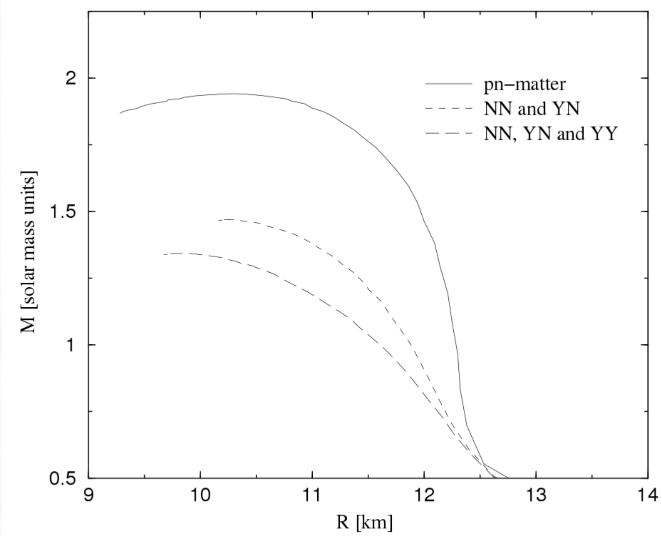
P. Russotto et al., PRC 94, 034608 (2016)

v_1,v_2 – probes for symmetric EoS

Ratios, differences v_2 – probes for Symmetry Energy

Hyperon Puzzle

Vidana et al., PRC 62, 035801 (2000)



$$e^- + n \rightarrow \Sigma^- + \nu_e$$

$$\mu_{\Xi^-} = \mu_{\Sigma^-} = \mu_n + \mu_e$$

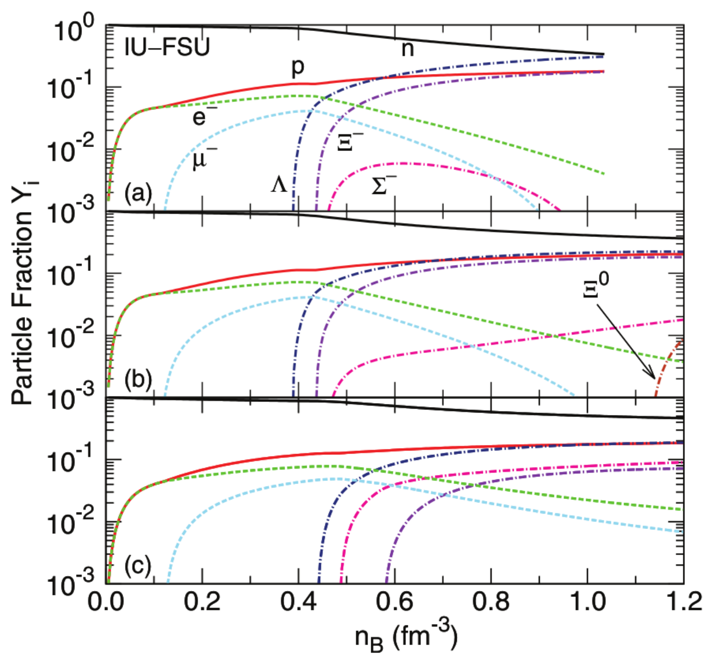
$$\mu_\Lambda = \mu_{\Xi^0} = \mu_{\Sigma^0} = \mu_n$$

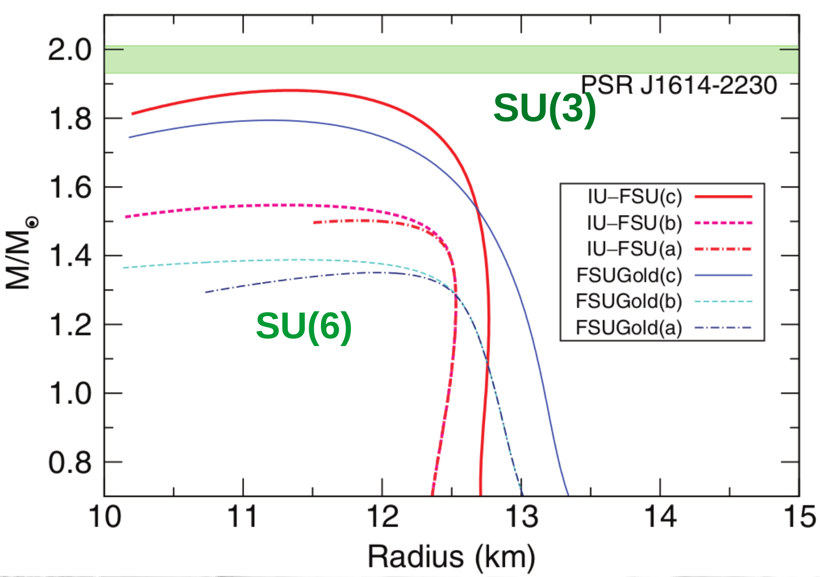
$$\mu_{\Sigma^+} = \mu_p = \mu_n - \mu_e$$

possible solution: SU(6) → SU(3)

FSUGold: L=60.5 MeV
IU-FSU: L=47.2 MeV

Fattoyev et al., PRC 82, 055803 (2010)





Miyatsu et al., PRC 88, 015802 (2013)