Introduction to the SM

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Yesterday

- Symmetries and the idea of local symmetry
- We can do it if we add a “gauge field”: a spin one massless particle
- Today: how we do it and move on

Gauge symmetry

New field A_μ. How we couple it?

- Recall classical electromagnetism

$$H = \frac{p^2}{2m} \Rightarrow H = \frac{(p - qA_i)^2}{2m}$$

- In QFT, for a local $U(1)$ symmetry and a field with charge q

$$\partial_\mu \to D_\mu \qquad D_\mu = \partial_\mu + iqA_\mu$$

- We get interaction from the kinetic term

$$|D_\mu \phi|^2 = |\partial_\mu \phi + iqA_\mu \phi|^2 \ni qA\phi^2 + q^2 A^2 \phi^2$$

- The interaction is proportional to q

The two aspects of symmetries

Thinking about E&M

- Charge conservation
- The force proportional to the charge

Q: Which of these come from the "global" aspect and which from the "local" aspect of the symmetry?

Accidental symmetries

- We only impose local symmetries
- Yet, because we truncate the expansion, we can get symmetries as output
- They are global, and are called accidental
- Example: $U(1)$ with $X(q=1)$ and $Y(q=-4)$

$$V(XX^*, YY^*) \Rightarrow U(1)_X \times U(1)_Y$$

  - X^4Y breaks this symmetry
- In the SM baryon and lepton numbers are accidental symmetries

SSB

Breaking a symmetry

SSB

- A situation that we have when the Ground state is degenerate
- By choosing a ground state we break the symmetry
- We choose to expend around a point that does not respect the symmetry
- PT only works when we expand around a minimum

What is the different between a broken symmetry and no symmetry?

SSB implies relations between parameters

SSB

Symmetry is $x \to -x$ and we keep up to x^4

$$f(x) = a^2x^4 - 2b^2x^2 \qquad x_{\min} = \pm b/a$$

We choose to expand around $+b/a$ and use $u \to x - b/a$

$$f(x) = 4b^2u^2 + 4bau^3 + a^2u^4$$

- No $u \to -u$ symmetry
- The $x \to -x$ symmetry is hidden
- A general function has 3 parameters $c_2u^2 + c_3u^3 + c_4u^4$
- SSB implies a relation between them

$$c_3^2 = 4c_2c_4$$

Partial SSB

Think about a vector in 3d. What is broken?

SSB in QFT

- When we expand the field around a minimum that is not invariant under a symmetry

$$\phi(x_\mu) \to v + h(x_\mu)$$

- It breaks the symmetries that ϕ is not a singlet under
- Masses to other fields via Yukawa interactions

$$\phi X^2 \to (v+h)X^2 = vX^2 + \ldots$$

- Gauge fields of the broken symmetries also get mass

$$|D_\mu \phi|^2 = |\partial_\mu \phi + iqA_\mu \phi|^2 \ni A^2\phi^2 \to v^2 A^2$$

Building Lagrangians

- Choosing the generalized coordinates (fields)
- Imposing symmetries and how fields transform (input)
- The Lagrangian is the most general one that obeys the symmetries
- We truncate it at some order, usually fourth

The SM

The SM

Input: Symmetries and fields

- Symmetry: 4d Poincare and

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

- Fields:
  - 3 copies of QUDLE fermions

$$Q_L(3,2)_{1/6} \quad U_R(3,1)_{2/3} \quad D_R(3,1)_{-1/3}$$
$$L_L(1,2)_{-1/2} \quad E_R(1,1)_{-1}$$

  - One scalar

$$\phi(1,2)_{+1/2}$$

Then Nature is described by

- Output: the most general $\mathcal{L}$ up to dim 4

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

- This model has a $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$ accidental symmetry
- Initial set of measuremnts to find the parameters
  - SSB: $SU(2)_L \times U(1)_Y \ \rightarrow \ U(1)_{EM}$
  - Fermion masses, gauge couplings and mixing angles

The SM pass (almost) all of it tests

The gauge interactions

The gauge part

$$SU(3)_C \times SU(2)_L \times U(1)_Y \;\to\; SU(3)_C \times U(1)_{EM}$$

Three parts, each look so different...

- QED - photon interaction: Perturbation theory
- QCD - gluon interaction: Confinement and asymptotic freedom
- Electro-weak: SSB and massive gauge bosons

$\mathcal{L}_{kin}$ and $SU(2) \times U(1)$

- Four gauge bosons DOFs

$$W_a^\mu \qquad B^\mu$$

- The covariant derivative is

$$D^\mu = \partial^\mu + igW_a^\mu T_a + ig'YB^\mu$$

  - Two parameters g and g'
  - Y is the $U(1)$ charge of the field D_μ work on
  - T_a is the $SU(2)$ representation
  - $T_a = 0$ for singlets. $T_a = \sigma_a/2$ for doublets
- Write D_μ for $L(1,2)_{-1/2}$ and $E(1,1)_{-1}$

Explicit examples

$$D^\mu = \partial^\mu + igW_a^\mu T_a + ig'YB^\mu$$

- Write D_μ for $L(1,2)_{-1/2}$ and $E(1,1)_{-1}$

$$D^\mu L = \left(\partial^\mu + \frac{i}{2}gW_a^\mu \sigma_a - \frac{i}{2}g'B^\mu\right) L$$

$$D^\mu E = \left(\partial^\mu - ig'B^\mu\right) E$$

- HW: Using $\phi(1,2)_{1/2}$ write $D^\mu \phi$

SSB in the SM

$$-\mathcal{L}_{Higgs} = \lambda\phi^4 - \mu^2\phi^2 = \lambda(\phi^2 - v^2)^2$$

- We measure the fact that $\mu^2 > 0$ by having SSB
- The minimum is at $|\phi| = v$
- ϕ has 4 DOFs. We can choose

$$\langle\phi_1\rangle = \langle\phi_2\rangle = \langle\phi_4\rangle = 0 \qquad \langle\phi_3\rangle = v$$

- It leads to: $SU(2)_L \times U(1)_Y \; \to \; U(1)_{EM}$
- We call the remaining symmetry EM
- Could we “choose” the vev in the neutral direction?
- We left with one real scalal field: the Higgs boson

QED

- Where is QED in all of this?

$$Q = T_3 + Y$$

- We can write explicitly for $L(1,2)_{-1/2}$ and $\phi(1,2)_{1/2}$

$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

- We can “tell” the differet component because we have SSB

Spectrum

Gauge boson masses

- $W_1, \; W_2, \; W_3, \; B$
- Gauge bosons masses from $|D_\mu \phi|^2$ (HW: do it)
- Diagonalzing the mass matrix the masses are

$$M^2_{W^+} = M^2_{W^-} = \frac{1}{4} g^2 v^2 \qquad M^2_Z = \frac{1}{4}(g^2 + g'^2) v^2 \qquad M^2_A = 0$$

- The mass eigenstates

$$W^\pm = \frac{1}{\sqrt{2}}(W_1 \mp i W_2) \qquad \tan\theta_W \equiv \frac{g'}{g}$$

$$Z = \cos\theta_W W_3 - \sin\theta_W B \qquad A = \sin\theta_W W_3 + \cos\theta_W B$$

- We have a θ_W rotation from (W_3, B) to (Z, A)

The $\rho = 1$ relation

We get the following testable relation

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 \qquad \tan\theta_W \equiv \frac{g'}{g}$$

The above is a signal of SSB

Experimental tests

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 \qquad \tan\theta_W \equiv \frac{g'}{g}$$

- High energy: Open your pdg and check W and Z decays to leptons. What do you expect to see?
- Z decays to lepton actually measures $\sin^2\theta_W \approx 0.23$
- HW: Calculate $\Gamma(Z \to \nu\bar{\nu})/\Gamma(Z \to e^+e^-)$, get $\sin^2\theta_W$ from the data and check the $\rho = 1$ prediction
- Also low energy data tests

Experimental tests of $\rho = 1$

- From the $\rho = 1$ relation

$$\frac{m_W^2}{m_Z^2} = \cos^2\theta_W \approx \left(\frac{80.4}{91.2}\right)^2 \approx 0.77 \quad \Rightarrow \quad \sin^2\theta_W \approx 0.23$$

- Z decays to leptons

$$\Gamma(Z \to \ell\ell) \ \sim \sum_{L,R} (T_3 - Q\sin^2\theta_W)^2$$

$$\Gamma(Z \to \ell^+\ell^-) \ \sim (1/2 - \sin^2\theta_W)^2 + (\sin^2\theta_W)^2 \sim 1/8$$

$$\Gamma(Z \to \nu\bar{\nu}) \ \sim (1/2)^2 \sim 1/4 \quad \Rightarrow \quad r \equiv \Gamma_\ell/\Gamma_{\rm inv} \sim 1/6$$

- PDG: $\Gamma_\ell = 3.37\%$ and $\Gamma_{\rm inv} = 20.00\% \quad \Rightarrow \quad r \sim 1/6$