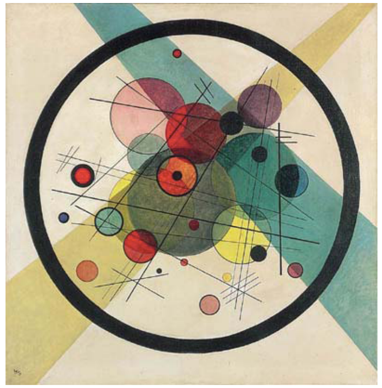
Andrea Vitturi

Role of continuum in the reaction mechanism in processes involving weakly-bound nuclei



Pisa, July 4, 2018

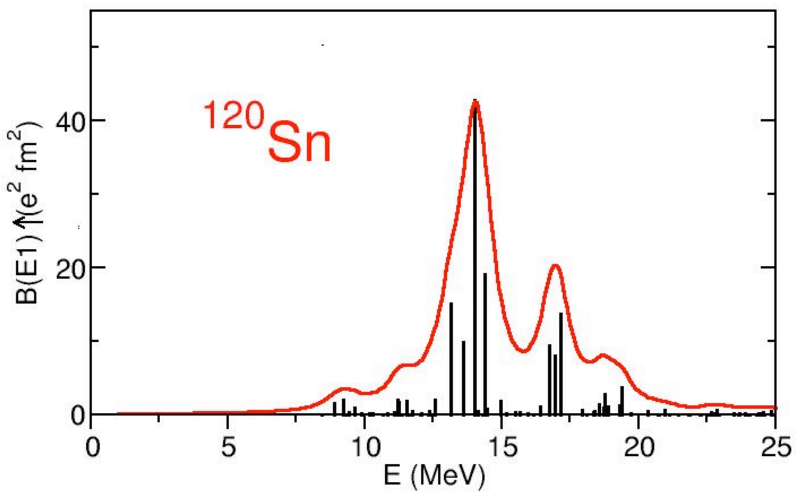
The possibility of reaching systems that are progressively closer to the drip lines (and that can eventually go beyond) has raised an increasing interest on the role of continuum states, both in structure and reactions, both theoretically and experimentally.

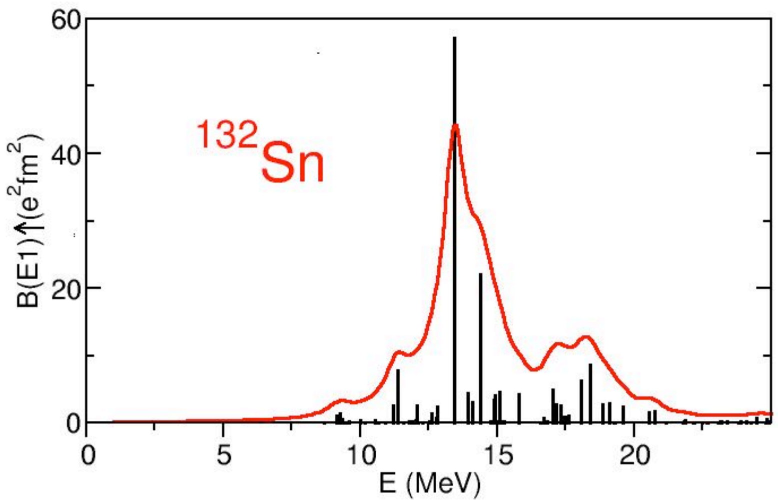
In nuclear structure one faces the situation not only of low-lying excited states that are above the particle emission threshold (that are therefore naturally in the continuum), but also the situation of bound states whose description and nature rely on the coupling to the close continuum states.

Similarly in nuclear reactions, due to the vicinity of the continuum, a ruling role is played by final states in the continuum (break-up channels). But again this dominant coupling to the continuum is strongly affecting also the characteristics of the population of final states that are in the bound sector.

The problem of continuum, and simple solutions, are however an old story.
For example in the description of giant resonance with DISCRETE RPA, where the strength of each high-lying state is distributed along a gaussian or lorentzian , as a simpler alternative to a full continuum RPA

Example: Dipole response, with GDR and PDR, with Skyrme HFB plus RPA





NB Of course, additional contribution to the full width comes from the coupling to 2p-2h states (mixing width)

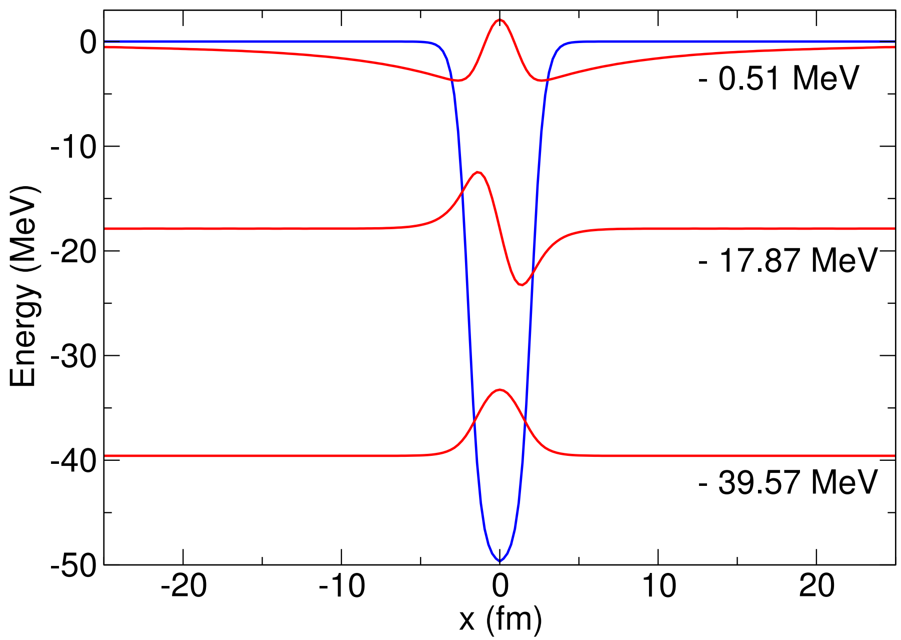
Many microscopic structure models (typical example the Shell Model) are based on an expansion starting from a single-particle basis (generally truncated). Leaving the stability valley the number of (Fermi allowed) available bound states reduces, and the continuum part of the spectrum has to be included. Generally one resorts to appropriate DISCRETIZATION procedure, in order to deal with a set of square-integrable states.

Typical methods (that provide different distribution and energy densities of state) are based on

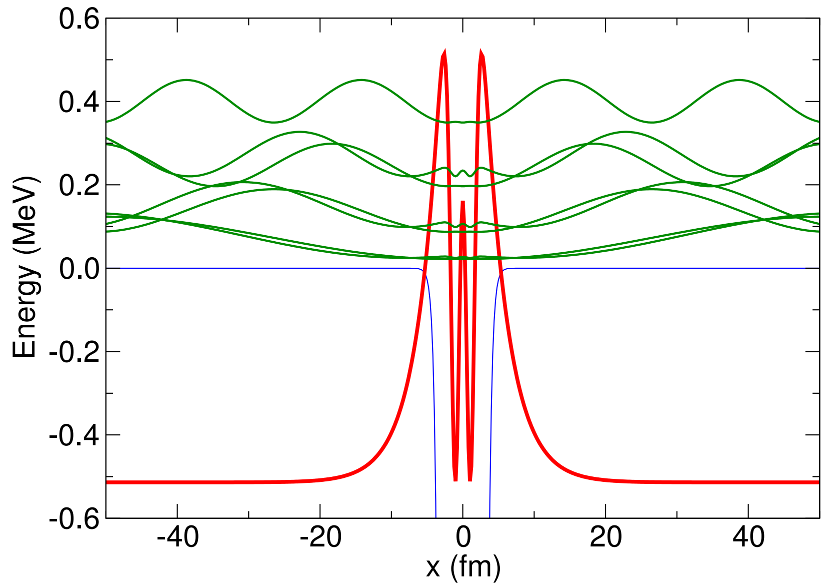
- Harmonic oscillator (HO)
- Transformed Harmonic Oscillator (THO)
- Box with infinite wells
- Gamow complex states
- Bunching of true continuum states into slices (CDCC)

All models are equivalent as long as one uses the full basis, but convergence may require extremely large bases, in particular for the tail and for weakly-bound states.

Example: a Woods-Saxon one-dimensional potential with a weakly-bound state, obtained by diagonalizing in a chosen basis

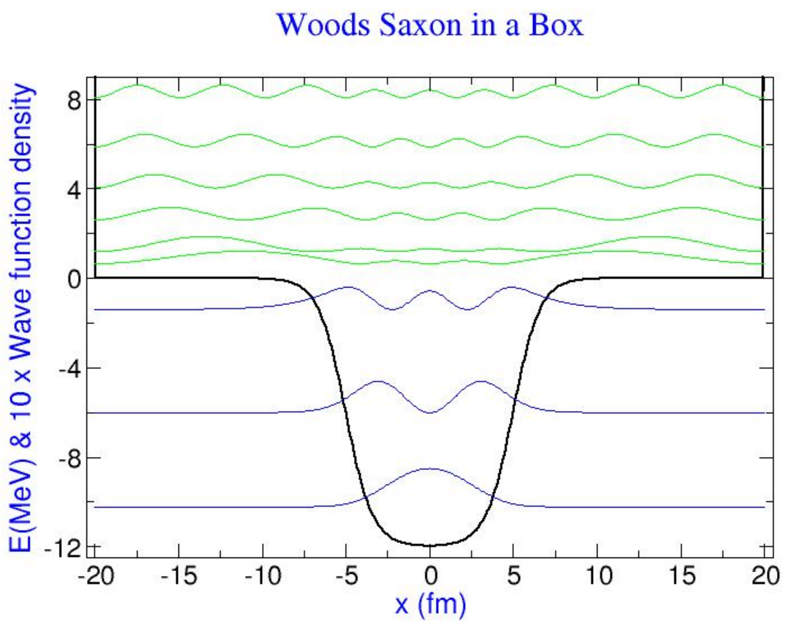


Box procedure, with R=200 fm.
In the figure the lowest discrete basis states



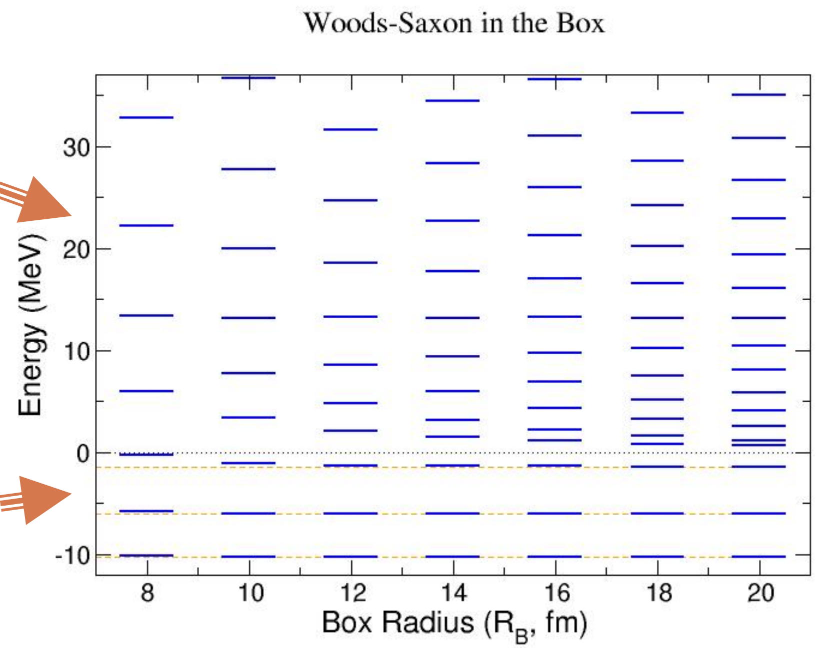
Example: Diagonalization in a box

WS single-particle states obtained imposing boundary conditions at a box (R=20 fm)

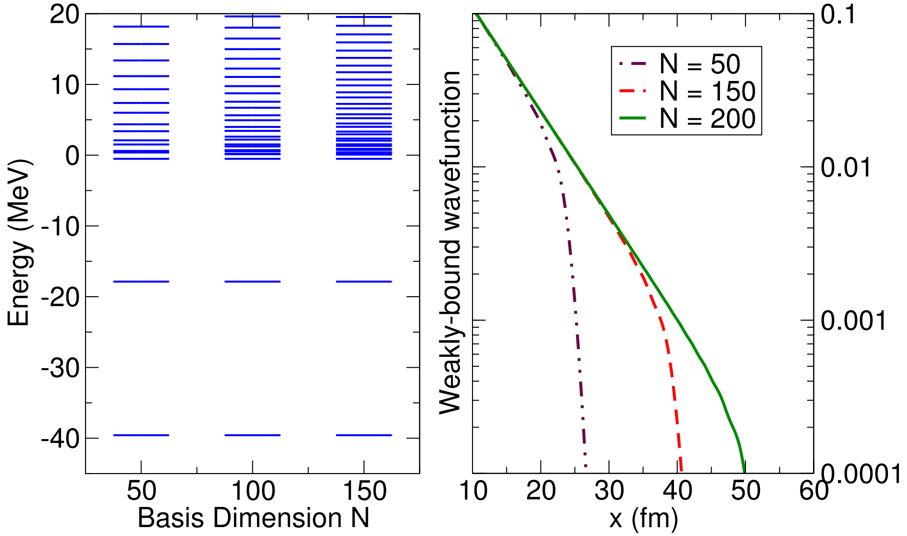


positive energy pseudo-states

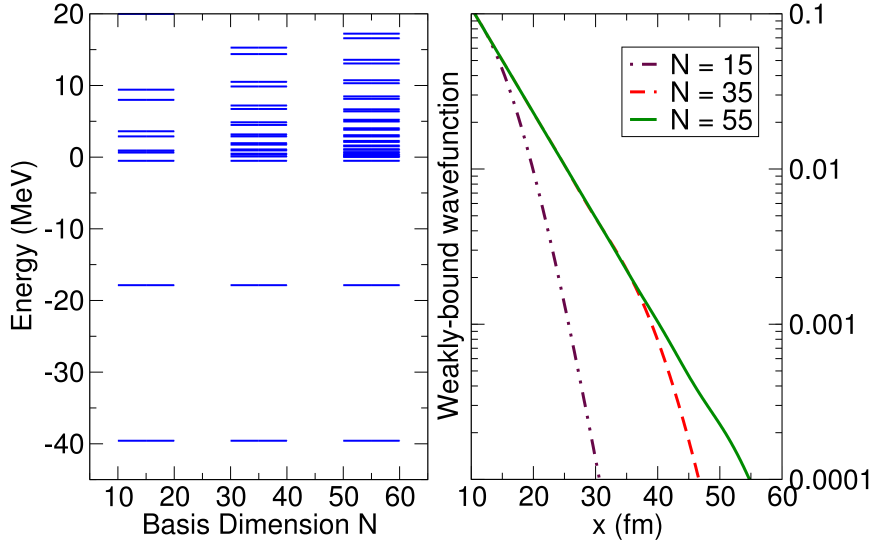
bound states



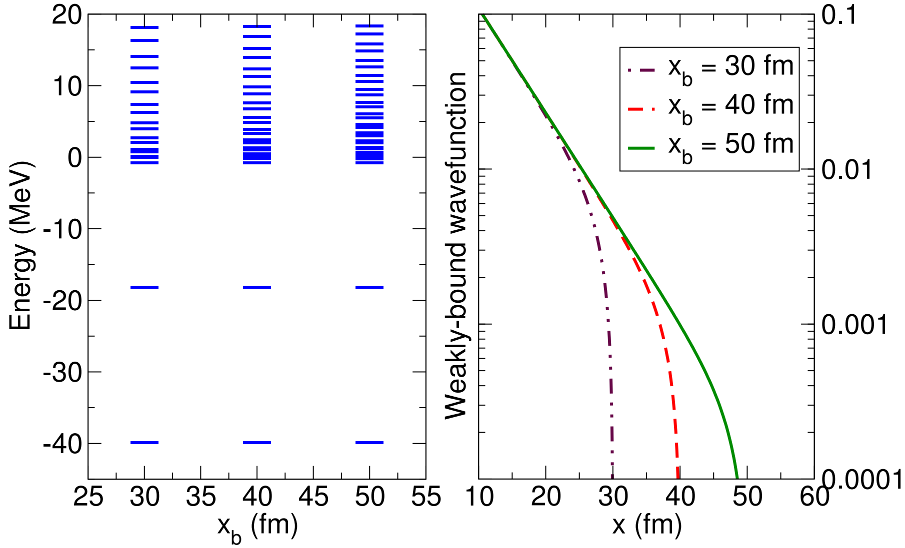
Energies of the bound states converge rather rapidly, but not the radial tails.
Large dimension are needed to get acceptable behaviors at large distances



HO, with different number of quanta



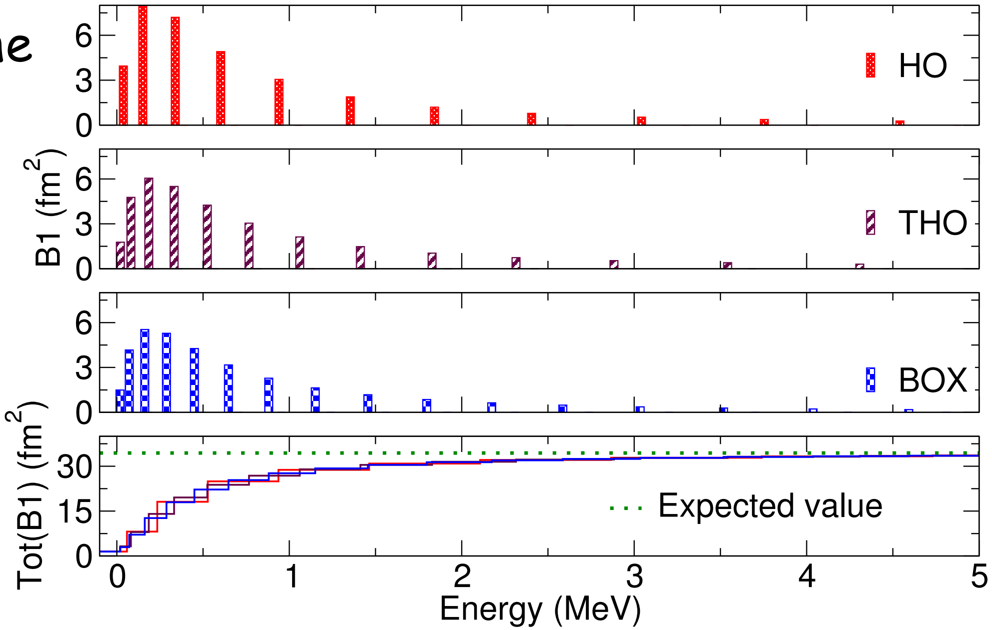
THO, with different number of quanta



Box, with different radial dimensions

Particle-hole response from the weakly-bound state to the "discretized" continuum

$$B1=|\langle\Phi_C\ |x|\ \Phi_b\rangle|^2$$

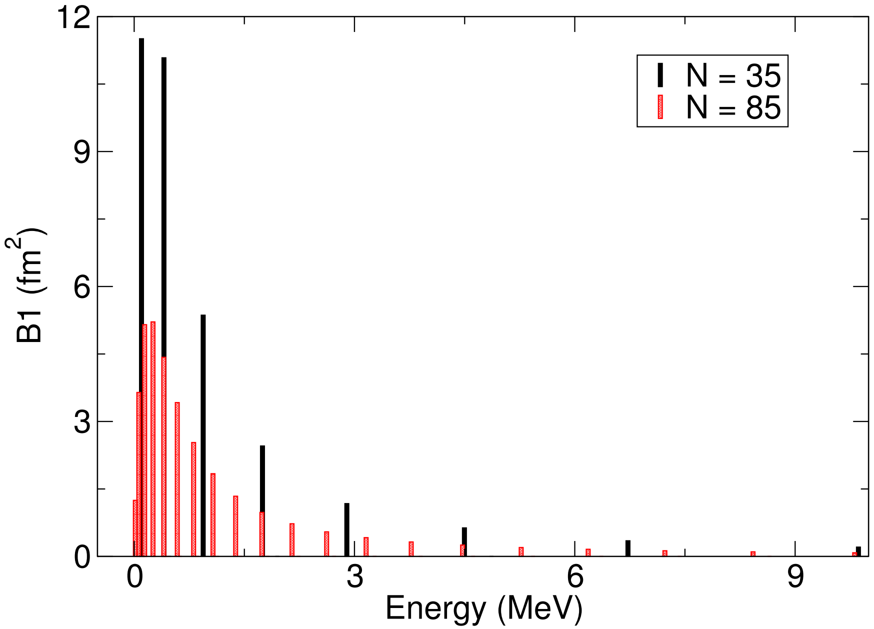


N_{HO}=250

N_{THO}=85

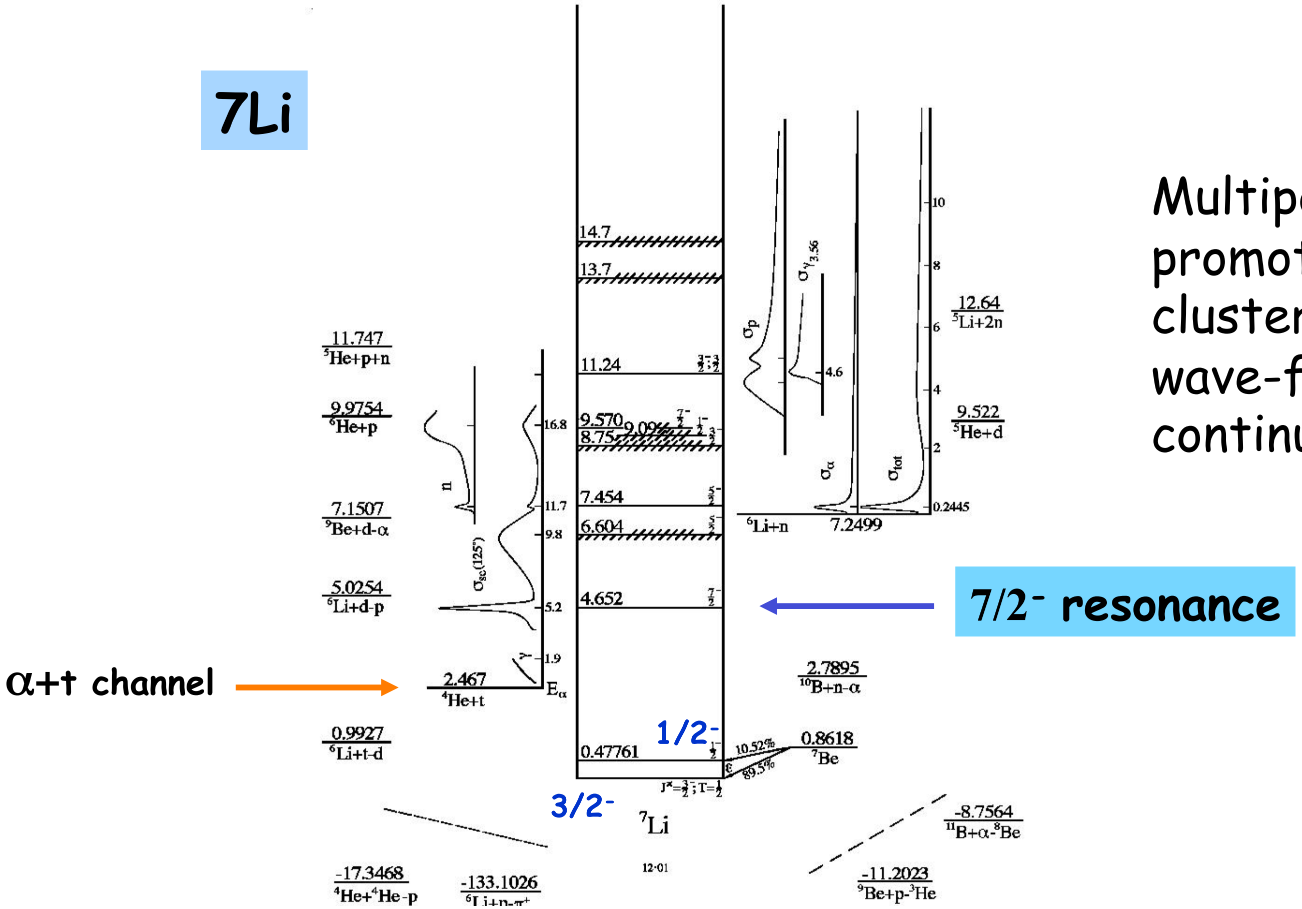
R_{BOX}=
100 fm

Example of convergence: the case of the THO basis with different number of quanta



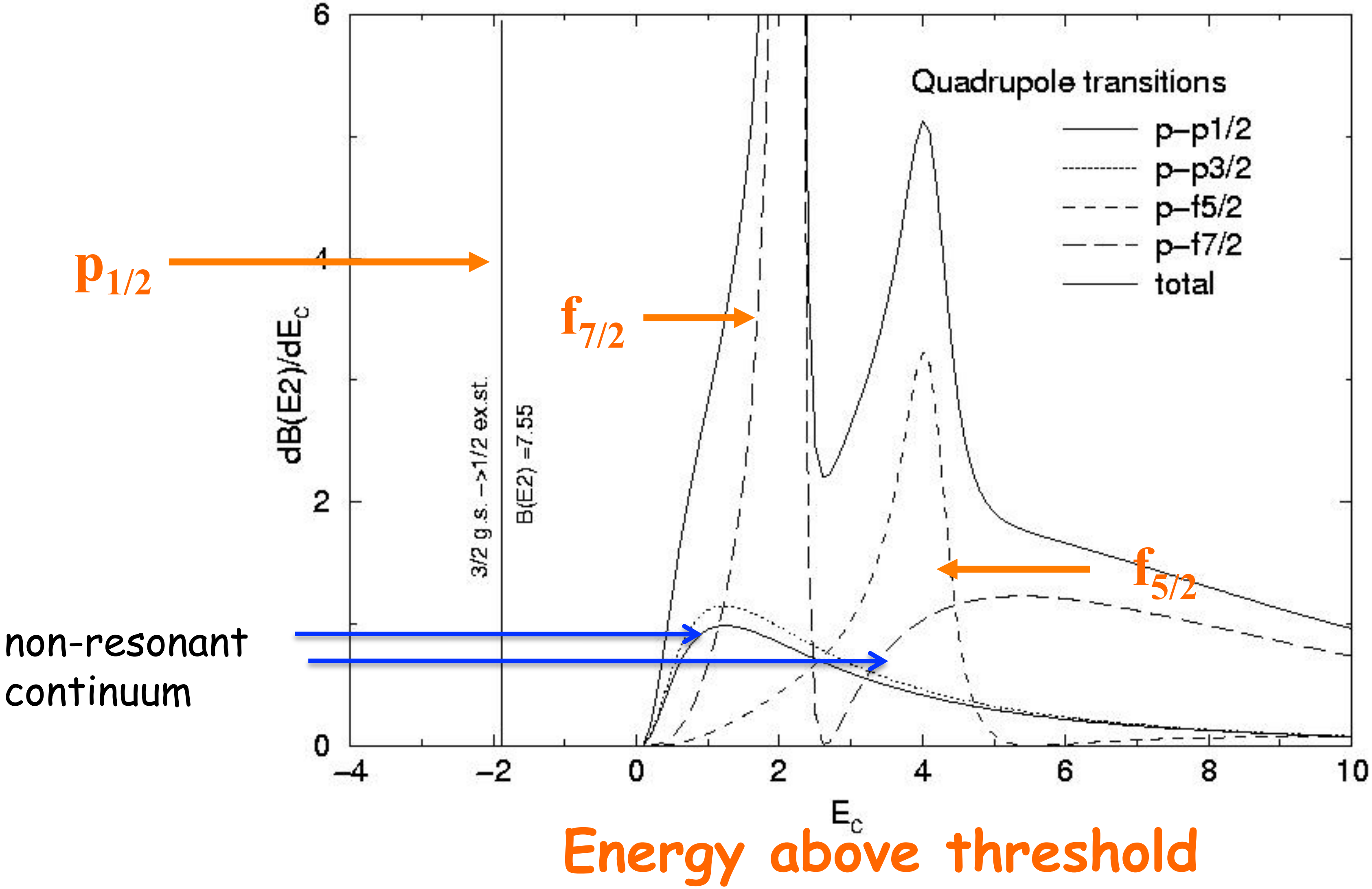
Interplay of resonant and non-resonant continuum
(leads to caution in the choice of energy step in discretization procedure)

An example: quadrupole excitation of 7Li described as alpha+triton model



Multipole strength obtained by promoting initial bound cluster-cluster p-state wave-function to cluster-cluster continuum wave functions

Quadrupole strength



Moving from the case of just one particle in the continuum to cases with more particles in the continuum. More interesting because it involves continuum-continuum couplings

Simple test case in structure

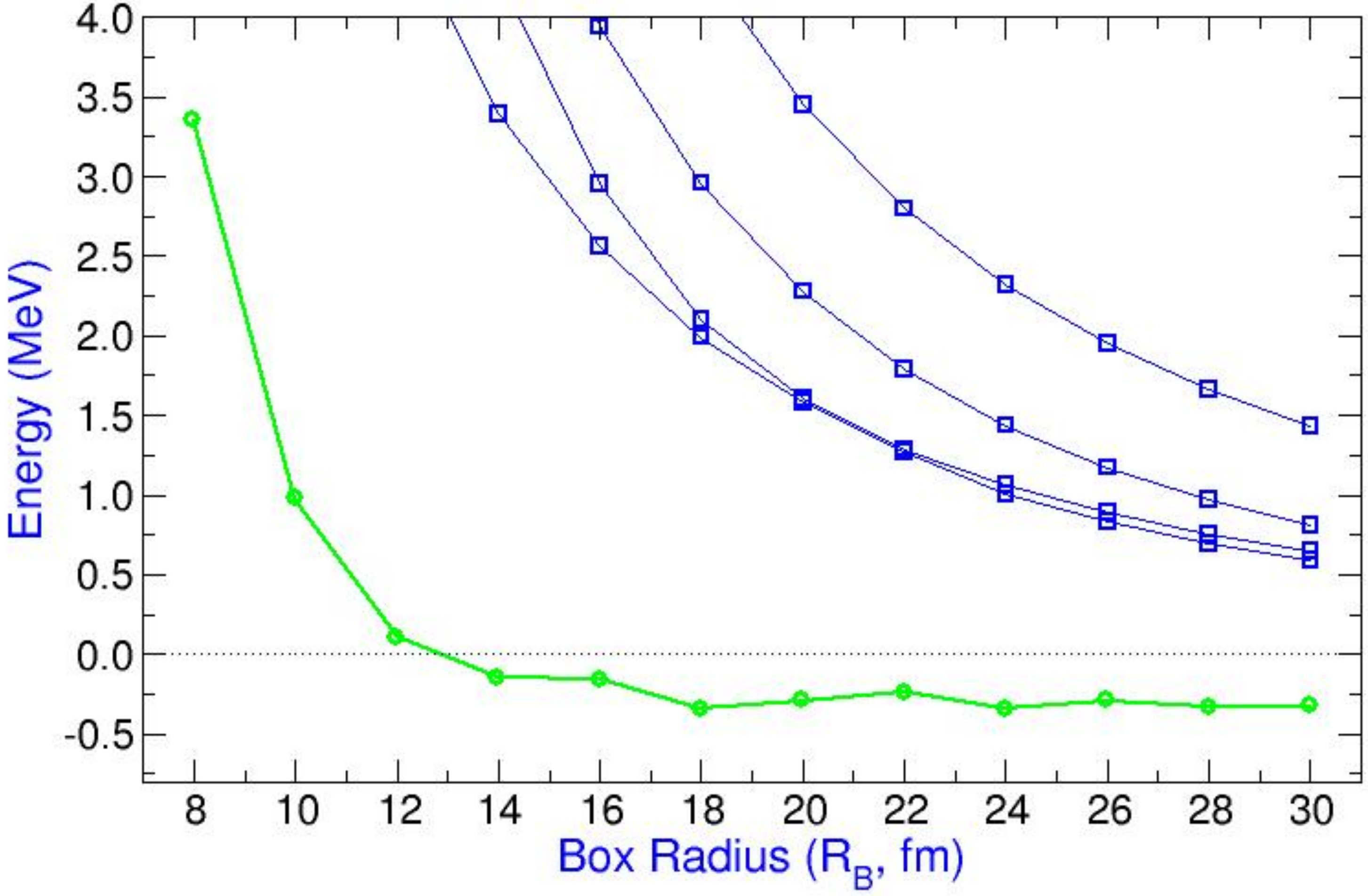
Two valence particles, moving in a one-dimensional Woods-Saxon potential V_0, interacting via a residual density-dependent short-range attractive interaction.

Modelling a drip-line system, one can choose the Fermi surface in such a way that there are no available bound states, and the two unperturbed particles must be in the continuum. The residual interaction

$$V(x_1,x_2) = V_0\, \delta(x_1-x_2)\, \rho((x_1+x_2)/2)/\rho_0$$

can be chosen in such a way that the final correlated wave function is however bound. Such a system is normally called "Borromean"

Correlated energy of the two-particle system
(as a function of the box radius)



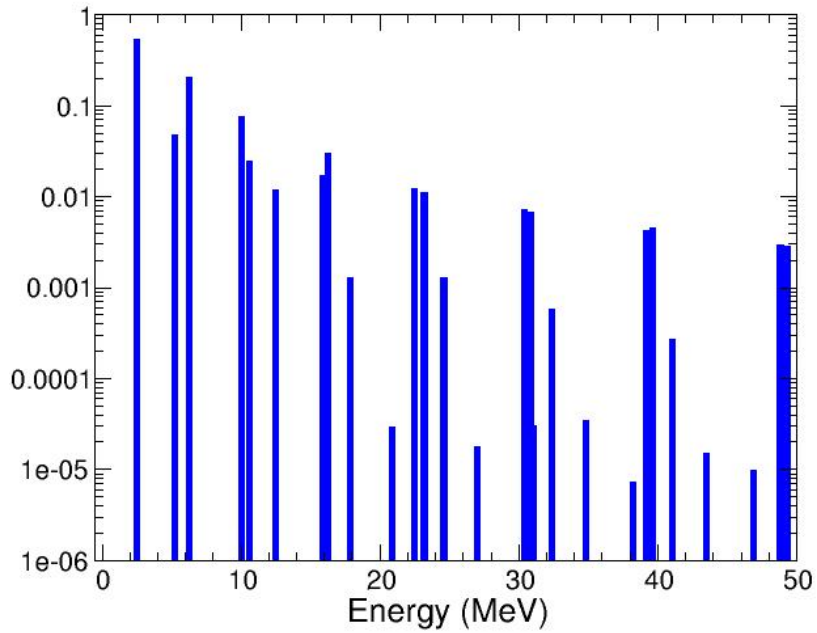
unphysical two-particle states (basis dependent)

physical two-particle bound state

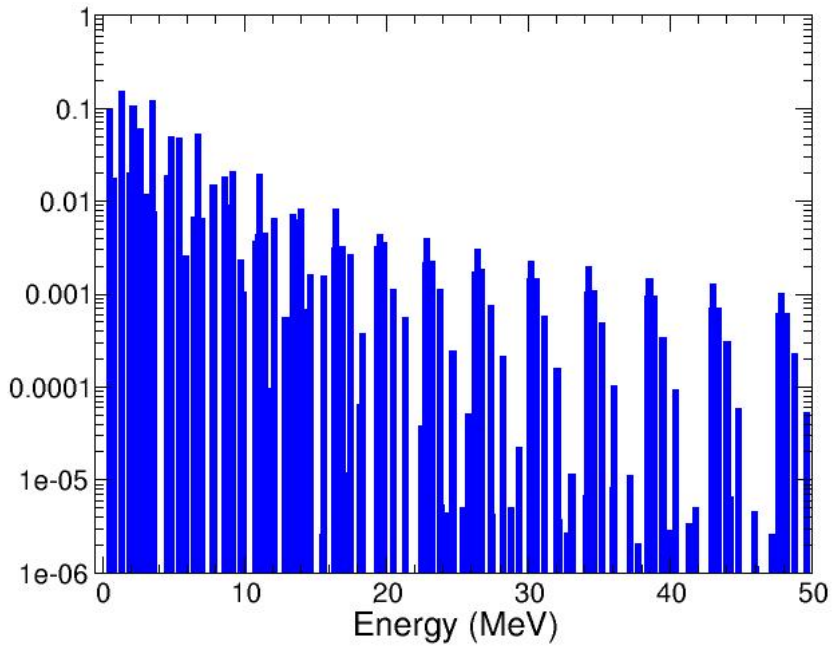
Correlated two-particle wave-function expanded over discretized two-particle positive energy states

OBS Enormous number of components

R=15fm



R=40fm



Application to the ground states of ^{6}He (bound), ^{22}C (bound) and ^{26}O (unbound)

Example: ^{26}O described as ^{24}O+2n with density-dependent pairing and continuum discretization

25O

lj	V_{ws}(MeV)	V_{ls}(MeV)	E_R	Γ
$d_{3/2}$	-47.100	29.187	0.740	0.0864
$p_{3/2}$	"	"	...	...
$f_{7/2}$	"	"	2.440	0.2060

Resonant and non-resonant continuum

Non-resonant continuum

Configuration mixing of ground state 0+ on 26O

ΔE	lj	DD (Present work)	Hagino
	d3/2	0.660	0.661
0.2	p3/2	0.089	0.105
	f7/2	0.250	0.183

Singh, Fortunato, Vitturi

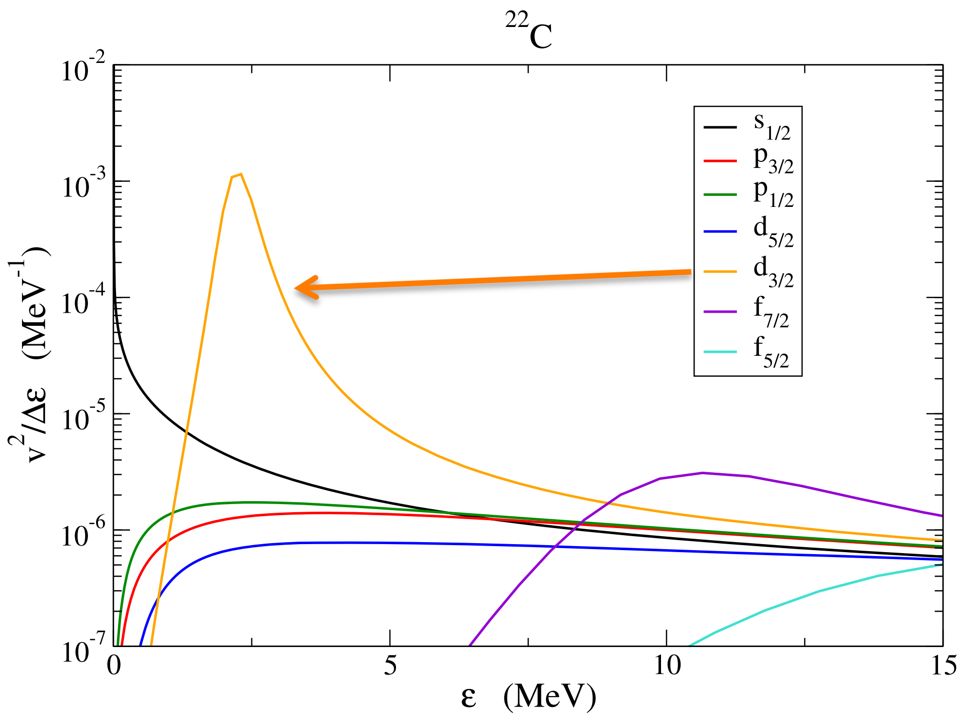
A different approach: BCS with discretized continuum states

Important points:

- what is the shape of the occupancy of the different continuum states?
- how changes the pairing gap as the Fermi energy crosses the continuum threshold?
- what is the interplay of resonant and non-resonant continuum?

v^2 occupancy factors

THO discretized pseudo-states



Lay, Alonso, Fortunato, Vitturi, J.Phys. G

Cf also Id Betan

Basic problem: how the properties of the pairing interaction eventually change as the Fermi surface crosses the continuum threshold

Example: proton knocked-out from ^{27}F

In ^{27}F the 9 protons create a neutron mean field that binds The last two neutrons. On the other hand, in ^{26}O, the remaining 8 protons cannot bind the two neutrons. But the correlations in the bound neutron pair is the same as in the unbound pair? How sensitive are the correlation to the form of the pairing interaction? Density dependent? Volume or surface pairing, or proper mixture?

How all this will be reflected on two-particle transfer processes?

Main model in well-bound systems: Sequential two-step process.

Each step transfers one particle

Pairing enhancement comes from the coherent interference of the different paths through the different intermediate states in (a-1) and (A +1) nuclei, due to the correlations in initial and final wave functions

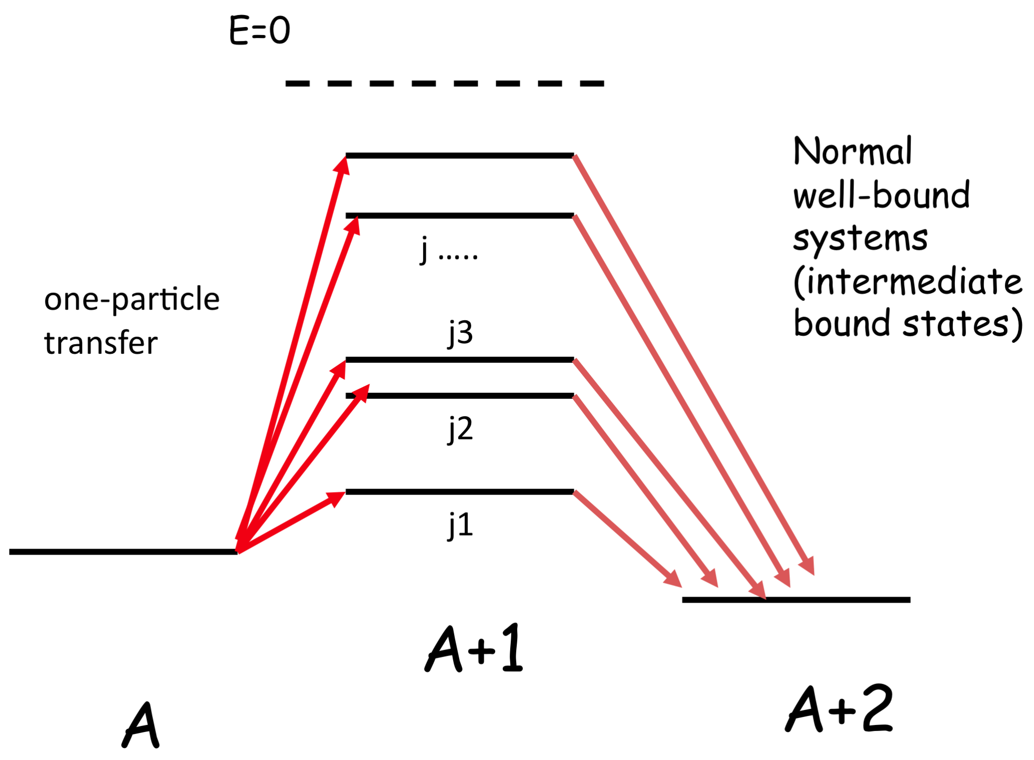
Basic idea: dominance of mean field, which provides the framework for defining the single-particle content of the correlated wave functions

Expansion to second-order in the transfer potential

Simultaneus	+	Sequential	+	not-orthogonality
(first-order)		(second-order)		(second-order)

this is <u>not</u> the cluster contribution

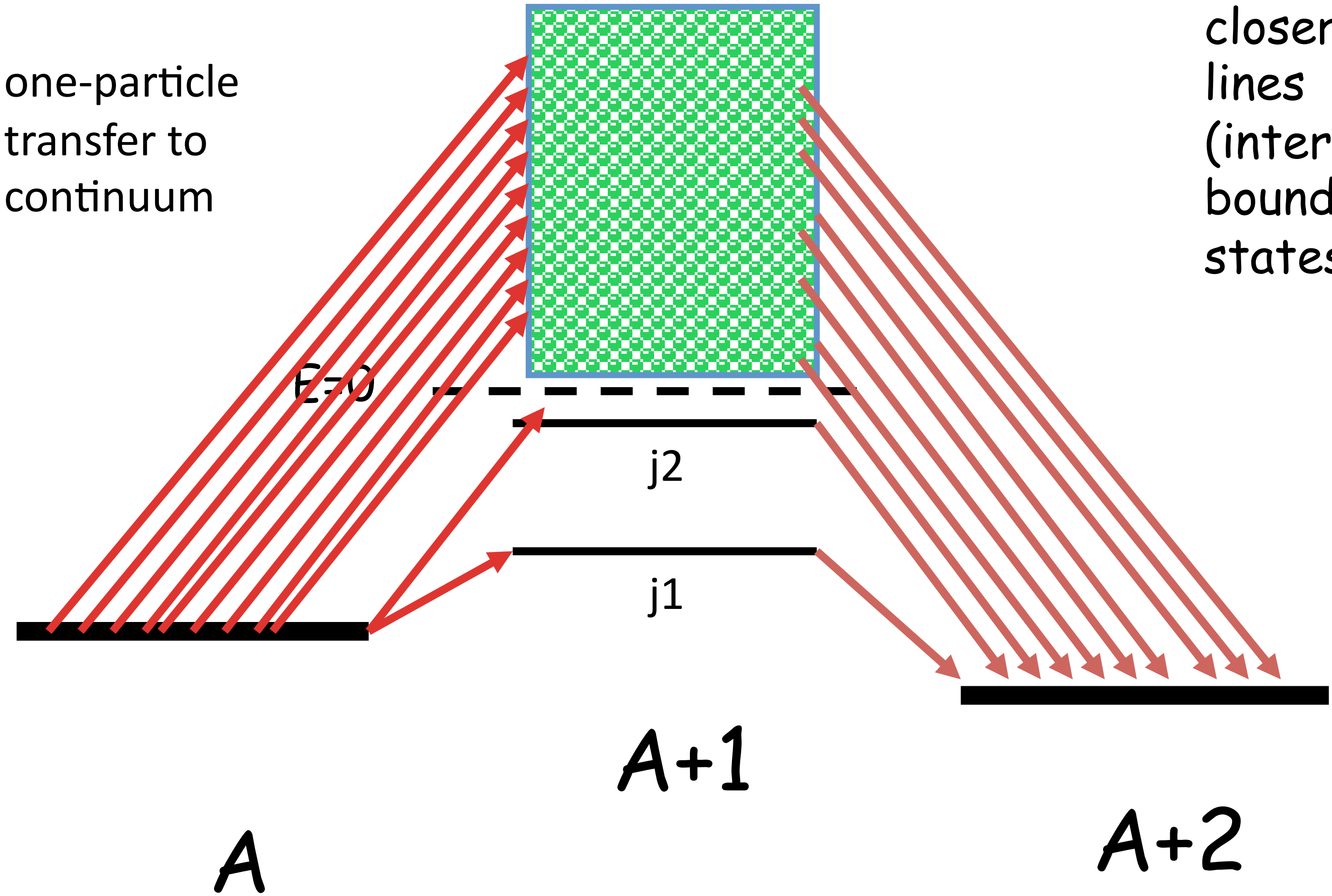
these two terms approximately cancel each other



Basic problem:

how is changed the picture as we move closer
or even beyond the drip lines?

one-particle transfer to continuum

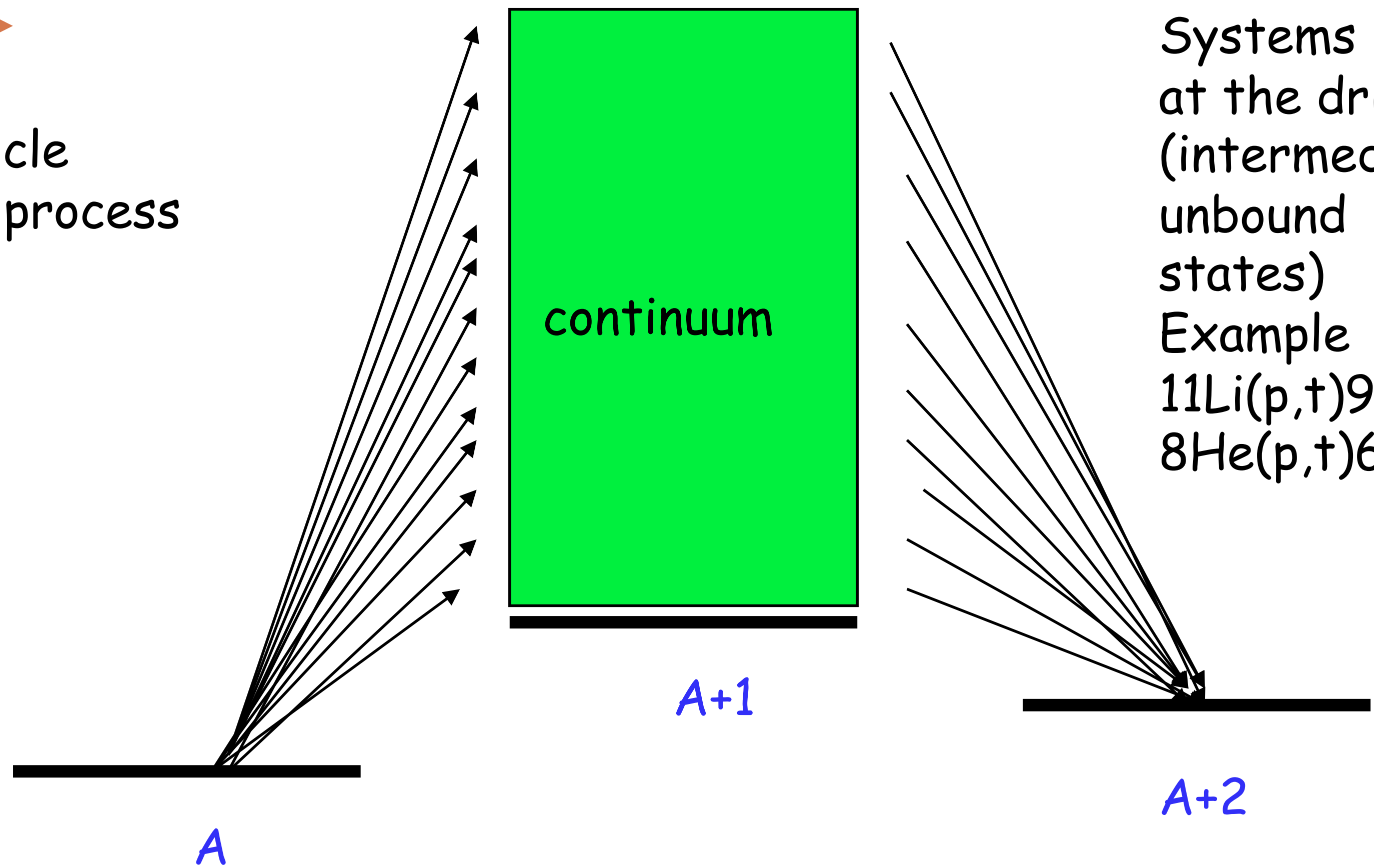


Systems closer to the drip lines (intermediate bound and unbound states)

Example

$$|A=2\rangle = \left\{ \sum_i X_i [a_i^+ a_i^+]_0 + \int dE\, X(E)\, [a^+(E) a^+(E)]_0 \right\} |A\rangle$$

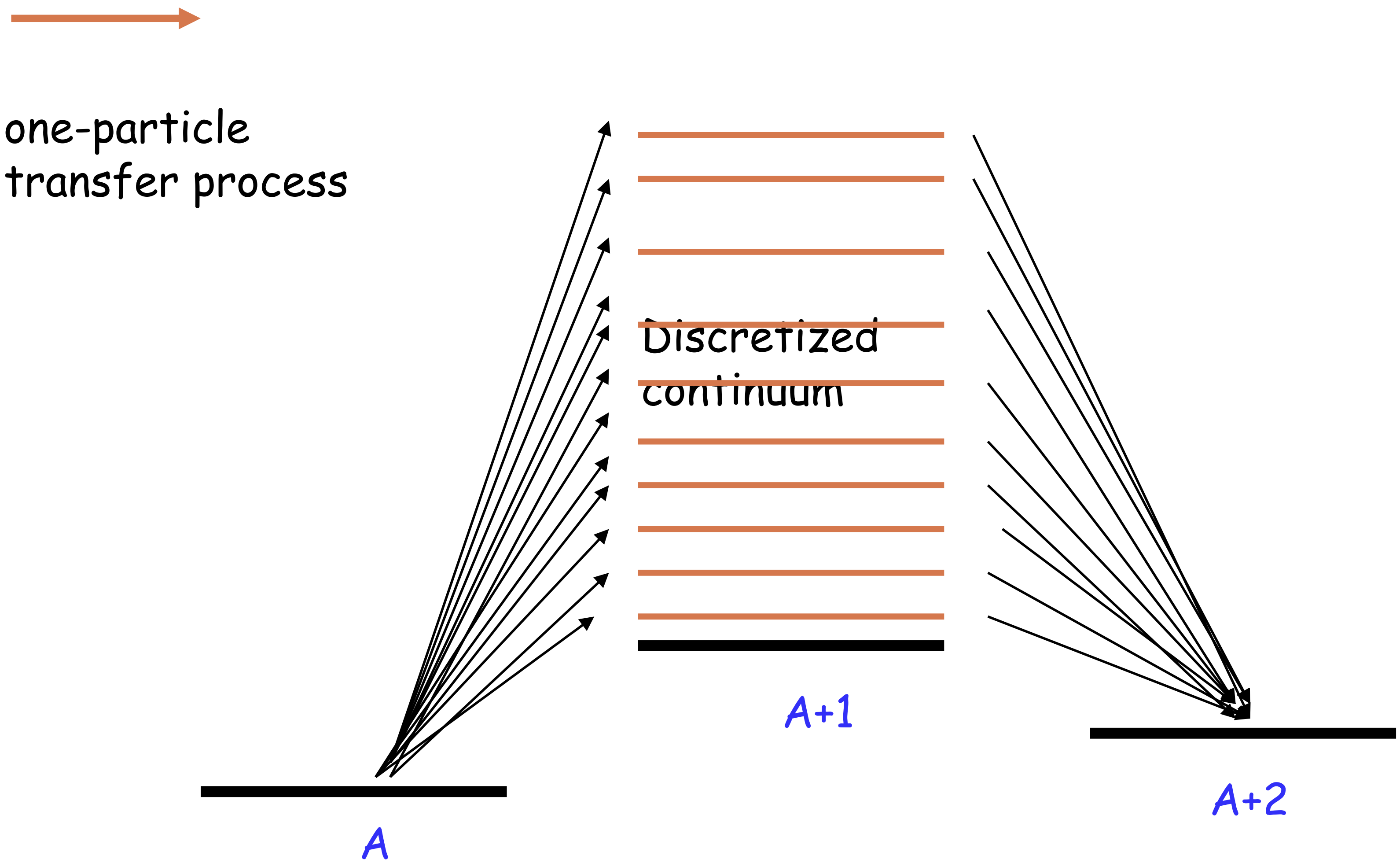
one-particle
transfer process



Systems
at the drip lines
(intermediate
unbound
states)
Example
11Li(p,t)9Li
8He(p,t)6He

$$|A=2> = \int dE\, X(E)\, [a^{+}(E)a^{+}(E)]_0\, |A>$$

Two-particle trasfer will proceed mainly by
constructive interference of successive transfers
through the (unbound) continuum intermediate states

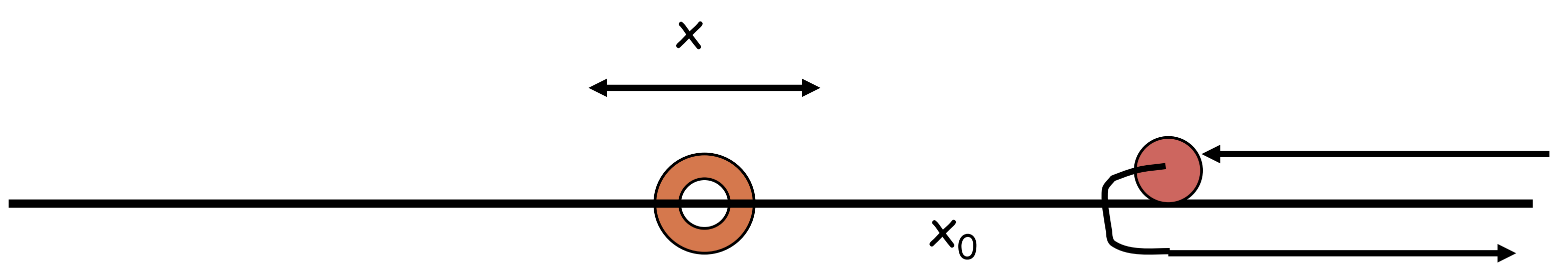


The integration over the continuum intermediate states can becomes feasible by continuum discretization:
but how many paths should we include? Thousands or few, for example only the resonant states?

Let us move now to the reactions.
For weakly-bound systems the break-up channel becomes dominant.
To describe the physical aspect let me consider a simple one-dimensional case

Single particle, initially moving in a one-dimensional Woods-Saxon potential V_0, perturbed by a time-dependent interaction $V(x,t)$, assumed to be of gaussiam shape

$$V(x,t)=V \exp(-t^2/\sigma_t) \exp(-(x-x_0)^2/\sigma_x)$$



Obs: simulation of the nuclear field generated in a collision with a heavy partner

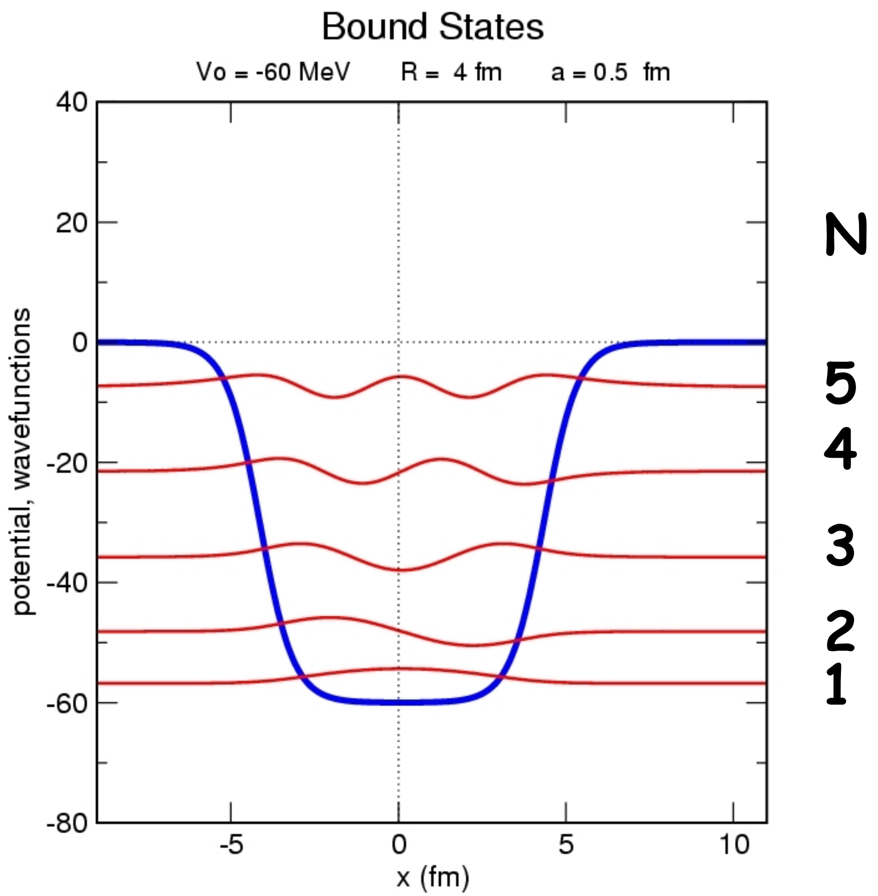
Exact full evolution of the system obtained by solving the time-dependent Schroedinger equation

$$i\hbar\partial\Psi(x,t)/\partial t = [H_0 + V(x,t)]\,\Psi(x,t)$$

with

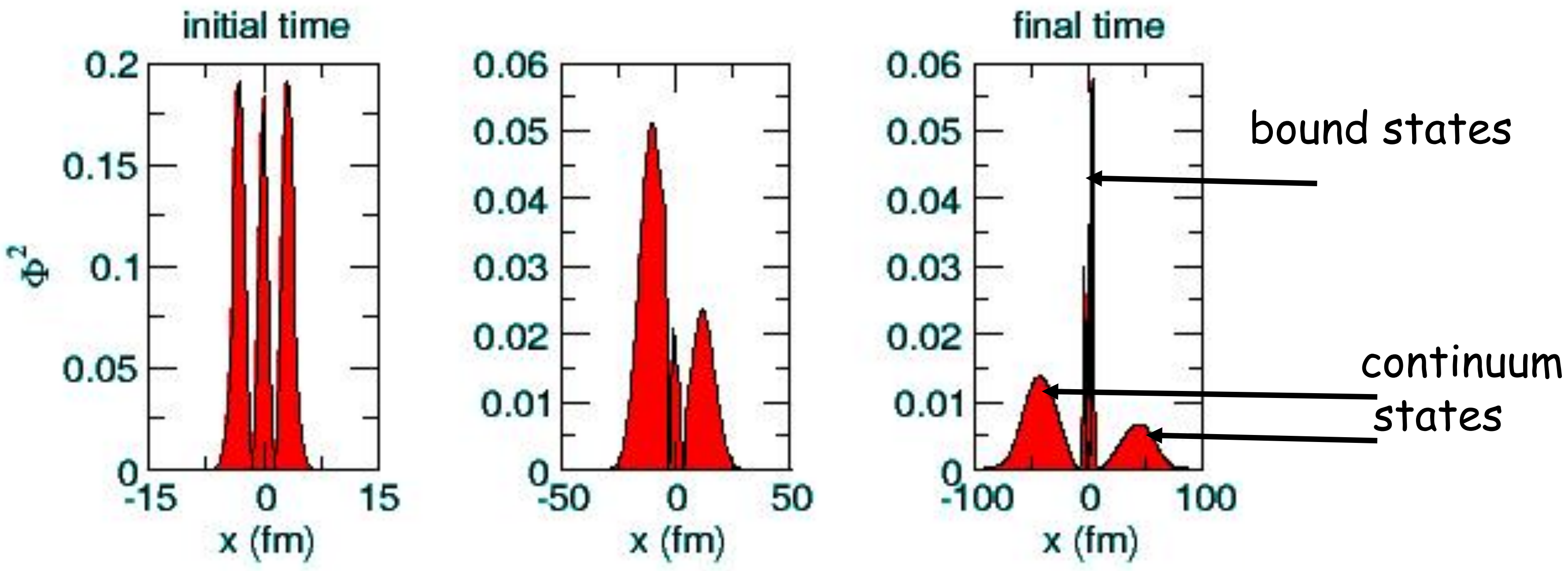
$$H_0 = -(\hbar^2/2\mu)\, d^2/dx^2 + V_0(x)$$

The particle is assumed to be initially in one of the bound states $\Phi_N(x)$ of V_0



initial bound state

N=3



Final wave function NOT confined in potential well

Obs: change of scale

Initial time

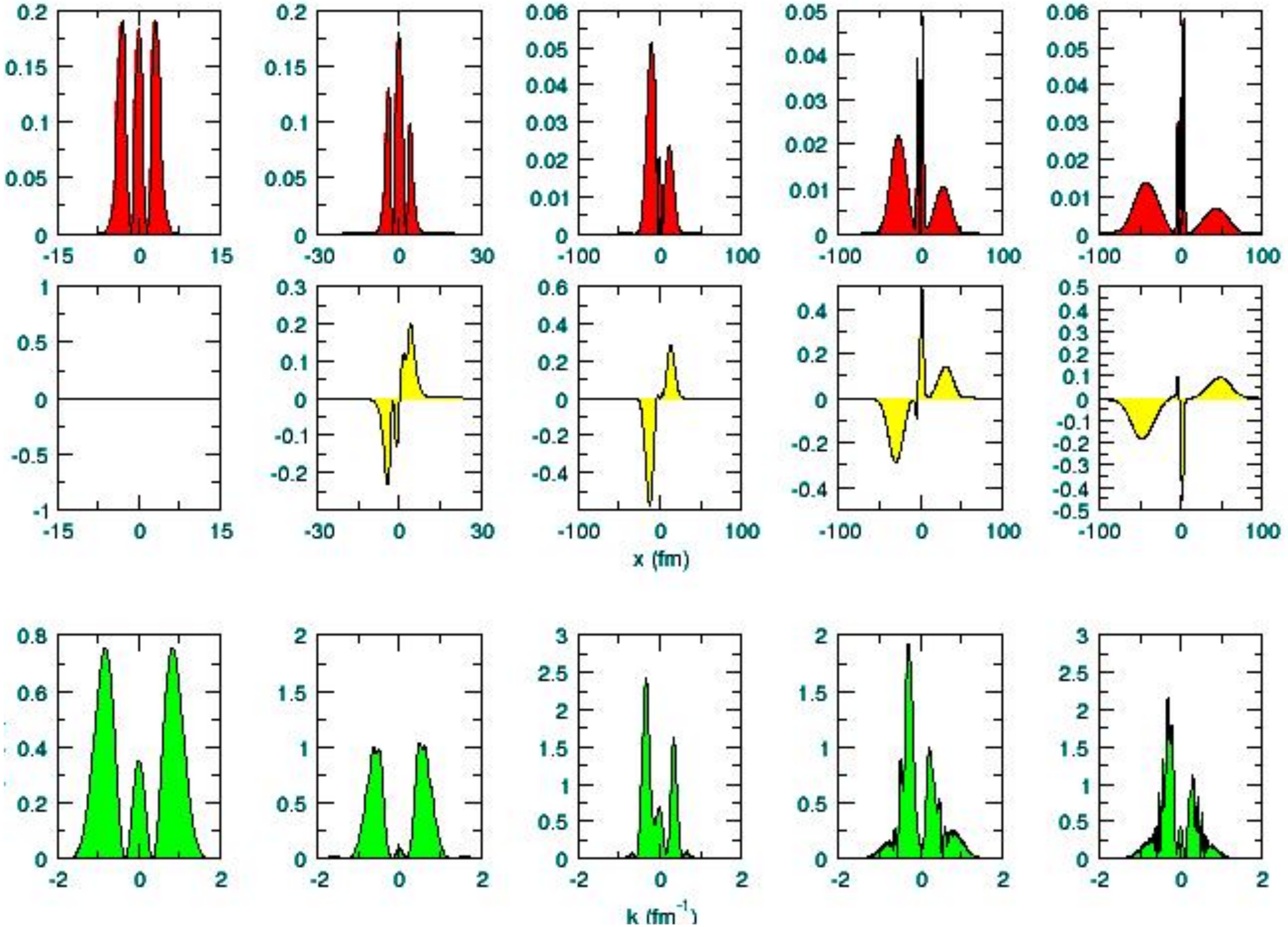
time

Final time

wave function $\Phi(x,t)^2$

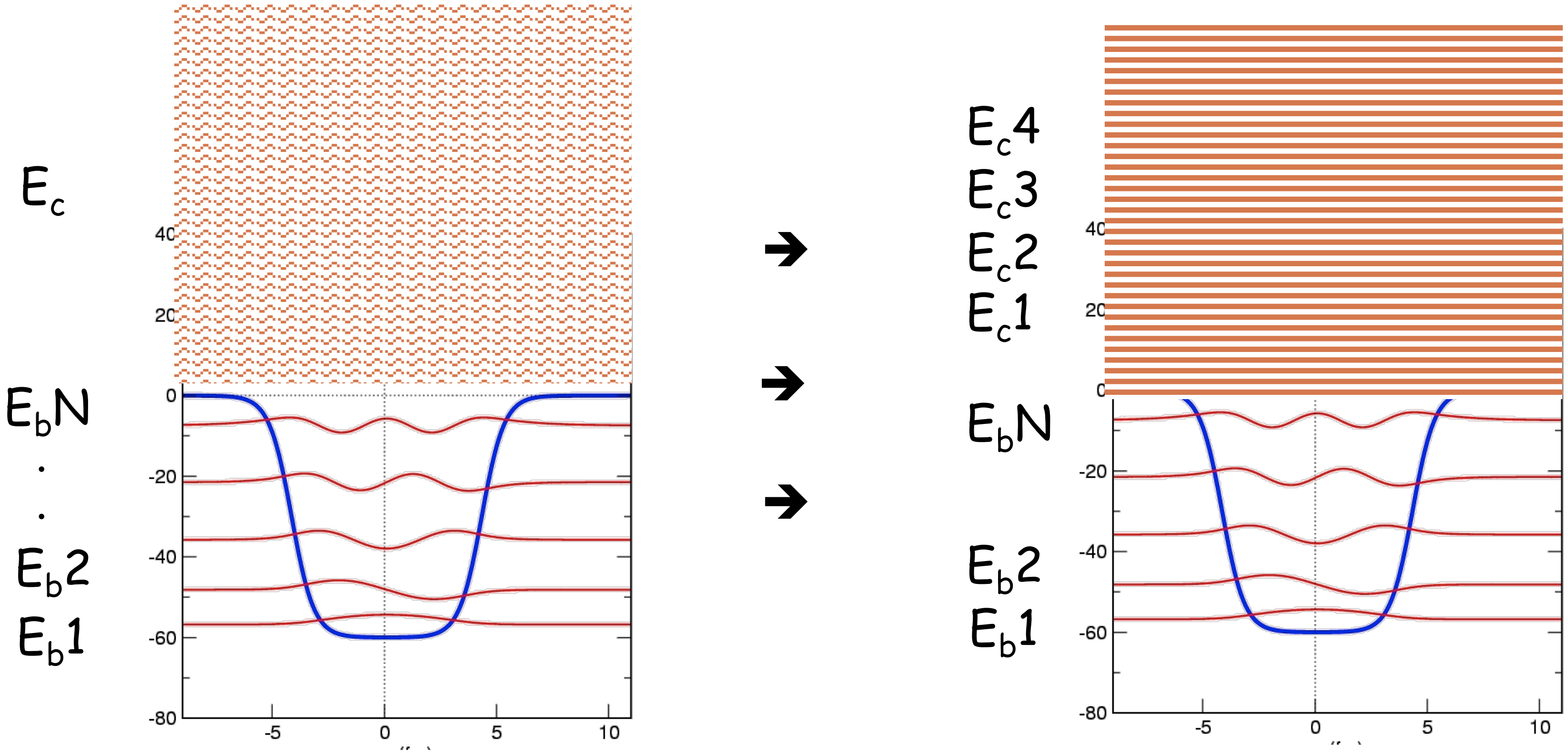
current distribution

momentum distribution $\phi(k,t)^2$



The same problem can be approached in
the ''standard'' coupled-channel formalism
where the Schrödinger equation is solved by
expanding the total wave function into a stationary
basis.
In this case the choice and the treatment of
continuum states are already essential in the
proper description of the evolution of the system

Slicing the continuum (ΔE)



$\Sigma\ \Phi i><\Phi i\ +$

$\int dE\ \Phi(E)><\Phi(E)$

$\Sigma\ \Phi i><\Phi i$ (bound)+

$\Sigma\ \Phi j><\Phi j$ (continuum)

Coupling matrix elements in the stationary basis

Continuum-continuum

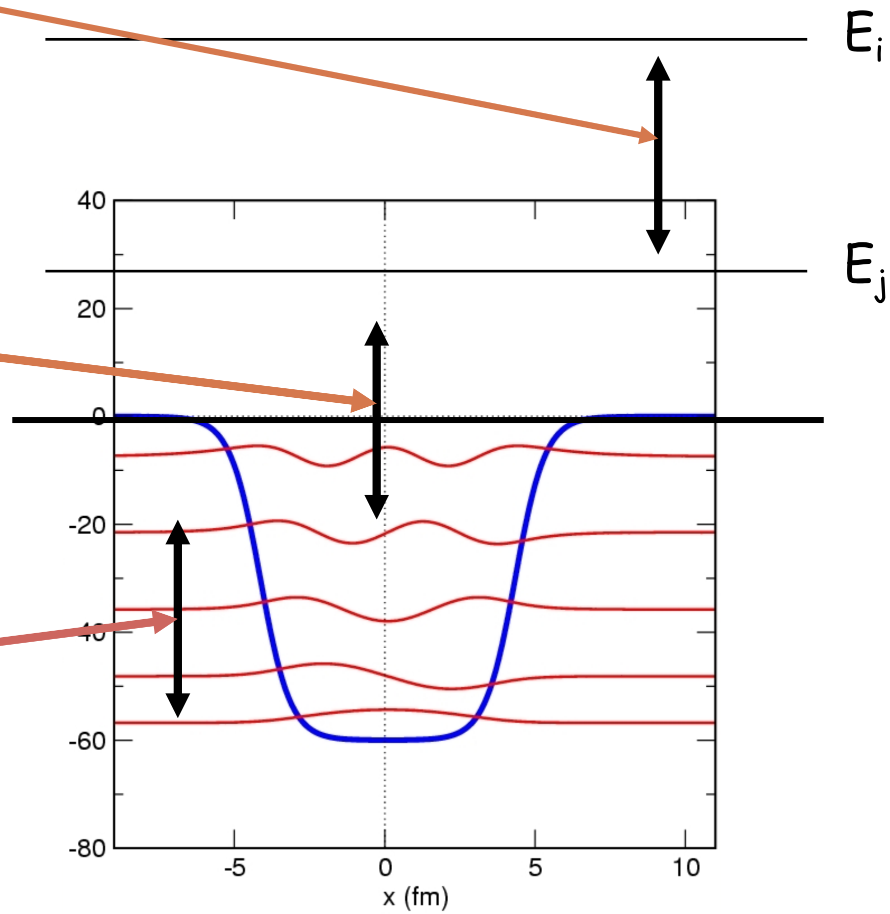
$F(Ei,Ej) = \langle \Phi(E_i;x)|v(x)|\Phi(E_j;x)\rangle$

Bound-continuum

$F_i(Ej) = \langle \Phi_i(x)|v(x)|\Phi(E_j;x)\rangle$

Bound-bound

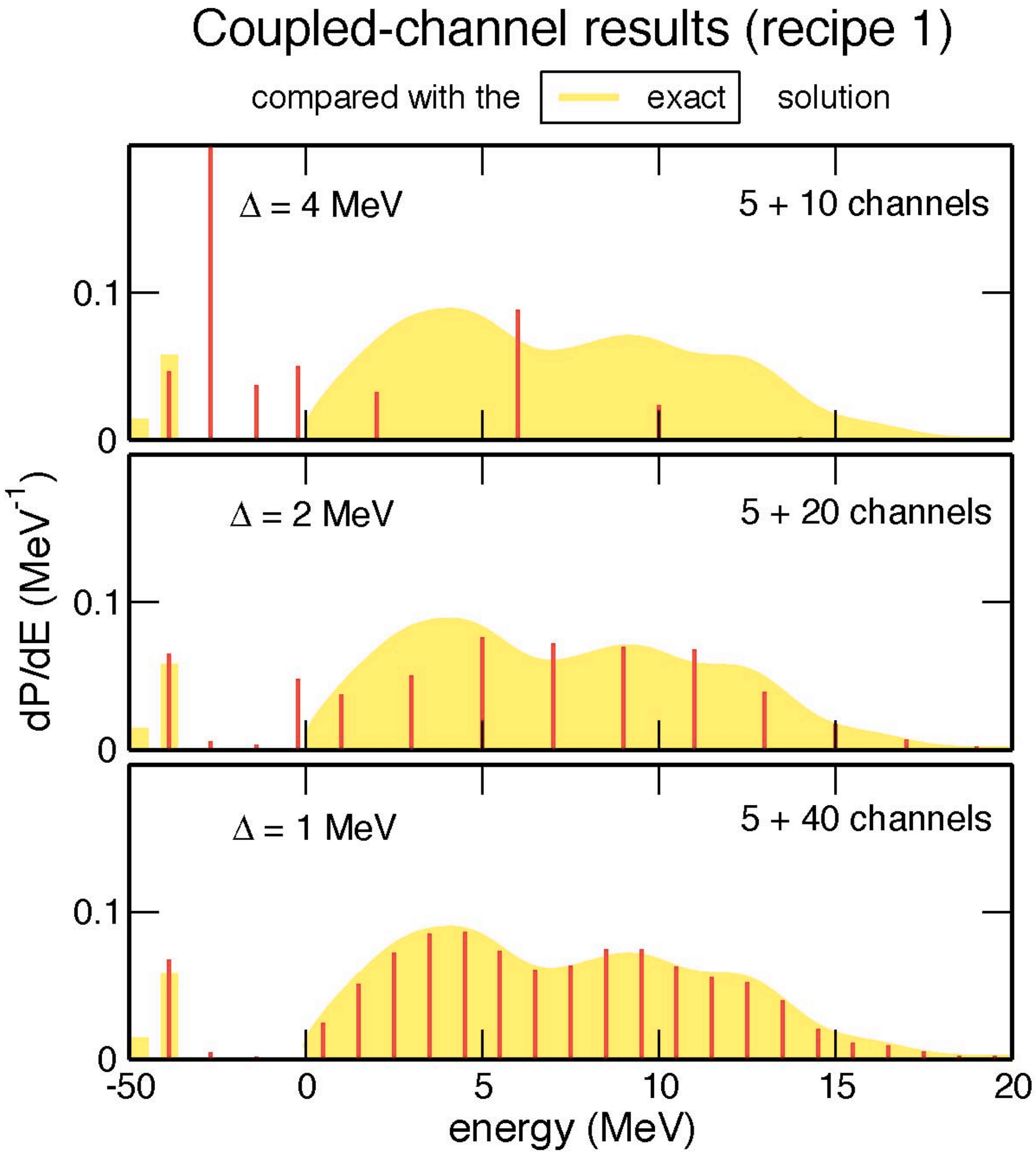
$F_{ij} = \langle \Phi_i(x)|v(x)|\Phi_j(x)\rangle$



Case of partial break-up starting from a deeply-bound orbital (N=3)

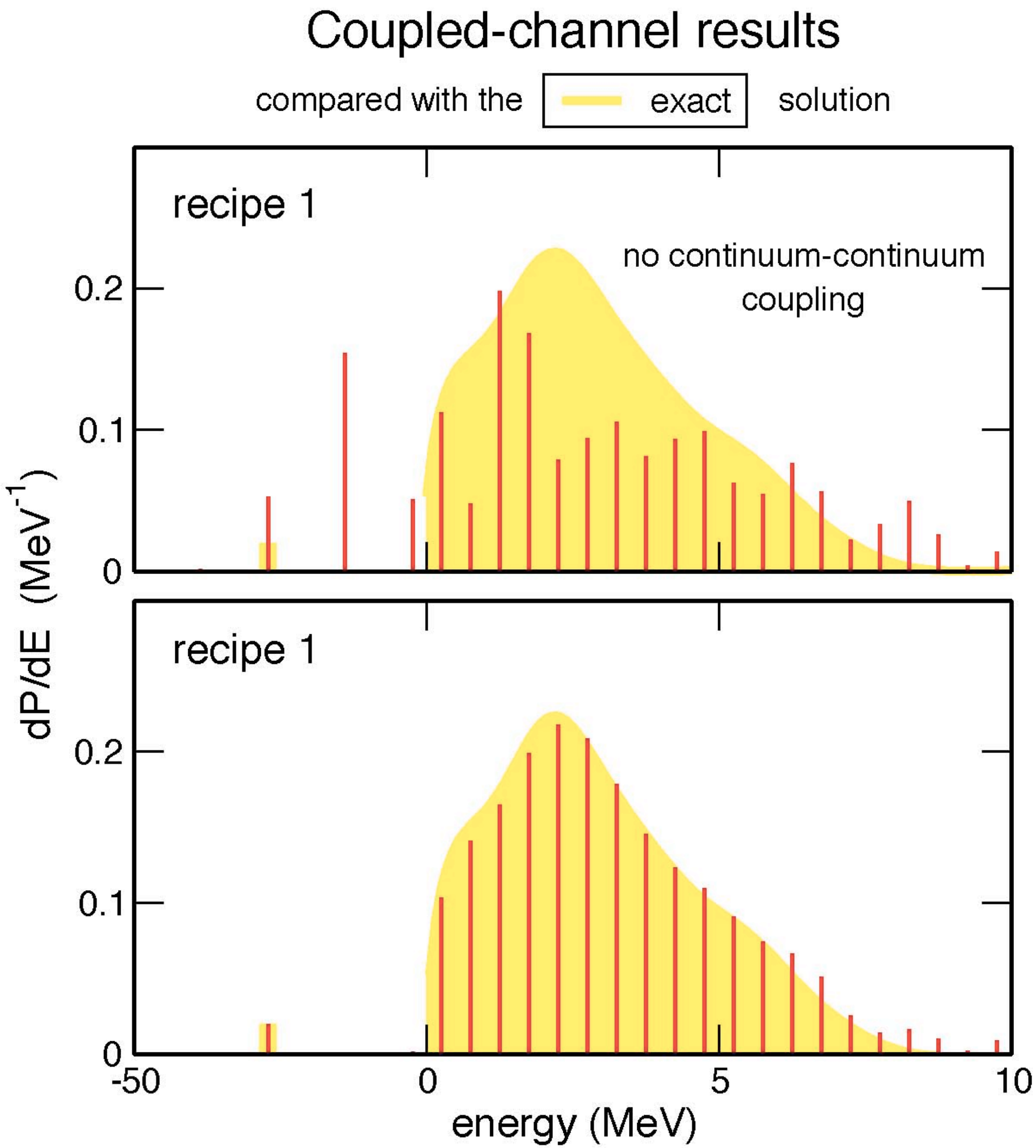
Final Q-value distribution

Different value of the energy mesh Δ



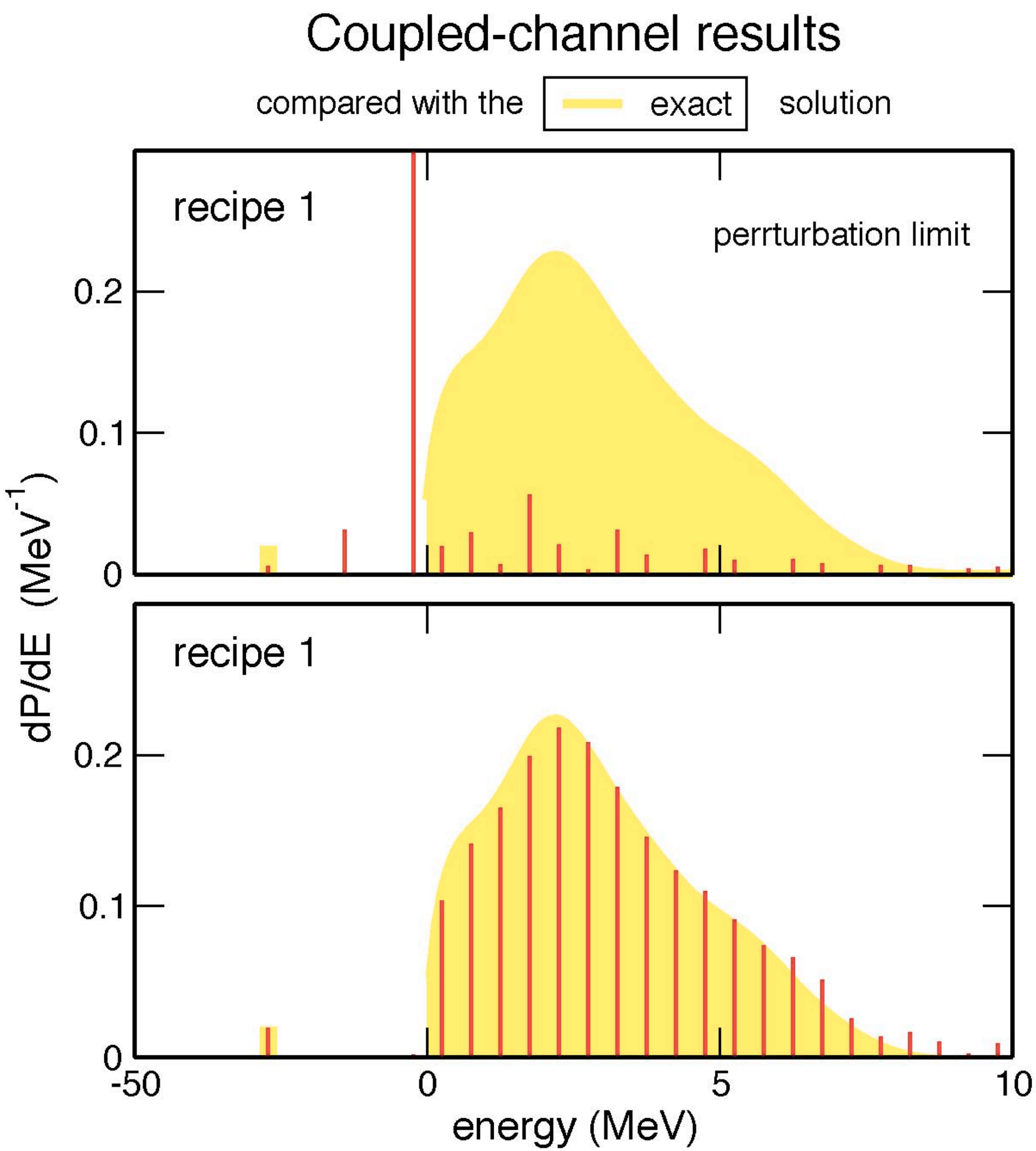
Case of partial break-up starting from a weakly-bound orbital

Fundamental role of continuum-continuum couplings



Case of partial break-up starting from a weakly-bound orbital

Role of multistep processes



Effects of continuum channels: another simple case.
Processes involving just one active particle
and two moving cores (possibilities of inelastic, transfer and break-up channels)

Our particle is initially sitting on a single-particle level of a one-body potential and feels the action of a second moving potential.

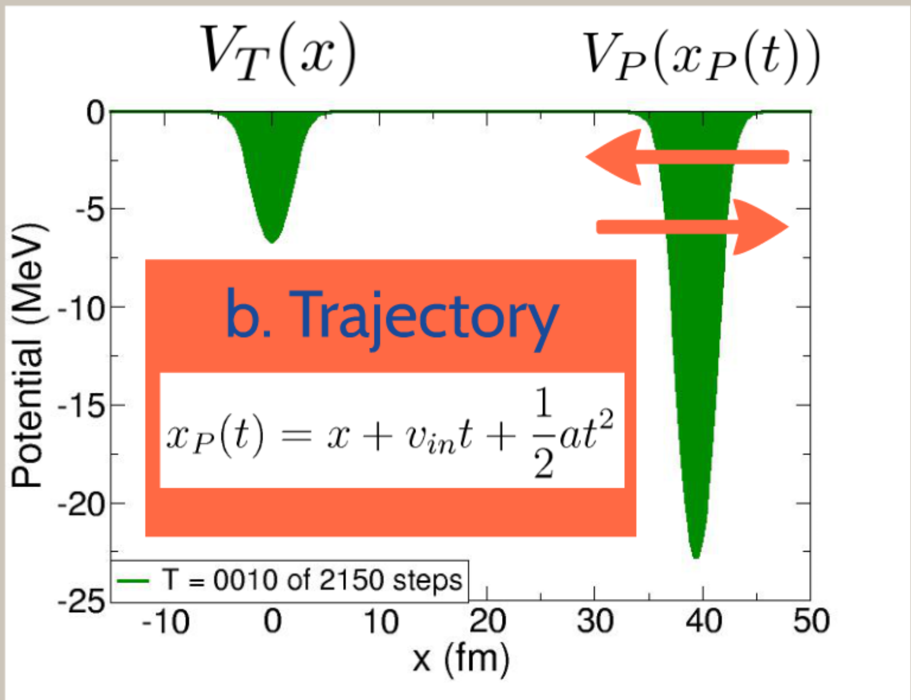
At the end of the process the wave function of the particle is
a) partly inside the initial well (elastic and inelastic processes)
b) partly inside the moving well (transfer process)
c) partly moving outside of the two wells (break-up process)

The different components can be obtained by projecting the final wave function on the eigenfunctions of the two wells, as well as on the continuum states

$$i\hbar \frac{d}{dt}\Psi(x,t) = \mathcal{H}(x,t)\Psi(x,t)$$

$$\mathcal{H}(x,t) = -\frac{\hbar^2}{2\mu}\frac{d^2}{dx^2} + V_T(x) + V_p(x_P(t))$$

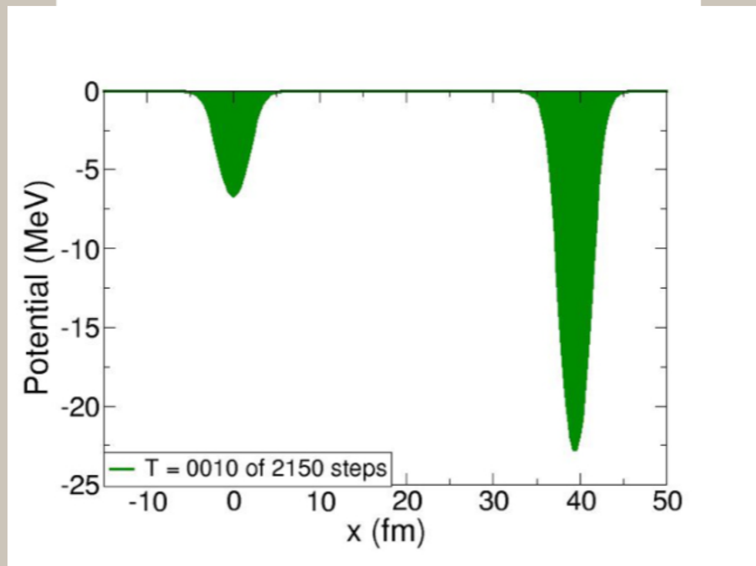
a. Potentials



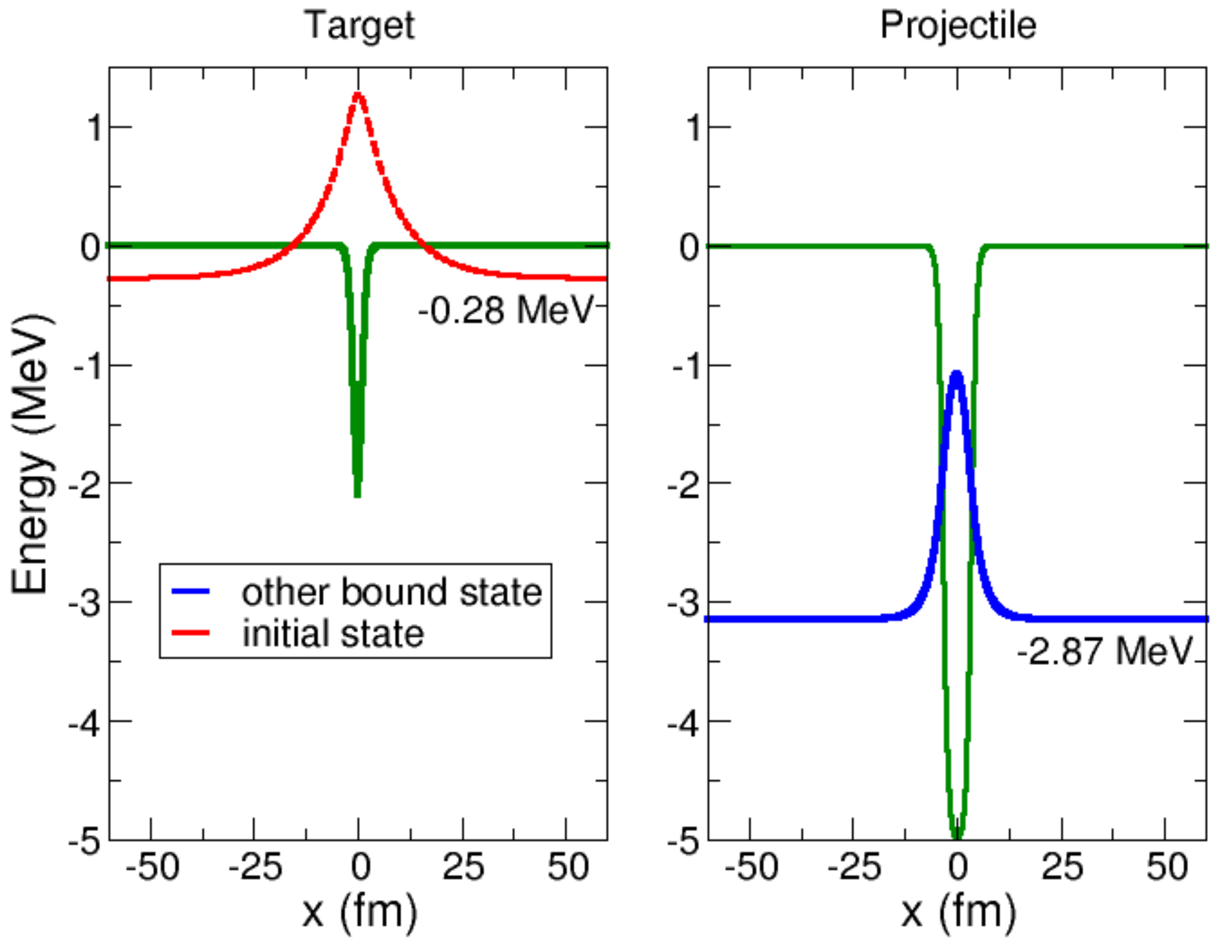
b. Trajectory

$$x_P(t) = x + v_{in}t + \frac{1}{2}at^2$$

Let's see it in action:



The two moving potentials

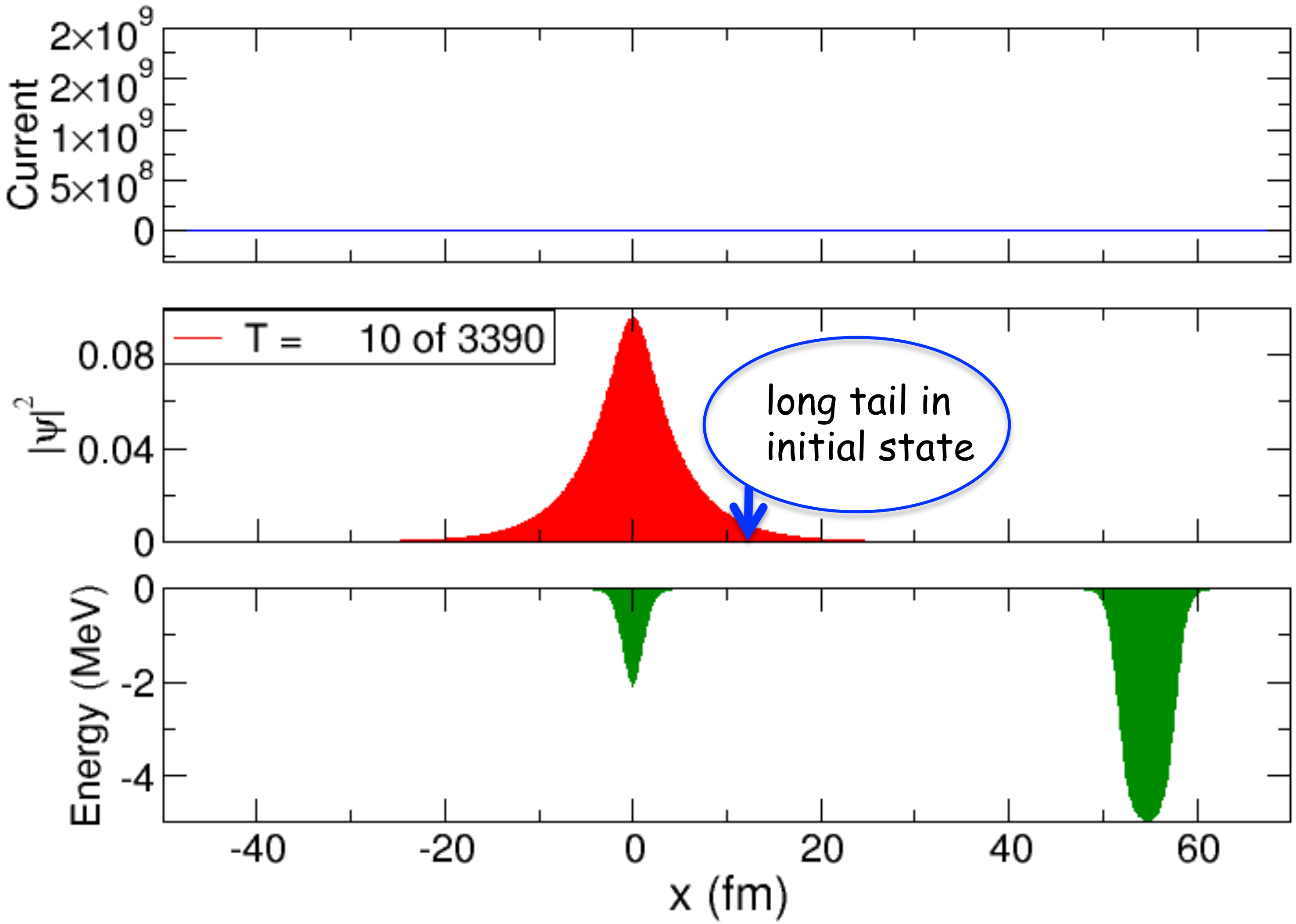


Time evolution

Current

Wave function

Potential

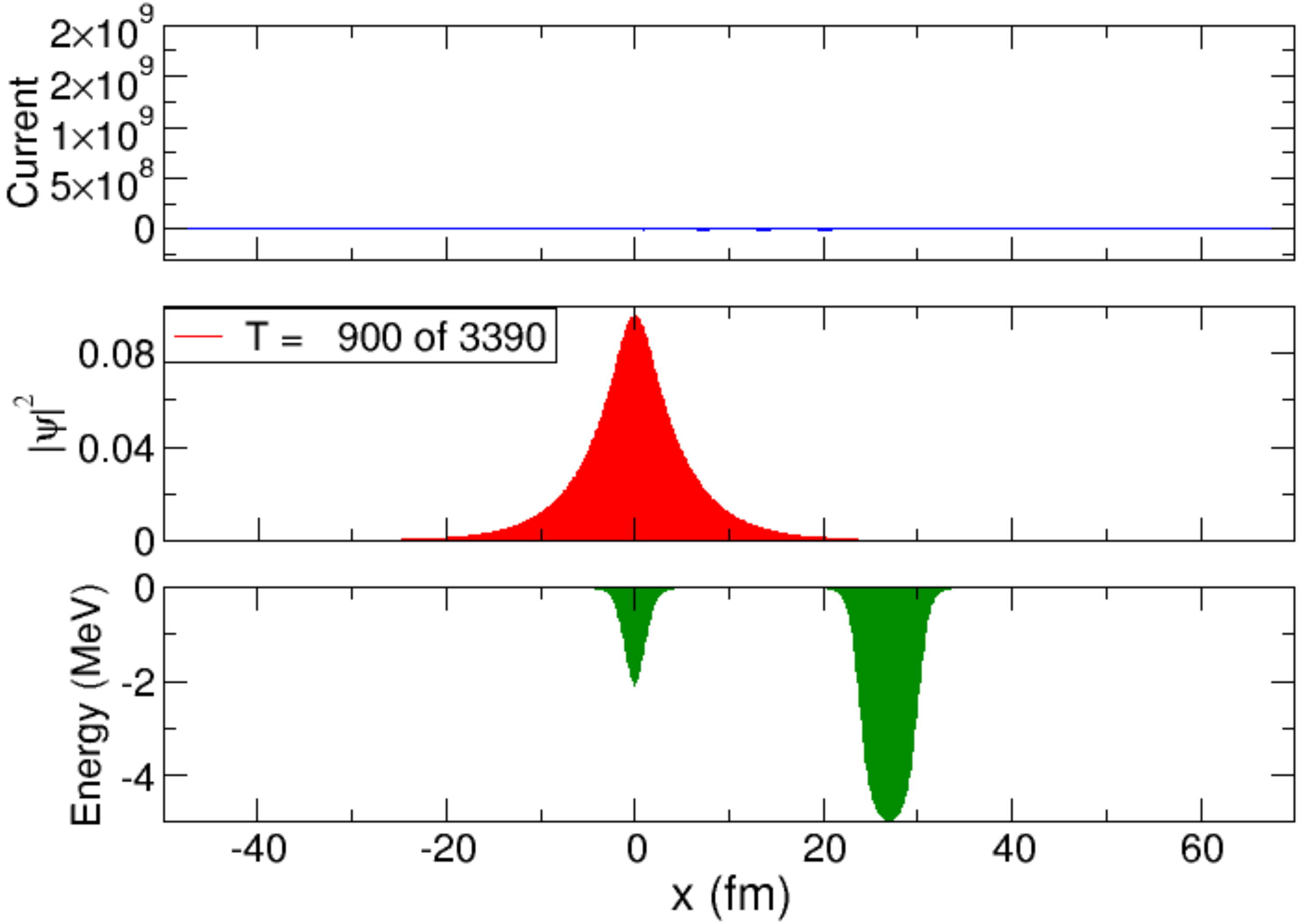


Time evolution

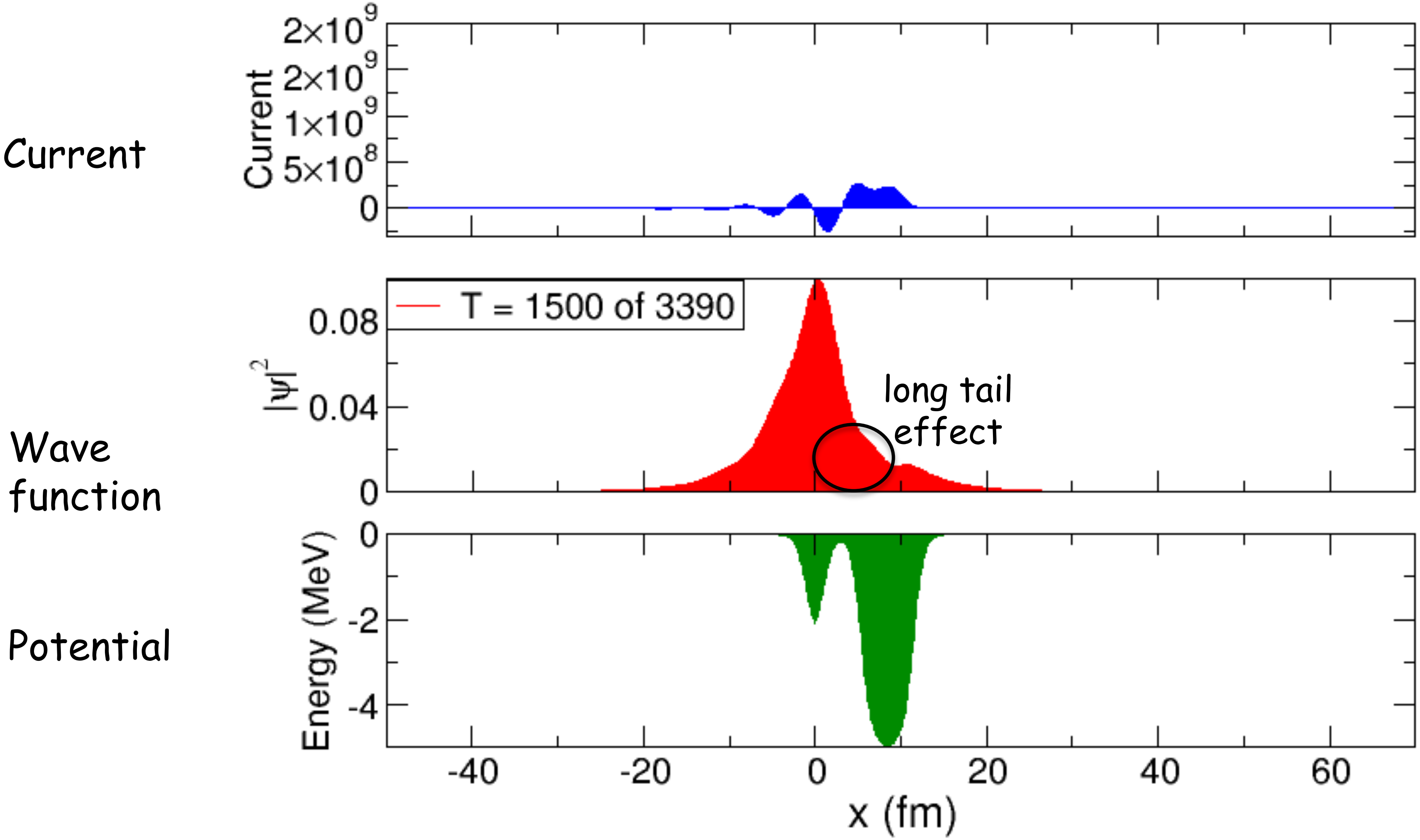
Current

Wave function

Potential



Time evolution

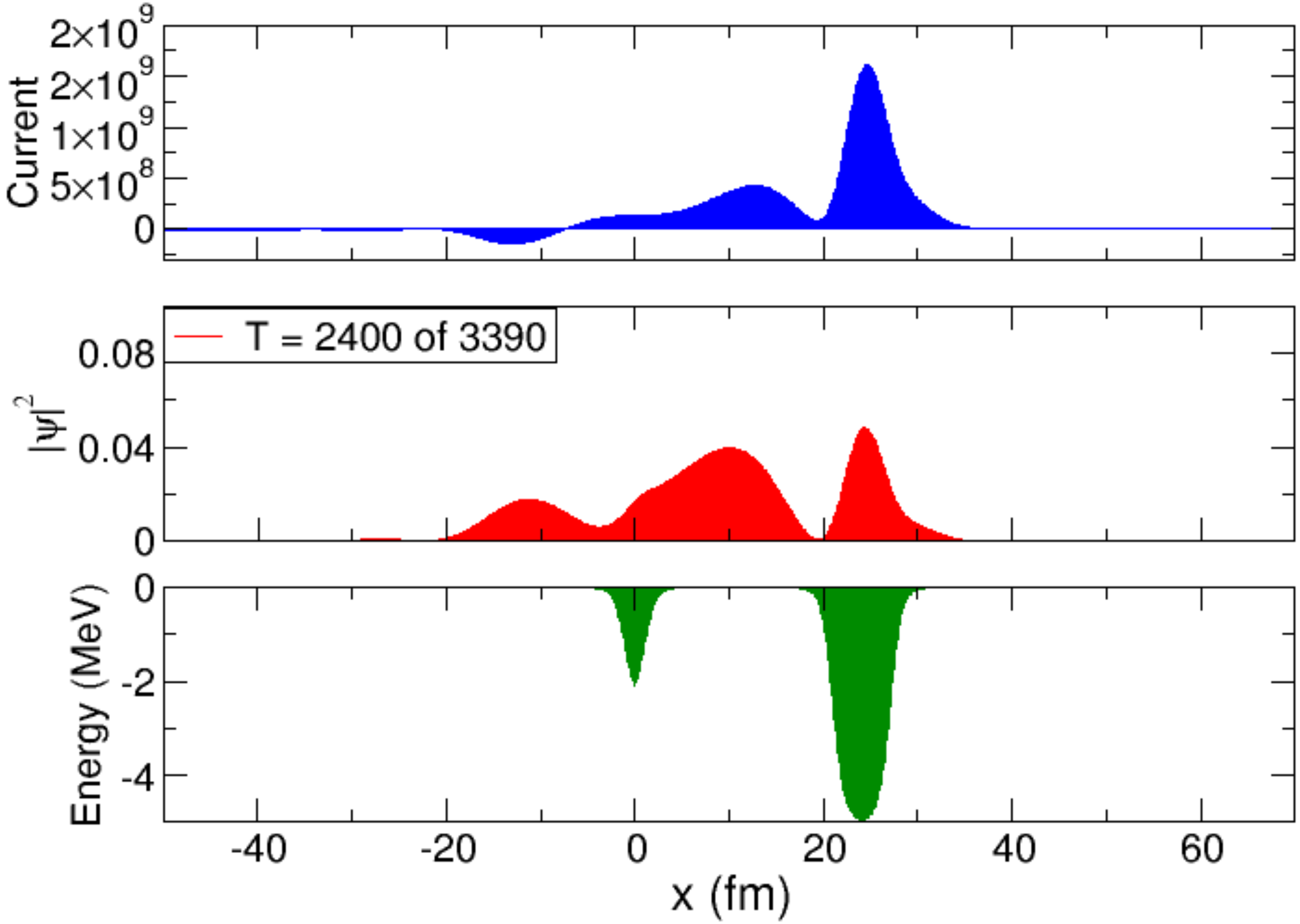


Time evolution

Current

Wave function

Potential

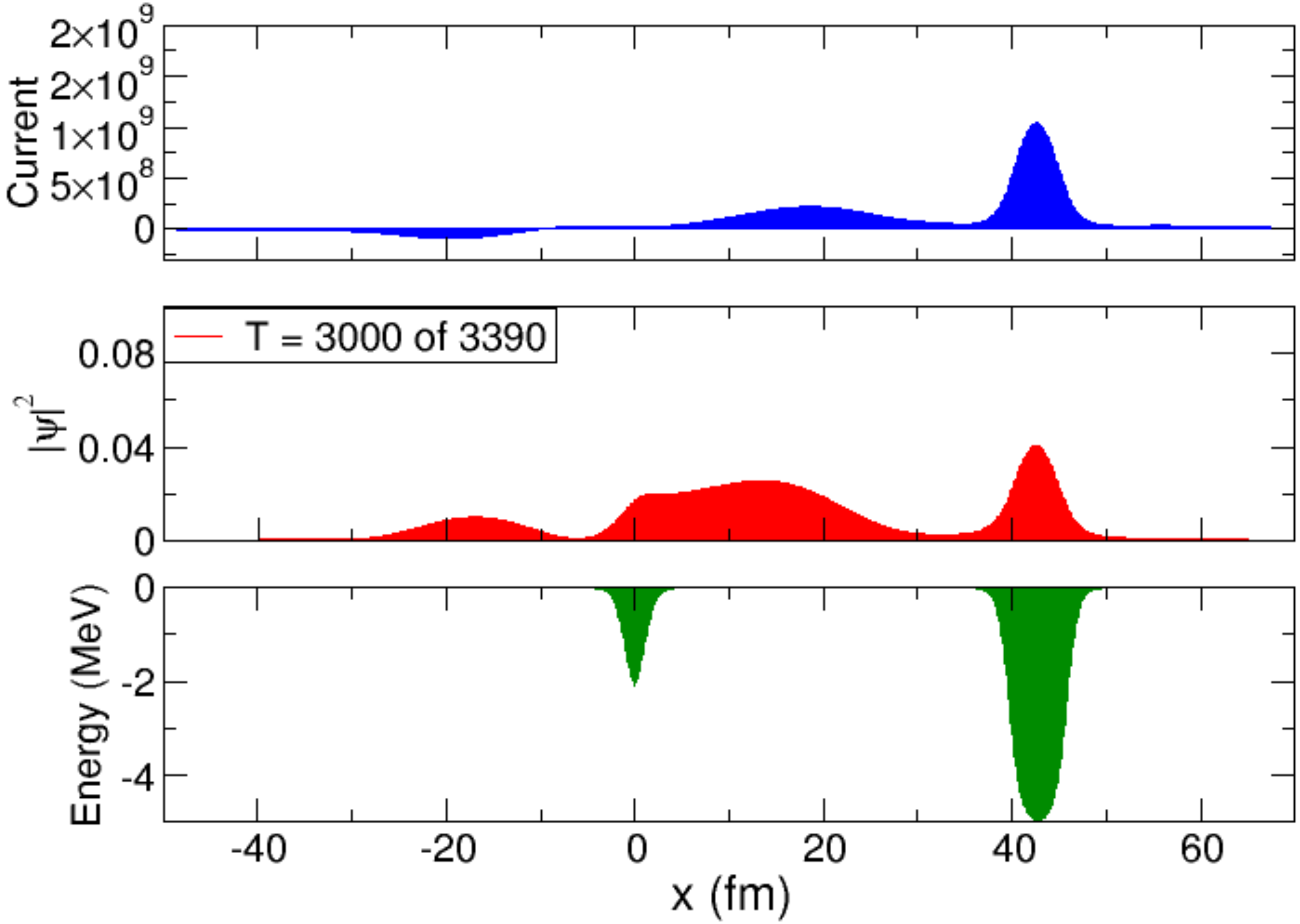


Time evolution

Current

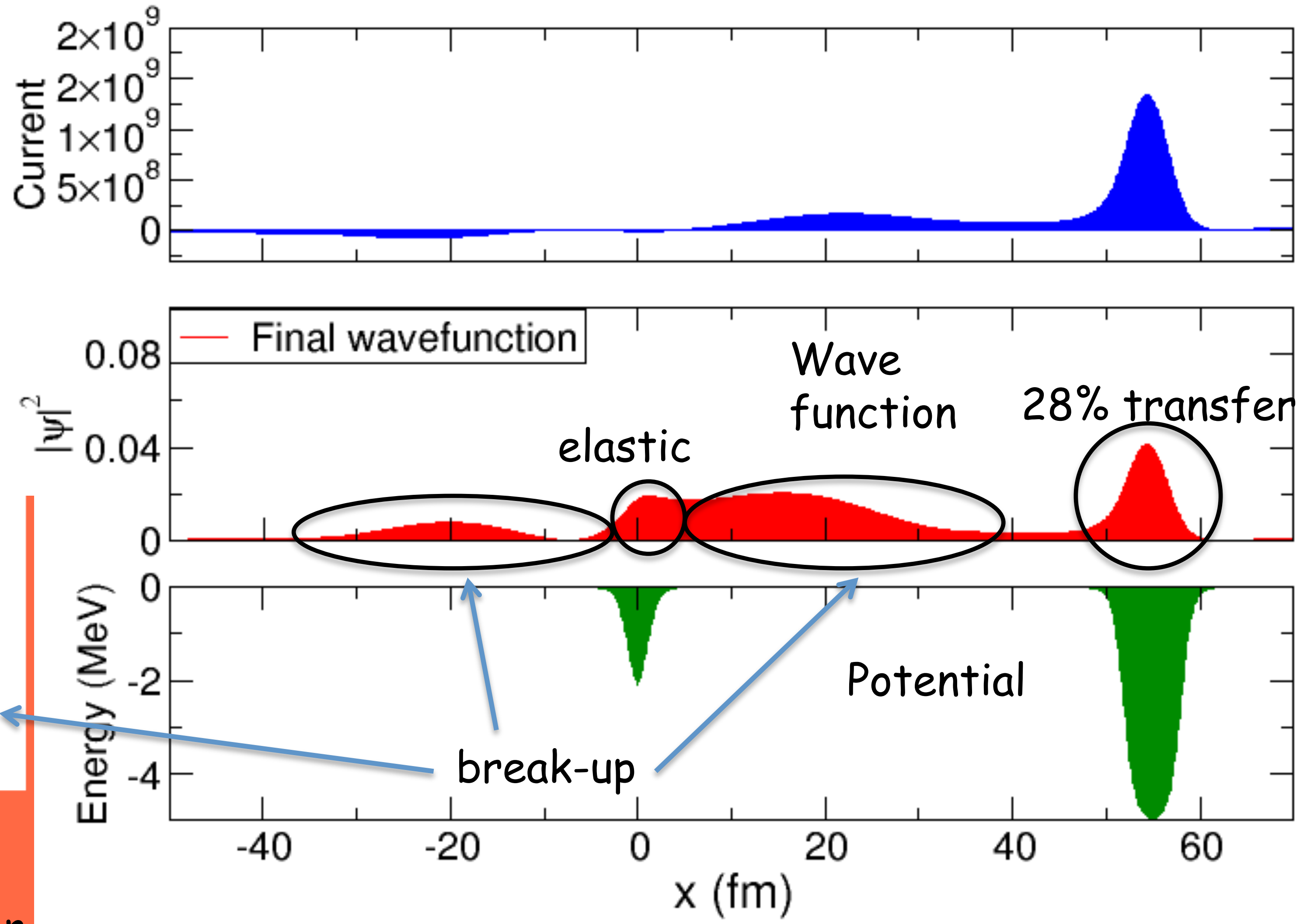
Wave function

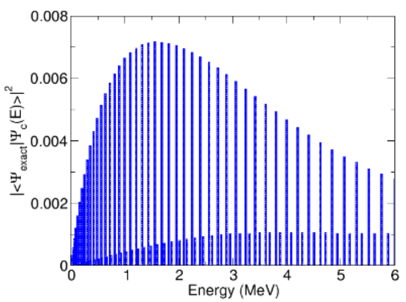
Potential



Time evolution

Current





Continuum Q-value distribution

Approaching the same problem in a coupled-channel approach
(including or not the continuum)

Final population	**Exact**	**CC (with only bound states)**	**CC (with bound and continuum target states)**	**CC (with bound and continuum states in both target and projectile)**
elastic	20.7 %	95 %	21.4 %	21 %
transfer	28 %	5 %		21 %
break-up	51 %		78.6	57 %

OBS: Proper definition of continuum states. Target continuum ? projectile continuum ? both continua?

The interest in haloes and weak-binding is not so much in the "static" behavior but rather in the dynamical effects in the response of these systems to different probes (B(E1) distribution etc). From the reaction point of view the weak-binding nature of halo nuclei favors the dominance of break-up channels, and the key question is the effect of the strong break-up channels and coupling to continuum states on the different collision processes (elastic scattering, direct reactions, fusion, etc).

It is well established the way in which coupling to excited bound channels effect, for example the elastic channel (cf. Feshbach theory). But will the effect change if the excited states are in the continuum? Continuum-continuum strong coupling will change the picture?

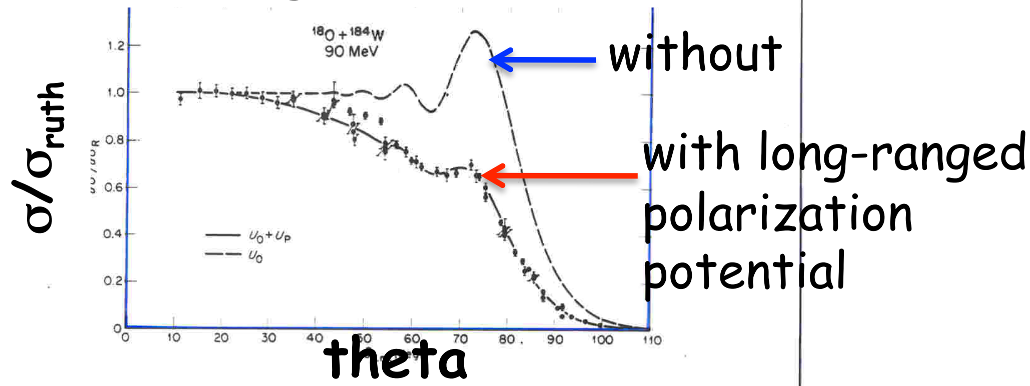
Polarization potentials due to nuclear coupling are normally short-ranged.
On the opposite, the contribution due to coulomb excitation is long-ranged ($1/r^5$).
In the case of large couplings (as the coupling to the rotational 2+ state in the deformed ^{184}W), this gives rise to characteristic patterns in the elastic scattering angular distribution

Text-book example

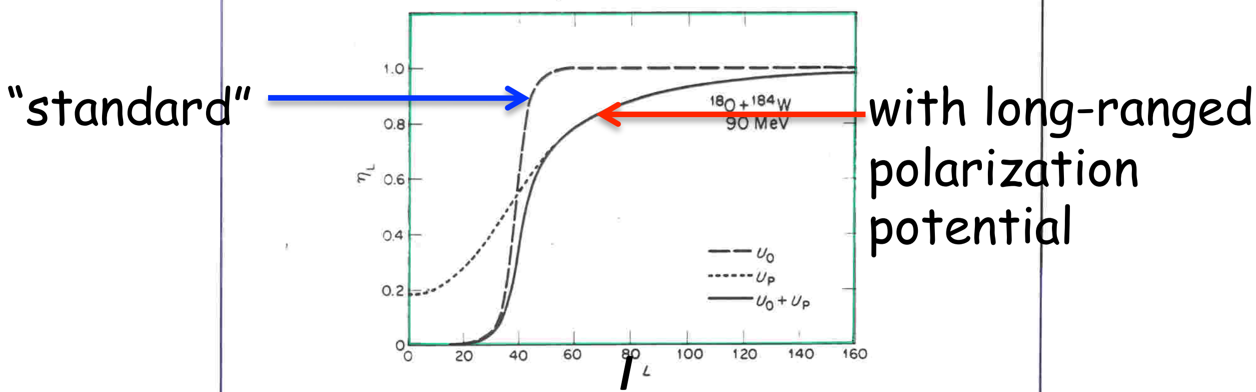
$^{16}O + ^{184}W$

Evidence for long-range Coulomb polarization potential

Elastic angular distribution

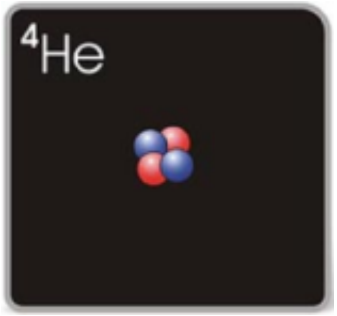


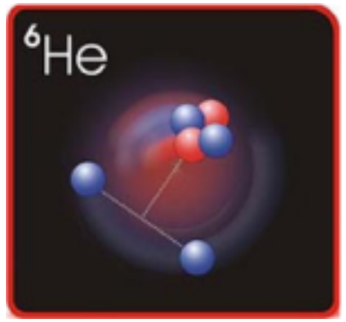
Transmission coefficient

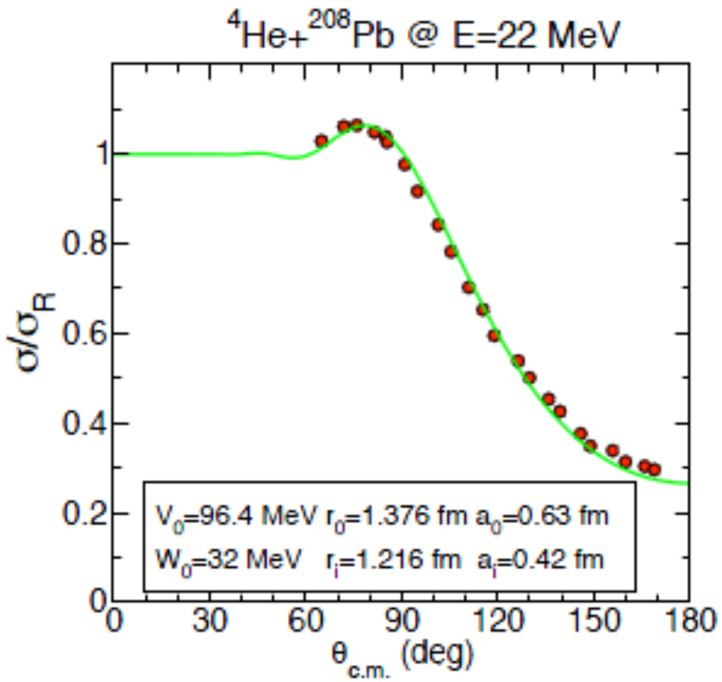


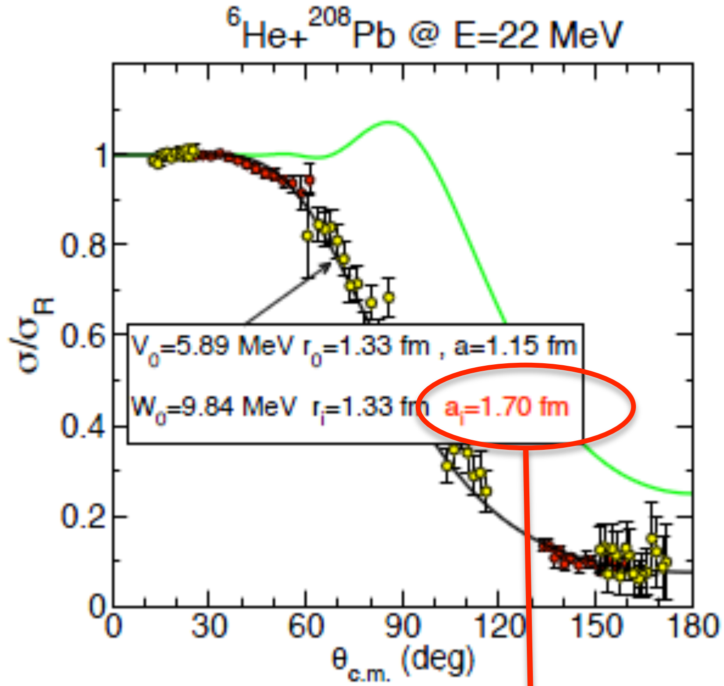
The coupling to continuum is reflected in the nuclear ion-ion potentials, absorptive potentials and couplings used in direct reactions that are normally short-ranged, with a shape that follows nuclear densities. The more striking effect in elastic scattering with weakly-bound halo nuclei is that one seems to need a long-ranged absorption that starts to be active also at bombarding energies well below the Coulomb barrier (and therefore at large distances), indicating the presence of long-ranged nuclear couplings in addition of the usual Coulomb interaction.
In addition the real part of the polarization potential seems to be repulsive for weakly-bound systems, at variance with standard case. Anomaly of the "threshold anomaly"? Peculiar nuclear-Coulomb interference processes?

Normal versus halo nuclei: the He case









- ^{6}He+^{208}Pb shows a reduction in the elastic cross section due to the flux going to other reaction channels (transfer, break-up or fusion?).
- ^{6}He+^{208}Pb requires a large imaginary diffuseness
  long-range absorption

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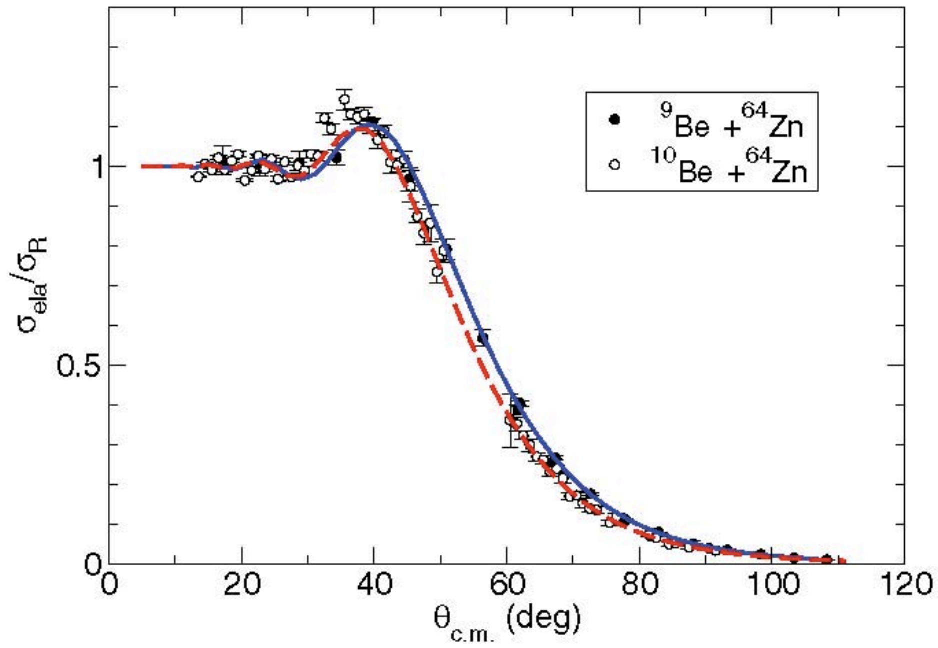
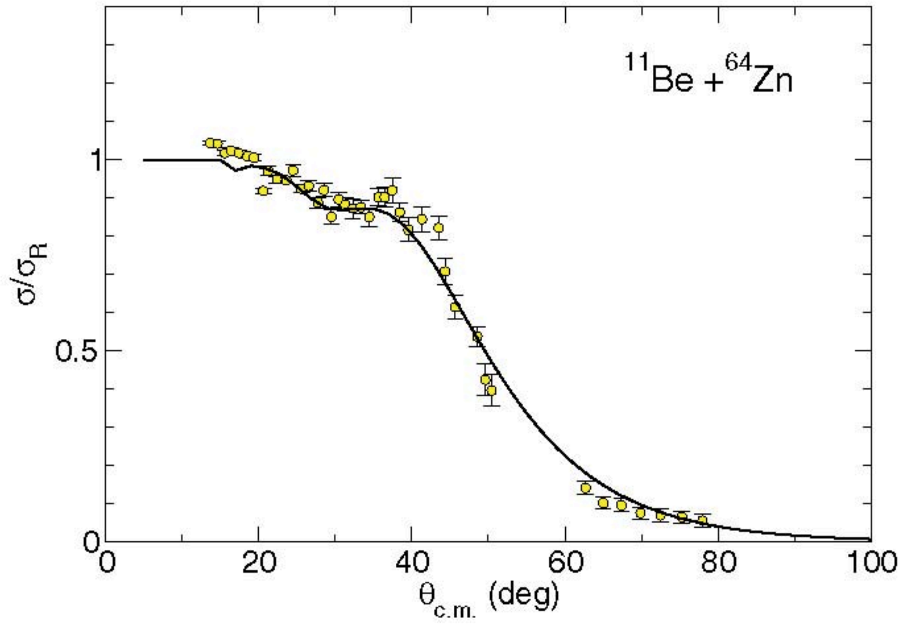
Best example: $^{9,10,11}Be + ^{64}Zn$ (Di Pietro etal, LNS)

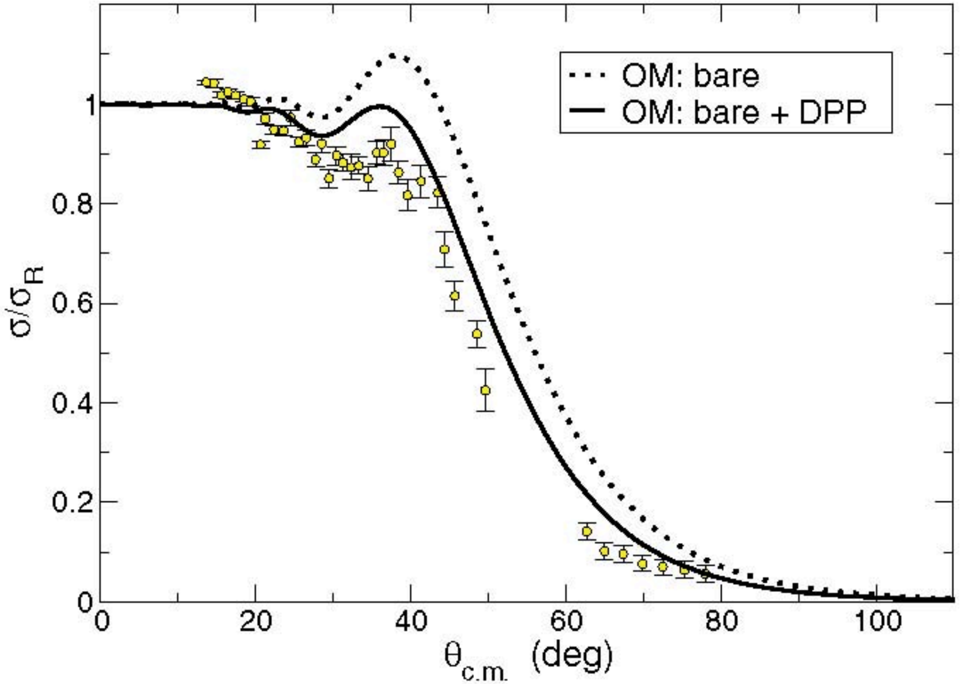
Optical model analysis

For $^{9,10}Be$ one can use a diffusivity a = 0.7 fm (standard value)



But for ^{11}Be one needs to add a term with a diffusivity a = 3 fm (unusually long range)



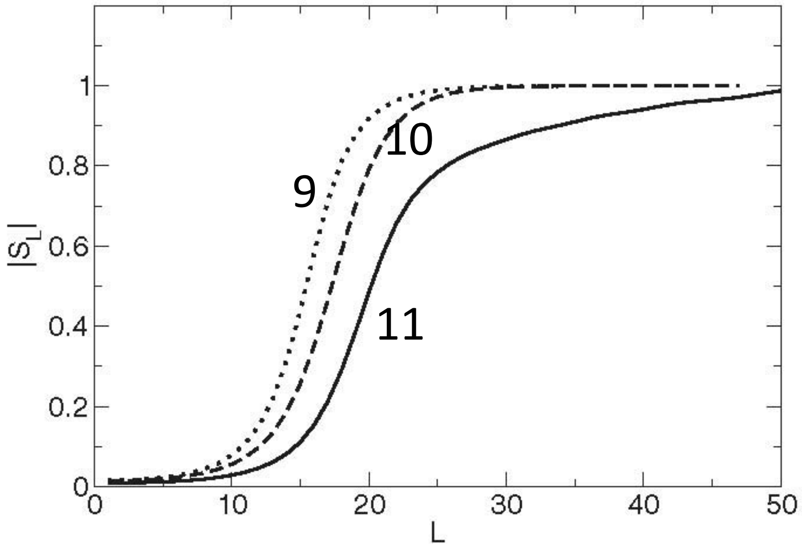


$^{11}Be + ^{64}Zn$

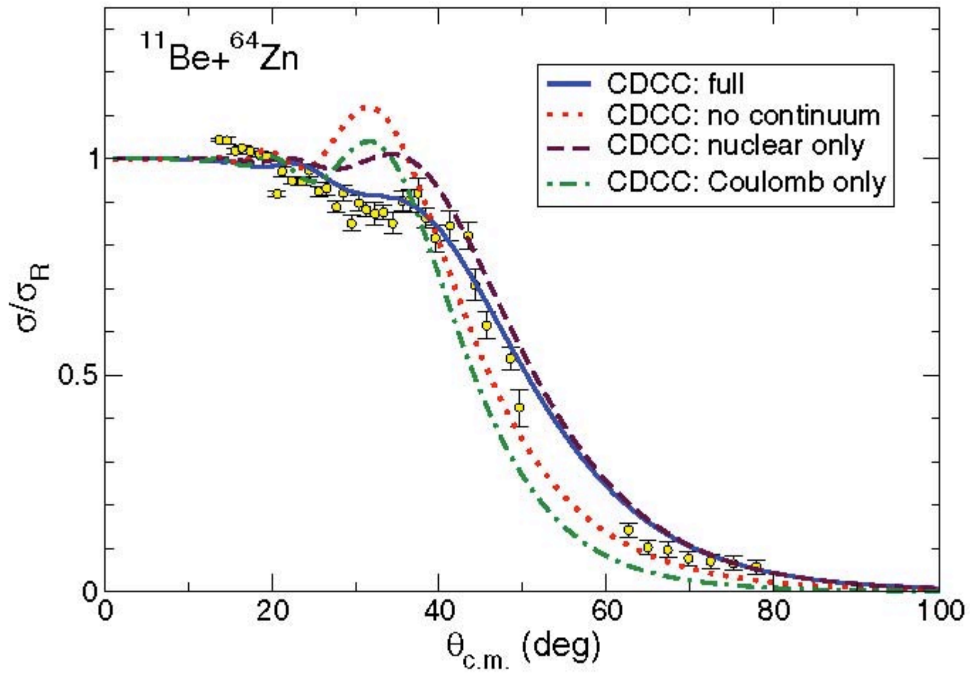
Effect of the long-range term
In optical potential

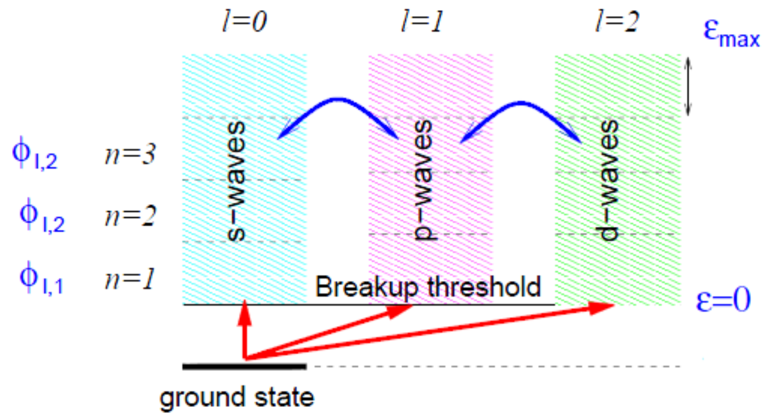
$^{9,10,11}Be + ^{64}Zn$

Transmission coefficient

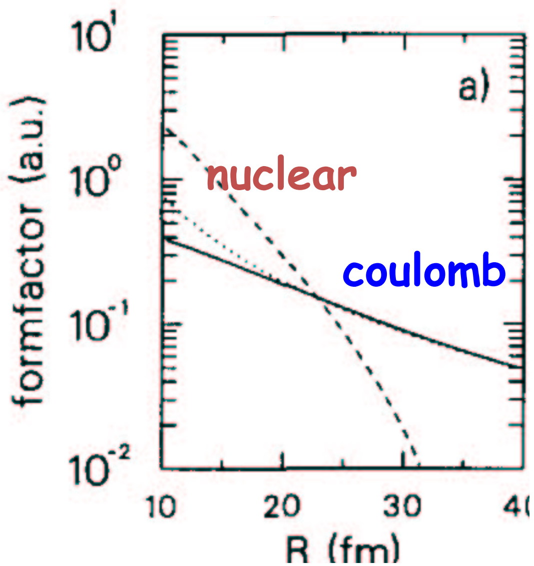


Origin of the long-ranged term from Coulomb
and nuclear couplings to continuum (break-up) states





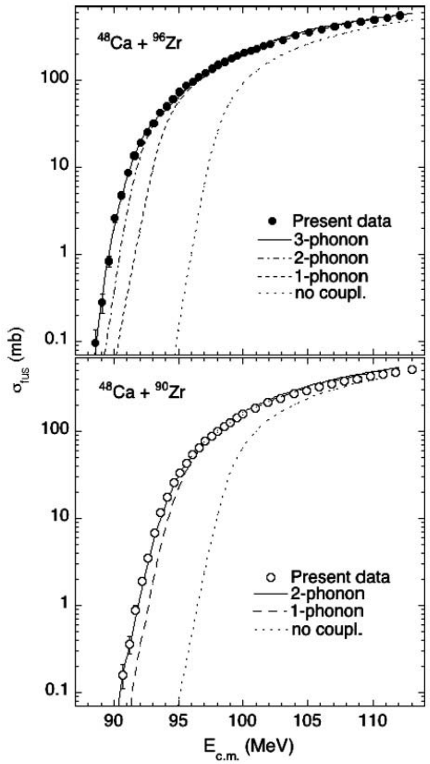
form factors to continuum states (theory, Dasso, Lenzi, Vitturi)



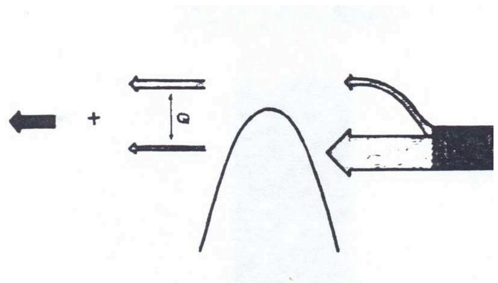
It is similarly well established the way in which coupling to bound inelastic or bound transfer channels effect subbarrier fusion reactions, leading to strong enhancement of the subbarrier fusion

$^{48}Ca + ^{90,96}Zr$
LNL data

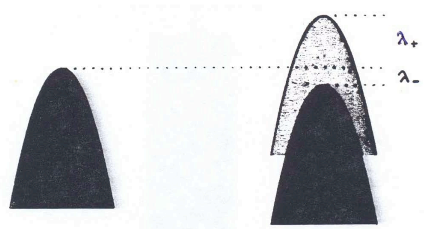
Coupling to vibrational states



Simple case: two channels
(costant coupling)



Effective barrier splitting



Will coupling to continuum channels change the picture?
Are break-up channels irreversible processes?
Complex inelastic form factors to continuum are responsible for a change of the behavior?

The challenging question:

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Does the presence of ^{11}Li breakup channels reduce the cross section for fusion processes?

C. H. Dasso[1,2] and A. Vitturi[3]

Whether the coupling to break-up channels enhances or hinders the subbarrier fusion is still an open question (after 20 years). Problem of reference (enhancement or suppression with respect to what?)?

Conclusions

Inclusion of continuum states in both structure and reaction studies is unavoidable . Coupling and dynamical interaction with these states strongly affect also the close bound part of the spectrum. This presents, however, novel features with respect to the standard coupling processes to excited, but still bound, states.